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Gaillard, J., Macpherson, N., Núñez, C. & Thompson, D. (2014). Dualising the baryonic branch: Dynamic <mml:math altimg="si1.gif" overflow="scroll" xmlns:xocs="http://www.elsevier.com/xml/xocs/dtd" xmlns:xs="http://www.w3.org/2001/XMLSchema" xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance" xmlns="http://www.elsevier.com/xml/ja/dtd" xmlns:ja="http://www.elsevier.com/xml/ja/dtd" xmlns:mml="http://www.w3.org/1998/Math/MathML" xmlns:tb="http://www.elsevier.com/xml/common/table/dtd" xmlns:sb="http://www.elsevier.com/xml/common/struct-bib/dtd" xmlns:ce="http://www.elsevier.com/xml/common/dtd" xmlns:xlink="http://www.w3.org/1999/xlink" xmlns:cals="http://www.elsevier.com/xml/common/cals/dtd" xmlns:sa="http://www.elsevier.com/xml/common/struct-aff/dtd"><mml:mrow><mml:mi mathvariant="italic">SU</mml:mi></mml:mrow><mml:mo stretchy="false">(</mml:mo><mml:mn>2</mml:mn><mml:mo stretchy="false">)</mml:mo></mml:math> and confining backgrounds in IIA. *Nuclear Physics B, 884*, 696-740.

http://dx.doi.org/10.1016/j.nuclphysb.2014.05.004

Available online at www.sciencedirect.com

ScienceDirect

Nuclear Physics B 884 (2014) 696–740

NUCLEAR PHYSICS B

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Dualising the baryonic branch: Dynamic $SU(2)$ and confining backgrounds in IIA

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Received 24 March 2014; accepted 5 May 2014

Available online 15 May 2014

Editor: Stephan Stieberger

Abstract

In this paper we construct and examine new supersymmetric solutions of massive IIA supergravity that are obtained using non-Abelian T-duality applied to the baryonic branch of the Klebanov–Strassler background. The geometries display $SU(2)$ structure which we show flows from static in the UV to dynamic in the IR. Confinement and symmetry breaking are given a geometrical interpretation by this change of structure. Various field theory observables are studied, suggesting possible ways to break conformality and flow in $\mathcal{N} = 1 T_N$ and related field theories.

1. Introduction and general idea of this paper

The notion of duality is of course quite old, going back to well-known examples like the Maxwell equations in vacuum. The true power of the idea became clear around 1940 with the

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http://dx.doi.org/10.1016/j.nuclphysb.2014.05.004

Kramers–Wannier [1] duality of the Ising model. In more recent times dualities have continued to be a driver of theoretical progress with examples including bosonisation [2], Montonen–Olive duality [3], S and T-dualities, Seiberg–Witten duality [4], Seiberg duality [5] and more general String dualities (U dualities). The duality conjectured by Maldacena [6], also called AdS/CFT or gauge–strings duality, is arguably the most powerful, widely applicable and conceptually deep duality of all known at present. All these dualities present common features: the degrees of freedom on both sides of the dual descriptions are in principle quite different; a strongly coupled (highly fluctuating) description of the system is characteristically mapped into a weakly coupled (semiclassical) one, in the same vein a phenomena that is 'local' in one set of variables becomes 'non-local' in the other (as exemplified by order–disorder operators and their typical 'uncertainty' relations), global symmetries are common to both dual descriptions, etc.

In this paper, we will mostly work with two dualities, the one conjectured by Maldacena and its extensions (see the papers [7] for a sample of representative work and reviews) together with what is called 'non-Abelian T-duality' [8]. Non-Abelian T-duality is the obvious extension of the more common T-duality of circles to cases where a target space supports a non-Abelian isometry group. In the NS sector, the rules for non-Abelian T-duality can be obtained in much the same way as for Abelian T-duality, by means of the Buscher procedure [9,10]. The string σ-model has a global symmetry group G corresponding to the isometries of the target space. This symmetry is gauged by the introduction of gauge fields which are however constrained by means of Lagrange multipliers enforcing a flat connection. Upon gauge fixing, the original σ-model is recovered. Instead, integrating out the gauge fields and again gauge fixing produces the non-Abelian T-dual σ-model. The Lagrange multipliers (or more accurately a $\dim G$ subset of Lagrange multipliers and the original coordinates) play the role of T-dual coordinates. The T-dual metric, NS two-form can be read off directly from the T-dual σ-model and the transformation law for the dilaton, a quantum effect, is obtained in a similar fashion to the Abelian case [9,10]. This was extended in [11] to include the action of non-Abelian T-duality on RR fluxes in Type II string theory. A detailed description of the implementation of this Buscher procedure together with a presentation of "rules" for dualisation can be found in e.g. Section 2 of [16].

It is anticipated that non-Abelian T-duality can be used as a solution generating symmetry of supergravity: a solution of Type II supergravity supporting a non-Abelian isometry group G can be mapped in to a new solution of Type II supergravity using this technique. This has been shown to be true in a wide variety of examples, but a comprehensive proof in all generality has not been presented. Following the implementation of non-Abelian T-duality as a solution generating technique of RR backgrounds in [11], there have been a number of recent developments in the use of non-Abelian duality, see [12–21]. We will make use of many technical tools developed in these various papers.

Despite the similarity in its derivation with the Abelian case, there are some important differences in non-Abelian T-duality. Firstly, as a result of performing the dualisation, the original isometry is often destroyed in part or completely. For this reason one can-not simply re-dualise the T-dual to recover the original model by means of a Buscher procedure. Secondly, there are a number of subtle global issues at play; in general it is not clear how to assign the periodicities to the T-dual coordinates meaning that global properties of the resulting geometries are not always clear. Related to this, the duality is not expected to hold, at least not without modification, at the level of string (genus) perturbation theory (as is also the case of fermionic T-duality or dualisation of non-compact isometries [22]). In this paper, we largely do not seek to address this issue, instead we will confine ourselves in the use of non-Abelian T-duality as a technique to generate new solutions to the equations of motion of Type II supergravity.

We will consider backgrounds of Type II supergravity that have a well understood (strongly coupled) field theory dual; we will then study the effect of this generating technique on the background. This will lead us to the construction of new solutions of ten-dimensional supergravity and, as advocated in [16], we will use these new backgrounds to define new field theories at strong coupling. All of our backgrounds will be smooth and minimal supersymmetry in four dimensions (four supercharges) will be preserved.

The system on which we will focus our study is the baryonic branch of the Klebanov–Strassler field theory [23–25]. This is perhaps, among the minimally SUSY examples known at the moment, the one that better passed test of the correspondence between geometry and (strongly coupled) field theoretical aspects. Besides, the baryonic branch field theory and geometry unifies the original Klebanov–Strassler system and the system of five branes wrapping a two cycle inside the resolved conifold [26]. Field theoretically, this unification can be thought as a Higgs-like mechanism and a particular limit where an accidental symmetry appears. See the papers in [27] for different geometric and physical aspects of this connection.

In this work, we will perform an $SU(2)$ non-Abelian T-duality on the baryonic branch geometry. A first result we will give is the complete specification, including the RR sector, of a new smooth background in massive Type IIA supergravity. Then, using the techniques of G-structures, we are able to show that this geometry is supersymmetric and in fact characterised by a dynamic $SU(2)$ structure. To explain this we recall that the IIB geometry corresponding to the baryonic branch is characterised by an $SU(3)$-structure, that is a couple of forms J_2, Ω_3 that also encode many aspects of the strongly coupled dual field theory. After the duality one finds that the geometry is characterised by forms j_2, w_1, v_1, ω_2 which together describe an $SU(2)$ structure. At a more technical level we find that whilst the large radius geometry has *static* $SU(2)$ structure (in which Killing spinors are perpendicular), once the IR effects are taken into account at small radius, the structure transitions to being *dynamical* and the Killing spinors become parallel. This is of interest since we provide an example of the most general type of supersymmetric flux backgrounds with dynamic $SU(2)$ structure of which relatively few other examples are known. Secondly, the phenomena of confinement and symmetry breaking are given a geometric description by the change in $SU(2)$-structure from static to dynamical.

We then connect this geometrical study to a number of properties of a field theory dual. We will show that the domain wall objects can be understood as D2 branes extended on $\mathbb{R}^{2,1}$ in this geometry. We show that a $U(1)_R$ symmetry is represented by a massless bulk gauge field in the non-Abelian T-dual of geometries dual to conformal field theories. This mode acquires a mass in the massive IIA geometry described in this paper and is interpreted as anomalous breaking of a $U(1)_R$ symmetry. This R-symmetry anomaly is also reproduced by considering Euclidean ‘instantonic’ branes. We additionally show the presence of a gravity mode corresponding to a spontaneously broken $U(1)_B$ symmetry inherited from the original baryonic branch. We suggest then a Euclidean E2 brane configuration extended along the radial direction and wrapping an S^2 may have the interpretation as a corresponding baryonic condensate.

The contents of this paper are organised as follows. In Section 2 we will briefly review the original background and field theory corresponding to the baryonic branch of the Klebanov–Strassler field theory (the *seed* background/field theory pair on which we will apply our generating technique). In Section 3 we will present explicitly the new solution. In Section 4, we demonstrate the supersymmetry of the background using the language of G-structures. In Section 5, we will discuss different aspects of the field theory dual to our new backgrounds. We close the paper with a list of possible future problems and conclusions.

A number of technical and useful appendixes complement our presentation.

2. Generalities on the baryonic branch

The Klebanov–Strassler field theory is a two-group quiver with bifundamental matter, charged under a global symmetry of the form $SU(2) \times SU(2) \times U(1)_R \times U(1)_B$. The ranks of the gauge groups are $(N, N+M)$ and the bifundamental matter A_1, A_2, B_1, B_2 self-interact via a superpotential of the form $\mathcal{W} \sim ABAB$. For a very clear explanation of many of the details of this quantum field theory, see [30,31]. One detail that will be crucial to our present work is the fact that the so-called 'duality cascade', a succession of Seiberg dualities, ends in a situation where the quantum field theory may choose to develop VEVs for the Baryon and anti-Baryon operators.

In the last step of the duality cascade the gauge group is $SU(M) \times SU(2M)$. This theory has mesons $\mathcal{M} = (A_a)_i^\alpha (B_b)_\beta^i$ and also baryonic operators [24]

$$\begin{aligned}\mathcal{B} = {} & \epsilon_{\alpha_1 \ldots \alpha_{2M}} (A_1)_1^{\alpha_1} (A_1)_2^{\alpha_2} \ldots (A_1)_{M-1}^{\alpha_{M-1}} (A_1)_M^{\alpha_M} \\ & \times (A_2)_1^{\alpha_{M+1}} (A_2)_2^{\alpha_{M+2}} \ldots (A_2)_{M-1}^{\alpha_{2M-1}} (A_2)_M^{\alpha_{2M}} \end{aligned} \tag{2.1}$$

and similar for $\tilde{\mathcal{B}}$ made out of $(B_i)_l^a$ fields. One can see that both baryons and anti-baryons are neutral under $SU(2) \times SU(2)$ transformations.

The moduli space consists of two branches – the *mesonic* and the *baryonic* [31]. On the mesonic branch the baryons are zero ($\mathcal{B} = \tilde{\mathcal{B}} = 0$) and the mesons satisfy $\det \mathcal{M} = \Lambda^{4M}$. The non-perturbative contribution to the superpotential means that the associated moduli space can be identified with a symmetric product of the deformed conifold. On the baryonic branch the mesons are zero ($\mathcal{M} = 0$) but the baryons acquire expectation values,

$$\mathcal{B} = i\xi \Lambda^{2M}, \qquad \tilde{\mathcal{B}} = \frac{i}{\xi} \Lambda^{2M}, \tag{2.2}$$

where Λ is the strong coupling scale of the group $SU(2M)$. Notice that both VEVs are equal only if $\xi = 1$. This corresponds to a $\mathbb{Z}_2$-symmetric point, represented by the exact solution in [23].

On this baryonic branch the $U(1)_B$ symmetry is spontaneously broken and the associated massless (pseudo-scalar) Goldstone mode corresponds to the phase of ξ. By supersymmetry this Goldstone lives in a chiral multiplet and comes along with scalar partner, the saxion, which corresponds to changing the modulus of ξ. As discussed in [31], the VEV of the operator,

$$\mathcal{U} = \mathrm{Tr}\big[A_i A_i^\dagger - B_j B_j^\dagger\big], \tag{2.3}$$

which contains the $U(1)_B$ current J_μ as its $\theta\sigma^\mu\bar{\theta}$ component, encodes the motion along the baryonic branch (the different values of ξ) according to

$$\langle \mathcal{U} \rangle \sim M\Lambda^2 \ln|\xi|. \tag{2.4}$$

Let us focus on the situation where the field theory chooses to move to the purely baryonic branch. In this case, there is a smooth solution of the equations of motion of Type IIB supergravity, that describes the strong dynamics of this field theory, including the spontaneous breaking of the $U(1)_B$ symmetry [24,25]. In the notation that we will adopt in this work, such background can be written compactly by introducing the (string frame) vielbein basis,

$$\begin{aligned} e^{x^i} &= e^{\frac{\Phi}{2}} \hat{h}^{-\frac{1}{4}} dx^i, \qquad & e^\rho &= e^{\frac{\Phi}{2}+k} \hat{h}^{\frac{1}{4}} d\rho, \\ e^\theta &= e^{\frac{\Phi}{2}+h} \hat{h}^{\frac{1}{4}} d\theta, \qquad & e^\varphi &= e^{\frac{\Phi}{2}+h} \hat{h}^{\frac{1}{4}} \sin\theta\, d\varphi, \end{aligned}$$

$$e^1 = \frac{1}{2} e^{\frac{\Phi}{2}+g} \hat{h}^{\frac{1}{4}} (\tilde{\omega}_1 + a\, d\theta), \qquad e^2 = \frac{1}{2} e^{\frac{\Phi}{2}+g} \hat{h}^{\frac{1}{4}} (\tilde{\omega}_2 - a \sin\theta\, d\varphi),$$
$$e^3 = \frac{1}{2} e^{\frac{\Phi}{2}+k} \hat{h}^{\frac{1}{4}} (\tilde{\omega}_3 + \cos\theta\, d\varphi), \tag{2.5}$$

where $\tilde{\omega}_i$ are the left-invariant forms of $SU(2)$. The metric $ds^2 = \sum_{i=1}^{10} (e^i)^2$ is supported by RR and NSNS fields:

$$B_2 = \kappa \frac{e^{\Phi}}{\hat{h}^{1/2}} \left[e^{\rho 3} - \cos\alpha \left(e^{\theta\varphi} + e^{12} \right) - \sin\alpha \left(e^{\theta 2} + e^{\varphi 1} \right) \right],$$
$$F_3 = \frac{e^{-\frac{3}{2}\Phi}}{\hat{h}^{3/4}} \left[f_1 e^{123} + f_2 e^{\theta\varphi 3} + f_3 \left(e^{\theta 23} + e^{\varphi 13} \right) + f_4 \left(e^{\rho 1\theta} + e^{\rho\varphi 2} \right) \right],$$
$$F_5 = \kappa e^{-\frac{5}{2}\Phi - k} \hat{h}^{\frac{3}{4}} \partial_\rho \left(\frac{e^{2\Phi}}{\hat{h}} \right) \left[e^{\theta\varphi 123} - e^{x^0 x^1 x^2 x^3 \rho} \right]. \tag{2.6}$$

We have defined

$$\cos\alpha = \frac{\cosh(2\rho) - a}{\sinh(2\rho)}, \qquad \sin\alpha = -\frac{2e^{h-g}}{\sinh(2\rho)}, \qquad \hat{h} = 1 - \kappa^2 e^{2\Phi}, \tag{2.7}$$

where κ is a constant that we will choose to be $\kappa = e^{-\Phi(\infty)}$. The functions are,

$$f_1 = -2N_c e^{-k-2g}, \qquad f_2 = \frac{N_c}{2} e^{-k-2h} \left(a^2 - 2ab + 1 \right),$$
$$f_3 = N_c e^{-k-h-g} (a - b), \qquad f_4 = \frac{N_c}{2} e^{-k-h-g} b'. \tag{2.8}$$

The system has a radial coordinate ρ, on which all functions depend, and we have set $\alpha' g_s = 1$. The functions (a, b, Φ, g, h, k) obey a system of BPS equations which can be arranged in a convenient form that decouples the equations (as explained in [33,34]). As a result, it can be shown that the whole dynamics of the string background is controlled by a single function $P(\rho)$, subject to a second order non-linear and ordinary differential equation. This function $P(\rho)$ can be determined numerically and has IR and UV behaviours

$$\text{UV:} \quad P = e^{4\rho/3} [c_+ + \ldots], \quad \rho \to \infty,$$
$$\text{IR:} \quad P = h_1 \rho + \mathcal{O}\left(\rho^3\right), \quad \rho \to 0. \tag{2.9}$$

There is only one independent parameter, $c_+ > 0$ (the constant h_1 is determined by c_+) and it is this parameter that can be identified with the baryonic expectation value

$$\mathcal{U} \sim \frac{1}{c_+}. \tag{2.10}$$

It is convenient to define a dimensionless quantity $\lambda = 2^{2/3} c_+ \epsilon^{-4/3}$ where ϵ may be identified with the conifold deformation. See the paper [29] for a good account of the logic and technical details.

2.1. $SU(3)$ structure of the baryonic branch

The supergravity background above is characterised by what is called an $SU(3)$ structure. That is, there exists a couple of forms $\hat{J}_2$ and $\hat{\Omega}_3$, in terms of which the BPS equations, the fluxes and various other quantities characterising the space can be written.

The observation of [28], it that the forms $\hat{J}$, $\hat{\Omega}$, describing the full baryonic branch can be obtained from the simpler ones describing a set of D5 branes wrapping the two cycle of the resolved conifold. We will not repeat the details of the derivation here, but we quote the results to the extent that we will find useful.

In general, an $SU(3)$ structure solution can be described by the following pure spinors in Type IIB [41],

$$\Psi_+ = -e^{i\zeta(r)}\frac{e^A}{8}e^{-i\hat{J}}, \qquad \Psi_- = -i\frac{e^A}{8}\hat{\Omega}_{hol}, \tag{2.11}$$

where e^{2A} is the warp factor of the metric. Let us define

$$e^{i\zeta(r)} = \mathcal{C} + i\mathcal{S}, \tag{2.12}$$

where $\mathcal{C}^2 + \mathcal{S}^2 = 1$. It is possible to show that for zero axion field, that is $F_1 = 0$, SUSY requires the following equalities to hold (these are the BPS equations previously mentioned)

$$\begin{aligned} &d\big(e^{-\Phi}\mathcal{S}\big) = 0, \qquad d\big(e^{2A-\Phi}\mathcal{C}\big) = 0, \\ &d\big(e^{3A-\Phi}\hat{\Omega}_{hol}\big) = 0, \qquad d\big(e^{4A-2\Phi}\hat{J}\wedge\hat{J}\big) = 0. \end{aligned} \tag{2.13}$$

The fluxes are determined as

$$B_2 = \frac{\mathcal{S}}{\mathcal{C}}\hat{J}, \qquad \frac{1}{\mathcal{C}^2}d\big(e^{2A}\hat{J}\big) = e^{4A}\star_6 F_3, \qquad d\big(e^{4A-\Phi}\mathcal{S}\big) = -e^{4A}\star_6 F_5. \tag{2.14}$$

The system of N_c D5 branes wrapped on the resolved conifold is supported by just F_3 flux and is a solution to these equations when $\mathcal{S} = 0$. The (string-frame) frame fields that describe this geometry can be obtained from those of Eq. (2.5) by setting $\hat{h} = 1$. In terms of these, the J_2, Ω_3 (denoted without hats to distinguish them from those of the baryonic branch) are given by

$$\begin{aligned} &J = e^{r3} + \big(\cos\alpha e^{\varphi} + \sin\alpha e^2\big)\wedge e^{\theta} + \big(\cos\alpha e^2 - \sin\alpha e^{\varphi}\big)\wedge e^1, \\ &\Omega_{hol} = \big(e^r + ie^3\big)\wedge\big(\big(\cos\alpha e^{\varphi} + \sin\alpha e^2\big) + ie^{\theta}\big)\wedge\big(\big(-\sin\alpha e^{\varphi} + \cos\alpha e^2\big) + ie^1\big), \end{aligned} \tag{2.15}$$

which obey the relations $J\wedge\Omega_{hol} = 0$, $\quad J\wedge J\wedge J = \frac{3i}{4}\Omega_{hol}\wedge\bar{\Omega}_{hol}$. The BPS equations for the functions h, g, k, a, b, Φ and the RR three-form flux, are

$$\begin{aligned} &d(J\wedge J) = 0, \qquad d\big(e^{\Phi/2}\Omega_{hol}\big) = 0 \\ &d\big(e^{\Phi}J\big) + e^{2\Phi}\star_6 F_3 = 0. \end{aligned} \tag{2.16}$$

Then the results of [28] show that the $\hat{J}$, $\hat{\Omega}$ of the full baryonic branch solution are obtained by introducing a non-zero phase or rotation parameter[1] $\zeta(r)$ in to (2.11) and defining:

$$\hat{J} = \mathcal{C}J, \qquad \hat{\Omega}_{hol} = \mathcal{C}^{3/2}\Omega_{hol}, \qquad e^{2A} = \frac{e^{\Phi}}{\sqrt{\mathcal{C}}}, \qquad \mathcal{S} = e^{\Phi-\Phi_\infty}, \tag{2.17}$$

where e^{2A} is the warp factor of the baryonic branch solution. For further details on the geometry and physics implied by this 'scaling of forms', we refer the reader to the original papers [28] and [27].

[1] This parameter can also be understood in terms of the boost parameter that enters in the duality chain that relates the wrapped brane geometries to the baryonic branch [27].

2.2. A useful gauge transformation

Let us comment on a small subtlety that will be important in what follows. The above rotation argument makes it quite clear that by sending $\zeta \to 0$, the geometry becomes that of the wrapped D5 branes. On the other hand taking $\zeta \to \frac{\pi}{2}$ accompanied with $\lambda \to 0$, the geometry becomes that given by Klebanov and Strassler, i.e. the $\mathbb{Z}_2$ point of the baryonic branch. Taking this limit is slightly delicate. One finds that $\sin\zeta \to 1$ and $\cos\zeta \to \frac{1}{\lambda}h_{KS}$ where h_{KS} is the Klebanov–Strassler warp factor. Expanding the functions (a, b, Φ, g, h, k) in the large λ limit and rescaling Minkowski coordinates $x_i \to x_i\lambda^{-1}$ one finds that leading term of the metric is independent of λ and reproduces the KS geometry. The limit applied on the NS two form is less trivial, in fact its expansion in inverse powers of λ is

$$B_2 = \lambda \frac{\epsilon^2 \sinh(2\rho)}{2\sqrt{3}\kappa P_1\sqrt{P_1'}} d\big(P_1(\tilde{\omega}_3 + \cos\theta d\varphi)\big) - B_{KS} + \mathcal{O}\big(\lambda^{-1}\big). \tag{2.18}$$

However the form of P_1 (the leading contribution of $P(\rho)$ in this expansion) ensures that the pre-factor on the first term in this expression reduces to a constant and one recovers the Klebanov–Strassler NS two form *modulo a pure gauge term*.

In fact it is going to suit our purposes to perform a similar gauge transformation across the whole baryonic branch (2.6). We do this by defining

$$B_2 \to B_2 + d\big(\mathcal{Z}(\rho)(\tilde{\omega}_3 + \cos\theta d\varphi)\big), \qquad \mathcal{Z} = -\frac{1}{2}\int_0^\rho e^{2k(\rho')+\Phi(\rho')}\mathcal{S}\big(\rho'\big)d\rho' \tag{2.19}$$

In the KS limit this reduces to exactly the gauge transformation required in (2.18) and it has the effect of removing certain mixing between the angular directions and the radial direction in the NS two-form.[2] This will greatly simplify matters upon performing a duality transformation.

3. Non-Abelian duality on the baryonic branch

In this section, we will present the result for the non-Abelian T-duality when applied to one of the $SU(2)$ isometries of the baryonic branch background in Eqs. (2.5)–(2.6). This study was initiated in [16] but here we make two essential new contributions that will allow our subsequent analysis. In [16] the NS sector was established however the geometry there displayed at first sight a mixing between angular and radial directions thereby creating difficulties for any field theory interpretation. Here we have established that this is a gauge artifact (details are presented in Appendix C). By making the gauge transformation introduced above in Eq. (2.19) to the seed geometry, as we do here, we remove this mixing and restore the expected asymptotic behaviour of the geometry.[3] Moreover we complete the specification of the geometry by providing the RR sector. Whilst this may seem laborious, it is essential in order to show the supersymmetry of the

[2] This transformation leaves unchanged the gauge coupling defined through the integral of B_2 however it is non-vanishing at infinity and so one should exercise appropriate caution.

[3] Alternatively one can perform the following coordinate transformation to the solution presented in [16] to obtain the solution presented here:

$$v_3^{there} \to v_3^{here} + \sqrt{2}\mathcal{Z}. \tag{3.1}$$

background and, in the absence of a *comprehensive* proof that non-Abelian T-duality is a solution generating map, required in order to verify that the equations of motion are satisfied. Moreover, much of the analysis of field theory properties that follows in Section 5 requires knowledge of the RR sector.

Our strategy is to perform a non-Abelian T-dualisation on an $SU(2)$ isometry that exists in the baryonic branch geometry. Here, in the interest of concision, we do not recapitulate the entire technology of non-Abelian T-duality and refer the reader who wishes to learn the details of the technique to the earlier work and in particular to Section 2 of [16]. Instead we present the results with some additional technical detail relegated to Appendix C.

We will perform the transformation described in [16] to the coordinates $(\tilde{\theta}, \tilde{\varphi}, \psi)$, present in the left-invariant forms of $SU(2)$, $\tilde{\omega}_i$, $i = 1, 2, 3$ of Eq. (2.5). Typically in the T-dual one finds that the Lagrange multipliers v_i introduced in the Buscher procedure play the role of T-dual coordinates. However there is some choice in how the T-dual geometry is parametrised since a gauge fixing must be invoked. Here, for reasons that will become apparent in Section 5, we will choose a gauge where the new coordinates after the duality will be (v_2, v_3, ψ).

We will start by specifying the vielbeins. The components

$$e^{x^i} = e^{\frac{\Phi}{2}} \hat{h}^{-\frac{1}{4}} dx^i, \qquad e^{\rho} = e^{\frac{\Phi}{2}+k} \hat{h}^{\frac{1}{4}} d\rho \tag{3.2}$$

do not change. The vielbeins in the (θ, φ) directions are also unchanged by the duality however we find it useful to introduce a rotation in $(e^{\theta}, e^{\varphi})$ such that the dual solution has no explicit ψ dependence.

$$e^{\hat{\theta}} = \sqrt{\mathcal{C}} e^{h+\Phi/2} \omega_1, \qquad e^{\hat{\varphi}} = \sqrt{\mathcal{C}} e^{h+\Phi/2} \omega_2, \tag{3.3}$$

where we have introduced left-invariant $SU(2)$ forms for the angles $\{\theta, \phi, \psi\}$. The vielbeins in the directions $\hat{1}, \hat{2}, \hat{3}$ and NS 2-form potential can be compactly written in terms of the quantities defined as,

$$\begin{aligned}
&\mathcal{H} = \frac{2\sqrt{2} v_3 + 4\mathcal{Z} + e^{2g+\Phi} \mathcal{S} \cos\alpha}{2\sqrt{2}}, \qquad \mathcal{Z} = -\frac{1}{2} \int_0^{\rho} \mathcal{S} e^{\Phi+2k} d\rho', \\
&\mu_1 = a e^{g} \cos\alpha + 2 e^{h} \sin\alpha.
\end{aligned} \tag{3.4}$$

The function $\mathcal{Z}$ was introduced as a gauge transformation to the seed solution already in (2.19). With these, we have

$$\begin{aligned}
e^{\hat{1}} = \frac{e^{g+\Phi/2}}{8\mathcal{W}} \sqrt{\mathcal{C}} \Big[& 4 e^{2k+\Phi} \mathcal{C} \mathcal{H} (a \mathcal{H} \omega_1 - v_2 \omega_3) - \sqrt{2} e^{2(g+k+\Phi)} \mathcal{C}^2 (d v_2 + a \mathcal{H} \omega_2) \\
& - 8\sqrt{2} v_2 (v_2 d v_2 + \mathcal{H} d v_3) \\
& + \frac{1}{2} \mu_1 \mathcal{S} e^{g+\Phi} \big(8 v_2^2 \omega_2 + e^{2k+\Phi} \mathcal{C} \big(e^{2g+\Phi} \mathcal{C} \omega_2 - 2\sqrt{2} \mathcal{H} \omega_1 \big) \big) \Big], \\
e^{\hat{2}} = \frac{e^{g+3\Phi/2+g}}{8\mathcal{W}} \mathcal{C}^{3/2} \Big[& 4 e^{2g} v_2 (d v_3 - a v_2 \omega_2) - 4 \mathcal{H} e^{2k} (d v_2 + a \mathcal{H} \omega_2) \\
& - \sqrt{2} \mathcal{C} e^{2k+2g+\Phi} (a \mathcal{H} \omega_1 - v_2 \omega_3) + \frac{1}{2} \mu_1 \mathcal{S} e^{g+2k+\Phi} \big(e^{2g+\Phi} \mathcal{C} \omega_1 + 2\sqrt{2} \mathcal{H} \omega_2 \big) \Big],
\end{aligned}$$

$$e^{\hat{3}}=\frac{e^{k+\Phi/2}}{8\mathcal{W}}\sqrt{\mathcal{C}}\big[4\mathcal{C}v_2e^{4g+\Phi}(v_2\omega_3-a\mathcal{H}\omega_1)-\sqrt{2}\mathcal{C}^2(dv_3-v_2a\omega_2)$$
$$-8\sqrt{2}\mathcal{H}(v_2dv_2+\mathcal{H}dv_3)+e^{g+\Phi}\mu_1v_2\mathcal{S}\big(\sqrt{2}\mathcal{C}e^{2g+\Phi}\omega_1+4\mathcal{H}\omega_2\big)\big]. \tag{3.5}$$

We will then have a metric that in terms of these vielbeins reads, $ds^2_{st}=\sum_{i=1}^{10}(e^i)^2$.

In terms of these vielbeins, the NS two-form B_2 reads,

$$\widehat{B}_2=-\frac{1}{4v_2}\big(2e^{-h}a\big(e^g v_2 e^{\hat{\theta}\hat{1}}+e^k\mathcal{H}e^{\hat{\theta}\hat{3}}\big)-4e^{k-g}\mathcal{H}e^{\hat{1}\hat{3}}+\sqrt{2}\mathcal{C}e^{g+k+\Phi}e^{\hat{2}\hat{3}}\big)$$
$$+\frac{\mathcal{S}}{\mathcal{C}}\bigg[\frac{\mathcal{H}e^k}{2v_2}\big(2e^{-g}e^{\hat{1}\hat{3}}-ae^{-h}e^{\hat{\theta}\hat{3}}\big)+\frac{e^{g+k+\Phi-h}}{4\sqrt{2}v_2}\mathcal{C}\big(\mu_1e^{\hat{\theta}\hat{3}}-2e^he^{\hat{2}\hat{3}}\big)$$
$$-\frac{e^{-h}}{2}\bigg(2e^{-h-\Phi}\frac{\mathcal{Z}}{\mathcal{S}}+2e^h\cos\alpha-ae^g\sin\alpha\bigg)e^{\hat{\theta}\hat{\varphi}}-\frac{e^{-h}}{2}\big(ae^ge^{\hat{\theta}\hat{1}}+\mu_1e^{\hat{\theta}\hat{2}}\big)\bigg]. \tag{3.6}$$

The dual dilaton is given by

$$\widehat{\Phi}=\Phi-\frac{1}{2}\ln\mathcal{W},\qquad \mathcal{W}=\frac{\mathcal{C}}{8}\big(e^{4g+2k+3\Phi}\mathcal{C}^2+8e^{2g+\Phi}v_2^2+8e^{2k+\Phi}\mathcal{H}^2\big). \tag{3.7}$$

And the RR sector is given by,[4]

$$F_0=\frac{N_c}{\sqrt{2}},$$
$$F_2=-\frac{e^{-\Phi}}{4\mathcal{C}}N_c\big[2e^{-2h}\big(1+a^2-2ab\big)\mathcal{H}e^{\hat{\theta}\hat{\varphi}}+e^{-g-h-k}\mathcal{C}(a-b)\big(\sqrt{2}e^{2g+k+\Phi}\big(e^{\hat{\theta}\hat{1}}-e^{\hat{\varphi}\hat{2}}\big)$$
$$+4e^k\mathcal{H}\big(e^{\hat{\theta}\hat{2}}-e^{\hat{\varphi}\hat{1}}\big)-4v_2e^ge^{\hat{\varphi}\hat{3}}\big)-8e^{-2g}\mathcal{H}e^{\hat{1}\hat{2}}-8e^{-g-k}v_2e^{\hat{2}\hat{3}}-2e^{-h-k}v_2e^{r\hat{\theta}}\big]$$
$$-\frac{\mathcal{S}e^{g-h}}{\sqrt{2}\mathcal{C}\sin\alpha}\big(N_cb+a\big(e^{2g}\cos^2\alpha-N_c\big)+e^{g+h}\sin 2\alpha\big)e^{\hat{\theta}\hat{\varphi}},$$
$$F_4=\frac{e^{-g-h-k-\Phi}}{8\mathcal{C}}N_c\big[\mathcal{C}\big(1+a^2-2ab\big)e^{\hat{\theta}\hat{\varphi}}\wedge\big(\sqrt{w}e^{2g+k+\Phi-h}e^{\hat{1}\hat{2}}+4e^{2g-h}e^{\hat{1}\hat{3}}\big)\mathcal{C}b'e^{r\hat{\theta}}$$
$$\wedge\big(4e^k\mathcal{H}e^{\hat{1}\hat{3}}-\sqrt{2}e^{2g+k+\Phi}e^{\hat{2}\hat{3}}\big)-8e^gv_2(a-b)e^{\hat{\theta}\hat{1}\hat{2}\hat{3}}e^{r\hat{\varphi}}$$
$$\wedge\big(4e^gv_2e^{\hat{1}\hat{2}}-b'e^k\big(\sqrt{2}e^{2g+\Phi}e^{\hat{1}\hat{3}}+4\mathcal{H}e^{\hat{2}\hat{3}}\big)\big)\big]$$
$$-\frac{2\mathcal{S}e^{-g-h-k-\Phi}}{\mathcal{C}^2\sin\alpha}\big(a\big(e^{2g}\cos^2\alpha-N_c\big)+\big(N_cb+e^{g+h}\sin 2\alpha\big)\big)$$
$$\times\big(\mathcal{H}e^ke^{\hat{\theta}\hat{\varphi}\hat{1}\hat{2}}+v_2e^ge^{\hat{\theta}\hat{\varphi}\hat{2}\hat{3}}\big). \tag{3.8}$$

3.1. Asymptotic behaviour

Using the semi analytic UV expansions that can be found, for example, in [29] it is possible to calculate the UV behaviour of the dual metric.

[4] *Warning on potentially confusing nomenclature*: The N_c appearing here originated as the number of D5 branes wrapping the resolved conifold which was then rotated to give the baryonic branch and then T-dualised to this solution. Prior to T-duality, N_c corresponds to the D5 charge which is also commonly denoted by M (which we will also use in Section 5 when we specialised to the Klebanov–Tseytlin geometry). We hope the reader will not get overly confused by this point.

The dual vielbeins at leading order in the UV are given by

$$e^{\hat{1}} = -\frac{c_+ e^{-2\rho/3}(24\rho-3)^{1/4}}{2^{3/4}\sqrt{N_c}(1-2\rho)}\omega_1, \qquad e^{\hat{2}} = \frac{c_+ e^{-2\rho/3}(24\rho-3)^{1/4}}{2^{3/4}\sqrt{N_c}(1-2\rho)}\omega_2,$$
$$e^{\hat{3}} = -\frac{2^{3/4}3^{1/4}}{\sqrt{N_c}(8\rho-1)^{1/4}}dv_3. \tag{3.9}$$

Thus the dual 3-manifold shrinks as one flows towards the UV, in line with our expectations from Abelian T-duality, where big circles are mapped to small circles.

One may worry that this vanishing manifold is a signal of a singularity in the UV, however, an explicit check shows that the curvature invariants: Ricci scalar, $R_{\mu\nu}R^{\mu\nu}$ and $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ are finite. In other words, both the g_s and the α' expansions are under control and the background is trustable in the far UV. Notice that there is a one-cycle, labelled by the coordinate ψ in ω_3, that shrinks to zero size in the large-ρ regime. This implies that strings wrapping this cycle will become light and will enter the spectrum of the dual QFT at high energies.

The dual dilaton is defined as $e^{2\hat{\Phi}} = \frac{e^{2\Phi}}{\mathcal{W}}$ where

$$\mathcal{W} = 3c_+ N_c\sqrt{12\rho - \frac{3}{2}}e^{8\rho/3} \tag{3.10}$$

asymptotically, and so the dilaton is UV vanishing.

Let us now study the small radius regime of the metric, corresponding with the low energy regime of the dual QFT. Things are a bit less-simple; at leading order, terms in the metric depend explicitly of the original IR-parameters of the baryonic branch solution, but they also depend on the values of the v_2, v_3 coordinates. Explicit expressions for the dual vielbeins in the IR are included in Appendix C.4.

Here again, it happens that the dilaton is bounded and the Ricci scalar and Ricci and Riemann tensors squared are finite. This was expected, as we are performing a duality transformation on a space that in the small-ρ regime was of finite size (the S^3 in the deformed conifold). Dualities typically invert ‘sizes’ (or couplings). This example is not an exception. One may start with a background solution where supergravity is a good approximation and obtain that in the far IR the new generated solution is still a trustable supergravity background.

A point that we want to emphasise again is that in the far IR, the parameter that was labelling the different ‘positions’ on the baryonic branch (that is the different baryonic VEVs) still appears in the small-radius expansion above. There is a still a one-parameter family of solutions. Indeed, notice the dependence on the integration constants $e^{\Phi(0)}$ and h_1 as defined in [28], both related to the number parametrising the baryonic branch.

4. $SU(2)$ Structure of the background

In this section we establish that the background obtained above is indeed supersymmetric and we give the associated G-structure. Again, we will postpone details to Appendix C. The geometry supports two pure spinors given by

$$\Phi_+ = \frac{e^A}{8}e^{i\theta_+}e^{-iv\wedge w}\big(k_\parallel e^{-ij} - ik_\perp\omega\big),$$
$$\Phi_- = \frac{ie^A}{8}e^{i\theta_-}(v+iw)\wedge\big(k_\perp e^{-ij} + ik_\parallel\omega\big). \tag{4.1}$$

In the case at hand we find

$$
\begin{aligned}
& e^{2A}=\frac{e^{\Phi}}{\mathcal{C}} \\
& \theta_{+}=0, \qquad \theta_{-}=\zeta(r) \\
& k_{\parallel}=\frac{\sin\alpha}{\sqrt{1+\zeta\cdot\zeta}} \qquad k_{\perp}=\sqrt{\frac{\cos^2\alpha+\zeta\cdot\zeta}{1+\zeta\cdot\zeta}} \\
& z=w-iv=\frac{1}{\sqrt{\cos^2\alpha+\zeta\cdot\zeta}}\big(\sqrt{\Delta}\tilde{e}^3+\zeta_2\sin\alpha\,\tilde{e}^{\theta}+i\big(\sqrt{\Delta}\tilde{e}^{\rho}+\zeta_2\sin\alpha\,\tilde{e}^{\varphi}\big)\big) \\
& j=\tilde{e}^{\rho 3}+\tilde{e}^{\varphi\theta}+\tilde{e}^{21}-v\wedge w \\
& \omega=\frac{i}{\sqrt{\cos^2\alpha+\zeta\cdot\zeta}}\big(\sqrt{\Delta}\big(\tilde{e}^{\varphi}+i\tilde{e}^{\theta}\big)-\zeta_2\sin\alpha\big(\tilde{e}^{\rho}+i\tilde{e}^{3}\big)\big)\wedge\big(\tilde{e}^2+i\tilde{e}^1\big).
\end{aligned}
\tag{4.2}
$$

Here the frames $\tilde{e}$ are obtained by a rotation, given by (B.19), of those in (3.5) and the parameters Δ, ζ_i which enter into this rotation are specified by (C.15).

There are various immediate things to observe. If we move to the large radius region of the geometry, the functions $\sin\alpha(\rho)\sim a(\rho)\sim b(\rho)\to 0$. The formulas simplify and we obtain, among other things that $k_{\parallel}\to 0$. This implies that, as happens in the paper [18], the two pure spinors are 'perpendicular' in the large radius regime of the solution and the $SU(2)$-structure is *static*. Similar behaviour was found in [20], where a dynamical $SU(3)$- structure in 7d becomes orthogonal in the UV. This changes as we evolve to the small radius regime of the background, the $SU(2)$-structure is said to become *dynamical*. In Section 5, we will discuss the physical effects that are associated with a change in the $SU(2)$-structure, from *static* in the far UV to *dynamic* in the IR.

5. Correspondence with field theory

In this section, we will connect our previous geometrical studies with aspects of the quantum field theory that our background is dual to. As it was anticipated in the paper [16], we believe that the field theory dual to our massive IIA background should be a non-conformal version of the Sicilian gauge theories presented in [35,36] or the linear quiver field theories studied in [37]. There are certain things that can be inferred immediately, like for example the confining character of the QFT. This follows from the fact that the calculation of the Wilson loop will proceed exactly as in the case of the baryonic branch field theory. Indeed, the $R^{1,3}\times\rho$ part of the geometry is unchanged, hence, the Wilson loop will give the same result as before the non-Abelian T-duality. Nevertheless, many calculations done with the Klebanov–Strassler/baryonic branch background involved the 'internal' five dimensional space. The purpose of this section will be to learn how some of those calculations for field theory observables change (or not) for the new geometries in massive IIA.

The idea that will guide us is that for a given correlation function or related QFT observable, that in the original background was calculated in a way that is 'independent' of the $SU(2)$ isometry used to perform the non-Abelian duality, will give the same result in the transformed background. We can think about those operators or correlators as 'uncharged' under the $SU(2)$ symmetry in question. Ideas of this sort already worked in other solution generating techniques, like T–s–T dualities. Similar ideas also appeared in large N_c (planar) equivalences between parent-daughter theories. The physics of the common or 'uncharged' sector goes through to the new field theory. The rest of the paper deals with observables that are, in principle 'charged' under the $SU(2)$ symmetry.

We will first examine the relation between the dynamical character of the $SU(2)$-structure and the field theoretical phenomena of confinement and discrete R-symmetry breaking. We will show how the presence of domain walls with an induced Chern–Simons dynamics on their world-volume follows as a consequence of the confinement and the dynamical character of the $SU(2)$-structure.

Then, we will make clear that the symmetry associated with changes in the ψ-direction is related with an anomalous $U(1)_R$ R-symmetry in the field theory. We will define an instantonic object using an Euclidean D0 brane; this will lead us to a possible definition for a Θ-angle and gauge coupling. We will find that this coupling has a non-conventional running in the far UV.

We will then move into studying different aspects of the 'baryonic branch', also present in our new backgrounds. We will find that a given fluctuation of the RR background fields can be put in correspondence with a global continuous symmetry that the IR dynamics breaks spontaneously. We will find the associated Goldstone boson and an expression for the conformal dimension of such a baryonic operator.

5.1. Dynamic $SU(2)$ and confinement

In this section, we will make more concrete the relation between the QFT phenomena of confinement and the dynamical character of the $SU(2)$-structure. The first observation is that the 'parallel projection' between both spinors, represented by $k_\parallel$ in Eq. (C.18), is proportional to the quantity $\sin\alpha$. This quantity is related to the background functions as can be read from Appendix B of the paper [32],

$$\sin\alpha(\rho) = \frac{4ae^{h-g}}{\sqrt{a^2 + 2a^2(4e^{2h-2g}) + (4e^{2h-2g}+1)^2}}. \tag{5.1}$$

This is compatible with the expression in Eq. (2.7) after following the algebra in Appendix B of the paper [32].

The presence of the functions $a(\rho), b(\rho)$ in the baryonic branch solution – see Eqs. (2.5)–(2.8) – are responsible for the de-singularisation of the space (the appearance of a finite size S^3) and the IR minimisation of the dilaton and warp factor. These have as a consequence the linear law, $E_{QQ} = \sigma L_{QQ}$ for large distance separations between the quark–antiquark pair. In other words, the functions $a(\rho), b(\rho)$ and their effects on the warp factor and dilaton 'produce' confinement. In the same vein, at the level of the metric, the presence of $a(\rho)$ implies the breaking of the symmetry $\psi \to \psi + \epsilon$ into $\psi \to \psi + 2\pi$. This is the remaining $\mathbb{Z}_2$ symmetry after the spontaneous discrete R-symmetry breaking. So, we see clearly that confinement and spontaneous R-symmetry breaking go hand-in-hand with the function $a(\rho)$. Hence, these phenomena in the dual QFT are closely related to the presence of $k_\parallel$, which as we made clear is related to the dynamical character of the $SU(2)$-structure. In the papers [44,45], the point was made that the functions $a(\rho), b(\rho)$ were directly related with the gaugino condensate. This suggests that in our massive IIA picture, there exists a relation of the form $\langle\lambda\lambda\rangle \sim k_\parallel$. Similar ideas will be discussed in the paper [54].

5.2. A comment on domain walls

It was proposed in [16], that domain wall objects were realised in the non-Abelian T-dual of the geometries we are considering, as D2 branes that extend on $R^{1,2}$. Indeed, the induced metric, action and tension of a $(2+1)$-dimensional object are,

$$ds^2_{ind} = e^{\Phi}\hat{h}^{-1/2}\left(-dt^2 + dx_1^2 + dx_2^2\right),$$
$$S_{\mathrm{BI}} = -T_{\mathrm{D2}}\int d^3x e^{\Phi/2}\hat{h}^{-3/4}, \qquad T_{\mathrm{DW}} = T_{\mathrm{D2}}e^{\Phi/2}\hat{h}^{-3/4}\big|_{\rho=0}.$$

If we also turn on a gauge field in the world-volume of this D2 brane, a Chern–Simons–Maxwell action will be induced, at leading order in α' on this D-brane,

$$S_{\mathrm{BIWZ}} = -T_{\mathrm{D2}}\int d^{2+1}x e^{\Phi/2}\hat{h}^{-3/4}\sqrt{1-\alpha' F_{\mu\nu}F^{\mu\nu}} + T_{\mathrm{D2}}\int d^{2+1}x F_0 A_1 \wedge F_2. \tag{5.2}$$

We have used that a new WZ-like term appears in massive IIA as explained in [46]. The Chern–Simons term is quantised, being proportional to $T_{\mathrm{D2}}N_c$.[5]

In the Type IIB baryonic branch solution(s), domain walls were realised by D5-branes extended on $R^{1,2}$ and the three-sphere $\tilde{S}^3 = [\tilde{\theta}, \tilde{\varphi}, \psi]$. Once a gauge field is turned on, a Chern–Simons terms was induced, proportional to $T_{\mathrm{D5}}\int_{\tilde{S}^3} F_3$. Naively, we can think that both objects are 'connected' by the non-Abelian T-duality, under which the directions on $\tilde{S}^3$ disappear and we are left with a D2 brane as described above.

Supersymmetry gives support to this. Indeed, around Eq. (6.19) of the paper [41], we are presented with the calibration form for a domain-wall like object, which is given by the real part of the pure spinor Ψ_+. Using that $|a|^2 = e^A = e^{\Phi/2}\hat{h}^{-1/4}$, we obtain that the BI action equals the calibration form. Notice also that this selects the $k_\parallel$ component of the pure spinor.

As it was shown in the paper [16], once the R-symmetry is broken in the Type IIB set-up, the non-Abelian T-duality maps these backgrounds to their partners in massive IIA. In a minimally SUSY quantum field theory, the presence of domain-walls is tied up with confinement and the spontaneous breaking of the $\mathbb{Z}_{2N_c}$-symmetry. As we emphasised, these phenomena are related to the 'dynamical' character of the $SU(2)$-structure, hence to the presence of the $k_\parallel$ part of the pure spinor.

5.3. The fate of the $U(1)_R$ anomaly

In the backgrounds presented in [16] and those of this paper it is somewhat natural to expect that the coordinate ψ is singled out as being related to an R-symmetry of any putative field theory dual. That this is true is by no means obvious, after all in the technical process of dualisation the fact that we retained the coordinate ψ was purely a result of a judicious gauge choice. Here we provide evidence that this is indeed the correct identification and furthermore that this $U(1)$ is afflicted with an anomaly, breaking it down to a discrete subgroup.

A robust understanding of how ∂_ψ plays the role of the R-symmetry in the holographic dual was given in [42] with several important details of the supergravity solution clarified in [43]. The essential point of [42] is to introduce a bulk 5d gauge field that gauges this $U(1)_\psi$ by making the replacement $d\psi \to \chi = d\psi - 2A$ in the metric. This must be supplemented with an appropriate ansatz for the fluxes. In the case of the Klebanov–Witten background one finds that the resultant gauge field is massless and is the dual fluctuation to the global $U(1)_R$ of the gauge theory. However, in the non-conformal cases, the correct ansatz for the fluxes actually yields a

[5] Note that it is the presence of an F_0 that allows D2 branes to be interpreted in this way, by way of comparison in [47] the relevant branes with Chern–Simons dynamics are D4 branes with a bulk F_2 turned on.

massive gauge field (the mass here comes from a Stückelberg rather than Brout–Englert–Higgs mechanism).

Let us begin our discussion with the non-Abelian T-dual of the Klebanov–Witten background. The NS sector of the geometry is given by

$$\begin{aligned} ds^2 &= ds^2_{\mathrm{AdS}_5} + \frac{1}{6}ds^2_{S^2} + \frac{6v_2^2}{\Delta}\sigma_{\hat{3}}^2 \\ &\quad + \frac{6}{\Delta}\left[\left(1+27v_2^2\right)dv_2^2 + 54v_2v_3dv_2dv_3 + \frac{3}{4}\left(\Delta - 54v_2^2\right)dv_3^2\right], \\ B_2 &= \frac{18\sqrt{2}}{\Delta}v_2v_3\sigma_{\hat{3}}\wedge dv_2 + \frac{(\Delta-54v_2^2)}{\sqrt{2}\Delta}\sigma_{\hat{3}}\wedge dv_3, \\ e^{2\Phi} &= 81\Delta^{-1} = 81\left(2+54v_2^2+36v_3^2\right)^{-1}, \end{aligned} \tag{5.3}$$

where $\sigma_{\hat{3}} = d\psi + \cos\theta d\phi$. This metric is supported by RR two and four form fluxes. The $U(1)$ acting as ∂_ψ can be gauged by making the replacement $\sigma_{\hat{3}} \to \tilde{\chi} = \sigma_{\hat{3}} - 2A$ in the NS sector above. The potentials corresponding to the correct modification of the RR forms that support this fluctuation are given by

$$\begin{aligned} C_1 &= -\frac{2\sqrt{2}}{27}(\cos\theta d\phi + A), \\ C_3 &= -\frac{2}{27}v_3\tilde{\chi}\wedge(\tilde{\omega}_2 - dA) + \frac{2}{9}v_3 \star_5 dA, \end{aligned} \tag{5.4}$$

where we introduce the volume form on the S^2, $\tilde{\omega}_2 = \sin\theta d\theta\wedge d\phi$ and $\star_5$ is the Hodge dual in the AdS_5 directions. This solves the linearised equations of motions, linearised Einstein equations and Bianchi identities provided that the gauge field obeys the equation $d\star_5 dA$. This, together with the fact that the Killing spinors of the geometry are charged under $U(1)_\psi$ identifies this as the dual to the R-symmetry. Upon substitution of this ansatz in to the action one finds all the gauge field dependence gives a field strength squared contribution,

$$\delta S = f(v_2, v_3)F_{\mu\nu}F^{\mu\nu} \tag{5.5}$$

for some function $f(v_2, v_3)$ of the internal coordinates that will be integrated over in a reduction to a five-dimensional theory.

Now we turn to the non-conformal geometry obtained by transformation of the Klebanov–Tseytlin geometry (since we are only interested in the UV behaviour we will not need the full Klebanov–Strassler or baryonic branch). The NS sector, with the $U(1)_\psi$ gauged, is given by

$$\begin{aligned} ds^2 &= h^{\frac{1}{2}}dr^2 + h^{-\frac{1}{2}}ds^2_{R^{1,3}} + \frac{r^2h^{\frac{1}{2}}}{6}ds^2_{S^2} + \frac{6r^4hv_2^2}{\Delta}\tilde{\chi}^2 \\ &\quad + \frac{6}{\Delta}\left[\left(r^4h + 27v_2^2\right)dv_2^2 + 54v_2\mathcal{V}_3dv_2dv_3 + \frac{3}{4}\left(\frac{\Delta}{r^2h^{\frac{1}{2}}} - 54v_2^2\right)dv_3^2\right], \\ B_2 &= \frac{18\sqrt{2}}{\Delta}v_2\mathcal{V}_3\tilde{\chi}\wedge dv_2 + \frac{(\Delta - 54r^2h^{\frac{1}{2}}v_2^2)}{\sqrt{2}\Delta}\tilde{\chi}\wedge dv_3 + \frac{r^5h'(r)}{54M}\tilde{\omega}_2, \\ e^{2\Phi} &= 81\Delta^{-1} = 81\left(2r^4h + 54v_2^2 + 36\mathcal{V}_3^2\right)^{-1}. \end{aligned} \tag{5.6}$$

Here $h(r)$ is the usual Klebanov–Tseytlin warp factor and $\mathcal{V}_3 = v_3 + \frac{r^5h'(r)}{27\sqrt{2}M}$. Without the gauging this is a solution of massive IIA with Romans' mass proportional to M. By examining how the non-Abelian T-duality transformation acts on the ansatz given by Krasnitz in [43], we can determine a suitable ansatz for the fluxes:

$$
\begin{aligned}
C_1 &= -\frac{M}{2} v_3 \cos\theta d\phi + \frac{M}{2} \psi d v_3 - 2\sqrt{2} K_1 - \sqrt{2} C_0 (\mathcal{V}_3 d v_3 + v_2 d v_2) \\
C_3 &= 2\mathcal{V}_3 K_3 - \frac{M\sqrt{2}}{4} \psi \tilde{\omega}_2 \wedge (v_2 d v_2 + v_3 d v_3) \\
&\quad + \frac{2\sqrt{2}}{M} f(r) C_0 \tilde{\omega}_2 \wedge (v_2 d v_2 + \mathcal{V}_3 d v_3) - 2 v_3 \tilde{\chi} \wedge d K_1 - 4 v_3 \tilde{\omega}_2 \wedge K_1 + \Theta_3. \qquad (5.7)
\end{aligned}
$$

The remaining term in the three-form potential is given implicitly by

$$
d\Theta_3 = \frac{1}{\sqrt{2}} M h^{\frac{1}{4}} \star_5 \left(C_0 dr + \frac{2}{3} r W \right) + \frac{3M}{\sqrt{2}} dr \wedge K_3. \qquad (5.8)
$$

Here W is a gauge-invariant 1-form that combines the gauge field A with a Stückelberg scalar $W = A - d\lambda$ though for practical purposes we may chose a gauge in which $W = A$. Then, this is a solution to the linearised flux equations and Bianchi identities provided the fields K_1, K_3, W introduced above obey a set of simple equations (the explicit form can be found in Eqs. (4.20)–(4.24) of the paper [43]). In particular, it was shown in [43] that the equations for K_1, K_3, W can be diagonalised and contain a mode corresponding to a massive gauge field whose mass is a result of the spontaneous (anomalous) breaking of R-symmetry. The mass of this mode is given by

$$
m^2 = \frac{4}{\alpha'(3\pi)^{\frac{3}{2}}} \frac{(g_s M)^2}{(\lambda N)^{\frac{3}{2}}}. \qquad (5.9)
$$

The interpretation is identical here and we conclude that the $U(1)_R$ symmetry is anomalously broken.

5.3.1. Dependence on ψ in the potentials and D0 brane instantons

To understand this breaking as an anomaly it is informative to look at the forms of the RR potentials. For the non-Abelian T-dual of the Klebanov–Witten we have following potentials

$$
\begin{aligned}
C_1 &= \frac{N\pi}{\sqrt{2}} \cos\theta d\phi, \\
C_3 &= -\frac{N\pi v_3}{2} \sin\theta d\theta \wedge d\phi \wedge d\psi. \qquad (5.10)
\end{aligned}
$$

For the dual of the Klebanov–Tseytlin (which has Romans mass proportional to M) we have

$$
\begin{aligned}
C_1 &= \frac{M}{2} v_3 \cos\theta d\phi - \frac{M}{2} \psi d v_3, \\
C_3 &= -\frac{\sqrt{2}M}{8} \left(v_2^2 + v_3^2\right) \sin\theta d\theta \wedge d\phi \wedge d\psi. \qquad (5.11)
\end{aligned}
$$

Note how the dependence on ψ in C_1 is quite different in the potentials in the conformal and non-conformal cases.

Let us now consider D0 branes. These D0 branes will move in the v_3 direction, leaving all other coordinates fixed, in particular we will choose $v_2 = 0$. We can then calculate using (5.6) the induced metric for this D0 brane, relevant gauge potential and its BIWZ action, that will read

$$ds^2_{ind} = g_{v_3 v_3} dv_3^2 = \frac{9}{2r^2 h^{1/2}} dv_3^2, \qquad C_1 = -\frac{M}{2}\psi dv_3,$$
$$S_{\text{BIWZ}} = -T_{\text{D0}} \int dv_3 e^{-\Phi} \sqrt{g_{v_3 v_3}} + T_{\text{D0}} \int C_1$$
$$= T_{\text{D0}} \int dv_3 \sqrt{\frac{r^2 h^{1/2}}{9} + \frac{2\mathcal{V}_3^2}{r^2 h^{1/2}}} - T_{\text{D0}} \frac{M\psi}{2} \int dv_3.$$

We use now that $T_{\text{D0}} = \frac{1}{g_s\sqrt{\alpha'}}$. Also, we call $\sqrt{\alpha'} L_{v_3} = \int dv_3$, the dimensionless length of the v_3 direction.

We will equate the BIWZ action of this Euclidean D0 brane with the gauge coupling and the Θ angle imposing that $S_{\text{BIWZ}} = \frac{8\pi^2}{g^2} + i\Theta$. In other words, we consider this D0 brane to be an instanton in the dual gauge theory.

Analysing the WZ term, we have that (like above, we choose $g_s = 1$),

$$S_{WZ} = \frac{M}{2}\psi L_{v_3} = \Theta. \tag{5.12}$$

Using that the theta angle should be periodic, we can impose that the allowed changes in the angle ψ get selected to be

$$\frac{M}{2}(\psi + \Delta\psi) L_{v_3} = \Theta + 2k\pi \tag{5.13}$$

which implies that

$$\Delta\psi = \frac{4k\pi}{M L_{v_3}}. \tag{5.14}$$

So, we see that there is a breaking of the global continuous symmetry into a discrete one. The residual discrete symmetry is determined by the domain of the coordinate v_3. In the case in which we would like to impose this discrete symmetry to be the same as before the non-Abelian duality we should impose that $L_{v_3} = 2$. One of the major challenges with understanding non-Abelian T-duality is to identify the periodicities of the coordinates of the T-dual geometry and it is encouraging that here we see a direct link between a field theory property (the anomaly) and the global properties of the geometry.

Let us look at the BI term. We have that the gauge coupling, associated is

$$\frac{8\pi^2}{g^2} = T_{\text{D0}} \int dv_3 \left[r^2 h^{1/2} + \frac{2}{r^2 h^{1/2}} \left(v_3 + \frac{r^5 h'}{27\sqrt{2}M} \right)^2 \right]^{1/2}. \tag{5.15}$$

We can perform the integral explicitly, but it is perhaps more illuminating to look at the large radius limit of the expression above. After all, we are doing this calculation in the non-Abelian dual of the Klebanov–Tseytlin solution, we should only trust the result in the far UV. We have then, considering the leading term in the large-r expansion,

$$\frac{1}{g^2} \sim (\log r)^{3/2} \tag{5.16}$$

this reproduces a result obtained by other means in [16].

5.4. The fate of $U(1)_B$

The Klebanov–Witten $SU(N) \times SU(N)$ conformal field theory coming from D3 branes at the tip of the conifold has a $U(1)$ baryonic number symmetry acting as $A_i \to e^{i\alpha} A_i$, $B_j \to e^{-i\alpha} B_j$. In the gravity dual this number current gives rise to a massless AdS$_5$ gauge field

$$\delta C_4 = \omega_3 \wedge \mathcal{A}, \tag{5.17}$$

where ω_3 is the usual closed three form on $T^{1,1}$. In the T-dual geometry given in Eq. (5.3), this $U(1)_B$ mode translates into a perturbation, which solves the linearised supergravity equations of motion, given by

$$\delta C_1 = \frac{1}{9}\mathcal{A},$$
$$\delta C_3 = W_2 \wedge \mathcal{A} + \frac{1}{9} u du \wedge \mathcal{F} + \frac{\sqrt{2}}{6} u dv_3 \wedge \star_4 \mathcal{F}. \tag{5.18}$$

The final two terms in δC_3 come from the a contribution from δC_6 under the T-duality transformation.[6] Although the two-form W_2 has a simple form

$$W_2 = \frac{v_3}{9} d\sigma_3 + \frac{\sqrt{2} v_2 e^{2\hat{\phi}}}{81} \sigma_3 \wedge (2v_3 dv_2 - 3v_2 dv_3) \tag{5.19}$$

it can-not easily be written in terms of the invariant tensors that define the $SU(2)$ structure of the geometry.

The existence of this mode is suggestive that the field theory duals corresponding to the conformal geometries constructed in [15] have a global $U(1)$ symmetry in addition to the preserved $U(1)_R$. In fact, the geometry T-dual to the Klebanov–Witten is closely related to those proposed in [36] as the gravity duals to $\mathcal{N} = 1$ SCFTs formed by wrapping M5 branes on Riemann surfaces (which in this case is genus zero giving rise to many subtleties). These SCFTs do indeed have $U(1)_R \times U(1)_F$ Abelian global symmetries which are seen geometrically as isometries of the corresponding eleven-dimensional supergravity solution. Upon reduction to ten-dimension one of these $U(1)$'s gets degeometrised corresponding to the above gauge field $\delta C_1 = \mathcal{A}$.

In this paper our main focus has been the cascading field theory where at the last step of the cascade when the gauge group is $SU(M) \times SU(2M)$ the baryons acquire expectation values,

$$\mathcal{B} = i\xi \Lambda^{2M}, \qquad \tilde{\mathcal{B}} = \frac{i}{\xi} \Lambda^{2M}. \tag{5.20}$$

On this baryonic branch the $U(1)_B$ symmetry is spontaneously broken. To see this from the gravity perspective it is sufficient to work with the Klebanov–Strassler geometry corresponding to the field theory at the $\mathbb{Z}_2$ symmetric point of the baryonic branch. As shown in [24], there is a massless glue ball corresponding to a Goldstone mode associated with changing the phase of ξ which is given by[7]

[6] For the AdS$_5 \times T^{1,1}$ we use $ds^2_{\mathrm{AdS}} = du^2 + e^{2u}(\eta_{ij} dx^i dx^j)$.

[7] Here and elsewhere use the standard notation for the deformed conifold and Klebanov–Strassler geometry which can be found e.g. in appendix of [24]. For the KS we stick with the notation τ as the radial coordinate but will use r elsewhere.

$$
\begin{aligned}
&\delta H = 0, \\
&\delta F_3 = f_1 \star_4 da - d\big(f_2(\tau) da \wedge g^5\big), \\
&\delta F_5 = f_1 \left(\star_4 da - \frac{\epsilon^{\frac{4}{3}}}{6K^2(\tau)} h(\tau) da \wedge d\tau \wedge g^5 \right) \wedge B_2.
\end{aligned} \tag{5.21}
$$

The linearised supergravity equations are solved when the pseudo-scalar is a harmonic function in $\mathbb{R}^{3,1}$ and the function $f_2(\tau)$ obeys a second order differential equation admitting a normalisable solution.

The non-Abelian T-dual geometries considered also admits a similar mode, which can be obtained simply by performing a T-dualisation of the ansatz for the scalar modes in the seed IIB solutions. The T-dual of the Klebanov–Strassler geometry was obtained explicitly in [16]. Performing a dualisation of the ansatz (5.21) gives rise to a perturbation δF_2 and δF_4. This perturbation solves the supergravity equations of motion when f_2 obeys the same differential equation as for the ansatz (5.21). The expressions for F_2 and F_4 are not particularly enlightening though for completeness let us provide a few details. Here we display the results in the UV regime where the geometry is given by (5.6). The corresponding deformations to the potentials are given by

$$
\begin{aligned}
&\delta C_1 = \big(2v_3 f_2(r) + f_3(r)\big) da, \\
&\delta C_3 = \left[f_4(r) - \frac{f_1}{\sqrt{2}} \left(v_2^2 + \left(v_3 - \frac{N\pi}{\sqrt{2}M} \right)^2 \right) \right] \star_4 da - \frac{f_2}{\sqrt{2}} da \wedge \sigma_3 \wedge d\big(v_2^2 + v_3^2\big) \\
&\qquad - \frac{f_3}{\sqrt{2}} da \wedge \sigma_3 \wedge dv_3 + da \wedge \sin\theta d\theta \wedge d\phi \left(f_5 - \frac{v_3}{\sqrt{2}} f_3 \right).
\end{aligned} \tag{5.22}
$$

The extra functions introduced above are completely determined by f_1 and f_2 according to

$$
\begin{aligned}
&f_1' = 0, \qquad 2r^4 f_2'' = -6r^3 f_2' + 16r^2 f_2 + 27M^2 f_1 \log r/r_0, \\
&f_3' = \frac{1}{6}\big(-3\sqrt{2} r f_1 h(r) \log r/r_0 - 2T(r) f_2'\big), \qquad f_4' = \frac{2\sqrt{2}}{3} r f_2, \\
&f_5' = \frac{1}{108}\big(-2\sqrt{2} r^5 f_1 h(r) = 18 M r f_1 h(r) T(r) \log r/r_0 - 3\sqrt{2} T(r)^2 f_2'\big),
\end{aligned} \tag{5.23}
$$

where $T(r) = \frac{9}{\sqrt{2}} M \log r/r_0$ and $h(r) = \frac{27}{32r^4}(3M^2 + 8N\pi + 12M^2 \log r/r_0)$.

The existence of this mode suggests a spontaneously broken global $U(1)$ in the field theories dual to the geometries obtained in Section 3. In the conformal case, the unbroken $U(1)$ becomes geometrised upon lifting to M-theory whereas these non-conformal backgrounds are solutions of *massive* IIA and so can-not be lifted. This further underlines the expectation that a $U(1)$ is broken.

In the same multiplet as the pseudo-scalar goldstone is a scalar perturbation corresponding to changing the magnitude of ξ. In the same vein as above, one could deduce the fate of this scalar perturbation under the T-duality transformation; it will give a similar, albeit complicated, perturbation in the dual IIA background. Since the full baryonic branch geometry found in [25] can be thought of as exponentiating such transformations to give arbitrary values of the baryonic vev, implicitly in the geometries presented in Section 3 we have already done just that.

5.5. The fate of the baryon condensate

In Klebanov–Witten theory the closest analogy to a baryon vertex – the object to which N *external* quarks can attach [38] – would be a D5 brane wrapping the $T^{1,1}$ space with world volume coordinates $\{x_0, \theta_1, \phi_1, \theta_2, \phi_2, \psi\}$ [39]. The primary reason for this identification follows the argument made in [38]; since we have

$$\int_{T^{1,1}} F_5 \propto N, \tag{5.24}$$

the WZ term induces a charge to the world volume $U(1)$ gauge field $\mathcal{A}$ via the coupling

$$\int_{\mathbb{R}\times T^{(1,1)}} \mathcal{A} \wedge F_5. \tag{5.25}$$

This introduces N units of charge which must be cancelled by some other source to give zero net charge in a closed universe. This cancellation is achieved by N elementary strings stretching from the boundary to the brane whose end points are external quarks. A perhaps naive approach would be to suggest in the IIA geometry dual to the Klebanov–Witten theory a similar role could be played by a D2 brane wrapping the S^2 with world volume coordinates $\{x_0, \theta, \phi\}$. Indeed, since in the case of T-dual to Klebanov–Witten we have $C_1 \propto \cos\theta d\phi$ the WZ coupling $\mathcal{F} \wedge C_1$ produces a charge contribution for the gauge field that could be cancelled with external quarks just as in the Klebanov–Witten scenario. It would be of some interest to study the baryon vertex in the massive IIA backgrounds.[8]

This baryon vertex should however be distinguished from the configuration representing the actual baryon condensate – which should be supersymmetric, gauge-invariant and not require BIon spikes. The configuration that describes the baryon condensate is a Euclidean D5 brane wrapping the $T^{1,1}$ and the radial directions [39]. This D5 has D3 branes dissolved within [40] which are traded for a world volume gauge field. Following the logic applied to the baryon vertex one might anticipate that in the IIA geometries presented here, the role of the condensate is played by a wrapped Euclidean D2 brane on the $S^2 \times \mathbb{R}$ with a world volume gauge field.

Here, to determine the existence of supersymmetric configurations, rather than calculate the kappa symmetry projectors, we will harness the power of the G-structure and the calibration techniques of [41]. The condition for a supersymmetric Euclidean p brane on a cycle Σ is essentially the same as that of a Lorentzian $p+4$ brane that is spacetime filling in the Minkowski directions. This condition is given by

$$e^{-\phi}\sqrt{-\det(g|_\Sigma + \mathcal{F})}\, d^p\sigma = 8e^{3A-\phi}\, \mathrm{Im}\, \Phi \wedge e^{-\mathcal{F}}\big|_\Sigma \tag{5.26}$$

where the world volume field strength is $\mathcal{F} = B|_\Sigma + 2\pi\alpha' d\mathcal{A}$ and the pure spinor entering the calibration form is given $\Phi = \Psi_+$ for IIB and $\Phi = \Psi_-$ for IIA. For reference, in Appendix F we re-derive some of the IIB embeddings found in [39] using this very efficient calibration technique.

Let us begin with the IIA non-Abelian T-dual of the Klebanov–Witten geometry given in Eq. (5.3). We find an E2 configuration extended along $\Sigma = \{r, \theta, \phi\}$ at the point $v_2 = 0$ but with

[8] Before duality in the cascading theories this is a D3 brane and it seems quite possible that D0 branes might play this role of the baryon vertex in the cascading massive IIA geometries. We thank O. Aharony and J. Sonnenschein for this suggestion.

a non-trivial embedding $v_3 = f(r)$. We search for a supersymmetric configuration solving the calibration condition (5.26) when supported by a gauge field

$$\mathcal{A} = \frac{1}{\sqrt{2}}\alpha(r)\cos\theta d\phi. \tag{5.27}$$

From the calibration condition one finds firstly that the embedding $f(r)$ and the gauge field should differ only by a constant c_0. The gauge field should then obey an equation

$$\alpha'(r) = \frac{1 - 18c_0\alpha - 18\alpha^2}{9(c_0 + 2\alpha)} \tag{5.28}$$

which can also be readily solved and one notices that when $c_0 = 0$ has the same form as Eq. (F.2) governing the configuration in IIB.

Now we turn to the non-conformal context. Evidently the geometry describing the full baryonic branch is rather involved so to make the analysis tractable we focus on the non-Abelian T-dual of the Klebanov–Tseytlin geometry given in Eq. (5.6). We search for an E2 configuration extended along $\Sigma = \{r, \theta, \phi\}$ at the point $v_2 = 0$ and now with $v_3 = \chi(r)$ and an ansatz for the gauge field

$$\mathcal{A} = \frac{1}{\sqrt{2}}\alpha(r)\cos\theta d\phi. \tag{5.29}$$

We take the square of the calibration equation (5.26) and first consider terms proportional to $\cos^2\theta$. From these one finds a first equation relating the gauge field and the embedding in v_3:

$$\alpha'(r) = \chi'(r). \tag{5.30}$$

We let c_0 be the additive constant between α and χ. Then from the remaining terms in Eq. (5.26) one finds a differential equation for the gauge field

$$r\alpha'(r) = \frac{1}{18(c_0 + 2\alpha)}\left(2r^4 h(r) - 6c_0 T + T^2 - 36c_0\alpha - 36\alpha^2\right), \tag{5.31}$$

where we remind the reader that $T(r)$ and $h(r)$ are given following Eq. (5.23). Changing variable to $t = \log r$ one can solve this equation on the exact logarithmic solution:

$$\alpha(r) = -\frac{c_0}{2} \pm \frac{r^{-3/2}}{8}\Big[64rc + r^3\big(16c_0^2 + 3M\big(8\sqrt{2}c_0 + 9M - 4(4\sqrt{2}c_0 + 3M)\log r \\ + 24M\log r^2\big)\big)\Big]^{\frac{1}{2}} \tag{5.32}$$

here c is an integration constant giving sub-leading contributions that we hence ignore.

Using Eq. (5.31) we find that the DBI action is given by

$$S_{\mathrm{DBI}} = \kappa\int\frac{dr}{r}\frac{1}{648}(c_0 + 2\alpha)^{-1}\left(2r^4h + (T + 6\alpha)^2\right)\left(2r^4h + \left(T - 6(c_0 + \alpha)\right)^2\right). \tag{5.33}$$

If we expand out asymptotically we find that

$$S_{\mathrm{DBI}} \sim \kappa\int^{t_{\mathrm{UV}}} dt\frac{27M^3t^2}{8\sqrt{2}} + \frac{9M^2t}{32}\left(3\sqrt{2}M - 4c_0 + 8\sqrt{2}\frac{N}{M}\pi\right) + \mathcal{O}(t^0), \tag{5.34}$$

which suggests an operator with a scaling dimension

$$\Delta = \frac{27\kappa M^3}{8\sqrt{2}}(\log r)^2 \tag{5.35}$$

where $\kappa = T_{D2}\,\mathrm{vol}(S^2) = \frac{1}{\pi}$. It would be interesting to pursue this line of reasoning further by extracting the value of the condensate across the baryonic branch. This is technically rather involved and we do not intend to do so in this report.

6. Conclusions and future directions

In this paper we have examined a new family of solutions of massive IIA supergravity. These new backgrounds were obtained by performing a non-Abelian T-duality on the geometry that describes the non-perturbative physics of the baryonic-branch of the Klebanov–Strassler field theory. We have explored the transition from $SU(3)$ structure, characterising the 'seed' backgrounds to the dynamical $SU(2)$-structure that describes the resulting massive IIA solutions. We made clear – at least for the type of backgrounds studied here – that the dynamical character of the $SU(2)$ structure is directly related to the phenomena of confinement and symmetry breaking. We believe that all these new features have not been discussed in previous literature, in a context as clear and unifying as the one presented here.

The new backgrounds discussed in this paper display a host of interesting non-perturbative phenomena that 'define' the dual field theory. Some of these are,

- The non-conformality of the geometry is enabled by a non-zero Romans' mass.
- Whilst the UV geometries proposed in [16] are characterised by *static* $SU(2)$ structure [18] the full IR complete geometry of this paper has *dynamic* $SU(2)$ structure.
- The transition to dynamic $SU(2)$ structure gives a geometric realisation of confinement and permits supersymmetric D2 branes that act as domain walls in the IR. This realises geometrically the relation between confinement, the spontaneous breaking of a discrete R-symmetry and the presence of domain walls.
- The $U(1)_R$ symmetry is realised by the vector ∂_ψ and the corresponding fluctuation, which is a massless gauge field in the conformal case, acquires a mass indicating an anomalous breaking.
- Euclidean 'instantonic' branes reproduce this anomaly of the R-symmetry and at the same time suggest a non-conventional running for a suitably defined gauge coupling.
- A further $U(1)$ (baryonic) symmetry is broken. In the conformal case of [16] this symmetry is unbroken and is realised geometrically by the M-theory circle. In our backgrounds, once conformality is broken by the addition of fractional branes, the symmetry is no longer geometrical as we are now in a massive IIA context. The $U(1)_B$ symmetry is spontaneously broken and we identified a corresponding massless glueball (the associated Goldstone boson).
- We give evidence that this $U(1)_B$ may be thought of as baryonic and that a baryonic condensate is given by a Euclidean D2 brane wrapping a two-cycle in the geometry.

Although we do not yet have a complete understanding of the field theory dual to this new geometry, the results of this paper together with those in [16] suggest that it may be a non-conformal and cascading version of the Sicilian theories of [35,36] or the linear quivers of [37].

We would like to close this paper on a forward looking note. We suggest that the features mentioned above may be prototypical of a wider class of holographic duals. The theories in

[35,36] and also the IIA linear quivers of [37], present a wide new class of interesting examples of $\mathcal{N}=1$ SCFTs. We anticipate that by a modification of these theories (this paper suggests that the modification will involve adding D8 branes in IIA) one can obtain a variety of non-conformal gauge theories. Some of the non-perturbative features of these new field theories should be the ones we are describing in this paper.

Aside from this and on a more geometrical note, we believe the backgrounds presented in this paper may serve as a prototype for new dynamical $SU(2)$ solutions of massive IIA supergravity that will be the corresponding string duals to the new field theories described above. This is, of course, in the same vein as the route from the conformal geometry of Klebanov–Witten to the non-conformal geometry of Klebanov–Strassler.

In our view, these represent the most interesting avenues of further investigation.

Acknowledgements

Discussions with various colleagues helped to improve the contents and presentation of this paper. We wish to thank: Ofer Aharony, Oren Bergman, Ben Craps, Tim Hollowood, Carlos Hoyos, Georgios Itsios, Zohar Komardgoski, Yolanda Lozano, Luca Martucci, Eoin O'Colgain, Diego Rodriguez-Gomez, Daniel Schofield, Kostas Sfetsos, Jacob Sonnenschein, and Brian Wecht. The work of J.G. was funded by the DOE Grant DE-FG02-95ER40896. N. Macpherson is supported by STFC Doctoral Training Grant ST/I506037/1. Carlos Nunez is a Feinberg Foundation Visiting Faculty Program Fellow, he thanks the hospitality extended at Weizmann Institute and The Academic Study Group for the Isaiah Berlin Travel award. D. Thompson is supported in part by the Belgian Federal Science Policy Office through the Interuniversity Attraction Pole P7/37, and in part by the "FWO-Vlaanderen" through the project G.0114.10N and through an "FWO-Vlaanderen" postdoctoral fellowship project number 1.2.D12.12N.

Appendix A. Conventions: supergravity and G-structures

A.1. Supergravity

We work in string and the 10d hodge dual is defined such that

$$F_n = (-1)^{\mathrm{int}[n/2]} \star F_{10-n}, \tag{A.1}$$

where F_n are the RR fluxes of either Type IIA or Type IIB supergravity. The fluxes may be used to define a polyform F such that

$$F = \begin{cases} F_0 + F_2 + F_4 + F_6 + F_8 + F_{10} & \text{for Type IIA} \\ F_1 + F_3 + F_5 + F_7 + F_9 & \text{for Type IIB} \end{cases}. \tag{A.2}$$

In terms of the polyform the Bianchi identities may be expressed as

$$(d - H\wedge)F = 0, \qquad dH = 0. \tag{A.3}$$

It is easy to show this is satisfied with the definition

$$F = (d - H\wedge)C + F_0 e^{B_2} \tag{A.4}$$

where C is a polyform constructed from the RR potentials in the same fashion as above and F_0 should be taken to be zero in Type IIB. The flux equations of motion are expressed as

$$(d + H\wedge) \star F = 0, \qquad d\left(e^{-2\Phi} \star H\right) = \frac{1}{2}\sum_n F_n \wedge \star F_n, \tag{A.5}$$

where the sum needs to me taken over the appropriate RR fluxes of Type IIA/IIB.

The dilaton must obey the equation of motion

$$d \star d\Phi + \star\frac{R}{4} - d\Phi \wedge \star d\Phi - \frac{1}{8}H \wedge \star H = 0, \tag{A.6}$$

while Einstein's equations are in Type IIA by

$$\begin{aligned} R_{\mu\nu} &= -2D_\mu D_\nu \hat{\Phi} + \frac{1}{4}H^2_{\mu\nu} \\ &\quad + e^{2\Phi}\left[\frac{1}{2}\left(F_2^2\right)_{\mu\nu} + \frac{1}{12}\left(F_4^2\right)_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\left(F_0^2 + \frac{1}{2}F_2^2 + \frac{1}{4!}F_4^2\right)\right], \end{aligned} \tag{A.7}$$

with an equivalent equation holding in Type IIB.

A.2. Pure spinors

Here we follow the conventions of [51] except for a difference in the self duality condition of the RR section which leads to a few sign differences. We work in string frame and consider solution with metrics that can be expressed as

$$ds^2 = e^{2A}dx^2_{3,1} + ds^2_6 \tag{A.8}$$

and preserve $\mathcal{N} = 1$ SUSY in 4d with non-trivial RR sector. This means that the internal space, with metric ds^2_6, must support an $SU(3) \times SU(3)$-structure [41]. We decompose the 10d MW spinors into a $4 + 6$ split as

$$\epsilon^1 = \xi_+ \otimes \eta^1_+ + \xi_- \otimes \eta^1_-, \qquad \epsilon^2 = \xi_+ \otimes \eta^2_\mp + \xi_- \otimes \eta^2_\pm, \tag{A.9}$$

where in ϵ_2 the upper/lower signs should be taken in Type IIA/B, the $\pm$ indicates chirality of both 4d and internal 6d spinors and we choose a basis for the internal spinors such that $(\eta_+)^* = \eta_-$. It is possible to define two *Cliff*$(6, 6)$ pure spinors on the internal space as

$$\Psi_\pm = \eta^1_+ \otimes \left(\eta^2_\pm\right)^\dagger \tag{A.10}$$

which may be identified with polyforms under the Clifford map. The internal spinors are decomposed as

$$\eta^1_+ = e^A e^{i\frac{\theta_+ + \theta_-}{2}}\eta_+, \qquad \eta^2_+ = e^A e^{-i\frac{\theta_+ - \theta_-}{2}}(k_{||}\eta_+ + k_\perp \chi_+) \tag{A.11}$$

where $k^2_{\|} + k^2_\perp = 1$, $\eta^\dagger_+\eta_+ = \chi^\dagger_+\chi_+ = 1$ and $\chi^\dagger_+\eta_+ = 0$. The $\mathcal{N} = 1$ SUSY conditions for such a $SU(3) \times SU(3)$-structure solution are given by the differential conditions

$$\begin{aligned} &(d - H\wedge)\left(e^{2A-\phi}\Psi_\pm\right) = 0 \\ &(d - H\wedge)\left(e^{2A-\phi}\Psi_\mp\right) = e^{2A-\phi}dA \wedge \bar{\Psi}_2 \mp \frac{1}{8}e^{3A} \star_6 i\lambda(\tilde{F}) \end{aligned} \tag{A.12}$$

where $\lambda(A_n) = (-1)^{\frac{n(n-1)}{2}} A_n$ and $\tilde{F}$ is the internal part of RR polyform in Type IIA/B where the RR forms are each decomposed such that

$$F_n = \tilde{F}_n \mp e^{4A}\,\mathrm{vol}_4 \wedge \lambda(\star_6 \tilde{F}_{10-n}). \tag{A.13}$$

As before upper/lower signs correspond to Type IIA/B.

Clearly in general η^2_+ is composed of a parts that is parallel and a part that is orthogonal to η^1_+. The $SU(3) \times SU(3)$-structure can categorised into 3 distinct cases depending on the values of the coefficients $k_\perp$ and $k_\parallel$:

A.2.1. $SU(3)$-structure

When $k_\perp = 0$ the internal spinors are parallel and the pure spinors define an $SU(3)$-structure in 6d such that

$$\begin{aligned} \Psi_+ &= -e^{i\theta_+}\frac{e^A}{8}e^{-iJ}, \\ \Psi_- &= -ie^{i\theta_-}\frac{e^A}{8}\Omega_{hol} \end{aligned} \tag{A.14}$$

where J and Ω_{hol} are the two and holomorphic three forms associated with $SU(3)$, they are defined as in terms of the 6d gamma matrices as

$$\Omega^{(hol)}_{abc} = -i\eta^\dagger_-\gamma_{abc}\eta_+, \qquad J_{ab} = -i\eta^\dagger_+\gamma_{ab}\eta_+, \tag{A.15}$$

and satisfy

$$J \wedge \Omega_{hol} = 0, \qquad J \wedge J \wedge J = \frac{3i}{4}\Omega_{hol} \wedge \bar{\Omega}_{hol}. \tag{A.16}$$

A.2.2. Orthogonal $SU(2)$-structure

When $k_\parallel = 0$ the internal spinors are orthogonal and the pure spinors define an orthogonal $SU(2)$-structure in 6d such that

$$\begin{aligned} \Psi_+ &= -ie^{i\theta_+}\frac{e^A}{8}e^{-v\wedge w} \wedge \omega, \\ \Psi_- &= ie^{i\theta_-}\frac{e^A}{8}(v + iw) \wedge e^{-ij} \end{aligned} \tag{A.17}$$

where the $SU(2)$-structure one forms v, w and two forms j, ω are defined as

$$\begin{aligned} w_a - iv_a &= \eta^\dagger_-\gamma_a\chi_+, \qquad j_{ab} = -i\eta^\dagger_+\gamma_{ab}\eta_+ + i\chi^\dagger_+\gamma_{ab}\chi_+, \\ \omega_{ab} &= \eta^\dagger_-\gamma_{ab}\chi_-, \end{aligned} \tag{A.18}$$

and obey the relations

$$\begin{aligned} j \wedge \omega &= \omega \wedge \omega = \iota_{(w-iv)}(\omega) = \iota_{(w-iv)}(j) = 0, \\ j \wedge j &= \frac{1}{2}\omega \wedge \bar{\omega}. \end{aligned} \tag{A.19}$$

A.2.3. Intermediate and dynamical $SU(2)$-structure

For intermediate $SU(2)$-structure $k_\parallel$ and $k_\perp$ are non-zero constants (the case of intermediate structure is described in [52] and further definitions including a helpful presentation of dynamic structure is given in the thesis of Andriot [51]), this and the previous example are also referred to as static $SU(2)$-structure. For dynamical $SU(2)$-structure $k_\parallel$ and $k_\perp$ are point-dependent. For both these cases the pure spinors are given by

$$\Phi_+ = \frac{e^A}{8} e^{i\theta_+} e^{-iv\wedge w}\big(k_\parallel e^{-ij} - ik_\perp \omega\big)$$
$$\Phi_- = \frac{ie^A}{8} e^{i\theta_-}(v+iw)\wedge\big(k_\perp e^{-ij} + ik_\parallel \omega\big), \tag{A.20}$$

where Eqs. (A.19) and (A.18) still hold.

In these conventions the SUSY conditions (here we consider Type IIA, details of Type IIB are given in Appendix E) may be split up as follows:

$$d\big[e^{3A-\hat{\Phi}}k_\parallel\big] = 0$$
$$d\big[e^{3A-\hat{\Phi}}\big(k_\parallel(j+v\wedge w)+k_\perp\omega\big)\big] - ie^{3A-\hat{\Phi}}k_\parallel H = 0$$
$$d\bigg[e^{3A-\hat{\Phi}}\bigg(\frac{1}{2}k_\parallel(j+v\wedge w)^2 + k_\perp v\wedge w\wedge\omega\bigg)\bigg] - ie^{3A-\hat{\Phi}}H$$
$$\wedge\big(k_\parallel(j+v\wedge w)+k_\perp\omega\big) = 0 \tag{A.21}$$

where the second of these gives a definition for H which can be combined with the first to give a definition of the NS potential, namely

$$B_2 = -\frac{k_\perp}{k_\parallel}\,\mathrm{Im}\,\omega \tag{A.22}$$

this is not the same as the NS potential generated by non-Abelian T-duality but must match it up to an exact.

The rest of the SUSY conditions are

$$\star_6 F_6 = 0$$
$$d\big[e^{4A-\hat{\Phi}}k_\perp(\sin\theta_- w - \cos\theta_- v)\big] = -e^{4A}\star_6 F_4$$
$$d\big[e^{2A-\hat{\Phi}}k_\perp(\sin\theta_- v + \cos\theta_- w)\big] = 0$$
$$d\big[e^{4A-\hat{\Phi}}\big(k_\parallel(\sin\theta_-\,\mathrm{Im}\,\omega - \cos\theta_-\,\mathrm{Re}\,\omega)\wedge w - k_\parallel(\sin\theta_-\,\mathrm{Re}\,\omega + \cos\theta_-\,\mathrm{Im}\,\omega)\wedge v$$
$$+ k_\perp(\sin\theta_- v + \cos\theta_- w)\wedge j\big)\big] + e^{4A-\hat{\Phi}}k_\perp H\wedge(\sin\theta_- w - \cos\theta_- v) = -e^{4A}\star_6 F_2$$
$$d\big[e^{2A-\hat{\Phi}}\big(k_\parallel(\sin\theta_-\,\mathrm{Re}\,\omega + \cos\theta_-\,\mathrm{Im}\,\omega)\wedge w - k_\parallel(\cos\theta_-\,\mathrm{Re}\,\omega - \sin\theta_-\,\mathrm{Im}\,\omega)\wedge v$$
$$- k_\perp(\sin\theta_- w - \cos\theta_- v)\wedge j\big)\big] + k_\perp e^{2A-\hat{\Phi}}H\wedge(\cos\theta_- w + \sin\theta_- v) = 0$$
$$d\bigg[\frac{1}{2}e^{4A-\hat{\Phi}}k_\perp j\wedge j\wedge(\cos\theta_- v - \sin\theta_- w)\bigg] + e^{4A-\hat{\Phi}}H\wedge\big(k_\parallel(\sin\theta_-\,\mathrm{Im}\,\omega - \cos\theta_-\,\mathrm{Re}\,\omega)$$
$$\wedge w - k_\parallel(\sin\theta_-\,\mathrm{Re}\,\omega + \cos\theta_-\,\mathrm{Im}\,\omega)\wedge v + k_\perp(\sin\theta_- v + \cos\theta_- w)\wedge j\big) = -e^{4A}\star_6 F_0$$
$$d\bigg[\frac{1}{2}e^{2A-\hat{\Phi}}k_\perp j\wedge j\wedge(\cos\theta_- w + \sin\theta_- v)\bigg] + e^{2A-\hat{\Phi}}H$$
$$\wedge\big(-k_\parallel(\sin\theta_-\,\mathrm{Re}\,\omega + \cos\theta_-\,\mathrm{Im}\,\omega)\wedge w + k_\parallel(\cos\theta_-\,\mathrm{Re}\,\omega - \sin\theta_-\,\mathrm{Im}\,\omega)$$
$$\wedge v + k_\perp(\sin\theta_- w - \cos\theta_- v)\wedge j\big) = 0 \tag{A.23}$$

from which it is possible to define the higher forms of the RR sector as:

$$F_6 = dC_5$$
$$F_8 = dC_7 - H\wedge C_5$$
$$F_{10} = dC_9 - H\wedge C_7 \tag{A.24}$$

Table 1
The table lists well known string backgrounds, their G-structure, and the structure of the background generated by the use of non-Abelian T-duality.

Seed solution	Seed structure	Dual structure
Klebanov–Witten	$SU(3)$	Orthogonal $SU(2)$
Klebanov–Tseytlin	$SU(3)$	Orthogonal $SU(2)$
$Y^{p,q}$	$SU(3)$	Orthogonal $SU(2)$
Klebanov–Strassler	$SU(3)$	Dynamical $SU(2)$
KS baryonic branch	$SU(3)$	Dynamical $SU(2)$
Wrapped D5s on S^2	$SU(3)$	Dynamical $SU(2)$
Wrapped D6s on S^3	$SU(3)$	Dynamical $SU(2)$
Wrapped D5s on S^3	G_2	Dynamical $SU(3)$

where the RR potentials are given by:

$$\begin{aligned}
C_5 &= e^{4A-\hat{\Phi}} \operatorname{vol}_4 \wedge k_\perp(\sin\theta_- w - \cos\theta_- v) \\
C_7 &= -e^{4A-\hat{\Phi}} \operatorname{vol}_4 \wedge \big[k_\parallel(\sin\theta_- \operatorname{Im}\omega - \cos\theta_- \operatorname{Re}\omega) \wedge w - k_\parallel(\sin\theta_- \operatorname{Re}\omega + \cos\theta_- \operatorname{Im}\omega) \\
&\quad \wedge v + k_\perp(\sin\theta_- v + \cos\theta_- w) \wedge j\big] \\
C_9 &= \frac{1}{2} e^{4A-\hat{\Phi}} \operatorname{vol}_4 \wedge k_\perp j \wedge j \wedge (\cos\theta_- v - \sin\theta_- w)
\end{aligned} \tag{A.25}$$

The calibration is given by

$$\Psi_{cal} = -8 e^{3A-\hat{\Phi}} \operatorname{Im}\Phi_- e^{\pm B_2} \tag{A.26}$$

where $\pm$ depends on our conventions in the WZ action. That $S_{\mathrm{DBI}} + S_{WZ} = 0$ is trivial because in these conventions we have:

$$C_5 + C_7 + C_9 = -8 \operatorname{vol}_4 \wedge e^{3A-\hat{\Phi}} \operatorname{Im}\Phi_- \tag{A.27}$$

This all works perfectly for the case $\theta_- = 0$ which is the dual of the wrapped D5 solution.

The action of non-Abelian T-duality on the G-structures has been studied in many backgrounds which we summarise in Table 1.[9]

Appendix B. Details of the non-Abelian T-duality on the D5 branes solution

The purpose of this section is to give some details of the $SU(2)$ isometry T-dual of Wrapped D5 branes on S^2. This was first derived in [16], but in slightly different conventions and the G-structure was not found. This is the $\mathcal{C} = 1, \mathcal{S} = 0$ limit of the full baryonic branch dual solution, and as the procedure for find the G-structure is the same in both case we hope that this more simple example will be instructive.

Solution of wrapped D5 branes on S^2 [32] has string frame metric given by

$$\begin{aligned}
ds^2 = e^{\Phi}\bigg(& dx_{1,3}^2 + e^{2k} d\rho + e^{2h}\big(d\theta^2 + \sin^2\theta d\varphi^2\big) \\
&+ \frac{e^{2g}}{4}\big((\tilde{\omega}_1 + a d\theta)^2 + (\tilde{\omega}_2 - a\sin\theta d\varphi)^2\big) + \frac{e^{2k}}{4}(\tilde{\omega}_3 + \cos d\varphi)^2\bigg)
\end{aligned} \tag{B.1}$$

[9] The details of the case of $Y^{p,q}$ are to appear in [53] and a detailed study of the D6 branes on S^3 will appear in [54].

where the functions a, b, g, h, k and the dilaton Φ only depend on the holographic coordinate r. The $\tilde{\omega}_i$ are $SU(2)$ left-invariant 1-forms which can be parametrised as

$$\begin{aligned}
\tilde{\omega}_1 &= \cos\psi\, d\tilde{\theta} + \sin\psi \sin\tilde{\theta} d\tilde{\varphi},\\
\tilde{\omega}_2 &= -\sin\psi\, d\tilde{\theta} + \cos\psi \sin\tilde{\theta} d\tilde{\varphi},\\
\tilde{\omega}_3 &= d\psi + \cos\tilde{\theta} d\tilde{\varphi}.
\end{aligned} \tag{B.2}$$

A convenient set of vielbeins is given by

$$\begin{aligned}
&e^{x^i} = e^{\frac{\Phi}{2}} dx^i, \qquad e^{\rho} = e^{\frac{\Phi}{2}+k} d\rho, \qquad e^{\theta} = e^{\frac{\Phi}{2}+h} d\theta, \qquad e^{\varphi} = e^{\frac{\Phi}{2}+h} \sin\theta\, d\varphi,\\
&e^1 = \frac{1}{2} e^{\frac{\Phi}{2}+g}(\tilde{\omega}_1 + a\, d\theta), \qquad e^2 = \frac{1}{2} e^{\frac{\Phi}{2}+g}(\tilde{\omega}_2 - a \sin\theta\, d\varphi),\\
&e^3 = \frac{1}{2} e^{\frac{\Phi}{2}+k}(\tilde{\omega}_3 + \cos\theta\, d\varphi),
\end{aligned} \tag{B.3}$$

with respect to which the non-trivial RR flux F_3 may be expressed as

$$F_3 = e^{-\frac{3}{2}\Phi}\big[f_1 e^{123} + f_2 e^{\theta\varphi 3} + f_3\big(e^{\theta 23} + e^{\varphi 13}\big) + f_4\big(e^{\rho 1\theta} + e^{\rho\varphi 2}\big)\big] \tag{B.4}$$

where the f_i are given by Eq. (2.8). In these conventions the projections the 10d Killing spinor ϵ obeys are

$$\Gamma_{12}\epsilon = \Gamma_{\theta\varphi}\epsilon, \qquad \Gamma_{r123}\epsilon = (\cos\alpha + \sin\alpha\,\Gamma_{\varphi 2})\epsilon, \quad i\epsilon^* = \epsilon, \tag{B.5}$$

with respect to the $4+6$ split we can define components of ϵ to be equal with positive chirality as

$$\epsilon_1 = \epsilon_2 = e^{A}(\xi_+ \otimes \eta_+ + \xi_- \otimes \eta_-) \tag{B.6}$$

where $2A = \Phi$. Once the usual decomposition of gamma matrices,

$$\Gamma_\mu = \hat{\gamma}_\mu \otimes \mathbb{I}, \qquad \Gamma_a = \mathbb{I} \otimes \gamma_a \tag{B.7}$$

is performed it is a simple matter to derive the $SU(3)$-structure forms of Eq. (2.15) using Eq. (A.15), where we have chosen $i\gamma_{r\theta\varphi 123}\eta_+ = \eta_+$. To do this it is helpful to perform a rotation in e^{φ}, e^2 which will also be useful later

$$\begin{aligned}
\hat{e}^{\varphi} &= \cos\alpha e^{\varphi} + \sin\alpha e^2\\
\hat{e}^2 &= -\sin\alpha e^{\varphi} + \cos\alpha e^2\\
\hat{e}^a &= e^a \quad \text{for } a \neq \varphi, 2.
\end{aligned} \tag{B.8}$$

The rotated 6d projections are then simply

$$\hat{\gamma}_{\varphi\theta}\eta_+ = \hat{\gamma}_{r3}\eta_+ = \hat{\gamma}_{21}\eta_+ = i\eta_+ \tag{B.9}$$

and the $SU(3)$-structure becomes canonical.

We want to T-dualise this wrapped D5-brane solution along the $SU(2)$ isometry parametrised by $\tilde{\omega}_i$. Section 2 and Appendix B of [16] give all the details of the algorithm one must follow to do this and so we direct the interested reader there for details of the NS sector. For the RR sector we only give details that will be relevant for later calculations.

The duality will drastically change the vielbeins that contain the $SU(2)$ left-invariant 1-forms e^1, e^2, e^3 and leave the others untouched. For the dual of the wrapped D5 brane solution gauge fixed such that the remaining dual coordinates are v_2, v_3 and ψ, the canonical vielbeins given by the procedure of [16] are

$$
\begin{aligned}
e^{\hat{1}'} &= \frac{e^{g+\frac{\Phi}{2}}}{8\mathcal{W}}\big[e^{2k+\Phi}\big(-\sqrt{2}e^{2g+\Phi}\big(\cos\psi(a\omega_2 v_3 + dv_2) + \sin\psi(a\omega_1 v_3 - \omega_3 v_2)\big) \\
&\quad - 4v_3\sin\psi(a\omega_2 v_3 + dv_2) + 4v_3\cos\psi(a\omega_1 v_3 - \omega_3 v_2)\big) \\
&\quad - 4v_2 e^{2g+\Phi}\sin\psi(a\omega_2 v_2 - dv_3) - 8\sqrt{2}v_2\cos\psi(v_2 dv_2 + v_3 dv_3)\big] \\
e^{\hat{2}'} &= \frac{e^{g+\frac{\Phi}{2}}}{8\mathcal{W}}\big[e^{2k+\Phi}\big(\sqrt{2}e^{2g+\Phi}\big(\cos\psi(\omega_3 v_2 - a\omega_1 v_3) + a\omega_2 v_3\sin\psi + dv_2\sin\psi\big) \\
&\quad - 4v_3\big(\cos\psi(a\omega_2 v_3 + dv_2) + \sin\psi(a\omega_1 v_3 - \omega_3 v_2)\big)\big) \\
&\quad - 4v_2 e^{2g+\Phi}\cos\psi(a\omega_2 v_2 - dv_3) + 8\sqrt{2}v_2\sin\psi(v_2 dv_2 + v_3 dv_3)\big] \\
e^{\hat{3}'} &= \frac{e^{k+\frac{\Phi}{2}}}{8\mathcal{W}}\big[\sqrt{2}e^{4g+2\Phi}(a\omega_2 v_2 - dv_3) + 4v_2 e^{2g+\Phi}(\omega_3 v_2 - a\omega_1 v_3) \\
&\quad - 8\sqrt{2}v_3(v_2 dv_2 + v_3 dv_3)\big]
\end{aligned}
\tag{B.10}
$$

with the remaining veilbeins still given by Eq. (B.3), that is $e^{a'} = e^a$ for $a \neq 1, 2, 3$. The ω_i are defined as in Eq. (B.2) but with $\tilde{\theta} \to \theta$, $\tilde{\varphi} \to \varphi$. It is possible to remove all the explicit angular dependence from the dual solution by performing a rotation in the θ, φ directions such that

$$
\begin{aligned}
e^{\hat{\theta}} &= e^{h+\Phi/2}\omega_1 = \cos\psi\, e^{\theta} + \sin\psi\, e^{\varphi} \\
e^{\hat{\varphi}} &= e^{h+\Phi/2}\omega_2 = -\sin\psi\, e^{\theta} + \cos\psi\, e^{\varphi},
\end{aligned}
\tag{B.11}
$$

and an additional rotation in $1', 2', 3'$ directions such that

$$
\begin{aligned}
e^{\hat{1}} &= \cos\psi\, e^{1'} - \sin\psi\, e^{2'} \\
e^{\hat{2}} &= \sin\psi\, e^{1'} + \cos\psi\, e^{2'} \\
e^{\hat{3}} &= e^{3'}.
\end{aligned}
\tag{B.12}
$$

Theses rotation make the expressions for the vielbeins and fluxes a lot more simple than they otherwise would be, they are given for the dual of the wrapped D5 solution as in Section 3 but with $\mathcal{S} = 0$, $\mathcal{C} = 1$. However, it is the $e^{a'}$ vielbeins rather than the $e^{\hat{a}}$ ones that are more suited to calculating the G-structure of the dual solution.

It was shown explicitly in [18] that the 10d MW Killing spinors transform under an $SU(2)$ isometry T-duality as

$$
\hat{\epsilon}_1 = \epsilon_1, \qquad \hat{\epsilon}_2 = \Omega\epsilon_2. \tag{B.13}
$$

where Ω is given by

$$
\Omega = \Gamma^{(10)}\frac{-\Gamma_{123} + \sum_{a=1}^{3}\zeta_a\Gamma^a}{\sqrt{1+\zeta^2}} \tag{B.14}
$$

and for the wrapped D5 background we have

$$
\begin{aligned}
&\zeta^1 = 2\sqrt{2}e^{-g-k-\phi}v_2\cos\psi, \qquad \zeta^2 = -2\sqrt{2}e^{-g-k-\phi}v_2\sin\psi, \\
&\zeta^3 = 2\sqrt{2}e^{-2g-\phi}v_3.
\end{aligned}
\tag{B.15}
$$

Starting from Eq. (B.10) we first rotate the veilbeins as in Eq. (B.8) so that the projections are canonical. The Ω matrix then becomes

$$\Omega = \frac{1}{\sqrt{1+\zeta\cdot\zeta}}\left(\cos\alpha\hat{\Gamma}^{123} + \sin\alpha\hat{\Gamma}^{1\varphi 3} + \zeta_1\hat{\Gamma}^1 + \zeta_2\cos\alpha\hat{\Gamma}^2 + \zeta_2\sin\alpha\hat{\Gamma}^\varphi + \zeta_3\hat{\Gamma}^3\right) \tag{B.16}$$

where we have used $\gamma^{1\varphi 3}\eta_+ = i\eta_-$. The new spinor $\hat{\epsilon}_2$ is:

$$\hat{\epsilon}_2 = e^{\Phi/4}\left(\zeta_+\otimes\hat{\eta}^2_- + \zeta_-\otimes\hat{\eta}^2_+\right) \tag{B.17}$$

where

$$\hat{\eta}^2_- = \frac{\cos\alpha\hat{\gamma}^r + \zeta_1\hat{\gamma}^1 + \zeta_2\cos\alpha\hat{\gamma}^2 + \zeta_3\hat{\gamma}^3 + \zeta_2\sin\alpha\hat{\gamma}^\varphi}{\sqrt{1+\zeta\cdot\zeta}}\eta_+ + i\frac{\sin\alpha}{\sqrt{1+\zeta.\zeta}}\eta_-. \tag{B.18}$$

It is clear here that, as long as $\sin\alpha\neq 0$, we are in the dynamical $SU(2)$-structure case, because $\alpha = \alpha(r)$. In order to simplify the expressions we perform another transformation of the vielbein basis:

$$R = \frac{1}{\sqrt{\Delta}}\begin{pmatrix} \cos\alpha & 0 & 0 & \zeta_1 & \zeta_2\cos\alpha & \zeta_3 \\ 0 & \sqrt{\Delta} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\Delta} & 0 & 0 & 0 \\ -\zeta_1 & 0 & 0 & \cos\alpha & \zeta_3 & -\zeta_2\cos\alpha \\ -\zeta_2\cos\alpha & 0 & 0 & -\zeta_3 & \cos\alpha & \zeta_1 \\ -\zeta_3 & 0 & 0 & \zeta^2\cos\alpha & -\zeta^1 & \cos\alpha \end{pmatrix} \tag{B.19}$$

where

$$\Delta = \cos^2\alpha + \zeta_1^2 + \zeta_2^2\cos^2\alpha + \zeta_3^2 \tag{B.20}$$

We define a new basis:

$$\tilde{e} = R.\hat{e} \tag{B.21}$$

where the order is $r\theta\varphi 123$. In terms of this new basis, the spinor is:

$$\tilde{\eta}^2_- = \left(\frac{\sqrt{\Delta}\tilde{\gamma}^r + \zeta_2\sin\alpha\tilde{\gamma}^\varphi}{\sqrt{1+\zeta\cdot\zeta}}\right)\eta_+ + i\frac{\sin\alpha}{\sqrt{1+\zeta\cdot\zeta}}\eta_- \tag{B.22}$$

And the projections in this basis are still:

$$\tilde{\gamma}_{\varphi\theta}\eta_+ = \tilde{\gamma}_{r3}\eta_+ = \tilde{\gamma}_{21}\eta_+ = i\eta_+ \tag{B.23}$$

Let us now express the forms of the geometric structure, following the conventions of Appendix A.

$$e^{2A} = e^\Phi$$

$$\theta_+ = 0 \qquad \theta_- = 0$$

$$k_\parallel = \frac{\sin\alpha}{\sqrt{1+\zeta\cdot\zeta}} \qquad k_\perp = \sqrt{\frac{\cos^2\alpha + \zeta\cdot\zeta}{1+\zeta.\zeta}}$$

$$z = w - i\,v = \frac{1}{\sqrt{\cos^2\alpha + \zeta\cdot\zeta}}\left(\sqrt{\Delta}\tilde{e}^3 + \zeta_2\sin\alpha\tilde{e}^\theta + i\left(\sqrt{\Delta}\tilde{e}^r + \zeta_2\sin\alpha\tilde{e}^\varphi\right)\right)$$

$$j = \tilde{e}^{r3} + \tilde{e}^{\varphi\theta} + \tilde{e}^{21} - v \wedge w$$
$$\omega = \frac{i}{\sqrt{\cos^2\alpha + \zeta\cdot\zeta}}\left(\sqrt{\Delta}\left(\tilde{e}^{\varphi} + i\tilde{e}^{\theta}\right) - \zeta_2 \sin\alpha\left(\tilde{e}^{r} + i\tilde{e}^{3}\right)\right) \wedge \left(\tilde{e}^{2} + i\tilde{e}^{1}\right) \tag{B.24}$$

which is a dynamical $SU(2)$-structure.

Appendix C. Details of the non-Abelian T-duality on the baryonic branch solution

In this section we give some details of the $SU(2)$ isometry T-dual of the baryonic branch of Klebanov–Strassler. This was originally derived in [16] with gauge fixing such that $v_1 = \varphi = \theta = 0$. The previous derivation indicated a departure in the T-dual from the log corrected AdS_5 asymptotics of the baryonic branch. Let as begin by giving some details of original calculation in our current conventions.

C.1. Dual of the baryonic branch without the shift in B_2

Once more we will start by specifying the dual vielbeins. The components

$$e^{x^i} = e^{\frac{\Phi}{2}}\hat{h}^{-\frac{1}{4}}dx^i, \qquad e^{\rho} = e^{\frac{\Phi}{2}+k}\hat{h}^{\frac{1}{4}}d\rho \tag{C.1}$$

do not change. The vielbeins in the θ, φ are also unchanged by the duality however we find it useful to introduce a rotation in e^{θ}, e^{φ} such that the dual solution has no explicit ψ dependence.

$$e^{\hat{\theta}} = \sqrt{\mathcal{C}}e^{h+\Phi/2}\omega_1, \qquad e^{\hat{\varphi}} = \sqrt{\mathcal{C}}e^{h+\Phi/2}\omega_2. \tag{C.2}$$

The vielbeins in the directions $\hat{1}, \hat{2}, \hat{3}$ can be compactly written in terms of the quantities defined as,

$$\begin{aligned}
&\mathcal{V}_3 = v_3 + \frac{e^{2g+\Phi}}{2\sqrt{2}}\mathcal{S}\cos\alpha,\\
&\Lambda = d\mathcal{V}_3 + \frac{e^{\Phi-2h}}{2\sqrt{2}}\mathcal{S}N_c\left(e^{2g} + 2e^{2h} - ae^{g}\left(be^{g} - 2e^{h}\cot\alpha\right)\right)d\rho,\\
&\mu_1 = ae^{g}\cos\alpha + 2e^{h}\sin\alpha.
\end{aligned} \tag{C.3}$$

With these, we have

$$\begin{aligned}
e^{\hat{1}} = {}& \frac{e^{g+\Phi/2}}{16\mathcal{W}}\sqrt{\mathcal{C}}\big[e^{2k+\Phi}\big(8\mathcal{V}_3(a\mathcal{V}_3\omega_1 - v_2\omega_3) - 2\sqrt{2}e^{2g+\Phi}\mathcal{C}(dv_2 + a\mathcal{V}_3\omega_2)\\
&- 2\sqrt{2}e^{g+\Phi}\mathcal{S}\mathcal{V}_3\mu_1\omega_1 + e^{3g+2\Phi}\mathcal{C}\mathcal{S}\mu_1\omega_2\big)\\
&+ 8v_2\big(e^{g+\Phi}v_2\mathcal{S}\mu_1\omega_2 - 2\sqrt{2}(\mathcal{V}_3\Lambda + v_2 dv_2)\big)\big],\\
e^{\hat{2}} = {}& \frac{e^{g+\Phi/2}}{16\mathcal{W}}\mathcal{C}^{3/2}\big[e^{2k}\big(-2\sqrt{2}e^{2g+\Phi}\mathcal{C}(a\mathcal{V}_3\omega_1 - v_2\omega_3) - 8\mathcal{V}_3(dv_2 + a\mathcal{V}_3\omega_2)\\
&+ e^{3g+2\Phi}\mathcal{C}\mathcal{S}\mu_1\omega_1 + 2\sqrt{2}e^{g+\Phi}\mathcal{S}\mathcal{V}_3\mu_1\omega_2\big) - 8e^{2g}v_2(-\Lambda + av_2\omega_2)\big],\\
e^{\hat{3}} = {}& \frac{e^{k+\Phi/2}}{16\mathcal{W}}\sqrt{\mathcal{C}}\big[e^{g+\Phi}v_2\big(\sqrt{2}e^{2g+\Phi}\mathcal{C}\big(ae^{g}\mathcal{C}\omega_2 + \mathcal{S}\mu_1\omega_1\big) - 4e^{g}\mathcal{C}(a\mathcal{V}_3\omega_1 - v_2\omega_3)\\
&+ 4\mathcal{S}\mathcal{V}_3\mu_1\omega_2\big) - \sqrt{2}\Lambda\big(e^{4g+2\Phi}\mathcal{C}^2 + 8\mathcal{V}_3^2\big) - 8\sqrt{2}v_2\mathcal{V}_3 dv_2\big],
\end{aligned} \tag{C.4}$$

where the rotation of Eq. (B.12) has been performed.[10] We will then have a metric that in terms of these vielbeins reads, $ds_{st}^2 = \sum_{i=1}^{10}(e^i)^2$. Notice that the quantity Λ in Eq. (3.4) will, when squared to construct the metric with the vielbeins above, imply the existence of crossed terms $g_{\rho v_3}$ and also the change of the asymptotic behaviour of $g_{\rho\rho}$ away from log corrected AdS$_5$.

In terms of these vielbeins, the NS two-form B_2 reads,

$$\begin{aligned}
\widehat{B}_2 = {} & -\frac{1}{4v_2}\big(2e^{-h}a\big(e^g v_2 e^{\hat\theta\hat 1} + e^k \mathcal{V}_3 e^{\hat\theta\hat 3}\big) - 4e^{k-g}\mathcal{V}_3 e^{\hat 1\hat 3} + \sqrt{2}\mathcal{C}e^{g+k+\Phi}e^{\hat 2\hat 3}\big) \\
& + \frac{\mathcal{S}}{\mathcal{C}}\bigg[\frac{\mathcal{V}_3 e^k}{2v_2}\big(ae^{-h}e^{r\hat\theta} - 2e^{-g}e^{r\hat 1}\big) + \frac{e^{g+k+\Phi-h}}{4\sqrt{2}V_2}\mathcal{C}\big(2e^{2h}e^{r\hat 2} + \mu_1 e^{\hat\theta\hat 1}\big) \\
& - \frac{e^{-h}}{2}\big(2e^h\cos\alpha - ae^g\sin\alpha\big)e^{\hat\theta\hat\varphi} + e^{\rho\hat 3} - \frac{e^{-h}}{2}\mu_1 e^{\hat\theta\hat 2}\bigg].
\end{aligned} \tag{C.5}$$

The dual dilaton is given by

$$\widehat{\Phi} = \Phi - \frac{1}{2}\ln\mathcal{W}, \qquad \mathcal{W} = \mathcal{C}\left(\frac{1}{8}e^{4g+2k+3\Phi}\mathcal{C}^2 + e^{2g+\Phi}v_2^2 + e^{2k+\Phi}\mathcal{V}_3^2\right). \tag{C.6}$$

And the RR sector is given by,

$$\begin{aligned}
F_0 = {} & \frac{N_c}{\sqrt{2}}, \\
F_2 = {} & -\frac{e^{-\Phi}}{4}N_c\mathcal{C}\big[2e^{-2h}\big(1+a^2-2ab\big)\mathcal{V}_3 e^{\hat\theta\hat\varphi} \\
& + e^{-g-h-k}\mathcal{C}(a-b)\big(\sqrt{2}e^{2g+k+\Phi}\big(e^{\hat\theta\hat 1} - e^{\hat\varphi\hat 2}\big) + 4e^k\mathcal{V}_3\big(e^{\hat\theta\hat 2} - e^{\hat\varphi\hat 1}\big) - 4v_2 e^g e^{\hat\varphi\hat 3}\big) \\
& - 8e^{-2g}\mathcal{V}_3 e^{\hat 1\hat 2} - 8e^{-g-k}v_2 e^{\hat 2\hat 3} - 2e^{-h-k}v_2 e^{r\hat\theta}\big] \\
& - \frac{\mathcal{S}e^{g-h}}{\sqrt{2}\mathcal{C}\sin\alpha}\big(N_c b + a\big(e^{2g}\cos^2\alpha - N_c\big) + e^{g+h}\sin 2\alpha\big)e^{\hat\theta\hat\varphi}, \\
F_4 = {} & \frac{e^{-g-h-k-\Phi}}{8\mathcal{C}}N_c\big[\mathcal{C}\big(1+a^2-2ab\big)e^{\hat\theta\hat\varphi} \wedge \big(\sqrt{w}e^{2g+k+\Phi-h}e^{\hat 1\hat 2} + 4e^{2g-h}e^{\hat 1\hat 3}\big)\mathcal{C}b' e^{r\hat\theta} \\
& \wedge \big(4e^k\mathcal{V}_3 e^{\hat 1\hat 3} - \sqrt{2}e^{2g+k+\Phi}e^{\hat 2\hat 3}\big) - 8e^g v_2(a-b)e^{\hat\theta\hat 1\hat 2\hat 3}e^{r\hat\varphi} \\
& \wedge \big(4e^g v_2 e^{\hat 1\hat 2} - b' e^k\big(\sqrt{2}e^{2g+\Phi}e^{\hat 1\hat 3} + 4\mathcal{V}_3 e^{\hat 2\hat 3}\big)\big)\big] \\
& - \frac{2\mathcal{S}e^{-g-h-k-\Phi}}{\mathcal{C}^2\sin\alpha}\big(a\big(e^{2g}\cos^2\alpha - N_c\big) + \big(N_c b + e^{g+h}\sin 2\alpha\big)\big) \\
& \times \big(\mathcal{V}_3 e^k e^{\hat\theta\hat\varphi\hat 1\hat 2} + v_2 e^g e^{\hat\theta\hat\varphi\hat 2\hat 3}\big).
\end{aligned} \tag{C.7}$$

We will now proceed to show that the bad asymptotic behaviour and off diagonal ρ terms of the metric are actually a gauge artefact.

[10] Actually this differs from [16] in orientation which can be compensated for via $\hat 1 \leftrightarrow \hat 2$.

C.2. The dual of the baryonic branch with the shift in B_2

The NS 2-from of the original solution contains the term

$$\tilde{B}_2 = -\frac{1}{2}e^{2k+\Phi}\mathcal{S}(\tilde{\omega}_3 + \cos\theta d\varphi)\wedge d\rho. \tag{C.8}$$

It is this term, when dualised, that gives rise to the undesirable behaviour as this will contribute to the dual metric in both $g_{\rho\rho}$ and $g_{\rho v3}$ via the dual vielbeins $e^{\hat{i}}$ which will have legs in ρ. This happens because of the $d\rho\wedge\tilde{\omega}_i$ term in $\tilde{B}_2$ which is not a spectator under the duality transformation.[11] However, one is always free to add an exact to the NS potential as this will not change the fluxes or metric of the original solution. Consider adding a closed form to the initial B_2

$$B_2 \to B_2 + d\big(\mathcal{Z}(r)(\tilde{\omega}_3 + \cos\theta d\varphi)\big) \tag{C.9}$$

This precisely cancels the effect of $\tilde{B}_2$ in the dual solution when $\mathcal{Z}' = -\frac{1}{2}\mathcal{S}e^{2k+\Phi}$ because

$$\begin{aligned}\tilde{B}_2 + d\big(\mathcal{Z}(r)\tilde{\omega}_3\big) &= -\mathcal{Z}(\tilde{\omega}_1\wedge\tilde{\omega}_2 + \sin\theta d\theta\wedge d\varphi)\\ &\quad + \frac{1}{2}\big(\mathcal{S}e^{2k+\Phi} + 2\mathcal{Z}'\big)d\rho\wedge(\tilde{\omega}_3 + \cos\theta d\varphi).\end{aligned} \tag{C.10}$$

As there is no longer a $d\rho\wedge\tilde{\omega}_i$ term in the NS 2 form before dualisation, the dual vielbeins will have no legs in ρ and so there will no longer be a modification to $g_{\rho\rho}$ and $g_{\rho v_3}$. The trade off is that the function $\mathcal{Z}$ will now enter into the dual solution.

We now once more follow the procedure of [16] with gauge fixing, as before, such that $v_1 = \varphi = \theta = 0$. We are lead to the dual vielbeins

$$\begin{aligned}e^{\hat{1}'} = \frac{e^{g+\frac{\Phi}{2}}}{8\mathcal{W}}\sqrt{\mathcal{C}}\Big[&e^{2k+\Phi}\big(-\sqrt{2}\mathcal{C}e^{2g+\Phi}\big(\cos\psi(a\omega_2\mathcal{H} + dv_2) + \sin\psi(a\omega_1\mathcal{H} - \omega_3 v_2)\big)\\ &- 4\mathcal{H}\sin\psi(a\omega_2\mathcal{H} + dv_2) + 4\mathcal{H}\cos\psi(a\omega_1\mathcal{H} - \omega_3 v_2)\big)\\ &- 4v_2\mathcal{C}e^{2g+\Phi}\sin\psi(a\omega_2 v_2 - dv_3) - 8\sqrt{2}v_2\cos\psi(v_2 dv_2 + \mathcal{H}dv_3)\\ &+ \frac{1}{2}\mu_1\mathcal{S}e^{g+\Phi}\big(8v_2^2\cos\psi\omega_2 + \mathcal{C}e^{2k+\Phi}\big(\cos\psi\big(\mathcal{C}e^{2g+\Phi}\omega_2 - 2\sqrt{2}\mathcal{H}\omega_1\big)\\ &+ \sin\psi\big(\mathcal{C}e^{2g+\Phi}\omega_1 + 2\sqrt{2}\mathcal{H}\omega_2\big)\big)\big)\Big]\end{aligned}$$

$$\begin{aligned}e^{\hat{2}'} = \frac{e^{g+\frac{\Phi}{2}}}{8\mathcal{W}}\sqrt{\mathcal{C}}\Big[&e^{2k+\Phi}\mathcal{C}\big(\sqrt{2}\mathcal{C}e^{2g+\Phi}\big(\cos\psi(\omega_3 v_2 - a\omega_1\mathcal{H}) + a\omega_2\mathcal{H}\sin\psi + dv_2\sin\psi\big)\\ &- 4\mathcal{H}\big(\cos\psi(a\omega_2\mathcal{H} + dv_2) + \sin\psi(a\omega_1\mathcal{H} - \omega_3 v_2)\big)\big)\\ &- 4v_2\mathcal{C}e^{2g+\Phi}\cos\psi(a\omega_2 v_2 - dv_3) + 8\sqrt{2}v_2\sin\psi(v_2 dv_2 + \mathcal{H}dv_3)\\ &+ \frac{1}{2}\mu_1\mathcal{S}e^{g+\Phi}\big(-8v_2\sin\psi\omega_2 + \mathcal{C}e^{2k+\Phi}\big(\big(\mathcal{C}e^{2g+\Phi}\omega_1 + 2\sqrt{2}\mathcal{H}\omega_2\big)\cos\psi\\ &- \big(\mathcal{C}e^{2g+\Phi}\omega_2 - 2\sqrt{2}\mathcal{H}\omega_1\big)\sin\psi\big)\big)\Big]\end{aligned}$$

[11] See Section 2 of [16] for details of how the initial B_2 enters into the definition of the dual vielbeins.

$$e^{\hat{3}'} = \frac{e^{k+\frac{\Phi}{2}}}{8\mathcal{W}}\sqrt{\mathcal{C}}\big[\sqrt{2}\mathcal{C}^2 e^{4g+2\Phi}(a\omega_2 v_2 - dv_3) + 4v_2\mathcal{C}e^{2g+\Phi}(\omega_3 v_2 - a\omega_1\mathcal{H}) - 8\sqrt{2}\mathcal{H}(v_2 dv_2 + \mathcal{H}dv_3) + \mu_1 v_2\mathcal{S}e^{g+\Phi}\big(4\mathcal{H}\omega_2 + \sqrt{2}\mathcal{C}e^{2g+\Phi}\omega_2\big)\big] \tag{C.11}$$

which upon rotating according to Eq. (B.12) give the vielbeins of Eq. (3.5).

A valid question at this point is whether there is a local diffeomorphism which maps us from the baryonic branch dual solution as defined in Appendix C.1 to the solution defined as in Section 3. The answer is yes, and it may be most easily found by comparing the dilaton as defined in Eqs. (3.7) and (C.6). Examining these makes it clear that one needs to transform $\mathcal{V}_3$ such that it is mapped to $\mathcal{H}$. This may be achieved with a transformation in v_3 only

$$v_3 \to v_3 + \sqrt{2}\mathcal{Z} \tag{C.12}$$

under this which

$$\mathcal{V}_3 \to \mathcal{H}, \qquad \Lambda \to dv_3 \tag{C.13}$$

and so vielbeins of Eq. (C.4) are mapped to those of Eq. (3.5). The map on the RR sector also follows trivially whilst the NS 2-form of Eq. (3.6) is mapped to that of Eq. (C.5) up to an exact.

So it is clear that one may "cure" the bad asymptotics and $g_{\rho v_3}$ mixing of Appendix C.1 either by a gauge transformation in the NS 2-from before dualisation, or by a local diffeomorphism on the dual coordinate v_3 after the duality procedure is performed.

C.3. Details of the dual baryonic branch structure

All that remains to compete the elucidation of the baryonic dual is to give supplementary details to Section 4 on the dynamical $SU(2)$ structure. Actually, the derivation of the structure is essentially the same as that of the dual of the wrapped D5 solution in Appendix B, so we will only focus on the differences here.

The 10d MW Killing spinors of baryonic branch obey the same projection as the wrapped D5 spinors (see Eq. (B.5)). However, whilst the internal spinors are still parallel, they now differ by a point-dependent phase $e^{i\zeta(r)} = \mathcal{C} + i\mathcal{S}$

$$\epsilon_1 = e^A\big(\xi_+ \otimes \big(e^{i\zeta(r)/2}\eta_+\big) + \xi_- \otimes \big(e^{-i\zeta(r)/2}\eta_-\big)\big),$$
$$\epsilon_2 = e^A\big(\xi_+ \otimes \big(e^{-i\zeta(r)/2}\eta_+\big) + \xi_- \otimes \big(e^{i\zeta(r)/2}\eta_-\big)\big) \tag{C.14}$$

where the Minkowski warp factor is now $e^{2A} = \frac{e^\Phi}{\mathcal{C}}$. We now follow the steps illustrated between Eqs. (B.7) and (B.9) such that the $SU(3)$-structure of the baryonic branch takes canonical form.

The dual 10d Killing spinors are given as in Eqs. (B.13), (B.14), however the ζ^a entering into their definition are now given by

$$\zeta^1 = \frac{2\sqrt{2}e^{-g-k-\phi}v_2\cos\psi}{\sqrt{\mathcal{C}}}, \qquad \zeta^2 = -\frac{2\sqrt{2}e^{-g-k-\phi}v_2\sin\psi}{\sqrt{\mathcal{C}}},$$
$$\zeta^3 = \frac{2\sqrt{2}e^{-2g-\phi}\mathcal{H}}{\sqrt{\mathcal{C}}}. \tag{C.15}$$

The new spinor $\hat{\epsilon}_2$ is:

$$\hat{\epsilon}_2 = \frac{e^{\Phi/2}}{\sqrt{\mathcal{C}}}\big(\zeta_+ \otimes \big(e^{-i\zeta(r)/2}\hat{\eta}^2_-\big) + \zeta_- \otimes \big(e^{i\zeta(r)/2}\hat{\eta}^2_+\big)\big) \tag{C.16}$$

where $\hat{\eta}^2_-$ is still given by Eq. (D.26).

The dynamic $SU(2)$-structure supported by the dual baryonic branch solution may be expressed as

$$\Phi_+ = \frac{e^A}{8} e^{-iv\wedge w}\big(k_\parallel e^{-ij} - ik_\perp \omega\big),$$
$$\Phi_- = \frac{ie^A}{8} e^{i\zeta(r)}(v + iw) \wedge \big(k_\perp e^{-ij} + ik_\parallel \omega\big). \tag{C.17}$$

The forms and functions entering into these expressions are given by

$$\begin{aligned}
e^{2A} &= \frac{e^\Phi}{\mathcal{C}}\\
e^{i\zeta(r)} &= \mathcal{C} + i\mathcal{S}\\
k_\parallel &= \frac{\sin\alpha}{\sqrt{1+\zeta\cdot\zeta}}, \qquad k_\perp = \sqrt{\frac{\cos^2\alpha + \zeta\cdot\zeta}{1+\zeta\cdot\zeta}}\\
z &= w - i\,v = \frac{1}{\sqrt{\cos^2\alpha + \zeta\cdot\zeta}}\big(\sqrt{\Delta}\tilde{e}^3 + \zeta_2 \sin\alpha \tilde{e}^\theta + i\big(\sqrt{\Delta}\tilde{e}^\rho + \zeta_2\sin\alpha\tilde{e}^\varphi\big)\big)\\
j &= \tilde{e}^{\rho 3} + \tilde{e}^{\varphi\theta} + \tilde{e}^{21} - v\wedge w\\
\omega &= \frac{i}{\sqrt{\cos^2\alpha + \zeta\cdot\zeta}}\big(\sqrt{\Delta}\big(\tilde{e}^\varphi + i\tilde{e}^\theta\big) - \zeta_2 \sin\alpha\big(\tilde{e}^\rho + i\tilde{e}^3\big)\big) \wedge \big(\tilde{e}^2 + i\tilde{e}^1\big),
\end{aligned} \tag{C.18}$$

with ζ^a defined by (C.15). Specifically the vielbeins $\tilde{e}$ that the structure is expressed in terms of a rotation of those in Eq. (C.11). First one preforms a rotation by α

$$\begin{aligned}
\hat{e}^\varphi &= \cos\alpha e^\varphi + \sin\alpha e^{2'}\\
\hat{e}^2 &= -\sin\alpha e^\varphi + \cos\alpha e^{2'}\\
\hat{e}^a &= e^a \quad \text{for } a \neq \varphi, 2',
\end{aligned} \tag{C.19}$$

and then rotates these vielbeins to get $\tilde{e} = R\hat{e}$, where the matrix R is given by Eq. (B.19) with ζ^a by Eq. (C.15).

C.4. Details of the UV and IR asymptotics

The dual vielbeins in the IR tend to

$$\begin{aligned}
e^{\hat{1}} = &-\frac{32 e^{\Phi_0/2}\sqrt{\mathcal{F}} h_1^{3/2}}{e^{2\Phi_0}\mathcal{F}^2 + 128 h_1^2(v_2^2 + v_3^2)}\bigg(v_3(dv_2 + v_2\omega_3) + v_2\bigg(v_2\omega_2 - \frac{1}{2\sqrt{2}} dv_3\bigg)\\
&- v_3^2(\omega_1 - \omega_2)\bigg)\\
e^{\hat{2}} = &-\frac{2 e^{\Phi_0/2}\sqrt{\mathcal{F}}\sqrt{h_1}}{e^{2\Phi_0}\mathcal{F}^2 + 128 h_1^2(v_2^2 + v_3^2)}\big(\sqrt{2} v_3 \mathcal{F} e^{3\Phi_0}\omega_1 - \sqrt{2} v_2 \mathcal{F} e^{\Phi_0}\omega_3\\
&+ 16 h_1\big(v_3 dv_2 - v_2 dv_3 + \big(v_2^2 + v_3^2\big)\omega_2\big)\big)
\end{aligned}$$

$$e^{\hat{3}} = -\frac{2e^{-\Phi_0/2}\sqrt{\frac{h_1}{\mathcal{F}}}}{e^{2\Phi_0}\mathcal{F}^2 + 128h_1^2(v_2^2+v_3^2)}\bigg(\sqrt{2}\mathcal{F}^2 e^{2\Phi_0}\bigg(\frac{1}{2\sqrt{2}}dv_3 - v_2\omega_2\bigg) - 16h_1v_2\mathcal{F}(v_2\omega_3 - v_3\omega_1) + \sqrt{2}128h_1^2v_3(v_2dv_2 + v_3dv_3)\bigg) \tag{C.20}$$

where we have defined

$$\mathcal{F}^2 = 4(2)^{3/2}\big(h_1^{5/2} - 2\sqrt{2}e^{\Phi_0}h_1\big) \tag{C.21}$$

for convenience. The function $\mathcal{W}$ tends to

$$\frac{\mathcal{F}e^{\Phi_0}}{512h_1}\big(\mathcal{F}^2e^{2\Phi_0} + 128\big(v_2^2+v_3^2\big)\big). \tag{C.22}$$

Appendix D. Details of the non-Abelian T-duality on D6 branes on S^3

In this appendix, we study another background, similar to the one described in the main part of this paper. We want to start with a solution of D6-branes wrapping a three-sphere in Type IIA supergravity, that preserves $\mathcal{N}=1$ supersymmetry. We first describe such a solution, then we apply a non-Abelian T-duality to find a new Type IIB supergravity solution. We study this transformation at the level of the geometric structure. We then take advantage of this example to make general statements on $\mathcal{N}=1$ Type IIB supergravity solutions.

D.1. The Type IIA solution

We are interested in finding a solution of D6-branes in Type IIA supergravity. For that purpose, we start by considering eleven-dimensional supergravity. Because we only want D6-branes, the M-theory solution is a background with no fluxes. Such a solution is described in [48] or [49] (we follow the notation of the latter). The metric of the solution is:

$$ds_{11}^2 = dx_{1,3}^2 + ds_7^2, \tag{D.1}$$

where the seven-dimensional internal space has the metric

$$ds_7^2 = dr^2 + a^2\big[(\S_1 + g\sigma_1)^2 + (\S_2 + g\sigma_2)^2\big] + b^2\big(\sigma_1^2+\sigma_2^2\big) + c^2(\S_3 + g_3\sigma_3)^2 + f^2\sigma_3^2, \tag{D.2}$$

with a, b, c, f, g, g_3 all functions of the radial coordinate r. Here the $\S, \sigma$ are left-invariant $SU(2)$ forms:

$$\begin{aligned} \sigma_1 &= \cos\psi_1 + \sin\psi_1\sin\theta d\varphi, & \S_1 &= \cos\psi_2 + \sin\psi_2\sin\tilde{\theta}d\tilde{\varphi},\\ \sigma_2 &= -\sin\psi_1 + \cos\psi_1\sin\theta d\varphi, & \S_1 &= -\sin\psi_2 + \cos\psi_2\sin\tilde{\theta}d\tilde{\varphi},\\ \sigma_3 &= d\psi_1 + \cos\theta d\varphi, & \S_3 &= d\psi_2 + \cos\tilde{\theta}d\tilde{\varphi}. \end{aligned} \tag{D.3}$$

The BPS equations of this solution give [50]

$$\begin{aligned} g &= -\frac{af}{2bc}, \qquad g_3 = 2g^2 - 1,\\ a' &= -\frac{c}{2a} + \frac{a^5f^2}{8b^4c^3}, \qquad b' = -\frac{c}{2b} - \frac{a^2(a^2-3c^2)f^2}{8b^3c^3},\\ c' &= -1 + \frac{c^2}{2a^2} + \frac{c^2}{2b^2} - \frac{3a^2f^2}{8b^4}, \qquad f' = -\frac{a^4f^3}{4b^4c^3}. \end{aligned} \tag{D.4}$$

To get a ten-dimensional solution, we reduce the solution above along a $U(1)$ isometry. To accomplish our goal of getting D6-branes wrapping a three-sphere, we choose the isometry generated by the Killing vector $\partial_{\psi_1}+\partial_{\psi_2}$. After some algebra, we get the following Type IIA solution in string frame:

$$
\begin{aligned}
ds_{10}^2 &= \alpha' g_s N e^{2A}\bigg[\frac{\mu}{\alpha' g_s N}dx_{1,3}^2+dr^2+b^2\big(d\theta^2+\sin^2\theta d\varphi^2\big)+a^2(\omega_1+gd\theta)^2\\
&\quad +a^2(\omega_2+g\sin\theta d\varphi)^2+h^2(\omega_3-\cos\theta d\varphi)^2\bigg],\\
h^2 &= \frac{c^2f^2}{f^2+c^2(1+g_3)^2},\\
e^{4\Phi/3} &= \frac{c^2f^2}{4(g_sN)^{2/3}h^2},\\
e^{4A} &= \frac{c^2f^2}{4h^2},\\
\frac{F_2}{\sqrt{\alpha'}g_sN} &= -(1+K)\sin\theta d\theta\wedge d\varphi+(K-1)\omega_1\wedge\omega_2-K'dr\wedge(\omega_3-\cos\theta d\varphi),\\
K &= \frac{f^2-c^2(1-g_3^2)}{f^2+c^2(1+g_3)^2},
\end{aligned}
\tag{D.5}
$$

where the ω are defined as §, replacing ψ_2 with $\psi=\psi_2-\psi_1$.

D.2. Non-Abelian T-dual

Let us now take the solution from the previous section, and apply a non-Abelian T-duality on the $SU(2)$ isometry parametrised by the ω. We follow Section 2 of [16] and fix the gauge as $\tilde{\theta}=\tilde{\varphi}=v_1=0$. We obtain a Type IIB supergravity solution. The metric, in string frame, is given by:

$$
\begin{aligned}
ds_{IIB,st}^2 &= e^{2A}\big[dx_{1,3}^2+Ndr^2+Nb^2\big(d\theta^2+\sin^2\theta d\varphi^2\big)\big]+\frac{1}{\det M}\big[2(v_3dv_2+v_3dv_3)^2\\
&\quad +4a^2e^{4A}N^2\big(g^2\big(a^2v_2^2(\hat{\omega}_2)^2+h^2v_3^2\big((\hat{\omega}_1)^2+(\hat{\omega}_2)^2\big)\big)-a^2dv_3(dv_3-2gv_2\hat{\omega}_2)\\
&\quad +2gh^2v_3\hat{\omega}_2dv_2+h^2dv_2^2-2gh^2v_2v_3\hat{\omega}_1\hat{\omega}_3+h^2v_2^2(\hat{\omega}_3)^2\big)\big],
\end{aligned}
\tag{D.6}
$$

where

$$\det M = 4e^{2A}\big(2a^4h^2e^{4A}+a^2v_2^2+h^2v_3^2\big), \tag{D.7}$$

and

$$
\begin{aligned}
&\hat{\omega}_1=\cos\psi\, d\theta-\sin\psi\sin\theta\, d\varphi, \qquad \hat{\omega}_2=-\sin\psi\, d\theta-\cos\psi\sin\theta\, d\varphi,\\
&\hat{\omega}_3=d\psi-\cos\theta d\varphi.
\end{aligned}
\tag{D.8}
$$

The dual dilaton $\hat{\Phi}$ is defined through

$$e^{-2\hat{\Phi}}=\det M e^{-2\Phi}, \tag{D.9}$$

and the two-form potential as

$$B_2 = -\cos\theta d\varphi \wedge dv_3 + \frac{4\sqrt{2}a^4 g h^2 e^{6A} N^3}{\det M}\big(\hat{\omega}_1 \wedge dv_2 + (g v_3\hat{\omega}_1 - v_2\hat{\omega}_3)\wedge\hat{\omega}_2\big)$$
$$\times \frac{2\sqrt{2}v_2 e^{2A} N}{\det M}\big(\hat{\omega}_3 \wedge \big(h^2 v_3 dv_2 - a^2 v_2 dv_3\big) + a^2 g\hat{\omega}_1 \wedge (v_2 dv_2 + v_3 dv_3)\big). \quad \text{(D.10)}$$

The RR sector has all possible fluxes turned on. F_1 and F_5 can be expressed as follows:

$$F_1 = 2N\big(v_3 dr K' + (K-1)dv_3\big),$$
$$F_5 = -\frac{2a^2 h U e^{6A} N^2}{b^2 \det M}\big(\sqrt{2}\det M\,\mathrm{vol}_4 \wedge dr - 4N^2 a^2 b^2 h v_2 \hat{\omega}_1 \wedge \hat{\omega}_2 \wedge \hat{\omega}_3 \wedge dv_2 \wedge dv_3\big),$$
$$U = g^2(K-1) - (K+1). \quad \text{(D.11)}$$

F_3 is considerably more complicated:

$$\begin{aligned}F_3 = \frac{\sqrt{2}N}{\det M}\big[&8N^3 a^4 h^2 e^{6A}\big(v_3\big(g^2(K-1)+K+1\big)\hat{\omega}_1\wedge\hat{\omega}_2\wedge dv_3\\
&+K'\big(g v_3(\hat{\omega}_1\wedge dv_2\wedge dr + g v_3\hat{\omega}_1\wedge\hat{\omega}_2\wedge dr + v_2\hat{\omega}_2\wedge\hat{\omega}_3\wedge dr)\\
&- v_2\hat{\omega}_3\wedge dv_2\wedge dr\big) + g(K-1)(\hat{\omega}_1\wedge dv_2\wedge dv_3 + v_2\hat{\omega}_2\wedge\hat{\omega}_3\wedge dv_3)\big)\\
&+4e^{2A}N\big((K-1)v_2\big(a^2 g v_2\hat{\omega}_1\wedge dv_2\wedge dv_3 + h^2 v_3\hat{\omega}_3\wedge dv_2\wedge dv_3\big)\\
&+a^2 v_2 K'\big(g v_2 v_3\hat{\omega}_1\wedge dv_2\wedge dr + g v_3^2\hat{\omega}_1\wedge dv_3\wedge dr\\
&- v_2(v_2\hat{\omega}_3\wedge dv_2\wedge dr + v_3\hat{\omega}_3\wedge dv_3\wedge dr)\big)\\
&+(K+1)v_3\big(a^2 v_2^2 + h^2 v_3^2\big)\hat{\omega}_1\wedge\hat{\omega}_2\wedge dv_3\big)\\
&+\det M(K+1)v_2\hat{\omega}_1\wedge\hat{\omega}_2\wedge dv_2\big].\end{aligned} \quad \text{(D.12)}$$

D.3. Spinors and structure

In this section, we follow the conventions of Andriot's thesis [51] (see also [52]) for the $SU(3)\times SU(3)$-structure. We start from the solution before T-duality, which has an $SU(3)$-structure. This is Type IIA supergravity so the spinors are of different chirality. The spinors of the original solution are:

$$\epsilon_1 = e^{\Phi/6}(\zeta_+\otimes\eta_+ + \zeta_-\otimes\eta_-),$$
$$\epsilon_2 = e^{\Phi/6}(\zeta_+\otimes\eta_- + \zeta_-\otimes\eta_+). \quad \text{(D.13)}$$

They define the following $SU(3)$-structure:

$$J = e^{r3} + \big(\alpha e^2 + \beta e^{\varphi}\big)\wedge e^{\theta} + \big(\alpha e^{\varphi} - \beta e^2\big)\wedge e^1,$$
$$\Omega = \big(e^r + ie^3\big)\wedge\big(\alpha e^2 + \beta e^{\varphi} + ie^{\theta}\big)\wedge\big(\alpha e^{\varphi} - \beta e^2 + ie^1\big), \quad \text{(D.14)}$$

where

$$\alpha(r) = \frac{ag}{\sqrt{b^2+a^2g^2}}, \qquad \beta(r) = \frac{b}{\sqrt{b^2+a^2g^2}}, \qquad \alpha^2+\beta^2 = 1, \quad \text{(D.15)}$$

in terms of the vielbein basis:

$$\begin{aligned}
&e^r = e^{\Phi/3}dr, \qquad e^\theta = e^{\Phi/3}b\,d\theta, \qquad e^\varphi = e^{\Phi/3}b\sin\theta d\varphi,\\
&e^1 = e^{\Phi/3}a(\omega_1 + g\,d\theta), \qquad e^2 = e^{\Phi/3}a(\omega_1 + g\sin\theta d\varphi),\\
&e^3 = e^{\Phi/3}h(\omega_3 - \cos\theta d\varphi).
\end{aligned}\tag{D.16}$$

Let us rotate this veilbein basis to put the structure in its canonical form:

$$\begin{aligned}
&\hat{e}^\varphi = \beta e^\varphi + \alpha e^2,\\
&\hat{e}^2 = \alpha e^\varphi - \beta e^2,\\
&\hat{e}^a = e^a \quad \text{for } a \neq \varphi, 2.
\end{aligned}\tag{D.17}$$

It is a rotation since $\alpha^2 + \beta^2 = 1$, but it reverses the orientation. With respect to this new basis, the structure is expressed as:

$$\begin{aligned}
&\hat{J} = \hat{e}^{r3} + \hat{e}^{\varphi\theta} + \hat{e}^{21},\\
&\hat{\Omega} = \left(\hat{e}^r + i\hat{e}^3\right)\wedge\left(\hat{e}^\varphi + i\hat{e}^\theta\right)\wedge\left(\hat{e}^2 + i\hat{e}^1\right).
\end{aligned}\tag{D.18}$$

That means that the spinors obey the following projections:

$$\hat{\gamma}_{\varphi\theta}\eta_+ = \hat{\gamma}_{r3}\eta_+ = \hat{\gamma}_{21}\eta_+ = i\eta_+,\tag{D.19}$$

where the $\hat{\gamma}$ matrices are defined in terms of the rotated vielbein basis.

Let us now look at the non-Abelian T-duality. We know that the spinors transform in the following way:

$$\tilde{\epsilon}_1 = \epsilon_1, \qquad \tilde{\epsilon}_2 = \Omega\epsilon_2.\tag{D.20}$$

Ω here is defined as:

$$\Omega = \frac{\Gamma^{(10)}}{\sqrt{1+\zeta\cdot\zeta}}\left(-\Gamma^{123} + \zeta_1\Gamma^1 + \zeta_2\Gamma^2 + \zeta_3\Gamma^3\right),\tag{D.21}$$

where the ζ_a are given by

$$\zeta_1 = -\frac{e^{-2\Phi/3}v_2\cos\psi}{\sqrt{2}Nah}, \qquad \zeta_2 = \frac{e^{-2\Phi/3}v_2\sin\psi}{\sqrt{2}Nah}, \qquad \zeta_3 = -\frac{e^{-2\Phi/3}v_3}{\sqrt{2}Na^2}.\tag{D.22}$$

We are now going to consider the space after T-duality. The value for Ω above is written in the vielbein basis obtained directly from T-duality of the original basis (D.16) without any rotation. To make things simpler, we are going to perform the same rotation with α, β on this basis as before the T-duality (see (D.17)), but we do not perform any rotation in ψ. We call this new basis $\check{e}$. It is defined in terms of the coordinate of the T-dual background as follows:

$$\begin{aligned}
&\check{e}^r = e^{\Phi/3}dr, \qquad \check{e}^\theta = e^{\Phi/3}b\,d\theta, \qquad \beta\check{e}^\varphi + \alpha\check{e}^2 = e^{\Phi/3}b\sin\theta d\varphi,\\
&\check{e}^1 = \frac{2\sqrt{N}e^{\phi/3}a}{\det M}\big[v_2\left(-\sqrt{2}v_3\cos\psi + 2e^{2\Phi/3}Na^2\sin\psi\right)dv_3\\
&\qquad - \left(\sqrt{2}v_2^2\cos\psi + 2e^{2\Phi/3}Nv_3h^2\sin\psi + 2\sqrt{2}e^{4\Phi/3}N^2a^2h^2\cos\psi\right)dv_2\\
&\qquad + 2e^{2\Phi/3}Ng\left(-v_2^2a^2\sin\psi\hat{\omega}_2 + v_3h^2\left(\sqrt{2}e^{2\Phi/3}Na^2\sin\theta d\varphi + v_3 d\theta\right)\right)\\
&\qquad + 2e^{2\Phi/3}Nv_2h^2\left(v_3\cos\psi - \sqrt{2}e^{2\Phi/3}Na^2\sin\psi\right)\hat{\omega}_3\big],
\end{aligned}$$

$$\begin{aligned}
\alpha\check{e}^{\varphi}-\beta\check{e}^{2}=\frac{2\sqrt{N}e^{\phi/3}a}{\det M}\big[&v_2\big(\sqrt{2}v_3\sin\psi+2e^{2\Phi/3}Na^2\cos\psi\big)dv_3\\
&+\big(\sqrt{2}v_2^2\sin\psi-2e^{2\Phi/3}Nv_3h^2\cos\psi+2\sqrt{2}e^{4\Phi/3}N^2a^2h^2\sin\psi\big)dv_2\\
&+2e^{2\Phi/3}Ng\big(-v_2^2a^2\cos\psi\hat{\omega}_2+v_3h^2\big(-\sqrt{2}e^{2\Phi/3}Na^2d\theta+v_3\sin\theta d\varphi\big)\big)\\
&+2e^{2\Phi/3}Nv_2h^2\big(v_3\sin\psi+\sqrt{2}e^{2\Phi/3}Na^2\cos\psi\big)\hat{\omega}_3\big],\\
\check{e}^{3}=\frac{2\sqrt{N}e^{\Phi/3}h}{\det M}\big[&-\sqrt{2}v_2v_3dv_2-\sqrt{2}\big(v_3^2+2e^{4\Phi/3}N^2a^4\big)dv_3\\
&+2e^{2\Phi/3}Nv_2a^2\big(-v_3g\hat{\omega}_1+\sqrt{2}e^{2\Phi/3}Na^2g\hat{\omega}_2+v_2\hat{\omega}_3\big)\big].
\end{aligned}\tag{D.23}$$

The projections obeyed by η_+ are still as in (D.19). In this new basis, the T-dual Ω becomes:

$$\Omega=\frac{1}{\sqrt{1+\zeta\cdot\zeta}}\big(-\alpha\check{\Gamma}^{1\varphi3}+\beta\check{\Gamma}^{123}+\zeta_1\check{\Gamma}^{1}-\zeta_2\beta\check{\Gamma}^{2}+\zeta_2\alpha\check{\Gamma}^{\varphi}+\zeta_3\check{\Gamma}^{3}\big)\check{\Gamma}^{(10)}.\tag{D.24}$$

So the new spinor $\tilde{\epsilon}_2$ is:

$$\tilde{\epsilon}_2=e^{\Phi/6}\big(\zeta_+\otimes\check{\eta}_+^2+\zeta_-\otimes\check{\eta}_-^2\big),\tag{D.25}$$

where

$$\check{\eta}_+^2=\frac{-\beta\check{\gamma}^{r}-\zeta_1\check{\gamma}^{1}+\zeta_2\beta\check{\gamma}^{2}-\zeta_3\check{\gamma}^{3}-\zeta_2\alpha\check{\gamma}^{\varphi}}{\sqrt{1+\zeta\cdot\zeta}}\eta_-+i\frac{\alpha}{\sqrt{1+\zeta\cdot\zeta}}\eta_+.\tag{D.26}$$

It is clear here that, as long as $\alpha\neq0$, we are in the general $SU(3)\times SU(3)$-structure case. In order to simplify the expressions, we are performing a transformation of the vielbein basis:

$$R=\frac{1}{\sqrt{\Delta}}\begin{pmatrix}\beta&0&0&\zeta_1&-\zeta_2\beta&\zeta_3\\0&\sqrt{\Delta}&0&0&0&0\\0&0&\sqrt{\Delta}&0&0&0\\-\zeta_1&0&0&\beta&\zeta_3&\zeta_2\beta\\\zeta_2\beta&0&0&-\zeta_3&\beta&\zeta_1\\-\zeta_3&0&0&-\zeta^2\beta&-\zeta^1&\beta\end{pmatrix}\tag{D.27}$$

where

$$\Delta=\beta^2+\zeta_1^2+\zeta_2^2\beta^2+\zeta_3^2.\tag{D.28}$$

We define a new basis:

$$\tilde{e}=R.\check{e}\tag{D.29}$$

In terms of this new basis, the spinor is:

$$\tilde{\eta}_+^2=-\left(\frac{\sqrt{\Delta}\tilde{\gamma}^{r}+\zeta_2\alpha\tilde{\gamma}^{\varphi}}{\sqrt{1+\zeta\cdot\zeta}}\right)\eta_-+i\frac{\alpha}{\sqrt{1+\zeta\cdot\zeta}}\eta_+\tag{D.30}$$

And the projections in this basis are still:

$$\tilde{\gamma}_{\varphi\theta}\eta_+=\tilde{\gamma}_{r3}\eta_+=\tilde{\gamma}_{21}\eta_+=i\eta_+\tag{D.31}$$

Let us now express the forms of the geometric structure, following the conventions of Andriot's thesis.

$$|a|^2 = e^{\Phi/3}$$
$$\theta_+ = \frac{\pi}{2}, \qquad \theta_- = -\frac{\pi}{2}$$
$$k_\parallel = \frac{\alpha}{\sqrt{1+\zeta\cdot\zeta}}, \qquad k_\perp = \sqrt{\frac{\beta^2+\zeta\cdot\zeta}{1+\zeta\cdot\zeta}}$$
$$z = w - iv = \frac{1}{\sqrt{\beta^2+\zeta\cdot\zeta}}\big(\sqrt{\Delta}\tilde{e}^r + \zeta_2\alpha\tilde{e}^\varphi - i\big(\sqrt{\Delta}\tilde{e}^3 + \zeta_2\alpha\tilde{e}^\theta\big)\big)$$
$$j = \tilde{e}^{r3} + \tilde{e}^{\varphi\theta} + \tilde{e}^{21} - v\wedge w$$
$$\omega = \frac{-i}{\sqrt{\beta^2+\zeta\cdot\zeta}}\big(\sqrt{\Delta}\big(\tilde{e}^\varphi + i\tilde{e}^\theta\big) - \zeta_2\alpha\big(\tilde{e}^r + i\tilde{e}^3\big)\big)\wedge\big(\tilde{e}^2 + i\tilde{e}^1\big) \tag{D.32}$$

In terms of those forms, the pure spinors are defined as:

$$\Phi_+ = \frac{|a|^2}{8}e^{i\theta_+}e^{-iv\wedge w}\big(k_\parallel e^{-ij} - ik_\perp\omega\big)$$
$$\Phi_- = \frac{i|a|^2}{8}e^{i\theta_-}(v+iw)\wedge\big(k_\perp e^{-ij} + ik_\parallel\omega\big) \tag{D.33}$$

Let us now look at the BPS equations of Type IIB supergravity in the general case of $SU(3)\times SU(3)$-structure, generalising the system of pure $SU(3)$-structure that exhibit a rotation.

Appendix E. BPS equations for a solution of Type IIB supergravity with a general *SU*(3) × *SU*(3)-structure

We again follow the conventions of Andriot's thesis in this section. We start with the following pure spinors:

$$\Phi_+ = \frac{e^A}{8}e^{i\theta_+}e^{-iv\wedge w}\big(k_\parallel e^{-ij} - ik_\perp\omega\big),$$
$$\Phi_- = \frac{e^A}{8}e^{i\theta_-}(v+iw)\wedge\big(ik_\perp e^{-ij} - k_\parallel\omega\big). \tag{E.1}$$

For Type IIB supergravity, the BPS equations are:

$$(d - H\wedge)\big(e^{2A-\phi}\Phi_-\big) = 0,$$
$$(d - H\wedge)\big(e^{A-\phi}\Re(\Phi_+)\big) = 0,$$
$$(d - H\wedge)\big(e^{3A-\phi}\Im(\Phi_+)\big) = \frac{e^{4A}}{8} *_6 (F_1 - F_3 + F_5). \tag{E.2}$$

Let us start with Φ_+. We have:

$$8e^{-A}\Re(\Phi_+) = k_\parallel\cos\theta_+\bigg[1 + (\tan\theta_+\chi + \lambda) - \frac{1}{2}\bigg(\chi + \frac{1-\sin\theta_+}{\cos\theta_+}\lambda\bigg)\wedge\bigg(\chi - \frac{1+\sin\theta_+}{\cos\theta_+}\lambda\bigg)\bigg],$$

$$8e^{-A}\Im(\Phi_+) = k_\parallel \sin\theta_+\biggl[1 - (\cot\theta_+ \chi - \lambda) - \frac{1}{2}\biggl(\chi + \frac{\cos\theta_+ + 1}{\sin\theta_+}\lambda\biggr) \wedge \biggl(\chi - \frac{\sin\theta_+}{\cos\theta_+ + 1}\lambda\biggr)\biggr], \tag{E.3}$$

where

$$\chi = j + v \wedge w + \frac{k_\perp}{k_\parallel}\Re(\omega), \qquad \lambda = \frac{k_\perp}{k_\parallel}\Im(\omega). \tag{E.4}$$

Notice that, because of the various relations between the structure forms ($j \wedge \omega = \omega \wedge \omega = 0$), we can use the following equations:

$$j \wedge \Re(\omega) = j \wedge \Im(\omega) = 0, \qquad \Re(\omega) \wedge \Im(\omega) = 0, \qquad \Re(\omega) \wedge \Re(\omega) = \Im(\omega) \wedge \Im(\omega). \tag{E.5}$$

Using those, we can get the following relation:

$$\lambda \wedge \lambda = k_\perp^2 \chi \wedge \chi. \tag{E.6}$$

From there, we derive our first set of BPS equations. $(d - H\wedge)(e^{A-\phi}\Re(\Phi_+)) = 0$ gives us

$$\begin{aligned}
&d\bigl[e^{2A-\phi}k_\parallel \cos\theta_+\bigr] = 0,\\
&d\bigl[e^{2A-\phi}k_\parallel \cos\theta_+(\tan\theta_+ \chi + \lambda)\bigr] - e^{2A-\phi}k_\parallel \cos\theta_+ H = 0,\\
&d\biggl[e^{2A-\phi}k_\parallel \cos\theta_+\biggl(\chi + \frac{1-\sin\theta_+}{\cos\theta_+}\lambda\biggr) \wedge \biggl(\chi - \frac{1+\sin\theta_+}{\cos\theta_+}\lambda\biggr)\biggr]\\
&\quad + 2e^{2A-\phi}k_\parallel \cos\theta_+ H \wedge (\tan\theta_+ \chi + \lambda) = 0.
\end{aligned} \tag{E.7}$$

From those, it is easy to see that $H = dB$ where:

$$B = \tan\theta_+ \chi + \lambda, \tag{E.8}$$

and the third equation simplifies into:

$$d\bigl[e^{4A-2\phi}\chi \wedge \chi\bigr] = 0. \tag{E.9}$$

Let us now turn to $(d - H\wedge)(e^{3A-\phi}\Im(\Phi_+)) = \frac{e^{4A}}{8} *_6 (F_1 - F_3 + F_5)$. We get

$$\begin{aligned}
&d\bigl[e^{4A-\phi}k_\parallel \sin\theta_+\bigr] = e^{4A} *_6 F_5,\\
&d\bigl[e^{4A-\phi}k_\parallel \sin\theta_+(\cot\theta_+ \chi - \lambda)\bigr] + e^{4A-\phi}k_\parallel \sin\theta_+ H = e^{4A} *_6 F_3,\\
&d\biggl[e^{4A-\phi}k_\parallel \sin\theta_+\biggl(\chi + \frac{\cos\theta_+ + 1}{\sin\theta_+}\lambda\biggr) \wedge \biggl(\chi - \frac{\sin\theta_+}{\cos\theta_+ + 1}\lambda\biggr)\biggr]\\
&\quad - 2e^{4A-\phi}k_\parallel \sin\theta_+ H \wedge (\cot\theta_+ \chi - \lambda) = -2e^{4A} *_6 F_1.
\end{aligned} \tag{E.10}$$

Using all the equations we have so far, we can rewrite the three-form ones as:

$$H = d\lambda + \frac{e^{\phi}\sin\theta_+}{k_\parallel}\big[*_6 F_3 + (*_6 F_5)\wedge\lambda\big] + \frac{e^{\phi}\cos\theta_+}{k_\parallel} d\big(e^{-\phi}k_\parallel \sin\theta_+\big)\wedge\chi,$$
$$e^{-2A}d\big(e^{2A}\chi\big) = \frac{e^{\phi}\cos\theta_+}{k_\parallel}\big[*_6 F_3 + (*_6 F_5)\wedge\lambda\big] - \frac{e^{\phi}\sin\theta_+}{k_\parallel} d\big(e^{-\phi}k_\parallel \sin\theta_+\big)\wedge\chi. \quad \text{(E.11)}$$

Those equations have been written in such a way as to make the limits for $\theta_+ \to 0, \pi/2$ obvious, and to give the equations of the rotation present in [28] when taking $k_\perp \to 0$, $k_\parallel \to 1$ (limit of $SU(3)$-structure). The last equation, involving $*_6 F_1$ can be rewritten in the following way:

$$\frac{1}{2}d\big(e^{-\phi}k_\parallel \sin\theta_+\big)\wedge\chi\wedge\chi = *_6 F_1 + (*_6 F_3)\wedge\lambda + (*_6 F_5)\wedge\lambda\wedge\lambda. \quad \text{(E.12)}$$

In summary, the BPS equations we get from Φ_+ are:

$$d\big[e^{2A-\phi}k_\parallel \cos\theta_+\big] = 0,$$
$$d\big[e^{4A-\phi}k_\parallel \sin\theta_+\big] = e^{4A} *_6 F_5,$$
$$H = d\lambda + \frac{e^{\phi}\sin\theta_+}{k_\parallel}\big[*_6 F_3 + (*_6 F_5)\wedge\lambda\big] + \frac{e^{\phi}\cos\theta_+}{k_\parallel} d\big(e^{-\phi}k_\parallel \sin\theta_+\big)\wedge\chi,$$
$$e^{-2A}d\big(e^{2A}\chi\big) = \frac{e^{\phi}\cos\theta_+}{k_\parallel}\big[*_6 F_3 + (*_6 F_5)\wedge\lambda\big] - \frac{e^{\phi}\sin\theta_+}{k_\parallel} d\big(e^{-\phi}k_\parallel \sin\theta_+\big)\wedge\chi,$$
$$d\big[e^{4A-2\phi}\chi\wedge\chi\big] = 0,$$
$$\frac{1}{2}d\big(e^{-\phi}k_\parallel \sin\theta_+\big)\wedge\chi\wedge\chi = *_6 F_1 + (*_6 F_3)\wedge\lambda + (*_6 F_5)\wedge\lambda\wedge\lambda. \quad \text{(E.13)}$$

Let us now look at the equations we get for Φ_-. We first define:

$$\xi = e^{i\theta_-}(v + iw),$$
$$\beta = j - \frac{k_\parallel}{k_\perp}\omega. \quad \text{(E.14)}$$

We get for the BPS equations, after some simplifications:

$$d\big[e^{3A-\phi}k_\perp \xi\big] = 0,$$
$$k_\perp(d\beta + iH)\wedge\xi = 0. \quad \text{(E.15)}$$

The equation we would get for the six-form is just the one for the four-form wedged with β, so it is not an additional independent equation.

It is quite easy to check that, taking the pure $SU(3)$ limit, that is $k_\parallel \to 0$, $k_\perp \to 1$, we recover the system we already knew from [28].

Finally, we want to explicitly specialise to the cases of $\theta_+ = 0$ and $\theta_+ = \pi/2$. First $\theta_+ = 0$:

$$d\big[e^{2A-\phi}k_\parallel\big] = 0,$$
$$F_5 = 0,$$
$$H = d\lambda,$$
$$e^{-2A}d\big[e^{2A}\chi\big] = \frac{e^{\phi}}{k_\parallel} *_6 F_3,$$
$$d\big[e^{4A-2\phi}\chi\wedge\chi\big] = 0,$$

$$\begin{aligned}
&*_6 F_1 + (*_6 F_3) \wedge \lambda = 0, \\
&d\left[e^{3A-\phi} k_\perp \xi\right] = 0, \\
&k_\perp (d\beta + iH) \wedge \xi = 0.
\end{aligned} \tag{E.16}$$

And, in the case $\theta_+ = \pi/2$:

$$\begin{aligned}
&d\left[e^{4A-\phi} k_\parallel\right] = e^{4A} *_6 F_5, \\
&H = \frac{e^\phi}{k_\parallel} *_6 F_3 + \frac{1}{e^{4A-\phi} k_\parallel} d\left[e^{4A-\phi} k_\parallel \lambda\right], \\
&d\left[e^{2A-\phi} k_\parallel \chi\right] = 0, \\
&d\left[e^{4A-2\phi} \chi \wedge \chi\right] = 0, \\
&\frac{1}{2} d\left(e^{-\phi} k_\parallel\right) \wedge \chi \wedge \chi = *_6 F_1 + (*_6 F_3) \wedge \lambda + (*_6 F_5) \wedge \lambda \wedge \lambda, \\
&d\left[e^{3A-\phi} k_\perp \xi\right] = 0, \\
&k_\perp (d\beta + iH) \wedge \xi = 0.
\end{aligned} \tag{E.17}$$

Those systems do not look much more complicated than the ones in the pure $SU(3)$ case, but there does not seem to be an easy transformation starting from either $\theta_+ = 0$ or $\theta_+ = \pi/2$ and recovering the full system.

Appendix F. Euclidean brane configurations in IIB geometries

Here we re-derive the embeddings corresponding to Baryon condensates in the IIB geometries found first in [39] using other means. We make use of the calibration condition (5.26). We first consider a supersymmetric configuration in the Klebanov–Witten theory. One finds the E5 configuration of a brane extended along $\Sigma = \{r, \theta_1, \phi_1, \theta_2, \phi_2, \psi\}$ with a world volume gauge field

$$\mathcal{A} = \frac{1}{3} \zeta(r)(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2), \tag{F.1}$$

obeys the calibration condition (5.26) provided that

$$\zeta \zeta' = \frac{1}{4} - \zeta^2, \tag{F.2}$$

which of course can be readily integrated.

Lets move up to the KT geometry working in the exact logarithmic solution.[12] Using the calibration technique one readily finds the E5 configuration is the same but with the gauge field equation of motion Eq. (F.2) modified to be

$$\zeta'(r) = \frac{2r^4 h(r) + T(r)^2 - 8\zeta(r)^2}{8r\zeta(r)}, \tag{F.3}$$

where $T(r) = \frac{9}{\sqrt{2}} M \log r/r_0$ and $h(r) = \frac{27}{32 r^4}(3M^2 + 8N\pi + 12M^2 \log r/r_0)$. This equation may be integrated to yield

[12] This is considerably simpler than the deformed conifold of the KS and reproduces all the main features of the calculation in [39] with the conformal dimension of the condensate agreeing to leading order.

$$\zeta(r) = \frac{9M}{8r\sqrt{2}}\left(c + 3r^2 - 4r^2\log(r) + 8r^2\log(r)^2\right)^{\frac{1}{2}}, \tag{F.4}$$

where c is a constant of integration which we now set to zero since its contributions are in any case sub-leading. Inserting this into to the DBI action one finds, changing variables to $t = \log r$,

$$S_{\mathrm{E5}} = \tau_5 \operatorname{vol}\left(T^{1,1}\right) \int^{t_{\mathrm{UV}}} dt \frac{27M^3}{64\sqrt{2}}\left(1 + 2t^2 + 8t^3\right)\left(3 - 4t + 8t^2\right)^{\frac{1}{2}}. \tag{F.5}$$

In [39], $e^{-S_{\mathrm{E5}}}$ was identified with the bulk field dual to the baryonic condensate. Using the standard asymptotic expansion the field theory scaling dimension can be extracted (at least in the large t regime) as

$$\Delta(r) = \frac{dS_{\mathrm{E5}}}{d\log r} = \frac{27}{16\pi^2}M^3(\log r)^2 + \mathcal{O}(\log r), \tag{F.6}$$

reproducing exactly the result of [39] notable for the scaling dimension dependence on the energy scale of the baryons as anticipated from the field theory.

References

[1] H.A. Kramers, G.H. Wannier, Phys. Rev. 60 (1941) 252.
[2] S.R. Coleman, Phys. Rev. D 11 (1975) 2088.
[3] C. Montonen, D.I. Olive, Phys. Lett. B 72 (1977) 117.
[4] N. Seiberg, E. Witten, Nucl. Phys. B 426 (1994) 19, arXiv:hep-th/9407087;
N. Seiberg, E. Witten, Nucl. Phys. B 430 (1994) 485 (Erratum).
[5] N. Seiberg, Nucl. Phys. B 435 (1995) 129, arXiv:hep-th/9411149.
[6] J.M. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, arXiv:hep-th/9711200.
[7] N. Itzhaki, J.M. Maldacena, J. Sonnenschein, S. Yankielowicz, Phys. Rev. D 58 (1998) 046004, arXiv:hep-th/9802042;
O. Aharony, arXiv:hep-th/0212193;
S.S. Gubser, C.P. Herzog, I.R. Klebanov, C. R. Phys. 5 (2004) 1031, arXiv:hep-th/0409186.
[8] X.C. de la Ossa, F. Quevedo, Nucl. Phys. B 403 (1993) 377, arXiv:hep-th/9210021;
E. Alvarez, L. Alvarez-Gaume, Y. Lozano, Nucl. Phys. B, Proc. Suppl. 41 (1995) 1, arXiv:hep-th/9410237.
[9] T.H. Buscher, Phys. Lett. B 201 (1988) 466.
[10] T.H. Buscher, Phys. Lett. B 194 (1987) 59.
[11] K. Sfetsos, D.C. Thompson, Nucl. Phys. B 846 (2011) 21, arXiv:1012.1320 [hep-th].
[12] Y. Lozano, E.O. Colgain, K. Sfetsos, D.C. Thompson, J. High Energy Phys. 1106 (2011) 106, arXiv:1104.5196 [hep-th].
[13] G. Itsios, Y. Lozano, E.O. Colgain, K. Sfetsos, J. High Energy Phys. 1208 (2012) 132, arXiv:1205.2274 [hep-th].
[14] Y. Lozano, E.O. Colgain, D. Rodriguez-Gomez, K. Sfetsos, Phys. Rev. Lett. 110 (2013) 231601, arXiv:1212.1043 [hep-th].
[15] G. Itsios, C. Nunez, K. Sfetsos, D.C. Thompson, Phys. Lett. B 721 (2013) 342, arXiv:1212.4840.
[16] G. Itsios, C. Nunez, K. Sfetsos, D.C. Thompson, Nucl. Phys. B 873 (2013) 1, arXiv:1301.6755 [hep-th].
[17] J. Jeong, O. Kelekci, E.O. Colgain, J. High Energy Phys. 1305 (2013) 079, arXiv:1302.2105 [hep-th].
[18] A. Barranco, J. Gaillard, N.T. Macpherson, C. Nunez, D.C. Thompson, J. High Energy Phys. 1308 (2013) 018, arXiv:1305.7229 [hep-th].
[19] E. Gevorgyan, G. Sarkissian, arXiv:1310.1264 [hep-th].
[20] N.T. Macpherson, J. High Energy Phys. 1311 (2013) 137, arXiv:1310.1609 [hep-th].
[21] Y. Lozano, E.O. Colgain, D. Rodriguez-Gomez, arXiv:1311.4842 [hep-th].
[22] N. Berkovits, J. Maldacena, J. High Energy Phys. 0809 (2008) 062, arXiv:0807.3196 [hep-th].
[23] I.R. Klebanov, M.J. Strassler, J. High Energy Phys. 0008 (2000) 052, arXiv:hep-th/0007191.
[24] S.S. Gubser, C.P. Herzog, I.R. Klebanov, J. High Energy Phys. 0409 (2004) 036, arXiv:hep-th/0405282.
[25] A. Butti, M. Grana, R. Minasian, M. Petrini, A. Zaffaroni, J. High Energy Phys. 0503 (2005) 069, arXiv:hep-th/0412187.

[26] J.M. Maldacena, C. Nunez, Phys. Rev. Lett. 86 (2001) 588, arXiv:hep-th/0008001;
R.P. Andrews, N. Dorey, Nucl. Phys. B 751 (2006) 304, arXiv:hep-th/0601098.
[27] J. Maldacena, D. Martelli, J. High Energy Phys. 1001 (2010) 104, arXiv:0906.0591 [hep-th];
E. Caceres, C. Nunez, L.A. Pando-Zayas, J. High Energy Phys. 1103 (2011) 054, arXiv:1101.4123 [hep-th];
D. Elander, J. Gaillard, C. Nunez, M. Piai, J. High Energy Phys. 1107 (2011) 056, arXiv:1104.3963 [hep-th].
[28] J. Gaillard, D. Martelli, C. Nunez, I. Papadimitriou, Nucl. Phys. B 843 (2011) 1, arXiv:1004.4638 [hep-th].
[29] E. Conde, J. Gaillard, C. Nunez, M. Piai, A.V. Ramallo, J. High Energy Phys. 1202 (2012) 145, arXiv:1112.3350 [hep-th].
[30] M.J. Strassler, arXiv:hep-th/0505153.
[31] A. Dymarsky, I.R. Klebanov, N. Seiberg, J. High Energy Phys. 0601 (2006) 155, arXiv:hep-th/0511254.
[32] R. Casero, C. Nunez, A. Paredes, Phys. Rev. D 73 (2006) 086005, arXiv:hep-th/0602027.
[33] C. Hoyos-Badajoz, C. Nunez, I. Papadimitriou, Phys. Rev. D 78 (2008) 086005, arXiv:0807.3039 [hep-th].
[34] R. Casero, C. Nunez, A. Paredes, Phys. Rev. D 77 (2008) 046003, arXiv:0709.3421 [hep-th].
[35] F. Benini, Y. Tachikawa, B. Wecht, J. High Energy Phys. 1001 (2010) 088, arXiv:0909.1327 [hep-th].
[36] I. Bah, C. Beem, N. Bobev, B. Wecht, J. High Energy Phys. 1206 (2012) 005, arXiv:1203.0303 [hep-th].
[37] O. Aharony, L. Berdichevsky, M. Berkooz, J. High Energy Phys. 1208 (2012) 131, arXiv:1206.5916 [hep-th].
[38] E. Witten, J. High Energy Phys. 9807 (1998) 006, arXiv:hep-th/9805112.
[39] M.K. Benna, A. Dymarsky, I.R. Klebanov, J. High Energy Phys. 0708 (2007) 034, arXiv:hep-th/0612136.
[40] O. Aharony, J. High Energy Phys. 0103 (2001) 012, arXiv:hep-th/0101013.
[41] L. Martucci, P. Smyth, J. High Energy Phys. 0511 (2005) 048, arXiv:hep-th/0507099.
[42] I.R. Klebanov, P. Ouyang, E. Witten, Phys. Rev. D 65 (2002) 105007, arXiv:hep-th/0202056.
[43] M. Krasnitz, J. High Energy Phys. 0212 (2002) 048, arXiv:hep-th/0209163.
[44] R. Apreda, F. Bigazzi, A.L. Cotrone, M. Petrini, A. Zaffaroni, Phys. Lett. B 536 (2002) 161, arXiv:hep-th/0112236.
[45] W. Mueck, J. High Energy Phys. 0302 (2003) 013, arXiv:hep-th/0301171.
[46] M.B. Green, C.M. Hull, P.K. Townsend, Phys. Lett. B 382 (1996) 65, arXiv:hep-th/9604119.
[47] B.S. Acharya, C. Vafa, arXiv:hep-th/0103011.
[48] A. Brandhuber, Nucl. Phys. B 629 (2002) 393, arXiv:hep-th/0112113.
[49] M. Cvetic, G.W. Gibbons, H. Lu, C.N. Pope, Phys. Lett. B 534 (2002) 172, arXiv:hep-th/0112138.
[50] M. Cvetic, G.W. Gibbons, H. Lu, C.N. Pope, Phys. Rev. Lett. 88 (2002) 121602, arXiv:hep-th/0112098.
[51] D. Andriot, PhD thesis, http://tel.archives-ouvertes.fr/docs/00/49/71/72/PDF/these5.pdf.
[52] D. Andriot, J. High Energy Phys. 0808 (2008) 096, arXiv:0804.1769 [hep-th].
[53] G. Itsios, K. Sfetsos, D.C. Thompson, in preparation.
[54] E. Caceres, N.T. Macpherson, C. Nunez, arXiv:1402.3294 [hep-th].