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# Geosynchronous inclined orbits for high-latitude communications

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## Abstract

We present and discuss a solution to the growing demand for satellite telecommunication coverage in the high-latitude geographical regions (beyond  $55^\circ\text{N}$ ), where the signal from geostationary satellites is limited or unavailable. We focus on the dynamical issues associated to the design, the coverage, the maintenance and the disposal of a set of orbits selected for the purpose. Specifically, we identify a group of highly inclined, moderately eccentric geosynchronous orbits derived from the Tundra orbit (geosynchronous, eccentric and critically inclined). Continuous coverage can be guaranteed by a constellation of three satellites in equally spaced planes and suitably phased. By means of a high-precision model of the terrestrial gravity field and the relevant environmental perturbations, we study the evolution of these orbits. The effects of the different perturbations on the ground track (which is more important for coverage than the orbital elements themselves) are isolated and analyzed. The physical model and the numerical setup are optimized with respect to computing time and accuracy. We show that, in order to maintain the ground track unchanged, the key

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parameters are the orbital period and the argument of perigee. Furthermore, corrections to the right ascension of the ascending node are needed in order to preserve the relative orientation of the orbital planes. A station-keeping strategy that minimizes propellant consumption is then devised, and comparisons are made between the cost of a solution based on impulsive maneuvers and one with continuous thrust. Finally, the issue of end-of-life disposal is discussed.

*Keywords:* Orbits, Geosynchronous, Perturbations, Station-Keeping, Maneuvers, Low-thrust, Disposal, Communications

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## 1. Introduction

We present a detailed analysis of a group of geosynchronous, highly inclined orbits for telecommunication satellites. The analysis includes the orbital propagation, a complete station-keeping strategy and an end-of-life disposal plan.

5 The motivations of the study reside in the growing demand for coverage in the high-latitude regions, approximately beyond  $55^\circ\text{N}$ . Geostationary (GEO) links work well at low and mid latitudes, with the coverage progressively degrading when moving northwards, for example through Canada, Alaska or the Arctic. The demand for communications service is not limited to land users (television

10 and radio signals, voice and data transmission), but includes aircraft on polar routes, UAVs and maritime navigation. Traditionally, global coverage has been guaranteed by constellations in Low-Earth Orbit (LEO) consisting of many satellites. For example, the Iridium constellation [1] is made up of 66 satellites orbiting at an altitude of approximately 800 km. Classical middle-to-high alti-

15 tude alternatives are the Molniya and Tundra orbits, with periods of 12 and 24 hours, respectively. Both types are characterized by critical inclination ( $63.4^\circ$ ) and argument of perigee of  $270^\circ$ , so that the apogee is at high latitudes. The eccentricity is moderate for Tundra ( $\sim 0.26$ ) and high for Molniya ( $\sim 0.75$ ). [2] analyzes the orbital evolution and lifetime of the Russian Molniya satellites.

20 The influence of several perturbing factors on the long-term orbital evolution is found to be related to the choice of the initial value of the right ascension of

the ascending node and the argument of perigee. [3] studies a constellation of satellites in Tundra orbits, and analyzes in detail the effect of each perturbation (Earth's aspheric potential, solar and lunar gravity, solar radiation pressure, atmospheric drag) on the orbital elements. Frozen orbital elements are identified using a double-averaged potential function for the third-body perturbation. This reduces station-keeping costs as far as the eccentricity and argument of perigee are concerned. The maneuver strategy consists in a bang-bang control method. [4] presents the orbital evolution and station-keeping corrections for a geosynchronous polar orbit, called Tundra Polar due to its similarities with the traditional Tundra in terms of orbital period and eccentricity. The orbit is intended to serve mission objectives such as weather monitoring for the North Pole and Canada. The physical model consists of a  $12 \times 12$  gravity field, solar and lunar gravity and solar radiation pressure perturbations. Thanks to its  $90^\circ$  inclination, the Tundra Polar orbit is not affected by the drift of the ascending node due to the second terrestrial zonal harmonic. The orbital parameters are selected so as to limit the variation of the right ascension of the ascending node and the inclination. In this way, only in-plane maneuvers for eccentricity and argument of perigee correction must be implemented. [5] is an extended analysis of satellite constellations for the Arctic communications system. A total of 15 solutions are considered, consisting in inclined, eccentric orbits with periods of 12, 16, 18 and 24 hours, respectively. The assessment is carried out on the basis of coverage, elevation, azimuth, launch cost, radiation exposure and station-keeping requirements. The physical model accounts for the Earth's gravitational potential developed to the second zonal harmonic and an estimation of the effects of the luni-solar third-body perturbations. The study identifies the best solution as that consisting in three satellites in three equally-spaced orbital planes and characterized by a 12 hours period. Other possibilities are the so-called responsive orbits subdivided by [6] into five types: Cobra, Magic, LEO Sun-Synchronous, LEO Fast Access, LEO Repeat Coverage, with periods ranging from typical LEO values (1.5-2 hours) to 8 hours (Cobra orbit) and requiring 6 to 24 satellites operating in different orbital planes. [7] describes the

Wonder orbit, a low-altitude, highly elliptical orbit with critical inclination and designed to obtain a non-drifting repeating ground track. To provide coverage at high latitudes ( $60^\circ\text{N}$ ) with minimum elevations of  $35^\circ$ , a constellation of ten satellites in two planes, with orbital period of 3.4 hours and perigee height of 600 km is suggested. [8] develops an extension of the critical inclination through the application of continuous low-thrust propulsion. The resulting orbit is called Taranis and enables continuous observation of regions beyond  $55^\circ$  latitude using only four spacecraft, and beyond  $50^\circ$  latitude using five spacecraft.

From 2000 to 2015, the Sirius constellation brought digital radio service to high-latitude users in the continental United States and Canada [9, 10]. Satellites Radiosat 1 through Radiosat 3 flew in geosynchronous highly-elliptical orbits with a 23 hours, 56 minutes orbital period (one sidereal day). The elliptical path of this constellation ensured that each satellite spent about 16 hours a day over the continental United States, with at least one satellite over the country at all times. The inclinations ranged from  $63.4^\circ$  to  $64.2^\circ$ . The advantage of a geosynchronous inclined orbit is that its period is equal to that of the Earth's rotation (repetition of orbital pattern), which gives the ground trace a characteristic figure-eight shape centered at the chosen reference longitude. Furthermore, if the orbit is eccentric and the perigee is placed at the point of lowest latitude, the apogee dwell occurs over the northern hemisphere and allows long contacts per orbit of the same satellite with the users. Full coverage is obtained by using three satellites on equally-spaced orbital planes (i.e, at  $120^\circ$  separation, see Fig. 1).

In this contribution, we consider a group of geosynchronous, inclined, eccentric orbits whose orbital elements secure communication service in regions affected by insufficient GEO coverage. This work is a continuation of the study developed in [11]. This paper provides an improved description of the numerical models and algorithms used. A more detailed analysis of the orbital perturbations and station-keeping strategy is presented. Specifically, corrections of the right ascension of the ascending node and their effect on the argument of the perigee are analyzed, both for continuous low-thrust and impulsive maneuvers.

Besides, even though their effect on the ground track is very limited, perturbations of the orbital eccentricity and inclination are also addressed. A section presenting a disposal strategy at end of life concludes the analysis.

The relevance and originality of the present work reside mainly in the treatment of the perturbations, whose degree of approximation is adjusted to the desired accuracy level, and in the station-keeping strategy, which aims at maintaining the ground tracks rather than the orbital elements themselves. **Besides, a procedure has been implemented which uses the coverage requirements and the population distribution in geographical coordinates to limit the number of orbital elements sets to be investigated.** We address the issue of the end-of-life orbit disposal, in observance of the current space regulation against the accumulation of space debris in the more populated orbits.

Section 2 illustrates the criteria adopted for the selection of the orbits. Section 3 describes the physical model, whereas Sect. 4 discusses the simulations setup. The orbital evolution is illustrated in Sect. 5. The station-keeping strategy and the result of its application are explained in Sect. 6. Section 7 illustrates the end-of-life disposal plan. The conclusions are drawn in Sect. 8.

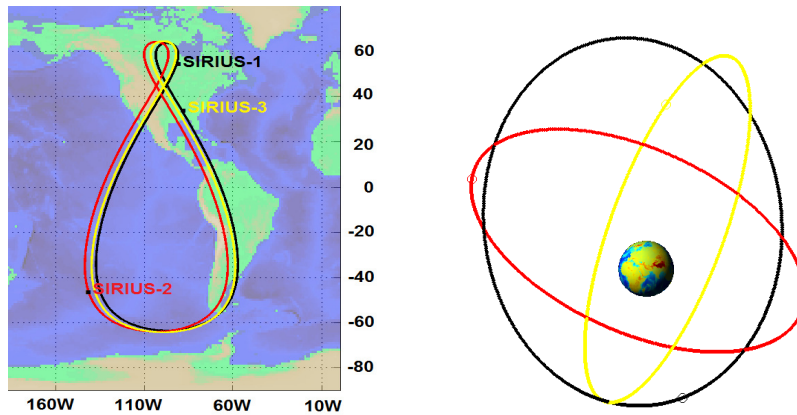


Figure 1: The Sirius constellation: ground track (left) and three-dimensional view of the orbits (right).

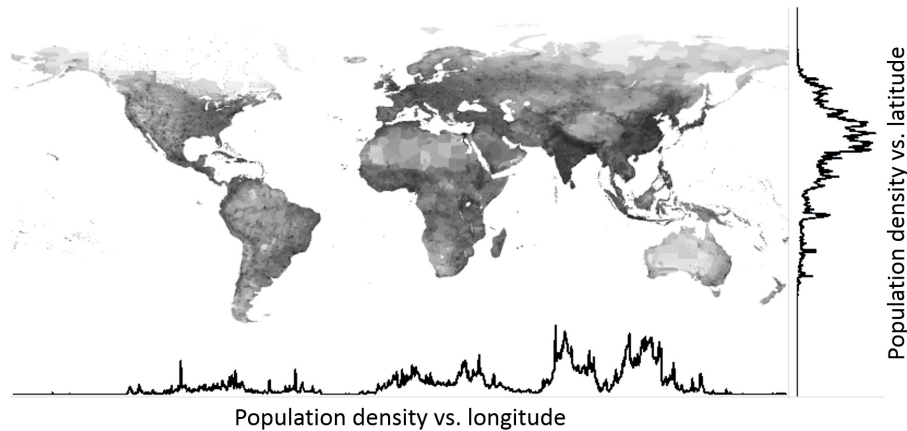


Figure 2: Distribution of world population by latitude and longitude as of 1995 [12, 13].

## 2. Orbits selection

Following [5] and inspired by the case of the Sirius satellites, we have considered orbits derived from the traditional Tundra. These orbits are geosyn-  
 105 chronous, hence their orbital mean motion  $n_0$  is equal to  $7.292 \cdot 10^{-5}$  rad/s  
 (i.e., the value of the Earth's sidereal rotation rate). As a consequence, their  
 orbital period  $T_0$  equals 86165.5 s and the semimajor axis  $a_0$  is of 42164.6 km.  
 Furthermore, these orbits are inclined and eccentric. In order to provide service  
 to high-latitude users, the perigee is positioned at the point of lowest latitude,  
 110 corresponding to an argument of perigee  $\omega_0$  of  $270^\circ$ . In this way, the apogee is  
 the point of highest latitude, and the service takes full advantage of the so-called  
 apogee dwell. We have chosen the values of eccentricity  $e_0$  and inclination  $i_0$   
 which bring satisfactory coverage to select regions, as described here below.

### 2.1. Coverage analysis: selection of eccentricity and inclination

115 We have examined the world population distribution in latitude and longi-  
 tude, and we have identified the most populated areas in the northern hemi-  
 sphere. According to Fig. 2, such areas are centered in longitude around  
 $\lambda_1 = 100^\circ\text{W}$  (North America),  $\lambda_2 = 10^\circ\text{E}$  (Europe) and  $\lambda_3 = 100^\circ\text{E}$  (Russia and

China). We have chosen these three values of the longitude as the most econom-  
120 ically sensible for operation of a constellation of communication satellites, and  
we have assessed the coverage provided by constellations of three satellites on  
geosynchronous orbits with several values of  $e_0$  and  $i_0$ . Following the example  
of the Sirius satellites, we have defined the visibility requirement as a minimum  
elevation of  $60^\circ$  over the horizon. The three orbital planes are equally spaced  
125 and the orbital phases (e.g., the mean anomaly or the true anomaly) of each  
satellite at the starting time are set by the requirement that the three ground  
tracks coincide. By tracking the three satellites of the constellation along their  
orbits, we have mapped the points on the Earth's surface in which the visibility  
requirement is satisfied by at least one satellite at all times. By trying different  
130 combinations of  $e_0$  and  $i_0$ , we have identified the values of these two parameters  
which yield satisfactory coverage over the area of interest. The population dis-  
tribution diagrams of Fig. 2 indicates scarcity of inhabitants above  $55^\circ\text{N}$ , with  
an extremely small percentage of people living in areas beyond  $65^\circ\text{N}$ . Therefore,  
coverage in regions above this value of the latitude has not been considered as  
135 it is not economically justifiable. Moreover, special applications like maritime,  
aircraft and UAV navigation can be served without the requirement for high el-  
elevation angles (tall obstacles blocking the view of the sky are very scarce in the  
Arctic region). As a result of our assessment, we have identified nine combina-  
tions of  $(e_0, i_0)$  to use for orbital simulations. These combinations are reported  
140 in Table 1, where they are referred to as sets. Initially, in view of the areas to  
cover, inclinations of  $55^\circ$ ,  $60^\circ$ ,  $65^\circ$  and  $70^\circ$  were considered. However, it was  
finally decided to use the critical inclination ( $63.4^\circ$ ) instead of  $65^\circ$  to allow di-  
rect comparisons with the evolution of a classical Tundra orbit. Evenly-spaced  
values of the eccentricity (0.25, 0.30, 0.35 and 0.40) have been chosen as they  
145 make it easier to interpret trends in the data. Coverage maps for each of the  
nine sets have been determined for each value of the reference longitude ( $\lambda_1$  to  
 $\lambda_3$ ). A sample of such maps can be found in Figs. 3 and 4.

To each  $(e_0, i_0)$  set we have assigned six equally-spaced values for the right  
ascension of the ascending node  $\Omega_0$ , namely  $0^\circ$ ,  $60^\circ$ ,  $120^\circ$ ,  $180^\circ$ ,  $240^\circ$  and  $300^\circ$ .



Table 1: The nine sets of initial eccentricities  $e_0$  (second column) and inclinations  $i_0$  (third column), and the values  $r_{\pi 0}$  (fourth column) and  $r_{\alpha 0}$  (fifth column) of the pericenter and apocenter radii of the corresponding geosynchronous Keplerian orbits.

Set	$e_0$	$i_0$ (degree)	$r_{\pi 0}$ (km)	$r_{\alpha 0}$ (km)
1	0.25	55.0	31623.5	52705.8
2	0.25	60.0	31623.5	52705.8
3	0.30	60.0	29515.2	54814.0
4	0.25	63.4	31623.5	52705.8
5	0.30	63.4	29515.2	54814.0
6	0.35	63.4	27407.0	56922.2
7	0.40	63.4	25298.8	59030.4
8	0.35	70.0	27407.0	56922.2
9	0.40	70.0	25298.8	59030.4

150 This results into 54 sets of orbital elements. These are used to initialize the orbit simulations.

### 3. The physical model

The physical model employed for orbit propagation accounts for the gravitational acceleration due to the terrestrial gravity field, the accelerations caused by the gravity of Moon and Sun as described by 155 [14] (with the positions of the two celestial bodies approximated as in [15, 16]), the effect of the solar radiation pressure (as in [14] with the shadow factor computed according to the double-cone model) and the term associated to the relativistic correction to gravity [17]. 160 The effect of atmospheric drag on the orbit has not been taken into account given the very high perigees (radial distances  $> 2.5 \cdot 10^4$  km) of the orbits considered.

As for the terrestrial gravitational acceleration, the traditional

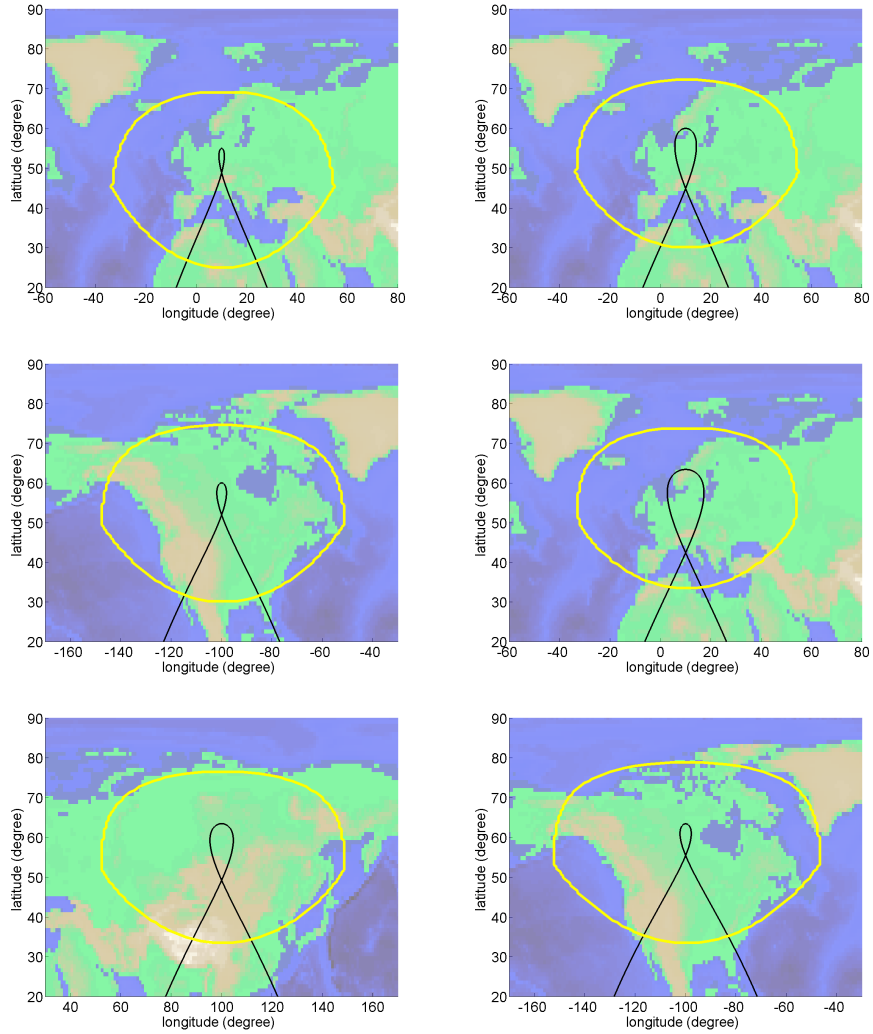


Figure 3: Geographical coverage at  $60^\circ$  minimum elevation for three-satellite constellations based on the orbital parameters of sets 1 (top left), 2 (top right), 3 (middle left), 4 (middle right), 5 (bottom left) and 6 (bottom right), respectively. The ground tracks are centered at the longitudes of interest, i.e.,  $100^\circ\text{W}$ ,  $10^\circ\text{E}$  or  $100^\circ\text{E}$ .

165 **formulation of the gravitational potential in outer space based on a spherical harmonic expansion in Associated Legendre Functions [18] has been replaced with a singularity-free representation in Cartesian Earth-fixed coordinates due [19] in which the spherical harmonics are**

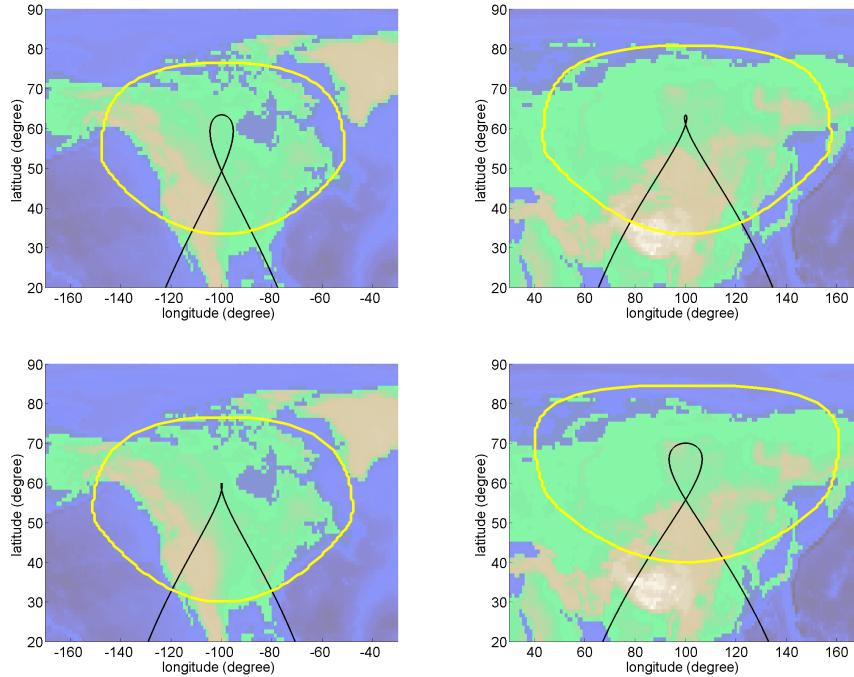


Figure 4: Geographical coverage at  $60^\circ$  minimum elevation for three-satellite constellations based on the orbital parameters of sets 5 (top left), 7 (top right), 8 (bottom left) and 9 (bottom right), respectively. The ground tracks are centered at the longitudes of interest (in this case,  $100^\circ\text{W}$  or  $100^\circ\text{E}$ ).

expressed in terms of Helmholtz polynomials (see [20, 21]). This formulation was improved by [22, 22, 23] with implementation of stable  
 170 recursion schemes on the Helmholtz polynomials and through the accumulation of so-called lumped coefficients (harmonic sums over the degree), which yields better performance. The adopted geopotential model is the Earth Gravitational Model 2008 [24] in its zero-tide version. It provides the fully-normalized, dimensionless, Stokes coefficients and their associated error standard deviations. The model is  
 175 complete to degree and order 2159.

#### 4. Numerical setup

The choice of the perturbations to model depends on the level of accuracy sought. For Tundra-like orbits, it has been decided that station-keeping to within  $1^\circ$  accuracy is sufficient for the intended applications. As a matter of fact, a satellite angular displacement of  $1^\circ$  corresponds to a shift in the ground trace of some 100 km. This is a small distance compared to the size of the coverage area (typically several thousand kilometers). This study considers the station-keeping requirements over two years. This time interval is twice the period of the slowest-varying cyclic perturbation (solar), and is thus considered representative of the long-term evolution. To be able to sense  $1^\circ$  errors in position after two years, the discretization error of the orbit propagator must be small compared with this value. A uniform drift of  $1^\circ$  over two years is equivalent to  $0.0014^\circ$  per orbit (i.e., per sidereal day). Given the altitude of the orbits under study (the lowest perigee height is of 19000 km, approximately), this corresponds to a distance of more than 450 m per orbit. Therefore, the error in the trajectory propagation must be at least 10 times smaller, i.e., 45 m/orbit. We set a conservative value of 40 m/orbit. The propagator tolerance and time step must be chosen so as to respect the target accuracy. It has been determined through numerical experiments that using a relative tolerance of  $10^{-6}$  and a maximum time step of 600 s produces errors well below the required 40 m/orbit threshold. The required accuracy also determines the magnitude of the perturbing forces to consider. A constant acceleration of  $10^{-8}$  m/s<sup>2</sup> acting for one day (the orbital period) causes a drift of 40 m. This means that perturbing accelerations below such value can be neglected without altering the quality of the results. Note also that most perturbations are likely to oscillate along the orbit instead of remaining constant, so their cumulative effect will be smaller (this is a worst-case scenario estimate). According to this criterion, the perturbations due to atmospheric drag and relativistic corrections (close to  $10^{-9}$  m/s<sup>2</sup>) can be ignored as they fall well below the  $10^{-8}$  m/s<sup>2</sup> level. For standard communication satellites with an area-to-mass ratio of 0.01 m<sup>2</sup>/kg, the

effect of solar radiation pressure in these orbits yields accelerations below  $10^{-7}$  m/s<sup>2</sup>, which is barely appreciable and with a marginal effect on the station-keeping requirements.

#### 210 4.1. Choice of the expansion degree of the geopotential

When performing the harmonic synthesis of the geopotential and its first-order gradient, the number of spherical harmonics to retain (i.e., the expansion degree) is determined by the accuracy requirement. Given that higher-order harmonics decay rapidly with altitude, the expansion degree necessary to meet  
215 the target accuracy decreases with height. To make the computations as efficient as possible, the number of degrees to retain must be determined dynamically while the trajectory is being computed. To determine the optimum expansion degree at each altitude, a sample of 36 longitudes and 17 latitudes has been analyzed (corresponding to points spaced  $10^\circ$  along a sphere). The expansion  
220 degree required to reduce the truncation error below  $10^{-8}$  m/s<sup>2</sup> for all the points of the grid has been determined, i.e., the degree for which the difference between the complete EGM2008 model and the truncated series is less than the threshold acceleration. This procedure has been repeated for several altitudes in geometric series with ratio 2 starting at an altitude of 250 km and covering  
225 the complete trajectory envelope. The table containing the required expansion degree as a function of altitude is used by the numerical integrator to compute the gravitational acceleration, guaranteeing the required accuracy while keeping the computational cost to a minimum (Fig. 5).

## 230 5. Simulations

The physical model has been coded in Fortran. The executable has been called CHEOPs which stands for Cartesian Harmonic synthEsis Orbit Propagator. CHEOPs incorporates a 7(8) Runge-Kutta-Fehlberg numerical integrator.

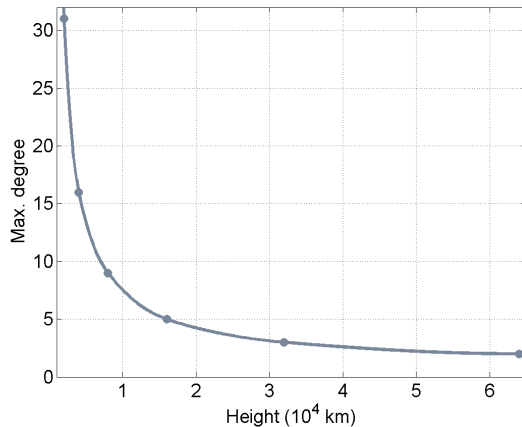


Figure 5: Expansion degree required to achieve the target accuracy for each height.

### 5.1. Preprocessing: removal of the linear drift of the ascending node due to $J_2$

235 It is well known (see e.g. [25] p. 347) that the longitude of the ascending node drifts at a uniform rate due to the second zonal harmonic  $J_2$  (0.0010826) of the terrestrial gravity field. Due to this effect, the orbital plane of the satellite precesses at a rate given by

$$\dot{\Omega}_{J_2} = -\frac{3}{2} \left( \frac{R_E}{a_0} \right)^2 \frac{n_0 J_2 \cos i_0}{(1 - e_0^2)^2}, \quad (1)$$

240 which, in turn, would cause the ground track to move eastwards at the same rate. On the other hand, the ground track of a Keplerian orbit with a slightly different mean motion  $n$  would move to the East at a rate  $n - n_0$ . Thus, the eastwards drift of the ground track due to  $J_2$  can be countered by changing the period of the (originally) geosynchronous orbit in such a way that both effects cancel out. That is,

$$n = n_0 - \dot{\Omega}_{J_2}. \quad (2)$$

245 Note that Eq. 2 assumes the change in orbital period is small, so there is no need to modify the rate of precession given by Eq. 1. Once the required period of the orbit  $T = 2\pi/n$  has been determined, the initial conditions for the orbit propagator must be obtained. Given that the real orbit is not Keplerian, the

semimajor axis cannot be computed directly from Kepler's third law. Defining  
 250  $T$  as the time between passages through the ascending node, its value can only  
 be determined after a set of initial conditions has been chosen and the orbit  
 has been numerically propagated. Therefore, the initial conditions must be  
 determined iteratively. From Kepler's third law we can derive the approximate  
 variation  $\delta T$  of period that corresponds to a given change  $\delta a$  in semimajor axis:

$$\delta T \approx \frac{6\pi^2}{GM_E} \frac{a^2}{T} \delta a. \quad (3)$$

255 Starting with an approximation of the semimajor axis  $a_i$ , an improved estimate  
 $a_{i+1}$  can be obtained by combining Eqs.2-3:

$$\frac{2\pi}{n_0 - \dot{\Omega}_{J2}} = T_i + \frac{6\pi^2}{GM_E} \frac{a_i^2}{T_i} (a_{i+1} - a_i), \quad (4)$$

leading to the following iterative formula:

$$a_{i+1} = a_i + \left( \frac{2\pi}{n_0 - \dot{\Omega}_{J2}} - T_i \right) \frac{GM_E T_i}{6\pi^2 a_i^2}. \quad (5)$$

For all the orbits tested, the change in period respect to the ideal geosynchronous  
 case,  $\Delta T = T - T_0$ , amounts to just a few seconds. Therefore, the approximation  
 260 made in Eq. 2 is acceptable.

## 5.2. Orbital evolution

The 54 sets of initial conditions presented in Sect. 2 have been propagated  
 over intervals of two years, to encompass all the cycles present in the model.  
 Due to the high altitudes of these orbits, the code automatically sets the expan-  
 265 sion degree to 4 or less (see also Fig. 5). The evolution of the orbits under the  
 forces accounted for in the model is summarized in Table 2, which provides the  
 maximum and typical (**i.e., representative of the median value**) variation  
 of the orbital parameters of interest as well as a reference (i.e., the set and the  
 initial value of the right ascension of the ascending node) to the representa-  
 270 tive orbit for each case. The orbital parameters given are the eccentricity  $e$ ,  
 inclination  $i$ , argument of perigee  $\omega$ , right ascension  $\Omega$  of the ascending node,  
 and the longitude LAN of the ascending node, which measures the East-West

Table 2: Maximum and typical variation of the orbital parameters of interest and a reference (i.e., the set and the initial value of the right ascension of the ascending node) to the representative orbit for each case.

Element	Maximum var.	Set	$\Omega_0$	Typical var.	Set	$\Omega_0$
$e$	0.07	9	$120^\circ$	0.03	5	$60^\circ$
$i$	$1.2^\circ$	1	$300^\circ$	$0.5^\circ$	4	$120^\circ$
$\omega$	$15^\circ$	9	$180^\circ$	$6^\circ$	8	$0^\circ$
$\Omega$	$11^\circ$	7	$0^\circ$	$9^\circ$	6	$300^\circ$
LAN	$150^\circ$	1	$60^\circ$	$100^\circ$	4	$300^\circ$

(E-W) motion of the ground track. Note that the semimajor axis is not shown because it is related to the orbital period, so its effect is lumped into LAN, i.e.,  
 275 in the loss of synchronism with respect to the rotation of the Earth. Figures 6  
 illustrates the evolution of  $e$ ,  $i$ ,  $\omega$  and LAN for the most representative cases of  
 the table, i.e., sets 1 and 9, with the six values of  $\Omega_0$ . The evolution of  $\Omega$  for  
 the worst case (set 7) and for a typical case (set 6) is given in Fig. 7. The most  
 important secular perturbation is due to the Moon, which causes variations of  
 280 the orbital period which in turn move the ground track in the E-W direction.  
 The displacement can reach hundreds of degrees over a two-year span and needs  
 a major axis correction in order to keep the ground track fixed. On the other  
 hand, the fluctuations of eccentricity and inclination have a very limited effect  
 on the ground track and do not require correction over a two-year period (as  
 285 far as the ground track is concerned). The maximum drift of the argument  
 of perigee observed is around  $15^\circ$  over two years and needs to be periodically  
 corrected.

## 6. Station keeping

Two different station-keeping strategies are devised. A classical one relying  
 290 on impulsive maneuvers, and another suitable for electric propulsion with the  
 corrections applied over thrust arcs spanning an important fraction of the or-



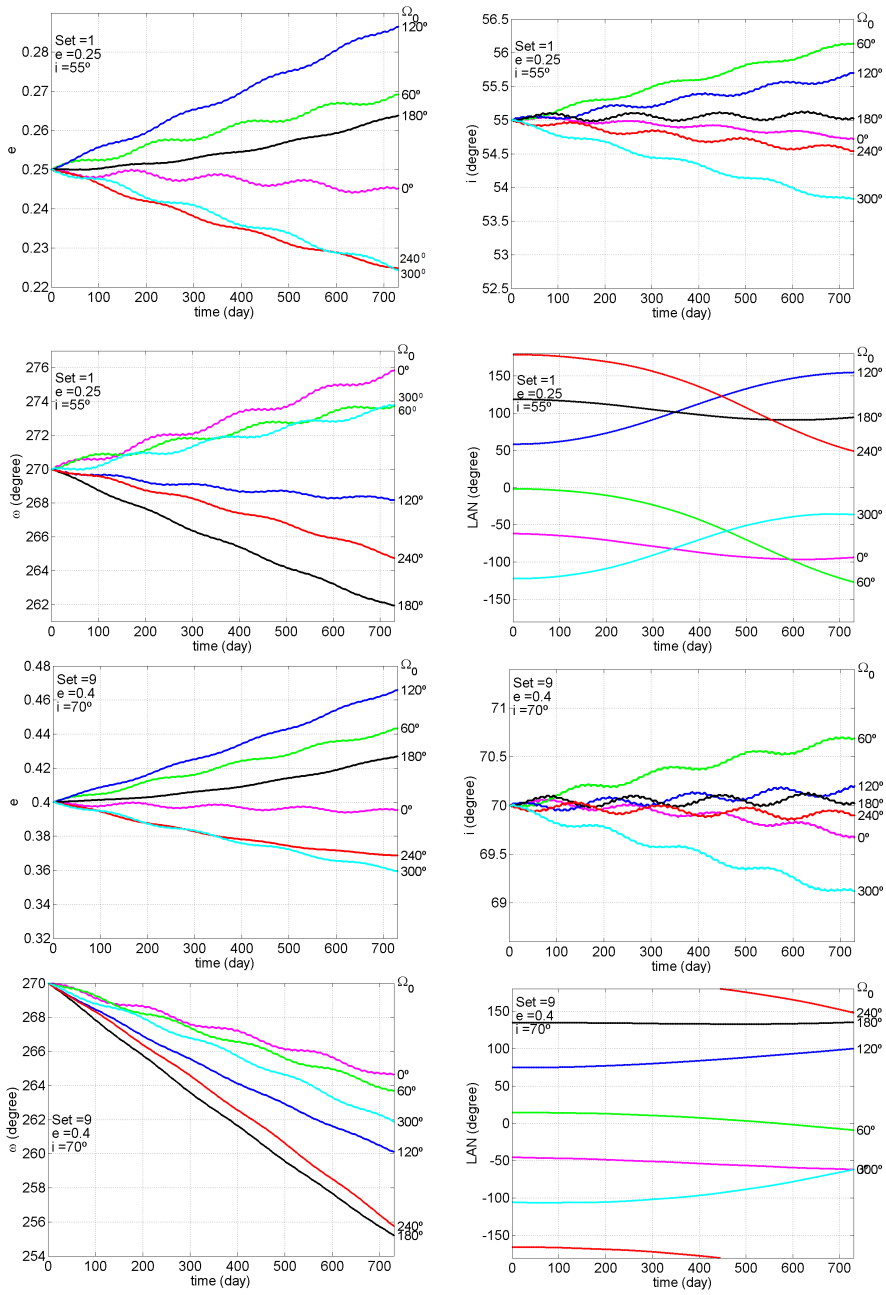


Figure 6: Evolution of the relevant orbital parameters of the orbits of sets 1 and 9: eccentricity  $e$ , inclination  $i$ , argument of perigee  $\omega$ , longitude of the ascending node LAN.

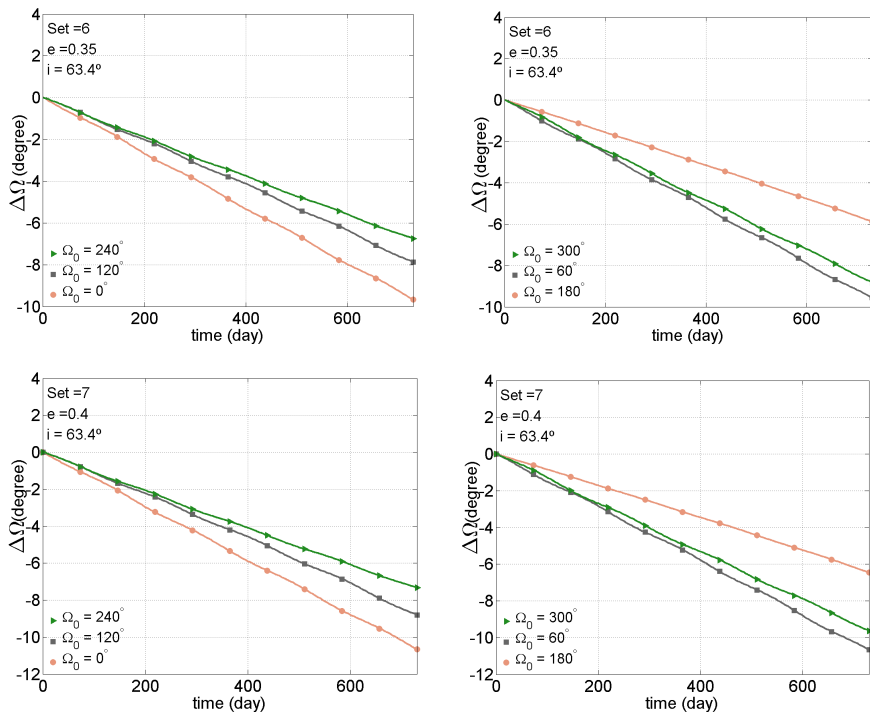


Figure 7: Drift of the ascending node: sets 6 (top) and 7 (bottom).

bital period. To determine the details of the low-thrust maneuvers (including  
 duration) the value of the acceleration of the spacecraft is needed. This, in  
 turn, requires knowledge of the vehicle mass and the thrust available. An initial  
 295 spacecraft mass of 3000 kg and maximum thrust of 79 mN have been assumed in  
 this study yielding an acceleration of  $2.63 \cdot 10^{-5}$  m/s<sup>2</sup>. The maximum available  
 acceleration increases as propellant is consumed (assuming no degradation of  
 the propulsion system). However, the acceleration has been taken equal to the  
 initial value at all times, which is a conservative assumption. This simplifies  
 300 greatly the interpretation of the charts relating the length of the thrust arcs to  
 the change in orbital elements. Moreover, the mass fraction of propellant in a  
 spacecraft using electric thrusters is likely small, reducing the changes in accel-  
 eration. The duration of the thrust arcs can be of several hours and may require  
 changes in spacecraft attitude. Therefore, to avoid disrupting normal satellite  
 305 operations, it has been assumed that all low-thrust maneuvers are applied near  
 the perigee (where the satellite does not provide coverage and can thus be safely  
 taken offline).

### 6.1. Correction of the longitude of the ascending node

It is important to realize that from the point of view of operation, the po-  
 310 sition of the orbital plane is of minor importance compared with the correct  
 placement of the ground track. The orbital perturbations cause the right ascen-  
 sion of the ascending node to change and therefore affect the E-W position of  
 the ground track. However, this effect is very small compared with the changes  
 caused by fluctuations of the orbital period. If the orbital period **of a geosyn-**  
 315 **chronous orbit** were to change by  $\Delta T$  for any reason, this would cause a drift  
 $\Delta \text{LAN}$  of the longitude of the ascending node,

$$\Delta \text{LAN} = -360 \left( \frac{\Delta T}{T_0} \right) \text{ degrees per orbit.} \quad (6)$$

The magnitude of this drift is so important that it overshadows the effect of  
 the orbital plane precession. Therefore, the station-keeping strategy must focus  
 on adjusting the orbital period to keep the ground track in place. Any E-W

320 drift of the ground track caused by the different perturbations is countered by  
 introducing an appropriate change in the orbital period. This is achieved with a  
 two-burn maneuver which changes the semi-major axis while keeping the other  
 orbital elements unchanged. The first burn takes place at the perigee of the  
 initial orbit and the second at the apogee of the final orbit. From Kepler’s third  
 325 law, the variation  $\Delta a$  in semimajor axis needed to change the orbital period by  
 a small amount  $\Delta T$  is

$$\Delta a = \frac{T_0 GM_E}{6\pi^2 a^2} \Delta T. \quad (7)$$

The impulse for the two-burn maneuver can be linearized (assuming small  
 changes in velocity) yielding:

$$\Delta V = \frac{GM_E}{4a^2} \left( \frac{1+e}{V_\pi} + \frac{1-e}{V_\alpha} \right) \Delta a. \quad (8)$$

where  $V_\pi$  and  $V_\alpha$  denote the perigee and apogee velocities, respectively. Using  
 330 Eqs. 7 and 8, the cost of the period control maneuver can be determined. For  
 the cases under study the fluctuations in orbital period are very small (35 s over  
 two years for the worst-case scenario, set 1,  $\Omega_0 = 60^\circ$ ). A change of orbital  
 period of 35 s can be accomplished with an impulse  $\Delta V_{LAN}^I$  of just 0.22 m/s.  
 Therefore, the contribution to the overall impulse budget (totaling hundreds of  
 335 m/s) can be safely neglected.

## 6.2. Correction of the right ascension of the ascending node

The motion of the ascending node is mostly due to lunisolar perturbations,  
 with the Moon accounting for roughly two thirds of the drift observed. Further-  
 more, numerical experiments show that the effect depends on the inclination of  
 340 the satellite orbit relative to the orbital plane of the Moon. The drift of the  
 ascending node is fastest when both orbits are almost coplanar and vanishes  
 when they are perpendicular. At the epoch of the start of the simulations (July  
 1<sup>st</sup> 2013), the ascending node of the lunar orbit has a right ascension of  $5^\circ$  and  
 an inclination of  $28^\circ$  respect to the equator. The minimum relative orbital incli-  
 345 nation (satellite respect to Moon) is obtained when the ascending nodes of the

Moon and satellite coincide. Therefore, the orbital planes needing the largest  $\Omega$  correction are those with  $\Omega_0 = 0^\circ$  and  $60^\circ$ .

As stated in the previous section, the E-W position of the orbital plane has a minor effect on the position of the ground track, which is controlled through very small adjustments of the orbital period. However, in order to have a properly working constellation, a  $120^\circ$  spacing between the orbital planes must be maintained. To optimize the propellant consumption, it was decided to correct only the drift  $\hat{\Omega}$  of each orbital plane with respect to the mean motion of the constellation. Let  $\Delta\tilde{\Omega}$  be the mean change in  $\Omega$  of the constellation:

$$\Delta\tilde{\Omega} = \frac{1}{3} \sum_{i=1}^3 \Delta\Omega_i. \quad (9)$$

Here,  $\Omega_i$  denotes the right ascension of the ascending node of plane  $i$  and  $\Delta\Omega_i$  its variation. The station keeping maneuvers are designed to correct the change in  $\hat{\Omega}$ , defined as

$$\Delta\hat{\Omega}_i = \Delta\Omega_i - \Delta\tilde{\Omega}. \quad (10)$$

By removing the mean drift of the constellation, the cost of the station-keeping maneuvers is reduced compared to a simpler strategy based in controlling each orbital plane independently. To illustrate the gain obtained with this strategy, Fig. 8 depicts the evolution of  $\hat{\Omega}$  for typical (set 3) and worst-case scenarios (set 9). While, as shown in Table 2, the maximum change in  $\Omega$  over 2 years is  $11^\circ$  (set 7,  $\Omega_0 = 0^\circ$ ), the largest change in  $\hat{\Omega}$  is just  $3^\circ$  (set 9,  $\Omega_0 = 180^\circ$ ). That is, the impulse budget is cut down by 70%. The typical correction, on the other hand, is reduced from  $9^\circ$  to just  $1^\circ$  (a 90% decrease).

### 6.2.1. Impulsive maneuver

**When the perigee argument is  $90^\circ$  or  $270^\circ$**  and in the limit of small corrections, the change in  $\Omega$  is achieved with a single burn near the apogee or perigee which rotates the velocity vector by an angle  $\gamma$  [26, 27]:

$$\gamma \approx \Delta\Omega \sin i. \quad (11)$$

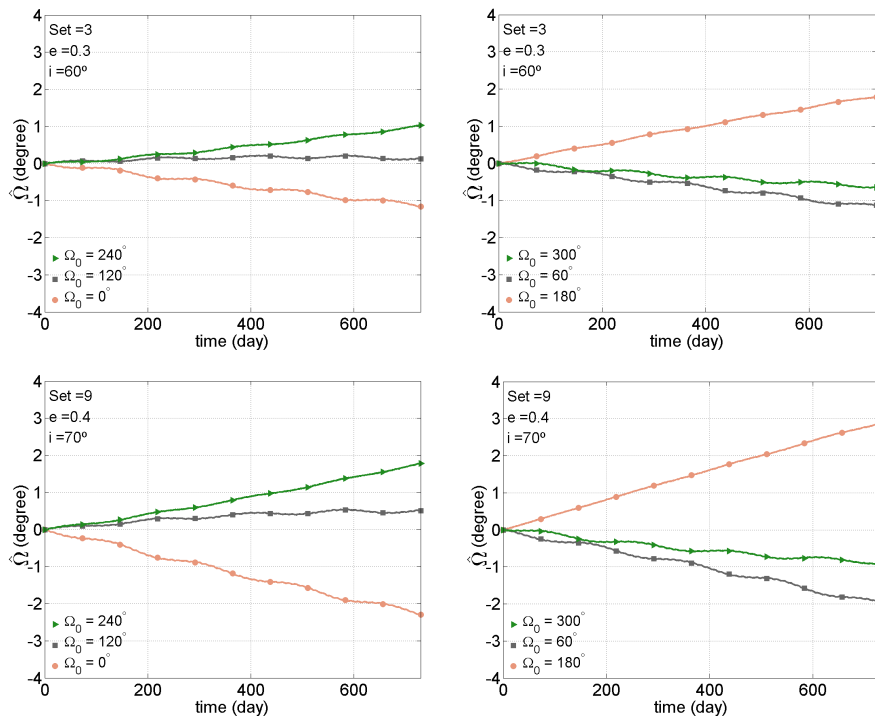


Figure 8: Drift of  $\hat{\Omega}_z$ : sets 3 (top) and 9 (bottom).

370 Note that the expression above is only valid if  $\Delta\Omega \ll 1$  rad. Given that the magnitude of the impulse is proportional to the satellite velocity at the time of the burn, it is preferable to apply the maneuver at the apogee. The impulse needed is then:

$$\Delta V \approx V\gamma \approx V_\alpha \Delta\Omega \sin i, \quad (12)$$

where small angles are assumed. For the orbits under consideration, the typical  
 375 magnitude of the impulse is 34 m/s for each degree of correction. For a worst-case scenario of  $3^\circ$   $\Omega$  correction over two years (set 9,  $\Omega_0 = 180^\circ$ ), the total impulse  $\Delta V_\Omega^I$  is 100 m/s. It is worth noting that this maneuver causes a change in argument of perigee as a side effect. For a burn near the apogee, the variation in  $\omega$  is approximately

$$\Delta\omega \approx \Delta\Omega \cos i. \quad (13)$$

380 This, depending on the sign of the relevant terms, reduces or increases the correction of the argument of perigee needed to compensate orbital perturbations. Given that the signs change for different orbits of the same set, the next section (correction of  $\omega$ ) assumes a worst-case scenario to obtain a conservative estimate of the impulse budget (i.e., the magnitude of the correction is always increased).

### 385 6.2.2. Low-thrust maneuver

To change the ascending node, thrust must be applied perpendicular to the orbital plane [25]. Following [28] and denoting with  $f$  the magnitude of the acceleration imparted by the motor, the execution of thrust arcs symmetric with respect to the perigee yields a variation in  $\Omega$

$$\Delta\Omega = -\frac{a^2 f}{GM_E \sqrt{1-e^2} \sin i} \left( -3e\bar{E} + 2 \sin \bar{E} + 2e^2 \sin \bar{E} - \frac{1}{2}e \sin 2\bar{E} \right). \quad (14)$$

390 As usual,  $a$ ,  $e$  and  $i$  are semimajor axis, eccentricity and inclination (all left constant by the maneuver), whereas  $\bar{E}$  is the half-span of the thrust arc (measured in terms of eccentric anomaly). If  $\beta$  denotes the thrust elevation over the orbital plane, then for a positive change in  $\Omega$  the thrust vector must be antiparallel ( $\beta = -90^\circ$ ) to the orbital angular momentum when the satellite is

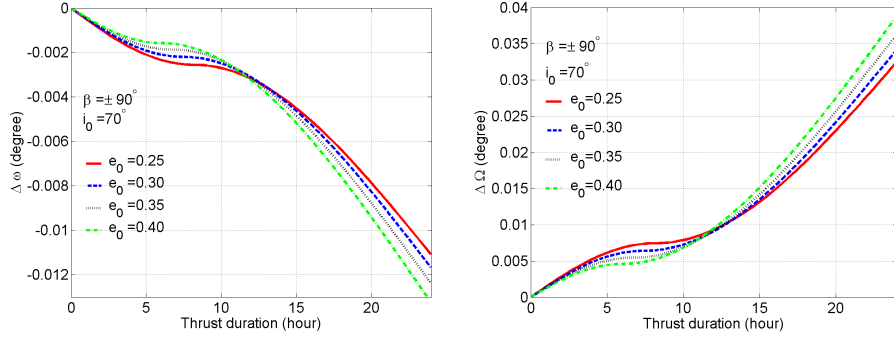


Figure 9: Right: change in  $\Omega$  per orbit achieved as a function of the eccentricity and total duration of the thrust arc for an orbit with  $70^\circ$  inclination by thrusting perpendicular to the orbital plane. Left: effect of the maneuver on the argument of perigee  $\omega$ .

395 in the southern hemisphere and parallel ( $\beta = +90^\circ$ ) in the remaining part of the orbit. Figure 9 (right) shows the change in  $\Omega$  per orbit achieved as a function of the eccentricity and total duration of the thrust arc for an orbit with  $70^\circ$  inclination. The thrust arc is symmetric with respect to the perigee. So, for example, a total duration of 10 hours corresponds to an arc spanning from 5 hours before perigee to 5 hours after perigee. Note that the curves show a point of zero slope. This happens when the satellite crosses the Equator, causing the net torque with respect to the line of nodes to vanish. Therefore, care must be taken not to apply the maneuver close to the nodes, as it would waste propellant. As was the case for the impulsive maneuver, there is a change in argument  
400 of the perigee associated with the correction of  $\Omega$ :

$$\Delta\omega = -\frac{f}{\cot i} \sqrt{1-e^2} \left( -3e\bar{E} + 2\sin\bar{E} + 2e^2\sin\bar{E} - \frac{1}{2}e\sin 2\bar{E} \right). \quad (15)$$

The magnitude of the coupling is depicted on the left graph of Fig. 9. For a worst-case scenario of  $3^\circ$   $\Omega$  correction over two years for the most eccentric orbit (set 9,  $\Omega_0 = 180^\circ$ ), the correction per orbit is  $0.0041^\circ$ . A thrust arc of 4.2 hours is required with an associated change in  $\omega$  of  $0.0014^\circ$  per orbit ( $1^\circ$  over two  
410 years). Note that the duration of the arc is within the acceptable range, as it takes place in the area where the slope of the curves is large (i.e., the satellite is far enough from the Equator). The total impulse is computed as the product



of the spacecraft acceleration times the duration of the thrust arc. This yields a change in velocity of 0.4 m/s per orbit or  $\Delta V_{\Omega}^{LT} = 290$  m/s over two years.

### 415 6.3. Correction of the argument of perigee drift

Computations show that, for the worst-case scenario, roughly 20% of the perigee drift is due to the harmonics of Earth’s gravitational field, 25% is caused by the Sun and 55% by the Moon. Classical Molniya orbits use critical inclination to minimize changes in  $\omega$  due to  $J_2$ . For Tundra-like orbits, however, 420 lunar perturbations prevail due to the higher altitude. Thus, the advantage of critically inclined orbits largely vanishes. As was the case with the ascending node, lunar perturbations depend on the relative orientation of the satellite and Moon orbits (i.e., the right ascension of the ascending node). In this case, however, the effect is maximum for the orbital planes with ascending nodes opposite 425 to the Moon (i.e.,  $180^\circ$  and  $240^\circ$ ). **Among the different sets analyzed, it is noteworthy that orbits with  $e_0=0.25$  and 55 degrees of inclination (see Fig. 6) experience the slowest average drift of the perigee (2.2 degrees per year when averaged across the six values of  $\Omega$ ).**

#### 6.3.1. Impulsive maneuver

430 To carry out an in-plane rotation of the major axis of the orbit, an optimized impulsive two-burn maneuver has been used. It involves an elliptical transfer orbit with the major axis oriented halfway between the initial and final orbits [29]. Due to the symmetry of the problem, the two burns have the same magnitude. The transfer has two degrees of freedom, the eccentricity of the transfer 435 orbit and the true anomaly of the first burn. These must be determined by minimizing the total impulse. As the optimization method described in the reference is very cumbersome to implement in software, an alternative, more computer-friendly strategy has been developed. The optimum combination is determined with a two-stage algorithm. For a given value  $\theta$  of the true anomaly 440 of the first burn, the eccentricity  $e^*$  of the transfer orbit yielding the minimum

impulse can be found by solving the equation

$$\frac{\partial \Delta V(e^*, \theta)}{\partial e^*} = 0. \quad (16)$$

This is easily accomplished numerically using the secant method, because the partial derivative has a simple analytical expression. This has been implemented in a function which calculates the minimum of  $\Delta V(\theta)$ . Next, this function is  
 445 passed as the argument of a robust minimization algorithm which does not require gradients (Brent's method [30]) and returns the optimum values of the true anomaly and impulse. The worst-case correction of the argument of the perigee over two years is  $16^\circ$ , of which  $15^\circ$  are due to orbital perturbations and  $1^\circ$  to the coupling with the  $\Omega$  correction. This requires a maximum impulse  
 450  $\Delta V_\omega^I$  of 182 m/s, with the typical impulse being some 50% lower.

### 6.3.2. Low-thrust maneuver

Let the components of the acceleration  $\mathbf{f}$  imparted by the motor be denoted with  $f_1$ ,  $f_2$  and  $f_3$  in a radial-transverse-normal reference frame. It is possible to change  $\omega$  while keeping the semimajor axis  $a$  and the eccentricity  $e$  constant,  
 455 by applying the following in-plane acceleration over thrust arcs symmetric with respect to the perigee [28]:

$$f_1 = f \left( \frac{\cos E - e}{1 - e \cos E} \right) \quad (17)$$

$$f_2 = -f \left( \frac{\sqrt{1 - e^2} \sin E}{1 - e \cos E} \right) \quad (18)$$

$$f_3 = 0 \quad (19)$$

Here  $E$  is the instantaneous value of the eccentric anomaly (which means the components of the acceleration vector change along the thrust arc). This acceleration causes a variation in  $\omega$

$$\Delta \omega = \frac{af\sqrt{1 - e^2}}{GM_E e} \left( -3\bar{E} + 2e \sin \bar{E} + \frac{1}{2} \sin 2\bar{E} \right), \quad (20)$$

460 where  $\bar{E}$  is the absolute value of the eccentric anomaly at either end point of the thrust arc. The length of the propelled arc is determined by the magnitude

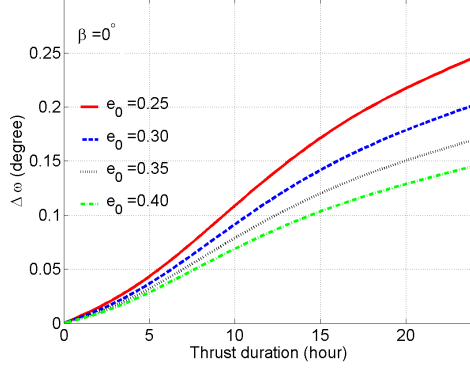


Figure 10: Change in  $\omega$  per orbit achieved as a function of the eccentricity and total duration of the thrust arc. Thrust vector is in the orbital plane.

of the correction  $\Delta\omega$  as shown in Fig. 10. The correction per orbit needed to compensate the worst-case perturbations is  $0.0205^\circ$ . Another  $0.0014^\circ$  must be added due to the coupling with the  $\Omega$  maneuver (see Sect. 6.2.2) for a total of  $0.022^\circ$  per orbit. Note that the central part of the arc (i.e., the neighborhood of the perigee) is not available, as it is used for the ascending node correction. Therefore, the thrust arcs must be applied before and after the  $\Omega$  correction maneuver which, as shown in Sect. 6.2.2, lasts 4.2 hours. To determine the bounds of the thrust arcs, the value of  $\Delta\omega$  for a duration of 4.2 hours is read from Fig. 10 ( $0.023^\circ$ ) and  $0.022^\circ$  are added obtaining  $0.045^\circ$ . The time corresponding to this value (7 hours) indicates the bounds of the thrust arcs. That is, the thrust arcs span 1.4 hours before and after the  $\Omega$  correction (e.g., from 3.5 to 2.1 hours before perigee for the first arc). The associated impulse is 0.27 m/s per orbit, which amounts to a total  $\Delta V_\omega^{LT}$  of 200 m/s over two years.

#### 6.4. Correction of the eccentricity

The change in eccentricity is due almost completely to the effect of the Moon. The variation over a two-year period is small and has no appreciable effect on the coverage footprint. Therefore, if left uncorrected it would not affect the primary mission of the satellites. However, due to operational reasons (e.g., to

480 avoid interfering with the orbits of other spacecraft) it may be desirable to keep  
 $e$  constant. The impulsive maneuver required is thus included for reference pur-  
 poses. The low-thrust option, on the other hand, has been disregarded because,  
 according to [28, 31], in order for the maneuver not to affect the semimajor axis,  
 thrust should be applied over the entire orbit, i.e., also around apogee when the  
 485 satellite is online for communication service.

#### 6.4.1. Impulsive maneuver

The maneuver consists in a pair of burns applied at the line of apsides. For  
 example, to increase  $e$  the apogee is raised with a forward impulse at the perigee,  
 while the perigee is lowered by the same amount with a reverse impulse at the  
 490 apogee. For small corrections, the total impulse can be linearized as follows:

$$\Delta V = \frac{GM_E}{4a} \left( \frac{1}{V_\pi} + \frac{1}{V_\alpha} \right) \Delta e. \quad (21)$$

For a worst-case correction  $\Delta e = 0.07$  (set 9,  $\Omega_0 = 120^\circ$ ) an impulse  $\Delta V_e^I$  of 60  
 m/s is required.

#### 6.5. Correction of the orbital inclination

The change in inclination is due almost entirely to the lunar perturbation.  
 495 The variation is small and has little impact on the constellation coverage. Given  
 that a change-of-plane maneuver requires a comparatively large impulse, it  
 should be carefully evaluated whether this correction is strictly necessary. For  
 the sake of completeness, the budget of the impulsive maneuver is included  
 (Sect. 6.5.1). The low-thrust option, however, has been discarded on the fol-  
 500 lowing basis: in order for the maneuver to cause the maximum rate of change  
 of  $i$ , thrust must be perpendicular to the orbit plane and must be applied over  
 an arc centered at a point whose true anomaly ( $\theta_i$ ) depends on eccentricity ( $e$ )  
 and argument of perigee ( $\omega$ ) [31]:

$$\sin(\theta_i + \omega) = -e \sin \omega. \quad (22)$$

For the orbits under consideration, such point falls in the northern hemisphere,  
 505 i.e., in the part of the orbit reserved for telecommunication service.

### 6.5.1. Impulsive maneuver

A change  $\Delta i$  in the orbital inclination not affecting the position of the ascending node is achieved by an impulse applied at either node:

$$\Delta V = 2V \sin\left(\frac{\Delta i}{2}\right). \quad (23)$$

For a worst-case correction  $\Delta i = 1.2^\circ$  (set 1,  $\Omega_0 = 300^\circ$ ) an impulse  $\Delta V_i^I$  of 62  
510 m/s is required.

### 6.6. Estimation of total station-keeping cost

The worst-case scenario (set 9,  $\Omega_0 = 180^\circ$ ) for the most relevant maneuvers, i.e.,  $\Omega$  and  $\omega$  correction, corresponds to velocity variations of 282 m/s ( $= \Delta V_\Omega^I + \Delta V_\omega^I$ ), and 490 m/s ( $= \Delta V_\Omega^{LT} + \Delta V_\omega^{LT}$ ), for the high and low thrust strategies,  
515 respectively. The perturbations of  $e$  and  $i$  for this scenario are 0.027 and  $0.17^\circ$ , respectively (observe these are not the worst-case values recorded in Table 2). The impulses required to correct these perturbations are  $\Delta V_e^I = 23$  m/s for eccentricity and  $\Delta V_i^I = 11$  m/s for inclination. Therefore, the total impulse budget over two years for the high-thrust maneuvers when all four parameters  
520 are corrected is  $\Delta V = 316$  m/s.

**From the economic point of view, the combination of orbital parameters yielding the minimum station-keeping cost is extremely interesting. The fact that each constellation includes three different orbital planes subject to vastly different lunar perturbations (which  
525 also change in time due to the precession of Moon's orbit) means that there is no single set of optimum orbital parameters. However, it is possible to find families of orbits that, over time, experience smaller-than-average changes of the orbital elements. The largest slice of the propellant budget (almost 60%) corresponds to  $\omega$  corrections, offering the maximum potential for cost reduction. As stated previously,  
530 when all the different values of  $\Omega_0$  in a set are considered, orbits with  $e_0=0.25$  and 55 degrees of inclination experience the slowest average**

drift of the perigee. They are thus the most economically advantageous.

## 535 7. End-of-life disposal

Once a satellite reaches the end of its operational life, measures must be taken to prevent it from interfering with the operation of active spacecraft. Low-altitude satellites are often removed by the action of atmospheric drag. For the orbits under consideration in this paper, artificially  
540 de-orbiting the satellite to let it burn in the atmosphere is out of question due to the enormous impulse required ( $\sim 1$  km/s). On the other hand, allowing the natural orbital evolution to act may decrease significantly the de-orbiting cost. Recent studies show that the orbital instabilities due to the lunisolar perturbations and the  
545 associated resonances, produce an increase of the orbital eccentricity and may cause atmospheric reentry over medium to long time scales (from decades to hundreds of years) [32, 33, 34]. However, if left uncontrolled, a satellite in an inclined geosynchronous orbit quickly enters the GEO protected ring. If this has to be avoided, a viable option  
550 is to move the spacecraft to a circular orbit far from the most congested bands (e.g., avoiding conflicts with GPS and GEO satellites) [35]. In this section we analyse the operational strategy and the cost associated to this type of disposal by considering both impulsive and continuous thrust maneuvers.

### *7.1. Impulsive disposal*

555 The orbit can be circularized to an altitude intermediate between the perigee and apogee with a two-burn impulsive maneuver. For example, the maneuver could start with a backward impulse at the perigee followed by a forward burn at the apogee of the transfer orbit to circularize the trajectory. Alternatively, the maneuver could start with a forward burn at the apogee followed by a backward  
560 impulse at the perigee of the transfer orbit. It turns out that the best strategy

is to start the transfer at the apogee of the original orbit. Fig. 11 shows the reduction in impulse achieved by starting the transfer at the apogee as a function of the radius of the final circular orbit (for an initial orbit eccentricity  $e_0 = 0.4$ ). Note that when the radius of the final orbit is the same as the perigee or apogee of the initial orbit (25299 km or 59030 km respectively) the difference vanishes because the transfer becomes a single-burn maneuver. To determine the optimal disposal altitude, the total impulse requirement is plotted against the radius of the parking orbit, as shown in Fig. 12. It is clear from the plot that the cost is minimum when the radius of the parking orbit is equal to the initial apogee. Observe that the cost of disposal increases dramatically if the radius of the final orbit is higher than the initial apogee or lower than the initial perigee. Note also that, from the operational point of view, the optimum circular disposal orbit is perfectly viable, as it lies well above the GEO ring. Table 3 summarizes the cost of impulsive disposal for the different eccentricities tested. The cost increases with the initial eccentricity, with a maximum of 586 m/s for  $e_0 = 0.4$ .

Table 3: Cost (third column) of the impulsive disposal in a circular orbit at initial apogee altitude (second column). The first column gives the initial orbital eccentricity.

$e_0$	Disposal orbit radius (km)	Disposal cost (m/s)
0.25	52706	368
0.30	54814	440
0.35	56922	513
0.40	59030	586

575

## 7.2. Low-thrust disposal

Circularisation at apogee altitude can be obtained by continuous in-plane thrust. Results are here provided for a solution consisting in uninterrupted thrust perpendicular to the major axis of the orbit (see also [28] and [36]).

580 Table 4 gives the performance of this maneuver for the four values of the initial

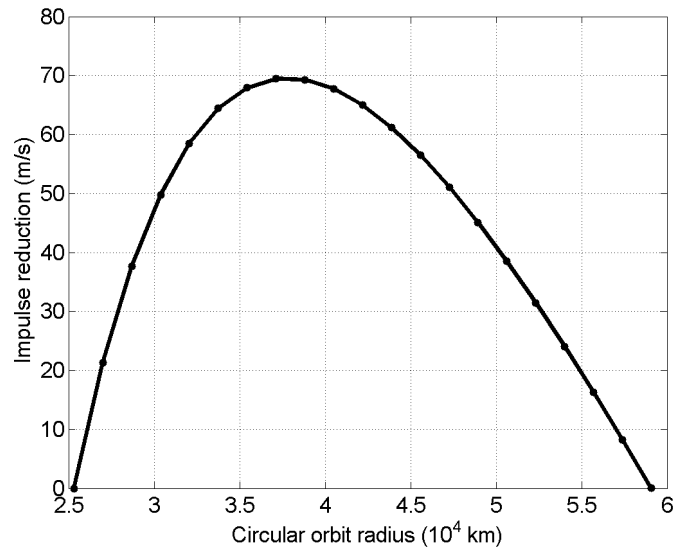


Figure 11: Reduction in impulse achieved by starting the disposal maneuver at the apogee instead of the perigee, as a function of the radius of the final circular orbit (for an initial orbit eccentricity  $e = 0.4$ ).

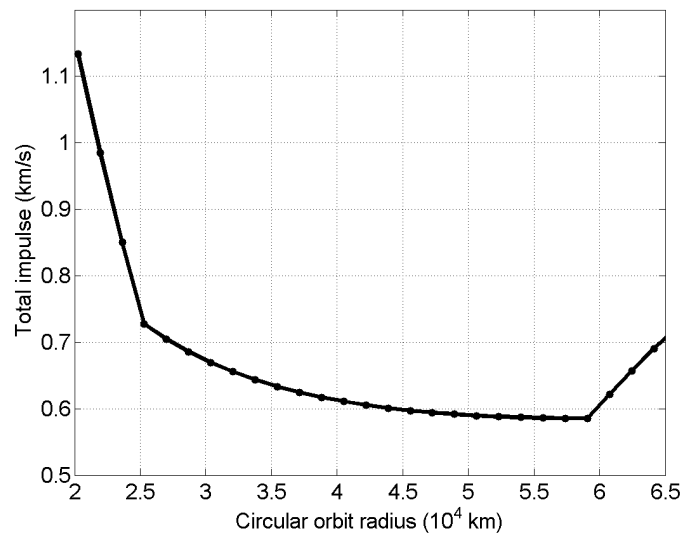


Figure 12: Impulsive disposal in a circular orbit: total impulse requirement against the radius of the final orbit ( $e_0 = 0.4$ ).



eccentricity chosen. The disposal of the most eccentric orbit ( $e = 0.4$ ) takes 344 days and corresponds to a velocity variation of 780 m/s.

Table 4: Characteristics (i.e., execution time and cost) of the low-thrust disposal in a circular orbit at initial apogee altitude assuming uninterrupted in-plane thrust perpendicular to the major axis of the orbit.

$e_0$	Execution time (days)	Disposal cost (m/s)
0.25	217	490
0.30	259	590
0.35	301	680
0.40	344	780

## 8. Conclusions

A study of orbital evolution, station-keeping maneuvers and disposal options for telecommunications satellites in highly-inclined geosynchronous orbits has been presented. A numerical model of the relevant physical effects has been developed which, considering the accuracy requirements of the mission and the instantaneous satellite altitude, dynamically adjusts the expansion degree of Earth's gravitational field to obtain maximum performance during orbit propagation. The model is applied to nine sets of constellations (defined by orbital inclination and eccentricity) that serve areas of maximum economic viability (i.e., housing the largest population). A station-keeping strategy has been devised where, instead of keeping the orbital elements constant, only the ground track of the constellation is maintained while reducing the propellant expense. The impulse budget for  $\Omega$  corrections can thus be reduced by as much as 90%.

Due to the high altitude of the perigee, the most important perturbation is due to the Moon with the harmonics of the gravitational

600 field of the Earth playing a minor role. The magnitude of the lunar perturbation depends strongly on the relative inclination of the orbital planes of the satellite and Moon, which is a function of the separation of their ascending nodes. Therefore, the three satellites of a constellation experience vastly different evolutions at any given  
605 time as their orbital planes are offset by 120 degrees. Note that, due to the dominance of lunar perturbations and the precession of Moon's orbit, the behavior of a satellite at an epoch different from that chosen in the discussion (01/07/2013) can be roughly approximated by a shift in  $\Omega$  equal to the motion of lunar ascending node. Thus, the  
610 data presented are also indicative of the evolution of spacecraft with arbitrary launch dates.

To provide an upper bound of the station-keeping cost, a worst-case scenario (requiring the maximum impulse over a two-year period) has been identified. For the sake of completeness, typical (i.e.  
615 representative of the median value) changes of orbital elements are also provided. The impulse budget has been computed for high and low-thrust maneuvers. The impulsive correction of the argument of the perigee uses an elliptical transfer arc which minimizes the propellant consumption. As there is no closed expression for the parameters  
620 of the optimum transfer ellipse, an efficient solution has been found using a combination of minimization algorithms with and without gradients. For the impulse strategy, the worst-case scenario requires a total impulse of 316 m/s. This includes 182 m/s for  $\omega$  corrections and 100 m/s for  $\Omega$ , with eccentricity and inclination accounting for  
625 around 10% of the total budget. The low-thrust maneuvers, for their part, have  $\Delta V$  requirements that are roughly twice as large.

Of the sets considered, orbits with  $e_0=0.25$  and 55 degrees of inclination experience the slowest rate of drift averaged over the different orbital planes. They are therefore the most attractive economically,  
630 as the cost of  $\omega$  corrections dominates the propellant budget.

Finally, an end-of-life disposal strategy has been outlined. Given the high altitude of the perigee, lowering it to the point where atmospheric drag de-orbits the satellite in a reasonable time frame is unfeasible (it calls for an impulse close to 1 km/s). Instead the spacecraft is parked in a circular orbit far from operational satellites. Minimum impulse is required when the radius of the circular orbit coincides with the apogee of the initial orbit. Cost of disposal increases with the eccentricity of the orbit. For impulsive maneuvers 586 m/s are needed for  $e_0=0.4$ . The orbit can alternatively be circularized with in-plane low-thrust perpendicular to the major axis at a cost of 780 m/s (the maneuver taking almost a year to complete).

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