



Adesso, Gerardo and Piano, Samanta (2014) Theory of genuine tripartite nonlocality of Gaussian states. *Physical Review Letters*, 112 (1). 010401-1-010401-6. ISSN 1079-7114

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Theory of Genuine Tripartite Nonlocality of Gaussian States

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(Received 15 July 2013; published 6 January 2014)

We investigate the genuine multipartite nonlocality of three-mode Gaussian states of continuous variable systems. For pure states, we present a simplified procedure to obtain the maximum violation of the Svetlichny inequality based on displaced parity measurements, and we analyze its interplay with genuine tripartite entanglement measured via Rényi-2 entropy. The maximum Svetlichny violation admits tight upper and lower bounds at fixed tripartite entanglement. For mixed states, no violation is possible when the purity falls below 0.86. We also explore a set of recently derived weaker inequalities for three-way nonlocality, finding violations for all tested pure states. Our results provide a strong signature for the nonclassical and nonlocal nature of Gaussian states despite their positive Wigner function, and lead to precise recipes for its experimental verification.

DOI: [10.1103/PhysRevLett.112.010401](https://doi.org/10.1103/PhysRevLett.112.010401)

PACS numbers: 03.65.Ud, 03.67.Mn, 42.50.Dv

Quantum mechanics defies our intuition every day: at the level of a single system, with superposition phenomena [1] and at the level of composite systems, with correlations that are incompatible with the classical notion of locality [2–4]. In a hierarchy of possible manifestations of the quantum fabric of the world, nonlocality is perhaps the strongest one. States violating a Bell-type inequality [5] are necessarily entangled [6], therefore marking their departure from a classical description [7], while the converse is not always true [8]. In recent years, nonlocality has come to be appreciated as an operational resource [9], which plays an essential role, e.g., for the implementation of secure quantum key distribution [10]. The concept of multipartite nonlocality is, however, significantly less understood than its bipartite counterpart [3–5,9,11–17]. For a tripartite system, the conventionally employed inequality, whose violation signals a genuine three-way nonlocality, is due to Svetlichny [12]. Such an inequality can be violated, e.g., by both Greenberger-Horne-Zeilinger (GHZ) and W classes of states for three qubits [14,15], and is essentially *the* unique witness of nonlocality when all three parties perform two measurements with two outcomes each [17]. More recently, a reassessment of tripartite nonlocality has led to a series of weaker inequalities, whose violation can reveal tripartite nonlocality even when the Svetlichny one is not violated [9,17]. According to this framework, it has been conjectured that all fully inseparable tripartite pure states are three-way nonlocal [17]. In general, the interplay between the quantitative violation of these inequalities and the degree of genuine tripartite entanglement in the tested states remains unclear [14]. We recall that multipartite entanglement, i.e., entanglement encoded in systems of more than two parties, can arise in many inequivalent forms [6]. In qubit systems, for example, GHZ and W states are

both fully inseparable, but in GHZ states only genuinely tripartite entanglement is present, while W states have maximum bipartite entanglement between any pair of qubits. Generally, multipartite entanglement obeys strict monogamy constraints on its distribution [18].

Nonlocality tests have been studied also for continuous variable systems [19–27], namely systems whose canonical degrees of freedom, which can be nonclassically correlated, have a continuous spectrum [28,29]. This is the case, for instance, for quadrature modes of light, phononic momentum modes of Bose-Einstein condensates, vibrational modes of mechanical resonators, or collective spin components of cold atomic ensembles [30]. In typical studies of continuous variable nonlocality, measurements on these systems are binned to return a dichotomic result, so that the traditional format of Bell-type inequalities can be straightforwardly adopted [19]. Continuous variable systems constitute a powerful setting for the unconditional demonstration of quantum communication and cryptography protocols, often outperforming their qubit counterparts thanks to high quality feasible measurements, despite the unavailability of arbitrarily large entanglement [28]. A central class of continuous variable states is represented by Gaussian states, which include coherent, squeezed, and thermal states of harmonic Hamiltonians [29–31]. Notwithstanding their limitations [32], Gaussian states and operations are preferred resources that can be realized in a plethora of experimental platforms [30]. On the theoretical side, equally, considerable effort has been devoted to characterize their informational properties and correlation structure [29,31,33].

To the best of our knowledge, nevertheless, no systematic study of multipartite nonlocality of Gaussian states has been reported so far: the purpose of the present Letter is to

fill this gap. Investigating the nonlocal character of Gaussian states is of special importance, as according to some criteria Gaussian states are essentially classical, being associated to a positive Wigner distribution in phase space [34]. This is often seen as an argument against their use as genuine quantum resources, since computations involving Gaussian states can be simulated classically [35]. Similarly, Bell tests involving homodyne detections cannot be violated by Gaussian states [19]. However, a breakthrough came when displaced parity measurements (which are non-Gaussian operations) were considered to design continuous variable nonlocality tests [20]. The valuable feature of these measurements is that their expectation value is directly proportional to the Wigner distribution of the state at a given phase space point [20]. Using these measurements, bipartite nonlocality of Gaussian states was predicted theoretically [20–22]. Recently, a prescription to extend the Svetlichny inequality to the continuous variable domain using these measurements has been proposed [27]. This formalism was, however, applied in the Gaussian setting only to sparse, specialized examples [22,26,27,36].

Here we perform a comprehensive analysis of the phase space Svetlichny inequality [12,13,27] for three-mode Gaussian states [37,38]. We provide analytical prescriptions for detecting the maximum violation of the Svetlichny inequality for symmetric pure states. For generic nonsymmetric mixed states, we investigate numerically the interplay between tripartite nonlocality, tripartite entanglement [33], and state purity. We further consider alternative tests of tripartite nonlocality which exclude signalling correlations [17], and we find violations for all tested fully inseparable pure states. These results advance substantially our understanding of entanglement and nonlocality in multipartite infinite-dimensional systems.

Three-mode Gaussian states.—We consider an n -mode continuous variable system; we collect the quadrature operators in the vector $\hat{R} = (\hat{q}_1, \hat{p}_1, \hat{q}_2, \hat{p}_2, \dots, \hat{q}_n, \hat{p}_n)^T \in \mathbb{R}^{2n}$ and write the canonical commutation relations compactly as $[\hat{R}_j, \hat{R}_k] = i(\omega^{\oplus n})_{j,k}$ with $\omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ being the symplectic matrix. A Gaussian state ρ is represented by a positive, Gaussian Wigner distribution in phase space,

$$W_\rho(\xi) = \frac{1}{\pi^n \sqrt{\det \sigma}} \exp(-\xi^T \sigma^{-1} \xi), \quad (1)$$

where $\xi \in \mathbb{R}^{2n}$, and σ is the covariance matrix (CM) of the second moments $\sigma_{j,k} = \text{tr}[\rho\{\hat{R}_j, \hat{R}_k\}_+]$, which completely characterizes the state ρ up to local displacements [31]. The purity of the state is $\mu(\rho) = \text{tr}\rho^2 = (\det \sigma)^{-1/2}$.

For a three-mode ($n = 3$) state ρ , we have $\xi = (\xi_1, \xi_2, \xi_3) = (q_1, p_1, q_2, p_2, q_3, p_3)$, and $\sigma \equiv \sigma_{123}$ given in block form by

$$\sigma = \begin{pmatrix} \sigma_1 & \gamma_{12} & \gamma_{13} \\ \gamma_{12}^\top & \sigma_2 & \gamma_{23} \\ \gamma_{13}^\top & \gamma_{23}^\top & \sigma_3 \end{pmatrix}. \quad (2)$$

A *pure* three-mode Gaussian state can be transformed by local unitaries in a standard form [37] characterized by $\sigma_j = \text{diag}\{a_j, a_j\}$, $\gamma_{jk} = \text{diag}\{g_{jk}^+, g_{jk}^-\}$, with $g_{jk}^\pm = \{\sqrt{[(a_i - 1)^2 - (a_j - a_k)^2][(a_i + 1)^2 - (a_j - a_k)^2]} \pm \sqrt{[(a_i - 1)^2 - (a_j + a_k)^2][(a_i + 1)^2 - (a_j + a_k)^2]}\} / [4\sqrt{a_j a_k}]$, where $|a_j - a_k| + 1 \leq a_i \leq a_j + a_k - 1$ and $\{i, j, k\}$ is a permutation of $\{1, 2, 3\}$. Any pure state with $a_i > 1 \forall i = 1, 2, 3$ is fully inseparable and its genuine tripartite entanglement, as emerging from the monogamy inequality [18,33,37,39], can be quantified by the residual Rényi-2 entanglement entropy $\mathcal{E}_{1|23}(\sigma)$, whose explicit expression in terms of $a_{1,2,3}$ is reported in [33]. For *mixed* three-mode Gaussian states, there are more layers of separability, as classified in [40]; see also [37,38] for further details.

Phase space Svetlichny inequality.—Quantum nonlocality can be revealed on composite systems by letting every party perform a selection of different measurements on their subsystems, each with two or more possible outcomes. Typically, a correlation function is constructed from the expectation values of the measured observables, whose value is bounded if a hidden variable theory is assumed. Correlations exceeding the bound signal the failure of local realism and the correctness of the quantum description of the world. We refer the reader to [3–5,9,17] and references therein for an updated account on the subject. In the case of a three-party system, when each party can perform two possible two-outcome measurements, Svetlichny derived an inequality whose violation reveals *genuine* tripartite nonlocality [12], in the sense that no hybrid local-nonlocal hidden variable model can be compatible with the measured data when the inequality is violated. This inequality builds upon bipartite Bell-CHSH and multipartite Mermin-Klyshko inequalities [3,4,11] and can be formulated as follows. Suppose Alice, Bob, and Charlie share a tripartite state ρ , and each party $j = 1, 2, 3$ measures the operator P_j in two possible settings, ξ_j and ξ'_j , where $\{P_j\}$ are operators with eigenvalues ± 1 . Then the Svetlichny parameter can be written as [12,13] (other expressions can be obtained by relabeling the measurement outcomes),

$$\begin{aligned} \mathcal{S}(\rho; \xi_1, \xi_2, \xi_3, \xi'_1, \xi'_2, \xi'_3) &= \langle P_1(\xi'_1)P_2(\xi_2)P_3(\xi_3) + P_1(\xi_1)P_2(\xi'_2)P_3(\xi_3) \\ &+ P_1(\xi_1)P_2(\xi_2)P_3(\xi'_3) - P_1(\xi'_1)P_2(\xi'_2)P_3(\xi'_3) \\ &+ P_1(\xi_1)P_2(\xi'_2)P_3(\xi'_3) + P_1(\xi'_1)P_2(\xi_2)P_3(\xi'_3) \\ &+ P_1(\xi'_1)P_2(\xi'_2)P_3(\xi_3) - P_1(\xi_1)P_2(\xi_2)P_3(\xi_3) \rangle_\rho. \quad (3) \end{aligned}$$

Assuming hidden variables for the measurements of any two parties, one gets the Svetlichny inequality [12]

$$|\mathcal{S}| \leq 4, \quad (4)$$

which can be instead violated quantum mechanically up to the value $|\mathcal{S}| = 4\sqrt{2} \approx 5.65$, reached for instance on three-qubit GHZ states [12,14,15]. Any violation of inequality (4) is a signature of genuine tripartite nonlocality on the state ρ , and implies that ρ is fully inseparable. Typically, one needs to optimize the measurement settings to single out the maximum violation achievable on a given state, $|\mathcal{S}_{\max}(\rho)| = \sup_{\{\xi_j\}, \{\xi'_j\}} |\mathcal{S}(\rho; \{\xi_j\}, \{\xi'_j\})|$.

In continuous variable systems, we choose displaced parity measurements as the operators to be measured on each mode j [20,21,27]. We have $P_j(\xi_j) = \sum_{n=0}^{\infty} (-1)^n |\xi_j, n\rangle \langle \xi_j, n|$, where $|\xi_j, n\rangle$ is the n th Fock state displaced by $\xi_j = (q_j, p_j)$. This operator can be measured by photon counting preceded by a phase space displacement, the latter implemented, e.g., by beam splitting the input mode with a tunable coherent field [20]. By definition of the Wigner function, one has $\langle P_j(\xi_j) \rangle_{\rho_j} = \pi W_{\rho_j}(\xi_j)$ for

an arbitrary single-mode state ρ_j . We can then redefine all the terms in Eq. (3) in terms of expectation values of the Wigner function of the three-mode state ρ evaluated at suitable phase space points. For instance, the first term reads $\langle P_1(\xi'_1) P_2(\xi_2) P_3(\xi_3) \rangle_{\rho} = \pi^3 W_{\rho}(\xi'_1, \xi_2, \xi_3)$, and so on [27].

In the case of ρ being an undisplaced three-mode Gaussian state, whose Wigner function is given by Eq. (1), then the Svetlichny parameter depends on the elements of the CM σ , as well as on the measurement settings $\{\xi_j\}, \{\xi'_j\}$.

Svetlichny nonlocality of Gaussian states.—We begin by investigating pure fully symmetric states [41], with CM $\sigma^s(a)$ given by Eq. (2) with $a_1 = a_2 = a_3 \equiv a$. In this case, the maximum of Eq. (3) can be found analytically, and is attained for phase space points $\xi_j = (0, p^*)$, $\xi'_j = (0, -p^*) \forall j = 1, 2, 3$, with $p^* = 0$ if $a \leq \sqrt{3/2}$ and $p^* = \sqrt{(a/f_a) \tanh^{-1}[f_a/(4a^2)]}$ otherwise, where $f_a = a^2 - 1 + \sqrt{9a^4 - 10a^2 + 1}$. The maximum Svetlichny parameter then becomes

$$|\mathcal{S}_{\max}(\sigma^s(a))| = \begin{cases} 4, & a \leq \sqrt{3/2}; \\ \frac{4(4a^2 + 3f_a - 4)(8a^2 - 2f_a - 5) \frac{3}{-8a^2 + 2f_a + 8}}{4a^2 + 5}, & \text{otherwise.} \end{cases} \quad (5)$$

Interestingly, despite these states being fully inseparable as soon as $a > 1$ [40], the Svetlichny inequality (4) is only violated for $a > \sqrt{3/2} \approx 1.22$. The value $|\mathcal{S}_{\max}(\sigma^s(a))|$ then increases monotonically with a , as does the tripartite entanglement of the considered states σ^s [33,37,41],

and saturates to the asymptotic value $|\mathcal{S}_{\max}^{\infty}| = 16/3^{(9/8)} \approx 4.65$ in the limit $a \rightarrow \infty$, when the state approaches the continuous variable GHZ state. We note that this achievable violation (generalizing the bipartite findings of Ref. [21]), is below the ultimate limit compatible with quantum mechanics, given by $4\sqrt{2} \approx 5.65$. We anticipate that $|\mathcal{S}_{\max}^{\infty}|$ is the absolute maximum Svetlichny parameter which can be reached by any three-mode Gaussian state via displaced parity measurements.

We investigate now the Svetlichny violation for arbitrary nonsymmetric pure Gaussian states with three-mode CM σ , Eq. (2). One can show that the maximum of Eq. (3) is attained for measurement settings $\xi_j = (0, p_j)$ and $\xi'_j = (0, -p_j)$. The remaining optimization over $\{p_j\}$ can be solved numerically for each given CM parametrized by $\{a_j\}$ ($j = 1, 2, 3$). It is interesting to analyze in detail how the maximum Svetlichny parameter $|\mathcal{S}_{\max}(\sigma(a_1, a_2, a_3))|$ compares with the genuine tripartite entanglement of the states as measured by the residual Rényi-2 entropy $\mathcal{E}_{1|2|3}(\sigma(a_1, a_2, a_3))$ [33]. We first observe that $|\mathcal{S}_{\max}|$ and $\mathcal{E}_{1|2|3}$ have quite similar fingernail-shaped profiles as functions of the local invariants a_j [Figs. 1(a) and 1(b)]. By running an extensive numerical investigation, we find that the maximum Svetlichny parameter admits tight upper and lower bounds as a function of the tripartite entanglement [Fig. 1(c)]. The lower bound is saturated by the fully symmetric states studied above, whose $|\mathcal{S}_{\max}|$ is given by Eq. (5), and whose tripartite entanglement is $\mathcal{E}_{1|2|3}(\sigma(a)) = \ln [8a^3 / (4(a^4 + a^2) - f_a(a^2 - 1))]$ [33]. The upper bound can be reached by bisymmetric states with $a_1 = a_2 = a$ and a_3 to be found numerically close to the boundary $a_3 \leq 2a - 1$. The analysis in Fig. 1(c) shows that all pure three-mode Gaussian states with $\mathcal{E}_{1|2|3} > \frac{1}{2} \ln(\frac{32}{27}) \approx 0.085$ necessarily violate inequality (4). The upper and lower bounds on the Svetlichny parameter close towards $|\mathcal{S}_{\max}^{\infty}|$ for diverging tripartite entanglement. An example of a nonsymmetric three-mode state which asymptotically reaches $|\mathcal{S}_{\max}^{\infty}|$ was provided in Ref. [36] for a driven Bose-Einstein condensate in a ring cavity.

We now turn our attention to mixed three-mode Gaussian states. A numerical investigation of $|\mathcal{S}_{\max}|$ versus the state purity μ is presented in Fig. 1(d). In general, the Svetlichny parameter is proportional to μ ; at fixed μ , $|\mathcal{S}_{\max}|$ is minimized by product states [which cannot violate inequality (4)] for which $|\mathcal{S}_{\max}^{\text{low}}| = 4\mu$, and is found to admit an upper bound as well, reached by fully symmetric mixed three-mode Gaussian states, described by a CM $\sigma^{ms}(a, \mu) = (\mu)^{-(1/3)} \sigma^s(a)$. The upper boundary in Fig. 1(d) is specifically obtained in the limit $a \rightarrow \infty$, which gives $|\mathcal{S}_{\max}^{\text{up}}| = \mu |\mathcal{S}_{\max}^{\infty}|$. Notably, this can fall below 4 whenever $\mu \leq 3^{9/8}/4 \approx 0.86$, which means that no mixed Gaussian state with purity below this threshold can ever violate the Svetlichny inequality.

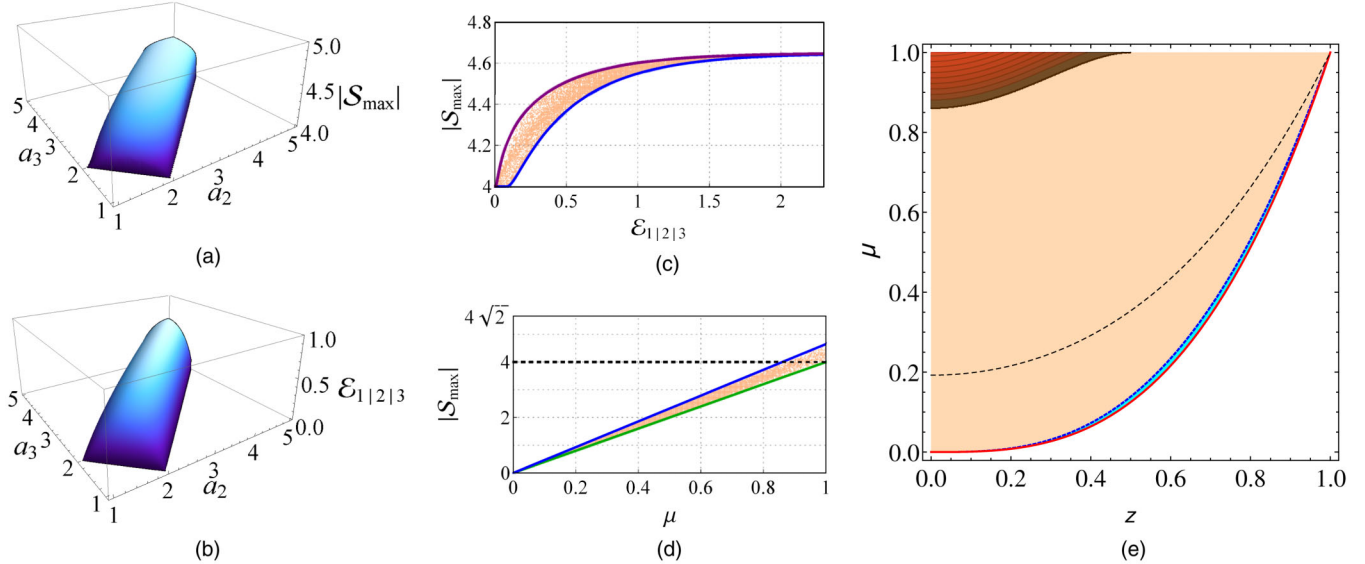


FIG. 1 (color online). Plots of (a) maximum Svetlichny parameter $|S_{\max}|$ and (b) genuine tripartite Rényi-2 entanglement $\mathcal{E}_{1|2|3}$ for pure three-mode Gaussian states with $a_1 = 2$. Panel (c) shows a direct comparison between these two quantities for 10^5 random pure three-mode Gaussian states. Panel (d) depicts the maximum Svetlichny parameter versus the state purity μ for 10^5 random mixed three-mode Gaussian states. Panel (e) is a classification of the inseparability and Svetlichny nonlocality properties of fully symmetric mixed three-mode Gaussian states with CM $\sigma^{ms}(a, \mu)$ in terms of their purity μ and of an inverse squeezing parameter $z = \frac{1}{2}\sqrt{12a^2 - 3f_a - 8}$. Details of the boundary curves and regions for the various panels are provided in the main text. All the quantities plotted are dimensionless.

The present study allows us to add one more layer to the diagram characterizing inseparability versus mixedness for symmetric three-mode Gaussian states (first investigated in Ref. [38]); see Fig. 1(e), which can be read as follows. From bottom-right to top-left: the unfilled region contains fully separable states. The tiny strip between the solid (red) line and the dotted (blue) line accommodates three-mode biseparable states, which exhibit tripartite bound entanglement [38,40]. All the states above the dotted line are fully inseparable. The subregion above the dashed line contains states whose tripartite entanglement has a promiscuous sharing structure [38,39], namely the reduced bipartite entanglements in all two-mode partitions are nonzero and enhance the genuine tripartite one. Eventually, in the top-left corner, we find the subset of fully inseparable states which display genuine tripartite Svetlichny nonlocality; the shading in such region reproduces the value of $|S_{\max}|$, ranging from 4 (darker) to $|S_{\max}^{\infty}| \approx 4.65$ (lighter). We recall that the states classified in Fig. 1(e), which can be generated, for example, by letting pure states $\sigma^s(a)$ evolve in Gaussian noisy channels [37], or by mixing three thermal squeezed beams at a three-mode beam splitter (tritter) [38,41], are useful resources for teleportation networks [41,42], Byzantine agreement [43], or quantum secret sharing [44].

Alternative tests of Gaussian three-way nonlocality.— Finally, we put to test recently developed criteria to reveal so-called tripartite NS_2 nonlocality [17]. This analysis reveals that a larger class of Gaussian states can exhibit three-way nonlocality even when inequality (4) is not

violated. A model is said to be NS_2 local if the correlations between any two parties can be simulated by no-signaling (NS) bipartite resources. The constraints imposed by NS_2 locality take into account an inherent timing of the local measurements (assumed instead as simultaneous in the Svetlichny analysis [12]), which excludes the possibility that outcomes on one or two parties can be used to determine measurement choices on the remaining one, violating causality [17]. Consequently, NS_2 nonlocality is a three-way form of nonlocality generally weaker than the Svetlichny one. To reveal NS_2 nonlocality, one needs to observe a nonzero violation in any one of a set of 185 inequalities [45] reported explicitly in the appendix of Ref. [17]. For a Gaussian state, we can reformulate them all in terms of CMs of the state and its marginals, adopting once more displaced parity measurement operators P_j and using their relation to Wigner functions.

We have tested 10^5 random pure three-mode Gaussian states, with particular emphasis on states with $a_j \lesssim 1.5$, including those not violating inequality (4). Remarkably, we have found violations of at least one NS_2 inequality for all tested fully inseparable pure states. As an example, in Fig. 2 we investigate the violation of NS_2 inequality #168 (in the notation of [17]) for fully symmetric pure states with CM $\sigma^s(a)$, demonstrating that it is violated for all $a > 1$, unlike the Svetlichny inequality, which is equivalent to the NS_2 inequality #185 [45]. We remark that these fully symmetric Gaussian states simultaneously embody normally competing characteristics, typical of both

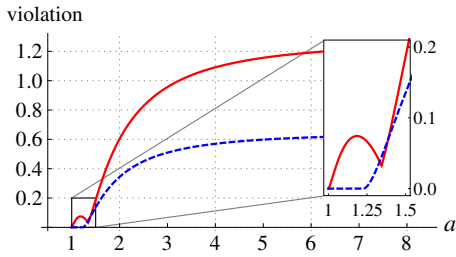


FIG. 2 (color online). Maximum violation of NS_2 inequalities #168 (solid red) and #185 (dashed blue, equivalent to Svetlichny inequality (4) [45]) as defined in [17], plotted for pure three-mode fully symmetric Gaussian states with CM $\sigma^s(a)$ versus the local mixedness parameter a . The inset displays a zoom into the region $1 \leq a \leq 1.5$. All the quantities plotted are dimensionless.

GHZ and W states [37,39]; in particular, W -like traits (significant pairwise entanglement in all two-mode reductions) dominate at low squeezing ($a \lesssim 1.5$) while GHZ-like traits (maximum genuine tripartite entanglement) dominate for $a \gg 1$. Consequently, different inequalities in the NS_2 set exhibit violations in different regimes [45]. This explains the nonmonotonic behavior observed in Fig. 2 (which results from the competition between the two forms of distributed entanglement), and the failure of the Svetlichny inequality (which is best suited to detect GHZ-type entanglement [14]) to be violated for small a [see Eq. (5)]. Our analysis reinforces the conjecture, raised in [17] based on a numerical test for three-qubit pure states, that *all* fully inseparable pure tripartite states in arbitrary dimensions are in fact three-way nonlocal, mirroring the seminal result that all pure bipartite entangled states violate a Bell-CHSH inequality [46].

Conclusions.—We have performed the first systematic study of three-way nonlocality for Gaussian states of continuous variable systems, providing unambiguous evidence for their nonclassical nature, and revealing close connections with genuine tripartite entanglement. Our findings yield practical recipes for the demonstration of multipartite continuous variable nonlocality in experiments [42,47,48], delivering precise prescriptions for the phase space points where maximal violations are detectable. A generalization to n -mode states will be the subject of further work.

We acknowledge fruitful exchanges with A. Acín, J.-D. Bancal, J. Li, L. Mazzola, M. Navascués, and M. Paternostro. We thank the University of Nottingham for financial support via a Nottingham Advance Research Fellowship (S. P.) and a Nottingham-Tsinghua Fund Grant (G. A.).

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