Thinking through Actions with Things:

A Systemic Perspective on Analytic Problem Solving and Mental Arithmetic

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Abstract

In solving everyday problems or making sense of situations, people interact with local resources, both material and cultural (Kirsh, 2009a). Through these interactions with the world, thinking emerges from within and beyond the boundaries of the mind. Traditional frameworks specify that problem solving proceeds from initial state to goal state through the transformation of a mental representation of the problem by the retrieval and manipulation of symbols and rules previously stored in memory. Information garnered through bodily actions or from transactions with the world is considered to be a passive input. As a result, classical models of cognitive psychology frequently overlook the impact of the interaction between an individual and the environment on cognition.

The experiments reported here were designed to inform a different model of problem solving that included the ubiquitous nature of interactivity in daily life by examining problem solving using artefacts. This research programme began with two experiments using an analytical problem, namely the river-crossing task. These experiments offered a platform to investigate the role of interactivity in shaping and transforming the problem presented. However, the problem space in the river-crossing task is relatively narrow and the research programme proceeded to three further experiments, this time using mental arithmetic tasks where participants were invited to complete long sums. These problems afford a much larger problem space, and a better opportunity to monitor how participants' actions shape the physical presentation of the problem.

Different task ecologies were used in the five experiments to contrast different levels of interactivity. In a low interactivity condition, solvers relied predominantly on internal mental resources; in a high interactivity condition participants were invited

to use artefacts that corresponded to key features of the problem in producing a solution. Results from all experiments confirmed that increasing interactivity improved performance. The outcomes from the river-crossing experiments informed accounts of transfer, as it was revealed that attempting the problem initially in a low interactivity condition followed by the high interactivity condition resulted in the most efficient learning experience. The conjecture being that learning of a more deliberative nature was experienced in the low interactivity version of the problem when followed by the opportunity to showcase this learning through the enactment of moves quickly in a second attempt that fostered a high level of interactivity. The mental arithmetic experiments revealed that a high level of interactivity not only produced greater accuracy and efficiency, but participants were also able to enact different arithmetic knowledge as they reconfigured the problem. In addition, the findings indicated that: maths anxiety for long additions could be mitigated through increased interaction with artefacts; trajectories for problem solving and the final solutions varied across differing interactive contexts; and the opportunity to manipulate artefacts appeared to diminish individual differences in mathematical skills.

The varied task ecologies for the problems in these experiments altered performance and shaped differing trajectories to solution. These results imply, that in order to establish a more complete understanding of cognition in action, problem solving theories should reflect the situated, dynamic interaction between agent and environment and hence, the unfolding nature of problems and their emerging solutions. The findings and methods reported here suggest that a methodology blending traditional quantitative techniques with a more qualitative ideographic cognitive science would make a substantial contribution to problem solving research and theory.

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"Accordingly, just as we say that a body is in motion, and not motion is in a body we ought to say that we are in thought, and not that thoughts are in us."

C.S. Peirce, 1868.

"Activity such as arithmetic problem solving does not take place in a vacuum, but rather, in a dialectical relationship with its settings."

J. Lave, 1988.

"Finding suitable replacements for the traditional dualistic images will require some rather startling adjustments to our habitual ways of thinking, adjustments that will be just as counterintuitive at first to scientists as to laypeople."

D.C. Dennett, 1991.

"The artifact is not a piece of inert matter that you *act upon,* but something active with which you *engage* and *interact*."

L. Malafouris, 2013.

Chapter 1

General Introduction

The Rodin thinker is the traditional image of effortful cognition. This image very much sums up important assumptions and research commitments of the prevalent cognitivist paradigm in problem solving research: thinking processes are mental processes that transform mental representations, that yield choices or solutions, that are then selected for action. The research reported in this dissertation questions these assumptions: it proceeds from a perspective on reasoning that casts it as a product of action. To understand thinking you have to understand the role of interactivity, that is, how a reasoner interacts with the physical environment in configuring a cognitive system. This thesis reports five experiments on problem solving that compare and contrast performance as a function of the degree of interactivity. In all cases, performance was substantially transformed by a high degree of interactivity. On the basis of these results, theoretical implications and methodological recommendations are outlined.

My interest in the notion of thinking as a product of action was initially sparked by Hollan, Hutchins, and Kirsh's (2000) theory of distributed cognition, where they proposed theoretical principles to distinguish their account on thinking from traditional accounts of cognition. Historically, cognitive psychology is based on the keystone elements of the information-processing model of human thinking, where cognitive processes are likened to those of a computer (e.g., Newell & Simon, 1972; Newell, Shaw & Simon, 1958). This is often referred to as an internalist account of cognition where thinking is purported to emerge from processes occurring within the brain; external interactions between the body in the world or knowledge drawn from the surrounding environment are regarded as passive inputs (see Gallagher, 2008; Menary, 2007; Sutton, Harris, Keil, & Barnier, 2010; Vallée-Tourangeau & Wrightman, 2010; Kirsh, 2013; Malafouris, 2013). Generally, a non-internalist[1], approach, such as the theory of distributed cognition, presupposes a systemic notion of cognition whereby thinking emerges from interactions between the mind, the body, and the external world. Through further investigation into systemic cognition and non-internalist theories it became apparent that explanations of human thinking excluding interactions with the world, paint an incomplete picture of cognition (at best; at worst, the accounts are misleading). As a consequence, I become increasingly convinced of the fruitfulness of formulating non-internalist explanations of cognition, where thinking is posited as an emergent product of the interaction between the individual and the environment. From this perspective on cognition, I became aware that generally, psychological research and many canonical theories on human thinking were inadvertently grounded in a disembodied approach to cognition. By way of illustration, in the Baddeley and Hitch (1972) model of working memory there is no representation of the external world as anything more than a passive input. This is not to be presumptuous, nor is it the aim of this thesis to suggest that such models and theories are necessarily invalid because of the exclusion of interaction between agent and environment. Rather, one of the goals of this research programme was to illustrate the validity of, and foster awareness for, the reporting of the impact of interaction and situatedness on experimental outcomes or when developing theoretical models of cognition.

[1] Although previous authors have used the term ‘externalist’ (e.g. Hurley, 2007; Menary, 2011) to describe theorists and researchers who do not ascribe to internalist theories, this thesis will adopt the term ‘non-internalist’. The approach subscribed to here is one of thinking emerging from both internal and external resources in a system of cognition, where the term externalist has the potential to imply exclusion of internal processes of the mind from cognition.

Zhang and Norman (1994) previously undertook innovative research into problem solving and external representations, however they did not specifically address interactivity with artefacts. Although their investigations revealed interesting insights into the effect of the external representation on performance in problem solving, in the ensuing years there was limited attention given to the topic from cognitive psychologists, with a handful of researchers pursing experimental approaches (e.g., Kirsh, 1995a; Ball & Litchfield, 2013; Vallée-Tourangeau, 2013; Vallée-Tourangeau, Steffensen, Vallée-Tourangeau, & Sirota, 2016;). Consequently, there remained an extensive opportunity to explore interactivity in the lab. While the research programme presented here was essentially constructed from a psychological perspective, the field for inspiration also modestly spanned other domains including ethnography and philosophy of mind. The reason for drawing on this literature will become apparent as the thesis unfolds. However in short, the primary driver was to present a well-developed argument in favour of systemic cognition, showcasing a history of theory and research on this perspective of human thinking that has spanned not a few decades but more than a century.

Much of the traditional psychological experimental literature on interactivity or that comparing concrete to abstract presentations of problems is predicated on research on the interaction between a person and a computer—the VDU and the keyboard (e.g., Kaminski & Sloutsky, 2009; Neth & Payne, 2001, 2011; Svendsen, 1991). To be sure, there is some degree of interaction when working with a computer, yet we are surrounded by a wealth of artefacts with which we interact—one of which is the computer. By framing interactivity in this narrow fashion, once again there is the possibility of only painting a partial picture of cognition. These computer-based activities are frequently centered on the affordances offered by the machine in a two dimensional context, they are not necessarily centered on the potential of the person acting in an artefact-rich world solving a problem through the manipulation and exploration of the resources in a multi-dimensioned environment (Norman, 1993). This thesis adopted a broader view of interactivity as a dynamical, sense-making process that co-ordinates and integrates human actions in the bi-directional coupling of the body with others, with artefacts and with practices (Steffensen, 2013).

The review of literature was divided into four chapters beginning with an overview of the traditional cognitive psychology account of thinking through to contemporary alternative theories on cognition. These chapters also include discussions on interactivity, artefacts and cognition, and the individual differences tasks used in the experiments presented in this thesis. The first chapter of the review, Chapter 2, is a foundation for discussion on the ecological turn of cognition, commencing with an outline of the traditional information-processing model, moving onto a situated account of cognition where thinking and knowledge are firmly tied to context and setting. Greeno's (1989, 1998) contribution in support of a situated perspective on thinking, as presented in this chapter, was considered a suitable starting point for the discussion, as he directly challenged the information-processing account of cognition. Greeno explained, that although a traditional cognitive science approach to thinking provided a strong framework for research, the impact of context and setting was disregarded. Greeno argued for a model that explained the interaction of abstract internal representations with physical external representations in order to understand the processes of everyday thinking. The exploration in Chapter 2 of the situated approach to cognition continued with Lave's (1988) *Cognition in Practice.* This is a rich source of examples and theoretical conjectures on cognition in the lived-in world as a precursor to subsequent approaches to cognition such as distributed cognition and extended mind. Chapter 3 introduces the 4E's—embodied, enacted, embedded, and extended cognition—with a more extensive treatment of Clark and Chalmers (1998) extended mind hypothesis finishing with a brief discussion of Material Engagement Theory (Malafouris, 2013). Chapter 4 moves on to the theory of distributed cognition, with a closer look at interactivity as the co-joining link in a dynamic cognitive system, and concludes by underscoring the role interaction with artefacts plays in problem solving. Chapter 5 presents a background of the individual difference measures employed in the experimental sessions presented as part of this research programme.

The empirical evidence is introduced in Chapters 6 and 7 with a series of five experiments focusing on problem solving performance as a function of interactivity. Various individual difference measures were also included in the suite of tasks as a means of exploring disparities in performance by participants when exposed to the same problem in different interactive contexts. In all five experiments the external problem presentations were altered in order to contrast performance in low and high interactivity contexts. In Experiment 1 and 2 a transformation problem, namely the river-crossing problem, was used to establish the impact of interactivity on performance. In addition learning transfer was investigated to determine the efficiency of learning across different problem presentations and levels of interactivity. The river-crossing puzzle provides a convenient platform for the study of problem solving with a well defined problem space bridging the initial state and the goal state. This particular transformation problem also lends itself well to presentations in different interactivity contexts from which to investigate situated and distributed perspectives of cognition.

However, these advantages come with limitations, including in this instance a narrow problem space, and tight constraints on the allowable moves, restricting the possible trajectories the solver can explore on the path to solution. To address these limitations, the research programme proceeded with three further experiments using mental arithmetic problems in the form of long additions of varying sum sizes. The aim of the three mental arithmetic experiments remained, at the core, to investigate the impact of interactivity in problem solving with additional hypotheses tested for each experiment as explained within the chapters. All mental arithmetic experiments included an investigation of the impact of maths anxiety across various interactive conditions. The first experiment contrasted problem solving performance in two interactivity contexts, with the second experiment employing four different interactive contexts (this was the only experiment with more than two levels of interactivity) and the third experiment explored the influence of expertise on problem solving across levels of interactivity.

Chapter 8 provides a summary of these experiments and methodological observations resulting from this research programme. In addition there is a snapshot of the theories on human thinking processes proposed by early pragmatists, Dewey and Peirce. As suggested by Gallagher (2014), these theories were precursors to many of the prevailing non-internalist views on cognition and potentially provide a basis for an integrated theory of systemic cognition.

The novel nature of the design of the experiments and the outcomes reported in this thesis have shown that it is possible to study interactivity by engineering research in the lab that allows participants to interact with the physical presentation of a problem. The argument presented here for a systemic account of human cognition promotes the importance of identifying and acknowledging the influence of different task ecologies and the effect of interactivity on thinking.

Chapter 2

Re-Thinking Thinking

Overview

Two significant approaches to cognition were being highly debated late last century. The well-established traditional approach of cognitive science based on a model of symbolic processing, and the emerging approach of situated cognition (Norman, 1993b). As an indication as to the importance of this debate to the study of human cognition, a special edition of the journal Cognitive Science was published with contributions from prominent advocates from both sides of the debate. Greeno and Moore (1993), Agre (1993), Suchman (1993), and Clancey (1993) presented a range of views on situativity in response to an article by Vera and Simon (1993) which suggested that this new approach of situated cognition could be subsumed into the existing symbol-system approach.

While this chapter does not pursue the debate of that special edition, it does explore both approaches to cognition. In order to establish the fundamental arguments that prompted the development of the situated perspective, this chapter begins with a modest outline of the information-processing model as the founding approach to the study of cognitive science. Those advocating the situated approach to cognition challenged the mainstream perspective that the individual is the locus of knowledge and thinking (Greeno, 1989; Hutchins, 1995a, 1995b; Lave; 1988). The review on situated cognition presented in this chapter showcases the work by Greeno (1989, 1998) and Lave (1988) as early researchers into learning and cognition in situ. Greeno (1989) was not dismissive of the information-processing approach as the model he proposed, in part, incorporated the concept of symbolic processing in representing the interactions between agents and social and physical systems. Greeno's research (1989, 1998) was chosen for discussion in this chapter as an introduction to the notion that cognitive processes consist of multiple representations—both internal and external to the mind—that led Greeno to doubt prevailing views of knowledge and learning transfer. Lave also questioned traditional approaches to learning transfer, motivating her to undertake ethnographic investigations similar to other influential researchers on situated thinking (e.g., Carraher, Carraher, & Schliemann, 1985; Hutchins, 1995a). However, Lave's (1988) study, The Adult Math Project, was chosen for in depth review in this chapter as her work was not only observational, it was also informed by experimental evidence. In comparing similar tasks across different settings, Lave's study illustrated how the environment in which an individual is acting does not simply provide stimuli for action; the interaction of the agent with the situation potentially impacts performance and strategies used in the problem-solving process. While Lave's ethnographic methodology was not used in experiments reported in this thesis, this early foregrounding of interactivity and the experimental evidence reported by Lave were influential in informing the methodological choices. This will be discussed in more detail in Chapter 8 as part of the reflections on methodology.

Subsequent chapters will illustrate, that as the debate unfolded situated cognition was not subsumed into the traditional model of cognition. Instead, by building on previous arguments in favour of the ecological perspective of cognition, investigation into other aspects of the role of the physical environment on thinking and the movement toward a better understanding of human cognition in situ flourished.

Introduction

Lave (1988) challenged conventional views that abstract knowledge learnt in the schoolroom was readily transferred to the ‘real’ world. Her book, *Cognition in Practice*, contributed to the emerging argument that to better understand problem solving it was remiss of researchers to exclude the interaction between individuals and their surroundings (Sawyer & Greeno, 2009). Lave was writing at the same time as a growing number of scholars worked on situated cognition and situated learning, with investigations spanning the classroom, the supermarkets, human computer interaction, the skies and the high seas (Brown, Collins, & Duguid, 1989; Greeno, 1989; Hutchins, 1995a, 1995b; Suchman, 1987, 2007). The term situated cognition first appeared in an article by Brown et al. (1989) and a chapter by Greeno (1989) in the same year, with Lave and Wenger (1991) discussing the term situated learning a few years later (Sawyer & Greeno, 2009). This situated approach to cognition challenged the paradigmatic approach to cognition of cognitive science, namely the computational theory of the mind, being that the mind-brain functions in a manner similar to that of a computer. The situationalists were not dismissive of a computational perspective on cognition rather that computations involve information that is both internal and external to the individual. Research inspired by the foundational work of Lave (1988) and Greeno (1989) did not challenge the computational perspective of problem solving, but rather questioned the strict internalist version of the computational perspective (Zhang & Norman, 1994; Wilson, 1994). Emerging from the situated perspective on learning and thinking came the development of many prominent alternative theories and interpretations to the traditional approach to cognition (e.g., Hollan, Hutchins, & Kirsh, 2000; Hutchins, 1995a; Zhang & Norman, 1994;) including the 4E’s

(Gallagher, 2008) embedded (e.g., Clark, 1997), embodied (e.g., Gallagher, 2005), enactive (e.g., Passoa, Thompson, & Noë, 1998; Varela, Thompson, & Rosch, 1991) and extended (e.g., Clark & Chalmers, 1998). The commonality between these theories, distinguishing them from the core principles of the cognitivists' computational model, is the inclusion of the external environment beyond the brain as part a dynamic system of cognition.

This chapter will introduce the basic premise of the computational or information-processing model as initiated by Simon and colleagues (e.g., Newell, Shaw, and Simon, 1958; Simon, 1973; Simon, 1996) as a foundation for progression to a discussion on the importance of considering cognition as a system that embraces the lived-in world. The discussion on the systemic view of cognition will begin with an overview of situated cognition, based on the premise that it was arguably the first key movement within psychology away from the predominant paradigm of human cognition as an information-processing system. This overview of situated cognition will focus on some of the work by Greeno (1989, 1998) and Lave (1988) to exemplify the emergence of the ecological turn through theories such as the situated perspective of cognition and learning in the late twentieth century.

The Information-Processing Model

Newell, Shaw and Simon (1958) were among the earliest cognitive scientists to propose and experimentally investigate the concept of comparing early versions of a computer's information-processing capabilities to that of the human mind. Essentially the information-processing model parallels the processes of a computer during problem solving with that of the human brain. This metaphor of the brain-as-a-computer equates computer data with information received by the brain as inputs; processing as carried out in the brain being similar to that of the central processing unit of a computer, drawing on memory to facilitate the manipulation of information (or data); with the solution or response being emitted as an output.

According to Simon (1996) building computer systems that are organised “somewhat in the image of man” (p. 21) would potentially provide solutions to questions posed by psychology on human behaviour. This argument was in part supported by his description of both computers and the human brain as “physical symbol systems” (Simon, 1996, p. 21), interchanging the phrases ‘information-processing system’ and ‘symbol system’, in explaining how information is encoded as symbols, stored in memory and retrieved as needed. Inputs encoded by the brain and stored as symbols form symbol structures serving as internal representations of the world, which in turn are manipulated by the system—the computer or the brain— to produce action within the external world (Simon, 1996). The Newell et al. (1958) information-processing metaphor equated much of the organisation of problem-solving components of the computer with that of the human brain, however it did not extend beyond behavioural to a biological comparison as they maintained that they were not directly comparing the electrical circuitry of the computer to the synapses of the brain. The aim was to explain any behaviours of a computer system that resembled the problem-solving strategies of a human. To facilitate this aim, Newell et al. (1958) developed a computer program called Logic Theorist (LT). This program was capable of working through a series of differently structured problems. The initial motivation for the artificial intelligence (AI) program, LT, was the belief that the construct of intelligence in humans could be extended to include the intelligence of the artificial systems of computers facilitating increased problem-solving capabilities of computers. The LT program developed by Newell, Shaw and Simon drew on psychological research and observations of human memory and problem-solving processes in order to better understand human behaviour, while at the same time providing the foundation to build programs that would enable machines to 'learn' (Newell et al., 1958; Simon, 1973; Simon & Kaplan, 1989). Newell and colleagues scrutinised the algorithmic processes of the computer in solving the problems, comparing these processes to those of humans solving the same problems. They believed there were sufficient similarities between the behaviours of problem-solving processes and learning by LT to that of a human to initiate a theory on human problem solving that would be driven by the notion that problems were solved as a hierarchy of processes. Under this approach, a problem was composed of a hierarchy of subproblems this proceeded in two stages: first, the breaking down of the problem into components; second, the generation of subproblems. Through these observations of similarity, Newell and colleagues concluded that information-processing systems, (i.e. the digital computer) would provide a valuable approach for the investigation of not only problem solving but, the cognitive processes of other domains in psychology such as learning and perception by both applying them to computer learning, and the understanding of human psychological processes (Newell et al., 1958). This early work on artificial intelligence became the basis for Newell and Simon's (1972) symbol system hypothesis. Here cognition was considered a computational process relying on schemas and symbolic structures drawn from memory, innate or learned, which are manipulated in the brain in an algorithmic manner not dissimilar to that of a computer program (Wilson & Clark, 2009).

Problem structure. A fundamental element in the explanation of the problem-solving process, according to the computational model proposed by

Newell and Simon (1972), is the internal representation of the problem by the solver. Newell and Simon conceptualised this as taking place through the solver's comprehension of the problem space. Essentially this problem space is considered a depiction of how the solver understands and mentally represents the problem within the task environment, which encompasses both the external and internal elements of the problem (Greeno, 1998; Kirsh, 2009a; Newell & Simon, 1972). This internal representation of the problem space is composed of three parts, an initial state, a goal state, and between these two states is the path to solution (Newell & Simon, 1972). According to this model, the reasoner therefore constructs a mental representation of the problem, developed through knowledge of the goal, subgoals, and constraints cuing certain operators that lead to the selection of a move or action to reduce the gap between the initial state and the goal state.

In attempting to analyse problems, the structure of a problem is frequently categorised as being either well structured or ill structured. In a well-structured problem, the solver searches or moves through the problem space guided by fixed operators and constraints creating this representation of the problem on the search path for a solution toward the clearly stated goal (Simon, 1973). The path to a successful solution consists of a series of simple, discrete moves that unfold as the task progresses. One such problem is the river-crossing problem, also known as hobbits and orcs or missionaries and cannibals (Knowles & Delaney, 2005; Reed, Ernst, & Banerji, 1974). Here the initial state is presented as six or ten 'travellers' (depending on the task variant) on one side of a 'river bank' with the goal being to move all travellers from one bank to the final position on the opposite river bank (e.g., Knowles & Delaney, 2005; Simon & Reed, 1976). The constraints are constituted by a set of simple rules, and the operators in this problem, as a means of transforming one state to another, are the permitted moves in transporting the travellers across the river on a raft (Knowles & Delaney, 2005). Within the problem space there are a minimum of eleven moves required to reach a solution. Given this apparently narrow problem space, solvers nonetheless frequently breach the rules or make moves ending in blind alleys resulting in unnecessary moves in the search for a solution. Well-structured problems, such as this river-crossing problem, are useful tools for investigating strategies used by solvers in reaching the goal state as the route taken to reach the solution is tightly constrained by a small number of rules, therefore the search and move selection is limited. In turn this makes it possible for a researcher to track the steps taken to solution, the latency to solution, and the frequency with which the solver visits the nodes of the problem state as legal moves or breaches the rules producing illegal moves.

In turn, problems may be deemed to be ill defined or ill structured where some aspect of the problem lacks the clarity of structure present in a well-structured problem. The initial representation of the problem may be misleading, though this may not be evident to the reasoner until an attempt to solve the problem has commenced. As the solver works toward a solution, an impasse may be reached where it is clear that the inappropriate initial representation cannot lead to the solution phase (Ash & Wiley, 2006). Here the problem features of an ill-structured problem may emerge as the problem unfolds resulting in a changing problem space as it is constructed and modified from subgoals, new operators and constraints by the solver (Greeno, 1998; Simon 1973). Thus to overcome this impasse, the representation of the problem must be restructured or altered to provide insight into discovering a new direction in searching for the solution. The moment when the impasse is overcome, or insight into a solution occurs, is coupled with the certainty that a satisfactory solution has been reached (Gilhooly & Fioratou, 2009). This type of problem, known as an insight problem, can be accompanied by an “aha” (Topolinski & Reber, 2010, p. 402) experience when there appears to be a sudden realisation of the correct route to the solution. However, restructuring of the problem may or may not produce insight, as the cognitive operators cued by this restructured representation may not promote the correct solution (Ohlsson, 1984).

According to Simon (1973), the majority of problems confronted in everyday life are ill-structured problems; he maintained there were possibly no well-structured problems, as such; only ill-structured problems that had been formalised for the problem solver to subsequently become well structured. Simon proceeded to decompose an example of problem solving in the real world using what he considered to be an ill-structured problem, an architect designing a house, in terms of units of well-structured subproblems that arise sequentially as the problem unfolds. In Simon’s example, during the creation of the design of the house, the architect tackles each problem as it emerges generating a series of well-structured subproblems. These well-structured subproblems are resolved as the architect draws on long-term memory or the use of experts or expert material within the reach of the architect. Simon described the process of designing the house as an organised system of production that unfolds from the architect’s memory sequentially, as a series of subprocesses where each step of the task is solved as the stimulus for a solution to the next element. Extending this example to a more generalised treatment of ill-structured problems, Simon (1973) proposed that by processing what may be considered as an ill-structured problem in components it becomes less complex. Thus, it may be perceived as ill structured when a large problem, but well structured when deconstructed as a series of smaller subproblems. In approaching problem solving from this perspective it provided a foil against arguments that artificial intelligence problem-solving systems would be unable to solve ill-structured problems, as the notion of breaking problems down into a series of subproblems could be readily transposed onto the design of more complex computer programs (Simon, 1973). However, this model of problem solving becomes very complex for human problem solvers by potentially creating multiple problem spaces (Greeno, 1998). It relies on the solver to invoke an increasing amount of information from long-term memory to either update or store at the same time maintaining all aspects of these subroutines, such as constructing new operators and updating constraints for assimilation into meeting the final goal (Greeno, 1998). Simon (1973, 1996) maintained the problem solver, machine or human, was capable of solving complex problems if a system is serial in operation, by processing only a limited number of inputs at any one time to generate correspondingly small outputs, ultimately transforming the larger problem space of the original ill-structured problem. This system proposed by Simon (1973) accounted for emerging changes in the landscape of the problem space. As the issues or subproblems arise and are dealt with in this manner of sequential subroutines drawing on long-term memory and what Simon refers to as the external-memory, such as models and drawings, with the features from the external-memory evoking familiar and relevant information from the internal memory of the solver (Simon, 1973).

Within this framework, the environment and changes presented by the environment in which the solver is embedded were considered as independent inputs. The external memory or the environment were not conceived as scaffolding new solutions, rather the solver was working within new constraints and new goals for the subproblem, in turn generating an alternative from long-term memory with the external environment essentially considered a resource and not a part of the thinking process. Simon (1996) did acknowledge in some respects, the interaction between individuals and the social world by discussing how those who are members of a society or an organisation are not passive instruments as they are using these social systems to advance their own goals. However, this was not an acknowledgment of the impact of social dynamics on cognition. Simon continued to express this interaction in terms of fundamental principles of logic, where the interaction is a series of problems solved sequentially as each member of an organisation performing his or her role, works toward achieving the over-aching goals of the organisation (Carroll, 2002). Some of the work on models for problem solving (e.g., chess) within the information-processing paradigm has been criticised for being based on problem spaces and environments that are stable, unlike many real-world situations where individuals are forming an understanding of a problem in a dynamic setting with features emerging from the unfolding problem-solving landscape (Greeno, 1988). In addressing Newell and Simon's theory on problem solving, Kirsh (2009a) expressed a similar criticism, noting how in developing their theory, Newell and Simon chose games and puzzles with a clear set of constraints and rules, thus creating an effective internal representation of the problem. In addition, the individual was assumed to require no specialist knowledge of the puzzle, with any individual differences presumed to affect the search within the confinements of the problem space, not across the entire task environment. The task environment was therefore considered abstract as the formal structure of a problem could be mapped across different presentations of the problem with any differences in the task environment being considered as irrelevant and not a part of the problem-solving task. Individuals were assumed to adapt behaviours to fit within the constraints of the formal problem space. Kirsh objected to the notion that the formal structure of problem solving, as proposed by Newell and Simon (1972), could be abstracted across all problem-solving landscapes or task environments. Kirsh believed that the solver's path to solution is situation specific and is inevitably enmeshed with the activity and the context.

The external environment. Within the task environment there may be diverse and alternative paths to solution, some pathways will be in sharp focus to the solver, other routes may not clearly lead to the target goal whereas others may lead to a blind alley, as exemplified by the river-crossing problem. Simon and Kaplan (1989) proposed that, in computers and people alike, the intelligent information-processing system undertakes a heuristic search of the task environment, using information stored in memory to filter out and select the most satisfactory paths to solution. Consistent with the information processing account of cognition, Simon and Kaplan's description of the search process is an internal one, comprising of mental computations. The search is driven by the need to solve a problem, learn, or in the longer time scale, undergo biological adaptation to changes in the environment. In terms of the computational capabilities of an intelligent system searching the task environment, human or computer, is bounded by the physiology or hardware of the inner environment and the complex external spaces of the outer environment (Simon & Kaplan, 1989). However, this view did not imply the outer, or external environment was part of the intelligent system; rather the external world was considered to be, in part, a burden, adding additional constraints and demands to be accommodated by internal cognitive processes. According to this model of problem solving, the individual encodes the stimulus from the external environment; this is then transformed into an internal mental representation of the problem that may be subject to further transformation as the solver searches the problem space for a more suitable internal representation in seeking an answer. Kirsh (2009a) offered an alternative perspective, by considering all actions involved in the problem-solving strategy as part of the task environment. This was in contrast to the narrow view of a task environment presented by Newell and Simon (1972) or Simon and Kaplan (1989), where essentially only the steps taken by the cognizer in reaching a solution were relevant to the cognitive process. The task environment according to Newell and Simon did not include the impact of all actions, like scratching one's head in a chess game, or the variation between artefacts, such as the difference between a chess game played on a computer or using wooden pieces (Kirsh, 2009a). According to Kirsh, this view not only restricted the interpretation of a task environment by excluding an account of actions and artefacts as part of the cognitive process, but the individual differences and creative possibilities of the solver. Kirsh's (1996) notion of a task environment was an abstract concept that offered a space of action possibilities, epistemic and pragmatic "laid over the interactions between an agent pursuing some goal, which we call its *task*, and the physical environment in which it is acting" (p.423). However Kirsh (2009a), in presenting a sketch of his theory on problem solving and situated cognition, did not entirely exclude the traditional cognitivist perspective, nor was he in full agreement with the situationalists. His proposal was a theory acknowledging both arguments, as situated cognition tended to focus less on search in the problem space than the social, cultural, material factors, where the search is of prime interest for cognitivists (e.g. Newell & Simon, 1972), with little interest in the external world. Kirsh (2009a) suggested that it would be useful in developing an understanding of problem solving to appreciate the role of both aspects in the cognitive process.

The "cognitive sandwich". At the risk of oversimplification of a complex paradigm, there appears to be two key components to the information-processing theory of cognition for problem solving: the first component being the sequential processing of information creating a series of perhaps well structured problems; the second being the access from and processing of information through long-term memory. As mentioned earlier, Greeno (1998) pointed out this approach assumes that the solver has sufficient internal mental capacity to deal with the increasing complexity of building subgoals and multiple problem spaces that emerge from ill-structured problem solving, in turn making analysis of this process complicated. This cognitive approach focuses on the informational content of the problem, task or activity rather than the interaction between the world and the individual (Greeno, 1998). Perception and action are considered as peripheral modules of the mind positioned either side of cognition, resulting in a classical cognitive sandwich, thus the mind is divided into components, with cognition being the interface between perception as the input and action as the output (Hurley, 2001).

Situated Cognition

The situated approach to cognition proposed thinking to be contingent on setting and context where knowledge and learning cannot be separated from the lived-in world (Norman, 1993a, 1993b; Robbins & Aydede, 2009). Situated cognition emerged as an alternative approach to the internal symbolic processing theory as part of the debate on whether or not cognition was the sole territory of the brain. Knowledge and learning within cognitive science are generally conceptualised in a symbolic manner within the mind of an individual. The traditional information-processing theory, when applied to solving a problem, maintains that knowledge learnt through experience has been stored as information in memory as symbols. This information is awaiting recall through a web of processes and associations leading to inferences within the mind that are in turn applied to an activity or solving a problem (Greeno, 1989). This mind-as-a-computer analogy (see Gigerenzer and Goldstein, 1996) had been the predominant cognitive theory in cognitive science and psychology for over 30 years when Greeno and others (e.g., Hutchins, 1995a; Lave, 1988; Suchman, 1987, 2007) began exploring the concept of situated activity.

Greeno and situated activity. Greeno (1989) questioned this treatment of knowledge within the information-processing approach, as there was limited consideration of the situation and context in which problems are located. Greeno (1989) was not dismissive of the information-processing paradigm nor the methodology used for investigating problem solving and reasoning, attesting to the strength and viability of the framework for the progression of research in cognitive science. Rather, he argued that in *only* dealing with cognition in terms of symbolic computation in the mind, the impact of situatedness of a problem or activity on reasoning was all but disregarded. Greeno suggested an alternative view “knowledge would be understood as a relation between an individual and a social or physical situation, rather than as a property of an individual” (p.286). In agreement with Lave (1988) on conclusions drawn from her study, the Adult Maths Project, where she investigated cognition in situ, he discussed how a better understanding of knowledge in context might be approached by analysing cognition in practice.

In support of his argument for a more situated approach to understanding cognition, Greeno began by briefly outlining various examples of previous research. These included Lave's (1988) reporting of de la Rocha's (1986) study on mathematics in every day life, which investigated the shopping and food preparation activities of individuals enrolled in a Weight Watchers programme. Greeno cited the actions of one participant in the study, where rather than using formal algebraic algorithms as taught in school when calculating the correct portion of food, the calculation was undertaken through the action of physically dividing the food by cutting it into sections on the kitchen counter. As described by Lave (1988), the weight watcher was preparing the required portion of three-quarters of two-thirds of a cup of cottage cheese for lunch, however the most suitable measuring device was not available. The dieter was perplexed at first, then in an 'aha' moment, he announced, "got it" (Lave, p. 165) filling a measuring cup to two-thirds with the cottage cheese. Tipping out the cheese onto a cutting board, the dieter proceeded to form the cheese into a circle. He marked a cross on the circle of cheese and took away one quarter, then served the remainder for lunch. The correct portion required of half a cup was derived by three-quarters multiplied by two thirds, without checking this against a solution worked out using pen and paper. Lave suggested that the checking process by which the dieter was able to assess the accuracy of the solution was thus enabled by the setting and enactment of the problem. In terms of O'Regan and Noë's (2001) sensorimotor account of vision, this checking process could perhaps be interpreted as the experience of seeing being a way of acting. This example of the dieter measuring out cottage cheese illustrated how an individual utilising the resources available in that particular situation, rather than brain-bounded symbolic computation alone, performed the computation of the portion of food required. Similarly, Greeno described a study by Carraher, Carraher, and Schliemann (1985) of street children with limited schooling, working as market vendors in Brazil, who were able to accurately calculate the prices of items in the market place. The street children used efficient computational techniques developed through actions and interactions with other vendors, customers, and the items in the market. However, Carraher et al. showed that these market vendors enacted maths differently from those children progressing through the formal education system (also see Carraher, 2008). These examples do not undermine the computational perspective of the information-processing model, rather they illustrate different kinds of computations also subserved by possibly different symbol transforming representational systems in situated activity and reasoning (Carraher, 2008). The solutions to the problems faced by the Weight Watcher and the Brazilian street children were built out of components in the form of resources offered by the situation, rather than symbolic computations alone. When acting in the world, an individual integrates the structural features and knowledge offered by that particular environment in which he or she is situated with the stored knowledge the individual brings to the situation (Greeno, 1989). Drawing on Gibsonian and Heideggerian concepts Greeno also expressed the view that when engaging in everyday activities with the world an individual may interact with the structural features of the environment directly, without forming and interacting with mental representations of these features. However, Greeno moderated this radical view on representations by adding that representations, or mental models, of the structural features of objects and events are generally constructed when direct interactions with the environment are not successful. These mental models integrate the features of the setting and the knowledge of the individual, thereby modifying and updating the knowledge available to the individual stored in the symbolic form. Closely related to this notion, O'Regan (1992) in arguing in favour of an alternative approach to the view that internal representations give rise to visual consciousness, proposed that people use the world in which they are embedded as an external memory. This proposal was further developed by Noë and O'Regan (2001) in suggesting, "the outside world serves as is its own, external, representation" (p.939).

The research of de la Rocha (1986) and Carraher et al. (1985) underscore Greeno's (1989) view, that the environment in which an individual is immersed is impactful on cognitive processes, with computations taking place across a cognitive landscape composed of internal and external features. Greeno (1989) believed that research such as de la Rocha's (1984) and Carraher et al. (1985) evidenced how answers to everyday problems are often built from the situation in which the activity takes place. The knowledge garnered from the environment in a particular situation may provide the potential for the solution for a particular problem; this is not the exclusive domain of the internal manipulation of knowledge as symbols drawn from biological memory stores. According to Greeno and others (see Lave, 1988; Neisser, 1976; Suchman, 1987, 2007) the information-processing model offered a static view of problem solving without taking into account the temporal interactions of an individual within the specific features of their environment. Greeno maintained that knowledge, and therefore reasoning should be considered as related to the situation and using resources to hand, rather than in the mind of an individual internally manipulating symbols as an entity detached from the external world.

In order to explain this theory of knowledge as a situated relational process and the issues of transfer from symbolic structures in the internal mind onto the external physical world, Greeno presented a simple diagrammatic representation of semantics (see Figure 2.1). This illustrated how the symbolic structures and notations as described by an information-processing theory (see left-hand side of Figure 2.1) are potentially disconnected from the objects and events in the physical, socially situated world (see right-hand side Figure 2.1). The lower left platform on the diagram denotes basic notations and symbols such as marks on a piece of paper with these forming organised symbolic structures shown on the upper left platform, this would include, for example, written and verbal instructions. On this upper platform the arrow (ψ) refers to how symbolic expressions may be mapped from one to another resulting in the same meaning, if transformed according to a prescribed set of rules, for example 2 x 2 can be expressed as 2^2. On the right-hand side the individual objects and events are on the lower platform, and the upper platform indicates the organisation of the objects or events in relation to daily activity with the arrow (μ) denoting actions on the objects within a setting. θ on both sides of the platform refers to the relationship between the lower platform and the corresponding upper platform. The arrow at the top (Φ) represents the mapping of symbolic expressions across to the objects and events denoted by the symbols. Greeno interpreted this diagram as exemplified by way of the studies from Lave (1988), de la Rocha (1986), and Carraher et al. (1985) discussed earlier, indicating how school-based knowledge may be separated from reasoning in everyday problem solving. It may be the case for students that ψ and μ exist as two separate systems thus difficulties may arise in the meaningful transposition of these symbols and resulting symbolic structures (see left-hand side Figure. 2.1) onto events or objects that do not closely resemble the same symbol or structure as stored in memory when applied beyond the classroom (see right-hand side Figure. 2.1; Greeno, 1989). This disconnect occurs by virtue of the notion that the symbols are encoded within the brain in a particular situation with a specific context, say within classroom. It may be problematic for an individual to map the symbolic structures onto real world problems as the language or notations learnt when mastering the symbolic structures do not correspond to the objects and activities. The semantics for the two sides are different, however, by viewing this from a situated perspective Greeno believed there would be an improvement in understanding "the separation between formulas and the interpretation of physical events that is evident in many students' understanding" (p. 297) and therefore a possibility for research to improve this bridge between the two sides. Although discussed more explicitly in terms of interacting with the world, Kirsh (2009a) also clearly identified the problem of mapping from one representational system to another. Well-defined entities in one representational system may be easily mapped onto another well-defined entity in another system. However, a problem becomes more difficult to attend to when there is a difference between the abstract internal representation and the concrete physical domain disrupting the mapping process (Kirsh, 2009a). The reasoner may now have difficulty linking the two systems potentially requiring a revision of the problem by exploring the physical world. Greeno's agenda was to provide a framework of situated thinking to describe and ultimately explain how successful transfer of knowledge between the two sides of the semantic equation could be possible by attempting to understand the co-ordination of interactions between physical representations and symbolic representations; he was not in any way proposing a complete solution. Greeno believed there were multiple representations at play during the reasoning process—mental representations in manipulating the symbols on the left side, and the physical representation of objects and events.

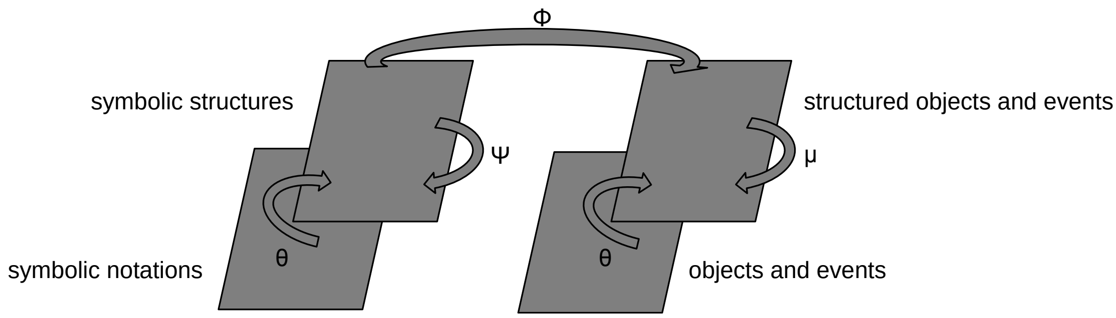

Figure 2.1. A view of semantics (adapted from Greeno, 1989). The left-hand side portrays symbolic expressions. The right-hand side indicates the entities (objects and events) referred to by the symbolic expressions.

The issue in understanding transfer of knowledge was to resolve difficulties in the interactive relationship between these multiple representations. However, the question is also whether there is in fact transfer of learning from the symbolic notations favoured by the cognitivists to the situated and structured events of everyday life. Greeno believed that successful transfer is not readily achieved, as situatedness is not addressed by cognitive science in models of reasoning and problem solving. The interaction on the left-hand side of Figure 2.1 is with the symbolic structure rather than the notations, the individual draws on the symbolic notations as constituents of the symbolic structures, then maps (ψ) from symbolic expression to symbolic expression to transform the structural feature (Greeno, 1989). Greeno used the example of 'John brewed some coffee' to exemplify a manipulation of the symbolic structures rather than notations, which can be rewritten as 'Some coffee is brewed by John'. Of course this takes place within a permitted set of rules, in this case the English language, it would not make or at least it would change the meaning to say, 'John was brewed by some coffee'. Similarly, manipulations may be said to take place in the relationship between objects and the way an individual interacts with these objects in a given situation; these would also be subject to constraints. The objects and events as depicted in Figure 2.1 are manipulated (µ) within the given environment. The issue for Greeno is there may be a disconnect in the mapping of symbolic expressions, as school-based knowledge (ψ) and the structural features of the environment in everyday reasoning (µ). To exemplify his view Greeno cites a study by Caramazza, McCloskey, and Green (1981) where students taught Newtonian principles of motion using formulas were unable to apply this school-learnt knowledge to a practical situation using objects including a weight and a pendulum. To the students, the knowledge they learnt was related to the situation of learning in the class and the symbolic expressions, such as a physics formula '*f=ma*', rather than how to apply the notations such as *f* to how a weight affects the swing of a pendulum. There appears to be a disconnect in the transfer of information between knowledge learnt in the class and the physical event or object in the world. Greeno used the research by Carraher et al. (1983), as described earlier, to illustrate how in that setting, there appeared to be no connection between traditional school-taught mathematics and the real-world reasoning about quantities as used by the Brazilian street vendors. This indicated the computations in calculating the price of goods by the street vendors were possibly carried out by manipulating representations provided by the world rather than internal representations alone. Although, Greeno did not discuss his model in terms of computations, there was no implication that he was attempting to undermine the computational perspective of traditional cognitivists. As he pointed out, there are occasions when computations take place in the head, for example mental arithmetic or the construction of sentences. Although, the Brazilian market vendors made computations using the items on the stalls as a reference point in calculating the amount to charge shoppers, it could be argued that the street vendors had their own internal symbolic notation for quantities that they mapped onto the real world situation. However, these symbolic notations would have been derived from their experience in the market, therefore indicating an interactive feedback loop of knowledge. The important point may be, in the case of the Brazilian market vendors, that there is an internal symbolism driven by and emerging from actions in the environment, therefore the computational system is distributed across the internal mind and the external environment.

Greeno provided an example taken from an unpublished study by Johnson (1988) showing how numerical concepts, such as more than or less than, interrelate with the environment in which an individual is situated, during the construction and interpretation of models. Here, young children completed various quantitative mathematical comparison tasks. They were given a set of problems to solve using tokens as counters and a similar set of problems as word problem tasks to solve without any artefacts. The children performed better in the artefact based maths task involving the interaction between various relational concepts using the tokens, than when asked to complete similar tasks as word only based problems. In analysing the strategies used, the outcome was interpreted as an illustration of how the children used relational concepts to obtain the correct answer when they were able to operate in the world by arranging the tokens. Whereas in the word problem, there appeared to be a lack of interpretation of the comparative nature of the problems as the children resorted to reworking the problem using the symbolic structure, locating the numbers and adding or subtracting them, resulting in errors. This illustrated how in the word problems the arrangement of the problem became a self-contained system depicted by ψ as in Figure 2.1 where the semantics of the symbolic structure of the problem on the left-hand side could not cross over to be mapped onto a representation of the problem on the right-hand side. The reasoning abilities of the young children in the study appeared to be enhanced by the opportunity to model the theory of general concepts in the world using objects. Not unlike Lave (1988), Greeno maintained that school-based knowledge was all too often not connected in a valuable manner, nor was it relational in a generative way to other real-world situations (Greeno, 1989; Suchman, 1987, 2007). Greeno and Lave were not suggesting that school-based knowledge was not situated. They proposed that knowledge learnt in school relates to the requirements of the setting of the classroom, students may learn knowledge in a symbolic manner, such as an algebraic equation; they are then tested on the manipulation of symbols in terms of the learning of the learnt algebraic equation. However, the student might not understand or interpret the knowledge in the way the teacher intended resulting in difficulty mapping these algebraic operations onto real-world settings (Greeno, 1989).

The situated perspective firmly placed enquiry into cognition and behaviour beyond that of the individual, to focus on larger systems of interactivity in which the individual is embedded as “behaving cognitive agents interacting with each other and with other subsystems in the environment” (Greeno, 1998, p.5). Greeno maintained, that toward the end of the twentieth century the framework for analysis of the behaviour and cognition of individuals and groups had developed into two streams—the cognitive strategy, and the situated strategy. The cognitive strategy used experimentation, usually in the lab, to better understand individuals and subsystems, using these as components from which to develop models and theories about larger more complex arrangements. As Lave (1988) pointed out psychologists frequently generalise about the population from observations of individuals. Whereas the situated perspective used tools such as discourse analysis and ethnography to investigate an activity of a person or people integrated in a larger system, for example Lave's (1988) supermarket study and Hutchins (1995a) study on navigation in *Cognition in the Wild*.

The situated perspective of learning and education as described by Greeno (1998), is one of participation and interaction with others, encouraging inquiry and discussion in the acquisition of skills by the individual, however there is very limited acknowledgement of the impact of interaction with artefacts. The study by Johnson (1988) of young children working with problems and tokens as previously described, offered no discussion on the use of the artefacts, the focus was on the mapping of symbolic representations onto physical real world events. Greeno briefly mentioned the use of pen and paper, calculators or computers as tools for the progression of skills and in the understanding of mathematical concepts. In one example presented by Greeno (1998), he described a study by Hall and Rubin (1998) where a student solving a maths problem drew a diagram, Hall and Rubin called "a journey line" (p.19), to represent the distance travelled as described in the problem. The student was asked to construct and explain this journey line to other students in a classroom situation which they used to calculate time, distance, and motion. The researchers found the explanation increased in fluency as the diagram unfolded over a number of presentations. The details of the experiment are not relevant here, however, what is interesting is the interaction in this classroom situation as a conduit for the social distribution of knowledge and cognition, which as Greeno explained, is an important component in the situated framework for the dissemination, clarification, and updating of knowledge. However, there was little attention given to the use of the timeline, pens, markers, and blackboards as functional components in the cognitive process. Despite closing comments that included reference to the focus of the situated perspective on not only interactions between people but also the environment, the interaction with artefacts does not appear to be acknowledged as having a significant role in a dynamic cognitive system within Greeno's interpretation of situated cognition.

Lave (1988) was also exploring the impact of situatedness and context on human thinking and behaviour, with an emphasis on learning transfer. Lave's study provided illustrative evidence for many of Greeno's (1989) views on cognition. Therefore, by way of expanding on Lave's influential contribution toward the establishment of a situated account of cognition, the following discussion on her book *Cognition in Practice* will consider her ethnographic and experimental evidence on the importance of considering the lived-in world in the understanding of human cognition.

Lave and situated cognition. The concept of Lave's (1988) book, *Cognition in Practice*, was based upon bringing the idea of "outdoor psychology" (Geertz, 1983/1993, p. 153) to reality, by combining observational and empirical techniques from ethnographic and experimental methodologies. Two studies, one by de la Rocha (1986) and a second by Murtaugh (1985) were used as foundations for the larger study that was to become the Adult Math Project, the basis for Lave's book. The result was an exploration of mathematics "*in situ*" (Lave, 1988, p. 5) investigating the multifaceted interactions and relationships between traditional cognitive theories, education, and the everyday practices of ordinary folk. Additional motivations for this study were driven by challenges from Lave of some prevailing cognitive theories and models. Following the findings of previous research, (Carraher, Carraher, & Schliemann, 1985; Lave, 1977; Reed & Lave, 1979) Lave was skeptical of the conventional learning-transfer theories, as held to be valid in the fields of psychology and education, where knowledge learnt in the classroom was considered to be readily transferred to everyday activities such as problem solving. She was also concerned that current internalist approaches to human cognition reduced individuals to disembodied, self-contained computational machines, which may result in distorted and unproductive characterisations of how people learn and think in the world outside the psychologist's laboratory. Lave argued that as people act in the world thinking is often stretched across the mind, the body, and the setting, therefore any analysis of human thinking should reflect the interaction between the person, the actions, and the setting of the activity. Finally, she also noted that the current debate on ecological validity of experiments in the laboratory had achieved very little in rethinking traditional experimental methodology, acknowledging how this was an easily identifiable problem but extremely challenging to solve.

This proposal by Lave to analyse the connection between cognitive theory, education, and everyday activities was an ambitious project, namely the Adult Math Project, which combined both observational and experimental enquiry across a variety of daily activities. In this investigation, researchers observed participants, with no particular mathematical skills, in everyday behaviours such as cooking, shopping, dieting, money management, in particular tasks within these activities requiring the application of any type of arithmetic. Lave explained that the term "everyday" (p. 15) as used in this study was not pertaining to a particular time of day, a set of activities, social events, social role or settings, rather it is what people, “just plain folks” (p. 4), do in daily, weekly, monthly cycles of routine activity using examples of the shopper in the supermarket and the scientist in the laboratory. She therefore assumed the substantial task of addressing these questions by going beyond theorising, beyond the laboratory and into the ordinary world. Lave’s work showed that it was possible to study cognition in practice using ethnographic and experimental techniques. However, the lengthy methodological processes undertaken by Lave (1988), Murtaugh (1984), and de la Rocha (1986) in the pursuit of this extensive project also underscored the complexity of taking an investigation outside the laboratory.

Arithmetic was chosen by Lave for this study of everyday practices as most individuals are exposed to maths through schooling, as professionals, laypersons, and in their day-to-day activities. These activities are also observable in daily life making maths a useful medium for the analysis of cognition as a social anthropological project. Maths is also frequently used for experimental research within cognitive psychology providing readily available examples with which to compare the results of the Adult Math Project. In addition, studying maths in-situ offers a rich opportunity to observe the impact and transfer of school-learnt maths on daily problem-solving activities. The aim of the study was to examine arithmetic and cognition—in particular problem solving—contextually and situated, as opposed to what Lave considered to be the more contrived environment of the laboratory setting.

Lave directed a number of criticisms at the limitations of functional approaches to cognition that separate thinking into emotional and rational states, with problem solving being considered as a normative process of rationality. This is a thinly veiled expression of the limitations of the information-processing model which not only ignores the impact of emotion on problem solving in daily life, it also gives the social-cultural aspects of the lived-in world a passive role in cognitive processing. Neisser (1976) was one of the earliest cognitive psychologists to convey one such concern. He stated that in committing to this model of cognition (i.e. the information-processing model) "…there may be trouble ahead. Lacking in ecological validity…such psychology could become a narrow and uninteresting field" (p.19) by not attending to the actions of people and interactions with the everyday world. Neisser was not advocating an end to laboratory-based experiments; however, he believed that to better understand cognition it was vital to consider the impact of cognition in context within its natural environment. Lave also stressed the importance of social custom and culture in everyday practice on cognitive processes, maintaining that cognition was not solely a reasoning task of the mind, but also impacted by the society in which an individual was embedded. It is worth noting that although not attributed to Lave, her view is reflected in subsequent theories such as that of enaction (e.g., Varela et al., 1991), cognitive integration (e.g., Menary, 2006) and material engagement (Malafouris, 2013).

One of the other motivations for the Adult Math Project was Lave's skepticism of accepted views within cognitive science on learning transfer in education. She began by questioning the common view within traditional psychology and education practices that arithmetic as learned in school is a transportable tool for direct application to practical everyday situations, with little consideration given to the setting or context of the learning environment. According to Lave, the cognitive process of learning transfer was therefore implicit as the primary device for connecting knowledge learnt in school with everyday living. The value attributed by traditional cognitive theorists of school-based learning as transferable to life beyond the classroom seemed a logical place for Lave to begin investigations into the relationship between cognition and everyday activities. Lave argued that the origins of learning transfer research rested with Thorndike's critique of the doctrine of formal discipline, where any form of mental discipline, from Latin to geometry, was considered to generally improve the minds of school pupils. Within this framework, the mind is like "a well-filled toolbox" (Lave, 1988, p.24) of knowledge whereby the individual carries the tools of knowledge, taking out the appropriate tools when required and returning them to be stored unchanged awaiting the next use. Two early theories of learning transfer and the mechanisms of learning transfer by Thorndike (1913) and Judd (1908) reflect this perspective in casting knowledge as a tool.

Lave presented the arguments of Thorndike (1913) and Judd (1908) as the two key concepts in early research on learning transfer. Thorndike's argument was that situations needed to share similar components for transfer of knowledge from one situation to another. In this case, the toolbox of knowledge contained special purpose tools with the appropriate tool for the relevant task. On the other hand, Judd proposed that generality of understanding was at the core of learning transfer; a few general-purpose tools could be used in a wide array of circumstances. General principles could then be applied to a new problem by recognising that it belonged to a class of problems previously encountered. Both Thorndike and Judd attempted to demonstrate their theories in the laboratory and in schools with limited success. Lave was not convinced that the word 'tool' was as an appropriate metaphor for knowledge-in-use, as this discounted any interaction between knowledge and the setting of the activity. The assumption in using this metaphor was that a tool was resilient to change and may be universally applied across unrelated settings, then returned to the toolbox unchanged. The use of this metaphor disregarded context and situation to which the knowledge was being applied as well as any updating of knowledge through interaction with the world.

In order to analyse the underlying assumptions of learning transfer within cognitive experimental research, Lave carried out what she described as an ethnographic enquiry into four papers—between them describing 13 experiments—all recognised for their seminal contribution to the discussion on learning transfer. The experiments described in each of the four papers were based on a series of isomorphic tasks where participants were requested to solve problems in the form of puzzles. The typical criteria for measuring learning transfer in these studies was increased efficiency, accuracy, or the evidence of the application of the same basic logic for solving one problem in finding the solution for other similar problems. In addition to ascertaining the effectiveness of learning transfer across time and settings, Lave was interested in what the researchers meant by ‘problems’ and ‘problem-solving activity’; how the notion of problem solving was used in the development of subsequent models, understanding and addressing issues of poor transfer; and finally whether or not context and settings were considered in interpreting the findings.

Learning transfer experiments. Lave described the experiments (see Table 2.1) in detail in her book, however for brevity within this thesis, the description of the experiments will be less extensive and based primarily on Lave’s interpretations.

1. Reed, Ernst, and Banerji (1974). Reed et al. (1974) studied the effect of transfer and the use of analogy in isomorphic problem solving. They used the missionary and cannibals problem (also isomorphic with the river-crossing problem) matched with the essentially isomorphic, although slightly more complex jealous husbands problem, comparing solution time, number of moves and illegal moves for each pair of attempts at solving the problems. In one experiment participants were told that the problems were analogous in the other they were not. Subjects were able to manipulate objects with comments being recorded for later analysis. There were some experimental design issues, however the outcome was essentially that transfer was only successful when subjects were told of the relationship between the problems.

2. Hayes and Simon (1977). Hayes and Simon used the tower of Hanoi problem replacing the disks with monsters or globes. They were concerned with exploring transfer between isomorphic problems and the sensitivity of problem solving activity, when small changes were made to the presentation. Half the problems used in these experiments were transfer problems with monsters or globes moving from one place to the other. The other half used were change problems with the size of the monsters or globes changing. Using this set up, they added other qualities to the monsters where they could transform or move things. In all, there were four types of problems. Although not an issue discussed by Lave, it is worth noting that 60% of subjects began by making a sketch of the problem, however planning was not mentioned as part the transfer process. This complex design and the set of hypothesis proposed, resulted in confusing and contradictory goals. Lave criticised their work, noting that at least one hypothesis was not about uncovering learning transfer but the limitations of transfer. Lave implied that the demonstration of positive transfer by Hayes and Simon was weaker than their conclusions suggested.

3. Gick and Holyoak (1980). Gick and Holyoak studied analogous problems that involved creative insight. They developed vignettes based on an experiment by Duncker (1945) with a common problem being how to “figure out how to destroy a tumor by radiation without destroying healthy tissue” (p.30). For example, in the first experiment, subjects were given a base analogy story and then the target story (the radiation problem). They were instructed to think aloud while working through the first problem and use the solution from the first problem to solve the second. Gick and Holyoak envisioned a type of cognitive mapping where the subject would use a representation of the base problem in evaluating the target problem in order to detect similarities between the two problems, thus generating a solution for the second problem from the first. Gick and Holyoak were exploring why people might not be able to use analogous situations to solve new problems. They appeared to establish that there were difficulties for many participants in solving analogous problems and that this required further research. To the apparent frustration of Lave, the conclusions of Gick and Holyoak ignored earlier comments in their discussion that alluded to the importance of context and situatedness of the activities.

4. Gentner and Gentner (1983). Gentner and Gentner also studied analogical problem solving, this time using an electronic circuitry problem. They described their research as testing “the generative analogy hypothesis, that analogies are an important determinant of the way people think about domain” in comparison to “the surface terminology hypothesis, that analogies merely provide a convenient vocabulary for describing concepts in the domain” (Gentner & Gentner, 1982, p. 1). They used analogies of moving water or teeming crowds as comparisons to the flow of electricity. Students were given problems based on batteries and resistors with half the subjects told the water analogy (analogous with battery circuitry), and the other half were given the moving-crowd analogy (analogous to resistor circuitry). Gentner and Gentner (1982, 1983) found some evidence of students using the appropriate analogies in solving the problems when students were given suggestions that analogies may be useful tools in the thinking process. However according to Lave, Gentner and Gentner (1983) cast some doubt over whether their findings could be translated into an explanation of the mechanisms of learning transfer for lay people who were unfamiliar with the concept of analogies. In other words, they believed this particular example of transfer relied on specialist knowledge for a successful result.

Table 2.1.

A summary of the characteristics of the four papers discussed by Lave (1988) in Chapter 2 (adapted from Lave, 1988).

Problem	Form of transfer expected	Transfer achieved	Rationale	Researchers	Year of publication
Missionaries and cannibals	algorithm	no	Understand problem solving	Reed *et al*.	1974
Tower of Hanoi	algorithm	(yes)[a]	Understand problem solving	Hayes and Simon	1977
Radiation	analogy	(yes)	Important to science	Gick and Hollyoak	1980
Electric circuits	generative analogy	(yes)	Important to science	Gentner *et al.*	1983

()[a] = partial transfer

Lave was not convinced the results reported by the four sets of learning transfer experiments provided robust evidence of transfer with only partial indications of learning transfer at best. In assessing the different problems used, she was critical of their validity as problem solving activities for these experiments; however, Lave was not explicit in her definition of a problem or problem solving in relation to these experiments. Although all experiments involved tasks that employed characteristics of a problem, she believed the problems were constructed in a manner that essentially suited the experimenter's requirements and ultimate expectations of normative models of thinking not reflective of the lived-in world. Lave believed that by agreeing to participate in the experiments, the individual implicitly had no choice but to attempt to solve the problems, with there being only a correct or incorrect solution, otherwise the data were deemed unsuitable for analysis by the researchers. Lave questioned the value of these problems as they were similar in structure to those presented to pupils at school, containing well defined constraints and goals unlike many everyday experiences, such as those described in the dilemmas facing shoppers and dieters in the Adult Math Project. The problems posed in these four experiments have a solution that presupposes that all problem solving activity involves the search for the one and only correct answer, which again does not necessarily apply to everyday problem solving. In addition, as with school-based learning, it was difficult to ascertain whether learning from these laboratory-based problems was transferable to situations and contexts that reflected problem solving in daily life. Lave believed the researchers implied one explanation for the limited success of learning transfer within these experiments, was that just plain folk were less accomplished at analogical transfer as a tool for everyday problem solving than the more skilled problem solvers in scientific and other academic professions. According to Lave, Gick and Holyoak suggested these techniques of learning transfer should be taught to those unaware of this problem-solving process.

Despite the findings falling short of providing evidence of learning transfer, there appeared to be no question raised by Gick and Holyoak that the concept of learning transfer as a normative process might be flawed.

Lave did concede that Gick and Holyoak briefly addressed context in terms of problem solving, however their definition of context was vague and was not inclusive of any specific discussion on everyday activity. The implication being that context was something to overcome in achieving learning transfer, rather than context providing structural features embedded in the topography of the problem-solving landscape, which in turn may alter the solver's perspective of the problem. Lave was disappointed that the context or situatedness of an activity was essentially overlooked or dealt with ambiguously in the research she reviewed. It was clear to Lave, that the traditional cognitivist perspective conceptualised learning from transfer as acquired knowledge to be applied unchanged to situations irrespective of context, interactions between people, activities, artefacts, or time. Therefore, under this model it follows that decontextualised teachings at school are readily transferable to unrelated situations in everyday life. According to Lave, the notion that knowledge could be acquired 'out of context' and mapped on to any setting informed the motivation and design for the four sets of experiments previously described. Following her assessment of these experiments on transfer across isomorphic problems, Lave believed there to be two shortcomings with the research efforts: One was the absence of social context; the second being the lack of identification of the motivations for problem solving and knowledge transfer, beyond static problems presented in the laboratory into the lived-in world.

Lave considered this dissociation of cognition from context within traditional psychology to be the result of insufficient theorising about cognition as an activity that is situated culturally or socially. The problems used in the experiments for the four papers on learning transfer as discussed by Lave were of a closed nature. As a result the research provided little evidence that addressed the dynamic unraveling of a problem and its solution that individuals might experience when faced with the multi-faceted dilemmas of everyday life. Despite these criticisms, Lave did concede that some of the learning transfer research was attempting to discover what it is that people do when they make a connection between similar problems in different settings, although no specific research was mentioned by Lave. However, the thirteen laboratory-based experiments she examined fell short of providing sound empirical evidence to support this type of context-independent learning transfer. The experiments also failed to address how knowledge garnered in the school environment was related to everyday problem solving. To remedy this shortcoming Lave believed it was necessary to observe people in their daily routines, activities, and settings, and utilise diverse empirical techniques to understand more about cognitive activity as part of the lived-in world. This undertaking by Lave to investigate just plain folk going about their daily lives took the form of the Adult Math Project.

The Adult Math Project. As part of the background to her own study, Lave discussed a selection of other research with a similar focus to her own investigation into maths and cognition in practice (see Lave pp. 63-68). This included Carraher, Carraher, and Schliemann (1982) and the Hutchins (1995a) study of navigation teams in the US Navy (Lave was aware of Hutchins's research, although it was yet to be published). Unlike the experiments previously discussed (i.e., Gentner & Gentner, 1983; Gick & Holyoak, 1980; Hayes & Simon, 1977; Reed et al., 1974), these studies focused on the actions of people engaged in everyday activities. According to Lave, in constructing transfer experiments to be conducted in the lab, the researcher first creates a target task with a target activity employing participants with no particular skills, so as not to contaminate the results. On the other hand, studies such as Carraher et al.'s (1982) research on Brazilian children working as market vendors, or Hutchins's (1995a) study on navigation teams, the activity was located around the experience of the participants and their everyday activity. The participants were active in only one setting and selected on the basis of the area of research to be studied. The Adult Math Project employed a different approach by selecting participants with no particular skills, observing them across a variety of settings, and for part of the study used a within-subjects design to enable comparisons between performance of individuals undertaking similar problems in different contexts.

There were three main components to the Adult Math Project, all investigating arithmetic performance and strategies in everyday practice. One component investigated supermarket best-buys; a second examined dieters as part of a Weight Watchers plan; and a third evaluated money management in the home. The third component revealed limited insight into maths in everyday practice, thus will not be part of the discussions here. The observations of all 35 participants (32 females, 3 males) in supermarket best-buy exercises and formal maths test were translated into a form of experimental design where 25 (one participant was later excluded due to health reasons) shoppers were involved in best-buy simulation problem sessions at home as a basis for studying every day activities that used well-learned routine maths practices. A second group of 10 Weight Watchers were monitored with the aim being to investigate the learning of new maths skills over a 6-week period in a setting outside a school-like environment. All participants were also tested in their home for formal arithmetic competencies.

Lave aimed to unite the methodology of ethnography with that of conventional laboratory-based experiments while at the same time addressing some of the criticisms of traditional experimental methodology. An approach Lave employed to address one of her criticisms, that lab-based experiments tended to investigate problems of a closed nature, was to focus on motivating the participants to reveal their problem solving skills by asking them to resolve dilemmas as they occurred in everyday settings. During observations in the initial stages of the Adult Math Project, Lave identified the activity that would be best suited as a more naturalistic problem-solving task for an experimental design. This task was the resolution of the dilemma by shoppers in determining the best-buy between two similar grocery items. In the second part of the study a different group of participants, Weight Watchers, were observed calculating food portions to satisfy the requirements of the diet programme. The Weight Watchers component of the study revealed strategies that illustrated the interaction between the individual and the environment when solving everyday problems. Thus, the shopper component of the study combined an ethnographic approach with experimental methodology; where the observation and analysis of the Weight Watcher component was essentially ethnographic.

The data gathered from the research on the problems involving best-buys were based on the performance of the same individuals undertaking real-world arithmetic tasks in varying situations. This made it possible to compare results across different settings, as opposed to obtaining results from laboratory testing then speculating on the effect when the situation is changed. Two motives supported the use of best-buys for further investigation in an experimental setting. First, there was the unexpected accuracy in calculations by participants in the supermarket, resulting in differences in problem solving techniques to those used in the formal test situations. Second, the tasks exposed the expertise of the participants in grocery shopping, suggesting they may not have integrated the school-taught algorithms into their daily life but developed their own algorithms. Lave also speculated, that as the design of this experiment was within-subjects, and the two tasks (the supermarket purchases and the formal maths tests) evaluated similar maths problems, the discontinuities between performance in the maths tests and the supermarket were related to the settings not individual differences.

Data collection. The procedure for gathering the data was lengthy, and underscored the complexity of undertaking studies that attempt to address criticisms of ecological validity in laboratory experiments. All participants were asked to keep a diary of their daily use of groceries with dieters also documenting details of food consumption. The first stage was to conduct two interviews: one being an extensive background interview with each participant, gathering general demographic information along with details of schooling such as years since leaving school, and a second detailed interview on shopping routines or diet strategies. Next, a researcher toured the participant's kitchens, observing meal preparation and asking questions about food management. The diaries were on hand during this stage providing a rich source for discussion. An inventory of calculators and measuring devices was also taken for each household. A system of observation was devised whereby participants were followed by researchers as unobtrusively as possible, fitting in with the schedules of the participants to observe shopping in the supermarket and storage of groceries in the kitchen at home. The initial observations of daily behaviours at home and in the supermarket of all participants were followed by simulation experiments. The shoppers participated in a one-off best-buy session in their homes while the Weight Watchers were given a variety of meal preparation tasks, also in their homes, across a period of six weeks.

Using the observations from supermarket behaviours in the initial stage of the study, researchers developed a series of best-buy problems for testing the performance and maths strategies of shoppers. In their home, the shoppers were presented with simulations of 12 best-buy problems. Some of the problems involved the use of the actual jars from the supermarket, while other problems were presented on cards informing the participant of the price and quantities of items for comparison (see Lave, pp. 104-105 for details of problems). The simulation experiment was devised to check the observational findings in the supermarket, enable comparison with the formal maths tests, and relationships with demographics. Due to difficulties in the supermarket environment with participants verbalising the problem-solving processes (e.g., shoppers felt self-conscious speaking into a tape recorder in the supermarket), it was not always possible to gather evidence of strategies used in the supermarket. However verbal protocols continued to be used in the form of direct discussions with the researchers, with more success in the simulation exercise as a way of confirming that shoppers had solved the problem not guessed the answer. Lave did not go into detail about the processes or strategies used when solving problems with either the cards or the physical items from the supermarket. Lave only noted that there was no statistical difference in the answers between those participants using cards and those using artefacts. Based on these results there was no investigation into how the participants used these items to solve the problems or the different strategies used with the cards as opposed to the supermarket items. This apparent indifference by Lave to the processes used with a range of external resources is disappointing. One of Lave's aims was to emphasise the point that different settings and external factors may impact problem solving and learning transfer, with the behavioural and cognitive processes in reaching solutions influenced by context and setting. Thus, it seems a logical assumption from Lave's argument that setting should indeed include the interaction with artefacts in situ, when drawing conclusions on learning transfer and problem solving.

Through observations of the same shoppers in the supermarket and the simulation exercises in the home, the researchers did however note that shoppers appeared to carry out similar mathematical practices in both settings. Although confusingly, Lave later noted that problem solving strategies were different from supermarket to the simulation in the home, explaining that this was not about the maths used but the other aspects of the unfolding action of the shopper in the supermarket. Actions that were carried out in the supermarket were not possible in the simulation exercises in the home, including the reading of the store unit price labels by shoppers, looking at the package size of comparable items, then using information on the packaging to compare value for money and searching shelves for alternative products. These actions were not possible, as the simulation exercises comprised of a set of specific items from which to make comparisons.

The study proceeded to the final stage, which involved all participants taking part in arithmetic exercises testing mathematical competencies. These exercises were representative of maths that would have been taught during formal schooling. The learning transfer theories of Thorndike and Judd, as discussed previously, guided the construction of five arithmetic tests split into three different components. The first component reflected Judd's theory of transfer to assess common maths competencies, consisting of a general maths knowledge test and a multiple choice test drawn from a standardised maths achievement exam. A maths fact test and a measurement fact test (based on weights and measures) were developed to assess school-learnt maths skills reflecting the Thorndike perspective. A third set of ratio comparison problems was presented to address conventional views of learning transfer. Generally, it was observed that shoppers used ratios between quantity and cost, rather than an exact calculation, when assessing the best-buy between two and three items. Therefore, this third test was produced following the analysis of these observations made in the supermarket and responses from participants of the best-buy simulation task. A test was then created that would assess school-learnt maths knowledge, at the same time enabling comparison between the formal maths test and the best-buy simulation tasks. Unfortunately, in the book Lave only reproduced the problems in the supermarket best-buy, the simulation exercise, and three out of five of the maths tests, excluding the ratio test. This makes it difficult to directly compare the content of the ratio tasks used in the supermarket and simulation with the ratio problems in the arithmetic test.

The shoppers. The scores for the shoppers on four of the arithmetic tests (excluding the ratio test, as this was devised as a comparison to the best-buy tasks and not considered directly related to school learning) were correlated. The results indicated that these measures of school-like maths were a good assessment of this type of maths knowledge, as they were either moderately or strongly correlated with each other. Years of schooling were highly positively correlated with maths performance, with a high negative correlation between age and years since leaving school, demonstrating these were good predictors of performance in school-like maths. However, there were no significant correlations between the three formal maths test scores and performance on the best-buy tasks, or between best-buy performance and schooling, age or years since completion of schooling. The surprising result was that the supermarket tasks were 98% accurate and the best-buy simulation resulted in 93% accuracy, as opposed to the performance on the formal arithmetic tests of 58%. Therefore, success in the supermarket and simulation best-buy tasks showed no relationship to schooling, age or years since leaving school. This suggested that the accuracy of performance in the supermarket and the simulation experiment in the home of the participant was not necessarily contingent on maths learnt in school. This added weight to Lave's argument that the setting has an influence on problem solving performance, and math strategy is not necessarily a reflection of school-based learning transfer but may be driven by the skills learnt from everyday experiences that are situationally specific.

Lave determined that the evidence from the Adult Math Project appeared to contravene the accepted logic of learning transfer between the setting of schooling and everyday practice in the supermarket. She argued that according to accepted cognitive theories, school algorithms when used in the supermarket should be "more powerful and accurate than quick, informal procedures" (p. 57). In addition, with the many distractions of a supermarket the expectation would be for greater cognitive demands on attention and loading on working memory than when undertaking the best-buy and maths tasks in the home, suggesting performance should be poorer in the supermarket. However this was not the case, with participants not necessarily using the maths strategies taught in conventional schooling, and performance generally being more accurate in the supermarket than in the formal maths tests.

In analysing the strategies used by shoppers it was observed that a number of characteristics of supermarket shopping calculations were dissimilar to those employed in the maths tests. When in the supermarket, the shopper would often use left to right calculations starting with the largest (hundreds) working down to the smallest (ones) number, using techniques such as rounding and easily divisible ratios in comparing and reconfiguring the problem. Notably, the environment was frequently used as a device for assisting in calculation of best-buys, such as the information on price per ounce supplied by the supermarket or differences in packaging of items. The environment also gave the shopper options not available in a traditional experimental situation. One example that may be the equivalent of an error in a traditional experimental session would be when a calculation was abandoned when the shopper felt it was too difficult. However, this was not considered an error in this study as the shopper frequently opted for alternatives by either postponing the purchase, substituted it for another, or "taking the big one because it won't spoil anyway" (Lave, 1988, p. 58). Strategies such as these led Lave to describe the problem-solving process in the supermarket as a transformation process, whereby the problem-solver must first transform the task of buying an item on the shopping list, or the dilemma of not knowing the best-buy in the supermarket into a problem requiring solution. The shopper would then work through the problem using the setting of the supermarket to offer strategies in order to reach a solution. While transforming the components of the problem as part of the computations, the solvers appeared to be very good at maintaining the relationship between the problem, the numbers, and the answer as indicated by the accuracy of the solution. In contrast to the techniques employed in the supermarket, participants used pen and paper for the maths test, with calculations made working right to left, using borrowing and carrying procedures, with long division problems checked using multiplication and subtraction. Transformation of numerical solutions in the maths tests was more problematic for participants than in the supermarket trials. One possible contributing factor may have been increased anxiety by the participants in the formal maths tests, as they may have perceived these as more of a school-like assessment, when compared to the informality of the best-buy problems factor. This anxiety by participants was noted by some of the researchers. Therefore, the context of the two tasks may have been perceived differently. Although the maths tasks were presented in the home environment, they were still considered a test of abilities, whereas purchases in the supermarket were considered as mundane everyday activities relatively unrelated to math skills. Participants also made different types of errors in the maths tests to the supermarket maths activities. Errors in the formal maths test were more common in problems that required transformation before solving, such as expressing fractions in terms of common denominators in fraction-subtraction problems ($3\frac{1}{3} - \frac{1}{2}$), rather than those that were directly solvable (36 + 98). Lave claimed analysis of techniques showed that the formulaic rules learned (often by rote) for problem transformation in school were very different from the successful transformation of problems in the supermarket, where there was little evidence for the use of formulaic rules. This claim appears to be primarily based on the inaccuracies of technique and answers in the more formal math tasks in comparison to the more ad-hoc methods resulting in more accurate responses in the supermarket and in-home simulation. Throughout the book Lave described in detail the methods used by a few shoppers and dieters in reaching solutions on specific problems. She also described the steps solvers used to break-down problems in the arithmetic tests, and an analysis of strategies used in best-buy problems for the in-home simulation and the supermarket. Unfortunately, there appears to be no description of an isomorphic problem by one shopper in the different contexts of supermarket, simulation, and maths test illustrating and comparing the problem solving strategies in both. However, the detailed examples included in the book did highlight the differences between strategies used in different situations.

Other studies of maths in practice as described by Lave (e.g., Carraher et al., 1985) provided evidence to support Lave's claims that rather than using traditional school-taught algorithms in day-to-day maths practices, the situation or setting might influence the strategies used by individuals when solving problems requiring arithmetic. Recalling the maths accuracy of the poorly educated children in Carraher et al.'s study on Brazilian market vendors, Lave noted that success in the lived-in world may not be dependent on the conscious application of school-learnt knowledge, nor is learning transfer as measured in a school-like test situation necessarily an indication of cognitive capacity. When investigating everyday maths strategies of those with formal maths education, it may be argued that it is difficult to evaluate whether the maths algorithms learnt in school have been the basis for self-taught algorithms in day-to-day maths practices. However, the argument could equally support a case for the impact of situatedness on cognition, as the setting and context when confronting daily problems has been instrumental in the development of these everyday self-taught algorithms and strategies, which may or may not have evolved from arithmetic learnt in school. Lave and Carraher (2008) were not advocating that individuals used a different kind of maths, or that everyday maths was necessarily superior to school-taught maths, as both methods resulted in successes and failures in the resolution of problems. They believed their research showed there were different maths strategies and possibly different symbolic representations being used by individuals for different settings.

The dieters. The Weight Watchers component of the study by Lave (1988) investigated the incorporation of new food measurement practices into the daily lives of participants. In part, this type of activity was chosen as formal measurement systems are generally taught in schools and therefore should be familiar to the participants. In this experiment, dieters were asked to prepare all meals according to the Weight Watchers programme. Maths competency as evaluated in the formal maths tests was not related to success in calculation accuracy in the kitchen while participating in this experiment, although Lave supplied no statistics for comparison. It also appeared that the more expert individuals became at the programme the less they used formal calculations, while maintaining accuracy and losing weight. Dieters also invoked different strategies, structured cooking activities, and rearranged the setting and artefacts in such a way as to simplify the preparation and consumption of meals. Tools such as measuring scales, measuring cups and spoons were used less over time as they learnt the portion sizes recruiting everyday items as aids to calculation. Therefore artefacts continued to play a vital role, with the objects used changing as the experiment progressed. One example was when measuring the correct portion of milk in the early stages of the study; the dieter began by using a measuring cup and transferring it to a glass. The participant quickly reduced the process to pouring the milk directly into the glass up to the line of blue flowers on the glass, resulting in the required portion. Thus the dieter soon moved from the formal prescriptions of the Weight Watchers programme to bespoke strategies in shaping the practices of meal preparation; formal maths became less salient in the structuring of resources for completion of the task at hand. This was often driven by dilemmas arising in order to reduce the time taken to prepare meals, accommodating family and work commitments. In some cases, as in the example described earlier by Greeno (1989), the dieter used an ad-hoc method to correctly obtain the portion size of cottage cheese when more precise measuring tools were not readily available. While the calculation may have been in part an internal process, the enactment of carrying out the calculation was an interaction between the unfolding problem and the setting. This exposed valuable insights into the impact of situatedness and evolving strategies employed in everyday problem solving. In addition this example exemplified Lave's suggestion of an alternative approach to the analysis of problem solving. By observing actions and resolution strategies unfolding organically during everyday activity, rather than the artificially simulated problems of the lab, the unit of analysis becomes the person in the activity and the environment—the person-in-action (Lave, 1988).

Psychology and the everyday. In Lave's opinion cognitive psychologists were inclined to ignore the settings of everyday life in the exploration of the human mind and in the quest to develop paradigms that describe thinking processes. In exploring why this may be the case, she believed the association of the term everyday with thinking, within the realms of cognitive psychology, to be steeped with negative connotations. Studying everyday thinking was not considered rational or normative enough for the development of theory, nor for the methodological rigor required for experimentation in the lab, even though one of the fundamental aims of psychology is to shed light on our thinking as we go about our daily lives. Subsequently, although not as a consequence of Lave's discussion, it has become more acceptable within psychology and other disciplines to give consideration to models that do not follow a normative paradigm, for example Kahneman and Tversky's (1979) prospect theory. Lave argued that there was a problem in cognitive research where the mind, in particular memory, was perceived to be a store for knowledge, being considered as the only possible source of continuity across situations, thus separating it from the sociocultural activities of everyday life. This was a problem for Lave, as she believed that through the observations and results from her own research, it was not conceivable to ignore the social when formulating an understanding of cognition. The "pejorative meaning of everyday thought" (Lave, 1988, p. 77) within cognitive psychology was an entrenched, long-standing view that the impact of everyday practice on thinking had little or no place in rational scientific investigation. Lave considered this to have emerged from the dichotomised classification of the modes of thought underpinned by the Western epistemologies of the dichotomy between mind and body, shaped by the positivist approach in science. Within this dichotomous framework, thinking modes are often divided into two classifications such as rational thought vs. primitive thought, domestic culture vs. wild culture, upper class vs. lower class. Over time, these classifications have transformed to reflect the differences between rational, abstract scientific knowledge and the simplistic, concrete thinking of the everyday. The essence being that primitive thinking is less cognitively demanding and rational than thinking processes of more developed societies. This translates into the possible view by cognitive psychologists that the activities of daily life taking place outside the rigor of experiments performed in the controlled environs of the laboratory are assumed simpler, not as demanding and lack rationality on which to develop sound and consistent theories. Lave (1988) even went as far as to suggest that the everyday is perhaps assumed simpler as it is considered to be only performed by "members of the lower classes and housewives" (p.82). This is somewhat of a paradox for psychology, as the assumptions made following hypothesis testing in laboratories are, in turn, frequently generalised to inform explanations for the activities of daily life.

Psychology and anthropology. In Lave's opinion, this view by traditional cognitive science of the everyday was also considered to be the foundation for one of the major divisions between psychology and anthropology, as the two disciplines interpret culture and cognition differently. Lave discussed differences between anthropology and cognitive psychology, initially pointing out two important commonalities between the disciplines which are often overlooked as the methodological approaches may be considered at opposite ends of the spectrum. Both are concerned with how people think and they share a theoretical positivist epistemology. She described how typically the knowledge offered by these disciplines has also been viewed as somewhat dichotomous, with psychology considered nomothetic (generalising about groups to find common rules of behaviour), whereas anthropology is viewed as ideographic (investigating the unique characteristics and peculiarities of an individual or groups). In these terms, psychologists seek a generalised description of cognitive processes, where anthropologists describe the peculiarities and the substance of culture to explain cognition. Traditional cognitive psychologists tend to investigate the processes that explain experiences abstracting theories using scientific testing, leading to hypotheses on the universal or macro view of cognition. On the other hand, anthropologists perceive studying culture as investigating the content of experiences leading to these particularities using more descriptive and observational techniques from which to draw conclusions. She suggested that the differences and similarities between psychology and anthropology are epistemological in terms of methodology, as this positivist approach to social science attempts to uncover rational explanations for human thinking and behaviour. However, the anthropologist tends to be a discreet observer or participant with very little experimentation involved in the investigative process, whereas the experimental techniques of psychologists focus on manipulating conditions to observe effects on behaviours.

Lave suggested that this dichotomous approach to studying human behaviour offered a similar distinction between the studying of cognition from a situated perspective and the more traditional cognitive theoretical methodologies. A balance between the approach of psychology and anthropology would potentially create an environment whereby the everyday actions of individuals and groups could be studied taking into account context and situatedness. This would require some relaxation in current approaches by both disciplines, where a reduction in laboratory controls is exchanged for a more rigorous approach to the alignment of observers' and subjects' perceptions of any given experience during observations of behaviour. Lave was not professing that these differences in methodology are easily reconciled in fact she notes that this debate is a difficult one to resolve.

Culture and cognition. Lave then turned to the relationship between culture and cognition within cognitive psychology. Despite changes in approaches within psychology such as the movement away from behaviorism toward information-processing, there was no change in the view of the relationship between the individual and the surrounding world. The default position continued whereby the environment was considered no more than a stimulus perceived by the cognizer evoking a response, resulting in an artificial division between the social world and the individual. Although the effect of culture and society over time in an evolutionary sense is acknowledged as altering human behaviour on a micro scale, such as over the span of a human life, culture is often excluded from the attempts to understand cognition or at best, limited. Lave considered the relationship between culture and knowledge was often considered only in terms of memory and retrieval, such as in the acquisition of expert knowledge. Traditional cognitive science conceives of expertise as an individual's ability to draw on a vast reservoir of knowledge accumulated over time and stored in long-term memory readily available, similar to a well-indexed encyclopedia (see Larkin, McDermott, Simon, & Simon, 1980). She maintained that within traditional cognitive psychology, the relationship between culture and cognition is always based in the past, not in the present. Consequently, in Lave's view there is very little account in traditional psychology for the relationship between people acting in the here-and-now as part of the world and the social domain they inhabit. Lave believed this carried through to traditional models of learning transfer, where the assumption is that people carry around a certain amount of knowledge and information in the mind, which is independent of the world in which the individual is situated. This stored information is retrieved as required and mapped onto interpretations of new situations or in solving new but similar problems as they arise in daily life. The assumption is, as a rational problem solver, that this knowledge is cognitively processed in a "context-free, value-free, body-free and factual" (Lave, 1988, pp. 88-89) manner. However, Lave referred to the results from the Adult Math

Project to conclude that in examining cognition the traditional approach was too narrow, and the agent should be considered as acting and situated when undertaking a task in the everyday world.

In Lave's view, undertaking everyday maths activities may or may not include formal maths knowledge, with resources not only drawn from the memory of the individual but the activity itself. The same maths problem may be solved in different ways, by using a pen and paper, calculator, or asking a friend, the answer is the same, but the process and strategy are different. The shaping or the structure of the problem solution is different, thus different activities give rise to this shaping of the solution to the problem. Lave applied similar logic to a maths algorithm in a classroom that was then applied to shopping problem in the supermarket; the actions and strategies to solution are performed differently in the classroom to the everyday purchase of a grocery item in the supermarket as both activities would be structured differently across the differing settings. In a visit to the supermarket to buy groceries, there are multiple things going on at once, not just the calculation of a retrieved mathematical algorithm. There is the physical layout of the supermarket to negotiate, the relationship between the needs of the buyer and what is available, what are the prevailing prices of the day, the time the buyer spends in shopping on that particular occasion. These factors influence the structure of the resources available to the buyer in shaping the solution to the problem of what is the best-buy, in the case of the Adult Math Project.

Ecological validity in research. This brings Lave to an issue mentioned earlier, echoing Neisser's (1976) doubts about the ecological validity of experiments in psychology. In comparing schooling to lab experiments she argued that one general way of solving a problem as taught in school is expected to be extrapolated for use with other similar problems, likewise, assumptions used to design one form of experimental practice are often used for all other similar experimental practices. Consequently, Lave questioned the validity of making inferences on everyday practices from findings in the lab and whether normative models are appropriate for the investigation into everyday lives. In approaching this issue of ecological validity in psychological research, Lave turned to a comparison between work by Capon and Kuhn (1979, 1982) and the Adult Math Project simulation experiment. Both experiments investigated consumer decision making at the supermarket using maths as a platform. According to Lave, Capon and Kuhn were concerned that no previous study had evaluated formal reasoning abilities in a naturalistic setting rather than in a formal testing situation, with the focus on the use of ratio comparisons by shoppers and whether strategies varied if the ratios became more difficult. The Adult Math Project followed up supermarket observations with simulations of best-buy problems in the home of the participant also using ratios as the basis for the problems. These problems were presented using cards or the actual items from the store, answers were announced to the researcher, with pen and paper or a calculator supplied to the participant as a last resort if the individual was having difficulty. As previously mentioned, Lave did not pursue the differences in presentation of the problem using jars and cards in the home to that of the supermarket environment, nor did she give any detail of when participants used different methods to calculate answers. Capon and Kuhn carried out two experiments at a card table set up outside a supermarket; one with 50 female participants, this was expanded by 100 to 150 female participants in a second. The shoppers were allowed to use pen and paper in calculating the answers. They were presented with two problems using two items: deodorant and garlic powder, two bottles of each in differing sizes and prices asking consumers to select the best-buy for each product. The disguised ratio for the garlic powder was 2:1, with the more difficult problem being the comparison between deodorants at 2:3. There were a number of similar facets in the Adult Maths Project and the Capon and Kuhn experiments that approached a more ecological methodology. First, they both based the research in a more naturalistic setting, in this case the supermarket, although Capon and Kuhn were situated outside the supermarket itself, and the second part of the Adult Math Project took place in the home. Second, both investigated people using maths in everyday problem solving. Third, artefacts were employed, for at least part of the study, rather than a series of school-like problems presented on paper. Fourth, there was a rudimentary recording by the experimenters of math strategies used by shoppers in reaching an answer. Finally, both used ratios between products to test and analyse participants' maths knowledge. Capon and Kuhn reported 44% of the150 participants were successful in solving both problems, this is in contrast to the Adult Math Project success rate by shoppers of 93%. After analysing the strategies used by their shoppers, Capon and Kuhn concluded that as individuals used the same strategies when solving the best-buy problems, indifferent to changes in the problem difficulty, there was consistency to the rationale used by individuals, reflecting stable attributes of individuals. The final conclusion being that even when given all the information, in this case weight and price, there was significant variability in logical reasoning abilities in the adult population. This was attributed to individuals not being aware of how to use the particular cognitive strategy tested here, again in contrast to Lave's study where awareness of strategy was not apparent either, however, performance was markedly better. This claim by Capon and Kuhn seems to be a bold one when there were only two problems to solve with only two ratios to compare. Lave suggested that the similarity between the strategies used by shoppers in the Capon and Kuhn study may have been due to the problems so closely resembling one another and being presented temporally close. The motivation for both experiments seems superficially similar, as they were both concerned with maths activities in a natural setting—the supermarket—and how basic maths ratio principles are applied in decision making. Given these similarities, Lave goes to great lengths explaining the differences between the experiments that lead to the differing results. Capon and Kuhn believed the issue could be found in individual differences of logical reasoning abilities, therefore consciously available strategies were required through consumer education to assist shoppers in efficiently selecting the best-buy. Lave disagreed believing that Capon and Kuhn used a simplistic model of the lived-in world unlike the complexities of everyday life that the researchers of the Adult Math Project attempted to inject into their model. In addition, according to Lave, Capon and Kuhn generalised the results across the adult population without an account of the demographics of their sample—all female from low to middle incomes—nor was any schooling investigated. Comparative criticisms of Lave's study could include that the sample size was not large enough at 24, and the majority were female, although the Lave study reported no significant difference in performance as a function of gender. She concluded that under the more complex simulations of the Adult Math Project shoppers were shown to be effective at selecting the best-buys utilising a number of strategies for varying problems in differing settings. Lave also suggested that participants in the Capon and Kuhn experiment would have perceived the task as more of a maths test than the Adult Math Project in store problems, possibly explaining why participants in the Capon and Kuhn experiment performed calculations using similar strategies to school-like procedures.

The differences in methodology resulted in disparities in the results between the Adult Math Project and the Capon and Kuhn experiment being indicative in Lave's view, of the shortcomings of the traditional approach to problem-solving research. Capon and Kuhn employed a more deductive approach typical of experimental psychology by initially identifying a theoretical model on which the experiment was based, then hypothesising about the outcome, followed by the experiment itself. The Adult Math Project was a hybrid of ethnographic and traditional psychological methodologies, starting with a more inductive approach to maths in practice undertaking observations of every day activities in grocery shopping. These observations informed a hypothesis aimed at exploring problem solving in a simulation experiment based on psychological experimentation techniques. According to Lave, Capon and Kuhn approached the research with a simplified view of the everyday world, with little evidence that the lived-in world was influential in constructing their hypothesis. In setting up a table outside a supermarket and using words such "as if" in posing questions, Capon and Kuhn did not capture the parameters of a naturalistic setting in which to study everyday approaches to problem solving. Lave concluded when comparing the two studies, that in different settings similar activities and problems may be subjectively construed as being the same, however these seemingly similar problems are transformed as are the resources available, cognitive or otherwise, in different contexts and settings. In her opinion these were not outcomes identified by the Capon and Kuhn study.

The difference in outcomes between Lave's study and that of Capon and Kuhn also demonstrated how in analysing purchasing decisions in the supermarket it is not possible to reduce the problem to a comparison of unit price alone, with the shopping dilemma infused with a multitude of factors influencing the decision (Kirsh, 2009a). As the consumer moves about the supermarket picking up packages and inspecting items on the shelf, the problem is not only framed and conceptualised by activity, but contextual factors such as information on labels, comparisons with other products, use by dates, family attitudes to the product, and storage of the item in the home (Kirsh, 2009). As Kirsh (2009a) suggested the layout of the supermarket and product display also have a dynamic impact on how the shopper frames the choice of product. In explaining the role of settings in problem solving Lave adopted the term "fields for action" (p. 130) from the sociological literature that she attributes to Bourdieu's field theory. Bourdieu's sociological field theory emphasised the relationships between the elements in the social world rather than the individual elements themselves. Within this framework, fields in the social world are viewed as structures where social agents and groups, think, act, and take relational positions (Hilgers & Mangez, 2014). Lave's fields for action depict this notion of the agent acting within a space that both defines and is defined by the activity of shopping or preparing food. The agent does not act in isolation, rather the actions are relational to other elements within the field for action; be it within the supermarket or the kitchen.

The activities within a setting are both created by and create fields for action[2]. In explaining how the setting in the supermarket was not merely a

[2] The notion of field theory is not unique to sociology, with field theories in place within the domain of mathematics, Newtonian physics and Gestalt theory. The epistemological commonality between these theories was the movement away from Aristotelian and Cartesian

mental map in the mind of the shopper. The supermarket had an independent physical character while at the same time it had a separate reality related to the activities of the shopper. In everyday situations the motivation for problem solving frequently arises from dilemmas occurring such as conflict of values in the supermarket—which is the cheapest, which is the most nutritious—they are created mid-action as the shopper moves about the supermarket. These dilemmas are not value free or context free, they are situational, value, and experience driven. The activity of the shopper may be shaped as she moves around the supermarket, updating choices and decisions through the interaction with the setting—which items to purchase, which items offer the best value, the item required may be unavailable therefore a new search is required for a substitute. Activity in solving shopping dilemmas is a result of the mutual relationship between the setting of the supermarket and the actions of the shopper, with fields for action both created by and facilitating problem solving activities. Lave described a simple example of how the supermarket setting was used to solve a pricing dilemma. A shopper was selecting cheese when he came across a block that appeared to be an unusually high price in comparison to other blocks of the same cheese. Rather than attempting to calculate the correct price from the information available on other packages of cheese, the shopper searched for a block of cheese of the same weight and compared the price. This problematic event experienced by the shopper was indicative of how within one field of action, the supermarket, an unexpected problem was

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principles of mind-body dualism toward relational systems of action (Hilgers & Mangez, 2014; Kadar & Shaw, 2000). In a similar undertaking Gibson and Crooks (1938) used the term field in their analysis of how drivers generally skillfully, almost seamlessly negotiate the road, perceiving and acting in the “field of safe travel” (p. 469), while this field is bounded it is also dynamical (Hodges, 2007). Lave’s notion of fields for action also has parallels to Gibson and Crooks ecological perspective of fields and Gibson’s later seminal work on visual perception where Gibson placed emphasis on the perception of the environment by the active observer. Similar to Lave’s observations, as the world shifts and changes the agent acts through this perception of the world (Gibson, 1986).

identified. The basket of cheese in the supermarket emerged as another field for action in which to search for a resolution where the shopper transformed the problem using cues from within that field for action. There are repeated interactions between the person enacting the shopping and the supermarket setting as a field for action with problems arising followed by resolutions. This suggests that the shopping activity is not necessarily structured by arithmetic; rather the arithmetic required is structured by the shopping.

Reflections, criticisms, and conclusions. In assessing the outcomes of the Adult Math Project, Lave was perplexed by two findings from the supermarket best-buy study. First, shoppers made errors in the formal testing situations but appeared to be almost error-free for similar problems in the supermarket. Lave proposed a possible explanation for the high accuracy of calculations in the supermarket, was that when a shopper abandoned a purchase involving difficult calculations, researchers did not record this as an error in calculation. In the supermarket, individuals were observed readily abandoning a potential grocery item in favour of another similar item if the calculations were particularly challenging. In this setting, there was little to be lost by abandoning the originally intended purchase when another similar purchase involving easier calculations was available. Second, although the study reported a high accuracy in best-buy tasks, shoppers made a number of attempts at calculation—on average 2.5 per grocery item—with errors being made in the intermediate steps in calculating best-buys. Again, these errors made on the path to solving the problem were not recorded as errors by the researchers. Lave explained why the errors made by a shopper prior to reaching a final best-buy decision were not included as in the analysis. The shopper generally recognised errors during the problem-solving process. The shopper then used these errors in iterative cycles of calculation, often ending in a final checking process as a scaffold to reaching a resolution. As Lave, Murtaugh, and de la Rocha (1984) pointed out, it is difficult to analyse these dialectically formed problems where the actions and the situation both create and change each other, with problems being generated and resolved during the ongoing activity of the individual. Lave believed the difficulty may lie in delineating the problem from the resolution, just as it is difficult to locate the problem as being in the head or on the supermarket shelf, suggesting once again that the unit of analysis should be the person-in-action rather than the problem itself.

Lave also took time to reflect on the method of observation and recording of shoppers' thought processes. Shoppers carried a tape-recorder over their shoulder and were asked to “think out aloud” as they progressed through the grocery shopping best-buy task with the researcher following behind noting observations. However, shoppers were more comfortable talking to a researcher than appearing to talk to themselves when using the tape-recorder in moving about the store. The procedure was altered after the pilot, abandoning the use of the tape-recorder, relying on the researcher to take notes on decisions made, ask questions of the shopper to expand on explanations for decisions and record verbalised calculations. Lave acknowledged that while the researchers attempted to ask questions or make comments in order to clarify the shoppers' thoughts rather than interpret or influence them, the interaction between the two cannot be ignored or eliminated. In order to acknowledge the impact of this interaction on decision making, transcripts of discussions between shoppers and researchers were analysed and a selection was reported. Lave reported how one shopper appeared to be establishing herself as a shrewd shopper when explaining her rationale for buying one brand of noodles over another to the researcher. Comments by the researcher appeared to cause this shopper to reconsider her rationale for the purchase, notably that the maths calculations in rethinking the purchase were correct. There was also evidence that when undertaking the calculations, the shopper used the dialogical relationship with the researcher to enact a process of transforming the maths into a form that was easier to manipulate. The shopper appeared to use this verbal interaction as a checking mechanism before arriving at the best-buy solution. In one example, the conversation with the researcher began with the shopper verbalising thoughts such as past experience and knowledge of the value for money the item to be purchased offered. In verbalising the rationale of the purchase by way of comparison between the usual buy and other options on the shelf, the shopper began to reconfigure the problem by transforming the prices and weight of the products using ratios in order to simplify comparisons. Using information on the packaging of the items, the shopper recognised over three or four assessments cycles of other products that, through the unfolding calculations, the purchase based on past decisions may not be the best-buy after all. The researchers observed similar behaviours by other shoppers of assessing and reassessing best-buys. Therefore, Lave deduced that a pattern had emerged where the shopper anticipated resolution to the problem; however further inspection of the available information by the shopper prompted reevaluation. This reevaluation, at times concluded in the shopper continuing with the original choice. On other occasions the shopper rejected the original anticipated purchase leading to an alternative, unexpected solution to the problem in the choice of a different item. It was also observed that there was an ongoing checking procedure during these partial forms of the final solution in comparing current knowledge and the prevailing resolution to the problem. This checking procedure was part of what Lave termed “gap closing” (Lave, 1988, p.164) whereby the shopper would solve the problem in a nesting process of gradually closing the gap between the dilemma and the solution over a number of back and forward interactions between the shopper, the items in the supermarket, and the researcher. The ongoing activity, interaction with products in the store and interactivity with the researcher shaped a resolution to the problem altering the structure of the problem enabling an improved solution to the original purchasing dilemma.

The work by Lave on everyday mathematics has been influential over the past three decades in the fields of cognitive science, cultural anthropology, and in maths education (Carraher, 2008; Greiffenhagen, & Sharrok, 2008). However, the work has not been without its critics, Greiffenhagen and Sharrok raised doubts about the validity of Lave’s critical interpretation of the empirical evidence presented in the study of everyday mathematics. In addition they were skeptical of the connection between Lave’s empirical findings and what Greiffenhagen and Sharrok considered as a criticism by Lave of traditional theories of reasoning, mathematics and cognition. They begin by explaining that, in their opinion, Lave exaggerated the achievements by the shoppers in the supermarket best-buy problems by implying that Lave (1988) and Murtaugh’s (1985) interpretation of the word calculation was too generous. According to Greiffenhagen and Sharrok this led to a larger number of ‘calculations’ being included in the analysis that, in their opinion, amounted to nothing more than basic arithmetic. However, it appears they may have misinterpreted Lave and Murtaugh’s definition, as Lave and Murtaugh were clear that the unit of analysis was arithmetic—that is, the use of basic arithmetic operations. While Greiffenhagen and Sharrok (2008) acknowledged that studies such as Lave's have been instrumental in updating models of teaching, thinking and rationality, they maintained that Lave and others in the field (e.g., Carraher, Carraher, & Schliemann, 1985) were attempting to show that everyday maths or informal maths, and school-taught maths or formal maths, were two different types of mathematics with formal school-learnt maths attempting to disregard informal, everyday maths. Greiffenhagen and Sharrok also questioned Lave's motivation for this study and other studies on everyday maths suggesting that it was more of a critical attack on the established theories on maths, rationality, and cognition, and that the findings in the studies were not particularly remarkable. In an article responding to the criticisms of studies on everyday mathematics, Carraher (2008) rebuked their skepticism over the motives and findings of Lave's studies on everyday maths. Carraher agreed that some of points made by Greiffenhagen and Sharrok may have some validity—the differences in difficulty between best-buys and some components of the formal maths tests, and the different treatment of errors in the formal maths tests to the supermarket best-buys tasks—cast some doubt over claims made by Lave in performance comparisons across differing settings. However, as emphasised by Carraher, while the descriptive component of any study plays an important role, the essential contribution of research is to make sense of the observations in order to move science forward. Carraher went to great lengths to explain how these studies into everyday cognition were not directed at showing that there were different kinds of maths, rather there were different ways to *enact* maths, with different strategies and symbolic representations used by individuals in maths outside the classroom. Carraher pointed out that the questions and arguments posed by Greiffenhagen and Sharrok actually reflected the changes over the decades in assumptions about maths education, symbolic representation and the impact on debates on context and situatedness since the publication of the studies by Lave and himself. Carraher dismissed the view by Greiffenhagen and Sharrok that Lave's work was motivated by an ideological critique of traditional views of cognition as being naïve. He suggested that deeper investigation into Lave's previous research would have shown them that her motivation was rooted in the exploration of learning and cognition from a cultural and situated context, not to promote any personal ideology. In brief, Carraher believed the authors to be labouring under the false impression that the focus of these studies was to show that everyday maths was superior to school maths. In doing so Greiffenhagen and Sharrok missed the point that everyday maths was not being promoted as a superior alternative to maths taught in schools, rather these studies on everyday maths shed light on the phenomena of different maths practices in everyday situations. Carraher suggested that alternative algorithms that appear to be created by individuals in the ongoing practices of daily life could ultimately be absorbed into mainstream education and research. This would contribute to understanding how these self-taught methods succeed and fail in everyday mathematics, resulting in the disappearance of the dualism that exists between these two approaches—one formal and one informal—to mathematical dilemmas.

The Adult Math Project provided a rich source of material for investigating the relationship between everyday problem solving and school-taught arithmetic. Support for Lave's argument that the setting in which a problem is actioned impacts the structure and strategies during the ongoing activity of an individual in searching for a resolution was evident in the analysis of the results from the ethnographic observations and the formal experiments reported.

Comparisons between more orthodox psychological experiments and the hybrid study presented by Lave stressed how differences in methodology influenced the results and therefore the inferences drawn. This confirmed Lave's hypothesis that by discounting the impact of situatedness, research by traditional cognitive psychologists was not accounting for the complexity of mental processes, sociocultural influences and the interaction between the individual and the dynamic environment in which the individual is embedded. Everyday activities such as grocery shopping are often perceived as unremarkable and routine, with the shoppers in this study themselves expressing such views. However, Lave has shown that these activities are frequently shaped by previous experience, transformed, updated, and altered across the setting as the activity progresses.

Conclusion

The contribution of Newell and Simon's (1972) canonical work on problem solving by information-processing systems, both human and machine, substantially changed approaches to research within the field of cognitive psychology. They offered a computational model of the mind based on the manipulation of symbolic notations and structures. This model, although widely accepted in the academic community, was challenged by a number of researchers and theorists for its limited account of the impact the external world has on cognitive processes. Greeno (1989, 1998) and Lave (1988) were among those influential in the early stages of the ongoing debate over the ecological turn in cognitive science away from a framework of cognition delimited by the brain. Motivated and influenced by previous work from others (e.g., Bartlett, 1958; Gibson, 1966; Neisser, 1976) Greeno and Lave both undertook research and contributed theoretical insight toward a greater understanding of the situated approach to cognition. This theory of cognition proposed that the environment in which an individual was embedded, including sociocultural influences and ongoing actions, shaped and guided learning and thinking. Consequently, this shifted the focus of analysis from the internal processes of the cognitive agent, where the outside world had been assigned a marginal role, to a wider system incorporating the agent and the external world in dynamic interaction.

The situated model of cognition not only integrated the role of the environment as part of the system shaping cognitive processes; it questioned accepted conceptions of knowledge and learning transfer. As previously discussed, Greeno suggested that knowledge should be considered as a relationship between the individual and the situation, rather than the exclusive domain of the internal mind of the individual. Lave also challenged the metaphor of knowledge as a tool to be accessed when required from a stored corpus of information, then replaced unchanged awaiting the next occasion for use. Through her analysis of everyday activity of shoppers in the supermarket she concluded that knowledge was characterised more as “a process of knowing” (Lave, 1988, p. 175) through interaction with the changing world, not bounded by an abstract domain within the mind of the individual. Lave’s study of everyday maths practices illustrated that problems frequently unfold organically during ongoing daily activities, with relations between the setting, the activity, and the individual being both determined by and creating these daily dilemmas. This was in contrast to the prefabricated tasks and problems presented in the classroom of traditional schooling and the laboratory of the experimental psychologist. Learning transfer as perceived in the traditional approach to cognition separated cognition from the social world. In analysing the four sets of traditional psychological experiments on learning transfer, Lave argued there was no clear evidence of learning transfer as a valid problem solving process. Nor were there any satisfactory explanations of how knowledge from formal learning situations might transfer to solving problems that unfold in everyday situations. This potentially undermined prevailing explanations and expectations of learning transferring from the setting of school to activities in different settings beyond the boundaries of the classroom (Lave, 1988). Lave and Greeno considered problems presented in the classroom and in the lab to be generally unrepresentative of problems in the lived-in world, as these formal problems offer a clear goal state with a well-defined and stable problem-space. Where in everyday problem solving the salient features of the problem-space are not as well defined, with these features frequently emerging during the interactions of the agent with the changing world (Greeno, 1998, Kirsh, 2009a). They did not explicitly deny the validity of results achieved within the context of the setting of the classroom or the laboratory; however, one argument posed by situative theorists was that all teaching and experimentation was itself situated and not context free. Therefore, it is essential to recognise the character of this situatedness and the context in which the activity takes place, this challenges the accepted practice of mapping inferences about cognition drawn from results in school and from the lab onto the lived-in world.

Lave's interest was in situatedness from a sociocultural and person-in-action perspective, consequently her investigation into the impact of the physical environment was only cursory. Lave did not explore any differences in maths strategy that may have arisen between the individual being in situ at the supermarket and at home for the simulation exercise, nor the use of different artefacts in the two settings. Greeno on the other hand tentatively considered the role of objects when expanding on the symbolic processing in the external environment. The unexplored aspects of the physical environment, in particular the role of artefacts in cognitive processes, exposed another essential line of investigation into problem solving in situ, instigated by the debate on situated thinking.

Lave’s project and Greeno’s framework for analysing cognition in situ were ambitious, pioneering and have been extensively cited in many fields beyond that of psychology. However, they also underscored the issues confronting researchers when taking research from the lab to the outside world—an unpredictable and complex world (Norman, 1993a). Issues include determining the best problems and methodologies in order to facilitate valid and reliable comparisons between the performance of a person across differing settings; recording both qualitative and quantitative data in a manner enabling the matching of observation of behaviour to empirical outcomes; separating the problem from the resolution in ongoing activities, when the problem-space is not stable and well defined; and identifying and addressing the factors that may influence variability in results that otherwise would be controlled in a laboratory setting. These challenges and the outcomes presented by Lave and Greeno, evolved and unfolded not unlike the situated process for problem solving they proposed, contributing to a platform from which to build an alternative model for re-thinking thinking.

Chapter 3

The Extended Mind

Overview

The ecological turn in cognitive research toward a greater emphasis on the role of the environment—sociocultural and physical—led to the development of a theory of situated cognition. Researchers such as Carraher, Carraher, and Schliemann (1985), Lave (1988) and Hutchins (1995a, 1995b) embarked on large-scale ethnographic studies, merging anthropology and psychology to investigate everyday thinking practices. At the same time, a growing number of psychologists (e.g., Greeno, 1989,1998; Zhang & Norman, 1994) and philosophers (e.g., Clark, 1997; Neisser, 1976) were also researching and developing theories supporting an ecological approach to cognition. In hindsight, it could be argued that the unintentional drawing together of this groundswell of ideas under the one umbrella of situated cognition focused the efforts of those researchers and theorists challenging the established views of traditional cognitive science. Consequently, the situated perspective in understanding human cognition gained momentum with the development of an increasing number of theories predominantly in psychology, philosophy, and artificial intelligence.

The aim of this chapter is to present a brief introduction to some of the predominant theories of cognition beyond the brain including the so-called 4E's—embodied, embedded, extended, and enacted cognition—along with cognitive integration and Material Engagement Theory. One of the 4E's, the extended mind hypothesis proposed by Clark and Chalmers (1998), is discussed in more detail in this chapter as it is arguably one of the seminal theories of a systemic view of cognition explicitly recognising that the products of a cognitive system emerge through interactivity.

Sutton (2010) and Menary (2006, 2010a, 2010b, 2015) further developed the Clark and Chalmers thesis in a second wave extended mind theory. They proposed a second wave, in part, to address criticisms of the parity principle as described in the original extended mind hypothesis. The chapter concludes with a stronger account of the extended mind thesis in Malafouris's (2013) theory of material engagement, offering further exploration of the relationship between objects and cognition by the attribution of agency to artefacts as an emergent property of interaction. The experiential and affective properties of things saturate the cognitive, social, and emotional lives of people. Therefore Malafouris argues, that artefacts should be studied as active, integral elements of human cognition rather than passive tokens within problem-solving activities, because things have the potential to shape thinking as people engage with the world. This emphasis on the importance of research into the cognitive role of artefacts as components of an emerging dynamical system of thinking in the world motivates the experimental work discussed in later chapters.

Introduction

In an endeavour to partially dissolve the boundaries surrounding cognitive processes as constructed by internalist theories, Hutchins (1995a) undertook a lengthy study of the behaviour of a ship's navigation team analysing the observations in terms of a naturally situated cognitive system. Hutchins's agenda was to shift the concept informed by cognitive science that human cognition was located somewhere inside the head, to understanding cognition as a process that extends "out beyond the skin of the individual" (p.287). A number of studies and theories emerged at the same time (e.g., Greeno, 1989;

Lave 1988; Lave & Wenger, 1991; Suchman, 1987) endorsing similar agendas, therefore underscoring the notion that knowledge and learning were relational to the environment. The contribution made by researchers (e.g., Greeno, 1989; Hutchins, 1995a; Lave; 1988) of situated cognition toward an explanation for cognitive processes that would potentially transcend traditional cognitivism, stimulated the emergence of theories founded on the premise that mental processes were dependent on the context or setting of an activity. The theories of embodied, embedded, extended, and enacted cognition, the so-called 4E's, represent a large body of the research that has contributed to this ecological turn in cognitive science by offering alternative frameworks to the traditional approach presented by cognitive science (Gallagher, 2008; Menary, 2010a; Menary 2015). The 4E's are generally considered complementary to the concept of situated cognition, proposing alternative interpretations of cognition to the mind-body dualism of traditional psychology, although offering differing methodological and philosophical strands to the non-cognitivist approach (Menary, 2010a; Robbins & Aydede, 2009). This chapter will discuss each of the 4E's with the focus on Clark and Chalmers's (1998) theory of the extended mind. This is not to imply that embodied, embedded, or enactive cognition offer any less to the debate on cognition than the extended theory, as all propose persuasive arguments in favour of their respective claims. However, extended cognition offers an important foundation for a discussion on theories and research that embrace the value of artefacts in varying degrees, within a broader interpretation of systemic cognition that is of particular interest within the framework of this thesis.

Three of the 4E's

Embodied cognition. Embodiment is clearly interwoven with situatedness, as the body is the contact point where physical action changes and impacts the external world in which an individual is embedded (Clark, 2008; Clark & Chalmers, 1998; Gallagher, 2008, 2009). Malafouris (2013) argued that while embodied cognition moves toward dispelling the demarcation between mind and body, the theory does not go far enough in dissolving the boundaries of the skin and the outside world. Gallagher (2008) explained how the physical construction and mechanics of the body influences the way an individual experiences the world, offering opportunities to explore and search the environment in activities and tasks. At the same time, the bodily shape constrains actions and the possible affordances offered by the world (Gallagher, 2008). The movement of the body is not just determined by neural activity alone, it is also enabled by the design and flexibility of bodily parts such as tendons, muscles, and joints to actively retrieve information from the world using sensory inputs as a source to take action and solve problems (Clark, 2008; Gallagher, 2008). The body interacts with the surrounding world, facilitating an intelligent interface between the internal mind and the outer environment (Gallagher, 2008). These actions in the world are not merely the result of some preconceived mental state; the embodied approach to cognition conceives of bodily actions as being integral in the shaping of abstract concepts, reasoning, and thinking (Hutchins, 2010b). Cognition comes from actions, with an individual's body influencing the way of thinking (Clark, 2008; Hutchins, 2010b).

Enaction. Enactivism as introduced by Varela, Thompson, and Rosch (1991) is a strongly embodied approach to cognition, placing the emphasis on experience in the lived in world, embracing the notion of bodily action of an organism on, and interaction, with the world it inhabits (Menary, 2015). The theory of enactivism offers a middle ground between strict cognitivism whereby experience is all but dismissed from cognitive theory, to the other end of the phenomenological spectrum defended by philosophers where experience is unquestionably accepted as part of cognitive practices. In proposing this middle ground enactivism is not a theory limited to human cognition, as the concepts offered extend to almost all living organisms (Menary, 2015; Stewart, 2010). Within this approach to cognition there are two inter-related key concepts: the autonomy of the living organism by generating and sustaining its own activity bringing about its own cognitive world; and the activity in this world must be sense-making by relating to the world in a cognitively adaptive manner (Thompson & Stapleton, 2009). Thus the proposal by enactivists is that through interactions with the environment in sense-making activities, an autonomous organism constructs its own world or its own reality (Thompson & Stapleton, 2009).

Embedded cognition. Just as the body is an active cognitive component situated in the world, individuals are embedded in the surrounding world, not as observers but as beings immersed in a world rich with artefacts and activities in which everyday engagement with these resources shapes plans and actions (Gallagher, 2008). In discussing various approaches to cognition, Gallagher (2008) does not differentiate between the terms situated and embedded noting that by virtue of existing and acting in the world, an individual is naturally embedded in the environment in which she is situated.

The Extended Mind

In keeping with the situated approach to cognition, but at the same time subsuming some of the tenets of the information-processing model, Clark and Chalmers (1998) advocated a further theory of cognition—the extended mind hypothesis. This hypothesis proposes that not only are an agent's body and the world in which the agent is situated impactful on cognition; both the body and the world should be considered as active components of the cognitive process. Drawing on the growing volume of work challenging the traditional information-processing conception of cognition, Clark and Chalmers took the position that, in addition to cognition being situated, embedded and embodied, the mind and therefore cognition was extended into the external world including, and beyond the body. Thus the extended mind hypothesis, proposed by Clark and Chalmers, asserts that cognition is not bounded by skull or skin, but integrates the mind, the body, and external resources into the cognitive system. Within this framework, there is no demarcation between the body, the mind, and the external environment; rather cognitive systems encompass individuals and their physical and social world, with the individual exploiting various aspects of the environment to contribute to thinking, decision making, and problem solving (Gallagher, 2008; Wilson & Clark, 2009). A keystone assertion is based on the notion of "active externalism" (Clark & Chalmers, 1998, p. 8) where the world is a causal factor playing an active role in cognition; that is, the agent acts on the world and the world acts on the agent in the world itself. Menary (2010b) believes that active externalism goes beyond a causal explanation to one where the features of the environment are not aids to cognition, rather these features can be constitutive components of the cognitive process. As an individual encounters a task, the world forms a functional component in the undertaking of that task, thus the setting and environment in which an agent is situated plays an active role in motivating and structuring the processes through which cognition both arises from, and creates actions in, the living world (Clark & Chalmers, 1998; Wilson & Clark, 2009). The active agent is immersed in a world surrounded by artefacts as complementary components in the cognitive system. Here the inner mind, the body, and external surroundings of an individual are coupled in a flexible and reliable manner in a "two-way interaction" (Clark & Chalmers, 1998, p. 8) creating a cognitive loop (Wilson & Clark, 2009).

The notion of 'a coupled system' is an important tenet in extended mind thesis, and ultimately many approaches to cognition from a systemic perspective. According to Clark and Chalmers (1998) the most reliable coupling takes place in the head, however this reliable coupling is not just brain-bound, it is evident between the individual and the surrounding environment. When using a diary to keep track of forthcoming events, if the diary is not to hand, the user may miss an appointment, in this case the removal of the diary from the cognitive system results in a loss of augmentation of the user's capabilities. Clark and Chalmers go as far as to state that such a loss of an external component of a cognitive system would be as if part of the brain had been removed. Clark and Chalmers (1998) proposed a parity argument where components, be they external or internal, can be considered to have a functional role in a cognitive system:

> If, as we confront some task, a part of the world functions as a process which, *were it done in the head*, we would have no hesitation in recognizing as part of the cognitive process, then that part of the world *is* (so we claim) part of the cognitive process. (p. 8)

This has become known as the Parity Principle (Clark, 2008). Clark and Chalmers point is further clarified by their much-discussed hypothetical Otto's notebook, with this artefact providing an interesting foundation for a thought experiment that demonstrates the extension of memory beyond the brain. To briefly describe the scenario, Otto is an Alzheimer's patient who carries a notebook with him in which he writes new information to assist in playing the role of his biological memory (Clark & Chalmers, 1998). While Otto carries the notebook with him, it is a permanent extension of his cognitive capabilities augmenting his thinking— without it, his competencies would be severely impaired. In consulting his notebook for say, the location of a museum, Otto not only relies on this portable artefact as a cognitive vehicle in order to find his way, but he also believes the information as he would believe his memory. The information in Otto's notebook takes on the role of a belief to Otto; Clark and Chalmers postulate that it is the role information plays in cognition that is important, not the location in which it is used that ultimately gives rise to information being considered a belief.

Wide computationalism. Consistent with the views of Greeno (1989,1998) on situated cognition, Wilson and Clark (2009) discussed cognition not in terms of an individualistic phenomenon as depicted by the information-processing model of cognitive science, but rather as cognitive systems reaching "beyond individuals into their physical and social environment" (Wilson & Clark, 2009, p. 58). While the cognitivist explanation does go beyond cognition being essentially neural, cognition nonetheless remains firmly in the mind of an individual as an internal computational process, with the surrounding environment offering information as a contributor of static inputs to this process (e.g., Newell, Shaw, & Simon, 1958). The notion of computationalism would then appear to be at odds with theories of situated cognition, and therefore extended cognition, as the inference is that cognitive computation is only possible in the head, as epitomised by the mind-as-a computer metaphor (Wilson & Clark, 2009). Wilson and Clark (2009) disagreed with this inference, as their theory on extended cognition embraced computationalism as being possible across a wider system beyond the skin of individual, reaching out to the surrounding milieu (Wilson, 1994). This position of cognitive processing as part of a “wide computational system” (Wilson, 1994, p. 351) concedes that some processing takes place only in the mind of the individual, however asserts that the brain of the individual is not the boundary of computationalism. In rejecting the traditionalist approach to computationalism, the process of cognition is broadened to include all components within the cognitive process that have computational properties. A component of the cognitive system however, does not necessarily need to be computational to be considered as an integrated part of the cognitive process. To be considered as an integral component to an extended cognitive system, any additional resource should enable improved efficiency to the existing system in reaching the anticipated goal or completing the action, therefore offering cognitive augmentation by its introduction to the system (Wilson & Clark, 2009). There may be components of this system that are part of the environment, but not part of the individual, such as pen and paper. Pen and paper together offer a computational module in itself for say calculating sums when required by the user, which in turn contributes to the wider computational system (Wilson, 1994). However, note that it may be that pen and paper separately are parts of the cognitive system but are not in themselves computational in this case. Wilson (1994) suggested that wide computationalism did not exclude the notion that computational cognitive states may be instantiated solely within the individual, he was making the case that these computations are not exhausted by the boundaries of the skull but are a part of the wider extended computational system (Wilson & Clark, 2009).

Three threads of cognition. The extended mind hypothesis characterises the brain, the body, and the world as “three threads” (Clark, 2008, p. 197) interwoven in a synthesis of available resources to solve a problem or complete a task with minimal effort. Clark called this the “Principle of Ecological Assembly” (p.13), indicating the significance of both embodied and embedded cognition within the extended mind thesis. The mind is not conceived of as a device that manipulates symbols according to a set of rules; rather the mind is an initiator of thinking as the cognitive process unfolds as “Cognition leaks out into body and world” (Clark, 2008, p. xxviii). Chalmers (2008) succinctly captured the role of the body and the world in terms of extended cognition when he describes how the body, including language and senses, should be considered a tool to extend thought and the relevant parts of the world not as mere aids to cognition but become part of the mind. Thus within this framework, all components in the cognitive system actively contribute in a causal manner—that is to say, if the environment changes or is changed by an individual, this has an effect on internal processes and vice versa (Clark & Chalmers, 1998). This interaction between the components creates a loop of activity, with behaviours of one component impacting other segments within the loop. This looping interaction between the internal and external constituents, mediated by the body, produces an active externalism resulting in online cognitive activity, creating a coupled system embedded in the world (Clark & Chalmers, 1998). This is not to say that all components have an equal weighting in the system—a frequently misunderstood interpretation of the Clark and Chalmers Parity Principle (Clark & Chalmers, 1998; Menary, 2010b, 2006, 2007; Sutton, 2010; Sutton, Harris, Keil, & Barnier, 2010; Wilson & Clark, 2009). Although Clark and Chalmers expressed some doubts about the conservativeness of the embodied or embedded approaches to cognition, it is clear that extended cognition incorporates many of the tenets of situated, embedded and embodied cognition (Clark, 2008; Clark & Chalmers, 1998; Menary, 2010b).

The structure of extended systems. By virtue of the notion that this paradigm includes brain, body, and the world, it is clear that extended cognition may take many forms, thus Wilson and Clark (2009) identified two dimensions to provide a structure in order to broadly establish the determinates of extended cognitive systems. The first dimension is the identification of nonneural cognitive resources—these may be natural, technological, or sociocultural; second, the resulting functional integration of the aforementioned resources with internal cognitive resources must be reliable and durable (Wilson & Clark, 2009). These dimensions track the nature of the resource along with the reliability and durability of the cognitive extension providing a framework that offers a graduated approach in considering the various forms of extended cognition (Wilson & Clark, 2009).

The first dimension sets out to define resources beyond the neural substrate that might potentially form part of the extended mind; these resources are either natural, technological, or socio cultural. Broadly speaking natural resources, when considered as part of extended cognition, are within the natural world of the cognizer and integrated functionally into the cognitive needs of the agent. For example, an empty shell used as protection by a crab, it is naturally occurring in the crab’s environment, it is required by the crab to continue to function in the world and in turn the shell functionally alters the actions available to the crab (Wilson & Clark, 2009). Non-natural or technological resources are those that are developed by human agents—examples by Wilson and Clark included a notebook and a prosthetic leg. The third nonneural resource, sociocultural systems, arguably could also be considered as either natural or technological resources. This resource draws on the social and cultural systems created and molded by prior generations, it may be a behavioural or a material product of the culture in which an individual is embedded, one example would be writing systems (Wilson & Clark, 2009). To be considered an integrated part of an extended cognitive system, a nonneural resource must play an active role in the cognitive operations of the agent, not merely a resource used by an agent with cognitive abilities (Wilson & Clark, 2009). By way of example, Wilson and Clark return to the example of writing systems. Here the individual does not generally write random words on a page just because he or she possesses the ability to write. Rather, writing is not a resource used by the individual to only augment existing capabilities such as working memory—individuals write to think and create (Oatley & Djikic, 2008; Wilson & Clark, 2009). The agent will mine the surrounding world for resources to solve problems or engage in tasks and to reduce the burden on the internal cognitive load such as working memory, in a manner that does not favour either internal or external resources (Fu & Gray, 2000; Lauri, 2013; Wilson & Clark, 2009). Resources are recruited by the cognizer to minimise effort while generating an acceptable outcome (Clark, 1989). Clark (1989) dubbed this the 007 principle:

> *The 007 principle.* In general, evolved creatures will neither store nor process information in costly ways when they can use the structure of the

environment and their operations upon it as a conventional stand-in for the information-processing operation concerned. That is, know only as much as you need to know to get the job done (p. 64).

Once a resource has been established as being functionally integrated within the cognitive system, the second dimension to consider is the reliability and durability of the system. This dimension of reliability and durability is perhaps less clearly captured than identifying nonneural resources and their integration into the extended cognitive system, as Clark and Chalmers do not specifically tease out the aspects of durability and reliability as separable elements. They briefly described Otto's notebook as having "enduring augmentation" (Clark & Chalmers, 1998, p. 67), suggesting that this resource is deeply integrated into the functioning of the cognitive agent, and is therefore more durable than a resource that only augments cognition on a single instance. However they do explore the aspect of reliability within an extended cognitive system more specifically as being determined by the relevance of the coupling between the internal and external resources. As discussed earlier, this coupling of resources must form a functional part of the cognitive activity, be portable and readily to hand, Clark and Chalmers (1998) then determined this constitutes reliable coupling. At times, relevant resources may be temporarily decoupled from the system just as sleep briefly interrupts the biological brain, consequently If Otto is not accompanied by his notebook occasionally, this does not permanently exclude it as a reliable cognitive component for coupling in the future. Therefore, if the relevant resource is generally available to the agent for integration as an augmenting component into the cognitive system as and when required, this would then be considered sufficient for reliable coupling to be established between the individual and the resource (Clark & Chalmers, 1998; Wilson & Clark, 2009).

Transient cognitive systems. Some cognitive systems that emerge as solutions to problems unfold, or as activities are undertaken, may be considered transient in nature, for example composing a piece of music or completing a tax return. In other situations, there may be more permanent systems in place, such as consulting an address book when sending Christmas cards every year. The pen and paper, computer, address book, or files of information created over the year for a tax return are “add-ons” (Wilson & Clark, 2009, p.64), both permanent and temporary, that integrate with the processes of the brain. Thus, some systems will make use of existing mental capabilities and integrate additional resources temporarily in the cognitive process, while others will make use of coupling between internal and external resources that is more permanent and stable (Wilson & Clark, 2009). In explaining more transient forms of extended cognitive system, Wilson and Clark (2009) began by drawing on the notion of a task-specific device (TSD), a theoretical approach proposed by Bingham (1988) to provide an organising framework for problems arising in the study of human physical action. Bingham dubbed this the Human Action System. The Human Action System involves the nonlinear linkage between the four subsystems of human action—the musculoskeletal system, link-segment system, the circulatory system, and the nervous system (Bingham, 1988; Wilson & Clark, 2009). Bingham considers the linkage of the subsystems to be nonlinear as it is not a case of studying the components of the subsystems in isolation as an action takes place, then adding the behaviour of the components together. Attempting to understand each of these four subsystems in isolation, then to investigate the nonlinear character of the dynamics of actions and the interactions between each component would prove overwhelmingly complex. Bingham proposed an inverse methodology by working backwards from the whole assembly that is the TSD to the complex dynamics of the subsystems. In essence a TSD is a dynamic system, it is a soft assembled (i.e. temporary and easily dissoluble) whole built from whatever resources are on hand consisting of both the individual and the surrounding environment (Bingham, 1988; Wilson & Clark, 2009). By focusing on the specifics of the task, the functionality and the goals, a TSD links together the intricate interactions of the high dimensional group of the four subsystems that enable an action to occur. Thereby reducing them to a more controllable lower dimensional structure, enabling the simpler dynamic assembly of the TSD (Wilson & Clark, 2009). This facilitates an approach where it is possible to look at the task-specific dynamics, then to establish the particular dynamics of the subsystems, which are acting together in creating the overall dynamical system (Bingham, 1988). Wilson and Clark (2009; also see Clark, 2008) used Bingham's notion of TSD as an analogy to their extended mind hypothesis, many seemingly disconnected parts making up a functional whole, thus providing a foundation for further exploration into extended cognitive systems. Clark (2008) explained how the central tenets of a TSD could also apply to larger problem-solving entities with similar transient features; consequently, these temporary forms of cognitive augmentation were labeled "transient extended cognitive systems" (p. 158). Transient extended cognitive systems (TECS) are also a soft-assembled unit bringing together the problem-solving attributes of the brain and the body with external resources that enable cognitive scaffolding (Clark, 2008; also see Wilson & Clark, 2009). TECS just like TSD, allow for the coupling and uncoupling of external resources with the brain and body in creating an extended cognitive system directed at a specific task. By considering transient cognitive systems in this manner, the durability of the extended cognitive system may be gauged by the form the system takes. When a TECS is perhaps a one-off system created for a task such as a new brainteaser, for example the river-crossing problem (see chapter 6 of this thesis for details on the river-crossing problem). A one-off TECS might be considered less durable and reliable than when a TECS is generated from a system that is frequently repeated such as a regular crossword solver completing a new crossword puzzle or simple maths additions. In the instance of the crossword, a solver may quickly employ re-assembled TECS in going through the clues that are easily completed first, rather than trying to solve each clue before moving onto the next: a system the solver may have used successfully previously. Other cases may fall between these two forms of TECS resulting in an intermediate form of TECS, such as solving a Sudoku puzzle (Wilson & Clark, 2009). There is the possibility of creating a range of TECS in succession, as a trekker might do when using a variety of equipment such as compass and map. As with the TSD, it may be useful to work back from the larger problem-solving ensemble by initially analysing the system as a whole, in the form of a TECS. This in turn may provide insights into the components and their interactions (Clark, 2008). Studying TECS may offer an understanding into the distinctly human ability of reasoning and problem solving as the reasoner exploits and interacts with the wide variety of artefacts offered by the external environment, thus providing a window into the investigation on how these additional resources augment problem solving (Clark, 2008; Wilson & Clark, 2009).

Where TECS are considered as soft-assembled, Wilson and Clark (2009) conceived the next step on from the concept of transient cognitive systems would be to consider the more permanent notion of the extended mind that would result in a more reliable and durable system than a TECS. Such a system could be exemplified in the fictional case of Otto's notebook where the notebook is an artefact of deep functional integration offering reliability and durability within an extended cognitive system. While the notebook obviously does not match the biological equivalent of the memory in the brain, the coupling of the artefact with the inner processes of the mind augments the cognitive processes of the individual.

Wilson and Clark proposed that the many forms of extended cognitive systems are on a continuum ranging from the transient, one-off and repeated systems through to the more permanent relationship between an individual and external cognitive resources. This extended cognitive system is therefore a hybrid process where the inner and outer components are different, nonetheless complementary and integrated into the system (Wilson & Clark, 2009). These components of the extended cognitive system might be recruited from anywhere, internal or external to the individual, taking any form, including processing and encoding, providing these functional links contribute to the computational activity of the task; Clark (2008) dubbed this "computational promiscuity" (p.106). This tenet of computational promiscuity fits well with the problem-solving activities of the everyday reasoner by reflecting the fluidity of the cognitive process, as resources are recruited by the individual from the artefact-rich environment and coupled with internal resources as complementary computational aids to the intrinsic contents of the extended cognitive system regardless of spatial location (Wilson & Clark, 2009).

Second-wave EM

In further developing an extended mind framework of cognition, Sutton (2010) defended a second line of thought that he claimed could be found in the original extended mind arguments (Clark, 1997; Clark & Chalmers, 1998). As Sutton (2010) explained, many critics of extended mind (e.g., Adams & Aizawa, 2001; Rupert, 2004) focused on the parity principle when attending to Clark and Chalmers's explanation for the nature of, and the relationship between components within the cognitive system. Wilson and Clark (2009) also noted there seemed to be a constant misinterpretation of the so-called claim of parity introduced by Clark and Chalmers in the extended mind hypothesis. In addressing this overemphasis, apparent misunderstanding and shortcomings of the parity principle by discussants of extended mind, Sutton (2010) proposed "*First-wave EM*" (extended mind) and "*Second-wave EM"* (p.193). The emphasis of first-wave EM was on the notion of parity between internal and external processes where these processes were considered to play a similar functional role in cognition (Menary, 2010b; Sutton, 2010). In contrast, second-wave EM as outlined by Sutton, was centered on what he labeled the complementarity principle; so-called as the external parts of the extended cognitive system may function differently, and at the same time are complementary to inner resources during cognition and action (Clark, 1997; Sutton, 2010). As Sutton pointed out, Clark (1997) not only addressed parity, but also the notion of resources being complementary in his (Clark's) writing as illustrated by this paragraph:

> …the brain's brief is to provide complementary facilities that will support the repeated exploitation of operations upon the world [and] to provide

computational processes (such as powerful pattern completion) that the world, even as manipulated by us, does not usually afford. (p. 68).

Wilson and Clark (2009) further addressed parity and complementarity in explicitly maintaining that the argument for extended cognition did not pivot on the requirement for inner and outer processes or states to be similar. Rather the internal and external resources should be *complementary* and deeply integrated into the cognitive whole. Therefore the focus in the extended mind thesis was on how the internal and external resources function together, not on their functional similarity. However, this response by Wilson and Clark to criticisms of the parity principle in the substantiation of the Clark and Chalmers argument appears to have been largely ignored. Clark and Chalmers were not claiming that in order to meet the principles of the extended mind theory, outer processes need to perform exactly like inner neural processes to be considered cognitive—rather the argument for extended mind was that the brain does not hold exclusive rights over the resources required for cognition. According to Sutton, (2010), when correctly understood, the parity principle and the complementarity principle are compatible. However, the parity principle was a more abstract notion, thus the argument frequently made by critics (e.g., Adams & Aizawa, 2001; Rupert, 2004) that internal and external resources must be isomorphic to meet the criteria of the parity principle and therefore extended mind, is better resolved through complementarity (Sutton et al., 2010). While these two principles are fundamental to the extended mind thesis, critics have often disregarded Clark and Chalmers outline of other conditions, such as reliability and durability—these “glue and trust” (Clark, 2010, p.83) conditions should also be in place to identify an extended mind (Wilson & Clark, 2009).

The arguments in favour of pursuing a complementarity approach to extended mind and possibly further distinct frameworks are not pursued in this thesis. However, Sutton (2010) believed that continuing to develop an extended mind hypothesis based on complementarity addressed many criticisms of parity and thus provided a basis for studying cognition from both a scientific and cultural perspective.

Cognitive Integration and Enculturation

Also with an eye to the second wave of extended mind, Menary (2006; 2007; 2010b) proposed an alternative approach to extended cognition in the form of cognitive integration (Sutton, 2010). The first wave or "extended-mind-style arguments" (Menary, 2010b, p. 228) of Clark and Chalmers (1998), Hutchins (1995a), and Gallagher (2005) integrate the features of the internal mind, body and external world into a hybrid cognitive process of looping interactions between manipulations by the mind and body on the environment in which the individual is situated (Menary, 2010b). The second wave or "cognitive-integration-style arguments" (Menary, 2010b, p.228) begin to unravel the nature of the integration in hybrid cognitive systems by examining how and where the integration takes place. In addressing the position of cognitive integration in terms of the 4E's, Menary (2015) placed the extended mind hypothesis in the category of strongly embedded, and enactivism as strongly embodied, where cognitive integration traverses both strongly embodied and strongly embedded approaches.

Menary (2007; 2010b; 2015) interpreted Clark's (2008) version of extended cognition as organism-centered, whereby thinking is extended out into the world in which the individual is embedded and situated, however the brain remains at the centre of the thinking process. Cognition begins with the body and extends out into the world, the focus then moves to the ways in which the brain and body interact with external resources as active components in a single dynamic cognitive system (Menary, 2007; 2015). The enactive approach focused on the role of the body of an organism, enacting with and in the world it inhabits as part of the cognitive process (Varela, Thompson & Rosch, 1991). This approach encompassed the spectrum of cognitive systems, from simple biological systems to the very complex, including memory and problem solving. The enactive approach of Varela et al. (1991) integrated cognition and emotion as part of sense-making, whereas the extended mind hypothesis as presented by Clark and Chalmers (1998) appeared to neglect emotion as part of the cognitive process (Malafouris, 2013; Thompson & Stapleton, 2009). Although beyond the scope of this thesis, it is important to note that Clark (1998) argued against relying on strong sensorimotor models believing that such models were insensitive to key information processing routines providing an incomplete picture of cognition. Clark also mentioned the role of emotion in cognition is worthy of discussion when concluding the chapter *Memento's revenge*, although he did not elaborate this point (Clark, 2010). While both the extended mind and enactive hypotheses proposed an expanded framework for cognition from the mind to the external world, neither emphasised the ways sociocultural practices impact the association between brain, body, and the external world in creating a distributed cognitive system (Menary, 2015). In line with Sutton's (2010) approach to extended cognition and the distinction between the first and second waves of extended mind, Menary (2010b) did not rely on the parity principle when explaining cognitive integration. He believed the question to be asked of the extended mind theory was not whether external processes are functionally similar but how do these external processes get to *function* similarly to those of the biological brain. Menary did not deny that some cognitive processes take place in the internal mind; however, the mark of the cognitive in the extended mind is when the interaction of the internal neural manipulations and bodily manipulations of "information-bearing vehicles" (p. 236; e.g., Otto's notebook) in completing a task transforms the cognitive process. Thus the focus of integrationists is on the type of bodily manipulations in the external world and how these integrate with neural processes (Menary, 2010b). The purpose of these manipulations is to achieve a particular goal, whether to solve a problem or complete some other task that might be considered a cognitive activity. There is, to some extent, acquisition and learning involved in manipulating these external vehicles that transform an individual's cognitive capacities. Menary provided an example of using a computer keyboard for the purpose of writing where the manipulation of the tool, in this case the keyboard, enables the writer to read, erase, rewrite, read, store and continue to develop new ideas or construct meaningful sentences. Without the keyboard, there would be a much-reduced level of competency; it would be an extremely difficult if not impossible task to complete in the head alone. This exemplifies the bodily manipulations of the artefact as an external process being complementary to, although different from, the internal process (Menary, 2010b; Sutton, 2010).

In addition to the concepts of manipulation and transformation, one of the defining aspects of cognitive integration theory as developed by Menary (2015) is the consideration of the embodied and embedded nature of cognition through the lens of enculturation. Menary's (2015) proposal of cognitive integration addressed this issue by describing how cognitive capacities are transformed through real time social interactions, interactions with artefacts and the inherited cultural practices of previous generations. The use of mathematical calculations by the Brazilian market vendors in Carraher, Carraher, and Schliemann's (1985) study, is an example of this cognitive transformation across time and space. Carraher et al. (1995) report how the market vendors rapidly calculated the cost of items on the market stalls without any formal mathematical knowledge, learning over time from the interaction with customers and other market vendors, as opposed to learning through the traditional education system.

Menary also proposed as part of the cognitive integration theory that, gradually over time, the plasticity of the brain permits neural changes initiated by cultural practices and constraints (see also Malafouris, 2013). This is illustrated by the ability to acquire new cultural capacities as a child develops. Humans are not born with the ability to read, it is acquired through learning. However, there is strong evidence that due to the neural plasticity of the brain, structural and functional changes to the brain occur as learning of words progresses and the child's brain develops (Menary, 2015). The argument proposed is that these changes are not part of normal brain development; rather the circuitry of the brain is changed as a result of learning and this learning only takes place as we interact with the world (Dehaene, Piazza, Pinel, & Cohen 2003). Learning is acquired through interaction with others, such as teachers, peers, family, and interaction with the physical world. In the case of reading, this interaction with the physical world frequently involves the use of artefacts, such as books, pens, paper, and computers. This argument suggests that artefacts are not only components of the ecological cognitive landscape as a scaffold for thinking online, but also for scaffolding functional changes to thinking determined by the plasticity of the biological brain (Menary, 2015). This in turn implies that interactivity with artefacts might be considered to both upload and download knowledge as part of a dynamic cognitive system. Artefacts 'upload' knowledge in the sense that interacting with artefacts may create new thoughts and new knowledge structures which is reflected in new neural circuitry. The looping back and forth process of interactivity between internal resources and artefacts can ‘download’ knowledge to facilitate the emergent thinking process within the extended cognitive system.

Material Engagement and Material Agency

As the boundaries of the mind were recast by theories arguing against an internalist view of cognition, Malafouris (2008, 2013) discussed an additional theoretical framework, Material Engagement Theory, grounded in the relationship between cognition and material culture. This framework continued with the agenda of breaking away from traditional cognitivism, drawing on work from disciplines such as philosophy of mind, archeology, anthropology, enactive, distributed, and extended cognition. Malafouris, in addressing the embodied approach to cognition, believed the paradigm had made great steps toward resolving the mind-body problem by conceptualising the body as an active, constituent element in cognition. However, he perceived limitations to this approach as the embodiment theory did not completely dispel the boundaries of cognition, rather it shifted them by failing to collapse the boundaries between the skin and the material world; thus the material remained external to cognition. While the extended mind thesis (Clark & Chalmers, 1998; Wilson & Clark, 2009) addressed this issue in part by dispelling the demarcation between the internal and external, Malafouris believed the theorists did not go far enough with the boundaries between the mental and the physical remaining. The theory of material engagement proposed that by moving to an approach whereby resources, internal and external, are dynamically configured and in synergy, boundaries do not apply. The mind is not found either inside or outside the head. Whatever is outside the head can still be part of the mind as, "cognition has no location" (Malafouris, 2013, p. 85).

Of particular interest is the active role attributed to objects in Malafouris's Material Engagement theory. One of the key tenets of this theory is that agency is not exclusive to humans; the relational properties of agency can also be applied to material things. This is not to ascribe agency to artefacts in an anthropomorphic sense; the agency of objects and things is an emergent product of material engagement of the individual with the world. Artefacts are not construed as passive objects that an individual acts upon, rather the artefact is something active with which the individual engages and interacts; thus the premise of Malafouris's proposal is that matter, things, artefacts, and objects can have agency. Malafouris is clear in stating that the agency of things is not to say things have human properties. Things are considered to have agency within the theory of material engagement, as the particular object or artefact exists as a distinct entity or component of an action. Therefore, it follows from this argument that studying the interaction between people and artefacts is fundamental in understanding the processes of human cognition.

Conclusion

Advocates of the situated approach to cognition were critical of traditional cognitive science for relegating the body and the lived-in world to a passive role in cognition proposing an alternative approach of situated cognition. This alternative way of thinking about cognition became a springboard for an increasing number of theoretical frameworks that were no longer bounded by the delineation of mind, body, and world. Subsequently, a new debate unfolded among those supporting the notion that cognition was not brain-bound. Arguably, the variety in interpretations of cognition proposed by these theories may offer independent claims about thinking in the world, or they may be inextricably linked by the notion of the agent and the world comprising a dynamic thinking system (Ward & Stapleton, 2012). However, irrespective of this argument, ensuing theories and research began to investigate in more detail the role of the environment, and the body in cognition; including sociocultural influences, sensorimotor experiences and artefacts. As illustrated by Lave's (1988) study on arithmetic in the lived-in world, it may prove difficult to isolate the various components of a problem solution entity. However, as part of Wilson and Clark's (2009) explanation for a possible methodology in analysing a problem-solving ensemble, they suggested it might be useful to analyse the system as whole in the form of TECS, in turn providing insights into how the solver exploits and interacts with the artefact-rich world in which the solver is embedded. Clark and Chalmers (1998) illustrated the extended mind using the hypothetical Otto's notebook. Gradually, as artefacts, objects, and things, being part a dynamic cognitive system, have come increasingly into focus within accounts of cognition, Malafouris (2008, 2013) developed the Material Engagement Theory proposing the radical notion that objects as distinct entities can have agency through action.

The active role played by artefacts in cognition through interactivity, has been alluded to in the first and second wave extended mind theory, with a more detailed treatment in Material Engagement Theory (Malafouris, 2013; Sutton, 2010). Cognition is spread across the mind and the external world, with artefacts within the milieu of the individual having the potential to change and manipulate thinking. Therefore, this underscores how an understanding of the role artefacts play in thinking is fundamental in defining and approaching cognitive properties within psychology and cognitive science.

Chapter 4

Distributed Cognition, Interactivity, and the Role of Artefacts in Cognition

Overview

In *Things That Make Us Smart*, Norman (1993a) recommended readers follow the debate surfacing from a controversial new approach to studying cognition called “situated action” (p. 265). As discussed in previous chapters of this thesis this new approach to cognition initiated innovative theories and research into human thinking processes. As an extension to the situated approach to cognition, distributed cognition emerged to address some of the apparent shortcomings of traditional cognitive science by extending the cognitive outside the mind of the individual to the wider external environment (Hollan, Hutchins, & Kirsh, 2000; Perry, 2003). Ensuing from Norman’s interest in the development of technology, and the potential impact on society and the human mind, he considered the role of artefacts in cognition as being under-explored with little understanding of the interaction between artefacts and users in daily activities (Norman, 1991). Norman (1993a) received the creation of artificial devices with both enthusiasm and trepidation, as he believed that they could make us “smart” (p.3) by expanding human cognitive capabilities, or “stupid” (p.3) by impeding creativity and human interactions with each other. Norman was working alongside other researchers, such as Hutchins (1995a, 1995) Kirsh (1995a, 1995b), in advocating a systems approach to cognition that extended beyond the individual. Hutchins (1995a) undertook an extensive ethnographic study of navigation on a US Navy ship. The resulting publication, *Cognition in the Wild,* was a significant contribution to the development of the systemic approach to cognition and the development of a theory of distributed cognition (Hollan, Hutchins, & Kirsh, 2000). This systemic perspective explained cognition through the tight coupling of the internal mind and the external world resulting in the spreading of memory and computational burden of thinking across a distributed cognitive structure (Hollan, Hutchins, & Kirsh, 2000). Interaction between the individual and artefacts as part of the distributed cognitive system alters the nature of the activity or problem, changing the cognitive properties of the system (Norman, 1991, 1993a). Although Norman (1991, 1993a) recognised the interaction between people and things, the interactivity per se, was not explored. This chapter will discuss interactivity with artefacts as facilitating emergent thinking in the distributed cognitive system, illustrating the importance of investigating interactivity between the individual and physical artefacts that configures a cognitive system.

Introduction

The theory of distributed cognition as part of the postcognitivist movement emerged toward the end of the last century, through empirical and ethnographic research (Sørensen, 2012). As a scientific enquiry into cognition, the distributed approach begins with the premise that cognitive activity is spread between and across the internal mind and the external world over space and time (Hollan, Hutchins, & Kirsh, 2000; Zhang & Patel, 2006). This distributed network functions as a cognitive system where the interaction between individuals, within groups, and with artefacts is guided and constrained by the physical, social and cultural context in which individuals are situated (Hutchins, 1995a; Zhang & Patel, 2006). Therefore, unlike traditional cognitive science, distributed cognition does not take the individual as the unit of analysis. Rather the unit of analysis is the distributed cognitive system as the cognitive properties of the larger system better reflect the entirety of the cognitive process than the properties of discrete elements taken in isolation (Hutchins, 1995b; Zhang & Patel, 2006).

Distributed cognition depicts thinking in terms of internal and external representations, where more recent interpretations of systemic cognition have adopted a nonrepresentational approach (e.g., Harvey, Gahrn-Andersen, & Steffensen, 2016; cf. Kirsh, 2009b, 2013; Zhang & Norman, 1994)[3]. This chapter will illustrate how changing the external representation influences the outcome of problem solving as the interaction with the physical structure facilitates changes to the internal representation, which in turn promotes novel thoughts and actions on the path to solution (Vallée-Tourangeau & Cowley, 2013; Zhang & Norman, 1994). In particular, the focus of this chapter will be on distributed cognition and the interactivity between individuals and artefacts, where interactivity enables the bidirectional flow of knowledge and information across the internal and external resources of the cognitive system (Zhang & Patel, 2006).

Distributed Cognition

Hutchins (1995a) and Norman (1993a) (see also Kirsh, 1995a; Saloman, 1997; Zhang, 1997) began developing a theoretical framework for a distributed approach to cognition through a series of extensive studies. These studies illustrated how the smooth operation of systems such as naval vessels, airplanes, and nuclear power stations require verbal and nonverbal

[3] The debate over representations as part of non-internalist theories of cognition is a somewhat contentious issue. For the purposes of this thesis it is considered a more philosophical debate and will not be addressed here (see Hutto & Myin, 2013 for radical embodied cognition; Malafouris, 2013 for an external representationalist approach in Material Engagement Theory; Harvey, Gahrn-Anderson and Steffensen, 2016, for a non-representationalist approach in researching interactivity; Thompson, 2007, for the enactive approach to representationalism). However, in keeping with work by Zhang and Norman (1994) and Kirsh (2009b, 2013) the approach adopted will be that representations play a role in cognitive processing whether distributed or not.

communication along with the synchronisation of activities between members of the team and the use of artefacts (Norman, 1993a). Hutchins (1995a) work in documenting the navigational skills of those on board the U.S. naval ship *Palau* is frequently attributed with expanding the framework in extensively demonstrating how cognitive processes may be distributed across social groups, across internal and external resources, and across time. While inclusive of the situated perspective, the distributed approach to cognition is receptive to the conventional approach of cognitive science using a computational basis for understanding human behaviour where cognition is viewed as a process of generation and transformation of representations (Hutchins, 2001; Perry, 2003). However, the distributed model of thinking goes further by proposing that cognition does not occur solely through representations of symbolic manipulations in the head, rather, cognition is shared across a system comprising of both internal processes and external resources (Hutchins, 2001; Saloman, 1997; Zhang & Norman, 1994).

This approach to cognition proposing the distribution of thinking across the internal mind and external world, offers an alternative and complementary perspective to some of the theories discussed in Chapters 2 and 3. Theories of situated, extended, distributed cognition, and embodied cognition advocate an ecological approach to cognition whereby cognitive processes span beyond the boundaries of the mind and body into the environment inhabited by the individual. As discussed in previous chapters the situated approach to cognition as introduced by researchers such as Lave (1988), Greeno (1989) and Hutchins (1995a) foregrounded the importance of context and setting in cognitive processes, becoming the foundation for many subsequent theories of ecological cognition. Within the distributed cognition framework, thinking is embodied and extended, however, this approach does not anticipate thinking as being initiated from a particular locus within the cognitive system—"Distributed cognition looks for cognitive processes, wherever they may occur, on the basis of functional relationships of elements that participate together in the process" (Hollan, Hutchins, & Kirsh, 2000, p. 175). Therefore, as with other non-internalist perspectives, the focus of attention is no longer on the individual, as cognition is not bounded by the manipulation of symbols within the head of the individual agent nor is the cognitive process confined to the boundaries of the skin and skull (Hutchins, 2001). The distributed cognitive system is dynamic and only constrained by the functional coupling of its constituent elements irrespective of location (Hollan, Hutchins, & Kirsh, 2000).

Distributed cognition is somewhat aligned with traditional cognitive science in a fundamentally representational approach to problem solving. The traditional approach to representational states is that all thinking takes place in the head with symbolic representations of the external world being manipulated by internal process of the mind. External representations are considered to be simply stimuli or inputs to these internal processes. A fundamental difference to this approach within the distributed cognition framework is that representations are not brain-bound. According to this account of cognition it is not necessary to construct an internal model of the world to facilitate action, as information from the external environment can be directly accessed by the cognizer acting and adapting as is appropriate within that particular situation and setting (Zhang, 1997). Not unlike the information-processing approach to problem solving, the theory of distributed cognition also conceptualises the problem space in terms of representational states, where the problem space consists of the initial state, a goal state and between these two states is the path to solution. However, the navigation of a problem space through the transformation of the problem between the transitory stages from the initial state to goal state is across internal processes and external representation of the problem, not exclusively through internal representations (Perry, 2003). Hollan, Hutchins and Kirsh (2000) discussed representations in general terms, however they did not specifically discuss distributed cognition in terms of internal or external representations. Kirsh (1995a, 2009b, 2013), on the other hand, discussed the distribution of cognition across external and internal representations empahising the role of external representations in augmenting cognition in making sense of the world.

By way of addressing some of the representational issues in the distributed cognition model, Zhang and Norman (1994) proposed a theory of distributed representations. They described three experiments using three puzzle isomorphs based on the Tower of Hanoi problem, although the three puzzles were formally the same they differed in terms of their physical presentations[4]. These experiments were particularly novel as participants were required to manipulate artefacts in solving the problem (although the focus of the analysis was not on interactivity per se). The participant was asked to imagine these were problems to be solved by a waiter or waitress in an unusual restaurant. One presentation of the problem used three different sized doughnuts and three pegs in a slight modification of the Tower of Hanoi puzzle; a second used three different sized oranges and three plates; and a third used three different sized coffee cups filled with real coffee and three plates (for details see Zhang & Norman, 1994). The coffee cup puzzle was a reverse

[4] Zhang and Norman (1994) generally referred to the problem presentations as external representations.

Tower of Hanoi problem with the cups initially stacked in size order, with the largest on top and the size of the plates were such that when more than one cup was on a plate it would have to be stacked on top of the other cups (see Figure. 4.1). This problem is of particular interest as it illustrates how some of the rules of the problem are coded in the physicality of the objects, affording some moves over others. The goal was to un-stack the cups onto the saucers, following specific rules so that the largest cup would be on the left, the midsized cup in the middle and the smallest on the right (see Figure 4.2). Essentially the first rule was only one cup could be moved at a time; the second rule was a cup could only be transferred to a saucer where it would be the largest; with the third rule being only the largest cup on the saucer can be transferred to another plate. The other two problems (i.e. one using doughnuts, the other oranges) were functionally the same as the coffee cup puzzle, with participants being shown the same three rules for each problem presentation, with slight adjustments to accommodate the appropriate artefacts. However, participants found the coffee cup problem to be the simplest to solve, next was the doughnut presentation with the oranges being the most difficult, producing more moves, more errors and more time required than the other two problems (Norman, 1993a). The disparities in performance could be explained by differing physical constraints offered by the properties of the artefacts as part of the problem's external representation. Essentially the participant did not want to spill the coffee so the second and third rules, although stated, were redundant as the coffee would be split if the small cup was placed inside the large cup and the saucers were not large enough to accommodate two cups side by side so there was no choice but to stack them. Therefore two of the three rules were built into the physical structure of the problem, this reduced the number of rules being maintained as part of the internal representation, making the problem easier to solve due to a reduction in the burden on internal resources such as working memory. As a result, this novel approach devised by Zhang and Norman illustrated how the physical artefacts were not just aids to cognition, the functionality embedded in the artefacts offered varying constraints on the movements in the problem space. This in turn changed the problem landscape facilitating different performances in solving three puzzles with the same three rules. While the rules for the problems were formally the same, they were distributed differently for all three problems across the internal and external representations of the problem. According to Zhang and Norman, the problem space was changed dependent on the combination of internal and external rules. This might be construed as encouraging a dichotomous approach to cognition which is not consistent with systemic theories. However, from a distributed representational perspective, the problem space is composed of an external problem space and an internal problem space, with these problems spaces co-joined to create an abstract problem space (Zhang & Norman, 1994). Zhang and Norman defined their notion of problem space, where an external problem space is made up of external rules; an internal problem space constructed by internal rules; and a mixed problem space of both internal and external rules (Zhang & Norman, 1994). On this basis, these three problems offered varying combinations of internal and external rules; thus creating different problem spaces. The coffee cup problem required the internalisation of fewer rules, as two of the three rules were implicit in the physical constraints of the artefacts as part of the external representation. This constrained the possible movements within the problem space providing an explanation for the differing results between the three problems.

*Figure 4.1***.** The initial state for the coffee cups problem (Zhang & Norman, 1994).

*Figure 4.*2. The goal state for the coffee cups problem (Zhang & Norman, 1994).

As Norman (1993a) also explained, in solving these three puzzles, despite being isomorphic, participants possibly did not perceive them as the same problem, rather as three different problems. This was similar to the point made by Lave (1988) when explaining that context and situatedness affect learning transfer from abstract knowledge acquired in the classroom to the mapping of this knowledge on to real world problem solving. The cognizer does not necessarily recognise problems as being similar when the problem presentation is different—this may be due to context, setting, or physical structure. In these experiments by Zhang and Norman the physical features—in particular the artefacts—offered affordances and constraints that assisted the reasoner, not only as an aid to memory and computationally, but in the perception of problem to be solved (Norman, 1993a).

Zhang and Norman (see also Norman, 1993a) also suggested that in analysing problem solving behaviour, there might be a third representation of the problem to consider. First the problem is represented internally in the mind of the solver, second the physical structure of the problem as an external representation, and third the problem is represented in the mind of the researcher. The third representation was an interesting observation from the perspective of the design of laboratory experiments. Norman (1993a) suggested that to the computer programmer or the research scientist all three problems were probably viewed as the same problem, where as suggested earlier, the problems might be considered as different to the solver. As these experiments revealed, while the formal structure of problems may be identical, changing the physical structure of a problem may alter the internal and external representation, in turn creating a new problem space affecting the approach to the problem by the solver. Essentially not only did the physical form of the artefacts influence the cognitive strategies of the solver, the tasks may have been perceived by the participant as quite different problems also impacting the ease with which the puzzles can be solved (Norman, 1993a; Vallée-Tourangeau & Krüsi Penney, 2005).

The distributed perspective shares another alliance with the traditional cognitive science approach in applying the notion of computationalism to cognition. Just as this distributed cognitive view conceives of the representation of a problem as not limited to the internal mind of the individual, so too are computations of the internal mind viewed as part of a larger computational system (Hutchins, 1995a). In discussing this computational account, Hutchins

(1995a) suggested that the same principles as those of the traditional information-processing approach to cognition could be applied to a larger system, with “computation realized through the creation, transformation, and propagation of representational states” (p. 51). The notion of systemic cognition as computation applies to computations such as maths inside the head of an individual; interactions between individuals; between people and groups; and people with artefacts. In *Cognition in the Wild* Hutchins (1995a) undertook an extensive examination into the navigational skills on board the US Naval ship *Palau* from a situated perspective. Similar to Lave, this was an exercise in studying cognition in context using ethnographic methodologies, however Hutchins was not investigating knowledge transfer or learning, as he was concerned with documenting the social interaction between people and the interaction of people with the physical world. Hutchins described in detail how the ship’s navigation system operated as a large nested computational system with the different components such as nautical charts created over time, the crew sharing experience and knowledge, the physical features of the vessel all acting and interacting in a broad dynamical process to steer the ship on its journey. As changes to the environment occur new information and interactions result between the crew, transforming the representational structure of the navigational plan. New elements were created, for example, in the form of pencil marks on the charts and logbooks as permanent records of the events (Hutchins, 1995a). Hutchins was not denying that some computations take place inside the head of the individual. However, those computations by one person would not suffice in the task of navigating the ship. Rather, it was the actions of the agent as one of the components of the distributed cognitive system across the material, cultural and social structure that effectively completed the task of steering the vessel safely into port (Hutchins, 1995a; Vallée-Tourangeau & Cowley, 2013). The coordinating of the properties of all elements, including the people and the artefacts, as part of the dynamic cognitive system, shaped and determined the successful pilotage of the *Palau.*

As Giere (2007) explained, a distributed cognitive system is 'cognitive' as the output if attributed to an individual would be considered as a cognitive product. This cognition can be distributed over a social setting with artefacts, as illustrated by Hutchins's (1995a) navigational system, or in the absence of others an individual with a pencil completing a Sudoku puzzle in *The Times* over breakfast. Kirsh (2006) believed that in scrutinising distributed cognition, the question is how the resources that comprise the elements of a dynamic, tightly coupled system are coordinated in order to achieve the required outcomes. This coordination is facilitated by interactivity as the individual makes sense of the world she inhabits (Kirsh, 2006). The unit of analysis is no longer the individual but the distributed cognitive system with the thinking distilled by the interaction between the elements of the system, rather than the distinct elements themselves driving the emergence of the thinking (Vallée-Tourangeau, 2013; Zhang & Patel, 2006). Therefore, interactivity is of particular interest in understanding a distributed system of cognition.

Interactivity

People are constantly interacting with their environment in a sense-making process, prodding the surrounding world to better understand it, to solve dilemmas, or perhaps to simplify a task (Kirsh, 2013). This interactivity with the world is a cycle of execution through actions on the world, and evaluation in comparing the outcome of actions to expectations and goals (Norman, 1991). It proceeds in a dynamic, looping back and forth flow within a tightly coupled system between the individual and the world; where the individual and world act and react to each other resulting in changes to both (Kirsh, 2013). Actions are frequently undertaken to make a task simpler, faster or reduce errors as the cognitive process migrates within the system to wherever the overall cost of cognition is lowest (Kirsh, 2013; Maglio, Matlock, Raphaely, Chernicky & Kirsh, 1999).

In Zhang and Norman's (1994) account of representational distributed cognition, while the role of artefacts in external representations was addressed, interactivity with the artefacts was largely ignored in their conclusions. A more reflective approach to interactivity from a distributed perspective was in word production tasks where participants were asked to produce words from a list of letters with and without the use of lettered tiles (Maglio et al., 1999; Vallée-Tourangeau & Wrightman, 2010). When undertaking these tasks, participants were unsurprisingly able to produce words generated by essentially internal processes; however overall, more words were produced when it was possible to manipulate the lettered tiles. In addition, the researchers found that the greater the task difficulty the more the solver engaged with the external environment by utilising the lettered tiles to create new words (Maglio et al., 1999; Vallée-Tourangeau & Wrightman, 2010). These results demonstrated that the components of the task environment played a pivotal role in the possibilities available to enable the reasoner on the path to solution (Kirsh, 1995a). By manipulating lettered tiles in a game such as Scrabble, a player will generally move the tiles to create the optimum letter combination (or any letter combination in the first instance). The player may push the letter 'q' next to a letter 'u' and then move vowels into the next position looking for a prompt for a suitable letter combination thus computing with the tiles what is the next best action for a suitable word output. This promotes a back and forward interaction within the distributed cognitive system between the individual's internal cognitive processes and the remaining tiles, by moving the tiles and through mental computations. Just as the external components impact the outcome reached by the solver, so do the internal capabilities of this solver, such as memory recall for learnt vocabulary in this case (Vallée-Tourangeau & Wrightman, 2010). Kirsh (1996) explained how introducing a tool or a physical artefact to a task environment changed the agent's "action repertoire" (p.438), for it was now possible to achieve outcomes that may not have been previously attainable without the additional resource integrated as part of the cognitive system. In rearranging the tiles to search for words, the external actions of the task complemented the internal resources in computing the final words resulting in a "complementary action or strategy" (Kirsh, 1996, p.443). In problem solving a complementary action occurs as part of a restructuring strategy. This complementary action interleaves physical and mental actions to restructure the environment or recruit external resources, improving the efficiency of cognitive processes in such a way that would not have been possible with mental or physical actions alone (Kirsh, 1995b, 1996). In the interactive process of searching for words, the physical actions in moving the tiles provided the opportunity for the cognizer to take advantage of the cost benefits to the overall cognitive process offered by the external environment, making the task simpler, faster, or more accurate.

Lave's (1988) study on adult maths in the lived-in world, also presented a number of examples illustrating interactive behaviour, as shoppers moved through the supermarket environment when making purchasing decisions, and dieters devised ingenious ways of measuring food in the kitchen. These interactions were observed through the relationship between ongoing activities of persons acting in a setting, with problems emerging and strategies for solution evolving through a self-generative process enacted through the setting. Lave explored the notion that the setting of the supermarket was not merely a mental map in the shopper's mind; it provided synergy between the experience and expectations of the shopper with the organisation of the commodities within the shopping arena. This synergy was evident in an example mentioned in Chapter 2 of this thesis, where the activity of a shopper performing a routine purchase of noodles is altered by actions and the packaging display in the supermarket. Although the shopper entered the supermarket prepared to purchase a particular brand of noodles, as she believed it to be the best value, the interaction in the supermarket prompted her to ultimately change her mind. The shopper approached the noodle display unable to remember which item she purchased previously, it was only upon scanning the packages that she recognised the exact product. She took the package from the shelf placing it in the trolley explaining to the researcher that she usually looked at different brands in the supermarket for items on sale, but this was the usual package purchased as this was generally offered the best value. By way of demonstrating to the researcher that she had made a shrewd decision the shopper then compared the intended purchase with some of the other noodles on display, calculating unit prices (it was unclear from Lave's account whether the shopper touched the packages as throughout the book, descriptions of the shoppers and dieters interactions with artefacts was not detailed by Lave). The shopper noted by physical observation of the packages on display that there were discrepancies in her assumptions about the weight of certain packages. This in turn required recalculation of the unit price, resulting in the resolution by the shopper that the noodles in her trolley were not the best value after all. Therefore, as a consequence of these actions and interactions, the shopper purchased a different packet of noodles than originally intended. The shopper's purchasing dilemma was both created, and resolved by the looping back and forth interaction between the individual's mental processes, the physical grocery display, and the researcher as part of a dynamic cognitive system. Lave's analysis of this shopping activity illustrated the transformation of the problem through the changing maths strategies offered by the setting. While acknowledging the interaction between the researcher and the shopper, Lave was not convinced that this interaction resulted in a changes to the decisions made by the shopper. Lave was of the opinion that the shopper assumed that the role of the researcher was to be more of an arbiter of normative maths practice. Therefore, the dialogue showcased the ongoing nature of the decision making activity rather than the interaction between the shopper and the researcher making a substantive difference to the unfolding decision making process of the shopper. Lave did briefly acknowledged how the items on the shelf may be relevant aspects in a supermarket setting for the initiation of the purchasing decision making process, although this line of explanation was not pursued. Thus, although Lave identified the synergies of the setting as impacting the decision of the shopper, interactivity between the researcher, artefacts, and the shopper or dieter were not specifically identified as part of the shopper's sense-making process.

Problem solving is situated, contextual and proceeds interactively in the world in which an individual is embedded. In problem solving an agent may not be able to mentally simulate the transformation of the problem to the goal state, however through interaction with the material world, problems and solutions frequently unfold with thinking distilled by interactivity rather than dictated by a plan (Vallée-Tourangeau, 2013). Actions and the manipulation of artefacts provide new information, unveil new affordances, and configure a more cognitively congenial problem presentation (Guthrie, Vallée-Tourangeau, Vallée-Tourangeau, & Howard, 2013; Kirsh, 1996). Interactivity, as revealed in activities such as playing Scrabble or shopping at the supermarket, once identified, is manifestly evident as coordinating the sense-making process in almost every human experience and activity (Kirsh, 2009b; Maglio et al., 1999; Vallée-Tourangeau & Wrightman, 2010; Steffensen, 2013). However, interactivity is invisible, often going unnoticed; this invisibility makes it challenging to test or isolate for analysis, as these interactions between internal and external resources are non-linear and looping in nature, it is difficult to separate the inner and outer processes (De Jesus, 2015; Norman, 1993a). The belief or hope of the information-processing approach to cognition was not to characterize the agent and environment relationship as transactional, rather to cast the agent as separate from the environment. Folk notions of intelligence and expertise are frequently modeled on this internalist perspective of human intelligence, perceiving of thinking as a process that takes place entirely in the head. This is illustrated in the popular television quiz programme Mastermind (https://en.wikipedia.org/wiki/Mastermind_(TV_series), where a contestant answers an impressive array of challenging specialist and general knowledge questions; the name itself fulfills the folk notion of intelligence as the individual is proclaimed to be a master of the mind. The contestant exhibits expert knowledge drawing on impressive internal mental capacities and capabilities including long-term memory storage, recall, and working memory in a hands-down, low interactive situation. In terms of folk psychology this may be sufficient evidence to label the contestant as an intelligent thinker. However, this success at answering a series of challenging quiz questions is not necessarily an indication that the contestant's knowledge is transferable to successfully thinking in or with the world. From a situated, extended mind, or distributed perspective this is not a complete picture of the cognitive system. Indeed, Greeno (1989) and Lave's (1988) accounts of knowledge transfer would suggest that learning in one domain does not necessarily readily transfer to another, as situatedness impacts the thinking process with the application of acquired knowledge is not necessarily being stable across contexts and settings. The performance in solving best-buy maths problems in the supermarket of the shoppers in Lave's (1988) Adult Math Project was generally far superior to when the same participants were asked to employ similar mathematical skills in a school-like maths tests at home. Returning to the example of the case of the Mastermind contestants, this is not to say that a Mastermind contestant might not perform equally well in situations away from the question and answer setting on of a television quiz show. However, working memory capacity, for example, may predict performance in one task ecology, but as the cognitive landscape changes so might the impact of working memory capacity as a predictor of performance. It may be tempting as cognitive psychologists to profile the cognitive abilities of the agent separately from the environment as this entails a far less complex explanation, however, this does not tell the entire story of the agent acting in the lived-in world. An impressive performance on the quiz programme Mastermind does not capture the cognitive system in its entirety, just as merely profiling the internal processes of an individual cannot accurately predict cognitive performance across all situations; this is to be blinded to the connectivity between the agent and the environment.

In attempting to provide a complete account of the human cognitive system, it is not possible to define one of the constituent components of the cognitive system without reference to the other. The dynamic system of thinking is constituted though bodily actions with artefacts, indicating a causal dependency between the body and material artefacts when people are thinking and acting in the world (Malafouris, 2013). The interactions and the physical association between the body and artefacts should be treated as important forms of thinking, not simply taken as being indicators of some unobservable internal mental processes; the cognitive processes happen in part through mental processes in the brain, however much of the thinking occurs as a result of interaction with the world (Malafouris, 2013). It follows then, that the ontology of thinking is relational: the agent—mind and body—and the environment are inextricably connected, with thinking emerging from the dynamic interactive processes occurring between them. Therefore, by separating agent from action in the world, as an internalist perspective would maintain, is to present a disconnected, disembodied, and static depiction of cognition. Thinking as an emergent property of interactivity between the individual and the world is not necessarily explained by profiling of the individual's mental capabilities alone. Malafouris (2013) proposed the human thinking process could be portrayed as a "*hylonoetic field*" (p. 226) where an ontologically of thinking proceeds "*through* and *with* matter" (p. 236). However, this depiction of the cognitive processes enacted between and within the constituent components of the distributed cognitive system, does not necessarily reflect the action that drives the emergence of thinking. The essential coupling of components through interactivity as the cognitive process unfolds suggests a *kinesionoetic* (from the

Greek *kinima* for movement, and *nous* for mind) ontology of cognition, where thinking emerges through action.[5]

Observations of interactivity with artefacts, such as gestures and with other people are possible, however the understanding of the dynamics of human interactivity as part of the distributed cognitive system are complex and an explanatory model of interactivity is, so far, elusive (Steffensen, 2013). Further obscuring the investigation of interactivity is the heteroscalar nature of environment-agent coupling (Harvey, Gahrn-Andersen, & Steffensen, 2016). Harvey et al. (2016) explained that the events, processes and patterns relevant to an individual carrying out one action over another might happen almost simultaneously but also exist on multiple timescales. A problem may be solved by drawing on experience from the past, the information available in the present context, and with an eye to the consequences for the future. This was illustrated by the actions of the navigation team on board the *Palau*, in Hutchins's (1995a) study of the ship's navigation system, as they plotted a safe course to the ship's final destination. They referred to previous professional experience, maps and charts produced through prior journeys, the prevailing weather conditions at the time of sailing, and projected ahead to possible situations that could affect weighing anchor. These events were also happening in timescales of varying magnitude; some segments of the problem-solving process might unfold in pico-scales of action while others were stretched across large-scale actions. The interaction may be affected or as a result of prior experiences or the interaction may be so remote from the output that resulting feedback from the world is delayed in time (Norman, 1991). Norman (1991) also pointed out that

[5] An original term developed through discussions with Professor Fred Vallée-Tourangeau and Dr. Elena Polycarpou.

interaction could be direct, like using a pen and paper, or indirect by asking someone else to write down thoughts on our behalf. Therefore interactivity brings together these resources across time and space. Without interactivity as the co-joining link, the coupling between the resources would not be possible wherever located in the system.

The Role of Artefacts in Cognition

Two of the distinctive characteristics of human beings as a species are the capability to first, create artefacts through which modifications to the surrounding environment are made, and second, to transmit this accumulation of modifications to future generations (Norman, 1991). As Norman (1991, 1993a) pointed out the world is saturated with artefacts that contribute to overcoming the many limitations of the body, the mind and constraints within the environment (see also Malafouris, 2013) Clothing, heating and housing enable communities to flourish in areas that would otherwise impose impossible climatic conditions for human habitation; cars, trains, planes allow travel faster than the unaided human body could accomplish; technologies help to make us smart with massive on-line databases that provide access to details in seconds (Norman, 1993a). Despite the importance and prevalence of artefacts in daily lives, the impact of artefacts on cognition has been largely ignored in the exploration of the human mind. When investigating, for example, memory, perception, language, and attention, the mind is generally studied within mainstream psychology unaided by or in isolation of external resources, and the use of artefacts frequently goes unacknowledged in analysis (Norman, 1993a; Sutton, 2010; Vallée-Tourangeau, 2014).

Norman (1991) coined the term “cognitive artefacts” (p. 17) to describe artificial, man-made or human-modified objects serving as aids to cognition, performing functionally as part of an external representation (Perry, 2003). Hutchins (1999) later described cognitive artefacts in broader terms as, “physical objects made by humans for the purpose of aiding, enhancing, or improving cognition” (p.126). Cognitive artefacts operate as more than memory aids that amplify existing human abilities, they transform the task at hand by offering the potential for reallocation of resources across the problem space to make the best use of the resources available to the reasoner—both internal and external (Norman, 1991, 1993a; Perry, 2003). The extended mind theory developed by Clark and Chalmers (1998) offers a similar view, where a nonneural resource must play a functionally active role to be considered as an integrated component of an extended cognitive system—not merely a resource used by the agent while carrying out a task (Wilson & Clark, 2009). Artefacts provide the prospect for cognition to be distributed in a number of ways: across time, setting, people, and divided across other artefacts as the opportunities for interactivity expand with the unfolding of the problem and scaffolding of solutions. In addition artefacts can transform the approach an individual takes to acting on the task by providing structure (Baber, 2003; Norman, 1993a; Zhang & Norman, 1994). For example, a shopping list is completed prior to the activity of shopping—perhaps written in the kitchen, enacted in the supermarket, it can be added to by other people, and the supermarket displays can prompt the purchase of other items in the store (Baber, 2003). The structure of the shopping list can, in turn, determine the path taken when purchasing the groceries in the supermarket. A cognitive artefact functions as part of the distributed cognitive system changing the nature of the task and the way an individual performs a task.

The agent adaptively structures and re-structures the environment in which she is embedded organising and re-organising artefacts and the physical world as part of the processing of information when undertaking a task or activity (Perry, 2003). Of course, for the artefact to function as part of the cognitive system it must have meaning-in-context for the user, this can be illustrated by the hypothetical Otto's notebook as described by Clark and Chalmers (1998). The notebook would only enhance Otto's cognitive abilities if it contained information required for the task at hand, if Otto needed to find his way to the museum but the notebook did not hold the relevant information then it would not be a constituent part of the cognitive process (Perry, 2003). Norman (1993a) considered that, in general, artefacts change the tasks people do, artefacts do not facilitate changes to cognitive abilities. Norman's point of view may be debatable as artefacts have potentially been shown to alter cognitive abilities and changes to neural substrates in activities such as learning mathematics and reading (Dehaene, Piazza, Pinel, & Cohen, 2003; Malafouris, 2013; Menary, 2007, 2015). Regardless of this debate, for researchers and theorists of a systemic view of cognition the overall performance of a task is not determined solely by the cognitive abilities of the individual or the properties of the artefact (Perry, 2003). According to Norman (1991,1993a) there are two representational functions for cognitive artefacts, one is from the personal viewpoint, and the second is the system viewpoint. From the personal viewpoint, that of the user of an artefact, the artefact has altered the way the task is performed or may offer a new set of tasks. From the system point of view the sum of the individual and the artefact is smarter than either component alone. Every artefact has a personal and system point of view, therefore the performance of the task for the individual can be altered and the wider cognitive system enhanced by the representational function of the artefact (Norman, 1991).

Norman (1991, 1993a) suggested artefacts might play a part in enhancing *performance* to make people smart, however in his opinion they do not enhance or amplify the *abilities* of the individual. From the perspective Norman suggested the example of a pair of spectacles, which could be considered an artefact for amplifying the inadequate capacities of vision for an individual, but not the abilities. However, a more accurate explanation of the functionally active role of spectacles in a cognitive system is as an object that redefines the information available to the wearer enabling a physical change to the external representation of an activity and a restructure of the task that would not have been possible to the individual without the spectacles (Baber, 2003). A tool, such as a hammer, is an artefact that is manipulated by a user, to bring about changes to some aspect of the surrounding environment (Baber, 2003). Although it may not have been designed to make people smarter, it also plays a cognitive role as it extends the existing capabilities of the user in reaching the required goal. In joining two pieces of wood together with a nail, the tool—whether it is a hammer or perhaps the heel of a shoe—is an active component of the cognitive system integral in reaching the final goal. The user's existing knowledge of working with a hammer is complemented by the action of the hammer, which in turn offers sensory feedback. The looping feedback between the user and the direct interaction with the hammer, and the subsequent indirect interaction with the nail, provides an opportunity for the user to perform and adjust actions with the hammer, such as pressure, until the nail is in place. Hutchins (1999) considered a piece of string tied around the finger as a reminder to complete a task, sufficient to be classified as a cognitive artefact, explaining that material cognitive artefacts are only of use when coordinated with the knowledge of how to use the object. Artefacts that are allocated an active cognitive function within a task should be considered an integral component of a cognitive system (Baber, 2003). Therefore artefacts emerge as having a cognitive role through interactivity in a distributed cognitive system, making humans smarter (Norman, 1991, 1993a).

Conclusion

Distributed cognition frames problem solving in terms of representations and processes that are not bounded by the brain (Perry, 2003). Cognitive processes are dynamically distributed, stretched, and constrained across the internal resources, body, and external representations of the world as perceived by the individual. The operations of thinking shift bidirectionally across and within this tightly coupled system of internal and external resources, migrating to wherever resources are most efficiently utilised (Fu, 2011; Gray, Sims, Fu, & Schoelles, 2006; Kirsh, 2013; Vallée-Tourangeau & Cowley, 2013). The execution of the task may not result in improved precision or accuracy, however the cost benefit evaluation undergone during the process is transacted within the capabilities of the individual given the resources available at the time. The resources are engendered with constraints and boundaries, along with opportunities for manipulation to maximise potential affordances within a world that is shaped by the experiences of the individual. When the external representation offers a greater opportunity for interactivity, the capabilities of the reasoner are frequently enhanced and transformed. This may encourage not only a quantitative change in performance in problem solving, but also a qualitative difference in the trajectories enacted in reaching a solution. The interaction between the artefact and the individual brings forth the potentiality of the affordances offered by the artefact as part of the external physical structure (Vallée-Tourangeau & Cowley, 2013). This interaction may be in the form of perception and action or it may be using the object as a referent to make sense of a situation (Kirsh, 2009b). Through this interaction, the material agency of the artefact emerges as a property of the tight coupling of the cognizer with the artefact that is deemed an integral component of the extended and distributed cognitive process (Malafouris, 2013). Thus, interactivity with artefacts can enact paths to solution that might have otherwise been elusive to the cognizer (Kirsh, 2013; Vallée-Tourangeau & Cowley, 2013).

When acting in the world individuals use artefacts as mediators with the environment, to execute actions resulting in changes to the world, and to perceive the state of the world through detection and interpretation of changes to the surrounding environment (Norman, 1991). Different artefacts, with the opportunities for action, constraints, and affordances offered, generate different representations, in turn affecting subsequent interactions (Norman, 1991). When individuals are presented with a task environment where it is possible to manipulate objects to solve the problem, the actions in the physical world result in interactivity as an interleaving process between internal and external resources—artefacts and other people—within the constraints bounded by the world and the body (Norman, 1993a). The intelligent actions of humans require a vast amount of knowledge, memory storage, and retrieval capabilities; executive functions such as planning, decision making, and problem solving are hugely complex and demanding on internal resources (Norman, 1993a). When there is tight coupling between the internal resources and the physical environment, the structure offered by this distribution of cognition assumes some of the memory and computational load, easing the internal cognitive burden (Norman, 1993a). As was evidenced by the work of Zhang and Norman (1994) and others (e.g., Baber, 2003; Guthrie & Vallée-Tourangeau, 2015; Kirsh, 1995a; Kirsh & Maglio, 1994; Maglio et al. 2003; Vallée-Tourangeau, 2013), it is not only identifying the artefacts utilised by the cognizer that is imperative to distributed cognition as a framework for analysis in problem solving, but how the artefacts are used and the changes made to the physical representation of the problem as the path to solution unfolds through interactivity (Perry, 2003). The combination of the informational processing capacities of the artefact with those of the user creates a system that expands, potentially enhances, and transforms the capabilities of the components within the system. However, without interactivity to coordinate this combination of individual and artefact, the artefact does take on the role of an integral component of the distributed cognitive system (Norman, 1991). The connectivity and coordination of interactivity is the glue that bonds the distributed cognitive system.

As people interact with the world, dilemmas or problems arise; solutions unfold though actions with the world in which the person is embedded. In solving a problem the person may not be able to simulate the path to solution mentally—as a traditional cognitivist interpretation presupposes—the emerging solution is a product of thinking in the world that is distilled through interactivity. This looping back and forth process between the internal mind and the external world encourages actions and the manipulation of artefacts, which in turn provides new information and unveils new affordances to configure a more cognitively congenial problem presentation. Therefore, artefacts are not just memory aids but provide opportunities to organise and re-organise the distributed cognitive system to make best use of internal or external processes.

In this chapter and in previous chapters, a number of non-internalist approaches to cognition have been discussed. Irrespective of which approach is followed, it is evident that interactivity is central to the systemic cognitive process. Naturally occurring interactivity often goes unnoticed as folk engage in daily life. Not only is this interactivity invisible to those in the everyday world, researchers and commentators on research often fail to seek and analyse it. This thesis attempts to bring some of this naturally occurring interactivity into the lab by investigating problem solving with artefacts.

Chapter 5

Individual Differences

Overview

Measuring individual differences offers a window onto explaining disparities in performance when people carry out the same task. In the experiments described in this thesis, the differences in cognitive abilities and affect of individuals is of interest as the level of interactivity available to people when undertaking a task may alter the impact of any individual differences on performance. This chapter outlines the various individual difference measures, including a measure of affect, across the five experiments reported in this thesis. Some modifications were made to the original tasks to better accommodate requirements of the experimental design, and not all measurements were part of the suite of tasks for every experiment. The details of modifications and tasks used will be included in the reporting of each experiment in the chapters to follow.

Introduction

Individual differences may moderate performance in thinking and reasoning tasks. Stanovich and West (1998) proposed that deviations from normative responses on reasoning and thinking tasks might not be solely accounted for by performance errors, such as momentary lapses in attention or memory. Individual differences also offer an explanation for the departure in behaviour from normative models. Within the framework of individual differences, they distinguished between cognitive capacities (e.g., working memory) and thinking dispositions (e.g., willingness to switch perspectives or maths anxiety). Cognitive capacities are considered more effected by long-term practice than instruction, whereas thinking dispositions or cognitive styles being related more to belief formation and decision making, are more malleable therefore potentially teachable (Baron, 1985). The results from four studies led Stanovich and West to argue that while cognitive abilities explained performance differences to some extent, after partialling out cognitive abilities, thinking dispositions continued to be a strong predictor of individual differences on reasoning tasks. They support suggestions by Baron (1985) that when an individual is unable to compute the normative model, it is usual to engage in cognitive strategies that approximate the normative model as closely as possible. Stanovich and West discuss the limitations of an individual's cognitive abilities, including a cursory acknowledgment of environmental and situational constraints that may also contribute to disparities in an individual's performance. However, they fail to pursue the potential influences of situatedness and environment on performance in their experiments. Therefore, this invites investigation into the elements of cognitive capacity and cognitive style, alongside situational and environmental settings as factors affecting fundamental cognitive abilities, by modifying the context within which thinking and reasoning tasks are presented to the participants.

Working memory, planning and the need for diligent thinking may impact a success result in problem solving activities. Factors such as anxiety, self-efficacy and expertise in mathematics have been shown to contribute to differing levels of arithmetic performance (Butterworth, 2006; Hembree, 1990; Hoffman, 2010; Moore, Rudig, & Ashcraft, 2015). In addition, a person's engagement when undertaking a task may also be relevant to problem solving performance; with a positive approach toward the task contributing favourably to the problem solving activity, possibly resulting in deeper comprehension

(Newmann, Wehlage, & Lamborn, 1992; Schiefele & Csikszentmihalyi, 1995; Storbeck & Clore, 2007).

Working Memory

Working memory is generally considered a fundamental theoretical construct, offering an explanation for the temporary memory processes required for many cognitive activities such as language, reading comprehension, mathematical calculations, decision making, and reasoning (Unsworth, Redick, Heitz, Broadway, & Engle, 2009). Working memory differs from other memory systems as it is responsible not only for storage, but also the simultaneous storage and processing of information, albeit limited in capacity (Salthouse & Babock, 1991). Although theories differ in the specifications of working memory, it is generally agreed that it consists of multiple subsystems working in unison to activate task-related information, and inhibit task-irrelevant information during cognitive tasks (Miyake & Shah, 1999; Yuan, Steedle, Shavelson, Alonzo, & Oprezzo, 2006). Baddeley and Hitch (1974) proposed an early multi-component model of working memory consisting of an overarching central executive system controlling two temporary memory systems. One of the systems, the phonological loop, processes verbally coded information; the second system, the visuo-spatial sketchpad, processes visual and spatially coded information (Baddeley & Hitch, 1974). A later modification to the three-component model of working memory was the addition of a fourth component, the episodic buffer (Baddeley, 2000). This fourth system, assumed to be controlled by the central executive, provides a temporary interface between long-term memory and the two slave systems, that is, the phonological loop and the visuo-spatial sketchpad (Baddeley, 2000). As with all components of working memory, the episodic buffer is conceived of as a temporary processing and storage system of limited capacity. Investigations into working memory have shown repeatedly that these limitations vary between individuals (Daily, Lovett, & Reder, 2001; Just & Carpenter, 1992). Individual differences in working memory limitations are tested using simple and complex span tasks. Simple span tasks assess the storage capabilities of an individuals working memory, where complex span tasks assess working memory capacity by measuring both storage and processing (e.g., Daneman & Carpenter, 1980; Turner & Engle, 1989).

Simple span tasks. Digit Span and Corsi Block tasks are examples of tasks that measure the storage aspect of an individual's working memory. These are often called simple span tasks where the individual being tested repeats a sequence of items in order of presentation (Redick et al., 2012). The Corsi block-tapping task has been used extensively as a nonverbal task in testing the capacity of the visuo-spatial sketchpad or as it is commonly termed, the visuo-spatial working memory (Berch, Krikorian, & Huha, 1998; Logie, 1995). The original task, as designed by Corsi (1972), was very simple: It comprised of a series of nine blocks irregularly arranged on a board; the researcher taps out a randomised sequence on the blocks, with the sequence increasing in length over the duration of the experiment. The participant is required to tap out the same sequence immediately after the researcher. This continues until the participant no longer produces as accurate replication of the sequence (Berch et al., 1998).

Complex span tasks. In complex span tasks the storage aspect of simple span tasks is interleaved with a processing task, such as reading a sentence or carrying out simple sums in order to assess working memory capacity (Daneman & Carpenter, 1980). Daneman and Carpenter (1980) developed a reading span task as a reliable tool to measure working memory capacity. In so doing they showed that complex span tasks measure working memory as a dynamic system of both processing and storage. Subsequently, other complex span tests were developed including computation span tasks where the participant is required to solve a series of arithmetic problems while remembering the last digit of each problem (e.g., Salthouse & Babock, 1991). In some tasks the participant is expected to answer the problem, in others the correct answer is selected from a choice of three. In either case, following the completion of a series of problems the participant must write a list of the last digit of each the problem (Salthouse & Babock, 1991).

Planning

The ability to effectively plan ahead has the potential to impact the performance of an individual in successfully solving a problem (Phillips, Wynn, McPherson, & Gilhooly, 2001). A widely used measure of planning ability in the clinical and nonclinical population is the three-disc and five-disc versions of the Tower of London (ToL) task (Phillips et al., 2001; Ward & Allport, 1997). In the original three-disc version of the puzzle as devised by Shallice (1982), three beads, one red, one green, and one blue, were positioned in an initial position on three vertical wooden rods of different lengths. The participants were asked to move the beads to a goal position in a minimum number of moves. The initial state was the same for all twelve problems in the experiment, however, the goal states for the problems were changed so as to increase the minimum number of moves required, thereby increasing the difficulty with each of the three groups of four problems. In reaching the goal state the easiest four problems needed at least two or three moves, the midrange problems required 4 moves and the most difficult was a minimum of five moves. As a measure of planning ability, the interpretation is that the closer a participant’s performance is to the minimum number of moves in reaching the goal state the greater the planning abilities (Phillips, Wynn, Gilhooly, Della Sala, & Logie, 1999).

Need for Cognition

Cohen, Stotland, & Wolfe (1955) identified the tendency by an individual to enjoy and engage in effortful thinking as the need for cognition. Cacioppo, Petty, Feinstein, & Jarvis (1996) suggested such a disposition by an individual in thinking diligence had the potential to impact performance in cognitively challenging activities, such as problem solving, decision making and reasoning. This led to Cacioppo & Petty (1982) developing a Need for Cognition scale (nCog). The fundamental difference between those high and low in the need for cognition, is those with a high need for cognition intrinsically experience greater satisfaction when engaging in effortful cognitive tasks than those with low need for cognition. As a consequence an individual with a high score on the nCog scale might be expected outperform someone with a lower score.

Maths Anxiety

The strain on working memory during mental arithmetic may be exacerbated when the individual experiences maths anxiety as this anxiety utilises cognitive resources that would otherwise be directed at the problem (Ashcraft & Kirk, 2001; Ashcraft & Ridley, 2005). Maths anxiety is typically associated with feelings of tension, uneasiness, confusion, and fear when faced with solving maths problems either in the classroom, workplace, or daily life (Ashcraft & Moore, 2009; Richardson & Suinn, 1975). Maths-anxious individuals have repeatedly been shown to perform less well in maths than their less anxious counterparts (Hembree, 1990; Lyons & Beilock, 2011; Ma, 1999). Ma (1999) conjectures that those exposed extensively to mathematics may have greater control over their anxiety, even suggesting that these feelings of anxiety may be channeled to an improved level of performance. In a study investigating maths anxiety and interactivity Vallée-Tourangeau, Sirota, and Villejoubert (2013) found that maths anxiety was highly correlated with calculation error in a low interactivity condition where participants could not modify the problem presentation nor use their hands to point at numbers. However, in a high interactivity condition where participants were able to shape and reshape the problem presentation, maths anxiety was no longer a predictor of calculation error. They argued that in the higher interactivity condition, a dynamic problem presentation wrought through action transforms working memory capacity, not only in terms of storage but also executive function skills, mitigating the impact of performance anxiety.

Maths Self-efficacy

In addition to maths anxiety, maths self-efficacy may also impede mathematical cognition (e.g., Betz & Hackett, 1983; Hoffman, 2010; Pajares & Kranzler, 1995; Parker, Marsh, Ciarrochi, Marshall, & Abduljabbar, 2014). The construct of maths self-efficacy is derived from Bandura's (1977) theory on self-efficacy expectations where one's own self-belief in the ability to successfully achieve an outcome in a particular task may influence behaviours (Bandura, 1977). Bandura proposed that efficacy expectations might mediate behaviours influencing outcome expectations, which in turn mediates the final outcome for the task. This may, for example, result in avoidance of a task or situation if there is a lack of belief in one's own ability to cope with that task or situation. Alternatively, strong belief in one's own capabilities may instill confidence to attempt a task that may otherwise be considered intimidating (Bandura, 1977). Bandura suggested that self-efficacy may be a learned behaviour and any interventions aimed at increasing self-efficacy expectations should be domain specific. Based on Bandura's self-efficacy theory and their own research into the relationship between self-efficacy and career related behaviours, Betz and Hackett (1983) developed a self-report scale to assess maths-related self-efficacy (Hackett & Betz, 1981). There is strong evidence that maths self-efficacy is a predictor of mathematical achievement, underscoring the importance of addressing maths self-efficacy in educational settings (Hoffman, 2010; Pajares & Kranzler, 1995; Parker et al., 2014). The relationship between self-efficacy and levels of anxiety has been shown to co-vary inversely, thus suggesting that any intervention successfully targeting a reduction in anxiety should also result in an increase in self-efficacy (Betz & Hackett, 1983; Hoffman, 2010; Jain & Dowson, 2009).

Expertise

Expertise in a particular domain is often attributed to innate aptitudes (Ericsson & Charness, 1994). Galton (1892) proposed that "intellectual powers" (p. 16), along with the enthusiasm for hard work were inherited gifts with these innate abilities almost certainly guaranteeing eminence. In response to his cousin's opinion, Darwin maintained, "men did not differ much in intellect, only zeal and hard work; I still think this is an *eminently* important difference" (Galton, 1908, p. 290). Ensuing research and theories have indicated that high levels of performance and expertise are mediated by ongoing acquisition and consolidation of skills (Ericsson & Charness, 1994; Sternberg, 1999). In the case of mathematical expertise, a number of factors have been identified as contributors to exceptional performance including deliberate practice, intrinsic reward in the success of solving a problem and working memory (Butterworth, 2006; Ericsson & Charness, 1994).

Working memory capacity may reflect the maths proficiency of an individual as the ability to, for example, retain interim totals for additions while reaching a solution may be enhanced or impeded by the limitations of working memory. Highly skilled individuals appear to have the capacity to manipulate greater amounts of information when performing tasks within the domain of their expertise than novices (Kirsh, 1995a). To account for this apparent larger working memory capacity and the rapid retrieval of skillful knowledge stored in long term memory (LTM) by experts, Ericsson and Kintsch (1995) argued for an extension to existing models of temporary working memory storage. This extension would encompass an additional mechanism in the hierarchy of retrieval of information within long term memory, namely long term working memory (LT-WM). LT-WM would offer a more stable storage component than that of short term working memory (ST-WM), which is generally acknowledged to hold around seven chunks of information for only a few seconds (Ericsson & Kintsch, 1995; Miller, 1956). A specific node of encoded information in LT-WM is subsequently associated with a retrieval cue held in ST-WM that can be called up when required. In short, the larger chunk of information is stored in LT-WM for rapid retrieval by a cue in ST-WM. This offers one account for an expert's apparent increased working memory capacity when performing tasks such as mental arithmetic.

Interactivity in problem solving has been attributed with diminishing the load on working memory as some of the limited internal memory storage is unburdened onto the external world (Kirsh, 1995a; Vallée-Tourangeau, 2013). Furthermore, other executive functions and strategy selection may benefit from the dynamic problem configuration enacted through interactivity. Experts and novices have been shown to devise shortcuts and procedures drawn from their interaction with the world in order to reduce the load on working memory (Butterworth, 2006; Kirsh, 1995a).

Affect: Task Engagement and Flow

The concept of flow was proposed by Csikszentmihalyi (1975/2000) to describe a state of consciousness when an individual is totally absorbed in an activity. Csikszentmihalyi (1990) considered flow to be the “optimal experience” (p. 3) in reaching a total sense of exhilaration and deep enjoyment when engaged in a task. According to Csikszentmihalyi’s model, to achieve flow there must be a positive balance between the challenge of the activity and the skill level of the individual. The challenge or opportunity for action in a task should be neither too easy nor too demanding. At the same time, the individual’s skills, or capabilities for action should not exceed the action opportunities, thus avoiding boredom and preventing anxiety if skills are perceived to be inadequate for the challenges of the task. When a person reaches the optimum point where the action opportunities in an activity match their capabilities it is possible to experience flow (Csikszentmihalyi, 1975/2000). In addition, the attention of the individual or “attentional involvement” (Abuhamdeh & Csikszentmihalyi, 2012, p. 257) must be focused on the activity at hand resulting in total absorption. This attentional involvement in the activity has been shown to be a potential mediator in reaching the optimal state of engagement of being in flow (Abuhamdeh & Csikszentmihalyi, 2012; Csikszentmihalyi, 1975/2000). Csikszentmihalyi (1990; 1975/2000) suggested that attentional involvement might also channel attention away from self toward the task at hand simultaneously increasing the salience of some objects used in the task while decreasing the salience of other aspects. This redirection of attention away from self may increase the value of the experience by devoting attentional resources toward satisfactory aspects of the task, or by positively offsetting any sense of negativity toward more unfavorable aspects of the activity (Abuhamdeh & Csikszentmihalyi, 2012). According to Nakamura and Csikszentmihalyi (2002) a fundamental aspect of the flow model is "interactionism" (p. 90), where the focus is not on the person enacting the task being detached from context, rather the emphasis is on the interaction between the individual and the situatedness of the task. Here the model underscores the importance of the act of being engaged in a task as a "dynamic system composed of person and environment" (Nakamura & Csikszentmihalyi, 2002, p. 90). Therefore, in terms of interactionism as a feature of the flow model, the experience of learning and achievement in formal education are influenced by factors that include active engagement by students in the performance of academic tasks conducted in the classroom. Students reported a greater sense of engagement, enjoyment, perception of control, and relevance to the real world when involved in academic and nonacademic subjects offering active participation with a balance between challenge and skill (Newmann, Wehlage, & Lamborn, 1992; Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2003). More active learning experiences such as participating in projects, building models, and performing in plays as opposed to the passive states of listening to lectures or completing worksheets maximised engagement for students (Newmann et al., 1992; Shernoff et al., 2003). Affective variables such as enjoyment, interest, and challenge have been associated with academic success; consequently, positive emotions elicited by the task experience have been shown to contribute to such factors as mathematical achievement (Fisher, Dobbs-Oates, Doctoroff, & Arnold, 2012; Hembree, 1990; Ma, 1997; Schiefele & Csikszentmihalyi, 1995). Conversely the relationship between affect and cognition suggests that difficulty in performing tasks may be experienced as a result of negative emotions (Storbeck & Clore, 2007). Improvements in task performance as a result of being in flow or engaged in a task appear to occur over time. This is explained by the desire of an individual to persevere and return to a task many times, stemming from the cumulative effects of the deep intrinsic rewards experienced, in turn encouraging the development of skills (Nakamura & Csikszentmihalyi, 2002).

Increasing the level of interactivity when solving a maths problem has been shown to positively impact the level of engagement (Guthrie & Vallée-Tourangeau, 2015). This implies that giving participants greater control over their environment may directly increase affect and engagement in the task compared to the level of engagement in a low interactivity environment. However, this greater of level of engagement may not necessarily translate into improvements in performance when the individual is only exposed once or twice to a task as there has been limited opportunity to hone skills. As Nakamura and Csikszentmihalyi (2002) suggested, any improvements due to flow may only be experienced gradually over time with increased exposure to the task. Therefore, although flow or engagement in the task may be improved by greater opportunity to interact with the world, it may not necessarily be the explanation for any changes in results with changes to mode of interaction with the world in one off tasks.

Chapter 6

The River-Crossing Problem

Overview

Outside the psychologist's laboratory, thinking proceeds on the basis of a great deal of interaction with artefacts that are recruited to augment problem solving skills. The role of interactivity in problem solving was investigated using a transformation problem, namely the river-crossing problem. In Experiment 1, participants completed the same problem twice, once in a low interactivity condition, and once in a high interactivity condition (with the order counterbalanced across participants). Learning, as gauged in terms of latency to completion, was more pronounced when the high interactivity condition was experienced second. When participants first completed the task in the high interactivity condition, the transfer to the low interactivity condition during the second attempt was limited. Participants thus showed greater facility to transfer their experience of completing the problem from a low to a high interactivity condition. Experiment 2 was designed to determine the amount of learning in a low and high interactivity condition; in this experiment participants also completed the problem twice, but the level of interactivity was manipulated between subjects. Learning was evident in both the low and high interactivity groups, but latency per move was significantly faster in the high interactivity group and this on both presentations; so-called problem isomorphs instantiated in different task ecologies draw upon different skills and abilities. A distributed cognition perspective may provide a fruitful perspective on learning and transfer.

Introduction

Problems are encountered frequently through everyday activity, varying in complexity and occurring across a diverse array of settings. In solving these problems, or indeed making sense of situations, people interact with local resources, both cultural and material (Kirsh, 2009a). Traditionally, problem solving has been cast and understood in terms of information-processing models of move selection in a clearly defined problem space (see Newell & Simon, 1972) or more recently of the shifts in excitatory and inhibitory activation in layered networks of "knowledge elements" (Ohlsson, 2011, p. 105) that result in the restructuring of a problem representation in working memory. An emphasis on mechanisms of information processing does not foreground the codetermination of an agent's representation of the problem and a problem's physical presentation wrought by interactivity (Kirsh, 2009a, 2009b; 2013).

Transformation problems have been the focus of research in cognitive psychology for the past 50 years. In these problems, a well-defined path connects an initial and a goal state. Legal moves are defined in terms of simple rules and enacted with simple operators. Participants must reach the goal state by transforming the initial state through a series of intermediate states. A well-studied class of transformation problems are river-crossing problems. In these problems, objects—people, animals, or things—must be carried from one 'riverbank' to another on a 'boat', but there are constraints (in the form of defined rules) on the moves that can be selected to reach the goal. A common version involves three missionaries and three cannibals (Reed, Ernst, & Banerji, 1974; or three hobbits and three orcs, Knowles & Delaney, 2005; Thomas, 1974). In transporting all cannibals and missionaries from one bank to the other, cannibals must not outnumber missionaries on either bank. The boat can take at most two passengers, and at least one. The problem space is relatively narrow since illegal moves cannot produce blind alleys of any depth (Reed et al., 1974) and the problem can be completed in a minimum of 11 moves (see Figure 6.1).

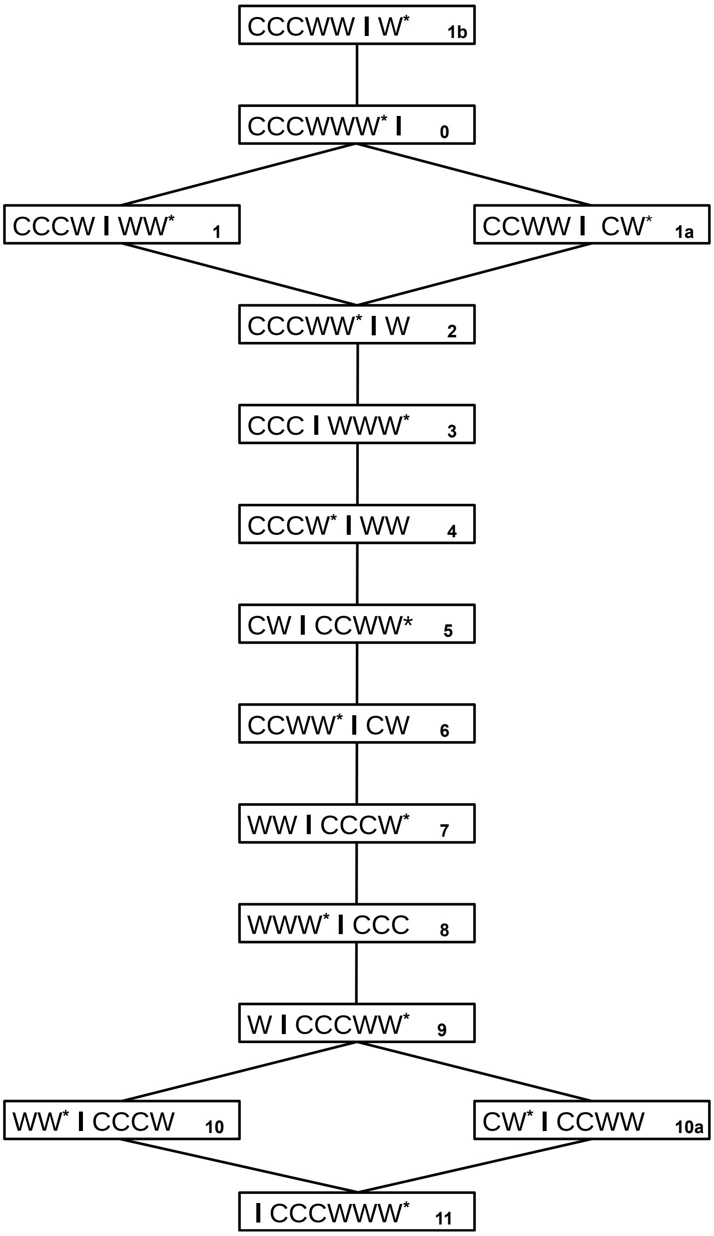

Figure 6.1. Depiction of the problem space for legal moves in the chicken (C) and wolves (W) version of the river-crossing problem. The states are labelled in the right hand corner of each rectangle with the initial state marked as 0 and the goal state marked as 11. The asterisk represents the position of the raft in each state. The vertical line is the river. (Adapted from Knowles & Delaney, 2005).

In different versions, problem difficulty is a function of the rules that constrain the number of objects that can be moved at any one time—which combinations of objects are allowed on the boat, the number of protagonists and which combinations can be left on either bank (e.g., Simon & Reed, 1976). The number of objects and the rules that govern their transport, map out a problem space that links the initial state with all objects on one side of the river to a goal state with all objects on the other riverbank. Cognitive psychologists have used this task as a window onto problem solving, particularly planning and search and move selection (Reed et al., 1974; Simon & Reed, 1976). As such river-crossing problems have been used as a testing platform for a number of process models of search and move selection, strongly influenced by developments in Artificial Intelligence (Simon & Reed, 1976).

Greeno (1974) suggested that individuals learn from repeated attempts at completing the river-crossing task, reflected primarily through a sounder appreciation of which move is correct in each state. Knowles and Delaney (2005) investigated ways to reduce the number of illegal moves generated by participants, reporting that with certain incentives, illegal moves could be reduced with repeated attempts. Reed et al. (1974) investigated the effects of experiencing this type of problem twice in a series of three experiments, examining transfer and learning using analogous problems (e.g., the river-crossing problem and the jealous husbands problem). They found that learning occurred with repetition of the same problem, however, transfer of knowledge between analogous problems was limited. Analogous or isomorphic problems are those with a similar structure having identical constraints, problem spaces, and goal structures, thus requiring essentially the same path to solution (Pierce & Gholson, 1994). The expectation is that once the solution has been learned for one problem it can be mapped onto the isomorphic problem. As described by Lave (1988) these problems have frequently been used to test learning transfer, however the results have been inconsistent. Greeno (1989) and Lave were also skeptical of successful learning transfer occurring across contexts and situations, particularly in the application of knowledge learnt in the classroom being readily mapped onto everyday situations. Examining the impact of interactivity may be valuable in contributing to the understanding of analogical transfer between isomorphs across differing problem presentations.

Interactive Problem Solving

The river-crossing task involves moving people or things across a surface, and as such, foregrounds the importance of interacting with a physical model of the task in problem solving. However, interactivity in solving the river-crossing problem has never been an explicit and systematic focus of investigation. The manner with which the river-crossing task has been implemented varies a great deal across studies. For example, Reed, Ernst, and Banerji (1974) used different types of coins to represent missionaries and cannibals. Jeffries, Polson, Razran, and Atwood (1977) developed a basic computer interface where participants typed in the objects they wanted to put in the boat on a given crossing. The interface accepted only legal moves and updated the simple representations (often with letters and numbers, such as '3M' for three missionaries) on either side of the riverbank. Participants continued typing in their moves until they managed to transport all objects from one bank to the other. Knowles and Delaney (2005) designed a more realistic interface with icons representing travellers against a backdrop of a river with two banks and a boat. Participants selected moves by clicking on the travellers, which then appeared next to the boat on the screen. In all these instances participants were never offered a three-dimensional work surface on which objects transparently corresponding to the scenario protagonists are manipulated and moved by hand. In contrast, developmental psychologists who worked with the river-crossing task, being less sanguine about formal operations presumably, have taken care to design rich interactive thinking environments with physical materials representing the boat, the river, and figurines corresponding to the cover story characters (e.g., Gholson, Dattel, Morgan, & Eymard, 1989).

A more explicit experimental focus on interactivity may unveil important aspects of problem solving performance, aspects that correspond more closely to problem solving performance as observed outside the laboratory. Previous research by Vallée-Tourangeau, Euden, and Hearn (2011) presented evidence that in another transformation problem interactivity substantially altered problem solving behaviour. They reported that mental set was significantly reduced in Luchins's volume measurement problems when participants interacted with a physical presentation of the problem. The actual manipulation of jars and water created a dynamic problem presentation revealing solutions that were not simulated mentally. The selection of moves was guided and governed by the pragmatics of manipulating real objects in a wet environment to achieve a goal, and participants were less likely to persevere in using a more complicated solution for the test problems. In a river-crossing task, interactivity may help participants work out the quality of different moves not by simulating their consequences mentally, but rather by simply completing the move and observing the results. Such moves are then "epistemic actions" (Kirsh & Maglio, 1994, p. 513): moves that may not, in themselves, necessarily help narrow the gap with the goal state, but rather provide information as to what to do next. Kirsh and Maglio (1994) demonstrated that it is faster and easier to physically rotate the tetrominoes in Tetris than to simulate their rotation mentally, leading to better and more efficient problem solving behaviour. Move selection in the river-crossing task can be opportunistic, although not necessarily mindless; rather the strategic consequences of a certain move can simply be observed. In a high interactivity context, planning need not take place 'in the head'—moves may not be premeditated; rather the trajectory through the problem space is enacted through the moves. Thus, in a high interactivity environment, there may be less pressure on reasoners to simulate mentally a path to a goal state and move selection may not be dictated by a plan (cf. Suchman, 1987). Problem solving performance could well be influenced by the ease with which reasoners can enact moves. In a context that favours interactivity, participants may produce more moves in solving the river-crossing problem, but do so more quickly than in a context in which implementing a move is slower and more costly in terms of mental planning effort.

Some have argued that as a result, high interactivity may retard the acquisition of a more abstract representation of the task and hence may not lead to the same degree of learning (O'Hara & Payne, 1998; Svendsen, 1991). With a river-crossing problem, a low level of interactivity may force participants to think longer before selecting a move and may encourage the development of a sounder appreciation of the logical structure of the task. This may help participants transfer their knowledge to a different presentation of the same or similar problems. These participants, once presented with the problem a second time, but in a high interactivity condition, may proceed to solve the problem much faster. In turn, solving a river-crossing problem first in a high interactivity condition, may promote a more procedural appreciation of the task that might be bound to the exact physical characteristics of the reasoning context. Hence any learning may transfer poorly when participants complete the problem a second time in a different context of lower interactivity. The goal of the present experiments was twofold: To determine the impact of interactivity on performance in the river-crossing problem and to determine the amount of learning across two presentations of the problem as a function of interactivity.

River-Crossing: Experiment 1

Experiment 1 examined performance in the river-crossing problem when presented with or without artefacts as an aid to solution. This was measured in terms of the number of moves to solution, latency to completion, and latency per move. In both conditions the problem was described on a piece of paper. In a high interactivity condition, the problem was presented with a board, a raft and six figurines: Participants were expected to solve the problem by moving the figurines on the raft across the board to register a move, continuing until they had moved all six figurines from one bank to the other. In a low interactivity version, participants were asked to keep their hands flat on the table and verbalise the moves they would make to reach the goal. They completed the problem twice, once with the high interactivity version and once with the low interactivity version; the order was counterbalanced across participants. Participants also undertook a number of other tasks in order to profile any relevant abilities or attitudes in determining any internal resources that might predict performance on the river-crossing problem. Therefore, the two attempts at the river-crossing problems were interleaved with two working memory tasks, the need for cognition scale and the task engagement scale. It is expected that working memory would be taxed in the low interactivity condition. However, the opportunity for interactivity with artefacts in the high interactivity condition would reduce the load on working memory thereby diminishing the impact of any individual differences in working memory storage and capacity (Vallée-Tourangeau, 2013). It is also expected that in the low interactivity condition those rating highly on the need for cognition scale would outperform those with a lower score, while any differences would dissipate with an increase in interactivity. With the opportunity to engage more in the task, it is anticipated that the river-crossing attempt with the greater interactivity will prompt greater feelings of being in the flow, resulting in higher scores on the task engagement scale for the high interactivity condition than the low interactivity hands-down condition.

Experiment 1 employed a mixed design with the interactivity level as the repeated measures factor and order—low interactivity first, high interactivity first—as the between subjects factor. As moves can act as epistemic actions, it is predicted that participants would produce more moves, would solve the problem faster, and hence latency per move would be shorter in the high compared to the low interactivity condition. In addition, it is predicted that participants would complete the second presentation of the task more quickly than the first since they would be familiar with the procedure and may well exploit an episodic record of their trajectory to help them select better moves, and select them more quickly. However, the nature of the experience during the first crossing as a function of interactivity level could influence the amount of learning. On the basis of the arguments formulated in O'Hara and Payne (1998; see also Svendsen, 1991), low interactivity forces participants to plan and contemplate moves and their consequences; the additional time and effort encourage more deliberation, and as a result participants are more likely to develop a sounder understanding of the problem and select fewer but better moves. When the problem is experienced a second time, this time in a high interactivity condition, performance improvements should be steep. In turn, experiencing the problem in a high interactivity condition first, may reduce the investment in deliberative efforts, perhaps mitigating the development of a more abstract, hence transferable, representation of the problem: There should be limited evidence of learning when the problem is experienced a second time in a low interactivity condition.

Sample Size. Reed, Ernst, and Banerji (1974) studied the effect of transfer between two problems with similar problem states, the Missionaries and Cannibals and the Jealous Husbands problems. The experimental design for their Experiment 2—the first experiment was inconclusive and the third addressing issues too dissimilar from the ones explored here—was a two factor mixed design, with problem type and order as the factors. They recruited a sample of 54 participants, with 50 successful solvers, 25 in each problem condition. An a priori power analysis was completed using G*Power 3.1 (Faul, Erdfelder, Buchner, & Lang, 2009) to estimate the sample size required to obtain a similar effect. The observed $\eta_p^2 = .149$ for the 2×2 interaction effect on latency per move in a pilot river-crossing experiment, not reported here, corresponded to a large effect size ($f = .42$, see Cohen, 1992), with a correlation between repeated measures of .016. Based on these estimates, the a priori power analysis indicated that a total sample size of 40 would be sufficient to detect a similar effect size. Given the sample depletion due to participants not completing one or both attempts, as well as the possibility of having to remove long latencies to control for skewness, it was estimated collecting between 60 - 70 participants would be acceptable.

Method

Participants

Sixty-five university undergraduate and postgraduate students participated in the experiment in return for course credits. Twelve participants did not complete the task and were excluded from further analysis; the final sample comprised of 53 participants (43 Females, M_{age} = 21.5, SD = 3.93).

Materials and Procedures

River-crossing problem. Chickens and wolves were the protagonists in a river-crossing scenario. The objective was for the six animals to be transported from the left riverbank to the right one. The selection of a move had to comply with the constraints and rules of the problem. The same instruction sheet explaining the objective of the task and the rules of the problem was used for both conditions and could be read by the participants throughout the duration of the task. The instructions read:

> "Three wolves and three chickens on the left bank of a river seek to cross the river to the right bank. They have a boat that can carry only two animals at a time, but there must always be an animal on the boat for it to move. However if at any time the wolves outnumber the chickens on either bank the wolves will eat the chickens. Thus you cannot move the animal(s) in a manner that will result in the wolves outnumbering the chickens on either bank. The goal of the task is to move all the animals from the left bank to the right bank."

In the low interactivity version of the task, the researcher transcribed each move as verbalised by the participant onto a record sheet. The record sheet was a simple representation of the raft between the left and right banks of the river, with slots to record the nature and number of the animals on either side (which was denoted with a "C" for chickens and "W" for wolves; see top panel of Figure 6.2); each page represented only one move. At any one time, participants could only inspect their previous move as they dictated their next move to the experimenter. As soon as the next move was dictated, the sheet with the previous move was turned over. Thus participants could not inspect a historical record of previous moves. Illegal moves proposed by the participant were noted, but the experimenter did not transcribe the nature of the illegal move on the recording sheet, nor were participants advised the nature of the illegal move. Rather, participants were invited to reread the task instructions to discover why such a move was not allowed.

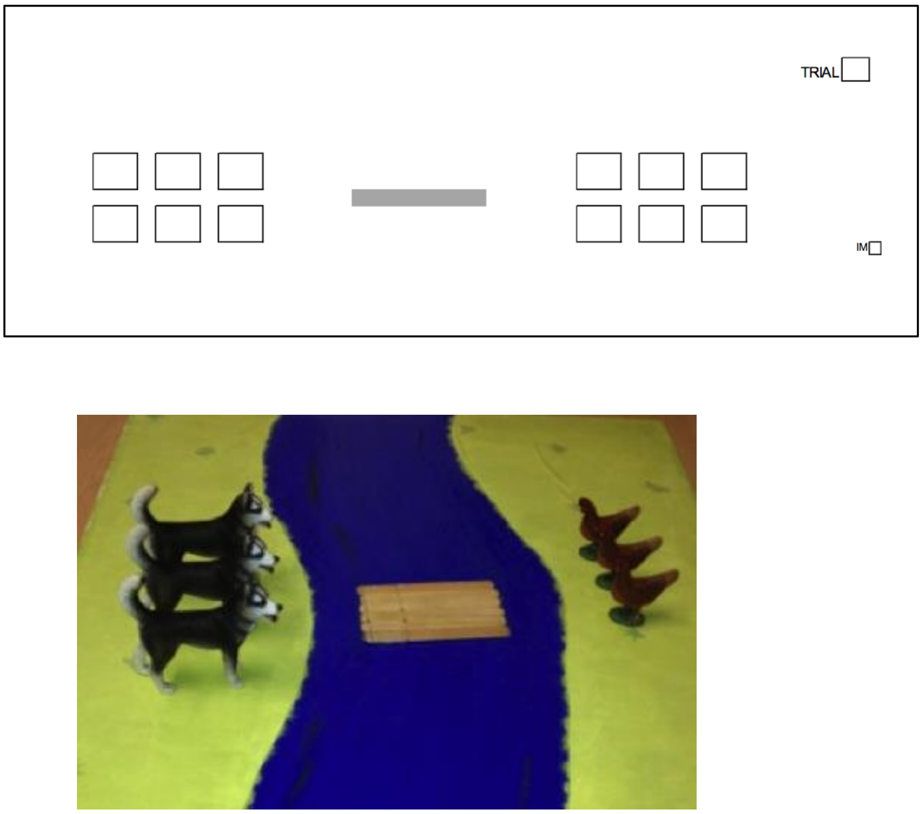

Figure 6.2. Top panel: The record sheet for the river-crossing moves in the low interactivity condition. The small box in the top right-hand corner is the move (trial) made within that attempt at solving the problem. That is, a “1” in the box would denote the first move, “2” the second move and so on. The small box in the lower right-hand corner is the notation of any illegal moves made in that trial. The bar in the middle of the sheet represents the raft. The researcher would draw an arrow on the raft indicating to which bank the raft was heading. The boxes on either side were used to indicate the number of wolves (W) or chickens (C) situated on each river-bank. Bottom panel: The board, raft, and figurines (wolves and chickens) for the high interactivity condition.

Legal moves were the moves made by the participant from the first move to the final move that met the constraints or rules as set out in the instructions sheet available to all participants throughout each attempt, whereas Illegal moves were denoted as any moves that did not meet these constraints. The decision to include all violations of the rules as illegal moves was made in order to measure the total number of moves completed by the participant during the entirety of the attempt. Knowles and Delaney (2005) did not include violations of the rules that negated the movement of the boat if either empty or carrying more than two passengers on the grounds that participants may make errors in using the computer interface or through lack of understanding of the rules. In the experiments presented here there was no computer interface to negotiate. In addition, the rules and instructions were available in a printed format for all participants throughout both attempts; in fact, participants were actively encouraged to refer to the rules throughout the task.

The high interactivity version of the task involved the use of six plastic figurines, three wolves (9cm x 7cm x 2cm) and three chickens (4cm x 5cm x 1.5cm), one pop-stick raft (9cm x 6cm) and a painted board (60cm x 45cm) representing the river and banks (see bottom panel of Figure 6.2). As the participants interacted with the artefacts, the experimenter recorded the moves, but this record was never shown to the participants; as with the low interactivity condition this ensured that participants could not review the problem-solving trajectory. An illegal move prompted the experimenter to instruct participants to move the raft and the animals back to the previous state and, as in the low interactivity condition, they were invited to reread the instruction sheet to determine which moves were possible. In both conditions participants were given up to 15 minutes to complete the river-crossing problem. Participants were not asked to prioritise the number of moves made or the time in making moves, nor were they explicitly told how long they would be given to complete the task. If the participant questioned the amount of time allowed to solve the problem, the researcher explained that a reasonable amount of time would be allowed within the confines of the experimental session time. However, any participant unable to finish one or both attempt within 15 minutes was excluded from subsequent analyses.

The experimental session began with one or two of the individual difference tasks as described below; these tasks were counterbalanced across the session. This was followed by one condition of the river-crossing problem (either low or high interactivity). After attempting the river-crossing problem in the first condition the participants completed another individual differences task. The river-crossing problem was presented again in the alternate condition to that which was presented first; the order was counterbalanced across participants. The remaining individual difference tasks completed the experimental sessions. The independent variables manipulated were condition (low interactivity, high interactivity) and order (low interactivity first, high interactivity first) in a 2×2 mixed design. Performance in both conditions was measured in terms of latency to solution, the total number of moves to solution, and latency per move. The latter offers the more interesting window onto problem solving performance across interactivity conditions since it provides a gauge of how quickly, on average, participants generate each move. In keeping with previous river-crossing studies legal and illegal moves are reported separately. The latency per move data was determined using the total number of moves.

Working memory. Two working memory tasks were included in the experimental session.

Computation span (C-span). The computation-span task (Ashcraft & Kirk, 2001; Salthouse & Babock, 1991), evaluates working memory capacity by testing both processing and storage of numbers. Participants were asked to answer simple arithmetic problems (e.g., 6 - 2 and 2 + 9) before recalling the second number of the problems (e.g., 2, 9). Sequences of sums began with one sum before recall, then two sums before recall and so on until there were seven sums before recall of the last digits. Therefore, participants had to correctly answer the sums, in the processing phase of the task, and list the relevant digits, as the storage component, to score a point. This was a PowerPoint presentation on a computer screen; all instructions were part of the on-screen presentation. The participants called out the answers to the sums and the list of digits to the researcher, who recorded the responses onto a preprinted answer sheet that was not visible to the participant at any time. The only feedback given to the participant after completion of the task was that the activity was difficult and it had been successfully completed.

Corsi block. Working memory storage was tested using a modified version of the Corsi block-tapping task (Corsi, 1972). This was a PowerPoint presentation on a computer screen with all instructions, including two practice sequences, as part of the on-screen display. Participants were shown a series of grids, each grid was divided into 4 x 4 cells, and only one grid would appear on the screen at any time. One cell (or block) on each grid was randomly blocked-out. After each sequence the participant was asked to complete a recall sheet by indicating the order and location of the blocks using numbers (see Figure 6.3). Participants were shown twelve sequences, with the number of cells blocked-out increasing from a two block series to a six block series. The total number of blocks correctly identified in order and position were used as the measure for working memory capacity. The maximum score possible was 40. Participants were not given any feedback upon completion of the task.

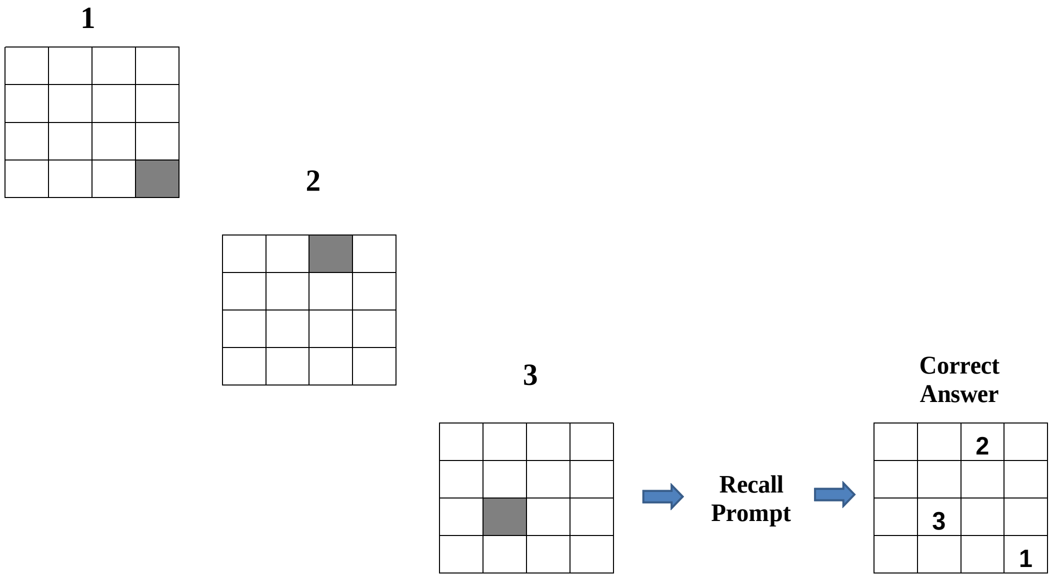

*Figure 6.3***.** Example of a sequence of grids from the modified Corsi task. Grids 1, 2, and 3 would be visible in sequence on the screen. The participant would then be prompted to recall the shaded cells by recording the order and position on a blank response grid sheet.

Task engagement scale (TES). This is a 9-item scale based on three key components of task engagement: concentration, enjoyment and interest (Shernoff, Csikszentmihalyi, Schneider and Shernoff, 2003). The scale was designed to assess a participant's flow through engagement and enjoyment during an activity or task (Csikszentmihalyi, 1990, 2000). Questions included "Did you enjoy the task?", "Did you feel challenged by the task?", and "Did you feel absorbed by the task?". Participants completed a paper-based questionnaire where they were asked to rate each item on an 8-point Likert scale, labeled from zero (definitely not) to seven (definitely yes): The higher the score the more positive the attitude toward the task. Each participant completed the TES scale twice, once following each attempt at the river-crossing task. The alpha reliability of the nine-item scale for both interactivity conditions was acceptable (Low, Cronbach's α = .83; High, Cronbach's α = .81).

Need for cognition scale (nCog). The Need for Cognition scale (Cacioppo and Petty, 1982) was used to test the trait of thinking diligence. It was a pen and paper activity where participants were requested to rate 18 statements on the satisfaction they gained from thinking. The scale was a six-point range from "completely false" to "completely true" and included questions such as "I would prefer complex to simple problems" and "I really enjoy a task that involves coming up with new solutions to problems".

Results

Performance Measures

Latency. Indices of skewness indicate that the latencies in the four experimental conditions were normally distributed. Latencies to solution are shown in Table 6.1; the pattern of findings closely replicated what was observed in Experiment 1A. The faster change in crossing latency was observed in the high interactivity condition when participants first completed the task in the low interactivity condition. A 2×2 mixed ANOVA showed the main effect of interactivity was not significant, $F(1, 51) = 3.45$, $p = .069$, $\eta_p^2 = .063$, while the main effect of order was significant, $F(1, 51) = 5.12$, $p = .028$, $\eta_p^2 = .091$; the interactivity condition by order interaction was also significant, $F(1, 51) = 9.76$, $p = .003$, $\eta_p^2 = .161$. Post hoc tests indicated that latencies in the low interactivity condition did not decrease significantly from the first to the second presentation, $t(51) = 0.358$, $p = .419$. In turn participants were faster in the second attempt at the problem than the first in the high interactivity condition, $t(51) = - 4.097$, $p < .001$. When participants completed the low interactivity condition followed by the high interactivity condition, they were significantly faster in the second attempt, $t(23) = 4.297$, $p < .001$. When participants completed the high interactivity condition first then the low interactivity condition there was no significant decrease in the time taken to complete the problem, $t(28) = .820$, $p = .419$.

Moves. The high interactivity condition elicited a greater mean number of legal moves compared to the low interactivity condition in the first attempt (see Table 6.1). With the number of legal moves in the second attempt being similar for both conditions. In turn, the mean number of illegal moves was higher in the high interactivity condition than the low interactivity condition when it was experienced first, but in the second attempt the number of illegal moves was lower in the high interactivity than the low interactivity condition. Overall, then, total moves were greatest in the high interactivity condition for the first attempt but in a 2×2 mixed ANOVA the main effects of interactivity, $F(1, 51) = 1.27$, $p = .265$, $\eta_p^2 = .024$, and order, $F(1, 51) = 2.70$, $p = .107$, $\eta_p^2 = .050$, were not significant, nor was the interaction, $F(1, 51) = 2.34$, $p = .132$, $\eta_p^2 = .044$.

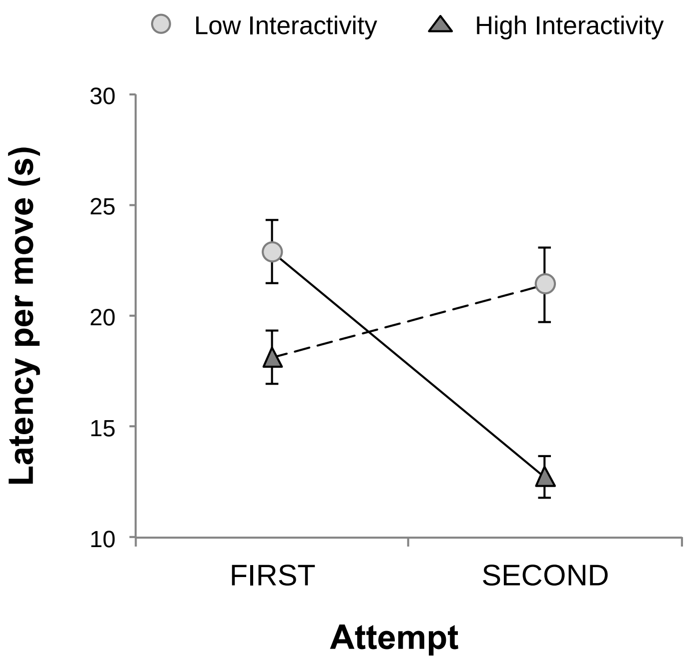

Figure 6.4. Mean latency per move in the two interactivity conditions (low, high) for the first and second attempt. The group that experienced the high interactivity condition first is represented by a broken line and the group that experienced the low interactivity condition first is represented by a solid line. Error bars are standard errors of the mean.

Table 6.1

Mean Latencies and Mean Number of Moves to Completion (with standard deviations for all means). Order Indicates the Order of Interactivity Undertaken in the Experimental Session (L = Low Interactivity and H = High Interactivity). First and Second Represents the First or Second Attempt in the Experimental Session.

					Moves											
	Latency (s)				Legal				Illegal				Total			
	First		Second		First		Second		First		Second		First		Second	
Order	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
L/H	463.92	236.71	246.04	139.77	17.46	7.27	17.17	8.18	3.04	4.41	2.50	4.19	20.50	10.42	19.67	10.51
H/L	493.97	267.51	438.62	271.15	19.07	7.22	16.97	7.66	7.55	4.61	4.14	6.87	26.62	10.14	21.10	13.29

Latency per move. Latency per move (see Figure 6.4) in the low interactivity condition appeared to be largely unaffected by order, however participants were faster at enacting moves in the high interactivity condition, and this was particularly evident on the second attempt. In a 2×2 mixed ANOVA the main effect of interactivity was significant, $F(1, 51) = 39.17$, $p = .001$, $\eta_p^2 = .434$, but the main effect of order was not, $F(1, 51) = 1.42$, $p = .238$, $\eta_p^2 = .027$; however, the condition by order interaction was significant, $F(1, 51) = 10.2$, $p = .002$, $\eta_p^2 = .167$. Post hoc tests indicated that the difference in latency per move was not significantly different between the two low interactivity attempts, $t(51) = .646$, $p = .521$, but the time taken to select a move during the second attempt in the high interactivity condition being significantly faster than during the first attempt, $t(51) = - 3.42$, $p = .001$. In addition, participants were faster in making moves in the high interactivity condition following experience in the low interactivity condition, $t(23) = 8.36$, $p = .001$; latency per move remained unchanged when the high interactivity preceded the low interactivity condition, $t(28) = 1.97$, $p = .059$.

Predictors of performance

To examine the possible influence of internal resources on performance in solving the river-crossing problem, the participant’s working memory, need for cognition and engagement in the task were assessed. There was no pattern of significant correlations revealed between the Need for Cognition scale and performance measures. However it was note-worthy that the computation span working memory test and the task engagement scale correlated with some performance measures.

Working memory. The measure for visuo-spatial working memory, Corsi, was highly correlated with computation span, r (22) = .457, p < .001 indicating that both tests measured some similar aspect of working memory. As the span task is a gauge of storage and processing capacities in working memory, it is possibly the storage aspect that contributed to the common variance. However, Corsi did not correlate with any other predictors of performance or measures of performance.

On the other hand, computation span (C-span), correlated with some of the performance measures in the low interactive condition when it was the first task; legal moves, r (51)= -.471, p = .025; total moves, r (51)= -.495, p = .014; latency, r (51) = -.485, p = .016. Correlation almost reached significance with illegal moves, r (51)= -.393, p = .058 and was not significant with latency per move or any other performance measures

Task engagement. When comparing engagement in the task across all participants, they reported feeling more engaged in the high interactivity condition (M = 46, SD = 9.6) than in the low interactivity condition (M = 40.8, SD = 9.9). This difference was significant, t (52) = -3.66, p = .001.

The TES results were analysed further to assess whether the order of the river-crossing task made a difference participants' feelings of engagement. When completing the low interactivity version first, TES was greater for the high interactive condition (M = 46.4, SD = 10.6) than the low interactivity condition (M = 42.8, SD = 10.5). This difference was significant t (23) = -2.3, p = .031. When the condition was reversed the results also showed participants reported greater engagement when undertaking the high interactivity token version of the task first (M = 45.6, SD = 8.9) then the hands-down version (M = 39.1, SD = 9.24) with the difference also significant, t (23) = -3.66, p = .001. In summary, regardless of the task order, participants always felt more engaged in the high interactivity condition.

The correlations between the performance measures and TES were only significant when the high interactivity condition was completed following the low interactivity condition, with the exception of latency per move, which was not significant (see Table 6.2).

Table 6.2

*Significant correlations including confidence intervals (**CI)** for the task engagement scale with performance measures (df = 22). **LI** = Low interactivity condition; **HI** = High interactivity condition; **L/m** = Latency per move.(df = 51).*

Order	Performance Measure	*r*	*p*	95% CI
LI first/Hi second	HI legal moves	-.451	.027	[-.722, -.059]
	HI Illegal moves	-.442	.031	[-.717, -.047]
	HI Total moves	-.527	.008	[-.767, -.157]
	HI Latency	-.556	.005	[-.783, -.197]
	HI L/m	.002	.993	[-.401, .405]

Discussion

This experiment investigated the impact of interactivity on problem solving performance for a river-crossing problem. Participants were required to solve the problem twice, once in a low interactivity context in which moves were simulated mentally and dictated to an experimenter and once in a high interactivity context where moves could be enacted through a three-dimensional display that corresponded to the main features and protagonists of the problem. A high level of interactivity generally encouraged participants to make more moves in reaching a solution than when they completed the low interactivity condition. The latency per move data indicated that participants were always quicker to select a move in the high interactivity condition, and were generally quicker to select a move during the second presentation of the problem. However, the more important pattern in these data was the interactivity condition by order interaction observed in this experiment: Latency per move dropped precipitously when the second presentation of the problem occurred in the high interactivity condition. Post hoc tests showed order did not affect latency per move in the low interactivity condition, but did so in the high interactivity condition. In addition, there was no difference in latency per move between the low and high interactivity conditions if the high interactivity condition was attempted first, but latency per move was substantially reduced when the high interactivity condition was attempted second.

The second presentation of the problem offered the opportunity to gauge the degree of learning and transfer from one interactivity context to another. There was much evidence of learning, when the second opportunity to solve the problem took place in a context that favoured a high level of interactivity: Participants completed the problem in less time and selected moves at a faster rate in the high interactivity context than when the second presentation of the problem was in the low interactivity condition. In fact, when the low interactivity condition was experienced second, performance reflected little learning and transfer. This pattern of results suggests two competing explanations: (i) the process and quality of knowledge acquisition is different as a function of the level of interactivity or (ii) interactivity is a performance facilitator and a high level of interactivity more clearly showcases learning. Each explanation is evaluated in turn.

First exposure to the problem without much interactivity might have fostered the acquisition of a sounder and more actionable representation of the task and appreciation of an efficient sequence of moves to solution. In contrast, experiencing the problem in a context that fosters a high level of interactivity might not be accompanied by the same investment in cognitive effort, proceeding primarily on the basis of procedural learning, which in turn might interfere with the development of an accessible and transferable conceptual representation of the problem. As a result, when the problem is encountered for the second time in a condition without much interactivity, the procedural knowledge does not facilitate transfer; however, when the second presentation occurs in the high interactivity condition, performance substantially benefits from the knowledge acquired on the basis of the experience in the low interactivity condition. The pattern of results from this experiment offers some support for this conjecture.

Alternatively, the substantial improvement in the high interactivity condition when participants are presented the problem a second time might not reflect differences in the type and quality of learning but rather release from a performance bottleneck. In other words, interactivity is a performance facilitator. Cognitive efforts and task demands are more exacting with low interactivity—as evidenced by the significantly longer latency per move. When participants encounter the problem a second time but this time can benefit from cheaper move selection by moving artefacts on the board, they experience a release from the cognitive demands of the low interactivity condition and are quicker at producing moves, and hence quicker to reach a solution. The river-crossing problem is narrow analytic problem with a tightly defined problem space:

participants cannot wander off-piste very far. Faster move selection and the production of more moves leads to reaching the goal state quicker.

The task engagement scale was included to assess whether participants' feelings about the different tasks affected their performance. It is clear from the results that, irrespective of the task order, individuals were more engaged with the task when presented with artefacts to manipulate than when asked to keep their hands on the table while attempting to solve the problem. Participants' performance was related to TES in the second attempt of the task, but only when interactivity was high. This indicates that even when given a second opportunity to solve the problem, the hand down condition was no more engaging than the first attempt. Therefore the order was not the sole factor that raised the levels of engagement; rather it was the chance to attempt the problem again with a greater opportunity to interact with the world.

The design of this experiment, however, cannot provide data to adjudicate the relative merits of these conjectures since attempt and interactivity level were not decoupled. Experiment 2 was designed to disentangle the effects of order and interactivity on learning by manipulating the levels of interactivity between subjects. Thus, as in experiment 1, participants completed the river-crossing problem twice, however they did so either in a low or a high interactivity context both times. In this manner, Experiment 2 could provide data to determine the magnitude of learning as reflected in improvement in performance across the two presentations. In light of O'Hara and Payne's (1998) conjecture that planning impacts performance for different levels of interactivity, Experiment 2 also sought to measure independently participants' planning abilities by having them complete a series of Tower of London problems during the experimental session. The Tower of London (ToL) is a transformation problem used to assess planning skills in healthy and neuropsychological patients (Shallice, 1982; Unterrainer, Rahm, Leonhart, Ruff, & Halsband, 2003; Ward & Allport, 1997). The inclusion of a planning task in this experiment sought to determine the degree to which planning performance with a series of three-disk ToL problems could predict river-crossing performance. Specifically, it was predicted that planning skills would correlate with river-crossing performance in the low interactivity condition; however, in a high interactivity context, the ease of selecting and implementing moves, should level off individual differences in planning abilities.

River-Crossing: Experiment 2

Method

Participants

Eighty-nine university undergraduates participated in exchange for course credits. Thirteen participants did not complete the river-crossing problem within the allocated time and were subsequently excluded from further analysis. Following tests for skewness in the latency data a further 6 participants were removed from the analysis to ensure the data were normally distributed. The final sample was composed of 70 participants (58 females, M_{age} = 22.9, *SD* = 5.5).

Materials and Procedure

The same procedure designed for Experiment 1 was employed in Experiment 2 save for two changes. First, participants were randomly allocated to either the low or high interactivity group, and therefore completed the river-crossing problem in the same interactivity condition twice. Second, the number of individual difference tasks was altered to accommodate a Tower of London planning task. In addition to a computation span task and the need for cognition, participants completed a series of Tower of London (ToL) problems during the experimental session. The task engagement scale and modified Corsi were not included due to time constraints. Half the participants completed the ToL before the first attempt at completing the river-crossing task, the other half after the second attempt. The ToL task was adapted from Shallice's (1982) version by using paper pegs printed on an A4 card and coloured paper disks that could be moved about on the card (see Figure 6.5). The rules were printed on an A4 sheet of white paper and read: (i) move only one disk at a time; (ii) move only the top disk; (iii) no more than two disks on the middle peg and no more than one disk on the shortest peg. The disks and pegs were placed in front of the participant in the initial state. The nature of the task was explained to the participant. It was also explained that the researcher would note the time taken and number of moves completed for each of the four problems. They were not given the opportunity to practice; no instructions were given on planning or time allowed for completion of the problems. Participants were then asked to read the rules before beginning the task, and to state that they understood the rules and the goal of the task. The rules were then removed from sight. Participants were shown four different goal-state configurations of the three disks on three pegs of different heights in an identical format to the card. At the start of each problem, the disks were set at the same initial state, participants were shown a new goal-state configuration and were then required to rearrange the disks one by one to match the goal state. Each problem could be completed in a minimum of five moves.

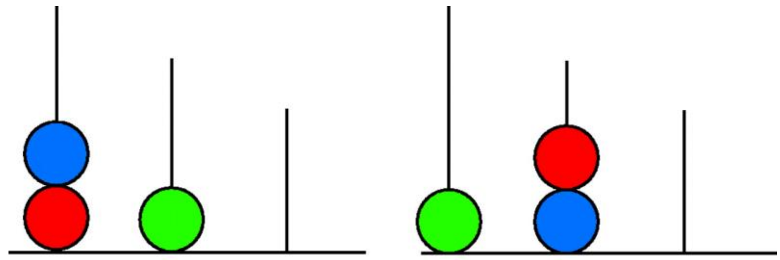

Figure 6.5. The three-disk Tower of London task adapted from Shallice (1982). All trials began with the same starting position (right-hand panel). The goal position varied on each trial (left-hand panel).

Results

Performance Measures

Latency. The latency data were skewed in three of the four experimental conditions. Removing the six slowest participants ensured that indices of skewness were within the acceptable range (Z in all conditions < 1.96). The mean latencies to completion in both groups for both attempts are reported in Table 6.3. Latency to completion declined considerably from the first to the second attempt in both interactivity groups. In a 2×2 mixed ANOVA the main effect of attempt was significant $F(1, 68) = 63.7$, $p < .001$, $\eta_p^2 = .483$; however, the main effect of group was not , $F < 1$, nor was the attempt by group interaction, $F < 1$.

Moves. Participants in the high interactivity group produced a greater number of legal moves than those in the low interactivity group during the first and second attempt, but participants in both groups selected fewer moves during the second attempt (see Table 6.3). The mean number of illegal moves was also greater in the high interactivity group than in the low interactivity group (see Table 6.3); however, both groups selected fewer illegal moves during their second attempt. A similar pattern was produced with the overall number of

Table 6.3

Mean Latencies and Mean Number of Moves to Completion (with standard deviations for all means). Order Indicates the Order of Interactivity Undertaken in the Experimental Session (L = Low Interactivity and H = High Interactivity). First and Second Represents the First or Second Attempt in the Experimental Session.

	Latency (s)				Moves											
					Legal				Illegal				Total			
	First		Second		First		Second		First		Second		First		Second	
Order	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
L/L	408.12	165.04	217.21	78.35	13.45	3.80	12.06	1.87	3.39	2.76	1.15	1.66	16.85	5.65	13.21	2.47
H/H	399.16	186.25	233.59	96.25	16.41	5.81	15.41	4.99	4.73	2.85	2.86	2.54	21.14	7.58	18.19	6.35

moves: More overall moves were made in the high interactivity group than in the low interactivity group, but fewer moves were made in the second attempt and this in both groups. In a 2×2 mixed ANOVA on total moves the main effect of attempt was significant $F(1, 68) = 11.0$, $p = .001$, $\eta_p^2 = .139$, the main effect of group was significant $F(1, 68) = 21.3$, $p < .001$, $\eta_p^2 = .238$, but the attempt by group interaction was not, $F < 1$.

Latency per move. The latency per move data are illustrated in Figure 6.6. There are two patterns of note. First, latency per move decreased during the second attempt in both groups; second, high interactivity participants were faster at selecting moves than the low interactivity participants during both attempts. In a 2×2 mixed ANOVA the main effect of attempt was significant, $F(1, 68) = 77.9$, $p < .001$, $\eta_p^2 = .534$, as was the main effect of group $F(1, 68) = 12.6$, $p = .001$, $\eta_p^2 = .157$; the group by attempt interaction was not significant $F(1, 68) = 2.15$, $p = .147$, $\eta_p^2 = .031$.

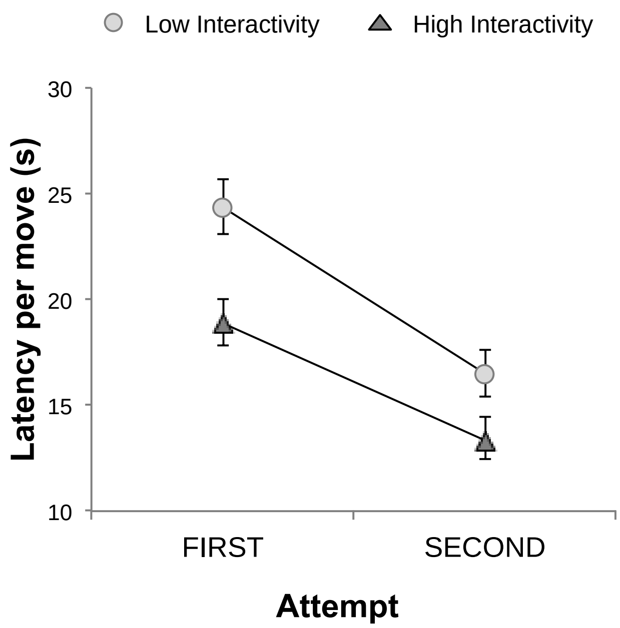

Figure 6.6. Mean latency per move in the two interactivity conditions (low, high) for the first and second attempt. Error bars are standard errors of the mean.

Predictors of Performance

Measures of working memory, the need for diligent thinking and planning were investigated in order to assess any influence internal resources might have on performance in solving the river-crossing task. Analyses of computation span and need for cognition revealed no pattern of significant correlations. However, analysis of the Tower of London planning skills task did reveal some significant results.

Planning skills. The mean latency to complete each of the four ToL problems was 26.9s (*SD* = 13.5) for participants in the low interactivity group, and 31s (*SD* = 18.9) for those in the high interactivity group; the difference between groups was not significant, $t(68) = -1.04$, $p = .302$. The mean number of moves for each ToL problem in the low interactivity group was 8.67 (*SD* = 2.63) and 9.94 (*SD* = 3.71) in the high interactivity group: the difference was not significant, $t(68) = -1.64$, $p = .105$. Thus, planning skills as determined by this measure did not differ between the two groups of participants. However, ToL latencies moderately correlated with the latencies for the first, $r(31) = .344$, $p = .05$, and strongly with the second attempt, $r(31) = .524$, $p = .002$, in the low interactivity condition. Thus, the faster participants were at completing the ToL problems, the faster they were at completing the river-crossing problems for both attempts in the low interactivity group. In contrast, participants' ToL latencies did not predict the time to complete the river-crossing problem in the high interactivity group, either for the first, $r(35) = .071$, $p = .675$, or the second attempt, $r(35) = -.151$, $p = .372$.

Discussion

Experiment 2 was designed to offer additional data to adjudicate the conjectures formulated in explaining performance during the second presentation of the task in Experiment 1. In experiment 1, latency and latency per move were substantially lower, suggesting significant learning, when the high interactivity but not the low interactivity condition was experienced second. Since interactivity was manipulated within-subjects in these experiments, the nature of the learning experience during the first crossing was not controlled across the two presentations of the problem. A couple of conjectures were offered that could only be tested with an experiment where the interactivity level was manipulated between subjects, hence controlling for interactivity level across the two presentations.

The data from Experiment 2 painted a relatively unambiguous picture: Learning was evident in both the low and high interactivity conditions. Thus, the substantial learning in terms of reduced latency and latency per move in the low interactivity condition in this experiment lends some support to the notion that, in Experiment 1, experience in a high interactivity condition first may have retarded transfer when the second attempt took place in low interactivity context. In addition, this performance improvement cannot be attributed to a performance bottleneck caused by the relative cost of move selection in the low interactivity condition. In Experiment 2 participants who completed the first crossing in the low interactivity condition were significantly faster completing the second crossing in the same condition. However, latency per move was faster in the high interactivity condition than in the low interactivity condition for both attempts at solving the problem. The absence of a significant interaction between interactivity and attempt in these data indicates that the performance improvement was similar in both interactivity conditions. Indeed, the average decrease in latency per move from the first to the second attempt at completing the problem was 7.86s (SD = 6.78) in the low interactivity group, and 5.62s (SD

= 6.06) in the high interactivity group, a nonsignificant difference, $t(68) = 1.47$, $p = .147$.

It was conjectured that learning in the high interactivity condition of Experiment 1 when experienced during the second presentation of the problem reflected a sounder appreciation of the task obtained through a more sustained planning effort during the first presentation with low interactivity. This hypothesis is not supported by the data obtained in the present experiment. Researchers such as O'Hara and Payne (1998; see also Svendsen, 1991) have argued that a low interactivity environment—in which it is relatively more costly in terms of efforts to produce a move—encourages people to plan more before selecting a move which results in a richer and more transferable problem representation. However, in the experiment reported here, participants who completed the problem in the high interactivity condition did so significantly quicker during the second presentation. Admittedly, a lower cost structure, where moves are cheap, encourages more moves; indeed participants in the high interactivity group always selected more moves than participants in the low interactivity group. However, latency per move was significantly faster in the second presentation, and indeed significantly faster than the latency per move for participants in the low interactivity group. Thus, the argument that learning is mitigated by the low cost structure is not supported by the latency per move data reported here.

Finally, participants in both groups did not differ in their planning abilities as reflected by the similar average latency to complete each of the four ToL problems. Of greater interest was the fact that ToL latencies were strongly positively correlated with latencies to complete the river-crossing task, and this for both attempts, in the low interactivity group. In contrast, the river-crossing latencies did not predict participants' performance on the ToL problems in the high interactivity group. To be sure, this pattern of correlational evidence indicates that higher interactivity may reduce the contribution of forward planning in the selection of moves. Perhaps more important, it suggests that high levels of interactivity may elevate the performance of participants with poor planning abilities, such that their performance is comparable to participants with higher planning abilities.

General Discussion

Outside the psychologist's laboratory, scientists and lay people alike naturally create and build artefacts or recruit existing ones to configure highly interactive contexts of reasoning and facilitate problem solving. Thus, solving jigsaw puzzles involves physically juxtaposing different pieces to gauge their fit; in Scrabble, letter tiles are physically rearranged to facilitate word production; in Tetris, tetrominoes are physically rotated to determine their optimal slot. Beyond puzzles and games, experts structure an external environment to support thinking. Scientists use physical objects and their arrangement in space to formulate and test hypotheses: Watson (1968) described how he cleared his desk, cut out shapes corresponding to the four nucleobases, and manipulated them until he saw which ones could be paired to hold the double helix together (see Vallée-Tourangeau, 2014).

The key driver of thinking in these examples is interactivity, how features of the world that configure a certain problem are arranged and rearranged dynamically over time to evince a certain solution, to produce a desirable goal state. The work reported here shares a number of theoretical commitments with the seminal characterisation of the role of external representations in problem solving outlined in Zhang and Norman (1994), as well as the elegant experiments and modeling efforts on soft constraints that determine when the world or internal memory is consulted in a wide range of tasks (e.g., Fu, 2011; Gray, Sims, Fu, & Schoelles, 2006). While Zhang and Norman demonstrated how problem solving performance is facilitated when certain rules and dimensions are externally represented, they said little about interactivity as such, although it is at the heart of their participants' performance. In turn, the interactive behavior described in the resource allocation experiments and models supporting the soft constraints hypothesis, is one that reflects the quick iterative sampling of information from either an internal source—memory—and the world (Gray & Fu, 2004; Gray et al., 2006). These researchers did not address the role of interactivity in modifying the physical arrangements of a problem, and the contingent spatio-temporal itinerary that maps the problem's solution.

In the case of the river-crossing problem, interactivity as designed in the high interactivity condition, did not change the nature of the problem or reconfigure it in a more cognitively congenial manner (unlike, for example, in Tetris or Scrabble, see Maglio, Matlock, Raphaely, Chernicky, & Kirsh, 1999). What interactivity did was to promote a more fluid way to explore the problem space, involving as it were limited cognitive resources to enact changes. The state of the world—as modeled by the artefacts—cues the next move. What is particularly interesting then is the tight coupling between the agent and the world. The raft, animal figurines, and river board are better thought as configuring a world that is representative of the real world, not a representation of it (see Noë, 2012): participants in the high interactivity condition directly manipulated the world not unlike how scientists manipulate three dimensional models of molecules (see Toon, 2011; Watson, 1968). This coupling may be maintained by perception-action loops that may not be mediated by complex representations. In addition, this level of interactivity may be accompanied by a greater degree of task engagement. Svendsen (1991) reports that participants who experienced a greater degree of interactivity in a low implementation cost condition of the Tower of Hanoi enjoyed the task more, were more likely to recommend the interface, and believed it was quicker to use and solve the problem. The problem space that described the river-crossing problem employed in the low and high interactivity condition was the same. However, from an ecological perspective, the problems were different in the two conditions. The two presentations afforded a different behavioural repertoire, supported by different perceptual and cognitive processes. Thus, the cognitive ecosystem (cf. Hutchins, 2010a) implemented in the low and high interactivity condition was different, and important questions about learning and transfer can and should be raised in these different ecosystems.

Previous work on the river-crossing problem demonstrated learning across repeated presentation of the same problem; however evidence of learning transfer across analogous problems is more equivocal (Reed et al., 1974). Knowles and Delaney (2005), using computer generated images, also investigated learning in the river-crossing problem by attempting to improve performance through the reduction of illegal moves. However, what was being learnt was not made clear beyond offering the conjecture that learning reflected “enhanced rule verification skills” (Knowles & Delaney, 2005, p.679). But these additional skills were not independently assessed and measured, and the conjecture did not offer much beyond a redescription of the data. Unlike Knowles and Delaney, the focus of the experiments reported here was not on improving performance in the number of moves made with a cost manipulation;

rather it was to investigate how different levels of interactivity using artefacts, not computer generated images, influenced problem solving performance and learning.

The experiments presented here indicated that learning proceeds in both interactivity contexts. In addition, Experiment 1 offered a potentially interesting window on the nature of the transfer of learning from low to high interactivity: Participants performed the task fastest in a high interactivity context when they had first experienced it in a low interactivity context. Over repeated presentations of the same problem, it would be expected that performance would improve, as it did. However, a change in problem solving mode may mitigate the learning effect in that participants must learn a new way to play the game, as it were (Norman, 1993a). The data reported here suggested that it was easier to adapt when the change was from a low to a high interactivity condition, rather than the reverse. The embodied immediacy of the contact with the problem, unmediated by symbolic representations, favoured a quicker selection of moves, which resulted in a quicker solution of the problem when going from the low to the high interactivity context. However, adapting to a new task environment was more challenging when participants moved from a high to a low interactivity condition. The transfer from a high interactivity to a low interactivity condition resulted in a slower adaptation to the change in the task ecology because move selection was mediated through an indirect symbolic representation of the task. The results of the second experiment made clear that both interactivity conditions promoted learning, although participants remained quicker in the high interactivity condition. As previously speculated, procedural learning in the high interactivity condition may not facilitate transfer. However, learning in the low interactive context was likely more deliberative, resulting in enhanced declarative learning. As a result, participants could rely on a more explicit understanding of the problem, which could be implemented quickly in the high interactivity condition during the second presentation of the problem. Problem solving performance was more efficient if participants experienced its abstract version first and then encountered the problem with the opportunity of a greater degree of interactivity on the second attempt.

Lave's (1988) ethnographic research investigated the transfer of knowledge learnt at school as it is applied to activities in everyday life, such as supermarket shopping, revealed the different heuristics people employed in solving maths problems in practical everyday situations. Lave questioned the common view within education and psychology, that arithmetic strategies as learned in school are carried away by students from the supposedly context-free learning environment of the classroom, as transportable tools for direct application to practical situations and problems (see Chapter 2 of this thesis). She suggested that this assessment of learning was characteristic of an information-processing view of the individual as a "self-contained, disembodied technology of cognition" (Lave, 1988, p.17), which excluded the impact of context and setting as an individual acts in a particular situation. Therefore, Lave was interested in whether knowledge transferred from abstract traditional teachings in school to more practical, concrete situations in everyday life. Lave's study showed that individuals did use maths learnt in school—additions, subtractions, ratios. However, through experience and actions in the lived-in world these basic maths capabilities were transformed into practices and strategies that did not appear to reflect the strategies learnt in school, rather the situatedness of the person acting in the setting of the activity. The transfer asymmetry observed in Experiment 1 supported Lave's findings indicating that knowledge acquired through traditional abstract teaching methods followed by an interactive experience using manipulable artefacts may result in improved transfer and learning beyond the expected learning of repeated problem presentations. In reflecting on Greeno's (1989) conundrum of devising a situated framework for learning transfer, the asymmetry of transfer may also indicate a way forward in connecting the gap he identified between mental processes and external physical representations of objects and events (see Chapter 2 of this thesis). In general, the results from Experiment 1 suggest that there are interesting implications for a better understanding of problem isomorphs especially as teaching and learning tools. The experiments reported here may lay a foundation to address these concerns with empirical evidence indicating that learning an abstract concept followed by consolidation with concrete three-dimensional recognisable artefacts embodied with the same rules and constraints of the original concept significantly enhances learning.

O'Hara and Payne (1998) discussed the planning-acting continuum in analytic problem solving, and investigated the task, environment, operator, and agent contingencies that influence the amount of planning. Clearly, a high degree of interactivity may encourage quicker and more fluid action that shapes and reshapes the problem presentation without much prior planning. A lower degree of interactivity may force reasoners to think more carefully in the process of identifying the best move in a sequence. While a higher level of interactivity enhanced the performance of an individual in terms of how quickly moves were made, when measuring the number of moves taken to complete the problem, low interactivity encouraged better performance. Therefore, determining which level of interactivity better promotes learning can only be answered relative to considerations of efficiency themselves relative to a particular context of reasoning. In other words, the cost structure for a particular task is relative to a set of situated parameters: sometimes it is useful to think long and hard (e.g., carefully planning a move in chess) and sometimes it is best to act quickly (e.g., moving a zoid in Tetris). The efficiency metric is dependent on the situation: If it is costly to make moves, then it is important to invest time into the contemplation of each move to be made; if the number of moves made is unimportant, but time is of the essence, then acting quickly is the efficient use of available resources. As Lave (1988) observed in the supermarket, shoppers would readily abandon or switch purchases requiring difficult calculations: if the supermarket shelf offered an alternative item where the calculations were less complex, then the benefit of picking up the alternative product off the shelf exceeded the mental cost of the calculations. Still, there remains an important challenge for research on interactivity: Namely, to determine the nature of the learning and the knowledge representation evinced by different levels and modes of interactivity.

In the experiments presented here, the functional constraints and rules of the problem do not change between problem presentations; however, interactivity in the physical world alters the cognitive landscape across the distributed system. In offering a more interactively congenial problem presentation with the board and artefacts, the external representation of the river-crossing problem differed from the hands-down presentation. While to the researcher the assumption might be that the problems are the same; however, in changing the physical structure, it is possible from the perspective of the solver the tasks are quite different, resulting in differing performance outcomes and thinking trajectories to solution (Norman, 1993a). These experiments have shown that performance outcomes vary with interactivity. People naturally interact with the surrounding environment; frequently this interactivity goes unnoticed, with commentators on research failing to observe the impact of interactivity. The exploration of the thinking trajectories of the problem solver in the lab, when encountering a changing cognitive landscape across different problem presentations could potentially shed light on naturally occurring interactivity in the world. In addition the amount of transfer and learning were seen to be contingent on the sequence with which the problem presentation was experienced. These results suggest that outcomes for learning and problem solving may differ when using the body and the concrete world to explore the problem space. Education is rapidly advancing toward the use of computer centered learning; coupling bodies to a dynamic and modifiable world during learning, problem solving and decision making poses important pedagogical questions (Kirsh, 1997; Klahr, Triona, & Williams, 2007; Moreno & Mayer, 2007; Renken & Nunez, 2013). Interactivity is now often couched in terms of interaction with a computer interface, knowledge and skills learnt from that mode of interactivity need to be assessed against interaction with the lived in physical world. Future research on this front would likely yield findings with important pedagogical implications as well as offering guidance to researchers working on the innovation and learnability of scaffolding interfaces (cf. Bolland, 2011).

Chapter 7

Mental Arithmetic

Overview

The two river-crossing experiments discussed in the previous chapter illustrated how, by offering solvers different physical presentations for analogous problems, performance may change as a function of interactivity. The experiments showed the extent to which interactivity impacted the number of moves and latency to completion of the problem as well as the differences in learning with repeated presentation of the problem. These results showcased how from a distributed cognition perspective, through interactivity, the search process shapes and reshapes the problem space (Kirsh, 2009a).

The three experiments presented in this chapter continued the investigation of problem solving from a distributed cognition perspective, examining the impact of interactivity on mental arithmetic. Similar to Lave's (1988) motive for using maths in the Adult Math Project, these maths problems were chosen as useful tools for research into the lived-in world as using arithmetic is ubiquitous in everyday life.

In all three maths experiments the problems were long arithmetic sums with the number of digits varying between experiments. In addition predictors of performance including maths anxiety, working memory and engagement in the task were measured. The initial experiment presented the participants with sums comprising of 7 or 11 digits. All participants were tested in two levels of interactivity; one being a high interactivity condition using numbered wooden tokens and the other a hands-down paper based low interactivity condition. The second experiment used 11-digit sums with all participants experiencing four conditions; the hands-down paper condition, the wooden token condition, a pointing condition and a pen with paper condition. In the final experiment participants with varying levels of expertise in mathematics were asked to solve problems of 11 and 17 digits. The results support the outcomes from the river-crossing experiments indicating that interactivity with the environment affords opportunities to solve problems and achieve results not necessarily attainable in relying on internal resources of the mind alone.

Introduction

Mathematical problems are embedded in everyday life in a variety of different shapes and forms, with mental arithmetic commonly construed as a mathematical operation to be completed in the head by virtue of the word 'Mental'. However, in practice individuals, adults and children alike frequently use the world around them to complete even simple maths tasks. When confronted with an arithmetic task, people often rearrange the physical display of the problem by interacting with the environment. They might move coins while counting their money, note subtotals with a pen, use their hands to gesture or fingers to point or count (Carlson, Avraamides, Cary, & Strasberg, 2007; Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001; Kirsh, 1995b; Neth & Payne, 2001). Scripture (1891) explained how the eminent "calculators" (p. 2) of the day used artefacts when learning the fundamentals of maths. These maths prodigies described learning arithmetic from pebbles, peas, marbles, shot, and dominoes, at times without any awareness of rudimentary terms such as multiply. Their expertise in completing large calculations using simple times tables was acquired through the physical arrangement of these manipulatives.

Children appear to learn to calculate by using their fingers in conjunction with repeating the names of the numbers aloud (Butterworth, 2005). In addition Alibali and DiRusso (1999) found that while gesturing is an aid to encouraging counting accuracy in children, touching items when counting facilitates more accurate performance than simply pointing to countable items. Classrooms frequently use interactive instruction when introducing mathematical concepts to children (Fyfe, McNeil, Son, & Goldstone, 2014; Martin & Schwarz, 2005). Martin and Schwartz (2005) explain how children use objects resembling pie wedges as partitions of a pie, when learning to calculate fractions. These pie wedges are used as tokens to indicate how to partition out or subdivide the pie into say quarters. Just as children use artefacts to progress through stages of development when learning complex maths skills, it is possible to draw parallels with Palaeolithic artefacts, such as calendars and clay tokens, as contributors toward the evolution of human numerical concepts (see Figure 7.1; De Smedt & De Cruz, 2011; Malafouris, 2013).

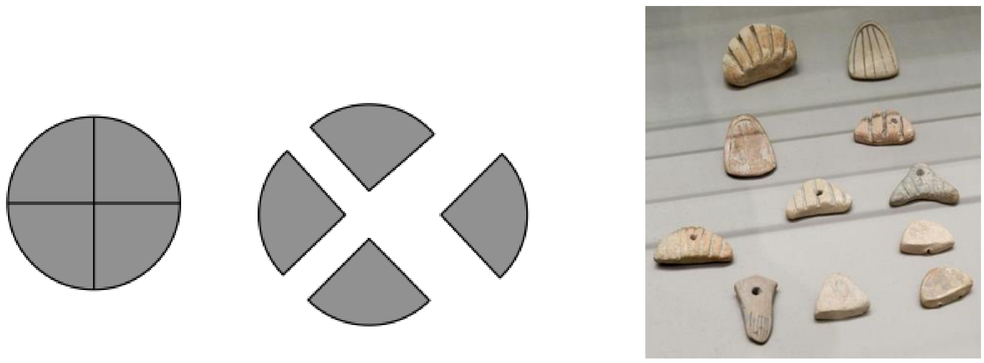

Figure 7.1. The images on the left and center provide representations of pie and wedges used as manipulatives when learning fractions in a classroom. The images on the right are clay accounting tokens from the Uruk period (4000 B.C.–3500 B.C). Copyright 2009 Marie-Lan Nguyen / Wikimedia Commons / CC-BY 2.5.

In tracing the evolution of human numerical cognition over millennia, it becomes apparent that sociocultural influences have gradually stimulated the development of innate abilities for basic approximation of magnitude and quantity toward highly developed skills in counting and complex calculations.

Pre-verbal human infants and nonhuman animals alike display strong basic numerical intuition for changes in quantities of up to four items (Butterworth, 2005; Malafouris, 2013; Spelke, 2000). However, it appears that humans alone have moved beyond this basic system of approximation to more exact numerical cognitive processes. It has been suggested that this innate number sense or approximate number system (ANS) is strongly linked to particular neural substrates (Dehaene, Piazza, Pinel, & Cohen, 2003; Malafouris, 2013). Neuroimaging techniques have identified three areas within the parietal lobe that are systematically activated during number processing: the horizontal segment of the intraparietal sulcus (HIPS); posterior parietal lobule (PSPL); and the left angular gyrus (AG) (Dehaene et al., 2003). The findings by Dehaene et al. (2003) were consistent across different participants, a range of nationalities and differing educational achievements in mathematics leading them to hypothesise that the internal mechanisms for numeracy are biologically determined as "pre-existing cerebral circuits" (p. 499). Further they postulate that these internal mechanisms have consequently served as foundations for the cultural construction of arithmetic over time.

In isolating the HIPS region, Dehaene and colleagues found there was consistent activation during mental arithmetic in particular with the emphasis on quantity processing. Although not conclusive, further testing identified this region as potentially being numerically domain specific. They proposed that management by this system of numerical quantities could be likened to an internal spatial map or a mental number line (MNL) on which "numbers are organised by their proximity" (Dehaene et al., 2003, p. 498). The other two regions, the AG and the PSPL, were identified as possibly supplementing the "core quantity system" (Dehaene et al., 2003, p. 487) identified in the HIPS.

Unlike the HIPS, increased activity in the PSPL did not indicate domain specificity for numbers, as this part of the brain also plays a central role in visuo-spatial tasks such as eye movements, hand movements (including reaching, grasping and pointing), and spatial working memory. Therefore the PSPL may play a role in the spatial orientation of number based attention on a MNL. In other words, focussing attention on say deciding which is the larger of two quantities, or putting numbers in order of magnitude.

In reassessing these conclusions, with an eye to the interactions between the mind, body, and environment, the evidence presented by Dehaene et al. (2003) supports the notion that interactivity has an impact on mathematics. Dehaene and colleagues conjecture that fundamental number mechanisms, such as the mental number line, may be in place within the brain. However, it was only through the cultural evolution of this innate numeracy that arithmetic came into being within societies, directly implicating the role of interaction with the world beyond the brain. In addition, the speculation of the existence of an MNL deduced from their evidence of the overlapping activation within the parietal lobe between numeric tasks and visuo-spatial tasks, suggests there is an interaction between the internal domain of mentality, the body, and the external environment. These pre-existing cerebral circuits appear to be activated by interfacing with the physical world (Malafouris, 2013).

From a cognitive archaeological perspective, Malafouris (2013) presents further evidence that interfacing with the material world drove the transition from an innate basic sense of numbers to the human ability for more exact and complex calculations. The development of explicit and complex calculations, Malafouris argues, is not attributable to any preceding physiological adaptations in cognitive structure, but rather to embodied experience supported by changes in the medium representing numbers and socio-temporal conditions. He identifies clay tokens and tablets in the Neolithic New Eastern accounting systems (7000-3000 B.C.) for recording and computing expenditure as one of the earliest examples of artefacts connected to the development of numerical thinking. The concept of numbers was yet to be developed in these early stages of counting, thus each clay token was a physical representation of an approximate quantity of a specific category of item. This made it possible to approximate the quantity of many different items beyond the narrowness of counting in just ones, twos, or threes. The clay tokens were manipulable, visible, and tangible—physical properties that provided impetus for the emerging process of counting. This exemplifies the role of materiality in affording the bridge between embodiment and the world as the clay tokens potentially provide the stimulus to build neural connections that otherwise may not have been possible. This interaction between artefacts and innate basic numerosity promotes a bringing together of the mind and the world to embody an “extended system of numerical cognition” (Malafouris, 2013, p. 116).

Thinking in the World

Mental arithmetic tasks often entail strategic, deliberate, and effortful thinking. Besides basic, well-rehearsed sums, computations can place high demands on limited working memory storage capacity and processes (Ashcraft, 1995; Butterworth, 2006; DeStefano & LeFevre, 2004). Numbers are held, added, and manipulated in order to solve the problem drawing on working memory resources and executive functions. In reducing the load on internal resources, people naturally mine their external surroundings in order to augment cognition (Kirsh, 2013). Cognitive processes migrate to wherever computations are most easily performed, extending to external resources in a dynamic distributed cognitive system (Kirsh, 2013). The physical actions of an individual within the environment are not only integral in distributing working memory load, they also provide a scaffold that can enact new strategies and expand the range of cognition (Gray & Fu, 2004; Kirsh, 2013; Vallée-Tourangeau, 2013). Thinking does not simply take place 'in the head' but rather emerges from an interaction with artefacts in the world. In other words, "cognition has no location" (Malafouris, 2013, p. 85), it is a dynamic interplay between internal and external resources.

The cognitive and physical resources deployed to tackle a problem may be also taxed by various features of the task—such as time pressure, level of difficulty, and fatigue. Reasoners naturally recruit artefacts and use the physical space in which they are situated to make thinking easier and more efficient (Kirsh, 1995a). This interplay between the cognitive and motor system has been associated with improvements in performance, indicated by increased accuracy and speed (Goldin-Meadow et al., 2001). Movement execution such as nodding, pointing and the manipulation of a problem's spatial arrangement help to surpass the original limitations of working memory capacity thus lowering the expense of internal resources necessary to solve the task and guide attention (Goldin-Meadow, Alibali, & Church, 1993; Vallée-Tourangeau, 2013). Kirsh (1995b) describes an organising activity that recruits external elements to reduce the load on internal resources as a complementary strategy to internal mental processes. Therefore, interacting with the environment and utilising artefacts can improve performance by distributing the storage and computational demands of the task across resources internal and external to the reasoner. Such distributed cognitive processes shift the cognitive load from the reasoner onto a system in which she is embedded (Vallée-Tourangeau, 2013).

The distribution of the computational cost across resources can enhance performance when solving a problem. However, this distribution of cognition concomitant with the dynamics of interactivity can also enact the application of different problem solving strategies, which can improve performance. Yet, it is not only the problem itself or its complexity that impacts how accurately or efficiently an individual performs in a mathematical task. The physical features of the problem presentation can guide behaviours and strategic choices in the path to a solution (Vallée-Tourangeau, Euden, & Hearn, 2011; Zhang & Norman, 1994). One of the outcomes of Lave's Adult Maths Project (1988) was to illustrate, through observations of everyday maths practices, how individuals employed different strategies when solving similar problems across differing settings. People solved best-buy problems in the supermarket, underwent school-based maths tests and solved simulated best-buy problems in their home. The setting of the supermarket offered an environment rich with opportunities to interact with the physical world, coupling internal capabilities with external resources as part of a dynamic problem solving system. The maths problems presented to shoppers for assessment of maths practice across settings were based on comparing ratios, although this was not transparent in the best-buy problems in the supermarket, the school-based arithmetic test ratio problems were not disguised. The solvers used different strategies and were more accurate in the supermarket best-buys, than when solving the same maths ratio problems in the maths tests where the problem presentation was altered. However, solvers were almost as accurate in calculating best-buys when undertaken in a simulation experiment in their home as that in the supermarket, and were more accurate than when completing the school-like maths tests also performed in their home. This suggested that the presentation of the problem as well as the setting might affect the strategy and performance of individuals. In turn, a hypothesis was extrapolated from Lave's findings that the same problem presented in a different way, in the same setting, might prompt different strategies on the path to solution. Therefore, the intention of the experiments discussed here is to show that the affordances offered by varying the cognitive landscape potentially alter the dynamic cognitive system.

Distributed thinking. Cognitive psychologists have long recognised the importance of the environment in shaping the content and function of cognitive processes with their investigations taking differing paths (e.g., Greeno 1989; Simon, 1996; Zhang & Norman, 1994). One research strategy is to explore problem solving performance across superficially different but structurally isomorphic problems. Using isomorphic problems makes it possible to retain the same problem space while at the same time changing the cover story by varying the rules or move operators. It has long been suggested that these changes can cause individuals to construct different (internal) problem representations resulting in differences in the solution process (Hayes & Simon, 1977). Zhang and Norman (1994) demonstrated the importance of external representations in an elegant series of experiments on transformation problem solving (using Tower of Hanoi isomorphs). Problems were structurally isomorphic but were presented with different objects. In using different artefacts the external rules and constraints for the problem were changed, which had a substantial impact on problem solving performance. However, in Hayes and Simon (1977) and Zhang and Norman (1994) the role of interactivity in shaping and reshaping the external problem presentation is largely ignored. Embedded in the problem presentation are the varying possibilities for interaction, the nature of these interactions having the potential to direct strategic choices (Neth & Payne, 2001; Kirsh, 2013). Through manipulations of artefacts, a dynamic loop of information and action flows between a person and the outside world, new perspectives are observed leading to new strategies improving the prospect of problem solving (Magnani, 2007). Processing of the problem is shared between the environment, and the body and mind of the agent, configuring a distributed thinking system (Kirsh, 1995a).

Mental Arithmetic

Previous research on mental arithmetic has investigated gesturing (e.g., Goldin-Meadow et al., 2001), interactivity, and additions utilising a computer interface (e.g., Neth & Payne, 2001; Neth & Payne, 2011), interactivity and working memory (e.g., Vallée-Tourangeau, 2013) and simple coin counting strategies (Kirsh, 1995b). Results indicated that interactivity influences performance and the ways by which participants achieve solutions. However, the picture is piecemeal, fragmented by different methodologies, and no study as yet has compared a wide range of different types of interactive behavior using artefacts. Consequently, the following experiments investigated the role of interactivity in adult participants using tangible artefacts with which the problem presentation could be modified through the completion of an arithmetic task in the form of simple additions. The decision to use simple additions for the exploration of interactivity in mental arithmetic was motivated by three considerations. First, a task was chosen requiring only fundamental arithmetic skills well within the expected capabilities of university undergraduates. This is particularly important since participants who experience mathematics anxiety might feel threatened by more difficult arithmetic calculations resulting in withdrawal from the experiment. Second, artefacts, such as tokens and pen and paper that closely resemble commonly used items, were artefacts that could be introduced in the experimental session with relative ease and efficiency. Finally, the relative length of the sums would make it possible to map the participants' progress in terms of interim totals and hence map the trajectory to solution.

As discussed earlier, 'Mental' arithmetic in common parlance is construed as mathematical operations accomplished 'in the head'. The use of the word 'mental' in the term 'mental arithmetic' when referring to maths tasks using artefacts may appear to be inaccurate when the process involves external resources. 'Mental' in the context of these problems is the calculation of sums without the aid of a device such as calculator, which would greatly diminish the load on internal resources, such as working memory, and indeed shoulder most if not all the computation complexity and cost. The calculator, in essence, would be doing the computations for the problem solver. However, in the case of the maths tasks presented here, interactivity relocates part of the computational load to the external world by offering a scaffold for the problem solver to shape and reshape the path to solution. The cognitive load is distributed between the agent and the world, thus the problem solver is computing the answer and not the artefact.

Fundamental to the understanding of interactivity as an element of distributed cognition, is that interaction is "essentially nonlinear and loopy, making it impossible to clearly demarcate an 'inner' domain of mentality from an 'outer' domain of environmental causal factors" (De Jesus, 2015, p. 393; Hutto & Myin, 2013). Thus all problems are solved within this looping process, meshing resources offered by brain, body, and environment.

Individual Differences

As discussed in Chapter 5 profiling individual differences provides an insight into determining different internal resources that might predict performance in problem solving and reasoning (Stanovich & West, 1988). Factors such as anxiety, self-efficacy, and expertise in mathematics have been shown to contribute to differing levels of arithmetic performance (Butterworth, 2006; Hembree, 1990; Hoffman, 2010; Moore, Rudig, & Ashcraft, 2015). It has been suggested that an individual's inclination toward the need to engage in effortful thinking may contribute to improved performance in problem solving, reasoning, and decision making (Cacioppo & Petty, 1982; Cacioppo, Petty, Feinstein, & Jarvis, 1996; Nair & Ramnarayan, 2000). In addition, a person's engagement when undertaking a task may also be relevant to problem solving performance; with a positive approach toward the task contributing favourably to the problem-solving activity possibly resulting in deeper comprehension (Newmann, Wehlage, & Lamborn, 1992; Schiefele & Csikszentmihalyi, 1995; Storbeck & Clore, 2007).

The Current Experiments

Here, a series of three experiments are reported investigating the effect of interactivity on mental arithmetic, with participants completing sets of simple sums in varying modes of interactivity. Increased levels of interactivity have been linked to better performance, possibly due to a stronger focus of attention and better distribution of the load on internal resources (Carlson, Avraamides, Cary & Strasberg, 2007; Goldin-Meadow, Nusbaum, Kelly & Wagner, 2001; Vallée-Tourangeau, 2013). In a previous experiment, Vallée-Tourangeau (2013) showed that in changing the physical presentation of a problem it is possible to improve an individual's performance when solving simple additions. In all three experiments participants completed the sets of simple arithmetic sums in at least two reasoning contexts: One being a low-interactivity context where participants were shown a random configuration of numbers, and were asked to calculate the sum with hands flat on the table. In a second, high-interactivity context, the same configurations were presented with number tokens, and participants were free to move them and rearrange the problem presentation as they calculated an answer. The sums comprised of varying combinations of the numbers 1 to 9, to create a mix of problems ranging in difficulty. The length of the sums for the different experiments ranged from seven single-digit additions to seventeen single-digit additions. Within each experimental session a number of individual difference measures were interleaved with the arithmetic tasks as potential predictors of performance.

The first experiment was designed to further explore the findings of Vallée-Tourangeau (2013) by examining the possibility that altering the problem presentation using two differing modes of interaction, namely low interactivity and high interactivity, may alter the performance in arithmetic of those folk that are maths anxious or have low sense of maths self-efficacy. The integration of artefacts in Experiment 1 offered reasoners the opportunity to reconfigure the physical presentation of the problem, enacting different arithmetic strategies. The artefacts appear to enable a shift in the affordance landscape as the problem trajectory is enacted through interactivity. A change in the type of interactivity or artefacts used may offer insight into whether it is simply an increase in the interaction with the environment or the affordances offered by different artefacts that affects performance. Experiment 2 builds on these findings exploring the affordances offered by alternative interactive modes. Two additional types of interactivity were introduced thus generating four modes of interactivity to assess performance and differing strategies prompted by varying the problem presentation. This experiment also investigated the effect of the differing modes of interactivity on participant engagement in the maths tasks using the task engagement scale (TES). The third experiment explores how expertise in mathematics, maths anxiety and task engagement contributed to mental arithmetic performance in a high and low interactivity context.

Measures of performance*.* Performance in each of the three experiments was measured in terms of four dependent variables: accuracy, latency, absolute deviation error and efficiency. Accuracy was measured as the number of sums correctly solved by the participant; latency being the time taken to complete the answer; absolute deviation error (ADE)[6] was calculated as the absolute deviation from the correct solution, for example if the correct solution was 40 an incorrect answer or 42 or 38 would result in an absolute deviation error of 2; efficiency, as explained in detail below, was operationalised as the ratio of the proportion of correct answers for a set of sums divided by the proportion of time (out of the maximum time) invested in completing the sums.

Latency**.** To gain an accurate measurement of latency the researcher placed a screen (a box measuring 24cm by 37cm) on the table between the sums and the participant. The researcher explained to the participant that this screen was used as an occluder behind which the researcher would prepare the maths task before revealing it to the participant. Once the maths task was ready the researcher removed the screen exposing the task and simultaneously asked the participant to “start”. The participant had previously been instructed to announce the answer to the researcher once a solution had been reached.

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[6]Absolute deviation error was termed the absolute error in Vallée-Tourangeau, 2013.

Thus, the latency was the number of seconds taken by the participant to announce the answer after the screen had been removed.

***Efficiency*.** Efficiency in cognition potentially offers a more complex window onto performance than the measures of accuracy, latency, and absolute deviation error. Hoffman (2012) defines cognitive efficiency as "qualitative increases in knowledge gained in relation to the time and effort invested in knowledge acquisition" (p. 133). Efficiency has the potential to be a useful tool for measuring performance as it provides a greater understanding of the amount of time and effort required to master a skill or complete a task. This understanding may help inform educators when improving teaching techniques in order to enhance the performance of learners. Researchers might also benefit from investigations into efficiency when attempting to determine the effect of working memory and executive functions on areas such as problem solving (Hoffman & Schraw, 2010). In determining efficiency for the purposes of these experiments it was calculated as a ratio of the proportion of correct answers for a given problem set over the proportion of time invested in solving that set (out of the longest time the slowest participants invested in solving that set). For each of the four conditions, participants were first ranked according to their averaged latencies. The average of the slowest 25% served as a reference point and represented the maximum effort one could expend in that condition. Thus the efficiency ratio denominator was a given participant's latency over the average latency for the slowest quartile; the numerator was that participant's proportion correct solutions in that condition. For example, a participant in a given condition may have solved three out of the five sums, for a proportion .6 correct. In turn, the participant's average latency for completing the five sums in that condition might have been 30 seconds. If the average latency for the slowest quartile was 40 seconds, then that participants invested 75% (30/40) of the total possible time for completing the sums in that condition. The efficiency ratio for that participant would then be .6/.75, or .8. Ratios at or above 1 reflect efficient reasoning; ratios below 1 reflect inefficient reasoning.

***Preferred measure of performance*.** To better focus on the predictors of performance one of the four measures, accuracy, latency to completion, absolute deviation error and efficiency was selected for final analysis. Absolute deviation error was chosen for further analysis in order to capture the most important aspect of arithmetic performance: the solutions themselves. Accuracy, although a rudimentary gauge of performance, as a binary measure producing a correct or incorrect solution was considered a comparatively coarse-grained measure. In addition, accuracy does not discriminate between small and large deviation errors and latency to completion does not reflect the participant's answers.

It is anticipated that different strategies enacted through interactivity would be observed, and hence performance would be best in the conditions that afforded rearrangement of the initial problem presentation. Therefore when individuals are given an opportunity to interact with a dynamic physical problem presentation performance may be enhanced. This will be manifested in the high interactivity condition by improved solution accuracy, slightly more time taken to completion due to token movement, fewer calculation errors, and increased efficiency.

Predictors of performance. The predictors of performance are also expected to reflect the benefits of increased interactivity. The reduced load on working memory should improve the performance of the maths anxious individuals by freeing up internal resources otherwise consumed by the rehearsal of anxious thoughts. In turn, a reduction in maths anxiety is expected to assist in alleviating low self-expectations of performance and improve maths self-efficacy, thereby yielding improved problem solving outcomes. Using artefacts to distribute the thinking may also afford the perception that a problem is easier to solve when using familiar objects thus raising the level of perceived self-confidence resulting in improved problem solving outcomes (Hoffman, 2010). An individual's need for cognition was expected to dissipate in the high interactivity condition, as the use of artefacts would possibly reduce cognitive load (Cacioppo & Petty, 1982). Expertise may predict deviation error in the low interactivity condition, that is, a higher degree of expertise may be linked with a lower deviation error; however, the high interactivity condition is expected to augment the skills of those with low expertise resulting in reducing any discernable difference in performance based on an individual's maths knowledge or experience. Finally, the measurement of flow is anticipated to predict greater task engagement in the high interactivity condition than the low interactivity condition as participants experience greater control over the environment in which they are working. It may also be the case that the negative feelings of those maths anxious folk when completing arithmetic calculations may be reduced when using tokens, as attention is redirected away from the self toward the artefacts. Although it is predicted that participants will report being more engaged in the task when the interactivity level is higher, it is not anticipated that any improvement in maths performance, when the level of interactivity is increased, would be attributed to flow.

Mental Arithmetic: Experiment 1

Mental Arithmetic and Interactivity

Introduction

This first experiment in a series of three mental arithmetic experiments was designed to provide a basis from which to explore the differences in an individual's performance in reaching solutions to simple addition problems when presented with two different modes of interactivity. Vallée-Tourangeau (2013) showed that an individual's efficiency and accuracy when completing simple 11-digit sums was elevated in a learning environment that encourages interactivity. In addition participants' levels of maths anxiety and maths self-efficacy were assessed to establish any relationship between performance in two differing modes of interactivity when completing short and long sums.

It is anticipated that a change in the mode of problem presentation by way of increased interactivity, when using the tokens over the more static, low interactivity presentation offered in the paper condition in calculating the solution, will foster an enhanced performance. Individuals are expected to solve more sums correctly, deviate less from the correct answer, and execute the problems more efficiently in the token condition. In reducing the load on internal resources such as working memory it is also expected that an intervention in the form of increased interactivity when solving the sums will benefit those with high maths anxiety and low maths self-efficacy when using the tokens. These anticipated improvements in performance emerge as the participants use the tokens to reach a solution, distributing the computation process across the internal and external domains of the problem space.

Method

Participants

Ninety-one undergraduate and postgraduate students (80 females, overall M_{age} = 24.0, *SD* = 7.53) received course credits for their participation.

Materials and Measures

Maths anxiety. Maths anxiety was measured using the Mathematics Anxiety Scale–UK (MAS-UK; Hunt, Clark-Carter, & Sheffield, 2011). This 23-item scale measures maths evaluation anxiety (e.g., “being asked to calculate £9.36 divided by 4 in front of several people”), everyday/social math anxiety (e.g., “working out how much time you have left before you set off to work or place of study”) and maths observation anxiety (e.g., “listening to someone talk about maths”). For each of these items, participants rate how anxious they would feel using a 5-point scale, anchored at 1 (“not at all”) and 5 (“very much”). The maths anxiety score for a participant was the sum of the ratings across the 23 items and could range from 23 to 115; the mean maths anxiety score was 55.5 (*SD* = 15.1; Cronbach’s α = .91).

Maths self-efficacy. This scale developed by Betz & Hackett (1983) consisted of 18 questions evaluating maths self-efficacy (MSE) by asking an individual how much confidence he or she would have in successfully undertaking particular maths related problems. Problems such as “Figure out how long it will take to travel from London to Cardiff driving at 40 mph” and “Add two large numbers (e.g., 5379 + 62543 in your head”. Participants were asked to rate their confidence on a 9-point scale (1 = “No confidence at all” and 9 = “Complete confidence”). The responses were summed to produce a maths self-efficacy score out of 162; the mean MSE score was 87.3 (*SD* = 27.4; Cronbach’s α = .91).

Numeracy. Numeracy was measured using a subjective numeracy scale and an objective numeracy scale.

***Subjective numeracy*.** The subjective numeracy scale was designed as a self-assessment measure of numeracy (Fagerlin et al., 2007) consisting of eight questions (such as ‘‘ how good are you at working with percentages”). Participants responded using a 6-point scale (1 = ‘‘ not good at all’’ and 6 = ‘‘ extremely good’’). The subjective numeracy score was computed as a sum of the responses out of a total of 48; the mean score was 29.4 (*SD* = 13.2; Cronbach’s α = .77).

Objective numeracy. A basic arithmetic scale (BAS) was used to test participants’ objective numeracy. It consisted of 45 simple arithmetic problems which included additions, subtractions, multiplications (such as 3 × 3 = ?), and did not include divisions. Participants were required to write the answers on the paper provided, in the order presented, completing as many as possible in 60 seconds. The score was calculated as the correct number of solutions with a mean score of 28.5 (*SD* = 10.8).

Working memory. The computation-span task (Ashcraft & Kirk, 2001) tests both processing and storage of numbers in working memory. Participants were required to answer simple arithmetic problems (e.g., 2 + 8 = ?, 12 – 4 = ?), before recalling the second number of these problems (e.g., 8, 4). Sequences of equations ranged from 1 – 7 and participants had to process each sum *and* recall the relevant digit correctly to score a point. The mean for computation-span was 26.1 (*SD* = 9.2).

Need for cognition scale. The thinking diligence scale by Cacioppo and Petty (1982) was presented as a pen and paper task. Participants were asked to rate 18 statements on the satisfaction they gained from thinking.

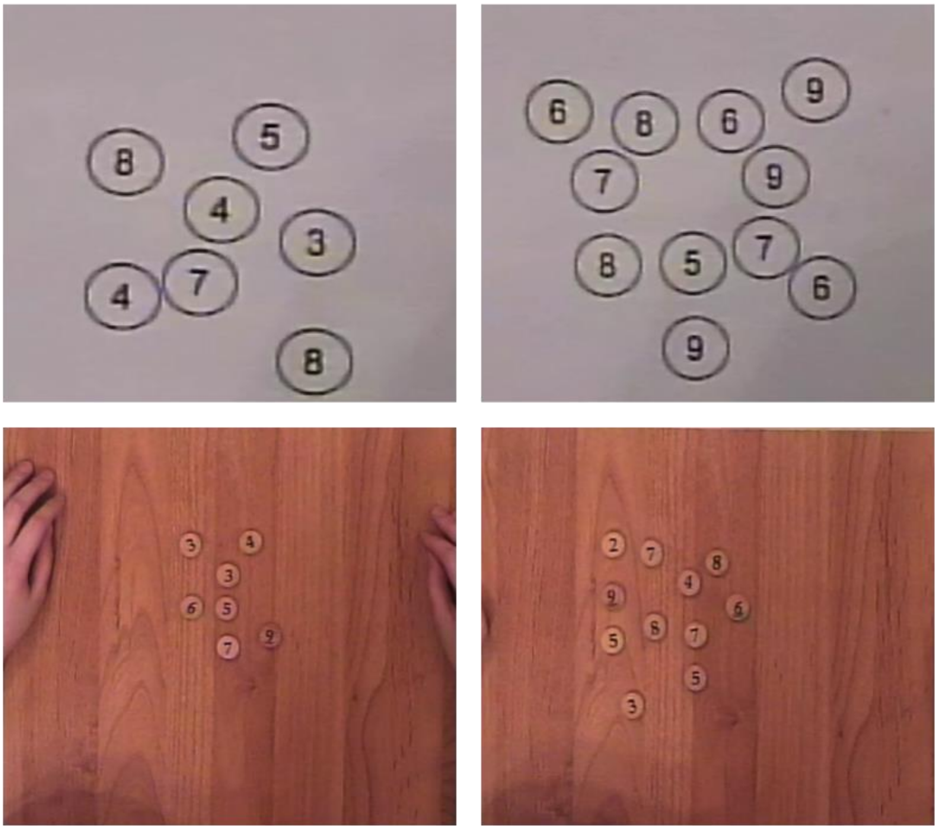

Figure 7.2. Examples of single-digit additions from the short sums (7 single-digit additions on the left panels) and the long set (11 single-digit additions on the right panels) shown in the low interactivity condition (top panels) and the high interactivity condition with the wooden tokens (bottom panels). Participants completed 5 additions from both sets for a total of 10 with problems presented in a randomized order for each individual.

Arithmetic task. All participants were presented with problems in two interactivity conditions. A low interactivity condition (see Figure 7.2, top panels) which consisted of the task printed on a single page of white A4 paper and a high interactivity condition where the format of the problem was presented using round numbered wooden tokens (see Figure 7.2, bottom panels). The mental arithmetic task was composed of 10 single digit additions consisting of five short sums (7 numbers; see Figure 7.2, left panel) and five longer sums (11 numbers; see Figure 7.2, right panel). Tokens were randomly distributed in order to reduce predetermined pathways to adding up the digits. The arrangement of the sums was designed such that the numbers were not presented in a grid or linear structure that would otherwise facilitate allocation of attention and help participants to scan rows or columns in a systematic manner. Adding up digits on the page in a random order rather than in a row would be a greater burden working memory, as not only did provisional totals need to be held, the digits already included in the addition would need to be retained.

As detailed previously performance was measured in terms of the accuracy, latency to solution, absolute deviation error, and efficiency.

Procedure

The arithmetic tasks and other scales (as described above) were embedded in an experimental session that lasted approximately 60 minutes. The presentation order of the MAS-UK was counterbalanced across participants such that it was completed either at the start of the session or at the end of the session with the arithmetic tasks separated by at least one of the scales. The five short and five long sums were mixed to create a set of 10 sums with the order of presentation being randomised for each participant. Participants completed these sums twice, once in a low interactivity condition, and once in a high interactivity condition. Thus, for one presentation participants performed the additions with their hands on the table facing them (the low interactivity condition) and announced their answer out loud. For the second presentation, round numbered tokens (2.2cm diameter) were used, arrayed in a manner identical to the presentations employed in the low interactivity condition; participants were encouraged to move the tokens in whatever manner to help them add the numbers. As in the low interactivity condition, once participants were done moving the tokens to arrive at a solution, they announced the solution for each problem out loud. The order of condition (low interactivity, high interactivity) was counterbalanced across participants. Thus, the length of the additions (7 or 11 digits) and level of interactivity (high, low) were repeated measures factors the factorial combination of which yielded four conditions.

Results

Accuracy

The mean number of correct answers (accuracy) is shown in the top left panel of Figure 7.3. In completing the 7-digit sums participants' accuracy appeared to be slightly improved by interactivity, with even greater accuracy in the high interactivity condition for the 11-digit sums than in the low interactivity condition. A 2 (interactive condition: high, low) by 2 (sum length: 7 digits, 11 digits) repeated measures analysis of variance (ANOVA) showed that the main effect of condition was significant, $F(1, 90) = 11.9$, $p = .001$, $\eta_p^2 = .117$. The main effect of sum length was also significant, $F(1, 90) = 39.8$, $p < .001$, $\eta_p^2 = .307$, however, the condition by sum length interaction did not reach significance, $F(1, 90) = 3.32$, $p = 072$, $\eta_p^2 = .036$.

Latency

Participants were slightly slower in the high interactivity condition for both sum lengths with the 11-digit sums taking longer to complete than the shorter 7-digit sums as would be expected (see Figure 7.3, top right panel). A 2×2 repeated measures ANOVA indicated that there was a significant main effect of condition, $F(1, 90) = 24.3$, $p < .001$, $\eta_p^2 = .212$, with the main effect of sum length also being significant, $F(1, 90) = 455$, $p < .001$, $\eta_p^2 = .835$. There was no condition by sum length interaction, $F(1, 90) = 2.64$, $p = .108$, $\eta_p^2 = .028$

Absolute Deviation Error

In the bottom left panel of Figure 7.3 it is evident that the magnitude of the errors made in calculating the correct answers in both conditions was very similar for the shorter sums. However, the high interactivity condition systematically produced smaller errors than in the low interactivity condition when participants completed the longer sums. A 2×2 repeated measures ANOVA revealed the main effect of condition to be significant, $F(1, 90) = 12.4$, $p = .001$, $\eta_p^2 = .121$. The main effect of sum length was also significant, $F(1, 90) = 40.2$, $p < .001$, $\eta_p^2 = .308$ as was the condition by sum length interaction, $F(1, 90) = 15.9$, $p < .001$, $\eta_p^2 = .150$. Post hoc tests confirmed there was no significant difference between interactivity conditions for the shorter sums, $t(90) = 1.11$, $p = .271$ however there was a significant difference between conditions when participants completed the longer 11-digit sums, $t(90) = 4.05$, $p < .001$.

Efficiency

The efficiency ratios are reported in the bottom right panel of Figure 7.3. When participants experienced short additions in the high interactivity condition they appeared slightly more efficient than when calculating the totals for similar sums in the low interactivity condition. This difference in efficiency increased with the longer sums individuals appeared considerably more efficient in the high interactivity than in the low interactivity condition. However, 2×2 repeated measures ANOVA showed that the main effect of interactivity was not significant, $F(1, 90) = 3.63$, $p < .060$, $\eta_p^2 = .039$ with the main effect of length being significant, $F(1, 90) = 29.9$, $p < .001$, $\eta_p^2 = .249$, and the condition by sum length interaction also reaching significance $F(1, 90) = 4.05$, $p = .047$, $\eta_p^2 = .043$.

Post hoc tests confirmed there was no significant difference between interactivity conditions for the shorter sums, $t(90) = .475$, $p = .636$, however there was a significant difference between conditions when participants completed the longer 11-digit sums, $t(90) = -2.47$, $p < .016$.

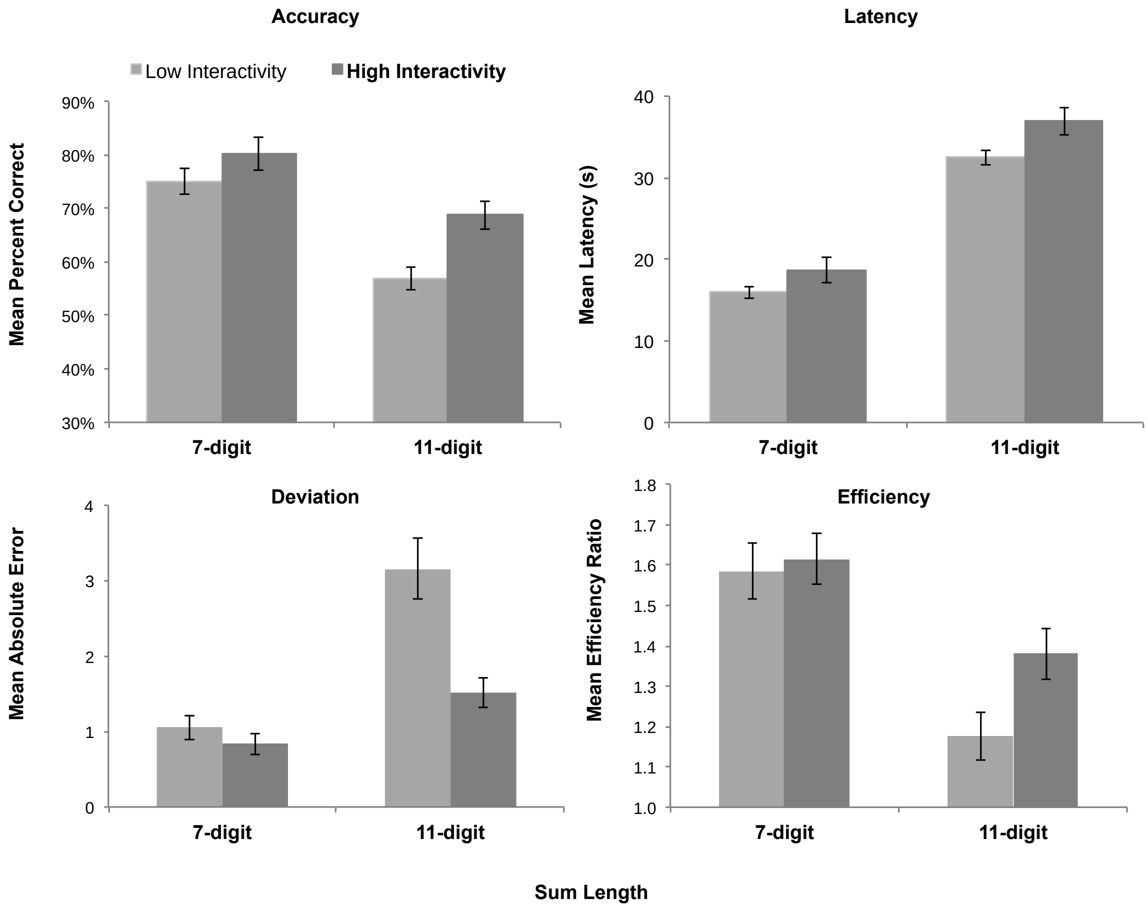

Figure 7.3. Mean percent correct (top left), mean latency in seconds (top right), mean absolute deviation error (bottom left) mean efficiency (bottom right) as a function of sum length (7-digit and 11-digit sums) in the low (light grey bars) and high (dark grey bars) interactivity condition. Error bars are standard errors of the mean.

Individual differences

In order to examine any possible influence of dispositions and cognitive capacities on performance, the correlations between performance measures and individual differences (viz. maths anxiety, maths self-efficacy, subjective numeracy, objective numeracy, and working memory) were calculated (see Table 7.1). There was no significant pattern of correlations for Need for Cognition with any performance measures.

The absolute deviation error was used as the preferred performance measure for investigation into the relationship with predictors of performance (as previously discussed in this chapter). Initial correlation analyses over all length and interactivity conditions revealed that correlations between absolute deviation error and individual difference measures were not influenced by sum length. Therefore the mean deviations in the 7-digit and 11-digit sums were averaged for each participant to create a new variable average absolute deviation error, which will be continue to labeled ADE but will no longer be segmented by sum size.

As would be expected, MAS-UK being a measure of anxiety in dealing with maths related problems in academic and daily life correlated moderately with the objective measure of numeracy. MAS-UK also correlated moderately with self-report measures of maths ability, MSE and subjective numeracy (see Table 7.2; Richardson & Suinn, 1972). While no significant relationship was detected between absolute deviation error and maths anxiety in either interactivity condition (see Table 7.1), scatterplots for all four experimental conditions revealed that an increase in sum length from 7 digits to 11 digits induced a trend toward a positive correlation between the two variables in the low interactivity condition (see Figure 7.4).

Notably MSE correlated with average absolute deviation error in the low interactivity condition, whereas there was an absence of correlation between this measure in the high interactivity. There was a similar pattern for subjective numeracy. In confirmation of the findings by Betz and Hackett (1983) maths self-efficacy also moderately inversely correlated with maths anxiety. Objective numeracy and the measure of working memory, C-Span, both correlated inversely with absolute deviation error in both the low and high interactivity conditions.

Table 7.1

Correlation matrix for average deviation error for 7-digit and 11-digit sums, individual difference measures of maths anxiety, maths self-efficacy, subjective numeracy, objective numeracy, working memory (computation-span) in both interactivity conditions (df = 89)

	1 MAS	2 MSE	3 Subj-N	4 Obj-N	5 C-Span	6 ADE-Avg-L	7 ADE-Avg-H
1	-	-.423 **	-.384 **	-.342 **	-.079	.132	.009
2		-	.190	.229 *	.026	-.253 *	-.118
3			-	.291 **	.260 *	-.288 **	.090
4				-	.370 **	-.448 **	-.336 **
5					-	-.322 **	-.245 *
6						-	-.231 *
7							-

Note. **MAS** = Maths anxiety; **MSE** = Maths self-efficacy; **Subj-N** = Subjective numeracy; **Obj-N** = Objective numeracy; **C-span** = Computation-span; **ADE-Avg-L** = Average absolute deviation error in the low interactivity condition; **ADE-Avg-H**= Average absolute deviation error in the high interactivity condition. * p < .05, ** p < .01.

Table 7.2

Significant correlations including confidence intervals for individual difference measures of maths anxiety, math self-efficacy, subjective numeracy, objective numeracy, working memory (computation-span) (df = 89).

Individual Differences				
Variable 1	Variable 2	*r*	*p*	95% CI
MAS	MSE	-.423	<.001	[-.578, -.238]
	Subj-N	-.384	<.001	[-.194, -.130]
	Obj-N	-.342	.001	[-.147, -.082]
MSE	Obj-N	.229	.030	[.025, .415]
	ADE-Low	-.253	.016	[-.436, -.050]
Subj-N	Obj-N	.291	.005	[.091, .468]
	C-span	.260	.013	[.058, .422]
	ADE-Low	-.288	.006	[-.466, -.088]
Obj-N	C-Span	.370	<.001	[.178, .535]
	ADE-Low	-.488	<.001	[-.630, -.314]
	ADE-High	-.366	.001	[-.531, -.174]
C-Span	ADE-Low	-.322	.002	[-.495, -.125]
	ADE-High	-.245	.019	[-.429, -.042]

Note. **MAS** = Maths anxiety; **MSE** = Math Self-efficacy; **Subj-N** = Subjective numeracy **Obj-N** = Objective numeracy (basic arithmetic skill); **C-span** = Computation-span; **ADE-Low** = Average absolute deviation error in the low interactivity condition; **ADE-High** = Average absolute deviation error in the high interactivity condition.

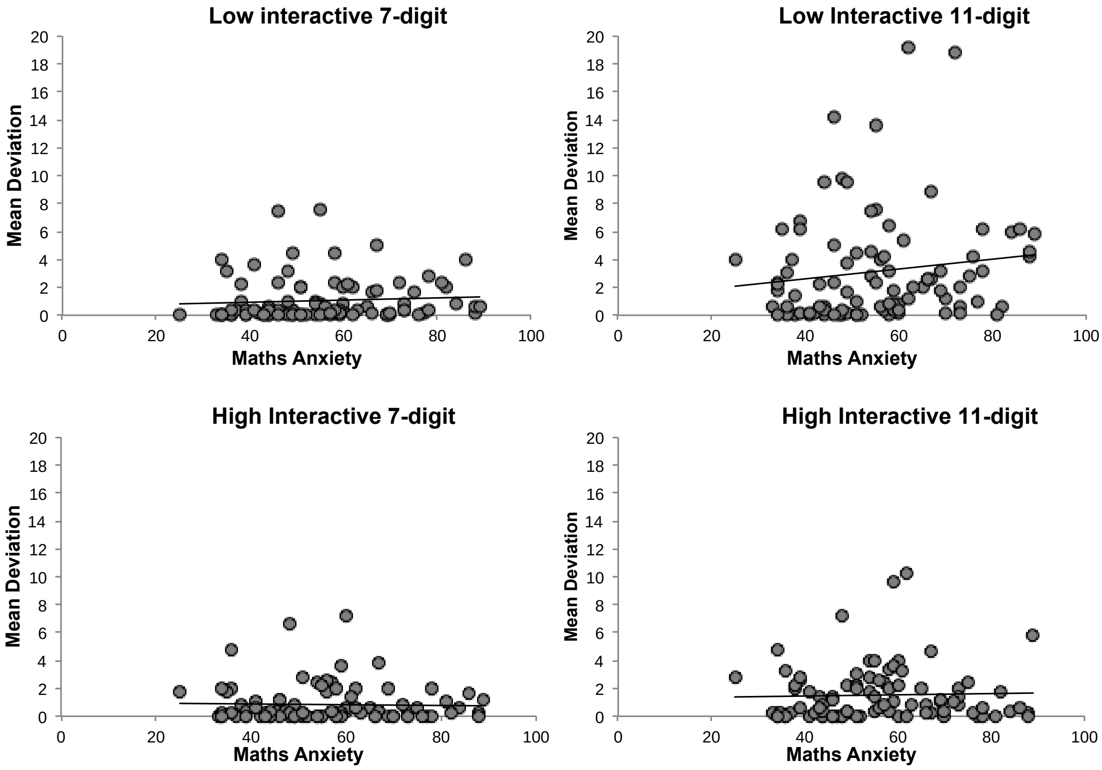

Figure 7.4. Scatterplots of maths anxiety and mean absolute deviation errors in the four experimental conditions revealed a discernable positive relationship between the two variables in the low-interactive11-digit condition only. The linear trends were not significant for any of the experimental conditions.

Discussion

This experiment investigated the impact on mental arithmetic performance when participants could modify the physical presentation of simple addition problems. All participants completed 10 sums in a low interactive condition, with hands flat on the table; and 10 sums in a high interactive condition using numbered wooden tokens. The 10 sums comprised of a mix of short (7 digits) and long (11 digits) sums. The presentation of the problems influenced the magnitude of the deviation error, with the sums completed using the wooden tokens, on average, resulting in smaller calculation errors, this was particularly evident in the longer sums. The number of sums a participant answered correctly was elevated in the higher interactivity condition as was efficiency, with the effect greatest in the 11-digit sums. These results principally reflect those presented in Vallée-Tourangeau (2013), endorsing the hypothesis that given an opportunity to interact with a dynamic physical problem presentation performance may be enhanced. As expected, the use of tokens in extending the affordance landscape of the problem space provided the individual with the prospect of using external resources to improve performance, not by solving the problem for them but as a scaffold for thinking.

There appeared to be limited influence of maths anxiety on the absolute deviation errors during the completion of these maths problems when averaged over low and high sums. However, maths anxiety significantly correlated with maths self-efficacy, subjective and objective numeracy, suggesting that it did indeed measure some level of affect in individuals when either exposed to maths problems or when asked to think about maths problems. Maths self-efficacy, subjective numeracy, objective numeracy and working memory all correlated with the performance measure of absolute deviation error in the low interactivity condition. However the correlations were nonsignificant in the high interactivity condition for MSE and subjective numeracy with the relationship between average deviation error and the measures of objective numeracy and working memory weakening. This indicates that altering the presentation of the problem may alleviate the lack of confidence in maths ability experienced by individuals.

It is possible to conjecture that the sums may have been too simple to illicit maths anxiety, while at the same time there is an indication that increasing the level of interactivity, even with these simple sums, ameliorated performance for those lacking confidence in their mathematical competency. Therefore it may be conceivable to speculate that in contradiction to Bandura's (1977) suggestion that the magnitude of the task impacts efficacy, with respect to maths self-efficacy, magnitude may not be not an influential factor. The thought of attempting a problem of any magnitude within the domain of mathematics may be sufficient to induce low personal expectations when approaching any maths related problem.

Turning to a more focussed analysis on the impact of sum length on performance, this first experiment revealed that even a small increase in sum length, from 7 digits to 11 digits, produced a decline in performance across all measures. Our participants were less accurate; the deviation from the correct answer was greater; as would be expected they were slower in completing the calculations; and as a result their efficiency suffered. In reviewing the relationship between maths anxiety and absolute deviation error, it was not significant for any of the four experimental conditions. However, further investigation indicated that as the sum size increased the performance of the maths anxious folk was impaired with the magnitude of errors increasing when asked to maintain a hands-down position but not when using the tokens in calculating the answers. Given these results the influence of sum length on performance appears to be worthy of further exploration. Thus in order to give further consideration to the impact of interactivity on maths anxiety the following experiments, while examining other factors, will implement an increase in the length of sums thereby incrementally increasing the difficulty of the sums.

Mental Arithmetic: Experiment 2

Mental Arithmetic and Problem Presentation

Introduction

In the experiment discussed here the external problem presentation tracks the dynamic interface between the agent's internal representation and the world. Interactivity and the potential to reshape the problem presentation were manipulated in terms of four conditions aimed at simulating the tools that might be used by individuals outside the psychologist's laboratory. The four conditions consisted of a low interactivity hands-down condition and three additional conditions whereby interactivity is increased; a pen and paper condition, a pointing condition, and a condition where numbered wooden tokens were available to use when calculating the sums.

In the first experiment, as in Vallée-Tourangeau (2013), participants certainly performed better with a greater degree of interactivity, but what is not shown in the first experiment is whether problem solving efficiency is enhanced by interactivity through the elaboration of qualitatively different strategies. Consequently, the analysis in this experiment profiles performance in quantitative terms—accuracy, latency, absolute deviation error, and efficiency—but also in qualitative terms by describing the different strategies enacted through different forms of interactivity. In addition this experiment explored the influence of interactivity on the attitude of participants to the tasks undertaken by measuring the engagement in the task or the flow and whether being in flow impacts performance (Csikszentmihalyi, 1990). As these three components address differing aspects of the one experiment they will be attended to in turn followed by a general discussion summarising the outcomes of the experiment.

It is anticipated that participants will perform best in the token condition as this offers the greatest degree of interactivity in all four conditions. In addition, by supplying participants with the familiar artefacts of pen and paper or letting them freely point and mark the paper using their fingers we will also observe a greater ability to calculate the sums correctly than with their hands flat on the table. Results from experiment 1 suggest that with a sum size of 11 digits, maths anxiety will be more evident in the low interactivity condition than the token condition; the expectation is that this will also apply to the other two conditions as they offer a greater level of interactivity than the hands-down condition. In assessing task engagement participants are expected to score more highly in the conditions offering greater interactivity rather than the more static, low interactivity condition as they experience greater engagement with those tasks. However it is not expected that performance will be anticipated by task engagement, as any improvement in performance will be due to the greater degree of interface with the world offered by any increase in interactivity.

Method

Participants

Sixty participants (40 females, mean age 23.3, *SD* = 4.41) were recruited for this experiment. Undergraduate and postgraduate students either volunteered or received course credits for their participation in the experiment.

Materials and Measures

Arithmetic task. Participants were presented with five problems in four conditions on a wooden board measuring 39cm by 34cm. In the first condition, participants were asked to add a sequence of single-digit numbers with their hands down and in a second they were allowed to point at the numbers. Thus in these two conditions, the problem presentation could not be modified, but participants could engage in some complementary actions in the latter. In the other two conditions participants could reshape the problem presentation. In the third condition, participants were given a pen. Using this pen and the paper on which the digits were presented, they could then recast the sum as they saw fit. In the fourth, the sums were presented as a set of wooden numbered tokens (2.2cm in diameter) that participants were invited to move around the board to arrive at the correct sum. Therefore a total of 20 sums were presented to participants across the four conditions; the sums were randomly allocated to each of the four conditions and the order of the conditions was counterbalanced across participants. Participants were requested to calculate each set as quickly and accurately as possible. Each unique sum consisted of eleven single digit numbers arrayed in a random configuration (see Figure 7.5). For the purpose of the present study, single-digit numbers between one and nine were first categorised as low (1-4) or high (5-9) in order to generate the range of possible sums in a more principled manner. Four groups of sums were created: Group I (5 low, 6 high), Group II (only high), Group III (3 low, 8 high) and Group IV (4 low, 7 high). A set of only low numbers was not included to reduce ceiling effects. The sets of sums presented to participants consisted of two sums from group I and three from groups II to IV. Each of these groups was assigned to one of the four interactivity conditions, and this assignment was counterbalanced across conditions. As a result each participant was presented with a unique set of sums in each condition.

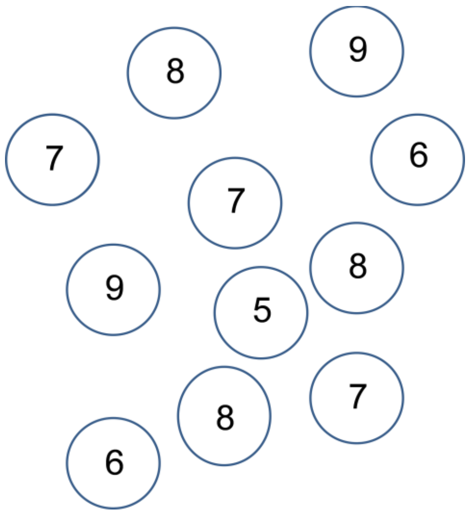

Figure 7.5. Twenty unique sums were created, each sum consisting of 11 single-digit numbers between one and nine. The numbers were categorised as low (1-4) or high (5-9) in order to generate a range of sums in a principled manner. A set of five sums was randomly allocated to each of the four experimental conditions.

In all four modes of interactivity the problems appeared in the same format with 11 randomly distributed circles (2.2cm diameter) on A4 size templates. For all but the token condition the templates were produced using white paper with the digits printed in the circles on the page. In the token condition the numbered wooden tokens were initially arranged using templates of tracing paper, created with the same configuration of the constituent numbers as the paper version of the other conditions (to ensure that the perceptual starting point was the same in each condition). Before each set of sums participants were shown an A4 page with the instructions for the forthcoming task. The wording was similar for all four conditions with changes made to reflect the interactivity opportunities and constraints. Other than in the low interactivity condition where they were instructed not to move their hands, participants were under no obligation to use the pen, the tokens or point, whichever was relevant to the current experimental condition. In order to maintain consistency of experience across conditions and participants, each sum was initially obscured from the view of the participant by a screen, with instructions from the researcher to start once the screen was removed. Performance was measured using the four dependent measures as described earlier in this chapter—accuracy, latency, absolute deviation error and efficiency.

Maths anxiety. Maths anxiety was measured using the Mathematics Anxiety Scale-UK (MAS-UK; Hunt, Clark-Carter, & Sheffield, 2011) as described in Experiment 1. The maths anxiety score for a participant was the sum of the ratings across the 23 items and could range from 23 to 115 (M = 50.7, SD = 16.4; Cronbach's α = .94).

Numeracy. Numeracy was measured using a subjective numeracy scale and an objective numeracy scale, also as described in Experiment 1. The subjective numeracy score was computed as a sum of the responses (M = 30.8, SD = 7.49; Cronbach's α = .79). The score for objective numeracy was calculated as the correct number of solutions.

Need for cognition scale. Cacioppo and Petty's (1982) scale for testing thinking diligence scale was presented as a pen and paper task, as in Experiment 1.

Task engagement scale. An eight-item scale to assess participants' attitude towards the familiar and novel tasks presented in this experiment was developed. The objective being to ascertain whether any performance differences were attributable to the use of artefacts with questions such as "Were you able to concentrate well on the task?" and "Did you find the task interesting?". The seven-point likert scale ranged from 0 – 7 with response of "definitely no" to "definitely yes". The scale was printed on an A4 page of white paper and was presented to participants after each condition, thus they completed four questionnaires in each experimental session. The scale was found to be highly reliable in all four conditions (Low interactivity, Cronbach's α = .80; Pen-paper, Cronbach's α = .77; Pointing, Cronbach's α = .78; Tokens, Cronbach's α = .77).

Mental Arithmetic Performance I:

Quantitative Results and Discussion

Results

Accuracy

The mean number of correct answers was greatest in the token (*M* = .69, *SD* = .22) and the pen-paper (*M* = .69, *SD* = .23) conditions (see Figure 7.6 top left panel). The pointing condition (*M* = .66, *SD* = .26) produced fewer accurate deviations, with the low-interactive condition producing the weakest performance (*M* = .60, *SD* = .30). A one-factor repeated measures ANOVA indicated a significant difference between conditions, $F(3, 177) = 3.12$, $p = .027$, $\eta_p^2 = .050$. Post hoc tests revealed a significant difference between the low interactive and the pen-paper conditions ($p = .006$) and the low interactivity and tokens conditions ($p = .020$), but no significant difference between the pen-paper and tokens conditions.

Latency

The lower left panel of Figure 7.6 shows that participants generally took about the same amount of time to complete the task across the four conditions (low interactivity *M* = 26.79, *SD* = 9.88; pen-paper *M* = 27.26, *SD* = 9.73; pointing *M* = 25.70, *SD* = 10.09; tokens *M* = 26.58, *SD* = 10.41). The main effect of interactivity in the one-way repeated measures analysis of variance (ANOVA) was not significant, $F < 1$.

Absolute Deviation Error

As illustrated in the top right panel Figure 7.6, the more interesting trends in performance were evident in the deviation from the correct answers: the best results being observed in the token condition and the worst in the hands-down low interactivity condition. Deviation from the correct answer was lowest when using the tokens (M = 1.41, SD = 1.69); the pen-paper (M = 1.61, SD = 1.65) and the pointing (M = 1.90, SD = 2.43) conditions produced higher deviations with the low interactivity condition (M = 2.64, SD = 2.39) eliciting the poorest results with the highest mean absolute deviation error. The one-factor repeated measures ANOVA revealed a significant difference between interactivity conditions, $F(3, 177) = 6.34$, $p < .001$, $\eta_p^2 = .097$, with post hoc tests indicating no significant difference between pen-paper and tokens, but again there was a significant difference between the pen-paper and the tokens conditions when compared to the low interactivity condition (p = .005, p < .001 respectively).

Efficiency

As these efficiency ratios were defined, higher ratios mean relatively better performance as a function of the resources invested in completing the problem. As the lower right panel of Figure 7.6 shows, performance was most efficient in the tokens (M= 1.20, SD = .62) and the pen-paper conditions (M = 1.15, SD = .60) with the low interactivity (M = 1.05, SD = .71) and the pointing (M = 1.12, SD = .59) conditions being least efficient. While the low interactivity condition produced the lowest level of efficiency, the main effect of interactivity was not significant however, $F(3, 177) = 1.39$, $p = .247$.

Individual Differences

There was no pattern of correlation between performance measures and the Need for Cognition scale. Maths anxiety correlated with the two numeracy measures indicating that maths anxiety reflects both the individuals self-report of their own maths ability and the more concrete maths ability assessed by the objective numeracy task (see Table 7.3 and Table 7.4). However, the results for predictors of performance for average deviation error did not concur with our expectations. As in Experiment 1, there were no significant correlations between maths anxiety and the calculation error, in fact the relationships now appear to be reversed whereby the low interactivity condition appears to induce slightly less maths anxiety than the other three conditions. Another conflicting result to Experiment 1 is that subjective numeracy correlated in a negative direction with the token condition but no longer with the low interactivity condition.

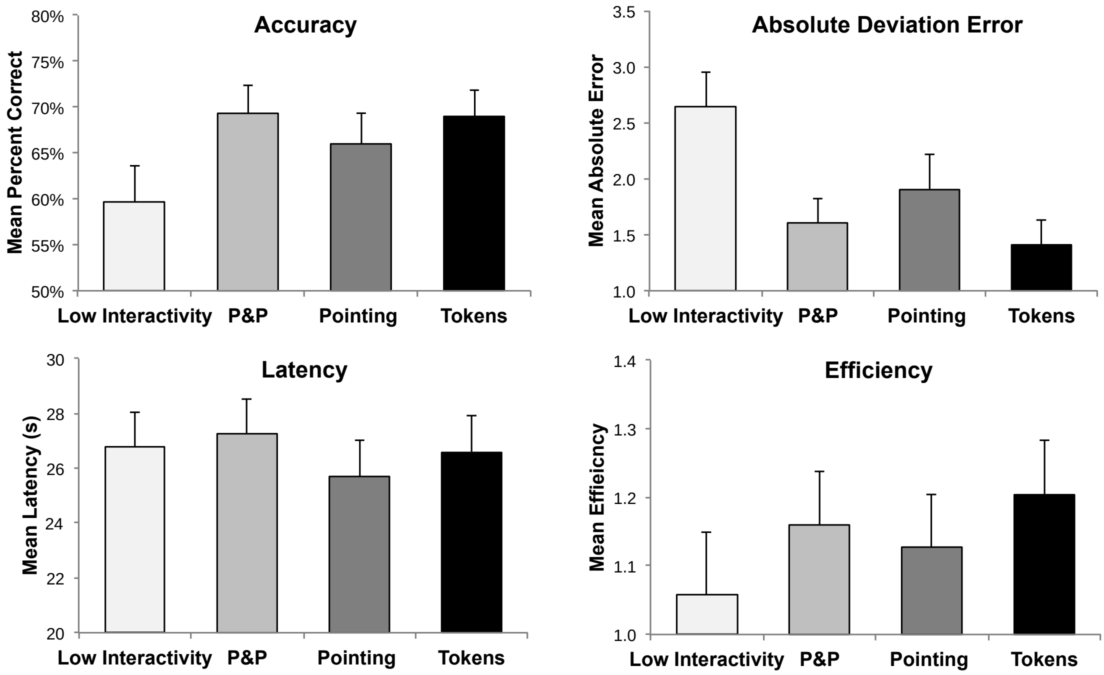

Figure 7.6. Mean percent correct answer (top left panel), mean deviation from the correct answer (top right panel), mean latency (bottom left panel) and mean calculation efficiency (bottom right panel) in the four experimental conditions. Error bars are standard errors of the mean.

Table 7.3

Correlation matrix for absolute deviation error for individual difference measures of maths anxiety, subjective numeracy and objective numeracy in all four interactivity conditions (df = 58).

	1 MAS	2 Subj-N	3 Obj-N	4 ADE-Static	5 ADE-P&P	6 ADE-Point	7 ADE-Tokens
1	-	-.452 **	-.353 **	.057	.180	.160	.228
2		-	.453 **	-.132	-.326 *	-.226	-.347 **
3			-	-.227	-.049	-.015	-.174
4				-	.125	.559 **	.456 **
5					-	.294 *	.314 *
6						-	.395 **
7							-

Note. **MAS** = Maths anxiety; **Subj-N** = Subjective numeracy; **Obj-N** = Objective numeracy; **ADE-Low I** = Absolute deviation error in low interactivity condition; **ADE-P&P** = Absolute deviation error in pen and paper condition; **ADE-Point** = Absolute deviation error in pointing condition; **ADE-Tokens** = Absolute deviation error in tokens condition * p < .05, ** p < .01.

Table 7.4

Significant correlations including confidence intervals for individual difference measures of maths anxiety, subjective numeracy, objective numeracy (df = 58)

Individual Differences				
Variable 1	Variable 2	*r*	*p*	95% CI
MAS	Subj-N	-.452	<.001	[-.633, -.244]
	Obj-N	-.353	.006	[-.556, -.109]
Subj-N	Obj-N	.453	<.001	[.225, .633]
	ADE-P&P	-.326	.011	[-.535, -.079]
	ADE-Tokens	-.347	.007	[-.552, -.103]

Note. **MAS** = Maths anxiety; **Subj-N** = Subjective numeracy; **Obj-N** = Objective numeracy; **ADE-P&P** = Absolute deviation error in the pen and paper condition; **ADE-Tokens** = Absolute deviation error in the tokens condition.

Discussion

The performance differences across the conditions revealed that even with these very simple sums, interactivity with artefacts helped to transform the execution of the calculations. While the participants used around the same amount of time to complete the sums regardless of the interactivity condition, they made more errors and the magnitude of the errors was larger when no artefacts were in use. Although there was no significant main effect of interactivity for efficiency, it is noteworthy that efficiency of performance between the hands-down, low interactivity condition and the high interactivity tokens condition approached significance (p = .060). The curious difference in the results between this experiment and Experiment 1 when investigating the relationship between maths anxiety and average deviation error, may suggest that it is not only sum length but the number of sums in each condition that impacts maths anxiety when calculating the sums. In this experiment there were only five sums in each block of sums, where there were ten sums in the previous experiment, the small number of additions may not be enough to produce a sense of anxiety. Next we turn to a more qualitative characterisation of performance to examine the potential strategies explored by participants in differing problem presentations, this may offer an interesting window onto how artefacts alter performance.

Mental Arithmetic Performance II: Qualitative Observations and Discussion

Observations

In the present study, the performance of a random selection of participants (n = 26) was captured on video in order to analyse the pathways to solution as they unfolded for these simple additions. Each participant consented to the video recording, which took place across the entire experimental session in a purpose-built laboratory with unobtrusive and noise-free cameras. The sample was determined by the availability of the audio-video observation lab and participants' consent to being filmed. The analysis of the participants' problem solving trajectory was based on two simple measures, namely the nature of the groupings of numbers, and whether these groupings were 'good provisional sums' (GPS; defined as Σ 5 MOD =0; Vallée-Tourangeau, 2013), offering congenial stepping-stones to promote more efficient problem solving. For example, grouping a '7' and an '8' to form an interim total of '15', may encourage participants to group a '6', a '4' and a '5' to create '30' as the next interim total on the way to the final solution.

Here the actions of one participant will be explored in detail, concentrating on a selection of the sums completed (see Figure 7.7 for an example of how the digits were arranged), for three of the interactive conditions. (The low interactivity condition is excluded from analysis as the video revealed no insight into the calculations). A number of video recordings were discarded from analysis due to technical reasons such as the hands or head obscured the camera's view of the participant's actions. From the remaining sample, the performance of a 21 year old, right-handed male undergraduate psychology student was selected for detailed discussion. His performance was considered typical to other participants as he utilised the pen and paper, pointed and grouped the tokens in a similar manner to those participants analysed frame by frame. In addition, as these behaviours were identifiable across all conditions for this individual, there is the opportunity to build a picture of how the presentation of the sums subtly alters his arithmetic strategy. The groupings will be labeled in keeping with the protocol established by Vallée-Tourangeau

(2013) as good provisional sums (GPS). In addition those groupings made up of the same digits, for example grouping all the nines together, will be called same digit groupings (SDG).

The Token Condition

The sums were presented as a random array of wooden tokens. All the token movements in this condition made by the participant were of a sliding nature. He generally moved the tokens into distinct groupings on different parts of the board before adding them together.

With the first presentation of the initial five sums the participant initially arranged some of the tokens into GPS: 2 and 8, then a separate group of 7 and 3, then 4 and 6. These three groups were moved together to make one large group equaling 30. Three further groups were created: 5 and 6, 8 and 3. The 5 and 6 were moved to the token ungrouped token 9 to make 20. The group of 8 and 3 was then combined with the 5, 6 and 9 to total 31. Thus 30 from the first large group added to 31 from the second large group summed to the correct answer of 61.The final order of groupings (with interim totals in brackets) was 2, 8 (10) + 7, 3 (20) + 4, 6 (30) and 5, 6, 9 (50) and 8, 3 (61). Thus, moving the tokens into the three groups of ten then the forming another of 5, 6 unveiled the opportunity to add this 9, bringing together the groups in a total of 50 which eased the way to add the final 11.

This pattern of using GPS was clearly evident in the next two problems and the fifth one. In the latter, the answer given was wrong by 10. Yet by that time, this participant was an efficient mover of tokens, reconfiguring the problem space into congenial subtotals of 10, and he generated an answer, albeit an incorrect one, in only 11 seconds. A careful analysis of the participant’s movements shows that he moved his hand across the groupings created, probably then tallying the GPSs, he missed a grouping that added to 10; hence the calculation error.

When summing the digits it may seem obvious to create good provisional sums from the layout of the tokens; however, the important observation is that the secondary rearrangement of tokens into the two large groups emerged from cues provided by moving the tokens into the original smaller groupings. This is a key observation that highlights the importance of interactivity: *the affordance landscape shifts as the problem trajectory is enacted through interactivity*. This was also discernable when the participant completed the fourth set of sums (see the left most panels of Figure 7.7). In this case the GPS were not as obvious and the participant had no choice but to employ a different approach. Initially the participant slid tokens into SDG's—5, 5 then 9, 9, the third 5 was moved but not grouped; 7, 7 followed by 8, 8 then 6 and 9. He then proceeded to rearrange the tokens into these altered groupings: 5, 5, 5 + 9 + 8, 8 were merged to form a group totally 40, then 9 was added and 6 to equal 55 then 9 to make 64, which made for an easy addition of 14 from 7 and 7. In changing the configuration, initial efforts of grouping cued and prompted a next level of grouping that simply could not occur without artefacts and interaction. In other words he could only create the final groups after reconfiguring the tokens from the first attempt.

The Pen and Paper Condition

The participant employed a strategy of crossing through the digits with the pen provided and writing numbers and additions at the bottom of the page using the paper on which the digits were printed as a worksheet. This system resulted in the largest mean latency across the four conditions. There was evidence of attempted grouping in provisional sums in groups of 10's; however he used his fingers and the pen to point to numbers in order to keep track of his additions.

The middle panels of Figure 7.7 illustrate a typical effort in this condition. The participant began by crossing off 2 and 8 to make 10, then 3 and 7 for another interim 10, then 6 and 4. He then pointed to 2, 5 and 4 and wrote 11. Next he added the 9 and 9, writing 18. He dragged his pen across the six digits crossed through earlier that made up the three groups of 10, and wrote 30. He added the transcribed numbers of 11, 18 and 30 to announce the correct total of 59. Note the additional tracking and mapping necessitated by the retranscription of the random number configuration into a more canonical columnar arrangement (something that was not required nor observed in the tokens condition).

Another sum in this condition (not illustrated) provided a window into how the formation of good groupings was not as obvious; the task of mentally rearranging digits was more challenging from a fixed arrangement unlike the manipulable token presentation. The participant began by crossing off 2 and 8, then 4 and 6 writing the number 20 below the printed digits. He then dragged his pen across the 4, 5 and 9 and wrote 18 on the worksheet. Again moving his pen across 7, 8 he added the figures 9, 9, 6 to the worksheet, then 18 again and crossed off the hand written numbers 9, 9 and 6 then wrote 6 above the 20. He scanned his pen over the written numbers 18 and then added 40, he crossed out 40, writing 36 on the right then below that 20 he penned a 6 and announced the answer of 62.

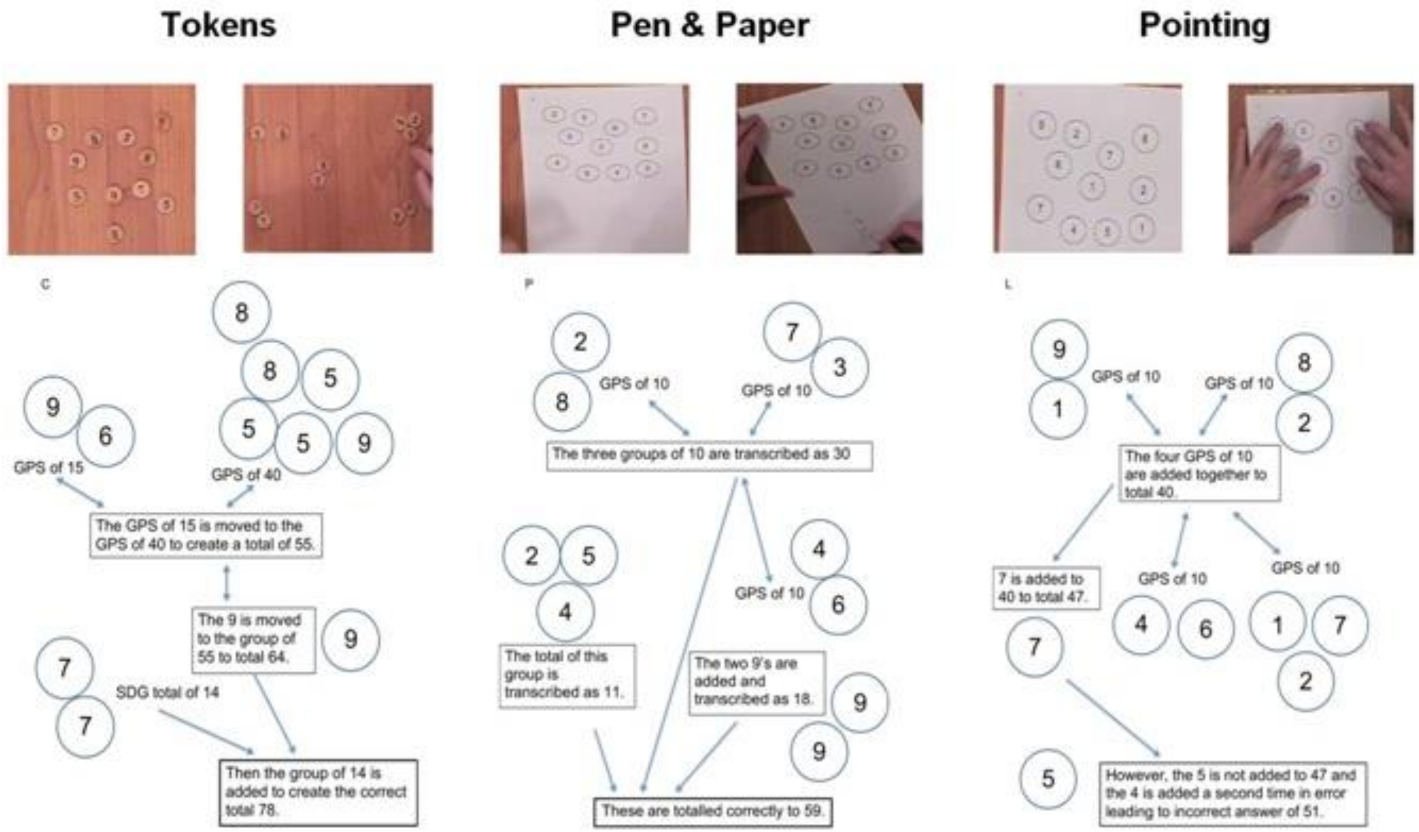

Figure 7.7. The three panels show the stages of progress as the participant solves simple math problems in varying conditions of interactivity. The first screen shot in each of the three panels shows the initial problem presentation. The second screen shot captures the participant part the way through the path to solution. The bottom image is a reconstruction of the final stages of the problem solving process as revealed in frame-by-frame analysis of the participant's actions.

Ultimately this convoluted series of crossing outs and adding on paper resulted in a deviation of four from the correct answer. Thus in the pen and paper condition, the participant sometimes retranscribed the additions, sought judicious pairings, but the mapping process was slow and the participant had to be particularly vigilant as he systematically converted struck-off digits into provisional sums.

Unlike the highly manipulable token condition, here the participant appeared to enact a form of iterative consulting by switching between his own reconstructed representation of figures or crossing-offs on the worksheet, and the random array of static digits, potentially increasing the chance of transcription error. The marking of the digits into GPS was more hesitant in the pen-paper condition than in the token condition, where the tokens were grouped together swiftly. It may be the case that the crossing off slowed him down; therefore it appears he may have rapidly switched to relying on internal memory rather than maximising the use of the external resources. It appears that the transcription of digits into calculations on the worksheet became more convoluted and time consuming. This indicated a possible cost-benefit trade-off occurring during the distribution of cognitive resources across the continuum between internal and external memory, even at the risk of making an error (cf. Fu and Gray, 2000).

The Pointing Condition

The participant used both hands and most fingers to point and to anchor counting points. At times throughout the task there was hovering with the fingers above numbers, pauses, counting and re-counting of digits. The strategy is mixed with some grouping at the beginning of each task, with more grouping of like numbers together than in the token condition.

In one set, the SDG's of 5, 5 then 7, 7 then 8, 8 were created then 9, 7 followed by 6. Therefore the totals were 10, 24, 36, 52, 68 and 74. In the final set, there are very few like numbers to group together therefore another strategy was required. In this case the participant used congenial totals of 8, 2, then 1, 9, then 6, 4 followed by 2, 7, 1. However, in order to achieve this he pressed his fingers on top of these 9 digits on the page, in doing so his hands obscured the remaining two numbers 5 and 7 (see the right most panels in Figure 7.7). The upshot is that upon taking his hands away he added the 4 that had already been accounted for and did not touch or point to the 5 that remained uncounted, as a result he announced an incorrect answer.

Discussion

In scrutinising the movements of the participant frame-by-frame, patterns emerge from what appear to be often random, unconnected strategies. However, what is particularly interesting is how the emergent pathway to solution differs between conditions of interactivity. The different types of interactivity transform the initial problem presentation with the agent behaving differently as a function of the artefacts offered and hence the distributed system reconfigures itself on the path to solution. In using the wooden tokens to add the numbers, shifting them into different groupings disclosed a number of ways to ease calculation. At times actions resulted in good provisional sums, other times same digit groupings were made or tokens were separated from the other tokens to ease thinking. This reconfiguration of the problem space expands the range of cognition revealing new ways to sum the numbers; ways that would not have been achievable without the possibility for arrangement and rearrangement of the artefacts offered by the level of interactivity. The pen and paper condition presented the opportunity to use a more traditional method of summing numbers. In every sum the participant crossed off some or all of the numbers and retranscribed figures onto the page. The examples scrutinised here showed that it was more effortful to keep track of groupings and that new strategies were not as easily enacted as with the tokens. Similar observations were made in the pointing condition, while GPS and SDG's appeared to be easily identified by the participant it was not an easy task to maintain the totals using his fingers. There was no evidence that this condition opened new pathways to alternative strategies for adding these digits. The frame-by-frame scrutiny of movements in problem solving afforded by different artefacts has emerged as an important tool in exposing strategies used by this participant. It may well be that in future studies verbal protocols or the measurement of eye movements in conjunction with filming could uncover different strategies employed in a low interactivity condition.

Task Engagement and Flow

The attitude of participants was more positive toward the pen-paper (M = 37.9, SD = 8.38) and the tokens (M = 37.7, SD = 8.94) conditions, than the pointing condition (M = 34.1, SD = 8.76) and least favourable for the low interactivity condition (M = 31.6, SD = 9.13). In a one-way repeated measures ANOVA, the main effect of interactivity was significant, $F(3,117) = 17.0$, $p < .001$, $\eta^2 = .231$. Post hoc tests further identified significant differences between the low interactivity and the tokens conditions and the low interactivity and pen-paper conditions ($p < .001$ for both comparisons). The low interactivity and pointing conditions were also significantly different ($p = .025$). Feelings toward the pointing condition differed significantly from those in the pen-paper (p < .001) and tokens conditions (p = .008).

It is noteworthy that the participants' attitude towards completing the sums in the different conditions paralleled the impact of interactivity on performance. Conditions involving external resources, pens or tokens, seemed to elicit a more positive, engaged attitude towards the simple arithmetic problems, than the restricted, low interactivity condition. Of course, participants were also more accurate in the interactive conditions. But the more positive attitudes towards the problems cannot be attributed to task success since participants were not given feedback about their performance, that is, after announcing each sum, the experimenter did not tell participants whether their answer was right or wrong. However, results also showed that as task engagement scores increased efficiency increased, with marginally significant correlations in the tokens (r(58) = .25, p = .056) and pen and paper (r(58) = .26. p = .045 (see Table 7.5 and Table 7.6). The token and pen-paper condition being the two conditions in which participants exerted some control over problem configuration. This suggests that engagement with the task tended to encourage more efficient performance. These findings are in keeping with the notion that higher levels of personal involvement positively affect performance (Shernoff et al., 2003). Also, changing the visual display may ease the task and thereby lighten the cognitive load, which increases effectiveness and alters attitudes (Vallée-Tourangeau, Sirota & Villejoubert, 2013). It is possible to speculate that this preference for the use of artefacts elicits a more positive attitude by participants, as utilising pen and paper is a familiar method of computation or the use of manipulatives is often considered a “fun” (Moyer, 2001, p. 175) learning technique. Therefore, one may argue that improvement in performance in the higher interactivity conditions may be explained by affect, since employing artefacts seemed to be preferred. However, there is good evidence in the correlational data that this cannot be the explanation; that is beyond task enjoyment, interactivity simply confers a distinct performance advantage. This is revealed in the pattern of correlations between engagement and calculation errors in the four conditions (as plotted in Figure 7.8). A negative correlation trend between task engagement and absolute deviation error was observed in all four conditions (as engagements increased, errors decreased) but the correlation between task engagement scores and deviation was only significant in the pen and paper condition (see Figure 7.8). The absence of correlations in the token condition when compared to the correlations for these measures in the pen-paper condition implies that the familiarity of using pen-paper may be a large component of performance whereas it is the manipulability of the tokens themselves rather than affect alone that accounts for the improvement in performance.

Table 7.5

Correlations between task engagement scores and the performance measures in all four experimental conditions (df = 58).

	Static	**P&P**	**Point**	**Tokens**
Accuracy	.171	.315 *	.062	.226
Deviation	-.215	-.386 **	-.156	-.137
Latency	-.175	-.156	-.264 *	-.270 *
Efficiency	.197	.260 *	.174	.248

Note. **P&P** = Pen and paper condition; **Point** = pointing condition. * $p < .05$, ** $p < .01$.

Table 7.6

Significant correlations including confidence intervals for task engagement scores for the conditions of interactivity pen and paper, pointing and tokens (df = 58)

Condition	Performance measure	*r*	*p*	95% CI
P&P	Accuracy	.315	.014	[.067, .526]
	ADE	-.386	.002	[-.582, -.147]
	Efficiency	.260	.045	[.007, .482]
Point	Latency	-.264	.041	[-.485, -.011]
Tokens	Latency	-.270	.037	[-.490, -.018]

Note. **P&P** = Pen and paper; **ADE** = Absolute deviation error.

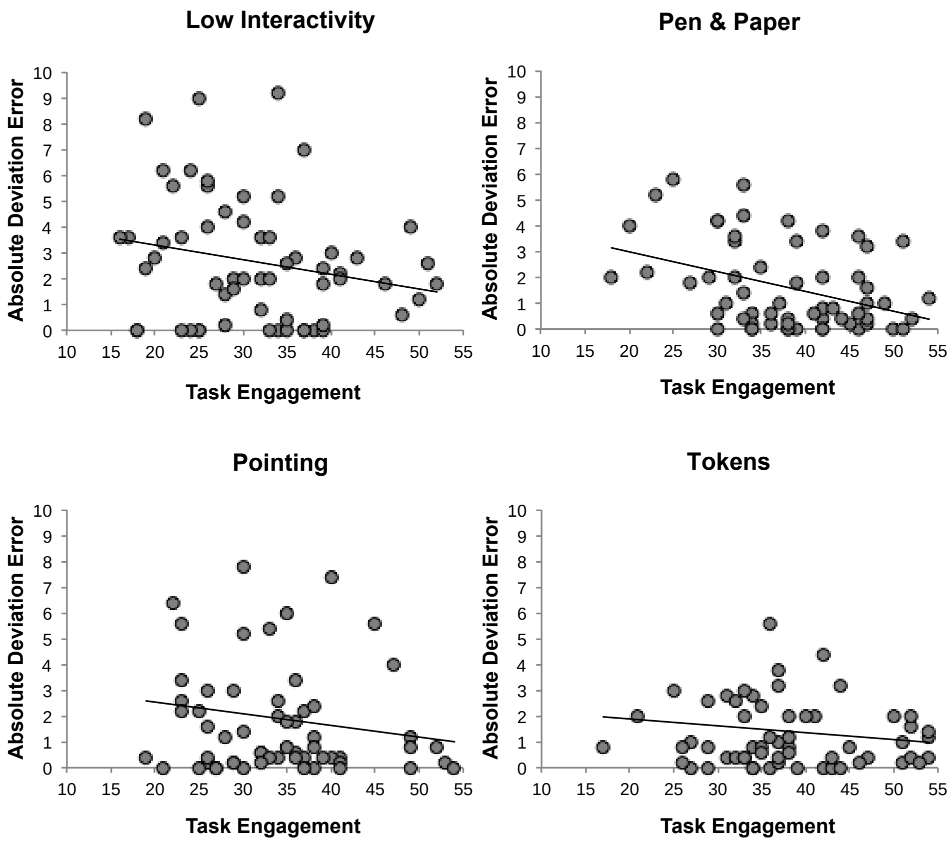

Figure 7.8. Scatterplots of task engagement scores and mean absolute deviation errors in the four experimental conditions. The slope of the linear trend in all four conditions was negative, but only significantly so in the pen and paper condition, *r*(58) = -.386, *p* = .002.

General Discussion

In calculating long arithmetic sums, individuals usually create opportunities to deploy a range of complementary strategies as a function of the level of interactivity that binds mental and physical resources. Studying systems rather than individuals poses theoretical and methodological challenges. Theoretically, the nature of the problem representation and the trajectory of the solution as it evolves from an embryonic to a fully formed answer, should perhaps be understood as being distributed and configured in terms of a transaction between the participants' internal resources and the shape and nature of the resources in the external environment. Attempting to segment and independently specify the components of a cognitive system, namely the thinking agent and his or her immediate environment, is not as productive as seeking to characterise the system as a whole (Baber, Parekh, & Cengiz, 2014). The methodological implications of this transactional perspective are important. Of course, systems can be more complex, and composed of a much wider range of functional elements, which challenges the traditional toolkit of experimental cognitive psychologists designed to deal with a cognitively sequestered individual in a laboratory environment that generally prevents interactivity. The findings and methods reported here suggest that a more qualitative idiographic cognitive science supported by an observational toolkit that can code at a much smaller time scale the evolution of a problem representation and its solution would make a substantial contribution to problem solving research.

In considering the impact of interactivity on problem solving it is not only the performance and task engagement that is altered with a changing problem presentation but also the affordances offered by the artefacts to hand. The qualitative analysis here clearly illustrated how the agent behaved differently as a function of the artefacts offered and hence how the system reconfigures itself on its path to solution. “An artifact is not a piece of inert matter that you *act upon*, but something active with which you *engage* and ‘*interact*’ ” (Malafouris, 2013, p.149). Interactivity encourages the reconfiguration of the problem space, opening windows to new strategies, improving efficiency and enjoyment.

Mental arithmetic: Experiment 3

Interactivity and Expertise

Introduction

Highly enactive approaches to maths have been shown to increase efficiency and accuracy, while reducing calculation error (Vallée-Tourangeau, 2013). In the current experiment, participants varying in maths expertise were invited to complete simple arithmetic additions, with sets of sums composed of 11 and 17 digits. While it is anticipated that this task does not challenge the arithmetic knowledge and skills of participants, accuracy potentially requires good working memory capacity and executive function skills especially when dealing with longer sums. The aim was to investigate whether any changes in performance were related to the mode of problem solving. The focus being on the effect of interactivity without and with artefacts, namely the wooden tokens, in facilitating an improvement in performance, in relation to expertise. In addition the size of the sums may have been a contributing factor to maths anxiety in the previous two experiments, therefore the number of digits in some of the sums was increased to 17. A selection of tests and questionnaires were also included in the experimental session to measure individual differences implicated in mental arithmetic such as maths anxiety, working memory, numeracy, maths expertise, and engagement. A dynamic, high interactivity environment using artefacts as opposed to a low interactivity quasi-static one may encourage more efficient calculations, reflecting better skills, through the dynamic reconfiguration of the problem. A high degree of interactivity may improve performance for participants with lower maths expertise. In contrast, the performance of participants with a higher degree of maths expertise may not vary greatly as a function of interactivity since their well-practiced internal resources may work efficiently and creatively when dealing with numbers. While individual difference measures in Experiment 1 were not significantly influenced by sum length, the trend for maths anxiety was such that an increase in sum length might impact maths performance if larger sums were introduced, as this would be expected to induce greater feelings of anxiety in the maths anxious. However, it is anticipated that this anxiety would diminish through interactivity with the tokens due to the shift in cognitive load from internal processes only, to one shouldered by the resources created thorough the agent-environment coupling. Therefore, as the length of the sums in this experiment have been increased to 17, both an average and a comparison between the two sum lengths for the individual difference measures will be undertaken.

Method

Participants

Sixty participants (38 women, M_{age} = 21.3, *SD* = 2.37) were recruited from a variety of academic backgrounds. Thirty-two psychology undergraduates participated in exchange for credits, 21 undergraduates from other disciplines volunteered to participate and seven additional participants either working in a highly numerical field (e.g., accounting), or recently graduated with a maths discipline degree also participated voluntarily.

Materials and Measures

Arithmetic task. Each participant was presented with two sets of simple additions, each composed of five 11 and five 17 single-digit numbers. As in the previous experiments, these additions were performed in two interactivity conditions. In the low interactivity condition, participants were given a sheet of A4 paper, with numbers to be summed distributed randomly on the page (see

Figure 7.9, left panel). While adding the numbers, participants were instructed to keep their hands flat on the table. In the high interactivity condition, participants were given a similar set of sums, with the same distribution, but presented as moveable numbered wooden tokens (1.2 cm in diameter; see Figure 7.9, right panel). On completing each sum participants were requested to announce the answer aloud to the researcher.

Low Interactivity **HIgh Interactivity**

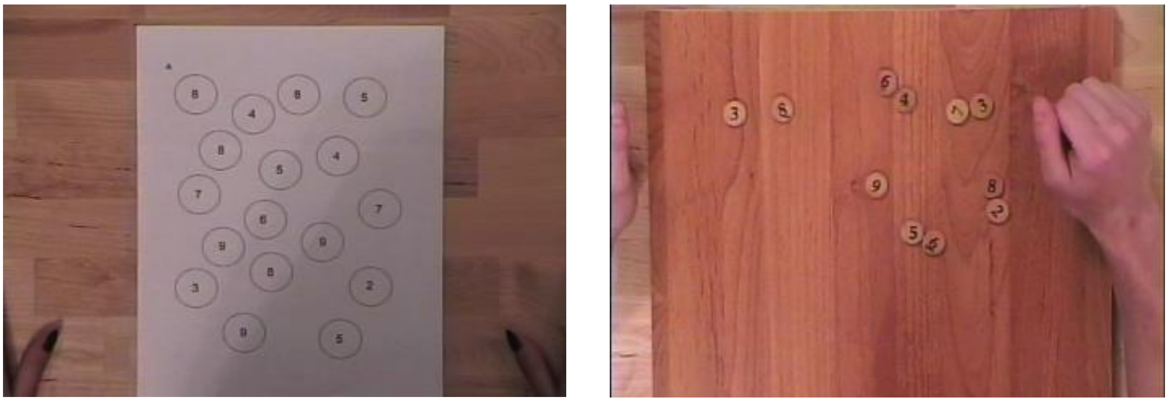

Figure 7.9. The sum was presented on a sheet of A4 as a random configuration of digits in the low interactivity condition (left panel); participants in that condition kept their hands flat on the tabletop. In the high interactivity condition, the sums were presented with movable wooden tokens (right panel) which participants touched, moved, grouped, as they saw fit.

Maths anxiety. Participants completed a 25-item Mathematics Anxiety Scale-UK (MAS-UK; Hunt, Clark-Carter, & Sheffield, 2011). The questionnaire consisted of a series of situations with participants asked to indicate how anxious they would feel in those situations, on a Likert-style scale, with 1 = "not at all" and 5 = "very much". Items included statements such as "Working out how much your shopping bill comes to" or "Taking a math exam". (M = 51.9, SD = 15.6; Cronbach's α = .93).

Objective numeracy. A basic arithmetic scale (BAS) was used to test participants' objective numeracy. It consisted of 60 simple arithmetic problems (such as 7×8 = ?). Participants were required to write the answers on the paper provided, in the order presented, completing as many as possible in 60 seconds (M = 29.8, SD = 16).

Working memory. Participants completed two working memory tasks. The computation-span task, testing both processing and storage of numbers, while a nonnumerical visuo-spatial task, the Corsi block task, testing the temporary storage of visual and spatial information.

Computation-span. The computation-span task (Ashcraft & Kirk, 2001) required participants to answer simple arithmetic problems (e.g., 2 + 8 = ?, 12 – 4 = ?), before recalling the second numbers of these problems (e.g., 8, 4). Sequences of equations ranged from 1 – 7 and participants had to process each sum *and* recall the relevant digit correctly to score a point (M = 23.6, SD = 8.8). This was presented on a computer screen with the responses recorded by the researcher.

Corsi block. In this version of the Corsi Block task participants were shown ten sequences of shaded blocks in a 4 × 4 matrix on a computer screen. The number of blocks to be remembered in each sequence increased from 2 to 6 blocks in length. The participant recorded their responses on an A4 sheet of paper by marking the squares on a grid preprinted on the paper resembling the grid that appeared on the screen. Participants scored one point for each correctly ordered block, thus the maximum score was 40 (M = 33.2, SD = 4.4).

Maths expertise. An instrument was developed to evaluate maths expertise based on experience. Four questions were related to maths grades at school such as, “Have you taken maths GSCE (or equivalent)?”, this was scored as a binary ‘yes’ = 1, ‘no’ = 0; “If yes, please indicate which grade”, this was scored as ‘A*/A’ = 4, ‘B’ = 3, ‘C’ =2, ‘<C’ =1 ‘N/A’ = 0. Three questions

asked for details on current university degree, any past university degree and current job if applicable. The responses were given a score from 1 - 4 where 4 = a maths-heavy degree or job and 1 = no degree or job. The highest score from these three questions was used to measure maths experience in terms of degree and employment. This score was added to the responses on maths education to provide a continuous numerical measure of maths expertise.

Task engagement scale. The task engagement scale (TES) was developed to gauge a participant's engagement and enjoyment during a task. The nine-item scale (a slightly modified version of the task used in Experiment 1) was based on Shernoff et al. (2003) who identified three key components of task engagement: concentration, enjoyment, and interest. The scale asked participants to rate how anxious they felt; how easy, pleasurable, fun, threatening, stressful, tiresome, or effortful the task was; and how motivated they were to perform well in the task. Each item was scored on an 8-point Likert scale, labeled from zero (definitely not) to seven (definitely yes): The higher the score the more positive the attitude toward the task. Each participant completed the TES scale twice once following each of the two sets of sums across interactivity conditions. The alpha reliability of the nine-item scale for both interactivity conditions indicated that the scale had good reliability (Low, Cronbach's $\alpha = .77$; High, Cronbach's $\alpha = .81$).

Procedure

The length of the additions (11 or 17 digits) and level of interactivity (low, high) were repeated measures factors yielding four conditions. The presentation order of these conditions was counterbalanced across participants. The sets of sums for each interactivity condition were separated by at least one other task (either the MAS-UK, BAS, Computation-span or Corsi Block). The other tasks were presented at either the beginning or the end of the session and their order was counterbalanced across participants. Each experimental condition was followed by the TES, and the experiment ended with a maths experience questionnaire. The working memory tasks were presented on a computer with all other tasks being presented on paper. The experimental session lasted approximately an hour.

In keeping with the previous two experiments mental arithmetic performance was measured in terms of accuracy (proportion of sums correct), latency to solution, absolute deviation error, and efficiency. Also in keeping with the previous two experiments mean absolute deviation was used for analysis except where stated.

Results

Accuracy

The mean percent correct, as reported in the top right panel of Figure 7.10, was greater in the high interactivity condition than the low interactivity condition for both sum lengths (11 digits and 17 digits). A 2×2 repeated measures analysis of variance (ANOVA) indicated a significant main effect of interactivity, $F(1,59) = 30.0$, $p < .001$, $\eta_p^2 = .34$ and sum length, $F(1,59) = 21.23$, $p < .001$, $\eta_p^2 = .265$. However, there was no significant interaction, $F < 1$.

Latency

While latency to completion was influenced by sum length, interactivity level resulted in very little difference in latency (see Figure 7.10, top right panel). A 2×2 repeated measures ANOVA produced no significant main effect of interactivity, $F(1,59) = 1.42$, $p = .239$, $\eta_p^2 = .02$. However, there was a significant main effect of sum length, $F(1,59) = 201$, $p < .001$, $\eta_p^2 = .78$ and a significant interaction, $F(1,59) = 6.68$, $p = .012$, $\eta_p^2 = .10$.

Absolute Deviation Error

The mean absolute deviation error from the correct answer as shown in Figure 7.10 (bottom left panel) was lower in the high interactivity condition than in the low condition regardless of the sum length. A 2×2 repeated measures ANOVA indicated a significant main effect of interactivity, $F(1,59) = 11.0$, $p = .002$, $\eta_p^2 = .16$ and sum length, $F(1,59) = 17.2$, $p < .001$, $\eta_p^2 = .23$. However, there was no significant interaction, $F < 1$.

Efficiency

Participants were less efficient when calculating the sums in the low interactivity condition than when using tokens across both sets of sums (see Figure 7.10, bottom right panel). The efficiency ratio decreased for longer sums, although it was still larger in the high interactivity condition. A 2×2 repeated measures ANOVA indicated a significant main effect of interactivity, $F(1,59) = 22.0$, $p < .001$, $\eta_p^2 = .27$ and sum length, $F(1,59) = 17.1$, $p < .001$, $\eta_p^2 = .22$. However, the interaction was not significant, $F < 1$.

Task Engagement

Participants were more engaged in the high interactivity condition (M = 44.1, SD = 9.2) than in the low interactivity condition (M = 37.8, SD = 8.8). This difference was significant, $t(59) = -6.16$, $p < .001$. There were no significant correlations between the task engagement scale (TES) and the measures of performance (see Table 7.7).

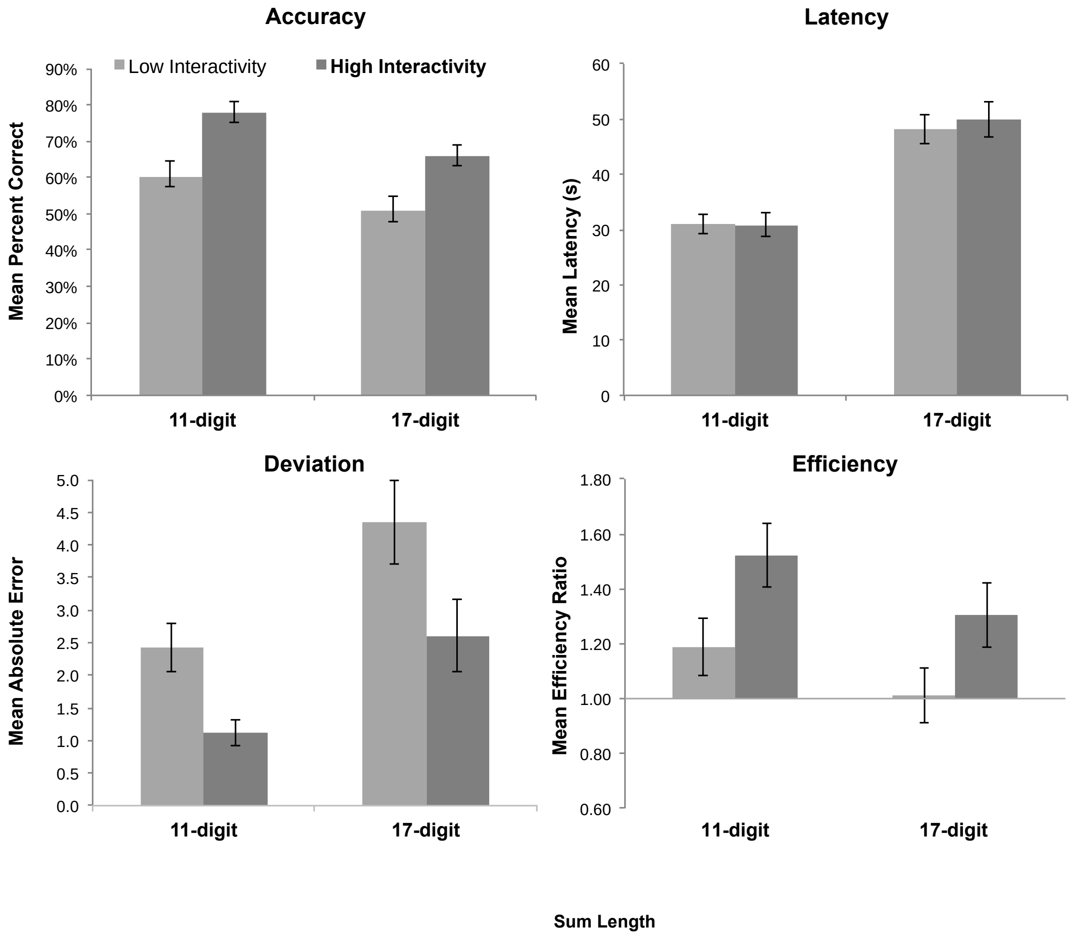

Figure 7.10. Mean percent correct (top left), mean latency (top right), mean absolute error (bottom left), mean calculation efficiency (bottom right) as a function of sum length (11-digit and 17-digit sums) and interactivity condition. Error bars are standard errors of the mean.

Table 7.7

Correlation matrix for average deviations, individual difference measures of maths anxiety, objective numeracy, working memory (computation-span and Corsi blocks), maths expertise and task engagement in both interactivity conditions (df = 58).

	1 MAS	2 OBJ-N	3 C-Span	4 Corsi	5 Exp	6 TES-L	7 TES-H	8 ADE-L	9 ADE-H
1	-	-.47 **	-.47 **	-.25	-.68 **	-.13	.08	.51 **	.27 *
2		-	.60 **	.30 **	.65 **	.18	-.02	-.48 **	-.28 *
3			-	.39 **	.59 **	.25	-.05	-.50 **	-.30 *
4				-	.36 **	-.04	-.08	-.17	-.10
5					-	.14	.02	-.52 **	-.11
6						-	.61 **	-.23	-.23
7							-	.04	.18
8								-	.42 **
9									-

Note: **MAS** = Maths anxiety; **OBJ-N** = Objective numeracy (basic arithmetic skill); **C-span** = Computation-span; **Corsi** = Visuo-spatial working memory; **Exp** = Maths expertise (continuous measure); **TES-L** = Task engagement in the low interactivity condition; **TES-H** = Task engagement in the high interactivity condition; **ADE-L** = Absolute deviation error in the low interactivity condition; **ADE-H** = Absolute deviation error in the high interactivity condition. * $p < .05$, ** $p < .01$

Expertise, Working Memory, and Maths Anxiety

The individual difference measures of expertise, working memory and maths anxiety were analysed as an average of the mean absolute deviations errors in the 11- and 17-digit sums in order to maintain consistency with experiment 1. As the sum length was increased to 17 for this experiment, the two sum lengths were also correlated with individual difference measures, as it was anticipated that the greater sum length would impact the outcomes.

As mentioned earlier in this chapter, relative to calculation error, accuracy is a coarse-grained measure; and accuracy does not discriminate between small and large calculation errors. Absolute deviation error was therefore chosen for further analysis above the other three performance measures for its resolution and for capturing the most important aspect of arithmetic performance: the solutions themselves.

Average sum length. In keeping with Experiment 1, the mean absolute deviation errors in the 11- and 17-digit sums were averaged for each participant creating two new variables, overall mean absolute deviation error in the low interactivity condition (M = 3.38, SD = 3.49) and in the high interactivity condition (M = 1.85, SD = 2.27). The mean absolute deviation error in the low interactivity condition was significantly greater than in the high interactivity condition, $t(59)$ = 3.31, p = .002.

In order to evaluate the influence of individual differences on performance, maths anxiety, numeracy, working memory, and expertise were correlated with mean absolute deviation error (see Table 7.7. and Table 7.8). There were a number of highly significant correlations observed in the low interactivity condition: Maths anxiety, $r(58)$ = .51, p < .001; objective numeracy, $r(58)$ = -.48, p < .001; computation-span, $r(58)$ = -.50, p < .001; expertise, $r(58)$ = -.52, p <

.001. While in the high interactivity condition the correlations were weak: maths anxiety, $r(58) = -.27$, $p = .04$; objective numeracy, $r(58) = -.28$, $p = .03$; computation-span, $r(58) = -.30$, $p = .02$. Expertise was not significantly correlated with average deviation in the high interactivity condition. The difference between the correlation for the high and low conditions with expertise was significant, $p = .013$. There were no significant differences between any other correlations for absolute deviation error.

The impact of sum length. Given the discussion in the previous experiments regarding sum length, it was also noteworthy that the average deviation in the larger sums of 17 digits was significantly correlated with maths anxiety in the low interactivity condition, $r(58) = .44$, $p < .001$, however maths anxiety was not significant in the high interactivity condition (see Table 7.9 and Table 7.10: Corsi and TES were excluded as the correlations produced no additional information). The difference between these correlations was not significant. Computation span was correlated with the 11- and 17-digit sums in the hands-down condition, $r(58) = -.49$, $p < .001$, $r(58) = -.41$, $p < .001$ respectively. However, computation span was not correlated with deviation in the high interactivity 11-digit sums, and the correlation was weak in the 17-digit high interactivity condition, $r(58) = -.28$, $p = .030$. There was a significant difference in correlations between the 11-digit high and low conditions for computation span, $p = .051$. Expertise was not significantly correlated to the high interactivity condition in either the short or long sums, with strong correlations in the low interactivity condition in both sum lengths $r(58) = -.46$, $p < .001$, $r(58) = -.06$, $p < .001$ respectively. The difference between the expertise correlations for both conditions was significant for the 11-digit sums, $p = .05$ and the 17-digit sums, $p = .02$.

Table 7.8

Significant correlations including confidence intervals for individual difference measures of maths anxiety, objective numeracy, working memory (computation-span and Corsi blocks), maths expertise (df = 58).

Variable 1	Variable 2	*r*	*p*	95% CI
MAS	Obj-N	-.466	<.001	[-.646, -.246]
	C-Span	-.464	<.001	[-.639, -.234]
	Expertise	-.681	<.001	[-.796, -.516]
	ADE-Low	.514	<.001	[.300, .679]
	ADE-High	.265	.040	[.012, .486]
Obj-N	C-Span	.598	<.001	[.409, .742]
	Corsi	.303	.019	[.050, .514]
	Expertise	.650	<.001	[.475, .775]
	ADE-Low	-.480	<.001	[-.654, -.258]
	ADE-High	-.282	.030	[-.500, -.031]
C-Span	Corsi	.386	.002	[.152, .585]
	Expertise	.591	<.001	[.396, .733]
	ADE-Low	-.495	<.001	[.282, .668]
	ADE-High	-.303	.020	[-.517, -.054]
Corsi	Expertise	.360	.005	[.117, .562]
Expertise	ADE-Low	-.519	<.001	[-.682, -.306]

Note. **MAS** = Maths anxiety; **Obj-N** = Objective numeracy (basic arithmetic skill); **C-span** = Computation-span; **Corsi** = Visuo -spatial working memory; **Expertise** = Maths expertise (continuous measure); **ADE-Low** = Absolute deviation error in the low interactivity condition; **ADE-High** = Absolute deviation error in the high interactivity condition.

Table 7.9

Correlation matrix deviations for 11-digit and 17-digit sums, individual difference measures of maths anxiety, objective numeracy, working memory (computation-span) and maths expertise in both interactivity conditions (df = 58).

	1 MAS	2 OBJ-N	3 C-Span	4 Exp	5 ADE-L-11	6 ADE-H-11	7 ADE-L-17	8 ADE-H-17
1	-	-.47 **	-.47 **	-.68 **	.49 **	.28 *	.44 **	.16
2		-	.60 **	.65 **	-.43 **	-.12	-.43 **	-.28 *
3			-	.59 **	-.49 **	-.17	-.41 **	-.28 *
4				-	-.46 **	-.13	-.46 **	-.06
5					-	.37 **	.56 **	.33 **
6						-	.15	.17
7							-	.33 **
8								-

Note: **MAS** = Maths anxiety; **OBJ-N** = Objective numeracy (basic arithmetic skill); **C-span** = Computation-span; **Exp** = Maths expertise (continuous measure); **ADE-L-11** = Absolute deviation error in the low interactivity condition for 11-digit sums; **ADE-H-11** = Absolute deviation error in the high interactivity condition for 11-digit sums; **ADE-L-17**= Absolute deviation error in the low interactivity condition for 17-digit sums; **ADE-H-17** = Absolute deviation error in the high interactivity condition for 17-digit sums. * $p < .05$, ** $p < .01$.

Table 7.10

Significant correlations for deviations for 11-digit and 17-digit sums including confidence intervals for individual difference measures of maths anxiety, objective numeracy, computation-span and maths expertise (df = 58).

Variable 1	Variable 2	*r*	*p*	95% CI
MAS	Obj-N	-.466	<.001	[-.646, -.246]
	C-Span	-.464	<.001	[-.639, -.234]
	Expertise	-.681	<.001	[-.796, -.516]
	ADE-Low-11	.489	<.001	[.269, .660]
	ADE-High-11	.282	.029	[.029, .498]
	ADE-Low-17	.437	<.001	[.206, .621]
Obj-N	C-Span	.598	<.001	[.409, .742]
	Expertise	.650	<.001	[.475, .775]
	ADE-Low-11	-.431	<.001	[-.617, -.199]
	ADE-Low-17	-.426	<.001	[-.613, -.193]
	ADE-High-17	-.282	.029	[-.500, -.031]
C-Span	Expertise	.591	<.001	[.396, .733]
	ADE-Low-11	-.495	<.001	[-.665, -.276]
	ADE-Low-17	-.411	<.001	[-.602, -.176]
Expertise	ADE-Low-11	-.463	<.001	[-.682, -.306]
	ADE-Low-17	-.462	<.001	[-.640, -.236]

Note: **MAS** = Maths anxiety; **Obj-N** = Objective numeracy (basic arithmetic skill); **C-Span** = Computation-span; **Expertise** = Maths expertise (continuous measure); **ADE-Low-11** = Absolute deviation error in the low interactivity condition for 11-digit sums; **ADE-High-11** = Absolute deviation error in the high interactivity condition for 11-digit sums; **ADE-Low-17**= Absolute deviation error in the low interactivity condition for 17-digit sums; **ADE-High-17** = Absolute deviation error in the high interactivity condition for 17-digit sums.

Discussion

Participants completed two sets of addition problems: one set was completed with restricted hand movement reducing interactivity; the other using round numbered wooden tokens increasing the opportunity to reconfigure the problem presentation as the sum was calculated. Generally, participants answered more sums accurately, achieved higher efficiency ratios and the calculation error was lower in the high interactivity condition. Latency, however, remained constant across the two levels of interactivity for the short and long additions, suggesting improvements in other measures were related to the mode of problem solving, rather than the time required in completing the addition. This improvement in performance could not be attributed to extraneous between-subject factors, such as individual differences, because of the repeated measures design employed in this experiment: all participants completed the sums in both interactivity conditions.

All participants completed a questionnaire in order to assess the individual's fundamental level of expertise in maths. The strong correlation between objective numeracy and expertise, $r(58) = .65$, $p < .001$, indicated that this measure of expertise reflected the arithmetic proficiency of the individual. With a low interactivity problem presentation and hands down on the table, participants' performance reflected their arithmetic and working memory skills. The lack of correlations with arithmetic performance and expertise in the high interactivity condition implied that the manipulation of the numbered tokens augmented the arithmetic skills of participants with less maths expertise such as to render their performance indistinguishable from those with greater expertise. This supports the notion that a greater degree of interactivity may improve the performance of those with less maths expertise in these simple arithmetic problems.

The influence of maths anxiety on performance was very different as a function of interactivity. When interactivity with the world was low, maths anxiety had a significant impact on performance: Higher maths anxiety scores were strongly related to greater calculation error. In turn, the high interactivity context reduced the variance in performance explained by maths anxiety, in fact when the sums were longer, high interactivity eliminated the impact of maths anxiety. This implies that even on these simple maths tasks, a dynamic presentation offering a greater level of interactivity may assist in reducing or controlling the impact of maths anxiety on mental arithmetic performance.

In the previous experiments sum length was identified as a possible factor in performance impacted by interactivity. Further exploration of performance when measured as the absolute deviation from the correct answer revealed that correlations with maths anxiety, numeracy, working memory capacity and expertise were lower for the high interactivity condition in both the long and short sums. Of particular note was when participants were asked to complete the long 17-digit sums in the high interactivity condition the influence of maths anxiety and expertise dissipated considerably, suggesting that as the arithmetic became more challenging, high interactivity was of particular assistance not only for the maths anxious folks but also for those with poorer maths skills.

The enhancements in performance of the lesser skilled individuals in the high interactivity condition may be attributed to off-loading working memory onto the external environment. However, a closer examination of the results revealed that cognitive artefacts potentially have a more complex role in enhancing the capabilities of internal resources such as working memory. In this experiment, computation-span correlated highly with numeracy and expertise supporting claims that working memory is a contributing factor to mental arithmetic skill (see Butterworth, 2006). The computation-span test used in this experiment was designed to reflect a conventional complex span task requiring some numerical skill; unsurprisingly, this correlated with measures of maths skill in the low interactivity condition, more interestingly the correlation with performance was weak in the high interactivity condition. The Corsi task, as a measure of visuo-spatial working memory was deliberately selected to reduce the reliance on numeracy. Corsi scores did not correlate with performance in either condition of interactivity. The findings here are consistent with previous mental arithmetic research conducted in lower interactivity environments indicating that the short-term storage component of the visuo-spatial sketchpad played a small role in mathematical performance (Lee, Ng, Ng, & Lim, 2004). The two measures of working memory, computation-span and the Corsi block task, were moderately correlated, indicating that they measure similar but not identical components of working memory. Span tasks, such as the computation-span assess an individual's working memory in both processing and storage, whereas the Corsi test as designed here gauges storage capacity only. These results suggest high interactivity does not simply function as a means for off-loading working memory storage, rather the manipulation of the tokens scaffolds thinking enhancing the participants' calculations (Kirsh, 2013).

Individuals were more engaged in the task when given the opportunity to use the tokens than when they had to maintain their hands on the table. This pattern in the level of engagement did not change as a function of math expertise. Notably performance, as measured by absolute deviation error, was not influenced by how engaged participants were in the task. Participants might have felt more engaged when completing the task with tokens, but the level of engagement did not in itself explain the improvement in arithmetic performance in the high interactivity condition.

Expertise in the domain of mathematics may be attributable to factors including practice, intrinsic reward and components of working memory. However, this chapter has shown that a systemic perspective on mental arithmetic helps us better understand how resources internal and external to the participants are configured dynamically to reflect expertise and skills at solving simple mathematical problems.

Chapter Summary and Conclusion

Over the centuries many devices have been created to ease the burden on internal resources when undertaking mental calculations—such as the abacus, the slide rule and the digital calculator (Moore, McAuley, Allred, & Ashcraft, 2015). There are good reasons for these inventions as mental arithmetic taxes internal resources including working memory and executive functions when strategies for commencement of the problem are devised then numbers are manipulated, held and carried forward during processing. However, these devices may not be to hand when a challenging arithmetic problem arises, in which case individuals may creatively mine the environment in order to share the cognitive load with other artefacts, such as coins, matchsticks or pebbles. Even simple sums similar to the ones presented in these three experiments, benefit from the interaction with objects, such as tokens, when arriving at the correct solution.

Whether it is everyday problem solving, school-based learning or experiments on cognition in the lab, individuals are interacting with the world they inhabit. Interactivity with the world is a dynamical, sense-making process that coordinates and integrates human actions in the bidirectional coupling of the body with others, with artefacts and with practices (Steffensen, 2013). The interaction with artefacts has been discussed in this chapter in terms of influencing the evolution of mathematics, speculation on the impact on biological changes in neural regions as a consequence of the plasticity of the brain, distributing the cognitive load and scaffolding cognitive processes (Dehaene et al., 2003; Kirsh, 2009a; Malafouris, 2013; Vallée-Tourangeau, 2013). Previous work on everyday problem solving based on ethnographic methodologies (Carraher et al., 1985; Hutchins, 1995a; Lave, 1988) has provided invaluable insight into the processes of distributed cognitive systems, however, the methodology is time-consuming and it is difficult to isolate variables influencing cognitive processes in real-world environments (Hollan, Hutchins, & Kirsh, 2000). By exploring one component of the ecology of the mathematical problem-solving environment in the lab, using artefacts that simulate everyday objects provides the opportunity to create a window onto the strategies and processes of interactivity between individuals and the world.

The Three Experiments

The experiments presented here contribute to the growing contention that thinking does not necessarily take place solely 'in the head' but rather emerges from an interaction with the world. As an individual's level of engagement with the world increased through changes in the task environment, the performance outcomes improved. Clearly illustrating that the task environment must be taken into consideration when researching cognition. The first experiment was part of a trilogy of experiments designed to investigate interactivity and problem solving. In the first experiment all participants completed a set of 10 sums, five 7-digit sums and five 11-digit sums in two conditions. One was a hands-down, low interactivity condition and the second was a high interactivity condition using numbered wooden tokens. The results of the first experiment using a low level and a high level of interactivity echoed a similar experiment by Vallée-Tourangeau (2013) where interactivity was shown to have a positive effect on the number of correct answers announced by participants. In addition previous research had shown that maths anxiety impacted maths performance (e.g., Ashcraft & Moore, 2009) of individuals even when completing simple sums. This informed the decision to measure the impact of levels of interactivity against maths anxiety and to assess the relationship between maths anxiety and other measures of performance. Although there was no significant relationship between maths anxiety and the levels of interactivity, scrutiny of the direction of the relationship between mean absolute deviation and maths anxiety revealed an interesting trend. In the 11-digit sums maths anxiety appeared to be reduced when individuals manipulated tokens while solving the problems, suggesting that feelings of anxiety in maths anxious folk may be eased by the opportunity to use artefacts in solving these problems.

The second experiment continued the investigation into maths anxiety and the effect of changing problem presentation on performance in solving simple maths problems. Evidence from previous research has shown, that when people use verbal explanations of maths problem solving, hand gesturing and pointing, aids learning in children; with hand gestures lightening the cognitive load for adults and children alike (Goldin-Meadow et al., 1993; Goldin-Meadow et al., 2001). Therefore, in addition to the two conditions used in Experiment 1—hands-down and tokens—a third condition was included, namely the pointing condition, giving individuals the opportunity to use their hands while adding the sums. As pen and paper are also common tools for easing the cognitive burden in simple and complex maths calculations, this was incorporated as a fourth condition. In this experiment all participants completed 20 sums, separated into four sets of five 11-digit sums of equal difficulty—one set for each of the four differing presentation conditions. In order to scrutinise the strategies used to solve the sums more closely, along with a quantitative analysis similar to Experiment 1, a video recording of one of the participants completing all four conditions was analysed from a qualitative perspective. Even in these simple sums, performance was enhanced when using artefacts as part of a distributed cognitive system. As anticipated, the video evidence revealed, differing strategies were used in each of the four presentations, supporting the conjecture that the affordances offered by changes to the components of the distributed thinking system may alter problem solving processes. The variety of unfolding strategies and trajectories enacted when participants were presented with similar sums in the differing interactive states indicated that cognition was a result of on-line activity taking place in the world (O'Regan & Noë, 2001; Wilson & Clark, 2009). The thinking was distilled through the interaction with the situation in which the reasoner was embedded, rather than dictated by a sequence of preplanned actions determined solely by an internal representation of the problem solution, where the world offers a dynamic representation (Vallée-Tourangeau, 2013). Although the maths problems were unambiguous there was evidence from the qualitative micro examination of the movements made by participant that the trajectory through the problem space was not as a result of a preselected fixed path formulated as an internal representation. In using the tokens the unfolding cognitive processes generated actions that were scaffolded by the physical environment, demonstrating the coupling of the internal and external resources in solving these simple maths sums.

Unlike Experiment 1 there was no evidence in Experiment 2 of a relationship between maths anxiety and absolute deviation error at any level of interactivity. This may have been a consequence of fewer sums in each set; as a consequence maths anxiety may not have been sufficiently discernible in the maths anxious folk for a significant result to emerge. Along with maths anxiety, the attitude of participants to the maths tasks was measured in terms of being in the flow and whether any changes in performance could be attributed to this feeling of being fully engaged in the activity (Csikszentmihalyi, 1990). Flow, as measured using the task engagement scale, was positively affected by a greater degree of interactivity, although it was not a predictor of performance. Experiment 2 illuminated the strategic differences in reaching a solution between various types of interaction; the pathway to solving the problem was altered in tandem with improvements in performance when the opportunity to manipulate the world was at its greatest.

The previous two experiments did not take into account the maths experience of individuals when assessing mathematical performance. The literature on expertise has shown that a number of factors may contribute to the higher levels of performance by experts over those less experienced, including genes, deliberate practice, intrinsic reward and working memory capacity (Butterworth, 2006; Ericsson & Charness, 1994; Galton, 1892; Sternberg, 1999). A novel proposal for this experiment was that the capabilities of those less skilled in maths would be enhanced when offered the prospect of greater interaction with the world. Conversely, it was expected that the simplicity of the sums would mean that changing modes of interactivity would not affect the performance of those proficient in maths. As in the previous experiments all participants completed sums in a low, hands-down condition, and a high interactivity condition using tokens. However, in this experiment the sums were 11 and 17 digits in length and information on maths experience for each individual was recorded. The results revealed that, as predicted, with performance measured as the absolute deviation from the correct answer, there was no significant difference in maths performance between experts and those less adept at maths when individuals where interactivity with artefacts was increased. Parallels can be drawn with the findings by Lave (1988) where maths performance for best-buys in the supermarket was not related to formal maths skills as taught in school. Lave concluded that through the shoppers actions in the physical environment of the supermarket, accuracy was improved as these actions gave rise to the shaping and reshaping of the problems as the shopper evaluated information in the supermarket setting. Therefore, both the results from mental arithmetic Experiment 3, and Lave's study provided evidence that in the dynamic coupling of the individual and the world, cognition is not reducible to knowledge stored in the head, but is enacted, and extended into the world shaped by the situatedness of the activity (Gallagher, 2014).

Unpacking the correlations as a function of sum length showed the increased difficulty of the task was evident in the decline in performance observed in all four of the performance measures (accuracy, latency, absolute deviation error, and efficiency) in the longer sums. However, maths anxiety dissipated when individuals used the tokens in completing the longer sums of 17 digits. On the other hand when interactivity was restricted the reliance on internal resources was greater, thus for the maths anxious individuals maths performance was hampered as working memory capacity was absorbed by anxious thoughts (Ashcraft & Kirk, 2001). This illustrates, as outlined in the discussion for Experiment 1 that as would be expected the length of the sum might have an impact on maths anxiety. Nonetheless these feelings, that otherwise interfere with successfully solving the problem, can be mitigated by stretching the cognitive load across mind, body, and the environment, in this case by increasing the opportunity to use physical artefacts in problem solving.

Consistent with findings in Experiment 2, the measure of flow indicated that participants were more engaged when using the tokens than when asked to solve the problems with restricted hand movements. There was no significant relationship with performance, nor was flow related to expertise or any other performance indicators; experts and less skilled mathematician's alike reported feelings of being engaged with the task. These findings also support the argument proposed in Experiment 2, that the feeling of flow is attributable to the problem presentation not the individual's change in capabilities. However, over time the positive effects of flow combined with the improvements in performance may be worthy of further investigation (Nakamura & Csikszentmihalyi, 2002).

Computationalism

Clearly participants utilised existing knowledge to complete the basic additions presented in the three mental arithmetic experiments, it would not have been possible to add a series of single digits together without the minimum of formal arithmetic knowledge. Wilson and Clark (2009) discussed the soft-assembled whole of transient extended cognitive systems (TECS), interweaving the problem-solving contributions of the internal resources of the human brain with the body and local elements available to enable cognitive scaffolding. The maths tasks here provide an example of where the cognizer draws on well-used, frequently accessed TECS in the form of simple additions knowledge enacted in the world coupled with manipulation of the artefacts to enhance performance in arriving at a solution. In addition to being indicative of this type of extended cognitive system, the tasks used in these experiments potentially present an empirical case showcasing wide computationalism in practice. In the micro-examination of movements in Experiment 2, it was apparent that the artefacts provided an opportunity to extend the computation beyond the internal manipulation of symbols in the mind. From an extended computationalist view the computational process was not restricted to processes in the head, the active manipulation of symbols, was spread across both the internal process and external representations of the problem (Wilson & Clark, 2009). Here, operators and symbols are retrieved from long-term memory, and are involved in producing interim totals in the computation of the sum. However in moving the tokens into groups of good provisional sums (GPS) and same digit groupings (SDG) the computations are also external to the cognizer as they are spread across the cognitive system. Therefore from a distributed perspective, the executive functions and enactment of strategies are best thought of as emergent properties of the agent-environment system, where the elements of the computational system are distributed.

Conclusion

All three experiments offered varying levels of interactivity. The interactive states ranged from a high level of interaction using tokens to a low level with only the numbers printed on a sheet of paper, where the interactivity was the back and forth process between the external presentation of numbers on the page and the internal processes of the brain. However, in all three experiments performance was clearly enhanced with a greater opportunity to extend the cognitive system into the world.

Adapting the cognitive psychologist's laboratory to permit the physical manipulation of a problem presentation offers a more representative window onto thinking outside the laboratory. To be sure, people can simulate and think in their head without physically interacting with the outside world—although this internal cogitation may well reflect the internalisation of much interactivity. However, as Clark (2010) points out it is worth taking note of how much cognitive activity takes place through engagement with the environment: " brains like ours will go to considerable lengths to *avoid* having to resort to …

fully environmentally detached reflection" (p. 24). The data presented here reveal the importance of engineering task environments in the lab that support distributed problem representations to better understand the engagement of individuals as they explore and manipulate the external world to solve problems.

Chapter 8

General Discussion

"Human life does not occur in a vacuum, nor is nature a mere stage setting for the enactment of its drama" (Dewey, 1916a, p. 267).

Individuals function and think in the world, adapting to changes in the surrounding environment, by interacting with things and the people around them. As Dewey (1916a) observed, humans do not live in a vacuum and the environment is not a passive backdrop to people living in the world, thus it follows that research and theories on human cognition should be inclusive of this interaction with the world. On this basis, as part of the research programme for this thesis, five experiments were designed for a laboratory setting, to investigate interactivity between the agent and the world. These experiments explored the effect on performance of differing levels of interactivity in problem solving by changing the nature of the task ecologies. Cognitive capacities and dispositions have been shown to impact performance in problem solving and reasoning (Stanovich & West, 1998); with this in mind, the effect of interactivity was also evaluated against a range of individual difference measures. The outcomes from the experiments reported here led to conclusions supporting existing research and theories that cognition emerges through a sense-making process of the coupling of agent with world in a dynamic interactive system.

Overview of Experiments

The river-crossing experiments. The specific aims in undertaking the first two experiments using the river-crossing problem were to determine the impact of interactivity on analogical problem solving, and if any learning occurred as a function of interactivity when completing the problem across different presentations. The problem was presented in two different formats one in a low interactivity context and the other offered a greater level of interactivity. The presentation for the lower level of interactivity was developed to maximise the burden on the internal resources of the participant, with the individual asked to verbalise the moves required in attempting to solve the problem. The researcher recorded the moves on paper; however, the only recording accessible to the participant was a simple graphical representation of the position of the animals on the riverbanks. In the high interactivity context, the participants were given artefacts to solve the problem in the form of a river painted on a board, a wooden raft and six plastic animals—three chickens and three wolves. The participant was free to move the animals and the raft in accordance with the rules. In the first experiment participants were asked to complete the problem in both the low and high interactivity condition, with the order counterbalanced across participants. Therefore, one group of participants completed the low interactivity condition first and one group completed the high interactivity condition first. When assessing performance based on the time taken for each move, overall the high interactivity performance was better than in the low interactivity condition irrespective of order. An effect was found when the low interactivity condition was completed first followed by the high interactivity condition; participants enacted the moves in the high interactive context significantly faster than any other attempt at the problem. Thus, in addition to showing that greater interactivity encouraged participants to make more moves at a faster rate than when asked to simulate a solution mentally, there were performance benefits to completing the low interactivity condition first, followed by the high interactivity condition. Two possible explanations were discussed in Chapter 6: (i) the process and quality of knowledge acquisition is different as a function of the level of interactivity or (ii) interactivity is a performance facilitator and a high level in interactivity more clearly showcases learning. In further evaluating the effects of interactivity and learning across the two presentations, a second experiment was designed to separate the effects of order and interactivity by manipulating the levels of interactivity using a between-subjects design. Similar to Experiment 1, in Experiment 2 participants were asked to complete the river-crossing problem twice, however, this time the participants either completed the high interactive version twice or the low interactive version twice. Performance mirrored the first experiment in so far as the greater level of interactivity using the artefacts resulted in more moves in a faster time than when moves were simulated mentally. Learning was also evident in the second attempt of the problem in both conditions. However, the results showed that the learning in both conditions was similar, that is, the performance improvement was the same when completing the task twice in the low interactivity condition or completing the task in the high interactivity condition. In disentangling the order from the learning, this experiment shed light on the results from the first experiment. The indication from these two experiments was that greater interactivity using artefacts enhanced the capabilities of the reasoner by encouraging the exploration of the world for a solution, as the solver enacted more moves faster than when interactivity was low. The epistemic actions in moving the artefacts around the board may not have been evidence of distributing working memory load. However, the implication may be that the manipulation of the artefacts provided a scaffold to enact new strategies, suggesting a fruitful avenue for future research in the detailed analysis of problem solving strategies. Learning transfer was evident, as the high interactivity condition, where moves were cheap, provided the opportunity to operationalise the rules learnt as a result of greater internal processing from a low interactivity environment.

This type of problem, while having the advantage of being adaptable to problem presentations of differing levels of interactivity—low and high—was also a puzzle with a narrow problem space and moves were tightly constrained by the rules. As Lave (1988) pointed out, a shortfall of this type of problem was that there were limitations as a representation of real world problem solving as it was based on normative models of formal problem solving. While participants could make many moves, there was only a narrow path to solution. However, the foremost reason for using this task was, not to compare the problem solving skills of individuals, rather to investigate the impact of changing the presentation. Lave (1988) also noted, and as discussed in Chapter 2, previous laboratory experiments did not take into account situation or context, resulting in an absence of an account of interactivity when investigating problem solving. The experiments presented in this thesis explored situatedness and external representations by altering the problem presentations, focusing on the changes in interactivity. The improvements in performance from the low to the high interactive contexts showed that interactivity with the artefacts augmented the capabilities of the individual by extending thinking into the world, with the artefacts becoming constituent components of the extended cognitive system. In a more specific focus on the extended mind hypothesis, the coupling of agent with the world in the river-crossing problem might be classified as a one-off Transient Extended Cognitive Systems (TECS) (Clark & Chalmers, 1998; Wilson & Clark, 2009). Here the thinking emerging in the coupling of the mind and the artefacts in this setting was only for the purpose of solving this particular problem.

It may be the case that using a different problem-solving task that draws on existing knowledge, could be considered as another type of TECS, which is more closely related to thinking in the world, possibly producing different results. Simple mental arithmetic tasks that required fundamental maths skills were chosen for the next experiment, as this was not a test of maths skills per se, rather to further the investigation into the effects of changing the problem presentation.

Mental arithmetic experiments. Evidence of problem solving strategies prompted through actions in the world was suggested by the increase in moves enacted in less time for the first two river-crossing experiments. In previous work on interactivity and mental arithmetic, Vallée-Tourangeau (2013) also reported an improvement in performance when the interactivity was greater. To investigate the possibility that interactivity with different problem presentations would affect performance, three experiments involving simple mental arithmetic were designed. The aim of these experiments was also to further explore complementary strategies, the role of artefacts in cognition, and wide computationalism as the distribution of computational cost was expected to spread across resources (Kirsh, 2013, Wilson & Clark, 2009). Investigating interactivity in varying problem presentations using mental arithmetic emphasises the concreteness of mathematics rather than an abstract manipulation of symbols taking place only in the mind as in the physical-symbol-system hypothesis within the classical framework of cognition (Hutchins, 1995a). This was not to reject the symbol-system hypothesis of traditional cognitive science, rather to recognise that the manipulation of symbols as part of the distributed cognitive system that encompasses internal and external processes. Returning to the images of the clay accounting tokens of the Uruk period (4000 B.C.-3500 B.C.) as discussed in the beginning of Chapter 7, these were external symbols manipulated in the physical environment as representations of the types and quantities of commodities being traded within the community (Malafouris, 2013). Therefore, it could be argued that the internal symbols as described by the traditional account of cognition, were formed as a consequence of interactions with the world. As Hutchins (1995a) pointed out the symbols were formed first in the world, only then were these symbol structures re-represented in the mind (also see Malafouris, 2013). In accepting the computer as a model of thinking, the assumption became that the manipulation of symbols as part of representations was an exclusively mental process. Therefore, as illustrated in the maths experiments presented in Chapter 6, internal symbol manipulation is only part of the story of the cognitive architecture, the representation of these symbols externally and the interaction between the constituent components of the dynamic cognitive system impact performance in solving a problem.

The three mental arithmetic experiments progressed from the investigation of interactivity across problem presentations through to testing the impact of interactivity on individuals with differing mathematical experience. All three experiments used as a basis for investigation a series of long sums in at least two conditions of interactivity: A low level of interactivity where the participant adds a series of numbers with hands on the table; and a second level of interactivity also adding up a series of numbers, however this time using numbered wooden tokens. Across the three experiments the length of the sums and the number of sums added was altered. In the first experiment all participants completed 10 sums—five sums of 7 digits and five sums of 11 digits—in the low interactivity condition and the high interactivity condition. In the shorter sums the performance, when measured as absolute deviation error, was very similar, however, in the longer sums performance was greatly improved when individuals were given the wooden tokens to manipulate. In the second experiment all participants were asked to complete 20 sums of 11 digits, these sums were split into four conditions: The low and high conditions, as previously described—hands-down and tokens—a pen with paper condition, and a pointing condition. The two conditions with the opportunity to use artefacts—tokens and pen with paper—resulted in a significantly better performance than the other two conditions—hands-down and pointing—with the token condition resulting in a slightly better performance than all four conditions. The qualitative frame-by frame examination of one participant's performance revealed the differing trajectories to solution in the three higher interactive conditions. In the third experiment participants with varying mathematical expertise were asked to complete 10 sums—five sums of 11 digits and five sums of 17 digits—in the low and high interactivity conditions. When assessing the performance of all participants, again measured as absolute deviation from the correct answer, performance improved when the task was enacted in the high interactivity context than in the low interactivity context in completing both the shorter and longer sums. The relationship between expertise and absolute deviation from error indicated that when participants relied more heavily on internal mental processes, the greater the level of expertise the better the performance, however when participants were able to use the tokens to calculate the solution the difference in performance dissipated. The results from the five experiments implied that when individuals were given greater opportunity to interact with the world, this adjustment to the dynamic coupling of individual with environment enhanced cognitive capabilities.

Cognition: A Circle Not Just an Arc

In the earlier chapters of this thesis, the discussion on situated and extended cognition indicated that the development of many contemporary approaches to cognition have some commonalities, although the philosophical backdrops may be different[7]. Some of the commonalties lay in the opposition to mind-body dualism of the internalist approach to cognition, the impact of situatedness, and the coupling of agent with the world in a dynamic cognitive system. However, as will be discussed further, this common ground between approaches to cognition such as situated, extended, enactive and distributed cognition was foreshadowed by some of the classical pragmatists a hundred years ago (Gallagher, 2014).

The initial foundations of contemporary cognitive psychology were influenced by research into artificial intelligence with claims that cognition was computational, as the mind essentially functions to manipulate symbolic representation as an internal information-processing device (Pylyshyn, 1989). According to Simon and Kaplan (1989), the human and the computer use similar symbol-manipulating processes with this invariant occurring as "computers were made in the image of the human" (p. 40). The ideas of cognitive science and artificial intelligence were based on the Cartesian assertion that human understanding comprises of the formation of symbolic representations; these representations, while modeled on external entities are manipulated inside the mind in accordance with sets of formalised rules built up

[7] For example, the foundations for enactivism can be found in phenomenology, extended mind has roots in analytical philosophy and distributed cognition has some beginnings in cognitive science and anthropology (Gallagher, 2016).

over time (Hutchins, 1995a). The physical-symbol system of traditional cognitive psychology, while indifferent to the perception-action of the body, does not entirely ignore the external world, with structures outside the brain—physical, social, cultural, historical, contextual—considered, at best as tools aiding internal cognitive processes functioning as passive inputs (Hutchins, 1995a). However, this over emphasises the capabilities of the internal processes, while disregarding the interaction that coordinates the internal processes of the mind, with the body and the outer world (Hutchins, 1995a). Hutchins argued that while computer programs are able to imitate many aspects of human thinking through the automated manipulation of symbols, the computer does not replicate human interaction within the situated environment. He described how the conception of Babbage's early hypothetical computer, and later the Turing machine, began with the notion of how a mathematician would break down a problem if acting in the material world. As Hutchins also argued, the computer was not made in the image of the way a person thinks in the world. Rather the computer was modeled on one component of the human cognitive process, namely that human thinking processes are patterns observed in the world resulting from the manipulations of symbols both internally and externally. Turing's hypothesis for his machine originated through the embodied actions of the individual generating a computation by the manipulation of symbols in the material world (Hutchins, 1995a). Dennett (1991) succinctly described the process of Turing's original rationale:

> He was thinking self-consciously and introspectively, about just how he, a mathematician, went about solving mathematical problems or performing *computations*, and he took the important step of trying to break down the *sequence* of his *mental acts* into their primitive components. "What do I

do," he must have asked himself, "when I perform a computation? Well, first I ask myself which rule applies, and then I apply the rule, and then write down the result, and then I look at the result, and then I ask myself what to do next, and…" (Dennett, 1991, p. 212; quoted in Hutchins 1995a, p. 361).

These concerns raised by Hutchins, regarding theories that the inner process of the computer reflect the totality of human thinking processes, were not dissimilar to those voiced by Peirce (1887) a century earlier when he discussed the reasoning capabilities of Weber's "adder up" and Babbage's hypothetical "analytical engine" (p. 165). Peirce doubted that the limited mechanical functions of the machine represented the full reasoning processes of the living mind. He believed these machines had two major deficiencies. The first being lack of initiative, thus the machine was incapable of developing original ideas resulting in the inability to create original problems other than those for which it was programmed. The second was the limitations of capacity, not unlike the human mind; however the human had the initiative and ability to draw on external resources such as pen and paper to extend and overcome cognitive limitations. Gallagher (2014) suggested that Peirce's views on human reasoning processes are identifiable as antecedents to non-internalist theories on cognition, such as the extended mind hypothesis. Therefore, it may be possible to locate the philosophical precursors to some of the contemporary theories of non-internalist cognition in the writings of Peirce and other early pragmatists such as Dewey (Gallagher, 2014, Menary, 2010b, 2011). Gallagher believed, that the identification of pragmatism as antecedent to the enactive and extended conceptions of cognition could potentially resolve some of the differences of opinion between these two approaches—enactive and extended cognition—resulting in the creation of a more integrated approach combining both approaches (see also Menary, 2010b; 2011).

Pragmatism

In order to appreciate Gallagher's (2014) position on the legacy of pragmatism as a possible foundation for an integrated theory of enacted and extended cognition, it will be useful to a provide a snapshot of some of the philosophical observations of two of the classical pragmatists, Dewey and Peirce, writing at the turn of the nineteenth century. The empirical evidence offered by the experiments in this thesis resonate with these observations, this will be discussed briefly as the theories unfold. This in turn will ally these philosophical observations with situated, distributed, and embodied cognition. While the non-internalist conceptions of cognition have different origins, as discussed previously, woven through many are the common thread of situatedness and interactivity.

Dewey (1896, 1916a, 1916b, 1938) with colleagues Mead, James, and Peirce, was an influential pragmatist, at the turn of the nineteenth century. Dewey's philosophical views on psychology were influenced by Darwin's evolutionary theories, and Hegelian philosophy of the self and object as mutually constituted (see Bredo, 1998, for a detailed account). In general, the pragmatists approach to psychology was to attempt a unified biological and sociocultural approach, in order to create a psychological middle ground between the associationist psychologists and the Idealist philosophers (Bredo, 1998). The associationists supported a reductionist approach to the mind where it was viewed as a mechanistic process, connecting rudimentary thoughts together; metaphorically it was described as a telephone switchboard device connecting different associated phone lines (Bredo, 1998). This mechanist approach was committed to mind-body dualism, separating the processes of the brain from bodily actions (Bredo, 1998). The other end of the spectrum in the account of human mentality was the more culturally holistic view of the mind—an idealistic representation of the world where human ideas shape society and culture, based on ideal principles, where reality is ultimately a mental perception (Bredo, 1998). Both these notions were diametrically opposite to the views of the pragmatists. The pragmatists were committed to an emphasis on the realities of the world, therefore it was not conceivable that the mind was a separate entity from the rest of the body and the environment, rather the mind was in dynamical interaction with the body and the surrounding world (Bredo, 1998; Gallagher, 2014). Dewey (1916a) could see no place for mind-body dualism when describing the role of the brain in cognition. He discussed how the brain, from a biological perspective, was essentially one organ in the body that interacted with other organs in the body, and functionally the brain co-ordinates both the interaction between other organs in the body and the reciprocal action between the body and the environment (Gallagher, 2014).

Dewey and sensori-motor coordination. Dewey's (1896) opposition to mind-body dualism was clearly set out in a critique of a psychological approach to human behaviour being proposed at the time, based on the physiological account of the mechanical functioning of sensory and motor systems in the human body—namely the reflex arc concept. This reflex arc concept emerged from physiological research into human reflexes in the early nineteenth century and was quickly mapped by psychologists on to the human psyche as an explanation for human behaviour.[8] The reflex arc is described as the reaction

[8] There is no mention by Dewey (1896) of a specific author proposing the reflex arc concept in psychology.

from a stimulus that results in a reflex action, such as the knee-jerk reaction when the knee is hit by a small hammer (Sherrington, 1892). According to the reflex arc concept when applied to psychology, human behaviour could be explained in a similar physiological manner as a series of stimulus response events. To some extent this critique by Dewey of the reflex arc concept in psychology was a response in reaction to the associationist model of psychology. The intention behind the reflex arc concept as applied to psychology was to overcome the dualistic conjectures of the associationists, as human behaviour could now be explained by a connection between the mind and the body, rather than a distinction between the two (Biesta, Miedema, & van IJzendoorn, 1990). However, Dewey (1896) was not convinced, believing that the old dualism had been replaced rather than "displaced" (p. 357). Dewey described how the "older dualism of sensation and idea is repeated in the current dualism of peripheral and central structures and functions; the older dualism of body and soul finds a distinct echo in the current dualism of stimulus response" (p. 358). Physiological research had shown how the nervous system was divided into a sensory component and a motor component with the parts being mediated by the brain via the spinal cord, therefore it was suggested mentality could be understood in a similar manner (Bredo, 1998). However, according to Dewey this new notion for explaining human behaviour was very similar to the prevailing associationist approach where complex thoughts were considered as a linear association by the internal mind of a series of simple ideas.

Dewey used James's (1890) child-candle example (Figure 8.1) to illustrate how the reflex arc concept separated sensation, idea, and action, portraying human behaviour as a disjointed, piecemeal series of events in an activity of "sensation-followed-by-idea-followed-by-movement" (p. 359). In his opinion what was required was a concept that did not perceive these constituent parts of sensory and motor as distinct entities rather as a members of a circle of "sensori-motor co-ordination" (p. 358). Dewey (1896) suggested that the reflex arc concept was a narrow view of the human mind, as the response-stimulus approach emphasised the physiological muscular reaction, ignoring the self-organising and the intentional goal directed behaviour of the individual acting in the world (Bredo, 1998; Dewey, 1916a). The reflex arc concept only captured a broken section—an arc—of the continuous circuit of action by an individual acting in the world (Dewey, 1896). The arc was considered by Dewey to be a meaningless fragment, where the context of the individual's action and the environment were excluded (Bredo, 1998). He conceived of this circuit of action as a continuous reorganisation of the process of experience, as the agent acted to change the world, and the agent was changed by the world as part of the process (Bredo, 1998; Dewey, 1896). In Dewey's view "this circuit is more truly termed organic than reflex, because the motor response determines the stimulus, just as truly as sensory determines movement" (p.363). There is no distinct sensory phase and separate motor act, only a "sensori-motor" (Dewey, 1896, p. 358) co-ordination of the members of the circle of activity—mind, body, and environment. To the pragmatists such as Dewey, the sensing of the stimulus was not a static internal cognitive state represented as a series of disconnected actions; the cognitive process was part of the activity dependent on bodily actions and shifted according to the situation. In the child-candle example, the mechanistic reflex arc account would portray this a seeing-reaching process, but this would only capture part of the unbroken sensori-motor coordination act. As Dewey explained, if the situation where the child reaches for the bright light was considered as a seeing-reaching process—where sometimes the bright light is a candle resulting in pain and sometimes it is not a candle with the reaching resulting finding a treat, the stimulus and the response would now be uncertain. The problem for the child is whether to reach for the bright light or not, with the encounter dependent on the interaction between the mind, body, and the environment as components of the sensori-motor coordinated circuit of activity, as the situation is different with each experience. This explanation also alluded to a view of problem solving not unlike that expressed in later theories of distributed cognition. The notion that problems and the solutions unfolding through action was evident in Dewey's (1896) reflections on thinking when he explained, "At one moment the various activities of reaching and withdrawing will be the sensation, because they are that phase of activity which sets the problem, or creates the demand for, the next act" (p. 368).

Stimulus 1 (Bright candle)	→	Response 1 (Reach)	→	Stimulus 2 (Pain)	→	Response 2 (Withdraw hand)

Figure 8.1. A representation of James's (1890) child-candle example, to illustrate the experience of the child reaching toward a candle when viewed linearly as described by reflex theorists (adapted from Bredo, 1989.)

In describing how the sensori-motor co-ordination was a unity of activity and not a series of disjointed events, Dewey's approach to psychology defined the unit of analysis, not as the individual but the person in action, initiated by what the individual was doing. For example, in the child-candle instance, Dewey's analysis began with the "act of seeing; it is looking, and not a sensation of light" (p. 358 -359). This was echoed in a similar conclusion drawn a century later by Lave (1988) following her ethnographic study into cognition in the lived-in world and others including Hollan, Hutchins, and Kirsh (2000) in developing the distributed approach to cognition.

Situatedness. Dewey (1938) proposed that it was the dynamic organism-environment interaction that was fundamental in explaining the process of cognition, with interaction inextricably linked with the situation: “The conceptions of situation and of interaction are inseparable from each other” (p.43). According to Dewey’s notion of ‘situation’ it was not the geographical location or physical setting in which an individual was acting—situation was considered as encompassing the environment. The individual and the environment are constituent components of the situation, as the individual does not think or act in isolation from the environment, but in a reciprocal circuit of interaction with the physical world of objects and people (Dewey, 1896, 1938).

Dewey’s (1896, 1916a, 1916b, 1938) awareness of the impact of the situatedness on cognition was evident in his explanation of how any analysis of behaviour should include the context in which the action was occurring. Dewey (1896) described the following scenario: if a noise is heard, the experience for a listener will differ depending on whether the person is reading a book, hunting, undertaking a chemical experiment or in alone in a dark place at night. He believed that this was part of the act of hearing that was not explained by the biological mechanics of the ear alone. Rather it was the whole organism acting in the world, with differing sensori-motor co-ordination arising dependent on the situation—such as the tilt of the head, the posture of the body, running from danger. As the individual acts to solve a problem by adjusting the environment, manipulating objects, using tools, and perhaps interacting with others, in turn the bodily actions and behaviours of the agent are also altered (Gallagher, 2014). Therefore, the situation is not the physical location of the individual during the activity rather it is the state of the agent-environment interaction. This is also evident in the results from the experiments described in this thesis, as the participants are in the same physical location of the laboratory for each trial of the experiment, however in changing the problem presentation the situation is changed. The agent-environment interaction has been altered resulting in changes to performance and trajectory to solution.

Thinking in doing. Dewey's notion of sensori-motor coordinated action is a precursor to not only enactivism (Stewart, 2010; Varela, Thompson, & Rosch, 1991), but also the extended mind hypothesis (Clark & Chalmers, 1998; Wilson & Clark, 2009). The evidence can be found in his strong objections to the dualism of classical philosophy separating the mind from activity, and in the idea of unity between the agent and the environment as constituent components in the sensori-motor circuit. This can be paralleled to the depiction of cognition used by contemporary cognitive theorists and researchers as agent-environment coupling within a dynamic cognitive system (e.g., Clark & Chalmers, 1998; Kirsh, 2005; Noë, 2012). Here the philosophy of pragmatism firmly assigned "the origin, place, and function of mind *in* an activity", a philosophy that "sees intelligence to be the purposive reorganization, through action, of the material experience" (Dewey, 1916a, p. 377).

Similar to Dewey, Peirce's (1931-1935) theories of cognition were a precursor to those of enacted, distributed and extended mind (Gallagher, 2014). He described Kant's philosophical views on thinking as "monstrous!", explaining how human reasoning "is not by simple mental stare, or strain of mental vision. It is by manipulating on paper, or in the fancy, formulae or other diagrams— experimenting on them, *experiencing* the thing" (Peirce, 1931-1935; vol.4, p.86; see Gallagher, 2014). Peirce's (1931-1935) views anticipated the conceptions of extended mind (Clark & Chalmers, 1998) and the notion of the close coupling of an individual with the world being essential to thinking processes. This was illustrated when Peirce (1931-1935, Vol. 5) described how a chemist dreamt of a complicated chemical process; to put this dream into practice he would "carry his mind into his laboratory… manipulating real things instead of words and fancies" (Vol. 5, p. 363). The solution to chemist's problem was distilled through interactivity with the world. Thinking becomes an intermeshing of the internal processes of the mind with the external manipulation of the laboratory equipment in an intentional sense coordinated activity. Peirce (Vol. 5) also described how problem solving could take place in the mind and in the world, with an example comparing a mathematician to a chemist. The mathematician might expend minimal mental cost in adding up a column of numbers in his head, with the computation being the same if produced with pen and ink. Where for a chemist a complicated experiment would require considerable mental cost with the added complexity of allowing for varying conditions, therefore experimentation in the world would potentially provide the better results. For Peirce and Dewey thinking was not an "armchair thing" (Dewey, 1916b, p. 14), the body and the physical world were as much part of cognition as the brain.

An Integrated Approach

Through this brief interpretation of Dewey and Peirce's work it is clear that their view of cognition was that of an active process that is situated, embodied, extended, and enacted. The pragmatists argued that an individual was not cognitively independent from the environment. The thinking process was conceived of as a dynamic coordinated interaction through the mutually constituted coupling of the individual and the environment, only then does the organism become a cognitive agent (Gallagher, 2014). The pragmatist's efforts in foregrounding the importance of analysing the actions of the individual in the world as a methodology of understanding the human mind would have clearly resonated with the situationalists (e.g., Greeno, 1989; Hutchins, 1995a; Lave, 1988) of the late twentieth century. However, despite the work of Dewey and other pragmatists, the influence of artificial intelligence on psychology in the 20th century, ushered in the return of a mechanistic information-processing approach to thinking (Bredo, 1998).

The pragmatist views on thinking that find purchase in many recent developments, also illustrate how the theoretical argument against mind-body dualism in psychology has gone full circle. Although the pragmatists' theories on cognition may not be widely credited with the foundations of contemporary anti-Cartesian philosophies, the insights might be worth revisiting to not only adjudicate, but to form the foundation for a cohesive approach to a non-internalist theory of cognition (See, Gallagher, 2014; Menary, 2007, 2010a). The issue of adjudicating the differences between the various contemporary approaches to anti-Cartesian accounts of cognition is beyond this thesis. However, what should not be overlooked is that the views of the classical pragmatists can be used as a resource to unite the various non-brain-bounded approaches to cognition in response to wider criticisms from the classical internalists (e.g., Adams & Aizawa, 2009; Baddeley & Hitch, 1974; Newell & Simon, 1972; Ohlsson, 2011; Vera & Simon, 1993; see Gallagher, 2014).

Theoretical Reflections and Methodological Observations

The methods, procedures, and materials for each experiment reported in this thesis were detailed in Chapters 6 and 7. However, given the unfolding nature of some methodological challenges arising when exploring the role of interactivity in cognition, it seemed prudent to briefly reflect on some of these prior to concluding the thesis. As these experiments were novel, although not entirely unique (e.g., Kirsh, 1995b; Vallée-Tourangeau, 2013) in approaching the investigation of interactivity in problem solving using artefacts, methodological antecedents for these experiments were limited. Initially the intention was to only undertake a quantitative approach to the analysis of data, however this evolved over the course of the experiments to include some qualitative analysis. The challenge became not only to identify whether interactivity with artefacts made a quantitative difference to performance, but also to investigate any qualitative change in behaviour. Therefore, the development of an observational toolkit using video resources was initiated. Although this analysis was only completed in detail for one experiment—Mental Arithmetic, Experiment 2—the results suggested this approach might be a springboard to better understand thinking trajectories in problem solving (e.g., Steffensen, 2013). In generic terms, cognitive theories attempt to understand the processes and organisation of cognitive systems (Hollan, Hutchins, & Kirsh, 2000). Theoretically, the cognitive process of problem solving is a representation-transforming activity involving the traversing of a problem space toward a goal (Perry, 2003). Problems arise as people act in the world with these problems varying from activity to activity, and resolutions emerging in context and situation specific ways (Kirsh, 2009a). Therefore, problem solving might be best understood as being situated and distributed in a transactional interactive process between a reasoner's internal resources and the external resources available in the environment in which she is embedded.

Theoretical frameworks of distributed cognition (e.g., Hollan, Hutchins, & Kirsh, 2000; Hutchins, 1995a; Kirsh, 1995a, 1995b; Vallée-Tourangeau, 2013) and the 4E's, particularly Clark and Chalmers's (1998) extended mind hypothesis, were influential in the methodological considerations for these experiments. Greeno (1998) advocated a mixed methodology as a framework for a situated approach to investigating cognition as a system, consisting of a synthesis between traditional cognitivism, where the focus is on the informational content of the mind and a behaviourist perspective centered on the actions and reactions of people to the environment. Hollan, Hutchins, and Kirsh (2000) also noted how studying cognition in the wild could offer the answers to many questions about the nature of human thinking in the real world. However, the richness of these settings also places limits and constraints on the observational methodologies. Hollan et al. (2000) suggested that by drawing on naturalistic settings it might be possible to design laboratory experiments with constraints not available to real-world ethnographic based studies. This would facilitate investigation into particular features or components of interactive behaviours, without losing sight of the systemic nature of cognition. As Perry (2003) pointed out, the framework offered by distributed cognition allows researchers to acknowledge all the constituent factors that are pertinent to the activity under scrutiny as a single unit of analysis. The work by early situated cognition researchers (e.g., Greeno, 1989; Hutchins, 1995a, 1995b; Kirsh, 2009a; Lave, 1988) was reflected in developing the aims of this thesis, by ensuring the focus of the unit of cognitive analysis was not the individual rather the person acting as part of a dynamic cognitive system. As Wilson and Clark (2009) pointed out it is important to study the cognitive system as a complex whole, however, this a challenging prospect as the interactions “may be highly complex, nested, and non-linear” (p. 73).

Lave (1988) and Hutchins (1995a) discussed the limitations of research in the lab when using games and puzzles, as they believed these tasks were designed to be challenging or difficult, testing the cognitive athleticism of individuals not how people act in the world. Hutchins (1988) described the use of puzzles as tasks to investigate problem solving as being “unrepresentative of human cognition” (p. 367). The point he made was that if performance on these tasks were of prime concern, then a focus would be on learning how to re-present the tasks in a manner that would make them easier for solvers to see the answer. In reviewing the literature on the river-crossing problem (e.g., Reed, Ernst, & Banerji, 1974; Simon & Reed, 1976; Thomas, 1974) this is a valid observation as performance was predominantly measured in terms of finding the solution. Whereas the research programme undertaken for this thesis, did not focus primarily on the participant solving the problem, rather in comparing the impact of different representations of the problem on the performance of the reasoner in reaching a solution. Despite the reservations by researchers such as Lave and Hutchins about the use of puzzles as the basis for designing experiments studying cognition, the river-crossing and the mental arithmetic problems, as used in the experiments reported here, have both provided useful scaffolds for the investigation of human cognition. Thus, the results from these experiments using varying presentations of the same problems support a case for the use of this genre of puzzles and problems in studying problem solving in the lab. While the ethnographic methodology employed by Lave was not used in this research programme, the experimental element of the Adult Math Project in the form of the best-buy simulation exercise supported the methodological choices used in the experiments reported here. Lave’s study foregrounded the potential difference not only in the setting of problem solving, but also the difference an external representation might make on performance outcomes and the strategies used in problem solving.

Preceding the experiments described in Chapters 6 and 7, there have been many experiments researching analytic problem solving and problem isomorphs in the lab (see Gentner & Gentner, 1982, 1983; Gholson, 1989; Gick & Holyoak, 1980; Hayes & Simon, 1977; Reed, Ernst, & Banerji, 1974). Very few have investigated the impact of different task ecologies (see Zhang & Norman, 1994) with none, to my knowledge, having considered both the performance outcomes and the interactive thinking trajectories in detail, in one experiment. Therefore, limited methodological guidance was available for the experiments planned as part of this thesis; a number of methodological challenges emerged as the experiments unfolded, with some of these addressed as the experiments proceeded. Subsequent to the completion of the experiments described in Chapters 6 and 7, research has been initiated in the development of detailed methodologies for the analysis of qualitative video data (e.g., Steffensen, 2013). The research methodologies described in this thesis will potentially assist future researchers in designing methodological protocols for use across a broad spectrum of interactive problem solving activities.

Methodological challenges. This thesis was essentially an investigation into interactivity between people and things in analytical problem solving using quantitative research methods. The initial aim was to establish any differences in performance as a result of changing the interactive nature of the task ecologies. The overarching challenge was to design experiments and employ a methodology that would encompass traditional cognitive psychological methods and analysis with a situated and distributed cognition perspective. In traditional cognitive psychology the individual as a problem solver is decoupled from the world where the thinking is considered to take place in the head, therefore logically, the emphasis when attempting to understand human cognition is to study the processes of the internal mind. However, by excluding the agent acting in the world, or not acknowledging the situatedness of cognition in action (as it were), the researcher would not be giving consideration to the cognitive system in its entirety. As Lave (1988) suggested, the validity of extrapolating results from such a narrow research perspective, then mapping them onto the wider world may be questionable.

Initially the decision on the appropriate methodology for all experiments was to employ research techniques modeled on a typical quantitative data analysis using experimental and correlational designs. This would also provide continuity with the experimental work previously undertaken on interactivity and artefacts, delivering a suitable baseline from which to make comparisons (e.g., Vallée-Tourangeau, 2013; Vallée-Tourangeau, Euden, & Hearn, 2011). The experiments were designed to compare the different conditions using various independent and dependent variables, with the correlational analysis employed to explore relationships with performance measures and individual differences. Therefore, to explore the impact of interactivity for different analytical problems, in different problem presentations and any relationship with individual differences, the analysis of the data and reporting of results would comprise of descriptive, inferential, and correlational statistics.

As the project unfolded it was increasingly apparent that it was not only important to describe *what* was happening by quantifying the actions of participants in terms of performance outcomes, but also to attempt to reveal *how* and *why* these differences were occurring. Thus, an additional challenge emerged: to attempt to identify the strategies and trajectories of participants in reaching a solution. To address this, in addition to the traditional experimental psychological methods for data collection and analysis, it was clear it would be beneficial to develop a toolkit of sorts to observe behaviours in order to offer a more accurate and illuminating method for investigating the differing paths to solution. The use of verbal protocols was piloted, however, this appeared to interfere with the problem-solving process as participants interpreted instructions in different ways and this verbalisation frequently encouraged the participant to attempt to interact with the researcher by, for example, asking questions about the rules of the river-crossing problem. This issue was also highlighted by Lave (1988) when researchers were drawn into conversations with the shoppers they were observing. Depending on the perspective taken, the researcher might be viewed as supporting the shoppers rational for decisions or becoming part of the decision making process on the path to solution. In addition, during the piloting of verbal protocols for the river-crossing and mental arithmetic experiments, in encouraging the participants to verbally express internal mental processes, they appeared to find it challenging to align the narrative with actions. This made it difficult for the researchers to identify actions that were premeditated or planned, from those that were taking place as the moves unfolded. Note taking by researchers detailing the moves made by participants in the high interactive simple maths condition was useful. However, this had many shortcomings, when participants moved artefacts swiftly or used both hands, it was difficult to be certain that the note taking was accurate. Although there were flaws with these methods of data collection and associated analysis, it was apparent that with different task ecologies interactivity with artefacts produced differing thinking trajectories, although individuals were attempting to solve problems that were functionally identical. To test this approach the decision was made to use a mixed approach, by continuing with the quantitative analysis and including qualitative video analysis for Experiment

2 of the mental arithmetic experiments. This experiment was chosen as participants were presented with the maths problems in four different presentations, potentially showcasing the difference in thinking trajectories when the task ecologies were altered for these problems that were functionally the same.

The experimental sessions for a random selection of participants in mental arithmetic Experiment 2 were captured in a purpose-designed laboratory for video observation. The participants were aware that the experiment was being filmed and signed specific video consent forms. However, the cameras were located unobtrusively within the room, in order to minimise any awareness by the participants of being videoed. The analysis of the video was completed using Elan, an open source software tool for creating detailed annotations from video and audio resources. The performance of one participant was randomly selected from the video resource. The actions of the participant in the three high interactive conditions—pen with paper, pointing, and token—were analysed, with the discrete moves regarded as indicative of directly contributing to the adding up process noted and used for further exploration of the trajectories to solution. For example, in the token condition, moving the wooden token was considered to be part of the adding up process. From these recorded actions a depiction of the trajectory of movements to the final solution was created for each of the conditions. The analysis of these videos was more time consuming than expected. As this was not part of the original timetable for thesis, it was not possible to complete more than one analysis. However, as discussed in Chapter 7, the results of the qualitative analysis revealed a rich of source information with which to evaluate the impact of differing task ecologies on problem solving. Although the problems used in the experiments were functionally the same, the problem presentations varied, resulting in the participant enacting distinctive trajectories to solution when confronted with these differing presentations. The results implied that different trajectories to solution should be recognised and accounted for, in order to enhance the understanding of cognitive processes involved in problem solving. Thus it would be fruitful for future researchers to undertake a similar detailed analysis of the problem-solving actions of a group of participants, in order to compare participants' thinking trajectories for differing task ecologies. Through experience with the analysis of only one participant, advice for any researcher when considering undertaking similar micro-detailed analysis would be to allow potentially hundreds of hours of time to complete the work. In addition, pilot the analysis for one or two participants prior to recording the performance of all participants to be certain that all information required is correctly captured. Subsequent to the analysis on the mental arithmetic experiment 2, Steffensen (2013) has been instrumental in the continued development of a technique for data analysis namely Cognitive Event Analysis (CEA), which has proved to be an invaluable tool for the analysis of fine-grained actions during problem solving tasks, both in the laboratory and the workplace (e.g., Steffensen, Vallée-Tourangeau, & Vallée-Tourangeau, 2016). It was not possible to video all participants in this experiment due to logistical issues, including time constraints on both the video lab used for observations and participant availability. However a future experiment of this type, with the experimental sessions designed to capture the problem-solving performance of all participants on video, would be useful in providing data for a full comparative analysis between and across the performance of a number of individuals. As a consequence of the results from these experiments, using quantitative methods in conjunction with the additional information that surfaced through qualitative observations of the participant's performance, it appears that it is possible to develop a mixed-methodology grounded in a synthesis of the two approaches. This could be viewed as a systemic methodology blending the nomothetic and idiographic methods drawn from across the social science spectrum. The nomothetic approach would be extracted from the macro view of performance outcomes and measures provided by the quantitative data, and the ideographic approach from the micro examination of the qualitative evidence (as discussed by Lave, 1988; also see this thesis, Chapter 2). Such a methodology proposes a direction for researchers to explore a more holistic approach toward the understanding human cognitive processes. This would also facilitate the identification and comprehensive reporting of interactivity in lab experiments guiding researchers, regardless of their approach to cognition, to be attuned to the impact of interactivity and an awareness of the effects of varying task ecologies on experimental outcomes.

General observations on methodology. The study of systems rather than individuals poses theoretical and methodological challenges. Theoretically, the nature of the problem representation and the trajectory of the solution as it evolves from an embryonic to a fully formed answer, should perhaps be understood as being distributed and configured in terms of a transaction between the participant's internal resources and the shape and nature of the resources in the external environment. Attempting to segment and independently specify the components of a cognitive system, namely the thinking agent and her immediate environment, is not as productive as seeking to characterise the system as a whole (see Baber, Parekh, & Cengiz, 2014). As Lave, Murtaugh, and de la Rocha (1984) pointed out, it is difficult to analyse these dialectically formed problems where the actions and the situation both create and change each other, with problems being generated and resolved during the ongoing activity of the individual.

Of course, systems can be more complex, and composed of a much wider range of functional elements, which challenges the traditional toolkit of experimental cognitive psychologists designed to deal with a cognitively sequestered individual in a laboratory environment that generally prevents interactivity. The findings and methods reported here suggest that a more qualitative idiographic cognitive science supported by an observational toolkit that can code at a much smaller time scale the evolution of a problem representation and its solution would make a substantial contribution to problem solving research.

Future Directions and Concluding Remarks

In leaving out the influence of situatedness and interactivity when explaining human cognition, as in an internalist approach, is to leave out a large portion of the account of how people think in the world. Dewey's (1896) critique of the reflex arc concept in psychology nicely illustrated how the physiological brain-bounded account of human behaviour was only part of the circuit of sensori-motor coordinated action. Dewey's work may not be widely acknowledged as having an influence on contemporary theories of non-internalist approaches to cognition, however the notion that thinking is not brain-bounded has continued to be a consistent theme in philosophy and psychology in the past century, despite the strong influences of traditional cognitive science on theories and research. As revealed by Lave's (1988) work on arithmetic, Greeno's (1989; 1998) discussions on situated methodologies, and in the reflections on methodology in this thesis, the task of including the lived-in world into both quantitative and qualitative investigations is easy to identify, but a difficult one to fulfill (Lave, 1988). However, work by researchers such as Lave (1988), Hutchins (1995a, 1995b), Zhang and Norman (2004) and Vallée-Tourangeau (2013) showed that this was not armchair theorising by providing empirically based evidence that cognition was situated and distributed.

Future Directions

The experimental work presented in this thesis was designed to further investigate the fundamental premise that thinking is not brain-bounded; rather it is a coupling of agent and environment as part of a dynamic situated system. The methodology employed was based on investigating situatedness and interactivity using problem solving and artefacts in a laboratory setting with the results laying the foundation for further investigation into interactivity using problem solving and artefacts. Additional testing of learning transfer enacted through interactivity could proceed over longer time frames—allowing days or weeks between the second attempt—rather than over one experimental session as in the river-crossing experiments described here. As computer centered education is on the rise, comparing learning through computer-based interactivity against the three-dimensional interactivity with the physical world as discussed in this thesis, would be useful from a pedagogical perspective (Klahr, Triona & Williams, 2007; Moreno & Mayer, 2007; Renken & Nunez, 2013). In addition, further development of the qualitative methodology, as discussed earlier in this chapter, to study the effect of changing the external representation on problem solving trajectories, would shed light on how interacting with artefacts alters the way people solve problems in the world. These micro-scale investigations might also reveal the nuances of interactivity exposing the various phases of problem solving, for example, identifying the stages in problem solving when individuals are near or far from reaching a solution by studying their interactions. Finally, these experiments have shown it is possible for research into cognition in the wild to be examined under laboratory conditions; but such research must be engineered to allow agents to interact with a physical presentation of the problem. In this way, the laboratory data offers a more representative window onto how people solve problems in the lived-in world.

Concluding Remarks: The Kinesionoetic Field

Against the backdrop of theories on cognition spanning over one hundred and twenty years, the experiments reported in Chapters 6 and 7 have shown that varying the situation by changing the problem presentation can affect performance in problem solving. Altering the situatedness does not necessarily mean locating the problem in a different physical location. In the case of the experiments discussed in this thesis, the same participants completed the same problems in the same environment; however changing the presentation of the problem affected the situation. The interaction between the agent and the world was altered by this change to the problem. This is evidence that the situation is not a geographical locale but is constituent of the agent-environment interaction, through this coupling of the individual and the world emerges the cognitive agent (Gallagher, 2014). With the many threads of commonality between the non-internalist approaches to thinking, a focus on interactivity and situatedness may offer the grounding for an integrated approach to ecological methodologies and theories on cognition. By way of illustrating the extended mind hypothesis and similar accounts of systemic cognition, the changes in problem presentation as described in the experiments reported here, have shown how artefacts are not just passive inputs to aid internal cognition. The artefacts in these experiments augmented cognition by modifying the cognitive system, altering the trajectory to solution, resulting in adjustments to the situation that improved performance (Clark & Chalmers, 1998; Wilson & Clark, 2009). Providing individuals with the opportunity to manipulate artefacts in these problems provided a scaffold to enact new strategies. Part of the investigation into interactivity also explored individual differences in performance by altering the components in an extended cognitive system. In the case of expertise in the final mental arithmetic experiment, the results revealed that reliance on internal processes was altered by adjusting the external representation, supporting claims by non-internalists that cognition functions as a holistic dynamic system not as discrete components as the classical internalist approach maintains.

Much of the research and theory on the role of artefacts in cognition examined in this thesis was inspired by Norman's research (1991,1993) and the Theory of Material Engagement (Malafouris, 2013). In discussing the impact of material engagement on human cognition, Malafouris proposed a "*hylonoetic"* ontology of thinking (p. 236; a neologism composed of the Greek word *hyle* for matter and *nous* for mind). Here cognitive processing is portrayed as emerging from a hylonoetic field comprised of the mind and material artefacts, where "thinking occurs through and with matter" (Malafouris, 2013,p.236). However the results from experiments reported here foreground action, rather than materiality, as the impetus for emerging cognitive processes in problem solving. This suggests that the field is *kinesionoetic*, (from the Greek *kinima* for movement, and *nous* for mind) where thinking unfolds and is shaped through movement or actions. Thinking is enacted as the individual and environment are situated in a cognitive system that emerges across time and space, with the properties of the agent-environment system configured through interactivity.

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