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Improving the design of high speed mechanisms through multi-level kinematic synthesis, dynamic optimization and velocity profiling

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ABSTRACT

This paper deals with the fundamental mechanical engineering challenge of mechanism design. While there is a significant body of research associated with mechanism design there are few, if any, approaches that consider kinematic synthesis and optimisation of dynamic performance in an integrated manner. To address this gap, this paper presents a layered (multi-level) design optimisation approach that enables kinematic and dynamic optimisation combined with velocity profiling of the motor/drive system. The approach is presented for both new design and redesign tasks, and is based on the use of inverse kinematic and inverse dynamic analysis, and a novel strategy for generating instantiations of spatial mechanisms that satisfy kinematic quality indicators but with improved dynamic performance. The experimental results validate not only the individual stages of the approach and the models but also the overall improvements achievable through the application of the method. In this regard, the experimental (practical) mechanism exhibited performance improvements in the peak-to-peak torque of 63%, which correlate closely with those predicted theoretically after kinematic and dynamic optimisation. The introduction of a velocity cam function is shown to improve the dynamic quality indicators further and results in an overall reduction in peak-to-peak torque demand of 85%.

1. Introduction

The design and optimisation of mechanisms and machines is a fundamental activity of mechanical engineering and has received considerable research attention over the last two decades. To date, much of the extant research has focussed on either the challenges of forward and inverse kinematic design [1,2], forward and inverse dynamic design [3,4], or techniques to enable optimisation with respect to various performance criteria [5–7]. While traditionally it may have been acceptable to consider the kinematic and dynamic response separately, or entirely ignore dynamics during the design process, particularly for low speed duty, this is no longer the case. For todays production environments and machine systems; speed, accuracy and reliability are critically important factors. This demands that mechanisms operate continuously or intermittently at high

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Nomenclature

$\mathbf{A}$	Matrix of Lagrangian function coefficients.
$\mathbf{D}$	Vector of $\mathbf{q}$, $\dot{\mathbf{q}}$ and t.
$\mathbf{F}$	Jacobian of constraint equations.
$\mathbf{Q}$	Vector of generalised inputs.
$\mathbf{q}$	Vector of generalised coordinates.
$\boldsymbol{\lambda}$	Vector of Lagrangian multipliers.
a	Coefficients of the Lagrangian function.
C_k	Kinematic cost function.
e_i	Shortest distance between the ith point and the desired orbit.
f	Constraint equations.
i,j	Integers.
L	Lagrangian function.
M	Number of constraint equations.
N	Number of generalised coordinates.
P	Number of points on the orbit.
Q	Generalised inputs.
q	Generalised coordinates.
t	Time.
U	Control input.
W_2	Weighting for the repulsion (control) point.
w_i	Weighting of the ith point.
λ	Lagrangian multiplier.
$(\cdot)^{-1}$	Inverse.
$(\cdot)^{T}$	Transpose.

speeds and do so without compromising their accuracy, without detrimental effect on their life or the life of the transmission and, importantly, without adversely affecting the wider system, e.g. through vibration. It has been shown that even small physical imbalances can induce harmonic content that can compromise performance [8,9]. Such imbalances caused by non-linearities in closed loop chain mechanisms, which become more energetic, the faster a mechanism is actuated. They also imply a direct link between the amount of harmonic content present in an output motion and the peak-to-peak torque magnitude, which a drive motor needs to exert [8]. They also describe methods of modifying the designs of existing mechanisms to reduce the amount of harmonic content present in their output motions by dynamically varying the length of some mechanism links using cams [8] or using smart materials such as piezoelectric stacks [9].

For the aforementioned reasons, it is desirable that both kinematic and dynamic responses are considered concurrently during the early stages of design. Not only should the behaviour of the mechanism be considered but the behaviour its drive system, taking into account parameters such as peak-to-peak torque demands of the drive motor. More specifically, for a spatial mechanism the kinematics and dynamics are interrelated. For example, small changes in the path (locus of motion of the mechanism) and scale (size) require modification of the properties of the mechanism linkages (size, mass and inertia) that can significantly affect the dynamic response, particularly at high speeds. However, their treatment in a unified manner is not straightforward from either a design, modelling formulation or computational perspective [10].

From a design perspective, deciding on mechanism topology, sometimes referred to as type synthesis [11], is arguably the most fundamental decision and will correspondingly limit the potential for dynamic optimisation. Further, in practice, hard constraints on topology will be imposed by machine footprint, internal space and the relative position of other sub-assemblies, which restricts mechanism topologies/types. Consequentially, to-date, research concerned with minimisation of harmonic content has primarily focussed on redesigning extant mechanisms to be more dynamically balanced. This has included modification of linkage lengths [12] and the addition of masses to linkages [13–15]. Methods exist in which, following the selection of a mechanism design, the kinematic and dynamic behaviour of the mechanism are analysed, considering such parameters as natural frequency as geometric parameters of the mechanism vary [16,17]. Using this information optimal physical parameters can be selected. One such example is proposed in [18] where the contour error of a parallel manipulator is minimised by considering the kinematics and dynamics at an early stage and using this information to switch between control modes as the mechanism is actuated.

From a modelling formulation perspective, there are a number of challenges. If a multi-objective cost function is to be employed, the issue of determining the relative weighting of the components of the cost function, and hence the trade-off between path accuracy and dynamic performance, must be resolved. Such an approach would necessarily require the designers input, as full automation is not possible due to the distinct situational constraints of each design problem (e.g. physical space, interfaces, kinematic limits and loads). A second, perhaps more important challenge, which also affects the former, concerns the modus operandi of the mechanism models/modelling environments themselves. Many environments for mechanism design allow the dimensions of the linkages (dimensional synthesis) to be modelled continuously, albeit over a bounded range [19]. This will frequently result in mechanisms which cannot perform continuous cyclic motions but instead have dead or limit configurations through which they cannot pass in continuous motion. While in the mechanism synthesis environment it is possible to detect the presence of limit configurations and respond accordingly, through for example, the introduction of a severe penalty into the cost function that will correspondingly heavily influence the objective function and the search strategy [20], the implication of a failed mechanism for the inverse dynamics modeller is catastrophic. Should this occur, it would not be possible to calculate a dynamic component cost value and thus, compromise the integrity of the multi-objective cost function [21]. This in turn frustrates the ability to apply a directed search method to optimise the system.

Finally, from a computational perspective, performing mechanism search, synthesis and dynamic analysis within a single optimisation function, which might involve 100,000s of iterations is both computationally expensive and time-consuming. Correspondingly, the vast majority of generalised mechanism dynamics analysis methods address the problem of mechanism dynamics by modifying the design of existing mechanism designs, which have been synthesised using a dimension synthesis method. The relatively few reported methods that do consider the problem of mechanism dynamic behaviour at the dimension synthesis stage of the process [13,22,23] are all based on parametric optimisation. Here complex cost functions are used with Fourier analysis to analyse the harmonic content in the output motion. While results have been demonstrated, the number of computational steps and implications of formulation of a single lumped objective function can be considered to limit the scope of design space search. Further, the use of Fourier analysis to analyse loci of motion that are, where mechanisms break during cycling, non-closed loop and comprised of discontinuities, is potentially misleading in any optimisation function. Current trends in addressing such parametric problem involve identifying more efficient methods of determining optimal parameter combinations. Many of the methods under consideration employ evolutionary, genetic algorithm type methods, many of which seek to exploit problem solving approaches inspired by nature [6,24–27]. Some methods combine established evolutionary search algorithms with other search methods for additional synergistic benefits [25,26].

Given the three challenges, their implications and previously reported approaches, it is contended that a lumped objective function may be less desirable than a layered strategy involving mechanism selection and kinematic optimisation, then dynamic optimisation. Further, such a layered strategy might also be followed by consideration of the design of the drive system and the use of techniques such as velocity profiling of the drive motor which have been identified by [9]. In addition, it is contended that a layered (multi-level) design strategy for combined kinematic synthesis and optimisation of dynamic performance is consistent with design theory concerning the modelling and reasoning about more complex systems where decomposition and subsystem modelling is necessary over a single holistic model [28]. Correspondingly the contribution of this paper is to present a layered (multi-level) method for Kinematic Synthesis, Dynamic Optimization and Velocity Profiling. The paper begins by presenting the three-level approach and how it can be applied to either a new or redesign task. The two modeling approaches for kinematic synthesis and inverse dynamics are described in detail. The proposed method is then demonstrated through application to a real world example. Throughout the results section the theoretical models and experimental results are compared and contrasted to validate the models and the potential overall benefits of the proposed method. In regard to the latter, the experimental rig is used to demonstrate the before and after cases. Conclusions and further work are then described.

2. A method for kinematic synthesis and optimisation of dynamic performance

As previously discussed, a layered design method has been created so as to enable a more integrated approach for kinematic synthesis and optimisation of dynamic performance of spatial mechanisms. The overall method is presented in Fig. 1 and comprises three stages, which can be used in different permutations according to the practical constraints.

2.1. Design Stage A: kinematic synthesis

This stage (DS A) can also be referred to as traditional mechanism synthesis, where a suitable geometry is generated to satisfy the motion requirements. This stage may involve the following steps:

A1. **Motion requirements and constraints (Kinematic Quality Indicators).** The critical and redundant portions of the motion are established and constraint rules on position, velocity, acceleration and jerk are defined. These might include key (control) points, portions of desirable motion e.g. approximately linear, maximal velocity or acceleration.

A2. **Mechanism type selection.** Possible mechanism topologies are investigated, such as crank-sliders; four-, five- and six-bar mechanisms, cams, Watt mechanism etc. Here the most promising topologies are defined and selected based on the kinematic quality indicators (KQIs) previously defined. For the purpose of mechanism type selection a catalogue function can be employed and is discussed later in Section 3.1.

A3. **Kinematic optimisation.** This is used to generate a candidate mechanism for the selected topology that best-satisfies the KQIs. The resultant mechanism will be used as the ‘seed’ mechanism for the instantiation of mechanisms for dynamic analysis. At this stage it is the parameters of the mechanism that are modified. These include linkage lengths, ground points and cam geometry. See [20] for a comprehensive overview of the model construction and optimisation processes.

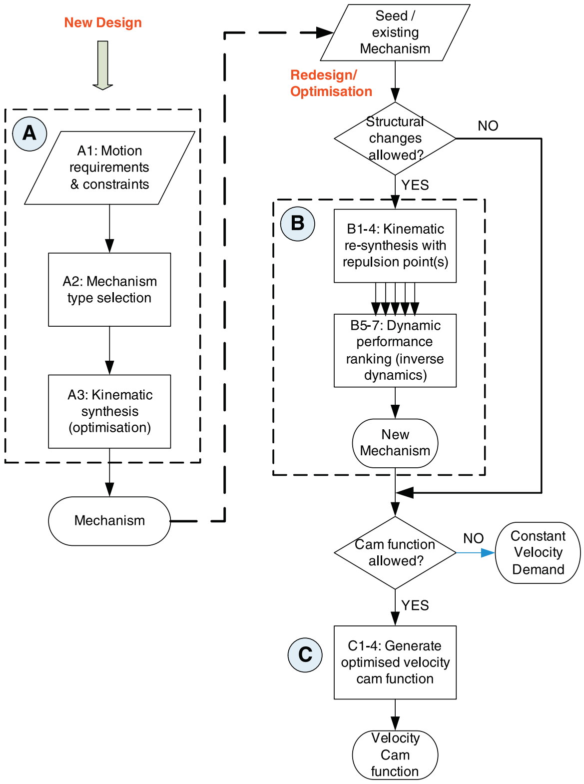

Fig. 1. Layered design method for kinematic synthesis and optimisation of dynamic performance, A: Kinematic synthesis, B: Kinematic re-synthesis with dynamic performance optimisation, C: Velocity cam function (for a new mechanism, design stages A+B+C should be followed, design stages B and C for redesign and optimisation)

2.2. Design Stage B: kinematic re-synthesis with dynamic performance optimisation

This stage (DS B) uses the mechanism designed in Stage A as a seed mechanism. Alternatively, if the mechanism already exists, then it can be used as a seed mechanism. It is assumed that the structure of the mechanism is allowed to change. Otherwise, Stage C can be performed on the kinematically optimised mechanism to improve the vibration characteristics (see Fig. 1). This stage re-synthesises the mechanism to take into account the dynamic performance requirements, and comprises the following seven steps:

B1. **Synthesis strategy.** A suitable strategy for generating instantiations of a mechanism type from the 'seed' has to be defined. For the purpose of improving dynamic performance the strategy adopted is to manipulate the redundant portions of the motion in order to reduce a dynamic cost function (e.g. jerk). This is achieved by altering one or more control points that can act to either attract or repel the locus of motion in a particular spatial region.

B2. **Synthesis.** Alternative instantiations of the seed mechanism(s) are generated, again using kinematic synthesis to satisfy critical motion constraints defined in stage A1. For the purpose of redesign stage A2 is not applied, rather A1 and A3 as the mechanism topology is already determined.

B3. **Compilation.** Instantiations of the seed mechanism are recorded including linkage lengths, and ground points.

B4. **Model creation.** Physical parameters are estimated and calculated for the mechanisms (mass, inertia and centre of mass) and dynamic models are created using a Lagrangian formulation to generate the equations of motion.

B5. **Dynamic performance requirements.** Suitable dynamic quality indicators need to be established. For the purpose of dynamic performance of mechanisms, parameters such as driving torque variation and peak-to-peak magnitude, and cyclic energy are considered important.

B6. **Dynamic analysis.** Here both forward and inverse dynamic analysis is conducted so that all possible quality indicators of dynamic performance can be generated and evaluated.

B7. **Rank mechanisms.** Here mechanisms are ranked based on the quality indicators of dynamic performance. The most suitable one(s) are then taken forward for further analysis (e.g. velocity profiling in Stage C) and final detailed design.

2.3. Design Stage C: velocity cam function

This stage (DS C) involves designing a velocity cam function to further improve the dynamic performance [29,30]. Most commercial motor controllers incorporate a velocity cam function facility, i.e. defining a velocity command signal as a function of the encoder position over a cycle. This stage involves the following four steps.

C1. **Analysis of simulated system response.** This step involves the creation of a simulated model for the overall system including the forward dynamics of the mechanism, motor and the controller. The parameters of the model should ideally be estimated by using experimental data. Initially a constant velocity demand response is used.
C2. **Selection of torque truncation limits.** Based on the previous step, initial torque truncation limits are specified and introduced in to the simulation.
C3. **Generate candidate cam function.** Here the steady state simulated velocity output alongside the torque truncations are recorded. The resulting velocity output is used as the basis for a velocity cam function.
C4. **Test cam function.** The aforementioned cam function is used in the simulation without the torque truncation limits to assess the dynamics of the system. If deemed necessary by the user, steps C2 to C4 can be repeated iteratively to improve the performance. Here user input is required to adjust the torque truncation limits.

3. Modelling environments

This section presents an overview of the two mechanism modelling environments used in this paper. The modus operandi of each environment is summarised and the key functionality that enables the overall approach is described. This includes mechanism type selection, kinematic optimisation (constraint satisfaction) and inverse dynamic analysis.

3.1. The kinematic synthesis environment

For the purpose of this study a mechanism modelling environment based up on constraint reasoning [1] is used for synthesis in Stage A and B2. This is because the environment supports both selection of mechanism type and kinematic analysis. The first of these activities is achieved by virtue of a 'catalogue' function. Here the user enters a discretised path in Cartesian coordinates (2D). A Fourier decomposition of the path in terms of the variation in Cartesian coordinates (x and y independently) is undertaken. The resulting coefficients are compared to a catalogue of Fourier path decompositions for a range of mechanisms [31]. These include, for example, crank-sliders, four-, five- and six-bar mechanisms, and Watt six bar. Within the environment the 'closest match' mechanisms are listed and the task is then to further refine the mechanisms through dimensional synthesis. The second activity of kinematic analysis is enabled through the use of a constraint-based formulation which supports computer-based dimensional synthesis to satisfy kinematic motion constraints. The approach is to attempt to minimise a kinematic objective function, C_k, which comprises the sum of the squares of the errors. Here,

$$C_k = \sum_{i=1}^{P} w_i e_i^2 \tag{1}$$

where w is a weighting with a default value of unity, e is a measure of falseness (an error term), and P is the number of constraints. The objective function is formulated in this manner so as to be able to accommodate two forms of constraint. The first form concerns the relations between geometry such as the distance between two points or a line and a point in space. This form of constraint is imposed through an inbuilt function called 'on' [1] which calculates the shortest distance. The second form of constraint rule relates to any algebraic formulation or derived value, such as velocity or acceleration and the constraint effectively imposes a limit or a bounded range.

With the constraint rules applied to a parametrically defined mechanism model, numerical optimisation can be employed to vary each parameter (lengths and ground points), cycle the resultant mechanism through its motion and evaluate its compliance with the constraints imposed. Importantly, for the proposed method, the formulation of the kinematic mechanism design task as a constraint satisfaction problem affords the opportunity to generate instantiations of spatial mechanisms that satisfy key kinematic constraints i.e. those imposed in step A1 but that possess different dynamic characteristics. This is possible because further geometric constraints can be applied to influence the form of the mechanism motion (orbit) in the regions where the constraint rules specified in A1, and hence KQIs, are not imposed. The goal being to generate variant mechanisms that all satisfy the KQIs but that produce altered motions in the regions of the orbit where the KQIs are not imposed. This layered or cascaded approach of satisfying first the kinetic quality indicators, generating variants and then optimising dynamic performance with input from the designer is fundamental to the proposed method. For reasons discussed in the introduction, modelling challenges can make the representation of the problem in a single holistic model very challenging. Further, adopting a single overall objective function would necessarily result in a series of comprises and trade-off being made or require the designer to specify *a priori* hard constraints and weights. Both of which remove the designer from the process and move the overall form of the process further away from a more natural design process. In addition, and as stated in the introduction, the nodus operandi of the modelling approaches means that a single holistic model and cost function cannot be applied.

In order to generate the variants, a synthesis strategy, i.e. the strategy that guides the modification of the mechanism to generate alternative instantiations, must be devised. For the example presented in this paper, a control point is introduced to repel the path in its vicinity meaning that it pushes the path away from its local origin. In order to be consistent with the form of the cost function given in Eq. (1) the reciprocal of the error term associated with the control point (CP) is used so that the error increases the closer the path is to the control point.

$$C_k = \sum_{i=1}^{P} w_i e_i^2 + W_2 \frac{1}{e_{cp}^2} \tag{2}$$

Here W_2 is the relative weighting and e_{cp} is the shortest distance between the orbit (represented as a closed curve) and the control point (CP). It follows that in order to generate alternative mechanism instantiations the relative weighting W_2 is varied. This relative weighting will depend on the total number of constraints; i.e. terms in the objective function and their relative weights which are set to unity other than W_2. The selection of this weighting value requires the designer's input and judgement so that a reasonable number (between 10 and 20) of alternative mechanisms is generated for the dynamic analysis and ranking in stages B6 and B7.

3.2. The inverse dynamics environment

Before the dynamic performance of any mechanical system can be analysed, a model of the system must be created and the equations of motion describing the system derived. In this investigation Lagrangian dynamics will be used to derive the equations of motion. For a M degrees of freedom system with N generalized coordinates of motion q_i, $i = 1 \cdots N$ ($N \geq M$), the constrained Lagrangian equation is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} + \sum_{j=1}^{N-M} \lambda_j \frac{\partial f_j}{\partial q_i} = Q_i, \quad i = 1 \cdots N \tag{3}$$

The definition of all the variables in Eq. (3) can be found in the Nomenclature. The Lagrangian function L is the difference between the kinetic and potential energy possessed by the system and can be written in a general form.

$$L = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{i,j} \dot{q}_i \dot{q}_j + \sum_{i=1}^{N} a_{i,0} \dot{q}_i + a_{0,0} \tag{4}$$

As superfluous coordinates exist in the system, $(N - M)$ constraint equations are needed. For holonomic constraints:

$$f_j(q_1, \cdots, q_N) = 0, \quad j = 1 \cdots (N - M) \tag{5}$$

Inserting Eq. (4) into Eq. (3) and double differentiating Eq. (5) results in the following $2N - M$ differential-algebraic equations.

$$\begin{bmatrix} \mathbf{A} & \mathbf{F}^T \\ \mathbf{F} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \end{bmatrix} + \begin{bmatrix} \mathbf{Q} \\ \mathbf{0} \end{bmatrix} \tag{6}$$

This can be solved to obtain the $2N - M$ unknowns, namely the second derivatives of generalized coordinates and Lagrange multipliers. The second derivatives can then be integrated twice to obtain the system forward dynamic response. The time history of Lagrangian multipliers is automatically calculated and can be used to calculate the forces of constraints if needed. The forward dynamic analysis will be utilised in the generation of a velocity cam function in Stage C. An in-house dynamic simulation package, *DYSIM*, is used for automatic generation and solution of equations of motions in *Matlab/Simulink* environment [3] for both forward and inverse dynamic analysis of the mechanism.

When performing inverse dynamic analysis, the M control inputs, $U_i(t)$, $i = 1 \cdots M$, are calculated to generate a desired output motion (specified in terms of second derivatives of any M generalised coordinates, $\ddot{q}_i = y_i(t)$, $i = \cdots M$). Various solution techniques for the inverse dynamic problem can be formulated as discussed in [3], but the following approach is utilised here:

The M unknown control inputs are added to the corresponding generalised inputs in Eq. (6). In the first stage, the M equations involving unknown control inputs are removed reducing the number of differential-algebraic equations to $2N - 2M$. Moving the specified M accelerations to the right hand side, allows the solution of remaining $2N - 2M$ equations for $2N - 2M$ unknowns, namely the $(N - M)$ Lagrangian multipliers and accelerations of $N - M$ generalised coordinates. The second stage involves using the M equations discarded in the first stage to calculate the M unknown control inputs.

Obviously in order for the inverse dynamics modelling to work, the mechanism must satisfy all the constraints, and also the specified motion must be kinematically achievable. These conditions may not be satisfied during each iteration of the kinematic synthesis process. Therefore, adding dynamic elements (that require dynamic simulation) to the kinematic cost function C_k will not work. Notwithstanding with this, the computational cost of the optimisation process is significantly reduced by separating kinematic and dynamic cost functions.

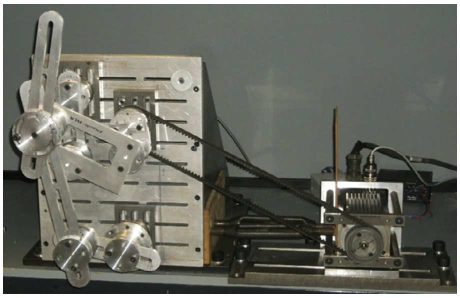

Fig. 2. The reconfigurable test rig.

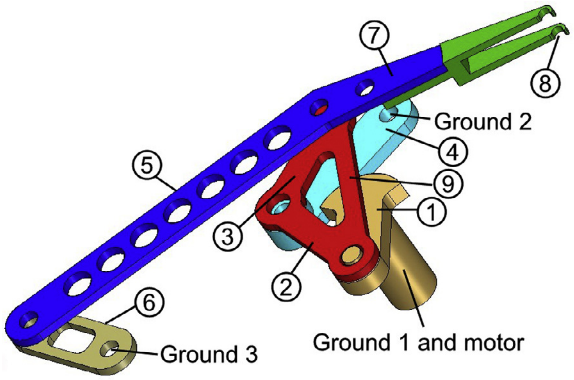

Fig. 3. A schematic view of the original mechanism.

4. Real world example and experimental rig

A reconfigurable laboratory test rig was designed and constructed as shown in Fig. 2 to validate the performance gains as a result of applying the proposed design method. The test rig was set up to mimic a six-bar mechanism in an existing packaging machine [29,30]. A schematic view of the original mechanism is shown in Fig. 3.

The test rig possesses the capability to be easily assembled in a wide variety of configurations. Using this test rig, it is possible to assemble mechanisms with varying geometries and styles without the need for the re-fabrication of parts. The number and length of the links, and the ground point connections can all be altered. The test rig consists of a series of slotted aluminium links, which are connected together using a two-part rolling connection assembly. In this way, adjacent links can be bound to each other but are still able to rotate relative to one another.

The main body of a rolling connection consists of two aluminium components, an axle component and a housing sleeve component which sits around the axle. Two SKF deep section ball bearing units are mounted on the axle shaft to permit free, rotary motion between the housing and the axle components. Spacers are placed inside the housing unit between the bearing units to locate them and to prevent the races of the inner and outer bearing units fouling on one another. Shoulders on the housing locate the outer race of the outer bearing. It is located on the axle shaft by an M5 $\times$30 mm screw and a washer, which lies flush against its inner race. The outer race of the inner bearing is located using an internal circlip. The inner race is located by a shoulder at the base of the axle shaft.

To attach a link to either the axle or the housing component, two M8 $\times$ 30 mm screws are passed through 8 mm wide plain clearance fit holes located in an end cap component. These screws then pass through a slot in the link and into tapped M8 holes in the large outer flanges on either the housing or the axle unit. Sufficient torque is applied on the screws to hold the link firmly in place. Spacing sleeves are positioned around the screws, between the end cap and housing or axle unit flange to locate the screw in the link slot. Using the slots in the link, the rolling link can be positioned anywhere along its length allowing the effective length of the link to be varied. An example of this assembly method is shown in Fig. 4.

The mechanism crank is driven by an Electro-Craft S-4075 industrial servomotor. The drive shaft of the motor is connected to the mechanism crank via a synchronous, toothed belt transmission system. The motor to crank gear ratio was 1:1. A belt-tensioning pulley is included to take up the slack in the belt.

Control of the servomotor is by virtue of a PC equipped with a Deva004 motion control card. The Deva004 was connected to a BRU-DDM30 servo drive via a series of connections made using the breakout board and communication ports on the Deva004 card. The BRU-DDM30 is connected to the servo motor and also connected to the control computer via an RS-232 connection. Suitable software is loaded onto the control computer to enable it to interface with the two devices. In the

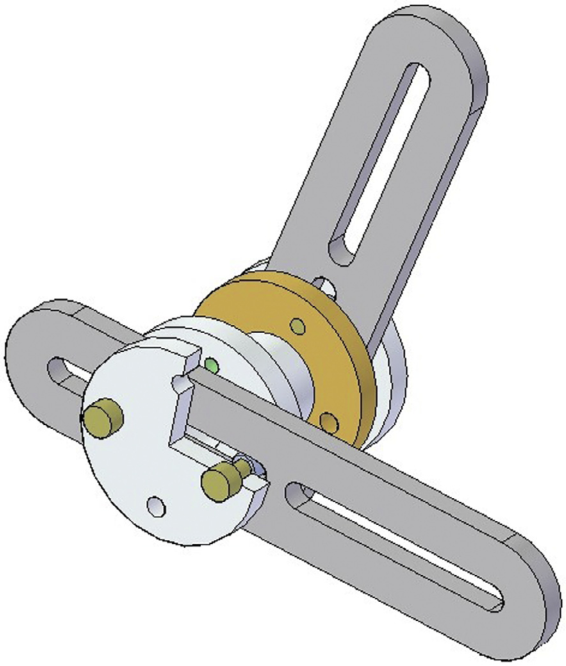

Fig. 4. A link attached to a rolling connection assembly.

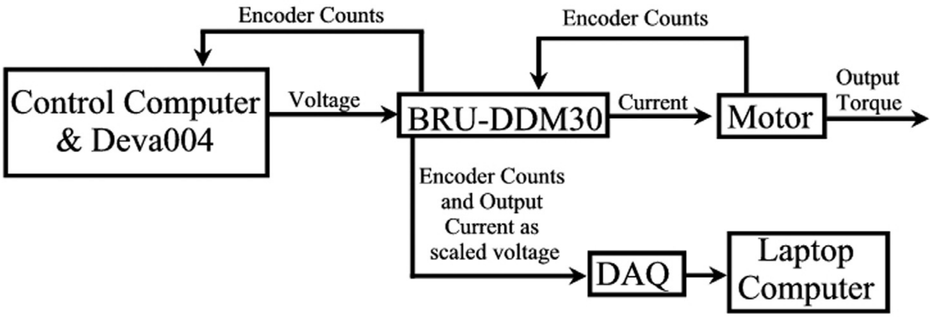

Fig. 5. An overview of the Woodpecker mechanism test rig control architecture.

case of the Deva004, this software comprises a series of system drivers and Visual Basic scripts [32]. To communicate with the BRU-DDM30, BRU Master software is used. Motor motion information, measured using optical encoders installed in the S-4075 servo motor, is fed back to the BRU Master control software via the BRU-DDM30. To log the data an external Data Acquisition unit (DAQ) is used. A summary of this strategy is depicted in Fig. 5.

5. Results

The original mechanism is a six-bar mechanism driven by a servomotor. Fig. 6 shows the orbit of the end effector in Cartesian space as a function of the angle of the crank shaft. The upper portion of the motion (from 180^0 to 60^0) is geometrically critical and cannot be altered significantly. This is due to the need to maintain the efficacy of the interaction with product (portion of motion that follows a straight line) and possible collision with other assemblies after interaction (portion of motion immediately after the straight line moving clockwise). In contrast the remainder of the portion is non-critical and can be altered.

At speeds of over 600 rpm the mechanism has been observed to exhibit significant noise and vibration. The redesign task is therefore to reduce the harmonic content while maintaining the kinematic motion constraints. Elicitation and clarification of these motion requirements and constraints fulfills the first step of the design method (Fig. 1).

This section discusses the redesign and optimisation of the mechanism given in Section 4 with respect to the design method presented in Section 2. The theoretical results are then applied practically to create a new mechanism. Fig. 7 shows a schematic view of the experiments carried out in this section.

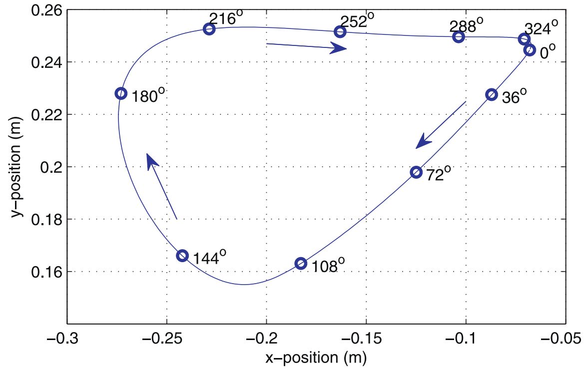

Fig. 6. Orbit of the end effector for the seed mechanism (numbers indicate corresponding crank angles).

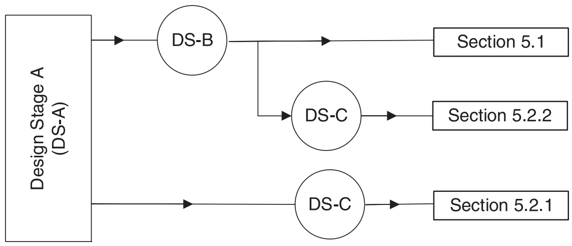

Fig. 7. Summary of the experiments carried out in Section 5 (DS denotes the Design Stage).

Table 1
The dimensional parameters of the seed mechanism design and the four selected synthesised mechanism designs. Note that Links 2, 3 and 9 form a tertiary link. Links 5 and 7 also form a tertiary link. The two links are offset at a fixed angle to one another. For ease of comparison, ground location coordinates have been normalised such that Ground 1 lies at the origin in each case.

Parameter	Seed	$W_2 = 100$	$W_2 = 200$	$W_2 = 300$	$W_2 = 390$
Link 1 (mm)	62.00	56.31	57	57.22	51.53
Link 2 (mm)	127.03	130.12	133.62	134,44	134.31
Link 3 (mm)	103.00	100.35	102.43	96.85	100.42
Link 9 (mm)	188.30	185.66	186.85	178.48	182.58
Link 4 (mm)	144.00	142.35	141.77	142.64	136.57
Link 5 (mm)	348.90	344.08	346.85	344.58	283.22
Link 7 (mm)	245.19	244.25	243.77	240.15	233.97
Link 6 (mm)	102.00	104.03	96.12	87.11	97.13
Link 5 to 7 offset	14.44°	17.98°	20.13°	22.68°	17.95°
Ground 1 (mm)	0, 0	0, 0	0, 0	0, 0	0, 0
Ground 2 (mm)	−121, 60	−124.24, 63.33	−131.25, 63.68	−86, 70.97	−128.95, 64.23
Ground 3 (mm)	7.5, −274	3.99, −274.67	0.27, −273.60	8.79, −266.94	−8.05, −276.58

For the purpose of redesign of an existing mechanism, Stage A of the design method is omitted. The parameter set of the current mechanism, as shown in Table 1, is used as the seed mechanism.

5.1. Design Stage B:

Stage B involves the development of a synthesis strategy, i.e. the strategy that guides the modification of the mechanism to generate alternative instantiations. As previously stated, in this paper, the mechanism model is formulated as a constraint satisfaction problem. This affords the capability to generate spatial mechanisms through the use of geometric constraints that influence the form of the motion. As previously stated in Section 2, a control point is introduced to repel the path in its vicinity i.e. to push the region of orbit closest to it away from its vicinity. In order to be consistent with the form of the error given in Eq. (2), the reciprocal of the error term associated with the control point (CP) is used so that the error increases the closer the path is to the control point, which was selected as the point on the desired path corresponding to the crank angle position of 130°, shown in Fig. 6.

It follows that in order to generate alternative mechanism instantiations the relative weighting W_2 is varied. Although each design problem will have distinct situational constraints, in this study W_2 was varied from unity to 1000 in order

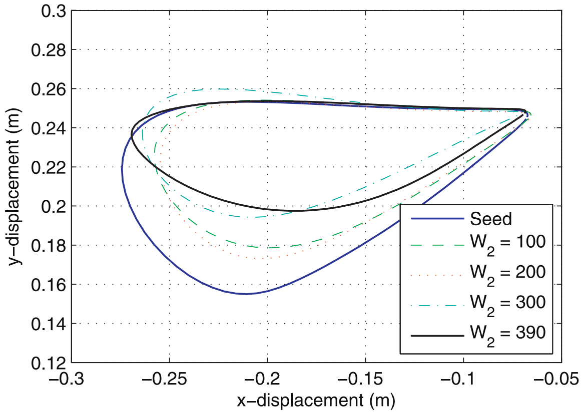

Fig. 8. The end effector path for mechanisms synthesised for different weighting W_2 values.

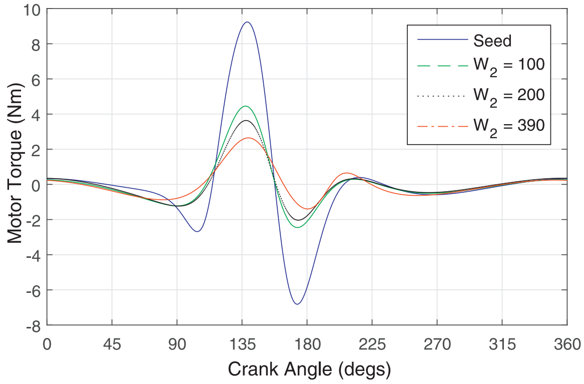

Fig. 9. The steady state driving torque needed to propel the crank for mechanisms synthesised for different weighting W_2 values.

to generate alternative mechanisms. In total more than 20 mechanisms were generated. However, of these only five were carried forward to the dynamic analysis. These were selected based on their compliance with the kinematic constraints (KQIs) which were checked for each mechanism generated. This was complemented by visual inspection. This conformance check is necessary as the objective function considers the sum of all error terms during kinematic synthesis. The inclusion of the control point and its high relative weighting means that some trade-off is made if no exact solution is determined. Hence, the need to trim these variants manually. Kinematic data for the four of the selected mechanisms are given in Table 1 with corresponding paths in Fig. 8.

Using CAD models it was possible to estimate single, unified mass and inertia values for each subassembly. A variety of mass and inertia values had to be derived due to variation in sub-assembly construction. For example some sub-assemblies incorporate truncated components, whilst others do not and so on, clearly leading to variations in mass and inertia. A more detailed formulation of mass and inertia values for an industrial manipulator can be found in [4], where an empirical formula was employed for the estimation of mass and inertia values of each link as a function of link lengths.

Using this data, inverse dynamic analysis was used to estimate the variations in torque which a motor would have to exert on the mechanism crank to propel each mechanism at a constant velocity of 100 rpm throughout a complete cycle of the crank. The results of this inverse dynamic analysis can be seen in Fig. 9.

These simulation results show that the dynamically most desirable mechanism design is that corresponding to $W_2 = 390$, which demonstrates a torque demand peak-to-peak magnitude reduction of 74.82% when compared to the same response of the seed mechanism.

These theoretical calculations were tested on the reconfigurable rig. The dynamic responses of all four mechanism configurations to a 100 rpm velocity step demand signal was recorded, and the steady state portion of each response established. The velocity and torque responses of the test rig in each mechanism configuration were then compared and analysed. A summary of the steady state torque and velocity responses of the four mechanisms, can be seen in Fig. 10.

Analysis of the experimental results, shown in Fig. 10, reveal that the most superior mechanism design corresponds to $W_2 = 390$. In response to a 100 rpm velocity step demand signal this mechanism design required a steady state demand torque variation with a peak-to-peak magnitude of 5.54 Nm. To generate the same motion, the seed mechanism required a demand torque variation with a peak-to-peak magnitude of 17.49 Nm. Thus, the alternative mechanism demonstrated a

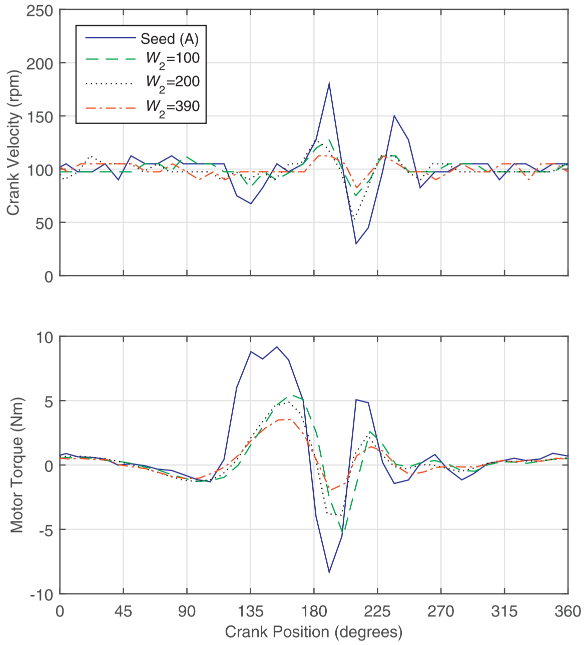

Fig. 10. Experimental steady state velocity and torque responses of the reconfigurable test rig when synthesised with different weighting W_2 values.

reduction in torque demand peak-to-peak magnitude of 68.3%. This experimental work confirmed the simulation results, which predicted that significant dynamic performance benefits could be achieved through the re-synthesis of the seed mechanism, using a method that takes into account both mechanism dynamics and kinematics.

5.2. Design Stage C: velocity cam function

The Cam Function Generation Method was developed by the authors [29,30]. Using this method, it is possible to derive motion and system specific variable velocity demand signals which can improve the dynamic performance of a test rig. To reiterate the effectiveness of the method, work was carried out to apply the Cam Function Generation method to the new reconfigurable test rig. The method was applied to the test rig in two mechanism configurations, namely that of seed mechanism and that of the most superior alternative configuration ($W_2 = 390$). In each case, a target steady state cyclic rate of 100 rpm was sought.

5.2.1. Applying cam function to the seed mechanism

The Cam Function Generation Method is first applied to the reconfigurable test rig in its seed mechanism configuration to assess the effect of the method on the dynamics of the system. Fig. 11 compares the steady-state velocity responses and torque demands of the test rig to a synthesised cam function generating a 100 rpm cyclic rate, with the equivalent responses to a 100 rpm constant velocity demand signal. The test rig was assembled in the form of the seed mechanism. Analysis of Fig. 11 shows that by using a cam function the magnitude of the peak-to-peak torque demand of the system was reduced from 17.49 Nm to 10.709 Nm, a reduction of 38.8%. Hence, using the cam function, the magnitude of this torque trough is greatly reduced.

5.2.2. Applying cam function to the dynamically optimised mechanism

The Cam Function Generation Method was also applied to the test rig when configured in the geometry of the mechanism corresponding to $W_2 = 390$. The mechanism configuration had been shown to be the dynamically most superior alternative design. A comparison between the velocity and torque responses of the system to a 100 rpm constant velocity demand signal compared to the equivalent responses to a cam function are shown in Fig. 12. This reveals that by using a cam function generating a 100 rpm cyclic velocity in place of 100 rpm velocity step demand signal, the peak-to-peak magnitude of the torque demand signal is reduced from 5.54 Nm to 2.609 Nm, a reduction of 52.9%.

Fig. 13 compares the torque and velocity responses of the reconfigurable test rigs in two different configurations and being actuated using two different velocity demand signals. The first set of responses depict the behaviour of the test rig in the configuration of the seed mechanism, being actuated by a 100 rpm constant velocity demand signal. The second set of responses show the behaviour of the test rig in the configuration of the dynamically most superior alternative mechanism, being actuated at a cyclic rate of 100 rpm by a cam function. From Fig. 13, it can be seen that by re-synthesising the seed mechanism and actuating the resultant alternative mechanism using a cam function instead of a constant velocity demand

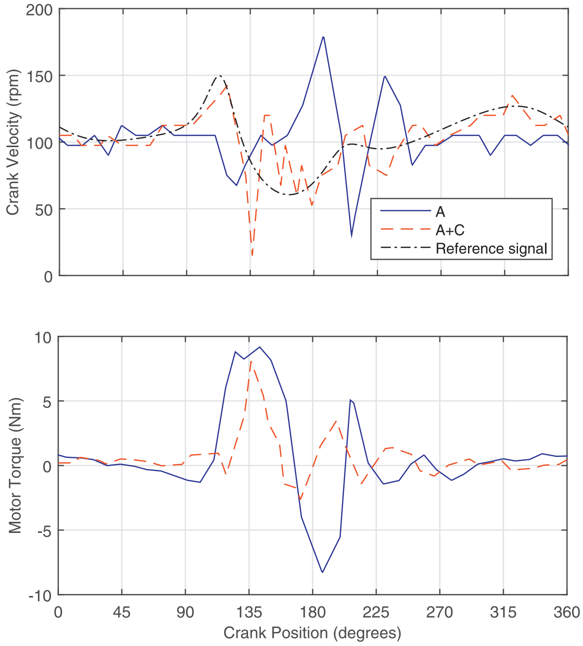

Fig. 11. The effect of cam function on the seed mechanism (blue solid line: response to constant speed reference (DS A), red dashed line: response to cam function reference (DS A+C), black dash-dot line: the cam function reference signal). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

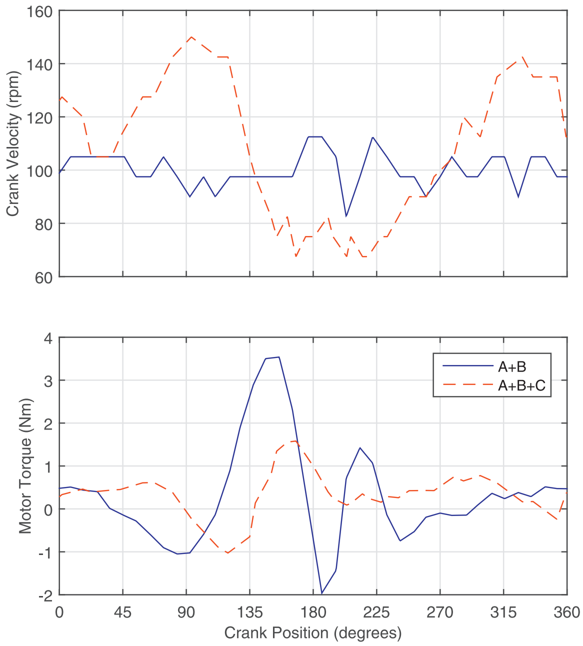

Fig. 12. The effect of cam function on the dynamically optimised mechanism (blue solid line: response to constant speed reference (DS A+B), red dashed line: response to cam function reference (DS A+B+C)). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

signal, the peak-to-peak magnitude of the torque demand is reduced from 17.49 Nm to 2.61 Nm, an overall reduction of 85.1%, a highly significant reduction. This clearly demonstrates the great benefits of synthesising a mechanism for both kinematic and dynamic performance and then propelling the mechanism using a variable velocity cam function generated using The Cam Function Generation Method.

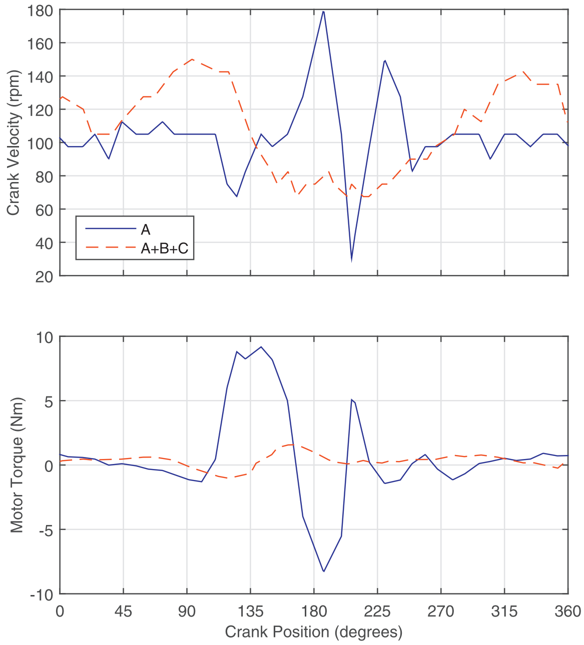

Fig. 13. The effect of dynamic synthesis and cam function (blue solid line: response of the seed mechanism to constant speed reference (DS A), red dashed line: response of the dynamically optimised mechanism to cam function reference (DS A+B+C)). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Summary of experimental results showing the percentage reduction in peak-to peak torque demand compare with DS-A.

Test scenarios	% reduction	Section
DS A+B	68.3%	5.1
DS A+C	38.8%	5.2.1
DS A+B+C	85.1%	5.2.2

6. Discussions

A layered design method is presented which integrates kinematic and dynamic design optimisation. Experimental results show significant benefits at different design stages as summarised in Table 2.

When applied to an industrial design problem the method resulted in a performance improvement of over 60% in the peak-to-peak torque demand. While the method does not guarantee that the mechanism is globally optimal it satisfies the key kinematic and dynamic quality indicators and is locally optimal given the topology selected and the strategy adopted for generating alternative instantiations. Such a layered approach is not inconsistent with the typical mechanism design problem or redesign task, where designers will adapt an existing design for a new or modified purpose or attempt to optimise the existing mechanism for improved performance. The proposed approach can therefore be considered to reflect the more natural mechanism design process, more closely aligned with practice, and likely to be more intuitive for the designer.

One of the main factors that influences the optimality of the final mechanism is the means by which alternative instantiations of a mechanism type are generated. Here a constraint-based environment is employed which allows spatial constraints on the redundant portion(s) of the motion to be applied in order to manipulate the path spatially. The rationale for this is to attempt to smooth the motion by reducing the rate of change of position. In this study this is achieved by the inclusion of a point that effectively act as a control point repelling or attracting the path in its vicinity. While this is shown to be successful there is a need to check that kinematic constraints are not violated. The reason for this is that the numerical optimisation method employed considers all components of the objective function collectively. There is hence a compromise sought when no exact solution is determined. For the case considered herein these cases were removed from the process resulting in a reduction from 20 to 5 variant mechanisms.

In this paper, a control point approach is employed to manipulate the motion. Other approaches such as light-weighting or profiling of acceleration, e.g. to follow a sinusoidal form, could equally be employed. For the purpose of dynamic performance of mechanisms, parameters such as driving torque variation and peak-to-peak magnitude, and cyclic energy are important. It has been shown that there exists a direct link between the amount of harmonic content in an output motion and the magnitude of the peak-to-peak variation in torque that a drive motor needs to exert [8]. Hence, reducing peak-to-peak torque will reduce the harmonic content and corresponding noise.

The industrial case study considered deals with the redesign and optimisation of an existing mechanism. Equally the 'catalogue' function [31] that is built in to the constraint modeller [1] could be used for new mechanism design problems. Further, for the case considered, peak-to-peak torque demand was the primary indicator of dynamic quality. Alternative measures such as total cyclic energy or maximal torque could be used.

7. Conclusion

The challenges of combined kinematic and dynamic optimisation have been discussed and include practical design considerations, modelling limitations and computational issues. To overcome these issues and address the lack of extant methods, a new design method has been presented. This layered (multi-level) method supports mechanism selection and kinematic synthesis followed by optimisation of dynamic performance from a set of mechanism instantiations. Further, it is contended that in comparison to a single lumped numerical optimization approach the proposed method mirrors more closely the natural mechanism design process. In addition to proposing a multi-level method, the method employs a novel spatial technique to generate alternative instantiations through inverse kinematic design that satisfy motion constraints (kinematic quality indicators) but possess observably different dynamic performance. A range of candidate mechanisms are used for the purpose of dynamic optimisation. Here the approach is to perform forward and inverse dynamic analysis for each mechanism, necessary to evaluate dynamic quality indicators such as peak-to-peak torque and cyclic energy. The benefit of the method is extended through the use of a velocity cam function to further reduce dynamic quality indicators.

The method is applied to a real world (industrial) example involving the redesign of a six-bar mechanism. Experimental results confirm the magnitude of predicted reduction yielding a reduction in peak-to-peak torque demand of just over 68%. The application of the velocity cam function to further improve dynamic performance revealed that a further reduction of almost 53% in peak-to-peak torque demand can be achieved. Hence, the application of the proposed method resulted in an overall reduction in peak-to-peak torque demand of 17.49 Nm to 2.61 Nm. A total reduction of 85% compared to the current mechanism. Therefore, the experimental testing validates the individual steps (models) and reveals that the overall approach can generate a substantially improved mechanism. It is important to reiterate that the proposed approach is practical given the nature of the problem and the two modelling tools.

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