A Sequential Monte Carlo Approach for Extended Object Tracking in the Presence of Clutter

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Abstract: Extended objects are characterised with multiple measurements originated from different locations of the object surface. This paper presents a novel Sequential Monte Carlo (SMC) approach for extended object tracking in the presence of clutter. The problem is formulated for general nonlinear problems. The main contribution of this work is in the derivation of the likelihood function for nonlinear measurement functions, with sets of measurements belonging to a bounded region. Simulation results are presented when the object is surrounded by a circular region. Accurate estimation results are presented both for the object kinematic state and object extent.

1 Motivation

Extended object tracking is an important application where the interest is in finding estimates of the centre of the area surrounding an object and the object extent/size. The extended object usually leads to multiple measurements. Different methods are proposed in the literature for dealing with this problem. Most of the methods separate the problem of kinematic state estimation from the problem of parameter state estimation such as in [KF09, Koc08]. The extent parameters are estimated separately from the states, for instance with the random matrices approach [KF09, Koc08]. A comparison between the approach with random matrices and the combined-set theoretic approach is presented in [BFF+10]. An approach with SMC method for extended object tracking is proposed in [VIG05]. Other related works are [SH07, SBH06, BH09a, BH09b, NKPH10].

In general the measurement uncertainty can belong to a hypercube or to another spatial shape. In our approach, we consider the general case with a nonlinear measurement equation. The main contributions of the work is in the derived likelihood function based on a parameterised shape and in the developed SMC filter for extended objects. Then we propagate this spatial measurement uncertainty through the Bayesian estimation framework.

In 2005, Gilholm and Salmond [GS05] developed a spatial distribution model for tracking extended objects in clutter, where the number of observations from the target is assumed to be Poisson distributed. Based on this approach Poisson likelihood models for group and extended object tracking were developed [CG07].

The rest of this paper is organised as follows. Section 2 introduces the SMC framework in the case of EOT. Section 3 gives the measurement likelihood in the presence of clutter.

Performance evaluation scenario is shown in Section 4 and conclusions based on these simulations are presented in Sections 5.

2 Extended Object Tracking Within the Sequential Monte Carlo Framework

This work considers the state estimation problem for extended objects with clutter noise. Such objects usually give rise to a set of measurements. The system dynamics and sensor can be described using the equations

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, \boldsymbol{\eta}_{k-1}), \tag{1}$$

$$\boldsymbol{z}_k = h(\boldsymbol{x}_k, \boldsymbol{w}_k), \tag{2}$$

where $\boldsymbol{x}_k = \left(\boldsymbol{X}_k^T, \boldsymbol{\Theta}_k^T\right)^T \in \mathbb{R}^{n_x}$, with $(\cdot)^T$ being the transpose operator, is the unknown system state vector at time step k, $k = 1, 2, ..., K$, where K is the maximum number of time steps. The vector $\boldsymbol{x}_k$ consists of the object kinematic state vector $\boldsymbol{X}_k$ and object extent, characterised by the parameter vector $\boldsymbol{\Theta}_k \in \mathbb{R}^{n_\Theta}$; $f(.)$ and $h(.)$ are respectively the system and the measurement functions, nonlinear in general; $\boldsymbol{z}_k \in \mathbb{R}^{n_z}$ is the measurement vector and $\boldsymbol{\eta}_k = (\boldsymbol{\eta}_{s,k}^T \ \ \boldsymbol{\eta}_{p,k}^T)^T$ and $\boldsymbol{w}_k$ are the system (kinematic state and parameters) and measurement noises, respectively.

Within the SMC approach the system state pdf is approximated by randomly generated samples and based on the sequence of measurements. According to the Bayes' rule the filtering pdf $p(\boldsymbol{x}_k | \boldsymbol{z}_{1:k})$ of the state vector $\boldsymbol{x}_k$ given a sequence of sensor measurements $\boldsymbol{z}_{1:k}$ up to time k can be written as

$$p(\boldsymbol{x}_k | \boldsymbol{z}_{1:k}) = \frac{p(\boldsymbol{z}_k | \boldsymbol{x}_k) p(\boldsymbol{x}_k | \boldsymbol{z}_{1:k-1})}{p(\boldsymbol{z}_k | \boldsymbol{z}_{1:k-1})}, \tag{3}$$

where $p(\boldsymbol{z}_k | \boldsymbol{z}_{1:k-1})$ is the normalising constant. The state *predictive* distribution is given by the equation

$$p(\boldsymbol{x}_k | \boldsymbol{z}_{1:k-1}) = \int_{\mathbb{R}^{n_x}} p(\boldsymbol{x}_k | \boldsymbol{x}_{k-1}) p(\boldsymbol{x}_{k-1} | \boldsymbol{z}_{1:k-1}) d\boldsymbol{x}_{k-1}. \tag{4}$$

The evaluation of the right hand side of (3) involves integration which can be performed by the PF approach [AMGC02] by approximating the posterior pdf $p(\boldsymbol{x}_k | \boldsymbol{z}_{1:k})$ with a set of particles $\boldsymbol{x}_{0:k}^{(i)}$, $i = 1, \ldots, N$ and their corresponding weights $w_k^{(i)}$. Then the posterior density function can be written as follows

$$p(\boldsymbol{x}_{0:k} | \boldsymbol{z}_{1:k}) = \sum_{i=1}^{N} w_k^{(i)} \delta(\boldsymbol{x}_{0:k} - \boldsymbol{x}_{0:k}^{(i)}), \tag{5}$$

where $\delta(.)$ is the Dirac delta function, and the weights are normalised so that $\sum_i w_k^{(i)} = 1$.

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Each pair $\{\boldsymbol{x}_{0:k}^{(i)}, w_k^{(i)}\}$ characterises the belief that the object is in state $\boldsymbol{x}_{0:k}^{(i)}$. An estimate of the variable of interest is obtained by the weighted sum of particles. Two major stages can be distinguished: *prediction* and *update*. During prediction, each particle is modified according to the state model, including the addition of random noise in order to simulate the effect of the noise on the state. In the update stage, each particle's weight is re-evaluated based on the new data. A *resampling* procedure introduces variety in the particles by eliminating those with small weights and replicating the particles with larger weights such that the approximation in (5) still holds. The residual resampling algorithm is applied here.

3   Measurement Likelihood in the Presence of Clutter

In the context of extended object tracking, the prediction step is generally well studied for various classes of interval and spatial representations of uncertainties. Ellipsoids, spheres and polytope families can be easily propagated when the system dynamics and sensor models are linear.

The update step with the likelihood calculation is less studied or often studied with a restriction to a particular class of a particular type of measurements. The aim of this paper is to derive general likelihood calculation procedures based on Monte Carlo methods without a restriction on the type of set of interest or the type of cluttered measurements available. For that purpose, the main idea of this paper is to introduce a sampling step for regions of interest in the extended object. This sampling aims to represents the probability of a given point, in the state space, to be the origin of a measurement. This section describes briefly the newly proposed method, details can be found in the references herein. As an illustration, the case of an extended object with circular form is considered.

Without loss of generality, assume that there is only one static sensor described by its state vector $\boldsymbol{x}_{s,k}$. Assume that at each measurement time k the extended object generates a matrix $\boldsymbol{Z}_k = \{\boldsymbol{z}_k^1, \ldots, \boldsymbol{z}_k^m\} \in \mathbb{R}^{n_z \times M_k}$ of $M_k = M_{T,k} + M_{C,k}$ measurement vectors. The number of measurements $M_{T,k}$ originating from the visible border of the source is considered Poisson-distributed random variable with mean value of λ_T, or $M_{T,k} \sim Poisson(\lambda_T)$. Similarly, the number of clutter measurements is $M_{C,k} \sim Poisson(\lambda_C)$, where λ_C is the mean value of the clutter measurements. The clutter measurements are modeled according to [DM01]. All of the measurements are assumed to be conditionally independent, i.e.

$$p(\boldsymbol{Z}_k|\boldsymbol{x}_k) = \prod_{j=1}^{M_k} p(\boldsymbol{z}_k^j|\boldsymbol{x}_k). \tag{6}$$

If at time step $k-1$ the posterior pdf $p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{k-1}^j)$ is known, then one can express the prior $p(\boldsymbol{x}_k|\boldsymbol{z}_{k-1}^j)$ via the Chapman-Kolmogorov equation:

$$p(\boldsymbol{x}_k|\boldsymbol{z}_{k-1}^j) = \int_{\mathbb{R}^{n_x}} p(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{z}_{k-1}^j)d\boldsymbol{x}_{k-1}. \tag{7}$$

The nearly constant velocity model [LJ03, BSL93, PMGA11] is used to model the motion

of the centre of the region surrounding the extended target.

3.1   Observation Model

Range and bearing observations from a network of low cost sensors positioned along the road are considered as measurements. The measurement vector is $\boldsymbol{z}_k^j = (r_k^j, \beta_k^j)^T$, where r_k^j is the range and β_k^j is the bearing of the measurement point j. The equation for the measurements originating from the target has the form: $\boldsymbol{z}_k^j = h(\boldsymbol{x}_k) + \boldsymbol{w}_k^j$, where h is the nonlinear function $h(\boldsymbol{x}_k) = \left(\sqrt{x_k^{j2} + y_k^{j2}}, \tan^{-1} \frac{y_k^j}{x_k^j} \right)$, x_k^j and y_k^j denote the Cartesian coordinates of the actual point of the source from where the measurement emanates in the case of two dimensional space. The measurement noise $\boldsymbol{w}_k^j$ is supposed to be Gaussian, with a known covariance matrix $\boldsymbol{R} = diag(\sigma_r^2, \sigma_\beta^2)$. The clutter measurements are considered uniformly distributed within the visible area of the sensor.

3.2   Introduction of the Notion of the Visible Surface

The measurement likelihood $p(\boldsymbol{z}_k^j | \boldsymbol{x}_k)$ of the extended target can be represented with the relation

$$p(\boldsymbol{z}_k^j | \boldsymbol{x}_k) = \int_{\mathbb{R}^{n_v}} p(\boldsymbol{z}_k^j | \boldsymbol{V}_k) p(\boldsymbol{V}_k | \boldsymbol{x}_k) d\boldsymbol{V}_k, \tag{8}$$

where $\boldsymbol{V}_k \in \mathbb{R}^{n_v}$ denotes a source of measurement in the state space. In practice, these visible sources $\boldsymbol{V}_k$ depend on the object position, nature and parameters $\boldsymbol{x}_k, \lambda_T$ or scatter characteristics λ_C and on the sensor state (e.g., the sensor position and angle of view). The pdf $p(\boldsymbol{V}_k | \boldsymbol{x}_k)$ represents the probability of a point in the state space to be a source of measurement given the extended object $\boldsymbol{x}_k$. The surface of the target with state $\boldsymbol{x}_k$ visible from the sensor with state $\boldsymbol{x}_{s,k}$ is denoted by $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$.

We assume that the sources of the true measurements at time step k, given the target state and the sensor prior state are uniformly distributed along the region $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$, visible from the sensor $x_{s,k}$, i.e.

$$p(\boldsymbol{V}_k | \boldsymbol{x}_k) = p(\boldsymbol{V}_k | \boldsymbol{x}_k, \boldsymbol{x}_{s,k}) = \mathcal{U}_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})}(\boldsymbol{V}_k), \tag{9}$$

where $\mathcal{U}_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})}(\cdot)$ represents the uniform pdf with the support $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$.

Inserting (9) into (8) gives

$$p(\boldsymbol{z}_k^j | \boldsymbol{x}_k) = \int_{\mathbb{R}^{n_v}} p(\boldsymbol{z}_k^j | \boldsymbol{V}_k) \mathcal{U}_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})}(\boldsymbol{V}_k) d\boldsymbol{V}_k = \frac{1}{||\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})||} \int_{\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})} p(\boldsymbol{z}_k^j | \boldsymbol{V}_k) d\boldsymbol{V}_k, \tag{10}$$

where $||\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})||$ denotes some measure of the region $\mathcal{V}_k(\boldsymbol{x}_k, \boldsymbol{x}_{s,k})$. That measure could represents the length of a curve (a one-dimensional concept), the area of a surface (a two-dimensional concept) or the volume of a solid (a three-dimensional topological

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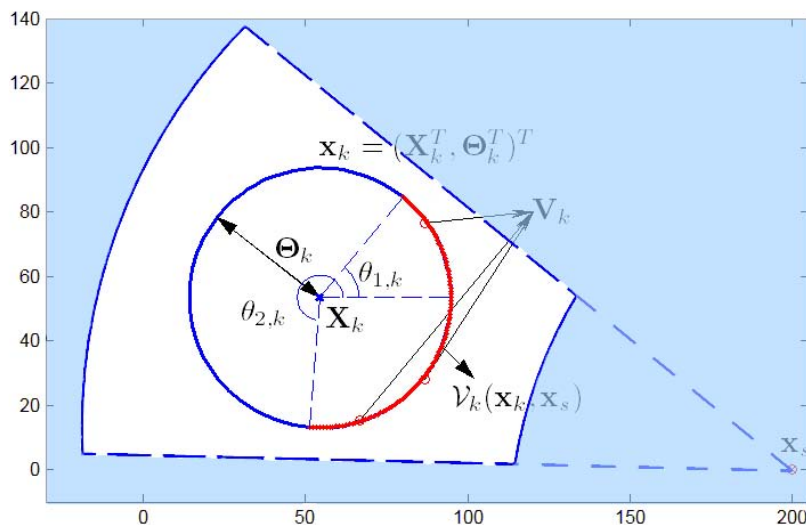
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Fig. 1: Notations and definitions in the context of an example with circular extent

manifold). Unfortunately that integral (10) is difficult to calculate. An particular example of this integral for circular shaped extent is considered in [PMGA11].

Gating is applied to the spatial area of the measurement sources. The gating region is defined by an angular field ($\beta_1 < \beta < \beta_2$) and minimum and maximum distance ($d_1 < d < d_2$) around the predicted object center with respect to sensor position. An example of such gating as well as some of the notations are shown for object with circular extent in Fig. 1

3.3 Parametrisation of the Visible Border

The calculation of the likelihood (8) can be performed using a Monte Carlo method in the following way. After the prediction step, a set of weighted particles $\{(\boldsymbol{x}_{k|k-1}^{(i)}, w_{k|k-1}^{(i)})\}_{i=1}^{N}$ is available (recall that each of the N particles can be seen as an extended object hypothesis). First, the visible border $\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k})$ is determined. Then for each particle $\boldsymbol{x}_{k|k-1}^{(i)}$ the likelihood function $p(\boldsymbol{V}_k|\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{z}_k^j)$ is defined from the support $\mathcal{V}_k(\boldsymbol{x}_{k|k-1}^{(i)}, \boldsymbol{x}_{s,k})$, taking into account the angular information of the measurement $\boldsymbol{z}_k^j$. The measurements $\left\{\boldsymbol{z}_k^j\right\}_{j=1}^{M}$ are generated according to $\mathcal{N}(\boldsymbol{z}_k^j, h(\boldsymbol{V}_k^{(i,\ell)}), \boldsymbol{R})$. Then sampling Q visible points from each of M_k measurements gives us a total of $S = QM_k$ samples per particle $\{\boldsymbol{V}_k^{(i,\ell)}\}_{\ell=1}^{S}$, for each particle i:

$$\boldsymbol{V}_k^{(i,j)} \sim \begin{pmatrix} r_{v,k}^{(i,j)} \\ \theta_{v,k}^{(i,j)} \end{pmatrix} = \begin{pmatrix} \mathcal{N}(r_{v,k}^{(i,j)}; r_k^{(i)}, \tilde{\sigma}_r^2) \\ \mathcal{N}(\theta_{v,k}^{(i,j)}; \theta_k^{(i,j)}, \tilde{\sigma}_\theta^2) \end{pmatrix}, \tag{10}$$

where $\tilde{\sigma}_r$ and $\tilde{\sigma}_\theta$ are the standard deviations for the radius and angle, respectively, chosen for generating the samples.

Once the points $\{V_k^{(i,\ell)}\}_{\ell=1}^S$ are available for each particle $x_{k|k-1}^{(i)}$, we can approximate the likelihood (see the equation (8)) according to

$$p(z_k^j|x_{k|k-1}^{(i)}) = \int_{\mathbb{R}^{n_x}} p(z_k^j|V_k)p(V_k|x_{k|k-1}^{(i)})dV_k, = \sum_{\ell=1}^M p(z_k^j|V_k^{(i,\ell)})p(V_k^{(i,\ell)}|x_{k|k-1}^{(i)}). \quad (11)$$

The term $p(z_k^j|V_k^{(i,\ell)})$ in (11) can be easily calculated in a classical way depending on the problem (for example using a Gaussian likelihood). The term $p(V_k^{(i,\ell)}|x_{k|k-1}^{(i)})$ depends very much on the visible set $\mathcal{V}_k(x_{k|k-1}^{(i)}, x_{s,k})$. It can be seen as

$$p(V_k^{(i,\ell)}|x_{k|k-1}^{(i)}) = p(V_k^{(i,\ell)} \in \mathcal{V}_k(x_{k|k-1}^{(i)}, x_{s,k})). \quad (12)$$

Here we will give an example for the calculation of the term $p(V_k^{(i,\ell)}|x_{k|k-1}^{(i)})$ over a circle. Let us denote with $x_k^{(i,\ell)}$ and $y_k^{(i,\ell)}$ the coordinates of $V_k^{(i,\ell)}$, i.e. $V_k^{(i,\ell)} = (x_k^{(i,\ell)}, y_k^{(i,\ell)})^T$. For circular object the border $\mathcal{V}_k(x_{k|k-1}^{(i)}, x_{s,k})$ can be given as

$$\mathcal{V}_k(x_{k|k-1}^{(i)}, x_{s,k}) = \{(x_{c,k|k-1}^{(i)} + r_{k|k-1}^{(i)}\cos(\theta)y_{c,k|k-1}^{(i)} + r_{k|k-1}^{(i)}\sin(\theta)), \theta \in [\theta_{1,k|k-1}^{(i)}, \theta_{2,k|k-1}^{(i)}]\}. \quad (13)$$

A simple expression of $p(V_k^{(i,\ell)}|x_{k|k-1}^{(i)})$ can be

$$p(V_k^{(i,\ell)}|x_{k|k-1}^{(i)}) = \mathcal{N}(\sqrt{(x_k^{(i,\ell)} - x_{c,k}^{(i)})^2 + (y_k^{(i,\ell)} - y_{c,k}^{(i)})^2}, r_{k|k-1}^{(i)}, \sigma_v^2) \quad (14)$$

where σ_v is the standard deviation for a Gaussian likelihood applied to the distance from $V_k^{(i,\ell)}$ to the center of the particle $(x_{c,k}^{(i)}, y_{c,k}^{(i)})$. Similarly to [GS05], in the presence of clutter the likelihood function of the PF can be calculated from the following equation

$$P(Z_k|x_k^{(i)}) = \prod_{j=1}^{M_k} \left(1 + \frac{\lambda_T}{\rho}p(z_k^j|x_{k|k-1}^{(i)})\right), \quad (15)$$

where $\rho = \lambda_C/A$ is the uniform clutter density and A is the area visible from the sensor.

4   Performance Evaluation

An example, similar to the one presented in [PMGA11] is considered with added clutter noise. Simulations are performed for 100 time steps each repeated for 500 iterations in the case of tracking of object with circular extent and cluttered measurements consisting of range and bearing. Scenarios with extended object particles from 80 to 500 are considered. Please note that the actual number of samples is much higher and depends on the number of measurements. For each of particles 5 Metropolis-Hastings proposals are generated. The initial position of the object is biased by Gaussian random noise with standard deviation
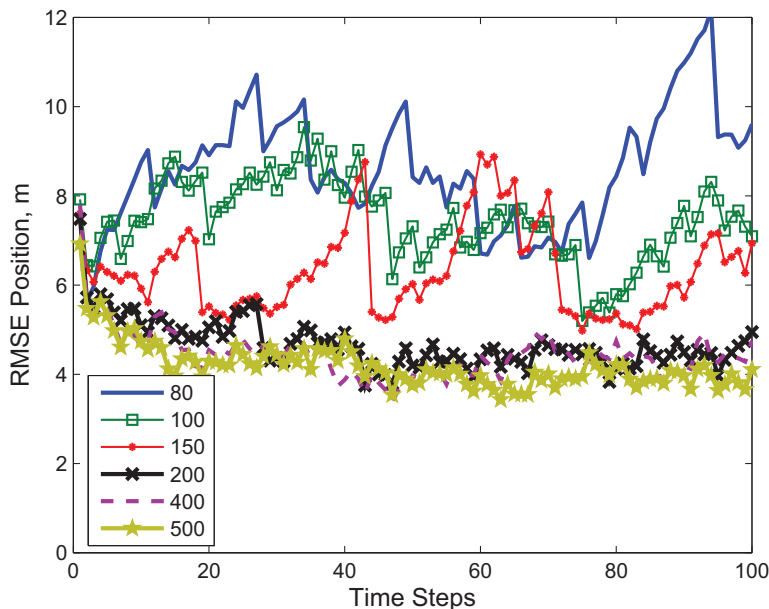
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Fig. 2: RMSE of the position for number of particles varying from 80 to 500

of $10\,m$ for both coordinates. The initial extent is altered by Gaussian noise with standard deviation of $5\,m$ in radius, and the initial velocity is generated with added Gaussian noise with standard deviation of $1.5\,m/s$ for each of the coordinates.

The number of both the target and the clutter measurements generated in each step is assumed to be Poisson distributed, i.e. $M_{T,k} \sim Poisson(\lambda_T)$, where $\lambda_T = 5$ and $M_{C,k} \sim Poisson(\lambda_C)$, where $\lambda_C = 13$. For each measurement and each hypothesis one random sample Q is generated according to (10). The radius of visibility of the sensor is assumed to be $200\,m$, therefore $\rho = 0.0001$. The standard deviations for the bearing and for the radius of these samples are respectively $\sigma_\beta = 3°$ and $\sigma_r = 1\,m$. The sensor line of sight for simplicity is determined by the angles $\alpha_{1,k} = 0$ and $\alpha_{2,k} = 2\pi$.

The real measurements are assumed to originate from random locations of the visible frontier of the extent where the angular position is uniformly distributed over the visible arc and the range has additive random Gaussian noise with standard deviation of $2\,m$. The clutter measurements are uniformly distributed in the visible area of the sensor.

The change of the size of the extent in the space-state evolution model is generated by adding/substracting (corresponding to extension/shrinkage) the absolute value of a Gaussian random variable with zero mean and standard deviation $2\,m$. The actual target trajectory is generated based on (1) with noise covariance equal to zero. The change of the size of the extent when generating the extended object particles is modeled a random walk using normally distributed random variable with zero mean and standard deviation $4\,m$. The tra-

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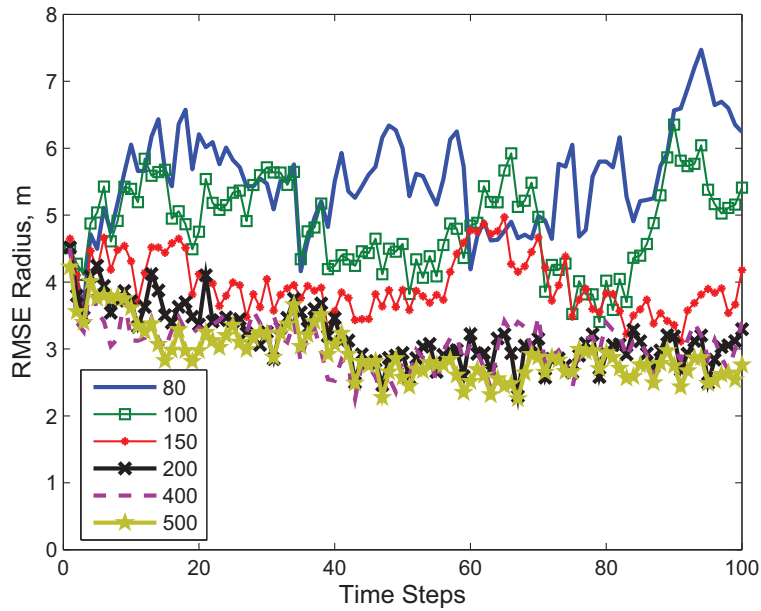
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Fig. 3: RMSE of the radius for number of particles varying from 80 to 500

jectory prediction in the filter is performed with standard deviation for the components of the system dynamic noise $\sigma_x = 1\,m/s^2$ and $\sigma_y = 1\,m/s^2$, respectively. The results from the simulations are presented in Figs. 2 - 5 and Table 1. The success rate refers to the percentage of successfully tracked trajectories.

Table 1: Success rate

N particles	Success rate, %
80	68.6
100	69.2
150	73.4
200	80.4
400	83.2
500	79.6

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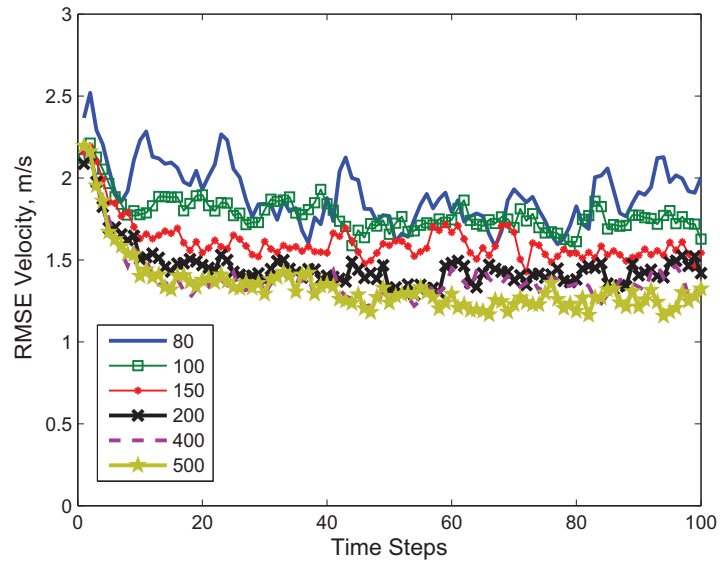
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Fig. 4: Comparison of the RMSE for velocity for number of particles varying from 80 to 500
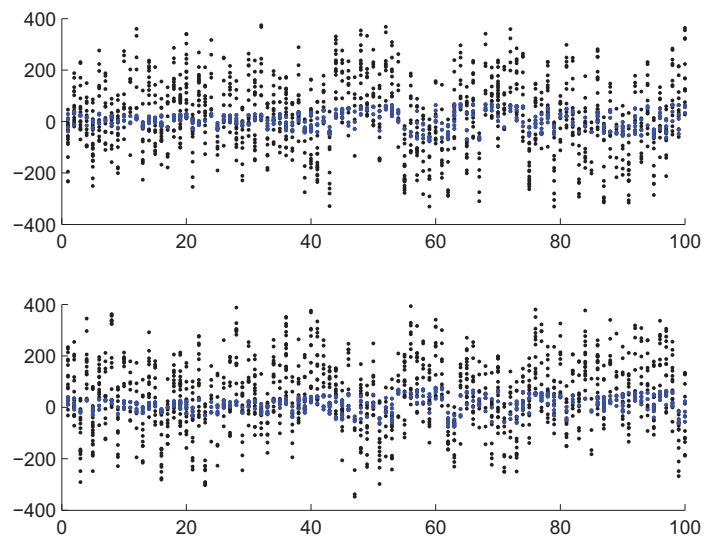
Fig. 5: Clutter/real measurements plot from a single run

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5 Conclusions

This paper presents an approach for coping with clutter noise when tracking extended objects. The simulation results show accurate performance results with more than 200 particles for the considered testing example.

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