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# EFFECT OF FRP WRAPS ON THE COMPRESSIVE BEHAVIOUR OF SLENDER MASONRY COLUMNS

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**Abstract.** In the last decade, Fibre Reinforced Polymer (FRP) wrapping technique has become a common method to retrofit masonry piers or columns with poor structural performances. The passive confinement effect induced by the external wrap allows increasing the compressive strength and ductility of the member. Several studies highlighted as the efficacy of this technique is affected by several key parameters, including the shape of the transverse cross section, stress intensification at the strength corner of sharp sections, amount and mechanical properties of adopted composite. Despite this technique has been widely studied from both theoretical and experimental point of view, most of studies focused on short columns and little information is available on the influence of second order effects on its structural efficacy. This paper presents a simplified method able to assess the effect of FRP confinement on slender columns. A preliminary evaluation of the constitutive law in compression of FRP confined masonry is made and the best-fitting model is adopted to model masonry in compression. Sectional analysis is performed by including the tensile strength of masonry and considerations are made on the increase of ultimate moment and curvature. Finally, the effect of column slenderness is considered using a simple numerical procedure, making it possible to calculate the allowable slenderness ratios as a function of the maximum drift, taking into account both strength and stability.

## Introduction

Confinement by Fibre Reinforced Polymer (FRP) is an efficient method to strengthen weak masonry members subjected to simple axial compression or combined axial force and bending moment. FRP wrappings applied along an existing member, increase the strength and ultimate strain capacity. This system proves to be very efficient, due to the great strength-to-weight ratio of the composite, which makes this technique suitable for structural retrofitting without increasing weights.

Several research works studied the capabilities of this technique, highlighting its features, especially referring to the achievable strength. One of the first studies was carried out by Krevaikas and Triantafillou (2005) [1], who tested forty-two clay brick masonry columns under compression and formulated some expressions for calculating ultimate stress and strain. Aiello et al. (2009) [2] tested masonry columns built with clay or calcareous blocks, and reinforced with external FRP wrapping. On the basis of the results achieved, they calibrated some design expressions for obtaining the strength of the retrofitted columns. Di Ludovico et al. (2010) [3] presented the results of an experimental investigation on the compressive behaviour of masonry columns confined with

Carbon or Glass FRP and proposed new expressions for evaluating the strength enhancement. Lignola et al. (2014) [4] summarized previous literature studies and proposed a theoretical approach based on the Mohr-Coulomb strength criterion for determining the strength increase due to FRP confinement. Micelli et al (2014)a,b [5] [6] studied the role of scale effect on the compressive behaviour of confined masonry columns, by testing medium and full-scale members wrapped with glass and basalt FRP systems. More recent experimental results can be found in Fossetti and Minafò (2017) [7], who performed a comparative analysis between different strengthening techniques for masonry columns, including FRP or fiber reinforced cementitious mortar (FRCM) wrappings, and collaring with steel wires. A more detailed literature review can be found in Minafò et al. (2016) [8], who presented also a reliability analysis of the main analytical criteria presented in the literature for obtaining the compressive response of FRP confined masonry columns, on the basis of a large experimental database.

All the works previously cited are based on the classical assumption of short column, in which the main external action is represented by a compressive axial force. However, real case studies refer to piers with proper geometrical slenderness, in which bending effects cannot be neglected. This case study is generally of interest, due to the fact that members needing retrofitting are usually affected by flexural cracks and therefore the slenderness can be greater than those evaluated geometrically.

In these cases, the beneficial application of FRP wraps for increasing the local strength and ductility could be limited by second order effects which increase the flexibility and reduces the load carrying capacity. Despite the importance in the field, the strength-buckling interaction in confined masonry members has not been yet investigated in the literature. So far research studies have focused only on unreinforced masonry columns. Initial studies on piers and walls [9] stressed the importance of taking into account the reduction of cross-sectional area due to progressive cracking. Consequently, the member stiffness decreases with an increase in the acting load and for larger deflections. Ganduscio and Romano (1997) [10] developed a numerical analysis for slender cracked masonry members, considering vertical and lateral concentrated and distributed loads, using finite element method (FEM) approach, and adopting a nonlinear monomial constitutive law for masonry in compression. La Mendola (1997) [11] adapted the model previously developed for a linear elastic material in compression to the case of a non-linear constitutive law. Specifically, the author studied the stability of prismatic fixed free-ended masonry columns, considering self-weight and vertical distributed loads. The material is supposed to have no tensile strength while the compressive behaviour is modelled by a non-linear constitutive law. Mura (2008) [12] analyzed the post-buckling behaviour of unreinforced load-bearing masonry piers subject to a combined load consisting of a uniformly distributed axial load and a concentrated eccentric load at the top end. The author analysed a cantilever column, taking into consideration no-tension material with a parabolic stress-strain law, and solved the problem by means of the finite difference method. Gurel et al. (2012) [13] adapted to circular sections the numerical model developed by La Mendola and Papia (1993) [14] for investigating the stability of masonry elements with rectangular cross-sections. More recently, Fossetti et al. (2015) [15] proposed a discretized beam analysis for calculating the axial load vs. lateral deflection curves for clay brick masonry columns. Geometrical non-linearity is considered by calculating the maximum allowable slenderness with an incremental procedure, while mechanical non-linearity is introduced by properly calibrated moment-curvature curves.

As stated above the interaction between confinement effects and buckling has not yet been investigated and is an open issue in the field of masonry structures. An analytical approach for obtaining the effective interaction between the two phenomena is still missing and needs to be addressed. In order to achieve this goal, buckling analysis should include all the nonlinearities, which arise from second-order effects and from the non-linear behaviour of masonry.

This paper presents a theoretical method for assessing the buckling curves of slender masonry columns confined by FRP wraps. On the basis of the model presented by Fossetti et al. (2015) [15], the effect of confinement is preliminarily evaluated by calibrating the stress-strain law of masonry in compression. Afterwards, moment-curvature curves are calculated via analytical integration of

equilibrium equations, the latter including the tensile behaviour of masonry. Finally, the effect of the column slenderness is considered by a simple numerical procedure, and allowable slenderness ratios are calculated as a function of the maximum drift and eccentricity. The proposed model requires a lower computational effort than other techniques, such as non-linear finite element or finite difference approach. Therefore, it can be an easy alternative to more complex methods for a preliminary safety evaluation of existing slender masonry members.

### Theoretical background: base model for linear elastic materials

The column model investigated is a cantilever beam having length  $L$ , with a concentrated load  $P$  placed at a distance  $e$  from the cantilever axis (Fig.1).

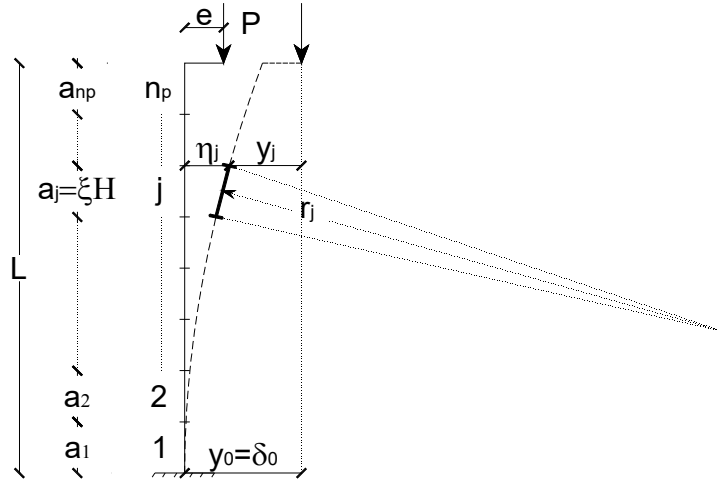


Fig. 1 Calculus model [15].

With reference to a linear elastic material with indefinite strength limit, the deformed shape of the member is given by the well-known Eulerian solution

$$y(x) = \frac{e}{\cos(\alpha \cdot L)} \cdot \cos(\alpha \cdot x) \quad (1)$$

$$\text{where } \alpha = \sqrt{\frac{P}{EI}}.$$

By imposing the boundary condition  $y(0) = \delta_0$ , the following expression is obtained

$$\frac{PL^2}{EI} = \left[ \arccos \frac{e}{\delta_0} \right]^2. \quad (2)$$

However, this approach is not applicable for columns made of non-linear materials with and whose constitutive law in tension is markedly different from that in compression (reinforced concrete or masonry). In these cases, the evaluation of the elastic modulus and moment of inertia to be introduced in Eq.(2) becomes problematic. Furthermore the Eulerian approach does not take into account the effective strength of the material and the limited tensile strength which induces distributed cracks along the member.

In the following a numerical procedure is developed, able to take into account these effects. The technique is firstly applied to linear elastic material in order to ensure its suitability by means of comparisons with the closed form solution expressed by Eq.(2).

If the cantilever beam (Fig.1) is divided into  $n_p$  elements having length  $a_j$  and for each portion of beam the curvature  $\phi_j = 1/r_j$  is assumed to be constant, the deflection  $\eta_j$  at the extremity of the generic  $j$ -th element can be expressed as a function of the curvature in the other elements by geometrical observations.

$$\frac{\eta_j}{H} = \xi^2 \sum_{i=1}^j \left( \frac{1}{2} \phi_i H + \sum_{k=1}^{i-1} \phi_k H \right). \quad (3)$$

Where  $\xi$  is a non-dimensional parameter defining the length of each portion ( $a_i=a=\xi H$ ) and  $H$  is the height of the cross-section.

Eq.(3) derives from the assumption that the curvature can be considered constant for each element; in this way, the deflection of the extremity of the  $j$ -th portion can be calculated as the sum of that related to the  $(j-1)$ -th piece and the relative displacement between the extremities of the  $j$ -th portion considered. As can be noted,  $\eta_j$  can be calculated once the curvatures of the beam parts from 1 to  $j$  are known.

The proposed approach gives the  $P$ - $\delta$  curves via two successive analysis steps. The first step is used to calculate the maximum allowable cantilever height  $L$  as a function of the top deflection  $\delta$  under given eccentricity and axial force. The main assumption is that the cross-section is constant along the column length, and the material behaves as linear elastic medium in both tension and compression with the same strength limit and elastic modulus. Under these assumptions the relationship between bending moment and curvature is

$$\varphi = \frac{M}{EI}.$$

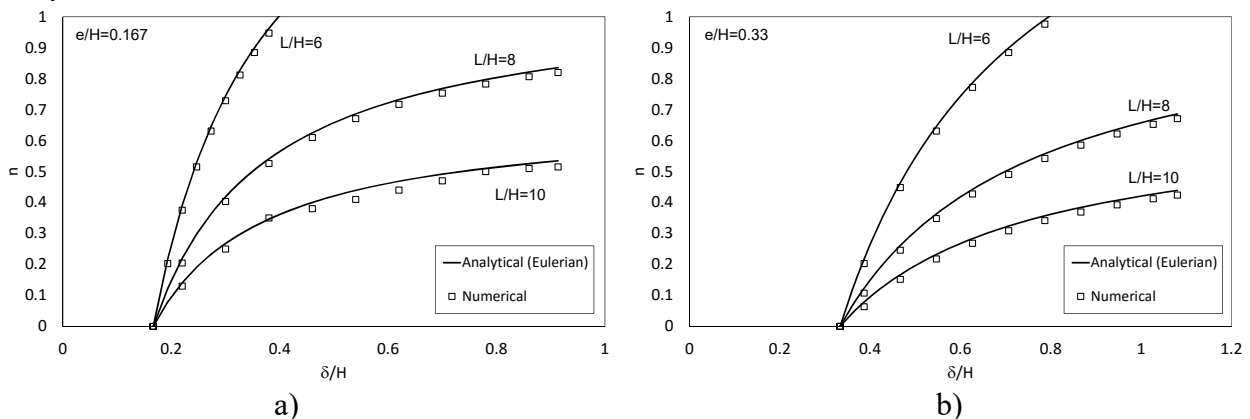
(4)

Eq.(6) can be expressed in non-dimensional form as  $m = \frac{M}{b \cdot H^2 \cdot \sigma_u}$ . Since  $\sigma_u = E \cdot \varepsilon_u$  the following

relation holds:

$$m = \frac{\varphi H}{12\varepsilon_u}. \quad (5)$$

The  $L/H - \delta/H$  relationship is calculated with the following step-by-step procedure: 1) a value is assigned to  $\delta_0$  and the non-dimensional base moment is calculated  $m_0 = n \cdot \frac{\delta_0}{H}$ , where  $n=P/bH\sigma_u$ ; 2) the curvature of the first portion of the column  $\varphi_1$  is evaluated by means of Eq.(5); 3) the top deflection of the same portion  $\eta_1$  can be calculated with Eq.(3); 4) the moment at the base of the portion of the second column is now computed as  $m_1 = n \cdot \frac{y_1}{H} = n \cdot \left( \frac{\delta_0}{H} - \frac{\eta_1}{H} \right)$ ; 4) the procedure is repeated from point 2, determining displacements for each part of the column, and it is interrupted when  $y_j < e$ ; 5) the length compatible with given eccentricity is calculated through linear interpolation.



**Fig. 2 Normalized load vs. drift curves for columns made with linear elastic material. a)  $e/H=0.167$ ; b)  $e/H=0.33$ .**

The  $L/H - \delta/H$  curves allow calculating the force-deflection response of a column under the assumption of the load patch shown in Fig.1. In fact, once that normalized length of the member  $L/H$  is defined, the different values of the normalized force  $n$  and the corresponding deflections can be read from the abovementioned  $L/H - \delta/H$  curves, the latter determined with an assigned value of eccentricity.

With reference to the L/H-  $\delta/H$  curves presented in [15], not here reported for the sake of brevity, Fig.2 shows the comparisons between the load-deflection curves calculated with the abovementioned numerical procedure and the Eulerian solution (Eq.(2)), for three different values of slenderness (L/H=6,8,10) and for two values of eccentricity (e/H=0.167, 0.33). The member was divided in 50 parts, being  $\xi=0.5$ . Good agreement is observed between numerical and closed form solutions. Moreover, it is stressed as the axial capacity is not reached for slender members and, as expected, more flexible responses are obtained for greater eccentricities and L/H ratios.

### Sectional analysis for confined masonry sections

The procedure presented in the previous section can be extended to include mechanical non-linearity by replacing Eq.(5) with a suitable non-linear moment-curvature relationships. To do so this section focuses on the development of a proper sectional analysis.

The characteristic response of a masonry section is provided by the moment-curvature law. In this study, moment-curvature curves are derived with the classical step-by-step analysis, which consists in assigning a value to the compressive strain of the top fiber  $\varepsilon_i$  of the section and calculating the corresponding curvature and moment by solving the two equilibrium equations. The latter, under the assumption of non-cracked section can be written in the following non-dimensional form:

$$\frac{S_1(\varepsilon_i)}{\varphi_i H} - \frac{S_{i1}(\varphi_i H - \varepsilon_i)}{\varphi_i H} = n \quad (6a)$$

$$\frac{S_2(\varepsilon_i)}{(\varphi_i H)^2} - \frac{S_{i2}(\varphi_i H - \varepsilon_i)}{(\varphi_i H)^2} = m + n \left( \frac{1}{2} - \frac{\varepsilon_i}{\varphi_i H} \right) \quad (6b)$$

where:

$$S_1(\varepsilon_i) = \int_0^{\varepsilon_i} \frac{\sigma(\varepsilon)}{f_m} d\varepsilon, \quad (7a)$$

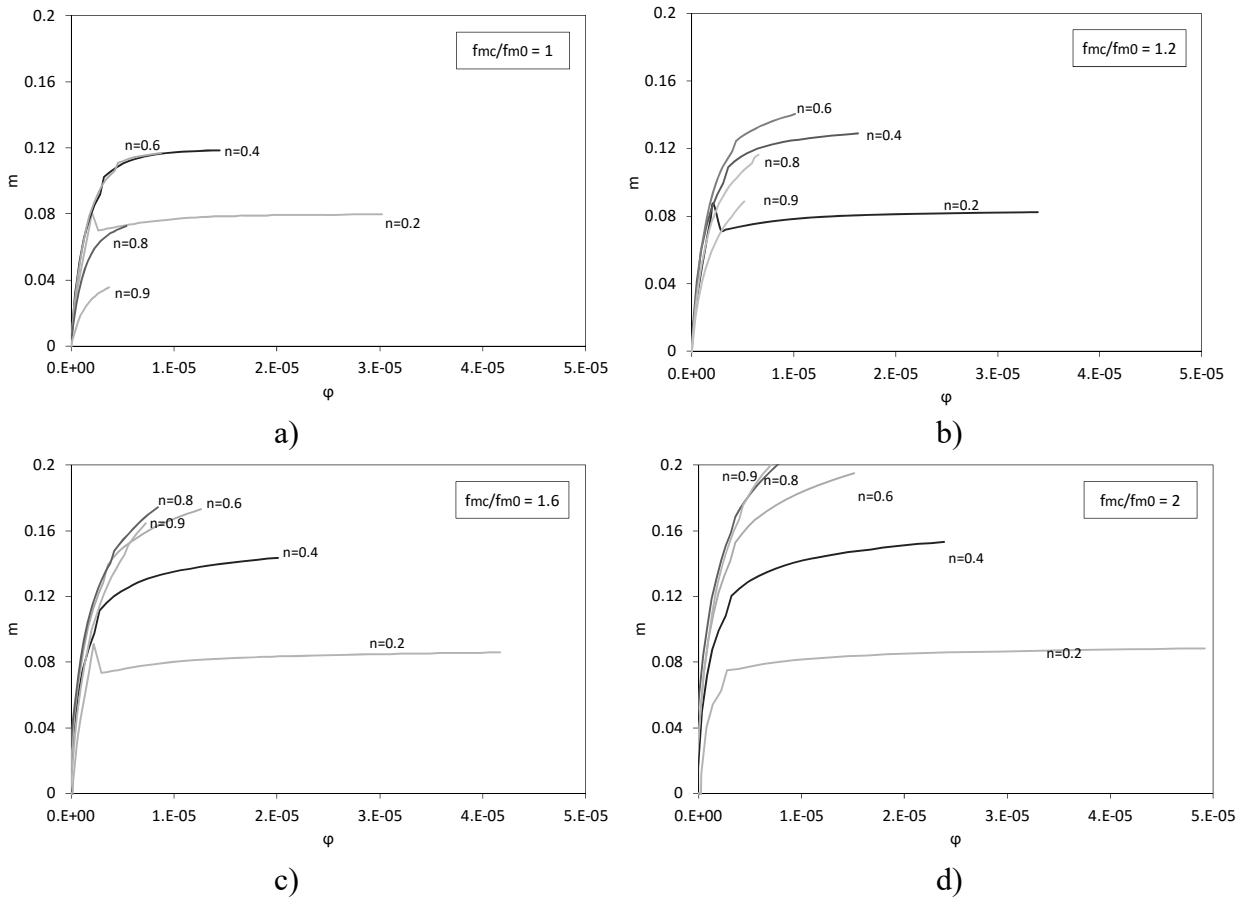
$$S_{i1}(\varphi_i H - \varepsilon_i) = \int_0^{\varphi_i H - \varepsilon_i} \frac{\sigma_t(\varepsilon)}{f_m} d\varepsilon, \quad (7b)$$

$$S_2(\varepsilon_i) = \int_0^{\varepsilon_i} \frac{\sigma(\varepsilon)}{f_m} \varepsilon d\varepsilon, \quad (7c)$$

$$S_{i2}(\varphi_i H - \varepsilon_i) = \int_0^{\varphi_i H - \varepsilon_i} \frac{\sigma_t(\varepsilon)}{f_m} \varepsilon d\varepsilon. \quad (7d)$$

with  $\sigma(\varepsilon)$  the stress-strain law in compression of the masonry and  $\sigma_t(\varepsilon)$  the stress-strain law in tension. In the present study,  $\sigma(\varepsilon)$  was assumed as proposed by the Italian Guidelines CNR-DT200 [16], while a linear relationship was considered for the tensile constitutive law  $\sigma_t(\varepsilon)$ .

Moment-curvature curves for square sections are generated using the above procedure for different levels of normalized axial force n. In particular, the procedure described above is implemented in Mathematica environment [17], and exact integration of Eqns.(6) is achieved by the solver adopted.



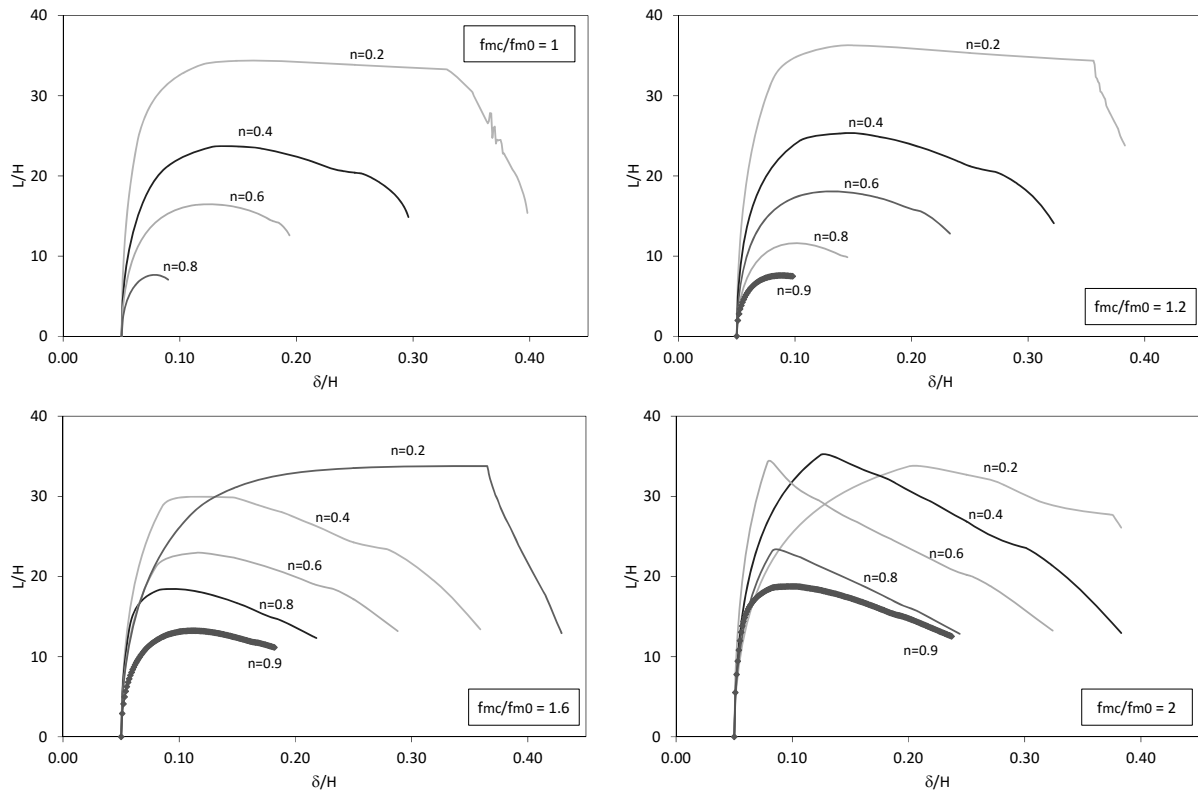
**Fig. 3 Moment-curvature response for different confinement levels.**  
**a)  $f_{mc}/f_{m0}=1$ , b)  $f_{mc}/f_{m0}=1.2$ , c)  $f_{mc}/f_{m0}=1.6$ , d)  $f_{mc}/f_{m0}=2$ .**

Fig. 3 shows the normalized moment  $m = \frac{M}{b \cdot H^2 \cdot f_{m0}}$  as a function of the curvature  $\phi$ , expressed in  $\text{mm}^{-1}$ , for different levels of axial force and confinement levels  $f_{mc}/f_{m0}=1, 1.2, 1.6$  and  $2$ . The non-dimensional axial force  $n$  is normalized by the unconfined masonry strength  $f_{m0}$ . It should be noted that the ultimate strain increase is neglected in the current analyses for highlighting the effect of the local strength gain. This assumption is also commonly adopted in practical applications for the sake of safety.

As it can be observed, ultimate curvature slightly increases for low levels of axial force and increasing confinement ratios, while ultimate moment does not change. When  $n$  increases, ultimate curvature tends to be the same with respect to the unconfined section, but ultimate moment raises and reaches very high values when  $n=0.8$  or  $n=0.9$ . This fact shows as high enhancements of flexural capacity are obtained only for high  $n$  values and high confinement ratios, highlighting the effectiveness of confinement for these cases.

### **Buckling curves for masonry columns retrofitted with FRP wraps**

The moment-curvature laws derived above can be numerically implemented to obtain the trend of the allowable  $L/H$  ratio as a function of maximum deflection.



**Fig. 4 Allowable L/H ratio as a function of normalised drift  $\delta/H$  for different confinement ratios.**  
**a)  $f_{mc}/f_{m0}=1$ , b)  $f_{mc}/f_{m0}=1.2$ , c)  $f_{mc}/f_{m0}=1.6$ , d)  $f_{mc}/f_{m0}=2$ .**

Fig.4 shows the L/H vs.  $\delta/H$  curves for the four values of confinement ratio adopted ( $f_{mc}/f_{m0}=1$ , 1.2, 1.6 and 2) and for a normalized eccentricity equal to  $e/H=0.05$ . The section under study has side length  $H=400$  mm and it is divided in 100 elements, assuming  $\xi=0.5$ . For each case analyzed, five axial force levels are considered ( $n=0.2, 0.4, 0.6, 0.8$  and  $0.9$ ).

It can be noted as for low values of axial force the curves tends to be the same also for very different confinement ratios. The maximum allowable L/H ratio is equal to 32 for unconfined section, while it is equal to about 35 for confined sections, independently by the local strength gain. In both cases, it is reached for an axial load equal to  $n=0.2$ . It is worth to note as higher maximum normalized slenderness can be reached for high axial load values, when high confinement ratios are ensured. In particular, for  $n=0.9$  the curve is negligible for  $f_{mc}/f_{m0}=1$ , while it is notably extended for confined members, with a maximum allowable normalized length equal to 18 for  $f_{mc}/f_{m0}=2$ .

## Conclusions

This paper presented a numerical approach for calculating the buckling curves in masonry members confined by FRP wraps. The target was to study the interaction between the local strength gain induced by confinement and the loss of load-carrying capacity induced by second order effects. The model presented is based on a sectional analysis that was preliminarily carried out in order to include the mechanical non-linearity. L/H vs.  $\delta_0/H$  curves were derived via a numerical procedure. From results obtained by the model and for the range of variables investigated, the following conclusions can be drawn:

- sectional analysis highlights as the efficiency of FRP wrapping technique in terms of flexural capacity is strictly related to the existing value of axial load. Good enhancement of ultimate moment can be observed only for  $n>0.4$  and for high confinement ratios;
- the maximum allowable slenderness ratio is only slightly influenced by confinement. It can be considered almost constant between different strength increases;
- FRP confinement influences buckling curves only for normalized axial force values over 0.4. The member could carry greater values of axial force, but with large deflections.



Further investigations should be carried out in the future to confirm obtained results. In particular, comparisons with non-linear finite element analyses and experimental results could clarify the role of several investigated parameters.

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