Mode-localized sensing in micro- and nano-mechanical resonator arrays

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***Abstract*— Micromachined resonant sensors have been researched for several decades for a variety of applications with potential benefits in terms of improved sensitivity and scalability relative to other transduction principles. Conventional implementations usually involve detection elements monitoring resonant frequency and/or dissipation shift in a single degree-of-freedom or several independent degrees-of-freedom. This paper discusses a complementary approach to resonant sensing in which the eigenstates in coupled array structures can be employed as a read-out mechanism offering the potential for both increased sensitivity and excellent common mode rejection. The technique, dubbed “mode-localized sensing”, is based on the spatial localization of vibration in an array of nearly identical weakly coupled resonators wherein the eigenstates are seen to be sensitive signatures of symmetry-breaking perturbations in structural parameters. The sensing methodology has been applied to a range of applications including gravimetric sensing, electrometry, inertial sensors and force sensors where in high accuracy approaches to physical transduction are often of interest. The paper concludes with a discussion of the current challenges in the field and an outlook on future developments.**

Keywords—resonators; resonant sensors; vibration mode localization; mode-localized sensing.

I. INTRODUCTION

Vibrations propagate evenly through a structure comprising of mechanically identical resonators coupled through weak springs. However, structural perturbations in any one of the resonators breaks system symmetry leading to confinement of vibration energy in certain spatial locations. The extent of vibration energy confinement or localization depends on the magnitude of the symmetry breaking perturbation as well as strength of the coupling between the individual resonators with weaker coupling resulting in stronger localization. The phenomenon of mode localization was first studied by Anderson in the field of solid-state physics and since applied to the areas of acoustics and structural dynamics [1]. By monitoring the shifts in eigenstates caused by such vibration localization it is possible to construct the basis for a new type of sensor where the measured shifts in eigenstate are made functions of the measurand. This technique, dubbed as *mode-localized sensing*, has received considerable research interest in recent years [2].

Measuring the shifts in the eigenstates caused by such vibration localization can, consequently, yield sensitivities orders of magnitude greater than corresponding fractional changes in resonant frequency under conditions of weak internal coupling [2]. Furthermore, assuming each of the coupled resonators in the array are subjected to approximately the same ambient environmental conditions while only one of the coupled structures can be designed for sensitivity to an induced load or measurand, an output that is highly sensitive to the differential load but relatively insensitive to common mode variables such as temperature and pressure can be obtained [3]. This technique can be combined with resonant frequency shift measurements across the same array to provide combined benefits of high accuracy and large dynamic range within the same device.

This paper reviews the theory underpinning the sensing principle, summarizes the results from emerging device applications and outlines the remaining technical challenges in order to fully realize the benefits offered by the sensing technique. An outlook for further research in the field is also discussed.

II. BACKGROUND AND THEORY

The simplest mathematical model for the system involves two weakly coupled nearly identical un-damped resonators ($k_1 = k_2 = k$; $m_1 = m_2 = m$) as shown in Fig. 1 (a) whose free vibration behaviour can be described as follows [2]-

$$\begin{aligned} m\ddot{x}_1 + kx_1 + k_c\left(x_1 - x_2\right) &= 0 \\ m\ddot{x}_2 + kx_2 + k_c\left(x_2 - x_1\right) &= 0 \end{aligned} \qquad (1)$$

The eigenvalues and eigenvectors for this system can be expressed as:

$$\begin{aligned} \lambda_{01} &= \sqrt{k/m}, u_{01} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} \\ \lambda_{02} &= \sqrt{\left(k+2k_c\right)/m}, u_{02} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix} \end{aligned} \qquad (2)$$

In the presence of structural perturbation (e.g. due to a change in stiffness in one of the resonators), the system describes symmetry-breaking and an estimate for the shift in eigenstates can be obtained using perturbation theory as [2]-

$$\frac{\Delta u_{0n}}{u_{0n}} \approx \left| \frac{\Delta k}{4k_c} \right| \tag{3}$$

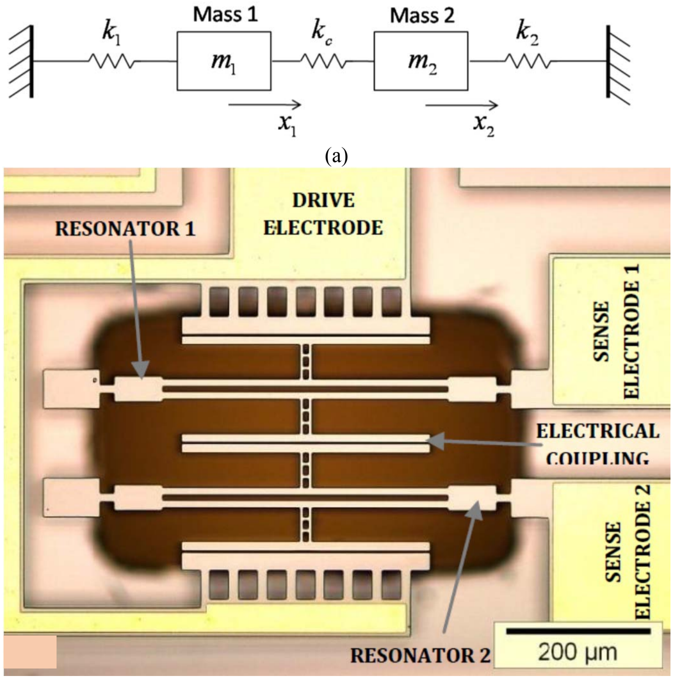
The associated fractional shift in resonant frequencies can be expressed as-

$$\frac{\Delta \lambda_{0n}}{\lambda_{0n}} \approx \left| \frac{\Delta k}{2k} \right| \tag{4}$$

It can be seen that the eigenstate sensitivity to structural perturbation can potentially be significantly larger than that for resonant frequency shifts if the coupling factor is such that-

$$k_c << k/2 \tag{5}$$

The relative shifts in eigenstate can be as large as approximately 3 orders of magnitude more sensitive as compared to shifts in resonant frequency as experimentally demonstrated in [2]. Furthermore by implementing electrical coupling between the resonators it is possible to allow for the scale factor to be tuned as a function of an input bias voltage.



(a)

(b)

Fig. 1. (a) A discrete model of two weakly coupled resonators; (b) An optical micrograph showing a microfabricated device whose response can be described by the model in (a).

Further, it is possible to estimate the limits to sensitivity imposed by intrinsic noise generation that represent the fundamental limits to measurement using this transduction approach. This minimum detectable stiffness perturbation can be expressed as [4]-

$$\frac{\Delta k_n}{k} = 8\frac{k_c}{k}\sum_{r=1}^{2}\sqrt{\frac{E_{th}\Delta f}{2E_c Q \omega_r}} \tag{6}$$

where E_{th} is the thermal noise energy, E_c is the energy stored in the resonator, Q is the Quality factor, Δf is the bandwith, and ω_r is the resonant frequency.

Furthermore, since the eigenstates are not sensitive to perturbations that preserve symmetry (e.g. due to common mode effects), the sensitivity to environmental variables such as temperature and pressure is minimized. This feature has been experimentally demonstrated in previous work [3] with upto two orders of magnitude passive immunity to temperature and pressure demonstrated.

III. DEVICE APPLICATIONS

Spletzer *et al* [5] first demonstrated the application of mode-localization to gravimetric sensing in a pair of coupled micro-cantilevers. They were able to show that the shifts in eigenstates could be as high as two to three orders of magnitude greater than the resonant-frequency variations induced by identical perturbations in mass. Thiruvenkatanathan *et al* [2] introduced the formalism of mode-localized sensing for both mass and stiffness perturbations in weakly coupled resonator arrays and the benefits of employing weak electrical coupling springs. An axial strain modulated electrometer comparing measurements of eigenstate variation with mode-localized sensing was demonstrated with sensitivity to charges in the range of > 100 fC demonstrated [6]. Thiruvenkatanathan *et al.* also subsequently demonstrated the application of this concept to displacement sensing [7] and force measurements. Zhao *et al.* extended this concept to a topology consisting of three coupled resonators and demonstrated a mode-localized micromachined force sensor [8]. More recently Zhang *et al.* demonstrated the application of mode-localized sensing to the measurement of acceleration [9] demonstrating the basis for new types of inertial sensors based on this transduction principle.

IV. SUMMARY AND OUTLOOK

Mode-localized sensing has emerged an alternative transduction approach to enabling high accuracy measurements using weakly coupled micro- and nano-mechanical resonator arrays. A number of device demonstrations have shown the practical realization of this physical transduction principle in formats that enable a myriad of sensing applications. However, current read-out techniques are typically based on open-loop swept frequency measurement using network analyzers. This considerably limits scalability and therefore techniques that involve closed-loop driving of the coupled resonators together with appropriately designed amplitude measurement circuits are essential to enable further development. Furthermore, such read-out techniques must also ensure that the associated electronic noise introduced through measurement does not compromise the performance enhancements achievable through the readout mechanism. Finally, it should be noted that the dynamic range associated with the measurement is usually considerably limited as compared to the resonant frequency shift readout principle. However, it is possible to adapt this technique to be used in concert with the resonant frequency shift-based readout approach to enable both high accuracy and large dynamic range measurements within a single device.

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