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(1+1)D calculation provides evidence that quantum entanglement survives a firewall

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We analyze how pre-existing entanglement between two Unruh-DeWitt particle detectors evolves when one of the detectors falls through a Rindler firewall in (1+1)-dimensional Minkowski space. The firewall effect is minor and does not wash out the detector-detector entanglement, in some regimes even preserving the entanglement better than Minkowski vacuum. The absence of cataclysmic events should continue to hold for young black hole firewalls. A firewall's prospective ability to resolve the information paradox must hence hinge on its detailed gravitational structure, presently poorly understood.

Introduction. — If black hole evaporation preserves unitarity, it has been argued from preservation of correlations that the horizon of a shrinking black hole must develop a singularity even when the evaporation is still slow and the black hole remains macroscopic [1–3] (for a selection of debate and reviews see [4–11]). Modeling the gravitational aspects of this proposed singularity has remained elusive. For instance, the emergence of a Planck scale shell near the horizon of an astrophysical black hole seems to be in tension with the gravitational dynamics predicted by general relativity [12]. Nevertheless, the nongravitational aspects of the “firewall” version of the singularity [3] can be modeled with a quantum field in Minkowski spacetime: a state in which correlations across a Rindler horizon are severed can be written down by hand [13], mimicking the severing that the firewall argument of [3] posits to develop dynamically during black hole evaporation. This severing of correlations has strong similarities to that which ensues on the sudden insertion of a reflective wall in a spacetime [14, 15]. The Rindler firewall state can be studied by usual quantum field theory techniques, and the conclusions should apply to young gravitational firewalls where the backreaction on the metric is still small.

In this letter we analyse the correlations between two particle detectors when one of them falls through a Rindler firewall. From the quantum field theory side, this question is motivated by the fact that any measurements of quantum fields are done through material particle detectors, and we may employ the Unruh-DeWitt detector model [16, 17] to capture the essential aspects of interactions between atoms and the electromagnetic field [18, 19]. From the firewall side, focusing on the correlations between two detectors is motivated by the central role of quantum correlations in the firewall argument [3]. The response of a single detector falling through the Rindler firewall is known to be sudden but finite [13]. For two detectors that are initially correlated, interaction with the quantum field will decohere the two detectors: might this decoherence be drastically enhanced when one

of the detectors goes through a Rindler firewall? A positive answer could be seen as indirect support of the firewall argument as given in [3].

Our main conclusion runs contrary to these expectations. The Rindler firewall turns out to have only a modest effect on the decoherence between the two detectors, and in certain regions of the parameter space the firewall even preserves the entanglement between the two detectors better than Minkowski vacuum. A firewall's prospective capability to resolve the black hole information paradox must hence hinge on its detailed late time gravitational structure, at present poorly understood.

Formalism: evolving an inertial detector pair. — We consider a pair of Unruh-DeWitt detectors [16, 17] coupled to a real scalar field ϕ and moving inertially in Minkowski spacetime without relative velocity [20]. (For an inertial detector and an accelerated detector, see, e.g., [21–23].) We evolve the system with respect to the Minkowski time t in a Lorentz frame in which the two detectors are at rest, and we denote t by τ as it coincides with the detectors' proper time. The interaction picture Hamiltonian is

$$H = \sum_{\nu} \lambda_{\nu} \chi_{\nu}(\tau) \mu_{\nu}(\tau) \phi(\mathbf{x}_{\nu}(\tau)) , \quad (1)$$

where the index ν labels the two detectors, $\mathbf{x}_{\nu}(\tau)$ are their worldlines, $\mu_{\nu}(\tau)$ are the monopole moment operators and λ_{ν} are the coupling constants. The real-valued switching functions χ_{ν} specify how the interaction is turned on and off. We assume that χ_{ν} either have compact support or have sufficiently strong falloff properties for the system to be treatable as asymptotically uncoupled in the distant past and future.

If the initial state (= density matrix) of the system is ρ_0 , the final state after the interaction has ceased is $\rho_T = U \rho_0 U^{\dagger}$, where U is the interaction picture time evolution operator. Assuming that each λ_{ν} is proportional to a formal perturbative parameter λ , U has the Dyson expansion $U = U^{(0)} + U^{(1)} + U^{(2)} + \mathcal{O}(\lambda^3)$ where

$U^{(0)} = \mathbb{1}$ and

$$U^{(1)} = -i \int_{-\infty}^{\infty} d\tau H(\tau), \quad U^{(2)} = - \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' H(\tau) H(\tau'). \quad (2)$$

Hence $\rho_T = \rho_0 + \rho_T^{(1)} + \rho_T^{(2)} + \mathcal{O}(\lambda^3)$, where

$$\rho_T^{(1)} = U^{(1)} \rho_0 + \rho_0 U^{(1)\dagger}, \quad (3a)$$

$$\rho_T^{(2)} = U^{(1)} \rho_0 U^{(1)\dagger} + U^{(2)} \rho_0 + \rho_0 U^{(2)\dagger}. \quad (3b)$$

When the initial state has the form $\rho_0 = \rho_{d,0} \otimes \rho_{\phi,0}$, where $\rho_{d,0}$ and $\rho_{\phi,0}$ are respectively the initial state of the two-detector subsystem and the initial state of the field, and assuming that $\rho_{\phi,0}$ satisfies

$$\text{Tr}_{\phi}(\phi(x)\rho_{\phi,0}) = 0, \quad (4)$$

we find that the final state of the two-detector subsystem is

$$\rho_{d,T} = \text{Tr}_{\phi}(\rho_T) = \rho_{d,0} + \rho_{d,T}^{(2)} + \mathcal{O}(\lambda^3), \quad (5a)$$

$$\begin{aligned} \rho_{d,T}^{(2)} = & \sum_{\nu,\eta} \lambda_{\nu} \lambda_{\eta} \left[\int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \chi_{\nu}(\tau') \chi_{\eta}(\tau) \right. \\ & \times \mu_{\nu}(\tau') \rho_{d,0} \mu_{\eta}(\tau) W[\mathbf{x}_{\eta}(\tau), \mathbf{x}_{\nu}(\tau')] \\ & - \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' \chi_{\nu}(\tau) \chi_{\eta}(\tau') \\ & \times \mu_{\nu}(\tau) \mu_{\eta}(\tau') \rho_{d,0} W[\mathbf{x}_{\nu}(\tau), \mathbf{x}_{\eta}(\tau')] \\ & - \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' \chi_{\nu}(\tau) \chi_{\eta}(\tau') \\ & \left. \times \rho_{d,0} \mu_{\eta}(\tau') \mu_{\nu}(\tau) W[\mathbf{x}_{\eta}(\tau'), \mathbf{x}_{\nu}(\tau)] \right] \quad (5b) \end{aligned}$$

where $W[\mathbf{x}_{\nu}(\tau), \mathbf{x}_{\eta}(\tau')]$ denotes the pullback of the Wightman function on the detectors' worldlines,

$$W[\mathbf{x}_{\nu}(\tau), \mathbf{x}_{\eta}(\tau')] = \text{Tr}_{\phi}(\phi(\mathbf{x}_{\nu}(\tau))\phi(\mathbf{x}_{\eta}(\tau'))\rho_{\phi,0}). \quad (6)$$

Detectors with a Rindler firewall.— We now specialise to (1 + 1)-dimensional Minkowski spacetime, $ds^2 = -dt^2 + dx^2 = -du dv$, where $u = t - x$ and $v = t + x$.

We take ϕ to be massless and $\rho_{\phi,0}$ to be the Rindler firewall state described in [13]. The one-point function of $\rho_{\phi,0}$ satisfies (4), as follows by extending the Wightman function discussion given in [13] to the one-point function. The Wightman function of $\rho_{\phi,0}$ is

$$\begin{aligned} W_F(\mathbf{x}, \mathbf{x}') &= \text{Tr}_{\phi}(\phi(\mathbf{x})\phi(\mathbf{x}')\rho_{\phi,0}) \\ &= W_0(\mathbf{x}, \mathbf{x}') + \Delta W(\mathbf{x}, \mathbf{x}'), \quad (7) \end{aligned}$$

where W_0 is the Wightman function in the Minkowski vacuum $|0\rangle\langle 0|$ and ΔW is the correction due to the firewall. For W_0 we have

$$W_0(\mathbf{x}, \mathbf{x}') = \frac{-1}{4\pi} \log[\Lambda^2(\epsilon + i\Delta u)(\epsilon + i\Delta v)], \quad (8)$$

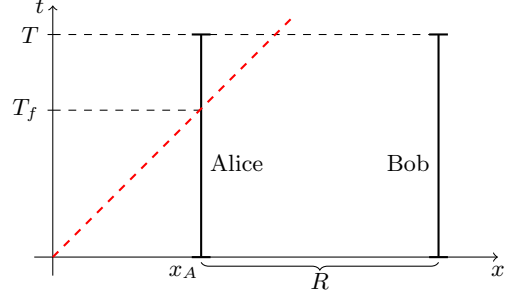


Figure 1. Spacetime diagram of the two-detector system with the Rindler firewall. The dashed line at $t = x$ is the firewall. The solid lines are the worldlines of the Alice detector and the Bob detector, switched on at $t = 0$ and off at $t = T > 0$. Alice crosses the firewall during the detectors' operation (at $t = T_f = x_A$ in the diagram) but Bob does not.

where $\Delta u = u - u'$, $\Delta v = v - v'$, the positive constant Λ is an infrared cutoff, the logarithm takes its principal branch and $\epsilon \rightarrow 0_+$. The full expression for $\Delta W(\mathbf{x}, \mathbf{x}')$ is lengthy but reduces for $v > 0$ and $v' > 0$ to

$$\begin{aligned} \Delta W(\mathbf{x}, \mathbf{x}') &= \frac{1}{4\pi} \left[\Theta(u)\theta(-u') + \theta(-u)\theta(u') \right] \\ &\quad \times \left(\log(\Lambda |u - u'|) + i\frac{\pi}{2} \text{sgn}(u - u') \right). \quad (9) \end{aligned}$$

In words, (8) and (9) show that when \mathbf{x} and \mathbf{x}' are to the future of the left-going Rindler horizon $t = -x$ but on opposite sides of the right-going Rindler horizon $t = x$, $W_F(\mathbf{x}, \mathbf{x}')$ is missing the contribution from the right-moving part of the field. This absence of correlations across the Rindler horizon models the absence of correlations that is argued to develop dynamically in an evaporating black hole spacetime [3].

For the detectors in the presence of the firewall, we take the worldline of detector A (Alice) to be at $x = x_A > 0$ and the worldline of detector B (Bob) to be at $x = x_A + R$, where $R > 0$ is the spatial separation. The detectors are switched on at $t = 0$, and they are switched off at a time when Alice has already crossed the firewall at $t = x$ but Bob has not, as shown in Figure 1.

We ask: If Alice and Bob are initially entangled, how does Alice's crossing the firewall affect this entanglement?

Methods.— We assume each detector to be a two-level system. We denote the respective energy gaps by Ω_{ν} , the ground states by $|g_{\nu}\rangle$ and the excited states by $|e_{\nu}\rangle$. The monopole moment operators are then $\mu_{\nu}(\tau) = \sigma_{\nu}^{+} e^{i\Omega_{\nu}\tau} + \sigma_{\nu}^{-} e^{-i\Omega_{\nu}\tau}$, where the nonvanishing matrix elements of the raising and lowering operators σ_{ν}^{\pm} are $\langle e_{\nu} | \sigma_{\nu}^{+} | g_{\nu} \rangle = \langle g_{\nu} | \sigma_{\nu}^{-} | e_{\nu} \rangle = 1$.

For each of the the individual detectors we may introduce a two-by-two matrix representation in which (sup-

pressing the detector index)

$$|g\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mu(\tau) = \begin{pmatrix} 0 & e^{-i\Omega\tau} \\ e^{i\Omega\tau} & 0 \end{pmatrix}. \quad (10)$$

For the two-detector system we employ the Kronecker product representation in which

$$|gg\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, |eg\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, |ge\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, |ee\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \quad (11)$$

where the first label in $|ij\rangle$ refers to Alice and the second label to Bob. It follows that

$$\mu_A(\tau) = \begin{pmatrix} 0 & e^{-i\Omega_A\tau} & 0 & 0 \\ e^{i\Omega_A\tau} & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-i\Omega_A\tau} \\ 0 & 0 & e^{i\Omega_A\tau} & 0 \end{pmatrix}, \quad (12a)$$

$$\mu_B(\tau) = \begin{pmatrix} 0 & 0 & e^{-i\Omega_B\tau} & 0 \\ 0 & 0 & 0 & e^{-i\Omega_B\tau} \\ e^{i\Omega_B\tau} & 0 & 0 & 0 \\ 0 & e^{i\Omega_B\tau} & 0 & 0 \end{pmatrix}. \quad (12b)$$

We take the initial state of the Alice-Bob system to be the maximally entangled state $|\psi_{\max}\rangle = \frac{1}{\sqrt{2}}(|gg\rangle + |ee\rangle)$, so that

$$\rho_{d,0} = |\psi_{\max}\rangle\langle\psi_{\max}| = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}. \quad (13)$$

In the final state $\rho_{d,T}$ (5), we separate the contributions to $\rho_{d,T}^{(2)}$ as

$$\rho_{d,T}^{(2)} = \lambda_A^2 \rho_{AA} + \lambda_B^2 \rho_{BB} + \lambda_A \lambda_B \rho_{AB}, \quad (14)$$

finding

$$\rho_{AA} = \frac{1}{2} \begin{pmatrix} -2\text{Re}(J_{-+}^{AA}) & 0 & 0 & -J_{-+}^{AA} - J_{+-}^{AA*} \\ 0 & I_{++}^{AA} & I_{++}^{AA} & 0 \\ 0 & I_{--}^{AA} & I_{--}^{AA} & 0 \\ -J_{+-}^{AA} - J_{-+}^{AA*} & 0 & 0 & -2\text{Re}(J_{+-}^{AA}) \end{pmatrix}, \quad \rho_{BB} = \frac{1}{2} \begin{pmatrix} -2\text{Re}(J_{-+}^{BB}) & 0 & 0 & -J_{-+}^{BB} - J_{+-}^{BB*} \\ 0 & I_{++}^{BB} & I_{++}^{BB} & 0 \\ 0 & I_{--}^{BB} & I_{--}^{BB} & 0 \\ -J_{+-}^{BB} - J_{-+}^{BB*} & 0 & 0 & -2\text{Re}(J_{+-}^{BB}) \end{pmatrix},$$

$$\rho_{AB} = \frac{1}{2} \begin{pmatrix} -2\text{Re}(J_{--}^{AB} + J_{--}^{BA}) & 0 & 0 & -J_{--}^{AB} - J_{--}^{BA} - J_{++}^{AB*} - J_{++}^{BA*} \\ 0 & I_{++}^{AB} + I_{--}^{BA} & I_{++}^{AB} + I_{--}^{BA} & 0 \\ 0 & I_{--}^{AB} + I_{++}^{BA} & I_{--}^{AB} + I_{++}^{BA} & 0 \\ -J_{++}^{AB} - J_{++}^{BA} - J_{--}^{AB*} - J_{--}^{BA*} & 0 & 0 & -2\text{Re}(J_{++}^{AB} + J_{++}^{BA}) \end{pmatrix}, \quad (15)$$

where

$$I_{\epsilon,\delta}^{\nu,\eta} = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' \chi_{\nu}(\tau') \chi_{\eta}(\tau) e^{i(\epsilon\Omega_{\nu}\tau' + \delta\Omega_{\eta}\tau)} W[x_{\eta}(\tau), x_{\nu}(\tau')],$$

$$J_{\epsilon,\delta}^{\nu,\eta} = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\tau} d\tau' \chi_{\nu}(\tau) \chi_{\eta}(\tau') e^{i(\epsilon\Omega_{\nu}\tau + \delta\Omega_{\eta}\tau')} W[x_{\nu}(\tau), x_{\eta}(\tau')]. \quad (16)$$

Finally, we characterise the entanglement in the Alice-Bob final state $\rho_{d,T}$ by the negativity \mathcal{N} [24]. For a two-qubit system this monotone provides a strict criterion of entanglement in the sense that it vanishes if and only if a state is separable. Working perturbatively to order λ^2 , the negativity can be computed in a straightforward way from (5a) and (11)–(16).

Results.— With the detector trajectories shown in Figure 1, we first consider switching functions with a

sharp switch-on and switch-off,

$$\chi_A(\tau) = \chi_B(\tau) = \Theta(\tau)\Theta(1 - (\tau/T)), \quad (17)$$

where Θ is the Heaviside function. Figure 2 shows a representative plot of the negativity as a function of x_A with the other parameters fixed. When $x_A > T$, Alice does not fall through the firewall during the operation of the detectors (see Fig. 1) and the entanglement degradation is just that in Minkowski vacuum [20], independent of x_A . When $x_A < T$, Alice's falling through the firewall does affect the negativity. Two outcomes are apparent from the figure.

First, the firewall effect on the negativity depends continuously on x_A and remains small in magnitude: the firewall does *not* wash up the Alice-Bob correlations as might have been expected from the gravitational firewall debate [1–11]. As a technical point, we note that the

smallness of the effect gives confidence in the reliability of our perturbative analysis.

Second, over most of the parameter range the firewall enhances the degradation of Alice-Bob entanglement, compared with the degradation in Minkowski vacuum. This is what one might have expected from the gravitational firewall debate [1–11]. However, if Alice crosses the firewall shortly before turning her detector off, the effect is the opposite: in this case the firewall helps Alice and Bob maintain their entanglement. Developing a qualitative explanation for this phenomenon could be an interesting challenge.

One might suspect some of the properties of the graph in Figure 2 to be specific to, and perhaps artefacts of, the sharp switch-on and switch-off. To alleviate this suspicion, Figure 3 shows results from a similar analysis with Gaussian switching functions,

$$\chi_A(\tau) = \chi_B(\tau) = e^{-(\tau-\tau_0)^2/\sigma^2}, \quad (18)$$

where the parameters τ_0 and σ are chosen as described in the figure caption to provide a smooth approximation to the sharp switching of Figure 2. The detectors now operate for $-\infty < \tau < \infty$, but the tails of the Gaussians are so small that this noncompact support of the Gaussian does not bring in new complications. The curve in Figure 3 is smoother but retains the qualitative features, including a regime where the firewall allows Alice and Bob maintain their entanglement better than in Minkowski vacuum. The conclusions drawn above from the sharp switching results hence apply also to the Gaussian switching.

Generalising our analysis from 1+1 to 3+1 dimensions would require new technical input at two steps. First, the firewall Wightman function must be evolved from the initial data at $t = 0$ using the (3+1)-dimensional field commutator. Second, recall that while the transition probability of a pointlike Unruh-DeWitt detector is well defined in 3+1 dimensions (an account that includes the switching effects can be found in [25, 26]), the evolution of the full density operator is singular even in Minkowski vacuum, as seen from (16) and from the nonintegrable coincidence limit singularity of the (3+1)-dimensional Wightman function [27, 28]. A decoherence analysis would hence require an additional regularisation of the detector model, for example by a spatial smearing, a standard and well documented procedure [16, 29–31]. However, crucially, the structure of (15) remains exactly the same in all dimensions. It is hence difficult to see how any reasonable input at either of these technical steps could lead to drastically enhanced singularities in the evolution across the firewall. In particular, correlations between a firewall-crossing detector and the outside world should still undergo only a modest change.

Conclusions. — Our main conclusion runs contrary to the vision of a firewall as a violently singular surface [3]: the Rindler firewall has only a modest effect

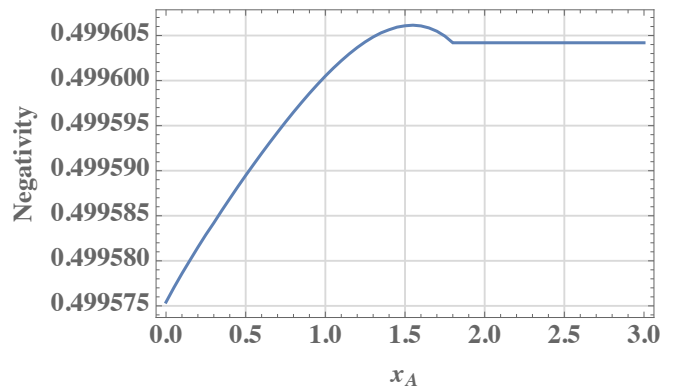


Figure 2. Negativity \mathcal{N} of the Alice-Bob final state as a function of x_A with the detector trajectories shown in Figure 1 and with the sharp switching functions (17), for $R = 4$, $T = 1.8$, $\Omega_A = \Omega_B = 1$, and $\Lambda = 10^{-2} \ll \Omega_\nu$ and $\lambda_A = \lambda_B = 0.01$. Before the interaction $\mathcal{N} = \frac{1}{2}$, and the evolution causes the entanglement to degrade. When $x_A > T$, Alice does not cross the firewall during her detector’s operation and the entanglement degradation is identical to that in Minkowski vacuum. When $x_A < T$, Alice does cross the firewall, and the entanglement degradation depends non-monotonically on x_A . Note that the leading order contribution to \mathcal{N} is a homogeneous quadratic in (λ_A, λ_B) : an overall scaling of the two coupling constants (within the perturbative approximation) leaves the plot invariant up to a rescaling of the vertical axis.

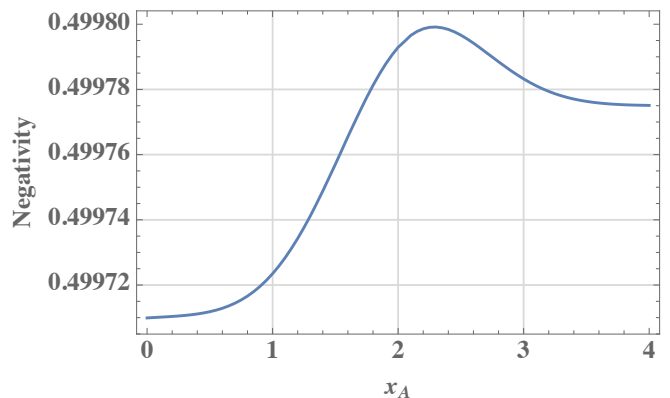


Figure 3. As in Figure 2 but with the Gaussian switching functions (18), with $\sigma = 1$ and $t_0 = 2$, so that the Gaussian switching provides a smooth approximation to the sharp switching of Figure 2. The firewall effect on Alice’s detector is now significant for $x_A \lesssim 3$. The qualitative properties are as in Figure 2, including the non-monotonicity in x_A .

on the entanglement between two inertial Unruh-DeWitt detectors when one of the detectors crosses the firewall. There is even a parameter range in which the firewall slows down the entanglement degradation, compared with the degradation that takes place in Minkowski vacuum.

Given that the Rindler firewall models the quantum field theory correlations in a black hole firewall [13], our results suggest that a similar conclusion should hold for

black hole firewalls at the early stages of the Hawking evaporation where the gravitational backreaction on the metric is not yet significant. As the Unruh-DeWitt detector captures the essential features of the interaction between atoms and the electromagnetic field [18, 19], the conclusion should further extend to systems of matter of which we and our experimental apparatus are built.

In summary, the key message of this letter is that we cannot think of a young firewall as a surface of cataclysmic events that erases all information about matter that crosses the firewall. If the matter is correlated with the outside world, these correlations will not be significantly altered by the crossing. We may not know why the chicken crossed the young firewall, but it did get to the other side, with most of its memories intact.

More broadly, our results push the burden of proof of the firewall's ability to resolve the black hole information paradox into the regime in which the detailed late time gravitational structure of the firewall is crucial. This regime is at present conspicuously poorly understood, and not exempt of problems [12] (a rare exception is a dilaton gravity model [9] in which the paradox turns out to be resolved by a remnant rather than by a firewall). While our results do not settle the viability of the firewall argument, they do identify the arena in which the viability will be settled.

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