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Frozen Quantum Coherence

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We analyze under which dynamical conditions the coherence of an open quantum system is totally unaffected by noise. For a single qubit, specific measures of coherence are found to freeze under different conditions, with no general agreement between them. Conversely, for an N-qubit system with even N, we identify universal conditions in terms of initial states and local incoherent channels such that all bona fide distance-based coherence monotones are left invariant during the entire evolution. This finding also provides an insightful physical interpretation for the freezing phenomenon of quantum correlations beyond entanglement. We further obtain analytical results for distance-based measures of coherence in two-qubit states with maximally mixed marginals.

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Introduction.—The coherent superposition of states stands as one of the characteristic features that mark the departure of quantum mechanics from the classical realm, if not the most essential one [1]. Quantum coherence constitutes a powerful resource for quantum metrology [2,3] and entanglement creation [4,5] and is at the root of a number of intriguing phenomena of wide-ranging impact in quantum optics [6–9], quantum information [10], solid-state physics [11,12], and thermodynamics [13–18]. In recent years, research on the presence and functional role of quantum coherence in biological systems has also attracted considerable interest [19–35].

Despite the fundamental importance of quantum coherence, only very recently have relevant first steps been achieved towards developing a rigorous theory of coherence as a physical resource [36–38] and necessary constraints have been put forward to assess valid quantifiers of coherence [36,39]. A number of coherence measures have been proposed and investigated, such as the l_1 norm and relative entropy of coherence [36] and the skew information [40,41]. Attempts to quantify coherence via a distance-based approach, which has been fruitfully adopted for entanglement and other correlations [42–52], have revealed some subtleties [53].

A lesson learned from natural sciences is that coherence-based effects can flourish and persist at significant time scales under suitable exposure to decohering environments. Recent evidence suggests that a fruitful interplay between long-lived quantum coherence and tailored noise may be in fact crucial to enhance certain biological processes, such as light harvesting [27,28,30,31]. This surprising cooperation between traditionally competing phenomena provides an inspiration to explore other physical contexts, such as quantum information science, in order to look for general conditions under which coherence can be sustained in the presence of typical sources of noise [54,55]. Progress on this fundamental question can lead to a more efficient exploitation of coherence to empower the performance of real-world quantum technologies.

In this Letter we investigate the dynamics of quantum coherence in open quantum systems under paradigmatic incoherent noisy channels. While coherence is generally nonincreasing under any incoherent channel [36], our goal is to identify initial states and dynamical conditions, here labeled freezing conditions, such that coherence will remain exactly constant (frozen) during the whole evolution (see Fig. 1).

For a single qubit subject to a Markovian bit-flip, bit + phase-flip, phase-flip, depolarizing, amplitude-damping, or phase-damping channel [10], we study the evolution of the l_1 norm and relative entropy of coherence [36] with respect to the computational basis. We show that no nontrivial condition exists such that both measures are simultaneously frozen. We then turn our attention to two-qubit systems, for which we remarkably identify a set of initial states such that *all* bona fide distance-based measures of coherence are frozen forever when each qubit is independently experiencing a nondissipative flip channel. These results are extended to N-qubit systems with any even N, for which suitable conditions supporting the freezing of all

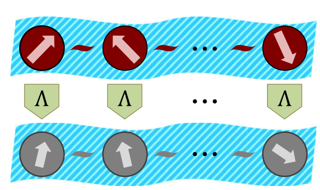

FIG. 1 (color online). Frozen quantum coherence for an N-qubit system subject to incoherent noisy channels Λ acting on each qubit.

distance-based measures of coherence are provided. Such a *universal* freezing of quantum coherence within the geometric approach is intimately related to the freezing of distance-based quantum correlations beyond entanglement [50,52,56–58], thus shedding light on the latter from a physical perspective. Finally, some analytical results for the l_1 norm of coherence are obtained, and its freezing conditions in general one- and two-qubit states are identified.

Incoherent states and channels.—Quantum coherence is conventionally associated with the capability of a quantum state to exhibit quantum interference phenomena [9]. Coherence effects are usually ascribed to the off-diagonal elements of a density matrix with respect to a particular reference basis, whose choice is dictated by the physical scenario under consideration [59]. Here, for an N-qubit system associated with a Hilbert space $\mathbb{C}^{2^N}$, we fix the computational basis $\{|0\rangle, |1\rangle\}^{\otimes N}$ as the reference basis, and we define incoherent states as those whose density matrix δ is diagonal in such a basis,

$$\delta = \sum_{i_1,\ldots,i_N=0}^{1} d_{i_1,\ldots,i_N} |i_1,\ldots,i_N\rangle\langle i_1,\ldots,i_N|. \tag{1}$$

Markovian dynamics of an open quantum system is described by a completely positive trace-preserving (CPTP) map Λ, i.e., a quantum channel, whose action on the state ρ of the system can be characterized by a set of Kraus operators $\{K_j\}$ such that $\Lambda(\rho) = \sum_j K_j \rho K_j^\dagger$, where $\sum_j K_j^\dagger K_j = \mathbb{I}$. Incoherent quantum channels (ICPTP maps) constitute a subset of quantum channels that satisfy the additional constraint $K_j \mathcal{I} K_j^\dagger \subset \mathcal{I}$ for all j, where $\mathcal{I}$ is the set of incoherent states [36]. This implies that ICPTP maps transform incoherent states into incoherent states, and no creation of coherence would be witnessed even if an observer had access to individual outcomes.

We will consider paradigmatic instances of incoherent channels which embody typical noise sources in quantum information processing [10,36], and whose action on a single qubit is described as follows, in terms of a parameter $q \in [0,1]$ which encodes the strength of the noise. The bit-flip, bit + phase-flip, and phase-flip channels are represented in Kraus form by

$$K_0^{F_k} = \sqrt{1-q/2}\,\mathbb{I}, \quad K_{i,j\neq k}^{F_k} = 0, \quad K_k^{F_k} = \sqrt{q/2}\,\sigma_k, \tag{2}$$

with $k=1$, $k=2$, and $k=3$, respectively, and σ_j being the jth Pauli matrix. The depolarizing channel is represented by $K_0^D = \sqrt{1-3q/4}\,\mathbb{I}$, $K_j^D = \sqrt{q/4}\,\sigma_j$, with $j \in \{1,2,3\}$. Finally, the amplitude-damping channel is represented by

$$K_0^A = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-q} \end{pmatrix}, \qquad K_1^A = \begin{pmatrix} 0 & \sqrt{q} \\ 0 & 0 \end{pmatrix},$$

and the phase damping channel is represented by

$$K_0^P = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-q} \end{pmatrix}, \qquad K_1^P = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{q} \end{pmatrix}.$$

The action of N independent and identical local noisy channels (of a given type, say, labeled by $\Xi = \{F_k, D, A, P\}$) on each qubit of an N-qubit system, as depicted in Fig. 1, maps the system state ρ into the evolved state

$$\Lambda_q^{\Xi\otimes N}(\rho) = \sum_{j_1,\ldots,j_N} (K_{j_1}^{\Xi} \otimes \cdots \otimes K_{j_N}^{\Xi}) \rho (K_{j_1}^{\Xi\,\dagger} \otimes \cdots \otimes K_{j_N}^{\Xi\,\dagger}). \tag{3}$$

Coherence monotones.—Baumgratz *et al.* [36] have formulated a set of physical requirements which should be satisfied by any valid measure of quantum coherence $\mathcal{C}$ as follows.

(i) $\mathcal{C}(\rho) \geq 0$ for all states ρ, with $\mathcal{C}(\delta) = 0$ for all incoherent states $\delta \in \mathcal{I}$.

(ii a) Contractivity under incoherent channels Λ_{ICPTP}, $\mathcal{C}(\rho) \geq \mathcal{C}(\Lambda_{\text{ICPTP}}(\rho))$.

(ii b) Contractivity under selective measurements on average, $\mathcal{C}(\rho) \geq \Sigma_j p_j \mathcal{C}(\rho_j)$, where $\rho_j = K_j \rho K_j^\dagger / p_j$ and $p_j = \text{Tr}(K_j \rho K_j^\dagger)$, for any $\{K_j\}$ such that $\Sigma_j K_j^\dagger K_j = \mathbb{I}$ and $K_j \mathcal{I} K_j \subset \mathcal{I}$ for all j.

(iii) Convexity, $\mathcal{C}(q\rho + (1-q)\tau) \leq q\mathcal{C}(\rho) + (1-q)\mathcal{C}(\tau)$ for any states ρ and τ and $q \in [0,1]$.

We now recall known measures of coherence. The l_1 norm quantifies coherence in an intuitive way, via the off-diagonal elements of a density matrix ρ in the reference basis [36],

$$\mathcal{C}_{l_1}(\rho) = \sum_{i\neq j} |\rho_{ij}|. \tag{4}$$

Alternatively, one can quantify coherence by means of a geometric approach. Given a distance D, a generic distance-based measure of coherence is defined as

$$\mathcal{C}_D(\rho) = \min_{\delta\in\mathcal{I}} D(\rho, \delta) = D(\rho, \delta_\rho), \tag{5}$$

where δ_ρ is one of the closest incoherent states to ρ with respect to D. We refer to bona fide distances D as those which satisfy natural properties [10] of contractivity under quantum channels, i.e., $D(\Lambda(\rho), \Lambda(\tau)) \leq D(\rho, \tau)$ for any states ρ, τ and CPTP map Λ, and joint convexity, i.e., $D(q\rho + (1-q)\varpi, q\tau + (1-q)\varsigma) \leq qD(\rho,\tau) + (1-q)D(\varpi,\varsigma)$ for any states $\rho, \varpi, \tau, \varsigma$ and $q \in [0,1]$. We then refer to bona fide distance-based measures of coherence $\mathcal{C}_D$ as those defined by Eq. (5) using a bona fide distance D: all such measures will satisfy requirements (i), (ii a), and (iii) [36]. Additional contractivity requirements are needed for a distance D in order for the corresponding $\mathcal{C}_D$ to obey (ii b) as well [60]. For instance, while the fidelity-based geometric measure of coherence has been recently proven to be a full coherence monotone [5], a related coherence quantifier defined via the squared Bures distance (which is contractive and jointly convex) is known not to satisfy (ii b) [53].

All of our subsequent findings will apply to bona fide distance-based coherence measures $\mathcal{C}_D$, which clearly include coherence monotones obeying all the resource-theory requirements recalled earlier. An example of a distance-based coherence monotone is the relative entropy of coherence [36], given by

$$\mathcal{C}_{\mathrm{RE}}(\rho) = \mathcal{S}(\rho_{\mathrm{diag}}) - \mathcal{S}(\rho) \tag{6}$$

for any state ρ, where ρ_{diag} is the matrix containing only the leading diagonal elements of ρ in the reference basis, and $\mathcal{S}(\rho) = -\mathrm{Tr}(\rho \log \rho)$ is the von Neumann entropy.

We can also define the trace distance of coherence $\mathcal{C}_{\mathrm{Tr}}$ as in Eq. (5) using the bona fide trace distance $D_{\mathrm{Tr}}(\rho, \tau) = \frac{1}{2}\mathrm{Tr}|\rho - \tau|$. For one-qubit states ρ, the trace distance of coherence equals (half) the l_1 norm of coherence [48,53], but this equivalence is not valid for higher dimensional systems, and it is still unknown whether $\mathcal{C}_{\mathrm{Tr}}$ obeys requirement (ii b) in general.

Frozen coherence: One qubit.—We now analyze conditions such that the l_1 norm and relative entropy of coherence are invariant during the evolution of a single qubit (initially in a state ρ) under any of the noisy channels Λ_q^{Ξ} described above. This is done by imposing a vanishing differential of the measures on the evolved state, $\partial_q \mathcal{C}(\Lambda_q^{\Xi}(\rho)) = 0, \forall\ q \in [0, 1]$, with respect to the noise parameter q, which can also be interpreted as a dimensionless time [61]. We find that only the bit and bit + phase-flip channels allow for nonzero frozen coherence (in the computational basis), while all the other considered incoherent channels leave coherence invariant only trivially when the initial state is already incoherent. We can then ask whether nontrivial common freezing conditions for $\mathcal{C}_{l_1}$ and $\mathcal{C}_{\mathrm{RE}}$ exist.

Writing a single-qubit state in general as $\rho = \frac{1}{2}(\mathbb{I} + \sum_j n_j \sigma_j)$ in terms of its Bloch vector $\vec{n} = \{n_1, n_2, n_3\}$, the bit-flip channel $\Lambda_q^{F_1}$ maps an initial Bloch vector $\vec{n}(0)$ to an evolved one $\vec{n}(q) = \{n_1(0), (1-q)n_2(0), (1-q)n_3(0)\}$. As the l_1 norm of coherence is independent of n_3, while n_1 is unaffected by the channel, we get that necessary and sufficient freezing conditions for $\mathcal{C}_{l_1}$ under a single-qubit bit-flip channel amount to $n_2(0) = 0$ in the initial state. Similar conclusions apply to the bit + phase-flip channel $\Lambda_q^{F_2}$ by swapping the roles of n_1 and n_2.

Conversely, the relative entropy of coherence is also dependent on n_3. By analyzing the q derivative of $\mathcal{C}_{\mathrm{RE}}$, we see that such a measure is frozen through the bit-flip channel only when either $n_1(0) = 0$ and $n_2(0) = 0$ (trivial because the initial state is incoherent) or $n_2(0) = 0$ and $n_3(0) = 0$ (trivial because the initial state is invariant under the channel). Therefore, there is no nontrivial freezing of the relative entropy of coherence under the bit-flip or bit + phase-flip channel either.

We conclude that, although the l_1 norm of coherence can be frozen for specific initial states under flip channels, nontrivial universal freezing of coherence is impossible for the dynamics of a single qubit under paradigmatic incoherent maps.

Frozen coherence: Two qubits.—This is not true anymore when considering more than one qubit. We will now show that any bona fide distance-based measure of quantum coherence manifests freezing forever in the case of two qubits A and B undergoing local identical bit-flip channels [62] and starting from the initial conditions specified as follows. We consider two-qubit states with maximally mixed marginals (M_2^3 states), also known as Bell-diagonal states [63], which are identified by a triple $\vec{c} = \{c_1, c_2, c_3\}$ in their Bloch representation

$$\rho = \frac{1}{4}\left(\mathbb{I}^A \otimes \mathbb{I}^B + \sum_{j=1}^{3} c_j \sigma_j^A \otimes \sigma_j^B\right). \tag{7}$$

Local bit-flip channels on each qubit map initial M_2^3 states with $\vec{c}(0) = \{c_1(0), c_2(0), c_3(0)\}$ to M_2^3 states with $\vec{c}(q) = \{c_1(0), (1-q)^2 c_2(0), (1-q)^2 c_3(0)\}$. Then, the subset of M_2^3 states supporting frozen coherence for all bona fide distance-based measures is given by the initial condition [50,52,57],

$$c_2(0) = -c_1(0)c_3(0). \tag{8}$$

To establish this claim, we first enunciate two auxiliary results, which simplify the evaluation of distance-based coherence monotones (5) for the relevant class of M_2^3 states.

Lemma 1.—According to any contractive and convex distance D, one of the closest incoherent states δ_ρ to a M_2^3 state ρ is always a M_2^3 incoherent state, i.e., one of the form

$$\delta_\rho = \frac{1}{4}(\mathbb{I}^A \otimes \mathbb{I}^B + s\sigma_3^A \otimes \sigma_3^B), \quad \text{for some } s \in [-1, 1]. \tag{9}$$

Lemma 2.—According to any contractive and convex distance D, one of the closest incoherent states δ_ρ to a M_2^3 state ρ with triple $\{c_1, -c_1c_3, c_3\}$ is the M_2^3 state δ_ρ with triple $\{0, 0, c_3\}$.

It then follows that any bona fide distance-based measure of coherence $\mathcal{C}_D$ for the M_2^3 states $\rho(q)$, evolving from the initial conditions (8) under local bit-flip channels, is given by

$$\mathcal{C}_D(\rho(q)) = D(\{c_1(0), -(1-q)^2 c_1(0)c_3(0), (1-q)^2 c_3(0)\}, \{0, 0, (1-q)^2 c_3(0)\}) = \mathcal{C}_D(\rho(0)),$$

which is frozen for any $q \in [0, 1]$, or equivalently frozen forever for any t [61]. The two lemmas and the main implication on frozen coherence can be rigorously proven by invoking and adapting recent results on the dynamics of quantum correlations for M_2^3 states, reported in Ref. [52]. A comprehensive proof is provided in the Supplemental Material [64]. This finding shows that, in contrast to the one-qubit case, universal freezing of quantum coherence—measured within a bona fide geometric approach—can in fact occur in two-qubit systems exposed to conventional local decohering dynamics.

Coming back now to the two specific coherence monotones analyzed here [36], we know that the relative entropy of coherence $\mathcal{C}_{\rm RE}$ is a bona fide distance-based measure; hence, it manifests freezing in the conditions of Eq. (8). Interestingly, we will now show that the l_1 norm of coherence $\mathcal{C}_{l_1}$ coincides with (twice) the trace distance of coherence $\mathcal{C}_{\rm Tr}$ for any M_2^3 state, which implies that $\mathcal{C}_{l_1}$ also freezes in the same dynamical conditions. To this aim we need to show that, with respect to the trace distance $D_{\rm Tr}$, one of the closest incoherent states δ_ρ to a M_2^3 state ρ is always its diagonal part $\rho_{\rm diag}$. The trace distance between a M_2^3 state ρ with $\{c_1, c_2, c_3\}$ and one of its closest incoherent states δ_ρ, which is itself a M_2^3 state of the form (9) according to Lemma 1, is given by $D_{\rm Tr}(\rho,\delta_\rho) = \frac{1}{4}(|s+c_1-c_2-c_3|+|s-c_1+c_2-c_3|+|s+c_1+c_2-c_3|+|-s+c_1+c_2+c_3|)$. It is immediately seen that the minimum over δ_ρ is attained by $s=c_3$, i.e., by $\delta_\rho=\rho_{\rm diag}$, as claimed. Notice, however, that the equivalence between $\mathcal{C}_{l_1}$ and $\mathcal{C}_{\rm Tr}$ does not extend to general two-qubit states, as can be confirmed numerically.

Similarly to the single-qubit case, we can derive a larger set of necessary and sufficient freezing conditions valid specifically for the l_1 norm of coherence. Every two-qubit state ρ can be transformed, by local unitaries, into a standard form [65] with Bloch representation $\rho = \frac{1}{4}(\mathbb{I}^A\otimes\mathbb{I}^B + \sum_{j=1}^3 x_j\sigma_j^A\otimes\mathbb{I}^B + \sum_{j=1}^3 y_j\mathbb{I}^A\otimes\sigma_j^B + \sum_{j=1}^3 T_{jj}\sigma_j^A\otimes\sigma_j^B)$. We have then that initial states of this form, with x_1, y_1, x_3, y_3, T_{33} arbitrary, $x_2=y_2=0$, and $T_{22}=uT_{11}$ with $u\in[-1,1]$, manifest frozen coherence as measured by $\mathcal{C}_{l_1}$ under local bit-flip channels; however, the same does not hold for $\mathcal{C}_{\rm RE}$ in general.

Frozen coherence: N qubits.—Our main finding can be readily generalized to a system of N qubits with any even N. We define N-qubit states with maximally mixed marginals (M_N^3 states) [58,66] as those with density matrix of the form $\rho = \frac{1}{2^N}(\mathbb{I}^{\otimes N} + \sum_{j=1}^3 c_j\sigma_j^{\otimes N})$, still specified by the triple $\{c_1, c_2, c_3\}$ as in the $N=2$ case. We have then that, when the system is evolving according to identical and independent local bit-flip channels acting on each qubit as in Eq. (3) with $\Xi = F_1$, the quantum coherence of the system is universally frozen according to any bona fide distance-based measure if the N qubits are initialized in a M_N^3 state respecting the freezing condition

$$c_2(0) = (-1)^{N/2} c_1(0) c_3(0), \tag{10}$$

which generalizes (8). This is the most general result of the present Letter [62], and its full proof is provided in the Supplemental Material [64]. We observe that, by virtue of the formal equivalence between a system of N qubits and a single qudit with dimension $d=2^N$, our results can also be interpreted as providing universal freezing conditions for all bona fide distance-based measures of coherence in a single 2^N-dimensional system with any even N. Naturally, one may expect larger sets of freezing conditions to exist for specific coherence monotones such as the l_1 norm, like in the $N=2$ case; their characterization is outside the scope of this Letter.

We further note that no universal freezing of coherence is instead possible for M_N^3 states with odd N, whose dynamical properties are totally analogous to those of one-qubit states.

Coherence versus quantum correlations.—The freezing conditions established here for coherence have been in fact identified in previous literature [50,52,56–58], as various measures of so-called discord-type quantum correlations were shown to freeze under the same dynamical conditions up to a threshold time $t^\star$, defined in our notation [61] by the largest value of q such that $|c_3(q)| \geq |c_1(q)|$, for M_N^3 states evolving under local bit-flip channels. Focusing on the two-qubit case for clarity, we note that for M_2^3 states with $|c_3| \geq |c_1|$, and for any bona fide distance D, the distance-based measure of coherence $\mathcal{C}_D$, defined by Eq. (5) and evaluated in Eq. (10), *coincides* with the corresponding distance-based measure of discord-type quantum correlations $\mathcal{Q}_D$, formalized, e.g., in Ref. [52]. Hence, the freezing of coherence might provide a deeper insight into the peculiar phenomenon of frozen quantum correlations under local flip channels (see also Ref. [67]), as the latter just reduce to coherence for $t \leq t^\star$ under the conditions we identified.

More generally, measures of discord-type correlations [46,68,69] may be recast as suitable measures of coherence in bipartite systems, minimized over the reference basis, with minimization restricted to local product bases. For instance, the minimum l_1 norm of coherence [36] yields the negativity of quantumness [48,70,71], the minimum relative entropy of coherence [36] yields the relative entropy of discord [45,70,72,73], and the minimum skew information [40] yields the local quantum uncertainty [74]. Our result suggests, therefore, that the computational basis is the product basis which minimizes coherence (according to suitable bona fide measures) for particular M_2^3 states undergoing local bit-flip noise Λ^{F_1} up to $t \leq t^\star$, while coherence is afterwards minimized in the eigenbasis of σ_1, which is the pointer basis towards which the system eventually converges due to the local decoherence [75]; similar conclusions can be drawn for the other k-flip channels [62].

We finally remark that, unlike more general discord-type correlations, entanglement [44] plays no special role in the freezing phenomenon analyzed in this Letter, as the latter can also happen for states that remain separable during the whole evolution, e.g., the M_2^3 states with initial triple $\{\frac{1}{4}, -\frac{1}{16}, \frac{1}{4}\}$.

Conclusions.—We have determined exact conditions such that any bona fide distance-based measure of quantum coherence [36] is dynamically frozen: this occurs for an even number of qubits, initialized in a particular class of states with maximally mixed marginals, and undergoing local independent and identical nondissipative flip channels (Fig. 1). We have also shown that there is no general agreement on freezing conditions between specific coherence monotones when considering either the one-qubit case or more general N-qubit initial states. This highlights the prominent role played by the aforementioned universal freezing conditions in ensuring a durable physical exploitation of coherence, regardless of how it is quantified, for applications such as quantum metrology [2] and nanoscale thermodynamics [17,18]. It will be interesting to explore practical realizations of such dynamical conditions [75–80].

Complex systems are inevitably subject to noise; hence, it is natural and technologically crucial to question under what conditions the quantum resources that we can extract from them are not deteriorated during open evolutions [81]. In addressing this problem by focusing on coherence, we have also revealed an intrinsic physical explanation for the freezing of discord-type correlations [52], by exposing and exploiting the intimate link between these two nonclassical signatures. Providing unified quantitative resource-theory frameworks for coherence, entanglement, and other quantum correlations is certainly a task worthy of further investigation [5].

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