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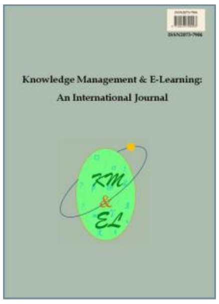

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A standardised, holistic framework for concept-map analysis combining topological attributes and global morphologies

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Abstract: Motivated by the diverse uses of concept maps in teaching and educational research, we have developed a systematic approach to their structural analysis. The basis for our method is a unique topological normalisation procedure whereby a concept map is first stripped of its content and subsequently geometrically re-arranged into a standardised layout as a maximally balanced tree following set rules. This enables a quantitative analysis of the normalised maps to read off basic structural parameters: numbers of concepts and links, diameter, in- and ex-radius and degree sequence and subsequently calculate higher parameters: cross-linkage, balance and dimension. Using these parameters, we define characteristic global morphologies: 'Disconnected', 'Imbalanced', 'Broad', 'Deep' and 'Interconnected' in the normalised map structure. Our proposed systematic approach to concept-map analysis combining topological normalisation, determination of structural parameters and global morphological classification is a standardised, easily applicable and reliable framework for making the inherent structure of a concept map tangible. It overcomes some of the subjectivity inherent in analysing and interpreting maps in their original form while also avoiding the pitfalls of an atomistic analysis often accompanying quantitative concept-map analysis schemes. Our framework can be combined and cross-compared with a content analysis to obtain a coherent view of the two key elements of a concept map: structure and content. The informed structural analysis may form the starting point for interpreting the underlying knowledge structures and pedagogical meanings.

Keywords: Concept map analysis; Concept map classification; Concept map morphology; Scoring schemes; Educational research tools

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1. Introduction

Concept maps as developed by Novak (2010) are a '*very powerful and concise knowledge representation tool*' (Novak & Cañas, 2006, p. 332). They were originally introduced during an investigation of science teaching for young school children (Novak & Musonda, 1991) as a means to visualise information gained from interviews in a compact way. As an alternative to the original hierarchical concept maps focussed on here, cyclic concept maps have also been introduced to better represent the dynamical relationships between concepts (Safayeni, Derbentseva, & Cañas, 2005). Nowadays, concept maps are increasingly used in both teaching and research in Higher Education. A range of such possible uses of concept-mapping have been discussed by Hay, Kinchin, and Lygo-Baker (2008): complementing expository teaching, concept maps have been invoked as an aid for educational design (Czarnocha & Prabhu, 2008; Darmofal, Soderholm & Brodeur, 2002), instruction (Czarnocha & Prabhu, 2008), diagnostic (Taber, 1994; Treagust, 1988) and formative (Austin & Shore, 1995) assessment. Alternatively, concept maps have also been employed to facilitate active (Hay & Kinchin, 2008), collaborative (Kinchin, De-Leij, & Hay, 2005) or dialogic learning (Hay, Dilley, Lygo-Baker, & Weller, 2009) and to foster reflective practice (McAleese, 1994). We have used concept maps both as a teaching and learning tool and to aid curriculum design. In the teaching context, we have found them useful to facilitate tutorial and revision sessions with students and to be particularly valuable when teaching across disciplinary boundaries.

Concept maps are also useful diagnostic and research tools. In these contexts, quantitative or qualitative techniques have been used to analyse and interpret them. As seen from Strautmane's (2012) review, quantitative measures can be divided into purely structural attributes such as the number of links (Conradty & Bogner, 2008; Austin & Shore, 1995; Novak & Gowin, 1984), cross-links (Miller & Cañas, 2008; Prosser, Trigwell, Hazel, & Waterhouse, 2000; Novak & Gowin, 1984) or hierarchical levels (Novak & Gowin, 1984) on the one hand and content-related criteria such as the correctness (Conradty & Bogner, 2008; Miller & Cañas, 2008; Prosser, Trigwell, Hazel, & Waterhouse, 2000; Novak & Gowin, 1984) and quality (Austin & Shore, 1995) of propositions and completeness (Miller & Cañas, 2008) on the other. Often, scoring schemes or criterion maps are used to aid assessment (Novak & Gowin, 1984).

While being easy to implement and relatively free of ambiguities, basic quantitative assessment schemes for concept maps often fail to capture important holistic aspects that the more interpretative, qualitative approaches can provide. The starting point for such approaches is often a morphological classification of concept maps in terms of their global structures. Kinchin, Hay, and Adams (2000) have identified spoke, chain and network as distinct morphological classes. Their scheme was extended by Yin, Vanides, Ruiz-Primo, Ayala, and Shavelson (2005) who added circular and tree classes. Based on their topological analysis, Koponen and Pehkonen (2008) have proposed an alternative classification as chains, loose and connected webs.

These qualitative approaches to concept-map analysis use graphic and topological analysis to generate morphological classifications and often go on to suggest links between these structures and characteristic learning 'attributes'. For instance, Hay and Kinchin (2006) have developed thinking typologies, suggesting that spoke structures are '*indicative of superficial and undeveloped knowledge*' (p. 139) or, in a more positive view, of '*learning readiness*' (Hay & Kinchin, 2006, p. 139). By contrast, chains are '*indicative of achievement, drive and goal-directed behaviour*' (Hay & Kinchin, 2006, p. 138), while networks represent '*a rich body of knowledge in which complex understanding is demonstrated*' (Hay & Kinchin, 2006, p. 138). Originally associating this latter type with expert knowledge, Kinchin and Cabot (2010) have later located expertise in the ability to dynamically transform between '*chains of practice and the underlying networks of understanding*' (p. 153). This notion relates back to Novak and Gowin's (1984) observation that learning involves a transition between '*written or spoken messages [that] are necessarily linear sequences of concepts and propositions*' (p. 53) and '*knowledge [which] is stored in our minds in a kind of hierarchical or holographic structure*' (p. 53). Thinking typologies provide an accessible and powerful framework for interpreting and comparing concept maps. However, one should bear in mind that such interpretation neglects influences on morphology other than the learner's knowledge structure, such as their graphical abilities and time spent producing a map.

While quantitative analyses of concept-map structural attributes are easy to perform and require little interpretation they fail to capture a holistic perspective of learning and are little more than descriptors of basic structure. The more interpretive and topological analysis extends this basic quantitative description towards interpretation, but can be subjective with a risk of limited reliability. This paper presents a systematic approach to topological analysis and morphological classification that builds on Kinchin's (2000) suggestion to take a combined approach to analysis, in an attempt to form a standardised and reliable basis for interpreting topology and morphology and provide a more consistent, reproducible and methodologically robust platform on which to build subsequent qualitative analysis and interpretation.

2. Methods

This paper describes this approach and is illustrated using recent work we have done with a total of 35 undergraduate and postgraduate students at Imperial College, London. The methodological description is illustrated using concept maps from Physics students asked to map the concept of 'light'. These concept maps were generated as part of the regular curricula. Students had received rules for generating concept maps, an incomplete example map on 'the universe' and some practice before they were asked to individually draw a concept map of 'light' on a sheet of A3 paper. They were told to work without time limit and indicate when they felt they had completed the task, which for this group was the case after around 20 minutes. This time frame aligns well with Hay, Kinchin, and Lygo-Baker's (2008) suggestions that most students will find 20-30 minutes sufficient to construct a reasonable map (p. 302).

The mapping task itself specified the root concept 'light' but without suggested or prescribed concepts, offering a high degree of freedom in content and structure (Cañas, Novak, & Reiska, 2012). The students were not given any prompts towards a desired map structure or to aim for deep or highly linked maps. The maps generated were varied and 'typical' of students mapping a core concept in their course, as such they served as a good exemplar. Being focussed on purely structural features, however, our method transcends the specific setting of discipline and topic. As a possible qualification, note that while exhibiting a range of expertise in terms of their subject knowledge, all students from our exemplar group were novice concept mappers.

3. Structural and topological normalisation of concept maps

We have developed an analysis of concept maps which proceeds in two stages: quantitatively and morphologically. The findings from the first stage inform the subsequent stage which is in turn substantiated by its predecessor.

As a preparation for both the quantitative and morphological investigations, we use a standardised approach to transform the original concept maps into normalised and comparable forms. This is achieved in two steps: First, concept maps are redrawn with all concept- and link-labels removed. This step results in a content-free map which is faithful in structure and geometrical layout to the original, see Fig. 1. In this example, the original concept map as drawn by the student is represented in type-set form and without link labels to aid clarity, Fig. 1(i). In the redrawn map, Fig. 1(ii), all labels are removed, concepts are represented by open circles and the root concept is indicated by a rectangle. To ensure greater comparability among the participants, boxes with multiple content may be split into two or more boxes to reflect their original multiple content; and if appropriate, multi-links with a single common label that appear to be concepts are elevated to concept status, Fig. 1(iii).

In a second step, the content-free maps are geometrically rearranged to facilitate an easier comparison of their structure (Fig. 2). During this topological normalisation, the source concept is placed at the top of the map and the other concepts are arranged on levels corresponding to their distance from the source concept. Fig. 2(i) shows the content-free concept map with the concepts numbered to illustrate their repositioning in the topologically normalised version shown in Fig. 2(ii). Such a procedure has been applied previously by Koponen and Pehkonen (2008) with the aid of automated graph-theoretical software. In this manually implemented normalisation, branches are further ordered from left to right according to their depth while striving for greatest possible balance among the branches in cases of ambiguous sub-branch assignments.

This topological normalisation procedure transforms the content-free concept map into a unique form which preserves the concept vertices and their links. Placing the source concept at the top, concepts which are once, twice etc. removed from the source are placed on subsequent hierarchical levels and linked as in their original form. Starting from the top, branches emerging from each concept vertex are ordered from left to right according to the following simple rules:

- Place the deepest (longest) branch first.
- For branches of equal length, place the branch with the largest total number of concepts first.
- For branches with an equal number of concepts, place the branch with the largest number of longest sub-branches first.
- For branches with an equal numbers of such sub-branches, place the branch whose uppermost concept has the largest number of sub-branches first.
- For branches with equal numbers of sub-branches of the uppermost concept, place the branch with the largest number of cross-links first.

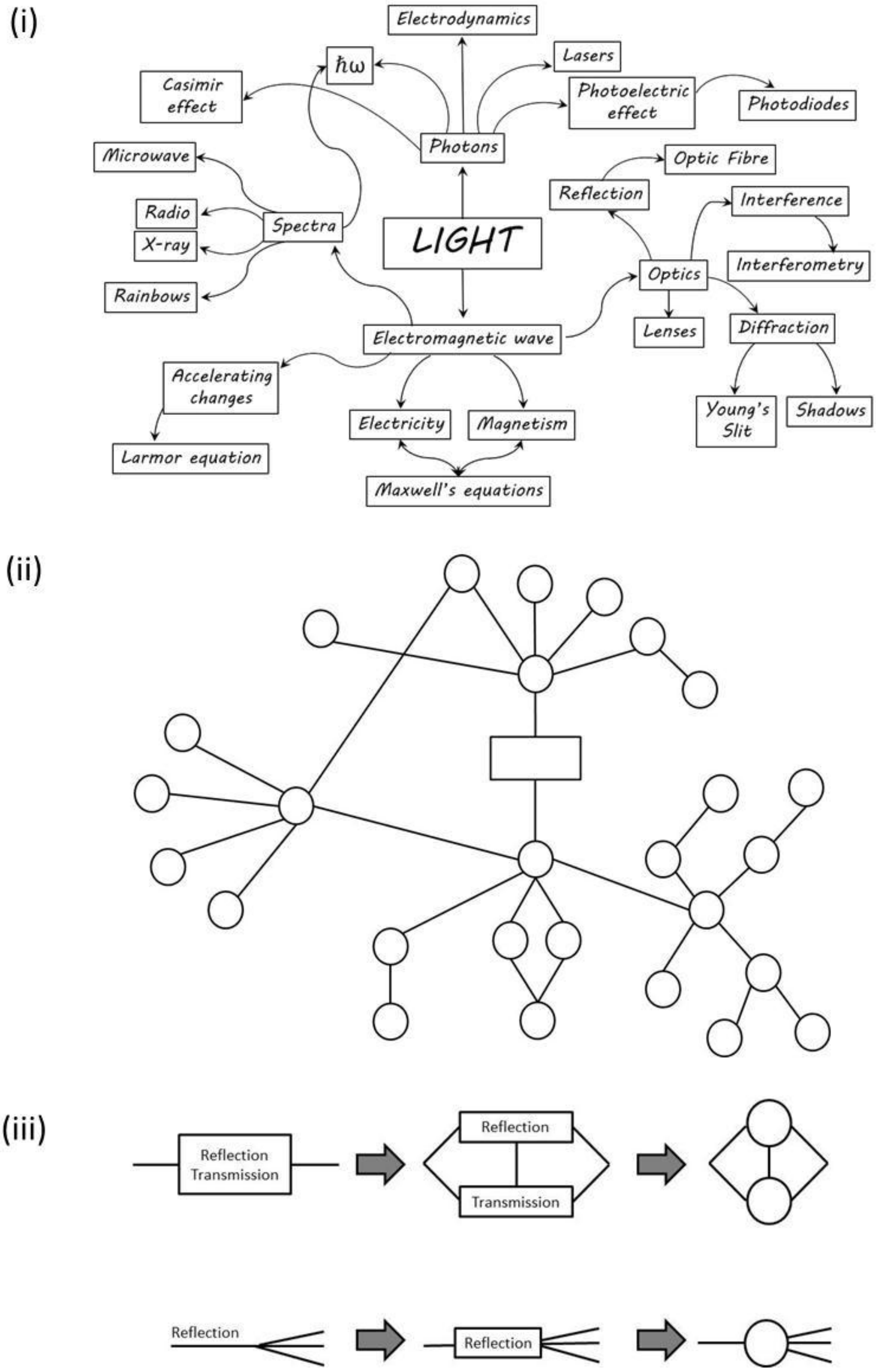

Fig. 1. The structural normalisation of concept maps. (i) Original student concept map (with link labels removed for clarity). (ii) Redrawn map, all labels removed, concepts represented by open circles and root concept by a rectangle. (iii) Splitting of combined concepts and elevation of links to concepts.

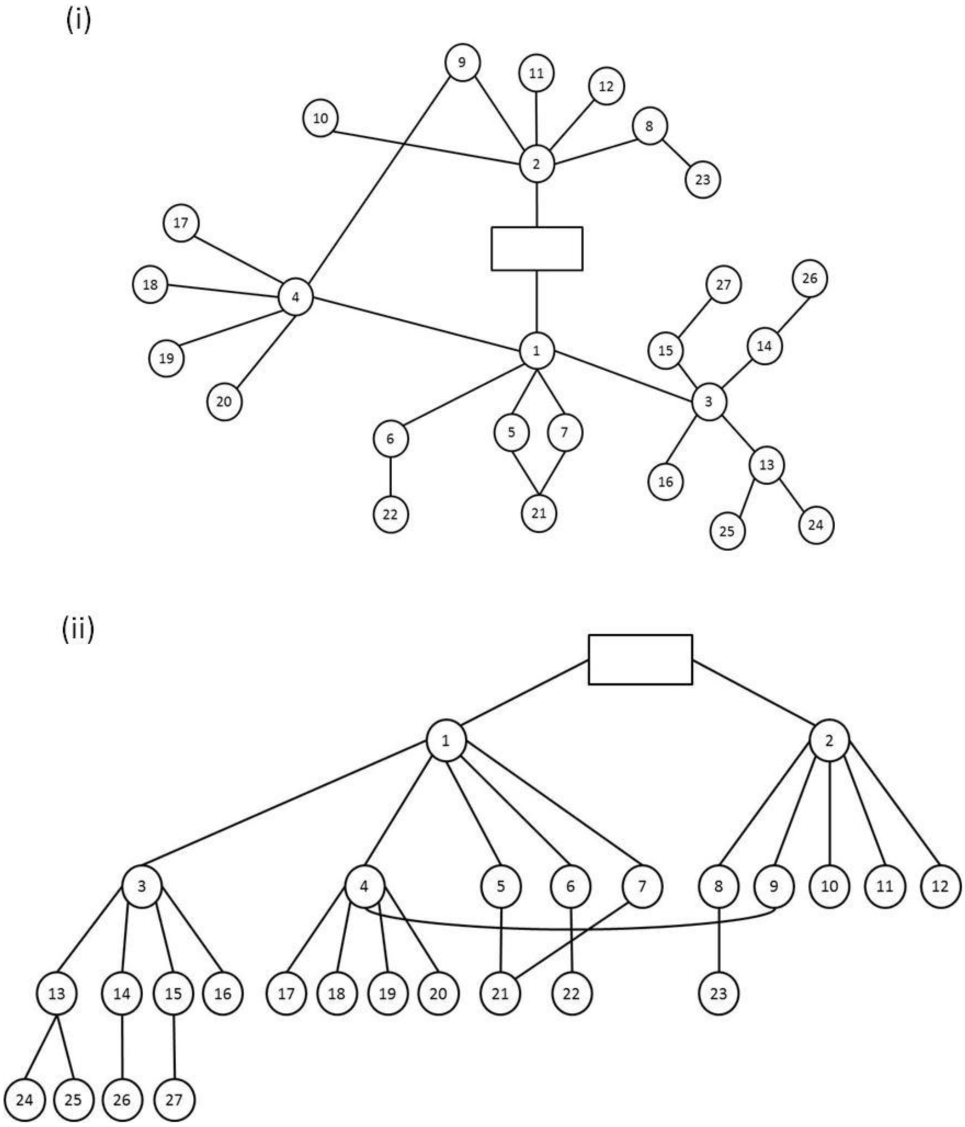

Fig. 2. Topological normalisation of content-free maps. (i) Content-free concept map with the concepts numbered to illustrate their repositioning in (ii) the topologically normalised version.

Due to the presence of cross-links, some concepts can alternatively be assigned to two or more branches. To render the procedure unique, a given concept is always assigned to the branch with the smaller number of existing concepts.

This topological normalisation results in a unique representation where concept maps appear as maximally balanced, skewed maps with longer, heavier branches on the left and shorter, lighter branches on the right.

The normalised maps provided a convenient starting point for the quantitative analysis. This step focusses on the pure structure of the maps without referring to any content or even specific geometrical layout.

4. Quantitative analysis

From a mathematical point of view, the normalised maps are graphs: collections of vertices (or concepts) and edges (or cross links) (Gould, 1988). Drawing on ideas of graph theory, the structural complexity of concept maps can be quantified via characteristic parameters. These parameters and their potential relevance will be introduced in everyday terms with some more precise mathematical definitions of the parameters as appropriate. We begin with some basic structural parameters which can be directly read off the topologically normalised concept maps. They are illustrated in Fig. 3.

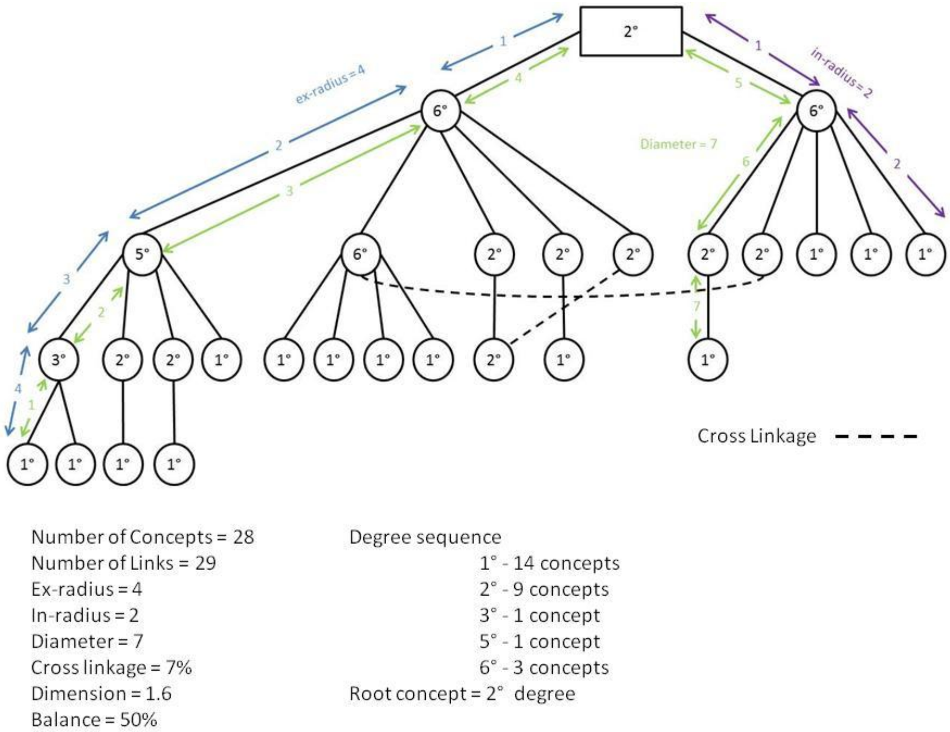

Fig. 3. Quantitative analysis of normalised concept maps

Number of concepts. For any mapping task without pre-given concepts, the most basic structural parameter is the number of concepts. Mathematically, it corresponds to the order of a graph which is defined as the number of vertices. This is simply counted directly from the normalised maps. To facilitate accurate comparison of even this simple parameter across different maps, care must be taken to consistently assign concepts in the case of duplication or elevation of labels to concepts, recall Fig. 1(iii). For the concept map used to illustrate this process, there are 28 concepts including the given source concept. The maps generated by our exemplar Physics students displayed a large variability having between 20 and 70 concepts. Different disciplinary contexts and map-creation settings may lead to different observed ranges.

Number of links. The number of links, mathematically defined as the size or number of edges, reflects the connectedness of the concept map. Again, this is simply counted directly from the normalised maps, and in the case of the concept map used to illustrate this process there are 29 links. Other maps of our exemplar students exhibited values for the number of links that were slightly higher than those for the number of concepts.

The number of links lies at the heart of content-based scoring criteria for concept maps (Novak & Gowin, 1984), where it represents the number of correct propositions. In this purely structural analysis, the emphasis is on the perception of a connection by a student, irrespective of whether the student has fully developed the corresponding precise proposition.

Diameter. The diameter is the greatest distance across the map in any given direction, with the distance between two concepts being the number of links along the shortest route connecting them (Sanders et al., 2008). Once again this is relatively easily determined for normalised concept maps by starting in the bottom left hand corner. This position represents the terminal end of the longest branch and by counting the links from here through the root concept at the top to the longest branch on the right of the map without doubling back through a concept gives the map diameter. Note that a large number of cross-links may complicate the identification of the appropriate starting and end points of the chain marking the diameter. In the case of the example map the diameter is 7 (Fig. 3), other observed diameters of our exemplar sample were between 5 and 10.

Radii. The in-radius and ex-radius measure the minimal and maximal distances from the root concept to the periphery of the map, respectively. In the normalised concept map, the in-radius is the number of links between the source concept and the terminal end of the right-hand chain; in the case of the example this is 2. The ex-radius is the number of connections between the source concept and the terminal end of the left-hand chain; in the example this is 4 (Fig. 3). We have found in-radii between 1 and 2 and ex-radii between 3 and 6.

Degree sequence. Finally, the degree of any given concept on a map is the number of other concepts to which it is connected. Koponen and Pehkonen (2008) refer to degree 1 concepts as outliers, degree 2 concepts as junctions and higher-degree concepts as hubs. The degree of the individual concepts on the example map is shown as the figure in each concept in Fig. 3. The relative number of outliers (degree =1), junctions (degree =2) and hubs (degree ≥3) in any map points towards the maps' connectivity and gross structural organisation. In the case of our example map, there are 14 concepts with a degree of 1 that could be called outliers (marked as 1°), 9 concepts with a degree of 2 that could be called junctions (marked as 2°), 1 concept with a degree of 3 (marked 3°), 1 concept with a degree of 5 (marked 5°) and 3 with a degree of 6 (marked 6°). Each of the 5 concepts with a degree sequence of greater than 3 could be considered hubs in the example concept map (Fig. 3).

The reliability of determining the degree sequence can be enhanced by using check-sums which are known mathematical identities for any connected concept map. One has:

$$\sum_k (\textit{number of concepts of degree } k) = \textit{number of concepts}.$$

The symbol $\sum_k$ stands for a summation over all relevant numbers k = 1, 2, 3 ..., meaning that we sum the numbers of concepts of degree 1, degree 2, degree 3 etc. In our example map of Fig. 3, the degree sequence adds to 14 + 9 + 1 + 1 + 3 = 28. This equals the number of concepts on the map, so the check-sum testifies that we have not omitted the degree of any concept on the map. A second useful identity reads:

$$[\sum_k k \times (\textit{number of concepts of degree } k)] / 2 = \textit{number of links}.$$

For our example, one calculates [1 × 14 + 2 × 9 + 3 × 1 + 5 × 1 + 6 × 3]/2 = 29, which is indeed the number of links. This confirms that we have not miscounted the degree of any concept.

The basic characteristic parameters described above are all read off directly from the normalised concept maps, with the normalised topology simplifying measurement. These basic parameters can then be used to calculate higher parameters which convey more complex structural information.

Cross-linkage. The cross-linkage is the number of links which are not required to hold the concept map together relative to the total number of links. Note that this value is unique whereas the decision as to which particular link is a cross-link is not. This is then the number of links that can be removed without leaving a disconnected concept or fragmented map. It can be calculated by means of the formula:

$$\textit{cross-linkage} = (\textit{number of links} - \textit{number of concepts} + 1) / (\textit{number of links}) \times 100\%.$$

In the example concept map 29 – 28 + 1 = 2 of the 29 linkages (for instance, those marked with dashed lines in Fig. 3), could be removed without leaving unattached isolated concepts. Thus this map has 2/29 × 100% = 7% cross-linkage. Cross-linkages across our exemplar samples varied between 0% and 30%. Note that the students of our exemplar sample were novice concept mappers; one might expect higher values for mappers with more experience with the tool.

The number of cross-links is again a central element of Novak and Gowin's (1984) original scoring scheme. In their content-based scheme, a cross-link has to connect distinct parts of a concept map. In our purely structural scheme, the ambiguity as to which link is a cross-link is lifted.

Dimension. The dimension is a parameter relating the number of concepts (i.e., volume) with the diameter of a concept map. This is based on the relation of diameter and volume in Euclidean space and inspired by the notion of fractal dimension (Mandelbrot, 1967). The formula that relates the diameter to the volume in this context is: $(\textit{diameter} + 1)^{\textit{dimension}} = \textit{number of concepts}$. Solving this equation, the dimension can be calculated from the formula:

$$\textit{dimension} = \log(\textit{number of concepts}) / \log(\textit{diameter} + 1).$$

The nature of this relationship means that a 'simple' chain has a dimension 1 while a concept map which is full of nearest-neighbour connections has dimension 2. Concept maps with a large number of cross-links beyond nearest-neighbour connections can easily reach dimensions larger than 2. A map of dimension 1.5 has a structure somewhat between a chain and a two-dimensional web. Maps with a high dimension have a small diameter in relation to their volume and are therefore more interconnected. Note the example map has a dimension of 1.6 and our exemplar student concept maps had dimensions between 1.4 and 1.7.

A simple interpretation of dimension is that maps of dimension 1 are dominated by linear, chain-like structures, maps of dimension 2 typically exhibit branches dominated by a high proportion of nearest-neighbour-links with few cross-links beyond this and higher-dimensional maps indicate a high degree of inter-connectivity.

Balance. The balance of a concept map is the ratio between its in- and ex-radii. It is a measure of how balanced the generally skewed topologically normalised maps are. This is calculated by:

$$\textit{balance} = (\textit{in-radius} / \textit{ex-radius}) \times 100\%.$$

A perfectly balanced map with all branches exhibiting equal depths would have a balance of 100%, the lower the balance percentage the more imbalanced or skewed the map is. The example map (Fig. 3) has an in-radius of 2 and an ex-radius of 4, giving a *balance* = (2/4) × 100% = 50%. Typical balances in our sample ranged from 20% to 100%.

To summarise, the quantitative analysis of the parameters described above (obtained from topologically normalised concept maps with the content removed) provides a robust and repeatable way of comparing the structure and topology of individual concept maps. These parameters and the standardised graphical structures of concept maps can then be used to identify differences in global map morphology.

5. Morphological analysis

The topological normalisation process described above not only facilitates quantitative analysis; it also lays the foundation for an inspection of characteristic global morphological appearance which is not biased by the mapper's individual geometrical layout. While the topographical normalisation produces a standardised gross morphology, it is possible to discern various characteristic map morphologies from the normalised maps. These are related to the maps' original structure and characterised by certain indicative quantitative parameters taken from those described above. Hence, the combined use of the normalised maps and the associated quantitative analysis to calculate the defining parameters facilitates the comparison of gross morphology. The use of the topologically normalised maps (which have a unique form that retains the concepts, the links and their relationship) allows fundamental morphology to be compared more easily; free from the complications of the infinite possible morphological variations of the original maps.

In order to investigate map morphology, one first normalises the maps and derives the parameters as described above. With this data for the maps under consideration the next stage is to look at the size of the maps, using the number of concepts to get the range of sizes and establish which maps are small and large in the range defined by a given set of maps.

Next, one uses the root degree, in- and ex-radius to place it on the Broad–Deep continuum: Broad maps are relatively wide compared to their depth. In their standardised form they generally have many links emerging from the root concept which consequently characteristically exhibits a high degree. They also are shallow rather than deep and so exhibit a characteristically small ex-radius. They can have a lot of content but little development away from the root concept, hence their broad structure with characteristic short chain development. A Broad map is illustrated in Fig. 4(i). Broad maps are dominated by spoke-like structures, so that this class is closely related to Kinchin, Hay, and Adams's (2000) spoke morphology class. In contrast, Deep maps exhibit long chains relative to their breadth. This is characterised by relatively large ex-radius with a high proportion of degree 2 concepts which are junctions in chains. A Deep map is illustrated in Fig. 4(ii). The dominant structural elements of such maps are long chains, making them closely related to Kinchin, Hay, and Adams's (2000) chain morphology.

The next stage is to use the balance data to determine how balanced the maps are. The topological normalisation of original concept maps results in generally skewed maps with longer, heavier branches on the left and shorter, lighter branches on the right. However, in a given range of concept maps, some will be more balanced than others and even with the normalisation some maps are completely balanced (balance =100%) with their left side (ex-radius) and right side (in-radius) of equal length. Imbalanced maps tend to be relatively small and undeveloped overall, but display regions of more detailed, deeply developed knowledge; for example, areas in which the mapper has a particular interest. The contrast between the more developed areas on the left of the normalised maps and the less developed areas results in a characteristically unbalanced map. An unbalanced map will therefore display a lower-than-average balance value; this is illustrated in Fig. 4(iii).

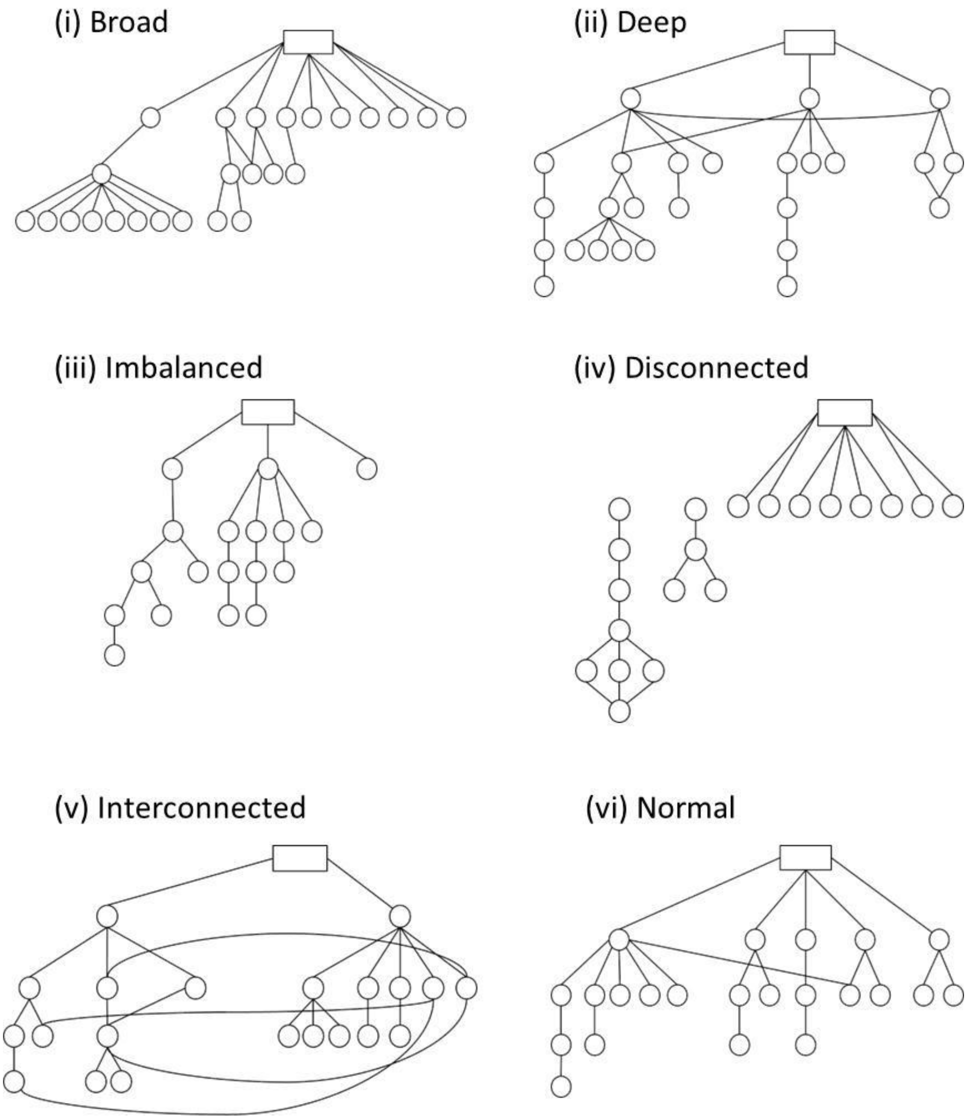

Fig. 4. Example student concept maps illustrating common morphological classes in topologically normalised concept maps

Lastly, one may examine cross-linkage and dimension data to determine how connected maps are. Again there will be a range in any given sample with some maps being relatively unconnected while others are more interconnected than average for the sample. Disconnected maps are examples of extreme disconnection in that they exhibit disconnected areas of knowledge. They tend to have large proportions of concepts with degree of 1 or 2 and few of above 3. They have few hub concepts and greater proportions of outliers. In particular, disconnected maps have small isolated groups of concepts not connected to the root concept. A disconnected map is illustrated in Fig. 4(iv). In contrast, Interconnected maps are morphologically distinguished by their high degree of interconnectedness. They may vary in size but are often relatively large and well developed. They are characterised by possessing many cross-links with a well connected network of concepts, characterised by relatively large cross-linkage and dimension values. An Interconnected map is illustrated in Fig. 4(v). Interconnected maps bear an obvious resemblance to both Hay and Kinchin's (2006) network class and Koponen and Pehkonen's (2008) connected webs.

While the calculated parameters should not be used in too rigorous a manner as indicators of characteristic morphology, they are useful in aiding the visual inspection of normalised maps to identify characteristic global morphological classes. The actual values of these parameters will depend on the context, what is being mapped and who is mapping it, but their relative values are useful together with appearance to identify important morphological classes.

The common classes:

- **Broad**, key indicators: root concept exhibits a high degree and the map has a small ex-radius.
- **Deep**, key indicators: a smaller root degree, large ex-radius and a high proportion of degree 2 concepts.
- **Imbalanced**, key indicator: a small balance value.
- **Disconnected**, key indicators: very low cross-linkage and dimension values, presence of isolated concepts which are not linked to root concept.
- **Interconnected**, key indicators: high cross-linkage and dimension values with large proportions of concepts with degree above 3.

These classes as illustrated using maps from our exemplar group are shown in Fig. 4. Table 1 shows the parameters derived from these maps to illustrate the relationship between these and the maps' characteristic appearance. It should be remembered that while the actual values will vary with context, their relative values are useful in defining these morphological classes for any given set of concept maps.

Of any given sample of concept maps, those that don't exhibit any of these more extreme morphologies can be considered 'normal' maps. These maps tend to be an intermediate between broad and deep, are relatively well balanced with an average chain length and number of cross-connections. Different areas of knowledge are equally well developed. Normal maps will vary with context, what is being mapped and who is mapping, but for capable students mapping a subject they know reasonably well they often correspond to the tree maps of Yin et al's (2005) extension to Kinchin, Hay, and Adams's (2000) classification scheme. They also correspond to the loose webs in Koponen and Pehkonen's (2008) alternative scheme. A 'normal' map from our exemplar group is shown in Fig. 4(vi) with corresponding data in Table 1 for comparison.

While we cannot claim that these morphological classes or patterns are universal, we have found them consistent when mapping different subjects (Physics, Pharmacology and Education) and with undergraduate students, postgraduates and staff. At the very least, this approach does enable a standardised and consistent morphological interpretation and comparison within any sample or context.

Table 1
Quantitative data for the example student concept maps shown in Fig. 4

	Concepts	Links	Diameter	Radius		Concepts of degree			Root degree	Cross-linkage	Dimension	Balance
				In-	Ex-	1	2	≥3				
Broad	25	26	6	1	3	16	3	5	10	8%	1.7	33%
Deep	27	29	10	2	5	11	10	6	3	10%	1.4	40%
Imbalanced	19	18	9	1	5	8	7	4	3	0%	1.3	20%
Dis-connected	21	20	5	1	1	12	5	4	8	0%	1.7	100%
Inter-connected	22	26	5	2	4	7	7	8	2	19%	1.7	50%
'Normal'	23	23	7	2	4	11	7	5	5	4%	1.5	50%

6. Discussion

This paper presents a standardised, systematic approach to topological and morphological concept map analysis to provide a consistent, repeatable and methodological robust platform on which to build subsequent content analysis and interpretation. The method presented removes the content and much of its interpretation from this stage of the analysis. The result is a standardised content-free version of concept maps (topologically normalised maps) that can be used to determine some relatively simple parameters which help to define and quantify the structural complexity of diverse concept maps in a reproducible standard way: number of concepts and links, diameter, in- and ex-radius, degrees, cross-linkage, balance, dimension. Combining visual inspection of the normalised maps with the information gained from the quantitative parameters, we have defined distinct characteristic global morphologies: 'Disconnected', 'Imbalanced', 'Broad', 'Deep' and 'Interconnected' compared to 'Normal' maps. We have briefly indicated how they may be related to existing morphological classifications of non-normalised maps.

The proposed new approach to concept-map analysis on the basis of topological normalisation of content-free maps, structural parameters and global morphologies makes the inherent structure of concept maps tangible in a reliable way. It combines the strengths of traditional quantitative and qualitative approaches to concept-map analysis by being informed by quantitative parameters without neglecting the global morphology. Our framework can then form the basis for comparison and any subsequent analysis. It should be emphasised that at this stage what is being analysed is the shape and complexity of the structure of the concept maps independent from content. A content analysis can then be undertaken as a necessary second step where structure and content combine into a coherent whole and further dimensions like abilities and intent of the map-creator also have to be taken into account. Our intentional focus on knowledge structures should not be taken as implying that the correctness of propositions is not of concern. Rather, we wanted to consider a systematic topological and morphological analysis of a learner's understanding as mapped. One might look at the 'correctness' of the content after examining the morphology of the students' conceptual structure. Indeed, by performing the described topological and morphological analysis before and after removing 'incorrect' concepts and links one may get an idea of how much the 'incorrect' content influences the overall map structure. This may provide insight into the nature of any misunderstanding and be useful in both diagnosis and in subsequent support of the development of a more adequate conceptual structure and understanding.

Possible limitations of our scheme originate in the context in which it was conceived and its focus on pure structure. In particular, our morphological classification scheme with its finite set of distinguishing structural characteristics and its emerging main types was developed using concept maps from a very scientific context with its strongly hierarchical disciplinary structure. One may wonder how our scheme might change for instance in Humanities disciplines with their less hierarchical and more open-ended subject organisation. For instance, concept maps of a work of literature may match the structure of the narrative, while concept maps of history may follow a linear temporal structure. Topological normalisation applied in these cases may not be as revealing as in our structured scientific examples, but it could potentially reveal differences in the mapper's perception of the subject matter in a similar way. When applied with an inside understanding for a range of different disciplines, our method might even reveal structural differences between the subject matters. We encourage investigations in other disciplinary areas to further explore these possibilities.

Another potential issue arising from our focus on structure and the fact that topological normalisation takes the structure in a given map at face value is that particularly important sub-structures of a map such as threshold concepts (Meyer & Land, 2003) are not given the special attention they deserve. We have found indications that the relevance of threshold concepts does translate to some extent into characteristic structural attributes such as a high degree in the normalised maps; indeed, the normalisation process may help reveal them as key vertices. However, one has to always bear in mind that structure is only one side of the coin that is a concept map and never lose sight of the other side: its content.

The extent to which our proposed scheme can be used for cyclic concept maps is an interesting question. For cyclic maps that are part of a hierarchical concept map or for hybrid cyclic-hierarchical maps, we believe our method would still apply, but the additional structural information inherent in the cyclic part of the map may not be adequately represented. To extend the scheme to cyclic maps, one would have to introduce additional structural parameters such as the number of cycles, their parities (i.e., positive/negative feedback loops) and sizes and include the cyclic links in the topological normalisation. This would probably lead to new types of morphological classes which have not been observed for the present purely hierarchical concept maps. Once again we would encourage further work exploring this interesting area.

The proposed systematic approach to concept-map analysis is a standardised and reliable basis for interpreting topology and morphology and provides a consistent, reproducible and methodologically robust platform on which to build subsequent qualitative analysis and interpretation of map content. It may be viewed as an informed point of departure for interpreting the knowledge structures and learning that underlie concept maps.

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