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Devising a fairer method for adjusting target scores in interrupted one-day international cricket

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One-day international cricket matches face the problem of weather interruption. In such circumstances, a so-called rain rule is used to decide the outcome. A variety of approaches for constructing such rules have been proposed, with the Duckworth-Lewis method being preferred in the sport. There are a number of issues to consider in reasoning about the effectiveness of a rain rule, notably accuracy (does the rule make the right decision?) and fairness (are both teams treated equally?). We develop an approach that is a hybrid of resource-based and so-called probability-preserving approaches and provide empirical evidence that this hybrid method is superior in terms of fairness while competitive in terms of accuracy.

keywords: cricket, Duckworth-Lewis, modelling, target adjustment

1 Introduction

The development of one-day international cricket from traditional five-day 'test cricket' was fuelled by an obvious demand for a shorter, faster version of the game which guaranteed a result. By limiting the number of bowls delivered to each team, one-day international cricket encourages more aggressive batting and leads to exciting finishes, making it an extremely popular spectator sport. It is however particularly vulnerable to inclement weather. As cricket requires dry conditions, rain interruptions often lead to one or both teams not having sufficient time remaining to complete their allotted overs. Neither

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abandonment nor postponement are satisfactory solutions as both contradict the basic objective of one-day cricket – to produce a clear winner within one day. Quantitative methods are instead used to adjust target scores in matches when one or both teams face reduced innings so that the match can still be concluded with a definite result obtained. Such methods are referred to as *rain rules*.

A number of rain rules have been proposed; however, none have proven entirely satisfactory. The various early attempts (for a summary, see Duckworth and Lewis (1998)) proved too simplistic and tended to dramatically favour one or other of the teams. Modern, more sophisticated rain rules have been more successful. They are characterised by two distinct aspects: the conceptual basis on which target score adjustments are made; and the implementation of that conceptual basis through statistical modelling of the run-scoring process. Concerning the conceptual basis underpinning a rain rule, two rival schools of thought prevail. *Resource-based* rules attempt to correct for a loss of 'run-scoring resources' occurring due to a rain interruption, while *probability-preserving* rules attempt to adjust target scores so as to preserve the probability of victory for each team as it stood before the interruption took place. In practice, resource-based rules have dominated. Owing to the technical quality of the modelling used in the implementation of the Duckworth-Lewis (DL) method (Duckworth and Lewis, 1998), it has remained the preeminent rain rule in world cricket since its introduction in 1998, being preferred by the International Cricket Council to its most prominent (also resource-based) competitor, the VJD method (Jayadevan, 2002). Even so, the DL method still receives regular criticism, primarily concerning the conceptual justification of its resource-based reasoning. However, to date no probability-preserving rule has emerged as a strong competitor.

The rest of the paper proceeds as follows. In Section 2 we outline the DL method and describe the criteria we use to assess the performance of the rule. We then present empirical estimates of these criteria using a data set of uninterrupted one-day international cricket matches. In Section 3 we construct a model for a probability-preserving rain rule and empirically compare its performance to both an existing probability-preserving rule and the DL method. In Section 4 we describe a further criterion for evaluating the performance of a rain rule and present empirical estimates for the three rules. Section 5 concludes.

2 The Duckworth-Lewis Method

The idea behind the DL method is to compensate teams for a loss of *run-scoring resources*. A team's ability to accumulate runs is constrained by two factors: overs remaining and wickets in hand. The process of setting a high target (equivalently, chasing a target) can be viewed as an exercise in trying to manage in an optimal way the depletion of these two resources. A team can employ a strategy of batting conservatively in order to minimise the risk of losing wickets, but in doing so increases the risk of setting a low target (equivalently, failing to reach the target by the end of the innings). Conversely, by batting aggressively a team can accumulate runs quickly but increases its risk of losing all 10 wickets with bowls left spare. At the beginning of an innings, with

50 overs remaining and 10 wickets in hand, a team has 100% of its run-scoring resources available. The DL method attempts to quantify any combination of overs and wickets remaining in terms of overall run-scoring resources remaining.

To do this, Duckworth and Lewis (1998) proposed an exponential decay function for Z, the average number of runs that will be scored with u overs remaining and w wickets in hand.[1] The model takes the following form:

$$Z(u, w) = A(w)\left[1 - e^{B(w)u}\right], \tag{1}$$

where $A(w)$ and $B(w)$ are parameters to be estimated, the forms of which have been updated periodically since the method's introduction to reflect changing patterns of scoring – see Stern (2016) for more details. Using (1), Duckworth and Lewis interpret the quantity

$$P(u, w) = Z(u, w)/Z(50, 10), \tag{2}$$

as the proportion of 'run-scoring resources' remaining with u overs remaining and w wickets in hand. When play resumes after an interruption, the number of overs remaining is reduced but not the number of wickets in hand. The DL method proceeds by adjusting the target score in line with the proportion of run-scoring resources lost due to the interruption. For example, suppose the team batting first (henceforth referred to as Team 1) experiences no interruption in its innings and scores S_1 runs. Suppose further that after 10 overs of the second innings, the team batting second (Team 2) has lost one wicket and rain causes the game to be shortened by 10 overs. We wish to calculate the percentage of resources the team has lost by going from 40 overs and 9 wickets remaining to 30 overs and 9 wickets remaining. Using (2), the total resources available to Team 2 is $R_2 = 1 - [P(40, 9) - P(30, 9)]$. As Team 1 faced no interruption, $R_1 = 1$ and so the target score should be rescaled to $T = S_1 \cdot R_2$. In general, the total resources available to each team is calculated and the target score is adjusted to $T = S_1 \cdot R_2/R_1$.

2.1 Assessing the Performance of the DL Method

Our data set comprises 741 uninterrupted one-day internationals which took place between 1 July 2007 and 30 January 2015. Ball-by-ball data was obtained from www.cricsheet.org. Using this data, we follow the work of Schall and Weatherall (2013) to empirically evaluate and compare the performance of rain rules. Schall and Weatherall (2013) suggest that interrupted cricket matches present a type of missing data problem: we cannot observe the outcome of the complete match, but observe instead the outcome of the interrupted match as determined (at least in part) by the rain rule. Viewing the outcome of the interrupted match as an imputation of the missing outcome of the complete match, Schall and Weatherall (2013) consider the joint distribution of the outcomes of the complete and interrupted matches to motivate criteria for the assessment of the performance of rain rules.

[1] Note that Duckworth and Lewis use w to denote wickets *lost*. For consistency of notation, we reserve w for wickets *remaining* and use it as such throughout the paper.

The application of a rain rule to deal with premature termination of a match provides the framework for empirically estimating these criteria. Suppose the first innings is completed without interruption and Team 2 begin their innings expecting to receive the full 50 overs. Subsequently, rain suspends play after i overs and persists sufficiently long for further play to become impossible and the match is terminated. In this case the outcome of the match is determined explicitly by application of a rain rule, based on the 'state of play' at the time of the interruption.

For any match in our data set we can impose artificial rain-terminations at any stage of the second innings and compare what the corresponding imputed outcome would be under application of a rain rule with the actual outcome of the completed match. This allows us to empirically estimate two properties of a rain rule:

Overall Accuracy: the probability that the imputed outcome of the match is the same as the unobserved outcome of the complete match. Estimates of overall accuracy are computed as the number of matches in which the second innings lasted at least i overs and the imputed outcome at the end of the ith over equals the actual outcome divided by the number of matches in which the second innings lasted at least i overs.

Conditional Accuracy: the probability that the interrupted match is won by Team t, $t = 1, 2$, *conditional* on the complete match being won by Team t. Estimates of Team t conditional accuracy are computed as the number of matches actually won by Team t in which the second innings lasted at least i overs and Team t won under the imputation divided by the total number of matches actually won by Team t in which the second innings lasted at least i overs.

International Cricket Council rules (ICC, 2015) specify that a minimum of 20 overs per innings must be played to constitute a valid match. As such, to assess how the accuracy is affected by the timing of the interruption, we compute our estimates of the overall and conditional accuracies of each rule at the end of each over from the 20th to the 49th over. Schall and Weatherall (2013) prove that for a rain rule to be *fair*, both the conditional accuracies should be equal, i.e. the rain rule should not systematically favour one team over the other.

Figure 1 shows how the overall and conditional accuracies of the DL method vary with the timing of match termination for our data set. It is immediately clear that the DL method is not fair, according to this definition. The conditional accuracy for Team 1 is consistently higher than that of Team 2, suggesting that the DL method systematically favours the team batting first.

3 Probability-Preserving Rain Rules and a New Hybrid Rule

Probability-preserving rain rules were first proposed by Preston and Thomas (2002) and Carter and Guthrie (2004), following the work of Clarke (1988) and Preston and Thomas

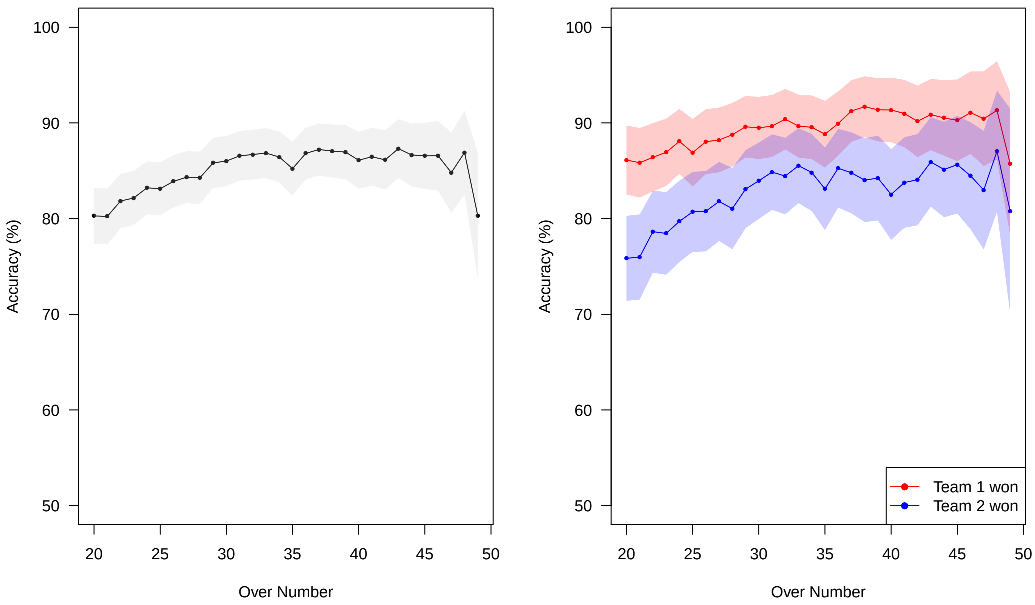

Figure 1: DL method overall accuracy (left) and conditional accuracies (right). 95% confidence intervals are indicated by the shaded areas.

(2000), who used dynamic programming to establish optimal run-scoring rates in either innings. They found that a team's objective is to maximise its probability of victory, and so "a rain rule which adjusts runs so as to preserve teams' advantage judged according to their own objectives appears. . . to capture well the notion of fairness" (Preston and Thomas, 2002, pg. 2). Furthermore, Preston and Thomas (2002, pg. 4) claim that "[i]t is only for rules within this class that the anticipation of rain will give batting teams no incentive to change strategy" and so the DL method is therefore not tactically neutral.

We agree with Preston and Thomas and believe that the probability-preserving approach is a conceptually superior basis for a rain rule in terms of fairness. This is reflected, in a sense, through our chosen method of assessment – we believe that in prematurely terminated matches, a rain rule should attempt to forecast which team would have won the complete match and so the performance of rain rules should be assessed based on an evaluation of the accuracy of such forecasts. It should be noted that the function of resource-based rules is not technically to forecast the actual winner; it is to award victory to Team 2 if the proportion of the target score achieved is greater than the proportion of run-scoring resources used (as measured by the rule) at the time of termination. Getting the 'right' result often is nonetheless a desirable property of any rain rule and one which will depend on the technical quality of the modelling employed

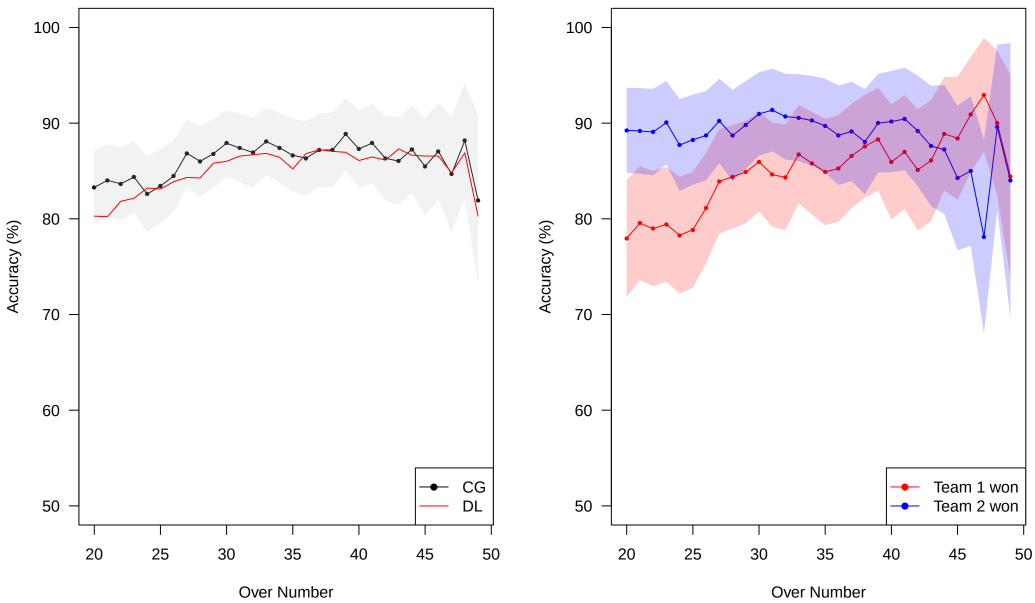

Figure 2: CG method overall accuracy (left) and conditional accuracies (right). 95% confidence intervals are indicated by the shaded areas.

by that rule. Prior to now, no probability-preserving rule has been shown empirically to outperform the DL method.

3.1 An Existing Alternative Method

The most prominent probability-preserving method is the Carter-Guthrie (CG) method (Carter and Guthrie, 2004). They use a dynamic programming procedure to estimate the cumulative distribution function $F(r\,;\,b,w)$ for the number of runs r a team will score with b bowls remaining and w wickets in hand; that is, a team will score r runs or fewer with probability $F(r\,;\,b,w)$. Using this, a team's probability of victory can easily be calculated at the point of interruption. When play resumes with a reduced number of bowls remaining, the target score is adjusted so that each team's probability of victory is unchanged. If play cannot resume, the match is awarded to the team with the higher probability of victory, provided a sufficient portion of each innings has been completed. The rule has been shown to readily provide sensible target score adjustments in the cases of single or multiple interruptions to either or both innings. For a detailed explanation of the procedure, see Carter and Guthrie (2004).

Schall and Weatherall (2013) compare a number of rain rules including the DL method and its most notable rival, the VJD method (Jayadevan, 2002). They conclude that the DL method is superior to its best competitors in terms of both accuracy and fairness. However, the CG method comes in close second, despite being calibrated using only a small number of matches from the 1999 World Cup. As such Schall and Weatherall (2013) suggest that the CG method may benefit from recalibration using a larger and more representative data set. We do just this, following closely the procedure detailed in Carter and Guthrie (2004, pg. 826-827) to estimate a cumulative distribution function $F(r\,;\,b, w)$. We split our data set of uninterrupted matches in half and use one half for estimation of parameters (the training set) and the other half for assessing the performance of the model (the test set). The loss of half our data for estimation purposes is not a concern; Carter and Guthrie estimated their model using data from only 26 matches.

Figure 2 shows the accuracy and conditional accuracies of the recalibrated CG model. We see that it achieves arguably superior levels of accuracy to the DL method although the difference is not statistically significant at the 5% level of significance. Fairness is still an issue however, especially early on in the innings.

3.2 A Hybrid Method

We build on the work of Carter and Guthrie to propose a 'hybrid' method which is probability-preserving in its implementation but which exploits the technical quality of the DL model (1) in its model estimation. Our model differs to that of Carter and Guthrie in two respects. First, we highlight and address an error in Carter and Guthrie's variable selection. Second, we extend the work of Carter and Guthrie by estimating separate distribution functions $F_1(r\,;\,b, w)$ and $F_2(r\,;\,b, w)$ for the run-scoring process in the first innings and second innings respectively to account for differing patterns of play.

3.2.1 Variable Selection

We note that to estimate the probability of losing a wicket or scoring $n = 0, 1, \ldots, 6$ runs on a given bowl as functions of balls remaining and wickets remaining, Carter and Guthrie (2004) employ probit and ordered probit models respectively, in both cases including w, wickets remaining, as a predictor variable. For example, in the wicket loss case, let y be an indicator variable which takes the value 1 if a wicket falls and 0 otherwise. Carter and Guthrie (2004) fit the probit model:

$$y^* = \beta_0 + \beta_1 b + \beta_2 w + \beta_3 b^2 + \varepsilon, \tag{3}$$

where $\varepsilon \sim \mathrm{N}(0, 1)$, and suppose that a wicket falls if and only if $y^* > 0$, which occurs with probability

$$\mathrm{P}(y = 1 \mid b, w) = \Phi(\beta_0 + \beta_1 b + \beta_2 w + \beta_3 b^2),$$

where Φ is the cumulative distribution function for the standard normal distribution.

In (3) and in a similar ordered probit model, w is treated as a continuous predictor variable, i.e. all other variables held constant, the estimated coefficient is interpreted as the effect of a one-unit change. By doing so, Carter and Guthrie fly in the face of all previous literature on rain rules. The great accomplishment of the DL method was to model the effect of losing a wicket on a team's ability to accumulate runs and, crucially, how this effect depends both on which wicket is lost and *when* in the innings it is lost. Asif and McHale (2016) point out the need therefore to allow the relationship between the dependent variable and w to be non-linear. We achieve this by replacing wickets remaining w with a new variable, *wicket resources* remaining, $\rho(b, w)$, where we define wicket resources remaining as the ratio of the remaining runs to be scored with b balls remaining and w wickets in hand to the remaining runs that would be scored if b balls were remaining but there were still all 10 wickets in hand. We use the DL model (1) to compute remaining runs to be scored, to yield

$$\rho(b, w) = \frac{Z(b/6, w)}{Z(b/6, 10)}.$$

Figure 3 shows a plot of the relationship between w and $\rho(b, w)$ as the innings progresses. While w only takes discrete values, $\rho(b, w)$ takes rational values between 0 and 1. One can clearly see that the relationship becomes less linear towards the end of the innings. For example, with only five overs remaining, the impact of losing a wicket on overall run-scoring resources depends strongly on which wicket is lost; it is much greater if a team has only 2 wickets remaining than if that team has 8 wickets remaining, say.

3.2.2 Modelling the Second Innings

There are a number of reasons to believe that the run scoring distribution differs in the two innings. Preston and Thomas (2002) argue that optimum strategies differ significantly between innings. Stern (2009) goes further and provides empirical evidence that the run-scoring pattern differs in the second innings from the first. Modelling the run-scoring process in the second innings is complicated however by the fact that play stops once Team 2 surpass the target score, even if there are balls remaining to be bowled. We cannot observe how many runs Team 2 would have scored had they carried on batting until they had used up all their overs or lost all their wickets. In order to overcome this problem, instead of modelling how teams tend to bat in the second innings on average, we attempt to model how teams in the second innings bat, *conditional on the target being chased*.

To do this we use weighted estimation for our probit and ordered probit models. In our first innings model, each observation (each delivery) in our training set has equal weight. To model the run-scoring process in the second innings, each observation is assigned a weight which is a function of the target which was being chased in the innings from which the observation came. We require a rule to assign weights such that every observation is given some positive weight but more importance is given to observations which are taken from games in which similar targets were being chased. We choose the probability density function of the beta distribution as it always assigns values only between 0 and 1.

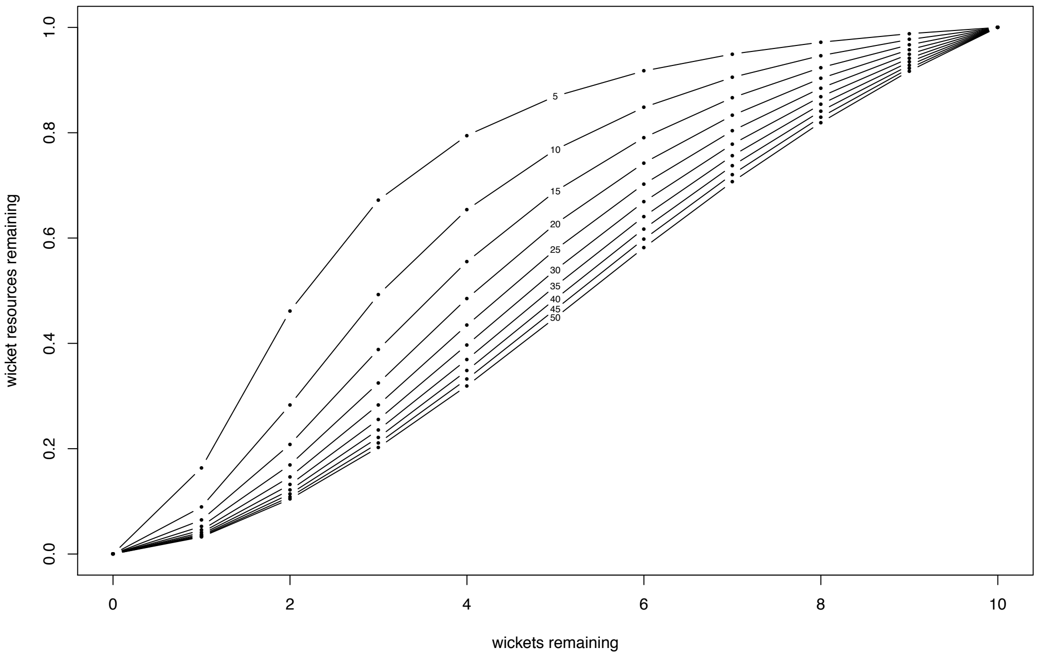

Figure 3: Relationship between wickets remaining and wicket resources remaining for each $u = 50$ (bottom line), $45, \ldots, 10, 5$ (top line) overs remaining.

We first reparameterise the density function so that it can be defined by its mode γ. Note that the mode of the beta distribution is a linear function of its two positive shape parameters. If one shape parameter is fixed at the mode, the value of the remaining parameter controls the distribution's variance. We can therefore define a single 'dispersion indicator' η, which controls how much the distribution collapses around the mode; large values of η correspond to small variance and vice versa. This allows us to parameterise the density function in terms of γ and η. Our beta weight function is thus

$$f(x\,;\,\eta,\gamma) = \begin{cases} \frac{1}{\mathrm{B}\left(\eta, \frac{1-\gamma}{\gamma}\eta - \frac{1-2\gamma}{\gamma}\right)} x^{\eta-1}(1-x)^{\frac{1-\gamma}{\gamma}\eta - \frac{1-2\gamma}{\gamma} - 1} & \text{if } \gamma \le 0.5 \\ \frac{1}{\mathrm{B}\left(\frac{\gamma}{1-\gamma}\eta - \frac{2\gamma-1}{1-\gamma}, \eta\right)} x^{\frac{\gamma}{1-\gamma}\eta - \frac{2\gamma-1}{1-\gamma} - 1}(1-x)^{\eta-1} & \text{if } \gamma > 0.5. \end{cases} \tag{4}$$

By rescaling the target scores in our data set to lie between 0 and 1, we can use this parameterisation to position a beta density on the target score of our choice and generate weights for data from all other games depending on the similarity of the corresponding target score, while controlling how narrow or wide we want the distribution to be. Figure 4 illustrates how this is applied. Overlaid on the histogram of target scores are seven beta distributions, all with $\eta = 8$, centred at various target scores. For each game on the histogram, the weight assigned to it is the density of the beta distribution. Note that

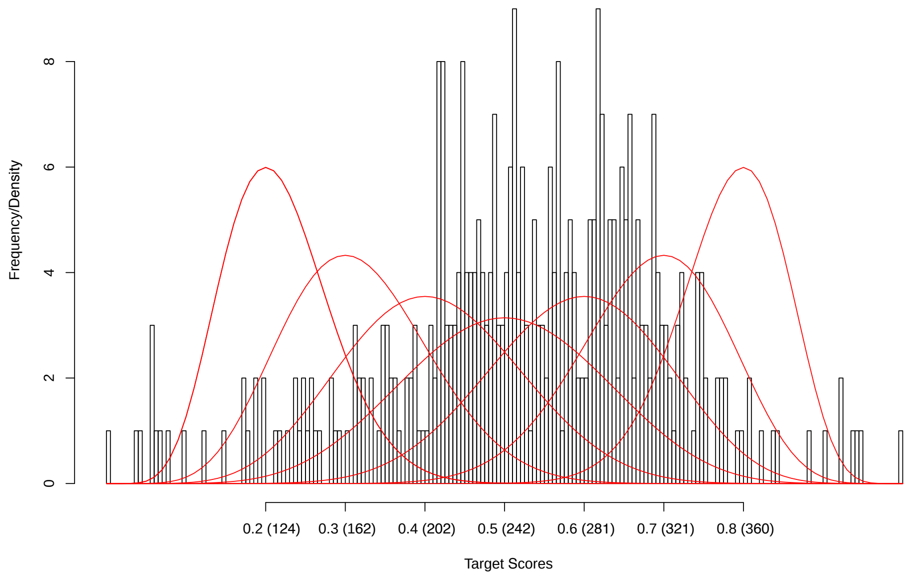

Figure 4: Using the beta density function to assign weights to observations from matches depending on the target score being chased. The target scores have been rescaled to lie between zero and one. On the horizontal axis the corresponding actual target score is given in parentheses. Overlaid on the histogram are seven beta distributions with $\eta = 8$ and $\gamma = 0.2, 0.3, \ldots, 0.8$.

for high scoring matches, say, only matches in which Team 2 were chasing similarly high target scores are given significant weight and low scoring matches are given almost zero weight. By adjusting η, we can control how the weights are distributed.

For any given η, we use locally weighted maximum likelihood estimation (Anselin et al., 2013, pg. 229) on our training set to estimate our probit and ordered probit models by assigning to each observation a (normalised) weight using (4). We construct cumulative distribution functions $F_{2,j}(r\,;\,b, w)$ for every unique target score j in our test set. When an interruption/termination occurs, Team 2's probability of victory is calculated using $F_{2,T}(r\,;\,b, w)$ for target score T. A cross-validation procedure can then be used to select the optimal η for the weight function which achieves highest accuracy. In practice, this is computationally very expensive. As such, we constructed cdfs for dispersion parameters 4, 8 and 12 and preferred 8 based on superior accuracy and conditional accuracy.

The accuracy and conditional accuracies are shown in Figure 5. We can see that our hybrid method achieves as high levels of accuracy as both the DL method and the original CG method but superior levels of fairness. The two conditional accuracies are less separated and their difference is not statistically significant at 5% significance level.

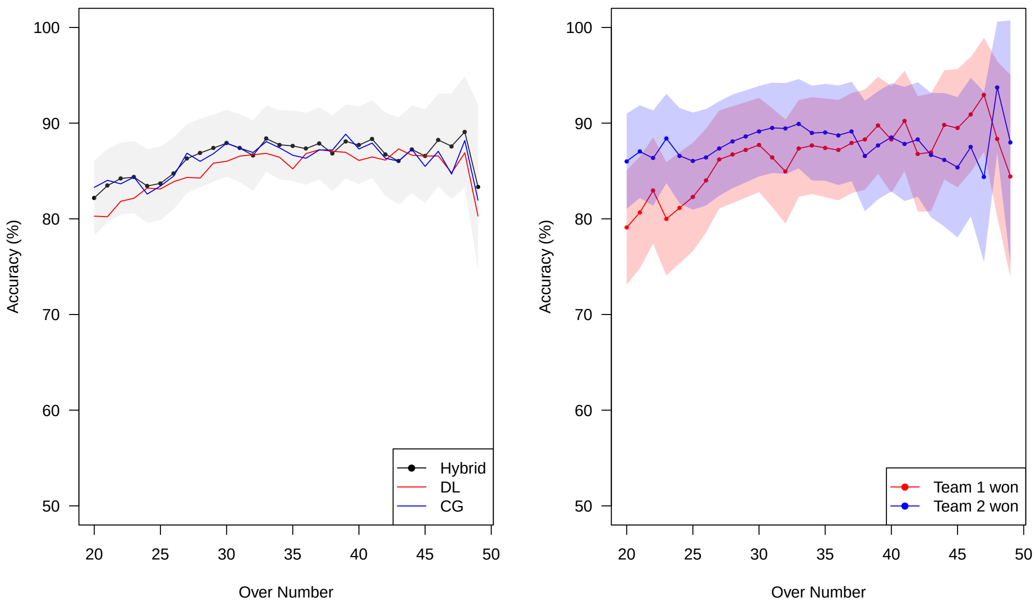

Figure 5: Hybrid method overall accuracy (left) and conditional accuracies (right). 95% confidence intervals are indicated by the shaded areas.

4 Predictive Value of Rain Rules

We have estimated the conditional accuracies of each rain rule; that is, the probability that the interrupted match is won by Team t, $t = 1, 2$, *conditional on the complete match being won by Team* t. In reality, when a match is interrupted we do not observe the winner of the complete match. We are interested therefore in a further property of rain rules:

Predictive Value: the probability that the complete match is won by Team t, $t = 1, 2$, *conditional on the interrupted match being won by Team* t. In binary classification, these conditional probabilities are referred to as the negative ($t = 1$) and positive ($t = 2$) predictive values.

Terminated games in which one team or the other is declared the winner by a narrow margin under application of a rain rule are typically the most controversial. We are further interested therefore in estimating the predictive value of a rain rule as a function of the winning margin of the interrupted match. We define the winning margin of the interrupted match as

$$d_i = R_{2,i} - T_i + 1,$$

where $R_{2,i}$ is the number of runs scored by Team 2 at the time of interruption and T_i is the corresponding target score calculated by the rain rule in question. We obtain non-parametric estimates of the positive and negative predictive value as a function of d_i by fitting a generalised additive model – for details see Schall and Weatherall (2013, pg. 2469).

The six panels of Figure 6 present the non-parametric estimates for hypothetical terminations at 20, 30, 35, 40, 45 and 48 overs respectively. The results are consistent with those from the same assessment carried out by Schall and Weatherall (2013). Neither the recalibrated CG method or the new hybrid method perform notably better than the DL method. The predictive value of the rain rules increases with increasing win margin and also increases as the premature termination occurs later. What is most concerning is how low the predictive value is for small win margins. In all instances, all three models achieve less than 50% predictive value for at least one of negative or positive win margins. For terminations at 40 overs, all three models have predictive values considerably less than 50% for matches in which Team 1 win under the rain rule. The same is true for terminations at 48 overs for matches in which Team 2 are declared winner. This suggests that in some instances, when any of these rain rules declares one team the winner by a small margin, that rule is wrong more often than it is right.

5 Conclusion

We have demonstrated that existing methods for constructing rain-rules, based on different paradigms, achieve comparable accuracy but have undesirable performance in terms of fairness. Under the currently used rule, the latter manifests as showing a preference for Team 1. We have developed a hybrid method, incorporating resource-based reasoning inside a probability-preserving approach that exhibits comparable accuracy with greatly improved fairness, such that neither team is disadvantaged. An outstanding problem relates to the issue of the performance of these methods in low-margin interrupted games. We have empirically demonstrated that the methods perform unreliably in this situation, and hence it is a promising area for future work. At the very least, this raises the question of whether a result should be declared when the rain rule puts either team marginally ahead following a premature termination.

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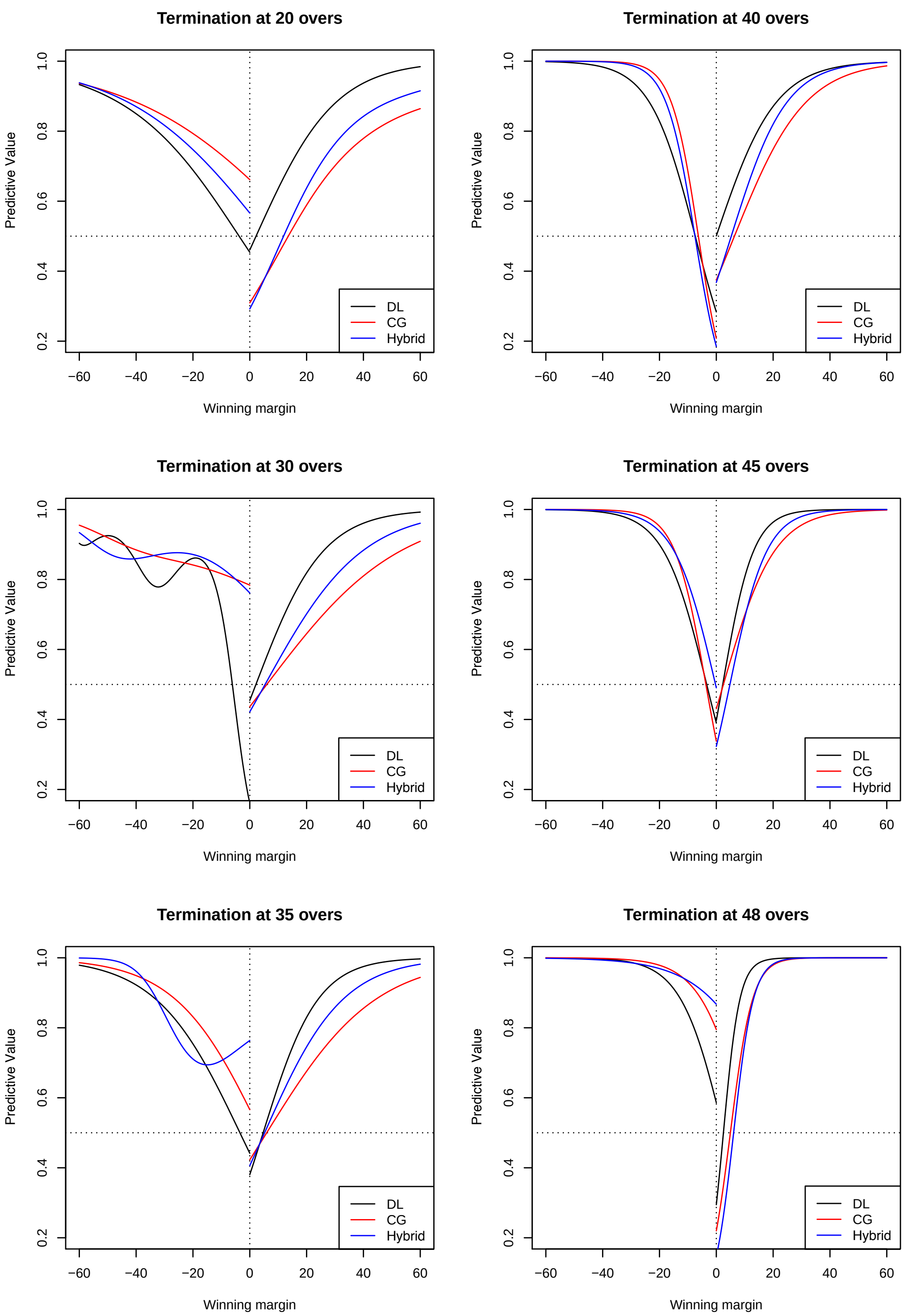

Figure 6: Predictive value vs winning margin for interrupted matches

References

Anselin, L., Florax, R., and Rey, S. J. (2013). *Advances in spatial econometrics: methodology, tools and applications*. Springer Science & Business Media.

Asif, M. and McHale, I. G. (2016). In-play forecasting of win probability in one-day international cricket: A dynamic logistic regression model. *International Journal of Forecasting*, 32(1):34–43.

Carter, M. and Guthrie, G. (2004). Cricket interruptus: fairness and incentive in limited overs cricket matches. *Journal of the Operational Research Society*, 55(8):822–829.

Clarke, S. R. (1988). Dynamic programming in one-day cricket-optimal scoring rates. *Journal of the Operational Research Society*, 39(4):331–337.

Duckworth, F. C. and Lewis, A. J. (1998). A fair method for resetting the target in interrupted one-day cricket matches. *Journal of the Operational Research Society*, 49(3):220–227.

ICC (2015). Standard one-day international match playing conditions. `http://icc-live.s3.amazonaws.com/cms/media/about\_docs/559921e3b7a49-2\%20\_Standard\_ODI\_July\_2015.pdf`. Accessed: 24/08/2016.

Jayadevan, V. (2002). A new method for the computation of target scores in interrupted, limited-over cricket matches. *Current Science*, 83(5):577–586.

Preston, I. and Thomas, J. (2000). Batting strategy in limited overs cricket. *Journal of the Royal Statistical Society: Series D (The Statistician)*, 49(1):95–106.

Preston, I. and Thomas, J. (2002). Rain rules for limited overs cricket and probabilities of victory. *Journal of the Royal Statistical Society: Series D (The Statistician)*, 51(2):189–202.

Schall, R. and Weatherall, D. (2013). Accuracy and fairness of rain rules for interrupted one-day cricket matches. *Journal of Applied Statistics*, 40(11):2462–2479.

Stern, S. E. (2009). An adjusted Duckworth-Lewis target in shortened limited overs cricket matches. *Journal of the Operational Research Society*, 60(2):236–251.

Stern, S. E. (2016). The Duckworth-Lewis-Stern method: extending the Duckworth-Lewis methodology to deal with modern scoring rates. *Journal of the Operational Research Society*. doi:10.1057/jors.2016.30.