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Article **Impacts of Sample Design for Validation Data on the Accuracy of Feedforward Neural Network Classification**

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Abstract: Validation data are often used to evaluate the performance of a trained neural network and used in the selection of a network deemed optimal for the task at-hand. Optimality is commonly assessed with a measure, such as overall classification accuracy. The latter is often calculated directly from a confusion matrix showing the counts of cases in the validation set with particular labelling properties. The sample design used to form the validation set can, however, influence the estimated magnitude of the accuracy. Commonly, the validation set is formed with a stratified sample to give balanced classes, but also via random sampling, which reflects class abundance. It is suggested that if the ultimate aim is to accurately classify a dataset in which the classes do vary in abundance, a validation set formed via random, rather than stratified, sampling is preferred. This is illustrated with the classification of simulated and remotely-sensed datasets. With both datasets, statistically significant differences in the accuracy with which the data could be classified arose from the use of validation sets formed via random and stratified sampling (*z* = 2.7 and 1.9 for the simulated and real datasets respectively, for both $p < 0.05\%$). The accuracy of the classifications that used a stratified sample in validation were smaller, a result of cases of an abundant class being commissioned into a rarer class. Simple means to address the issue are suggested.

Keywords: cross-validation; multi-layer perceptron; remote sensing; classification error; sample design; machine learning

1. Introduction

Artificial neural networks are widely used for supervised classification applications. In these applications, cases of known class membership are used to train the neural network in order to allow it to predict the class membership of previously unseen and unlabeled cases. This type of analysis is common in, for example, the production of thematic maps, such as those depicting land cover, from remotely-sensed imagery [\[1–](#page-13-0)[3\]](#page-13-1). The imagery contain data on the remotely-sensed response of the land surface that is converted into information on land cover class via the classification analysis and a wide variety of approaches and applications have been investigated, e.g., [\[2](#page-13-2)[,4](#page-13-3)[,5\]](#page-13-4). Neural networks have become a popular method for image classification as numerous studies have shown that they can yield more accurate maps than a variety of other alternative approaches to classification [\[6](#page-13-5)[–8\]](#page-13-6). The relative performance of neural network classifiers in relation to a range of alternative methods, including standard statistical classifiers, machine learning methods, and decision trees, is discussed in the literature (e.g., [\[1\]](#page-13-0)). As with all classifiers, the quality of the final classification is, however, in part a function of the classifier and the nature, notably the size and quality, of the ground reference dataset on the class membership used $[9-12]$ $[9-12]$. Ground reference data are used to provide data to train the neural network and to evaluate the quality of its predictions. The latter is typically expressed in terms of the overall accuracy of the classification output from the network.

In remote sensing applications, the class membership of the cases in the training set is typically determined by ground-based observation or interpretation of very fine spatial resolution imagery [\[1](#page-13-0)[,13\]](#page-13-9). Clearly, the nature and quality of the ground reference data used will impact upon the accuracy of the predictions obtained from the neural network. Classification accuracy is, for example, influenced by the size, composition, and quality of the ground reference dataset used in training [\[2,](#page-13-2)[9,](#page-13-7)[10,](#page-13-10)[14–](#page-13-11)[16\]](#page-13-12). The way the ground reference data are used is also important, especially if part of it is used for the purpose of validation [\[10\]](#page-13-10). The use of some reference data for validation purposes, to indicate the quality of the trained model generated, is commonplace in defining a supposedly optimal neural network. In common remote sensing applications, the optimal approach would be the one that yielded the most accurate thematic map when applied to the image data.

In a typical remote sensing application, the aim of the classification is to accurately map the land cover classes in the region of study. Throughout there are a set of basic assumptions made in classification analyses. These include the need for the set of classes to be mutually exclusive and exhaustively defined. Failure to satisfy these assumptions will result in errors. Cases of an untrained class will, for example, typically have to be allocated erroneously to one of the defined classes [\[17\]](#page-13-13). Although there are instances when a neural network may be used to obtain a non-standard classification, such as soft classification when classes intergrade or are mixed together [\[17\]](#page-13-13), the standard hard classification in which a case belongs fully to a single class is the focus on this article. In all cases, the ground reference data used in training, validating, and testing the neural network have a key role to play in the production of an accurate map. Hence, the design of these datasets is important.

Good practices for classification accuracy assessment for the evaluation of the quality of thematic maps derived via remote sensing have been defined and include guidance on the construction of the testing sample upon which the assessment is to be based. Typically, for example, the use of a reference dataset acquired following a probabilistic sampling design is recommended [\[18,](#page-13-14)[19\]](#page-13-15). This allows a rigorous design-based assessment of classification accuracy, typically based upon the analysis of an error or confusion matrix in which the predicted class label obtained from the neural network is cross-tabulated against the label in the reference dataset for the sample of cases under consideration [\[1,](#page-13-0)[18\]](#page-13-14).

The goal of training a classifier is different to that of testing its output predictions and, hence, the nature of the ideal training set may be very different to that of the testing set. In training, the aim is, essentially, to guide the neural network to learn the identity of the classes from their remotely-sensed response. Thus, the observed remotely-sensed response for the training sample of known class membership is used in network learning to ultimately form decision rules to accurately label cases of unknown class membership in order to map the region of interest [\[1\]](#page-13-0). Much conventional guidance on the design of the training set in remote sensing applications is based upon historical work undertaken with conventional statistical classifiers, such as the maximum likelihood classification. This advice typically calls for the sample size to be estimated following basic sampling theory in order to derive a representative and unbiased description of each class to allow cases of unknown class membership to then be allocated the label of the class they had greatest similarity to. Alternatively, a simple heuristic, such as the use of a sample of cases for each class, the size of which is at least 10 times the number of discriminating variables, such as the spectral wavebands, are used [\[10,](#page-13-10)[20,](#page-14-0)[21\]](#page-14-1). In essence, this type of approach is calling for the use of a stratified sample design in the formation of the training set. Despite the development of new classifiers, such approaches are still widely used even though the nature of the ideal training set varies between classifiers.

With neural networks and machine learning methods such as the support vector machine (SVM) and relevance vector machine (RVM) attention in training is focused more on individual cases than broad statistical summary statistics that are central to statistical classifiers, such as the maximum likelihood classification [\[1\]](#page-13-0). The individual training cases in the training set can vary greatly in value to an artificial neural network classification [\[22](#page-14-2)[,23\]](#page-14-3). In addition, different classifiers may ideally focus upon different cases in the training set. For example, a SVM may require only a very small training sample and, ideally, cases that lie in the boundary region between classes, while a RVM might also

require only a small number of training cases, but these are anti-boundary in nature [\[12\]](#page-13-8). Considerable research has, therefore, focused on how the properties of the training dataset impact on a classification, typically with a desire to maximize the final mapping accuracy (e.g., [\[9,](#page-13-7)[10,](#page-13-10)[24](#page-14-4)[–26\]](#page-14-5)). Sometimes it may be possible to predict the location of useful candidate training cases for a classification [\[27\]](#page-14-6) or, contrary to conventional approaches, even deliberately use mixed cases in training and the analyst can seek to form a training set intelligently for a given application scenario [\[22,](#page-14-2)[28](#page-14-7)[,29\]](#page-14-8). As such, there is no single ideal way to define a training set that is universally applicable. Given the popularity of consensus or ensemble methods that use a variety of different classifiers [\[7](#page-13-16)[,30\]](#page-14-9) it is common to see relatively large training sets acquired following conventional guidance. While this approach may sometimes be inefficient, notably in that the sample may be larger than needed, it has the capacity to provide useful training data for a wide range of classifiers. It is, therefore, common to see either a training sample designed explicitly for a specific task, which could be a small and highly unrepresentative sample [\[29,](#page-14-8)[31\]](#page-14-10), or the use of a stratified design that seeks to ensure each class can be described well [\[1\]](#page-13-0). The design details are, however, important. The nature of the training set has a significant impact on the accuracy of predictions by a neural network [\[2](#page-13-2)[,15](#page-13-17)[,32\]](#page-14-11). For example, the size of the training set, notably in relation to the complexity of the network, can have a marked effect on classification by a neural network [\[9,](#page-13-7)[16,](#page-13-12)[33\]](#page-14-12). The composition of the training set in terms of relative class abundance is also important [\[15,](#page-13-17)[34\]](#page-14-13). Variations in class abundance can yield imbalanced datasets that, as in other classifiers, can have substantial impacts on the final classification.

In a supervised classification with a conventional feedforward neural network it is common for part of the training sample to be used for validation purposes [\[1](#page-13-0)[,2,](#page-13-2)[10,](#page-13-10)[33](#page-14-12)[–35\]](#page-14-14). In this data splitting approach part of the training set is used in the normal way to provide examples of the classes upon which the classifier may learn to form rules to classify cases of unknown membership. The remaining part of the training set forms the validation set and is used to evaluate the performance of the network in terms of the accuracy with which the validation set is classified, as well as help determine when to stop network learning [\[10,](#page-13-10)[36](#page-14-15)[–39\]](#page-14-16). A variety of approaches exist for the splitting of the training cases to form the training set and the validation set. If reference data are plentiful the training set and validation set could be completely separate and independent samples but if this is not the case other approaches to cross-validation, such as the leave one out approach, may be used [\[34\]](#page-14-13); for simplicity, the focus here is on the use of a completely independent validation set. The use of a validation set is important in neural network-based approaches to classification as there is a desire to avoid overfitting to the training data and there are a variety of network parameters that require definition. The latter includes the basic structure of the network (e.g., number of hidden layers and units) and the learning algorithm and parameters (e.g., momentum, learning rate and number of iterations). There is an extensive literature on this topic $[2,10]$ $[2,10]$. The basic idea is that a range of different networks can be defined and one that is optimal for the task at hand, defined with the aid of the validation dataset. Thus, for example, a range of different networks may be generated and the one that classifies the validation set to the highest accuracy is selected for the final classification analysis to produce a thematic map. This analysis might also suggest ways to further enhance the classification by, for example, indicating redundant discriminating variables that could be deleted in order to allow more rapid computation [\[28](#page-14-7)[,40\]](#page-14-17). It is also useful as some approaches used in training may artificially inflate class separability and be unhelpful [\[23\]](#page-14-3). Given that the ultimate aim of the analysis in a typical remote sensing application is the production of an accurate thematic map via a classification analysis the nature of the validation set can be important. Commonly, with classifiers that use validation data, the validation set is formed by simply taking cases randomly from the entire set of data acquired for training activity $[38,41]$ $[38,41]$ or a separate, often stratified, sample of cases is obtained (e.g., $[42]$). The size of the validation set is important. As in other aspects of the analysis, the literature contains guidance on the way the reference data should be divided up. For example, Mas et al. [\[43,](#page-14-21)[44\]](#page-15-0) suggests that half of the labelled cases be used for training, a quarter for validation and the final quarter for testing.

The composition of the sample of cases forming the validation set, notably in terms of the relative abundance of the cases of the classes may vary as a function of the design used to acquire the training dataset. If the validation sample is formed with a simple random sample design it is likely to be imbalanced in composition, with the number of cases of a class reflecting its relative abundance in the region being mapped. The use of a stratified sample design in the generation of the validation sample will act to give a balanced dataset but this need not be an ideal approach. Indeed, the use of a stratified design could result in the selection of a network that was sub-optimal for the task if the classes vary in abundance and separability. For example, standard approaches to the assessment of overall classification accuracy weight errors equally and could inflate the importance of rare classes while deflating that of abundant classes. Sometimes it may be possible to account for the sample design if there is information on class abundance [\[18\]](#page-13-14). Alternatively, if the study aim is focused on a single class, which is often the case, the accuracy assessment used in the validation could be focused on that class at the expense of the others. However, for a general purpose map, it may be more appropriate to follow the guidance on sampling that is typically used in the formation of the testing set as this typically allows for variations in class abundance to be accounted for.

The effect of different sample designs for the formation of the validation dataset is explored in this paper. Specifically, the focus is on the use of samples acquired by simple random and stratified random sampling designs (both without replication). Due to the way overall accuracy is typically estimated with a validation sample, it is hypothesized that the overall accuracy of the final classification, evaluated using the testing set, will be larger for a neural network trained using a validation sample generated via simple random as opposed to stratified random sampling. Indeed a series of outcomes may be predicted as having the potential to arise as a function of the validation dataset used. For example, in relation to two classes that overlap, it would be expected that the hyperplane to best separate the classes fitted when a balanced validation set was used would migrate away from the more abundant class if an imbalanced training set was used. As a consequence of this, there is an opportunity for the accuracy with which the more abundant class is classified to rise as fewer cases of it will be omitted from it and fewer of its cases commissioned by the rarer class (es). These trends arise as classification errors are weighted equally in standard assessments of overall accuracy. The overall trend expected would be for the accuracy with which the abundant class is classified to increase while the accuracy of the classification of the rarer class would decline. As such it is hypothesized that the use of a stratified sample design may not be ideal as its use relative to a randomly-defined validation dataset would be associated with a decrease in overall accuracy, arising noticeably through a decrease in the accuracy for the abundant class (es) as a result of an increase in the commission of cases of abundant class (es) by the set of rarer classes.

2. Data and Methods

Two datasets were used. First, a simulated dataset was used to illustrate the issues and also to facilitate, if desired, replication. Second, a real dataset consisting of remotely-sensed data and associated ground reference data on land cover class labels was used.

A very simple multi-class classification scenario was simulated. This comprised data on four classes acquired in two dimensions, or bands. The data for each class were formed using a random number generator using analyst provided values for the class mean and standard deviation on the assumption that the data for each class were normally distributed. For each class the standard deviation was set equal to 5 and the mean values used to generate the data in each band are shown in Table [1.](#page-5-0) In the scenario generated, class 1 was the most abundant class. Specifically, class 1 was five times more abundant than each of the other classes. Most attention was focused on class 1 and class 2, which exhibited a degree of overlap in their distributions with class 3 and, especially, class 4 was highly separable (Figure [1\)](#page-5-1).

For the analyses of the simulated dataset, training validation and testing sets were generated (Table [2\)](#page-6-0). These were used in two series of analyses, one using a validation dataset formed via simple

random sampling and the other formed via a stratified random sample design. In brief, a single training set was generated using a stratified random sample of 400 cases per-class. Similarly, a single testing set was used to evaluate the accuracy of the classifications from the neural networks selected as optimal in each series of analyses undertaken. This testing set was simulated to represent a sample acquired by simple random sampling and comprised 800 cases. Due to the way the scenario was designed, class 1 was five times more abundant in it than the other classes. The two validation datasets used each contained 800 cases, but one was formed via a simple random sample in which class abundance varied as in the testing set, while the other was formed with a stratified random sample in which each class as in the testing set, while the other was formed with a stratified random sample in which each class
was equally represented. Each series of neural network analyses used a software package that sought to generate an optimal network, with optimality defined as the maximization of the overall accuracy
with which the validation set was classified. with which the validation set was classified.

The remotely-sensed data were acquired by an airborne thematic mapper (ATM) sensor for a test The remotely-sensed data were acquired by an airborne thematic mapper (ATM) sensor for a test
site near the village of Feltwell, Norfolk, UK. The latter is located approximately 58 km to the northeast of the city of Cambridge. The land around the village was topographically flat and its land cover of the city of Cambridge. The land around the village was topographically flat and its land cover
mosaic was characterized mainly by large agricultural fields. At the time of the ATM data acquisition these fields had also [typ](#page-6-1)ically been planted with a single crop type (Figure 2).

The ATM used was a basic multispectral scanning system that acquired data in 11 spectral The ATM used was a basic multispectral scanning system that acquired data in 11 spectral wavebands from blue to thermal infrared wavelengths (Table 3). Given the relatively low altitude wavebands from blue to thermal infrared wavelengths (Table [3\).](#page-6-2) Given the relatively low altitude
of airborne data acquisition (~2000 m), the spatial resolution of the imagery was very much smaller than the typical field size, approximately 5 m. As a result, image pixels tended to represent an area than the typical field size, approximately 5 m. As a result, image pixels tended to represent an area
composed of a single class (i.e., pure pixels) and, hence, were appropriate for hard image classification analysis; boundary pixels were ignored. Attention focused here on the six crop classes that dominated analysis; boundary pixels were ignored. Attention focused here on the six crop classes that dominated
the region at the time of the ATM data acquisition. These classes and their approximate coverage (%) of the study area at the time were: sugar beet $(S, 30.3%)$, wheat $(W, 30.0%)$, barley $(B, 16.0%)$, carrot $(C,$ 10.3%), potato (P, 7.8%), and grass (G, 5.4%).

Band 1 50 35 83	Class 4	Class 3	Class 2	Class 1	Band
		50	60	50	Band 2

Table 1. The classes in the simulated dataset; note: units are arbitrary. **Table 1.** The classes in the simulated dataset; note: units are arbitrary.

Figure 1. Simulated data (training, validation (random), and testing). Note: class 2 is shown overlaid **Figure 1.** Simulated data (training, validation (random), and testing). Note: class 2 is shown overlaid on top of part of class 1 and the area of overlap may be inferred. on top of part of class 1 and the area of overlap may be inferred.

Figure 2. Data for Feltwell. (a) The location of Feltwell, © OpenStreetMap contributors; (b) airborne thematic mapper (ATM) image extract in waveband 4 and (**c**) ATM image extract in waveband 7. thematic mapper (ATM) image extract in waveband 4 and (**c**) ATM image extract in waveband 7.

Table 3. The 11 spectral wavebands of the ATM sensor used. **Table 3.** The 11 spectral wavebands of the ATM sensor used.

Ground reference data for the purposes of network learning (training) and evaluation were for training, 25% for validation, and 25% for testing. The composition of these datasets, however, sometimes varied (Table 4). acquired. Following [\[43,](#page-14-21)[44\]](#page-15-0), the ground reference dataset was partitioned such that 50% was used

Conventional guidance on the design of the training set was followed with 110 cases of each class obtained for training purposes; meeting the often stated requirement of a sample of at least 10 times the

number of discriminating variables (wavebands) used as input. The training set, therefore, contained 660 cases acquired by a stratified random sampling design. This training set was used throughout.

Testing sets should, ideally, be acquired using a probability sampling design [\[18,](#page-13-14)[19\]](#page-13-15). Here, a simple random sample (without replication) was used to acquire 330 cases to use for testing. Note that this sample size exceeds that required for accuracy estimation of a map with an accuracy of 85%, a standard if contentious target accuracy in remote sensing, with an allowable error of 4%. This testing set was used in all analyses of the ATM data. This latter issue impacts on comparisons of accuracy estimates and requires the use of a technique suited for related samples [\[45\]](#page-15-1).

Table 4. Class composition of the datasets used for analyses of the remotely sensed data.

As with the analyses of the simulated dataset, the search for an optimal neural network was undertaken twice. In each case the training and testing sets were the same, the only difference was the composition of the validation set used to identify the optimal network from a set of candidate networks generated for the task. In one set of analyses, the validation set was generated by simple random sampling and thus the number of cases of each class tended to reflect the actual abundance of the classes in the region to be mapped. Indeed, here, the sample was selected to ensure that the class composition of the validation set equalled that of the testing set. This sample comprised 330 cases. In the other set of analyses, a validation sample of the same size, but acquired following a stratified random sample such that each class was equally represented was used. The nature of the datasets used is summarized in Table [4.](#page-7-0)

In the search for an optimal neural network to classify the ATM data, optimality was defined in relation to the maximum overall accuracy of the classification of the validation data. The accuracy of each classification was calculated from the confusion or error matrix generated for it which shows a cross-tabulation predicted and actual class label for each case in the dataset analysed $[1,18]$ $[1,18]$. Using the layout and notation defined for the confusion matrix shown in Table [5,](#page-8-0) which is used throughout the article, overall accuracy, *O*, was calculated using Equation (1):

$$
O = \frac{\sum n_{ii}}{n} \tag{1}
$$

In addition to the global estimate of classification quality conveyed by overall accuracy, the producer's, *P*, and user's accuracy, *U*, were calculated with reference to each class [\[1](#page-13-0)[,18\]](#page-13-14). These were obtained for class *i* from Equations (2) and (3) respectively:

$$
P_i = \frac{n_{ii}}{n_{\cdot i}}\tag{2}
$$

Note that, as is common, the calculations of these measures of accuracy are based on the raw counts of cases shown in the elements of the confusion matrix. These approaches to accuracy assessment were used to support all analyses, whether based on the training, validation, or testing datasets. Most focus is, however, on the accuracy values arising from analyses of the validation and testing datasets.

To determine if the use of different validation samples impacted on the accuracy of the final thematic classification, the statistical significance of differences in the accuracy of the classifications of the testing set were assessed. Standard approaches for the comparison of accuracy values that are popular in remote sensing projects are unsuitable here as the same sample of testing cases was used throughout. To accommodate for this situation, the statistical significance of differences in accuracy was assessed using the McNemar test [\[45,](#page-15-1)[46\]](#page-15-2). The latter is a non-parametric test that is based on a binary confusion matrix which shows the cross-tabulation of the cases that have been labelled correctly and incorrectly by the two classifications being compared. The test focuses on the discordant cases, those which were classified correctly by one classifier, but incorrectly by the other. Without continuity correction, the test is based on the normal curve deviate, *z*, as expressed as:

$$
z = \frac{n_{CI} - n_{IC}}{\sqrt{n_{CI} + n_{IC}}}
$$
\n
$$
\tag{4}
$$

where n_{CI} indicates the number of cases in the relevant element of the matrix with the subscript *C* indicating if the classification was correct in its labelling or *I* if it was incorrect and order of the subscripts indicates the specific classification from the pair under study. For a standard two-tailed test at the 95% level of confidence, the null hypothesis of no significant difference is rejected if the calculated *z* exceeds the critical value of |1.96|. Similarly, for a one-tailed test, if the hypothesis under test has a directional component, the direction (sign) needs consideration and the magnitude of the critical value of *z* to indicate that a significant difference exists at the 95% level of confidence is 1.645.

Class			\cdots	q	Total
	n_{11}	n_{12}	\cdots	n_{1q}	n_1 .
2	n_{21}	n_{22}	\cdots	n_{2q}	n_2 .
÷	$\ddot{\cdot}$	٠ ٠		٠ ٠	$\ddot{\cdot}$
q	n_{q1}	n_{q2}	\cdots	n_{qq}	$n_{\rm q}$.
Total	$n_{\cdot 1}$	$n_{\cdot2}$	\cdots	n_{q}	n

Table 5. The confusion matrix based on raw counts of cases for a classification of q classes. Matrix columns show the label in the reference data and rows the label in the classification.

3. Results and Discussion

With the simulated dataset, two sets of analyses were undertaken to identify optimal neural networks, one using the validation set formed via simple random sampling and the other formed using a stratified random sample. The key properties of the selected networks and their ability to classify the datasets are defined in Table [6](#page-9-0) with a full set of confusion matrices for each set of analyses shown in Tables [7](#page-9-1) and [8.](#page-10-0)

Attention focused especially on the accuracy with which the testing set was classified, as this reflects the accuracy of the final product obtained. It was evident that the accuracy of classification obtained with the use of the simple random sample (98.37%) was slightly higher than that arising from the use of the validation set formed via stratified sampling (96.62%). Although small, this difference was significant (*p* < 0.05), with the calculated value of *z* from Equation (4) given the 26 discordant cases observed being 2.745 (Table [9\)](#page-10-1). It was also evident that the predictions of what might happen as moving from a balanced stratified to random validation sample outlined in the introduction occurred.

Note, for example, that relative to the classification obtained with a stratified validation sample, the use of the validation set formed by random sampling resulted in a higher overall accuracy of the testing set. This arose because of a higher accuracy with which the abundant class was classified due to a reduction in omission error (from 27 to 7 cases) and the accuracy of the rarer class it was confused with declined (Tables [7](#page-9-1) and [8\)](#page-10-0). Critically, the results suggest that the use of a balanced training set acquired via a stratified random sample may produce a sub-optimal final output. As hypothesized, the use of a stratified sample resulted in a lower accuracy than was achievable because the accuracy with which the abundant class was classified declined, associated with a large omission error due to cases being commissioned into a rarer class.

A series of analyses was undertaken to define an optimal neural network using the two validation datasets in the analyses of the remote sensing data. The core focus here is on the classification results from the network determined to be optimal from each series of analyses. Although the precise details of the neural networks are relatively unimportant as the classifications that arise from them are the focus of attention, key details on the networks selected are summarized in Table [10](#page-10-2) and the confusion matrices for the testing set are shown in Tables [11](#page-11-0) and [12.](#page-11-1)

Table 6. Key characteristics of the networks selected for the analyses of the simulated data; note: network architecture is expressed as input:hidden:output.

Validation Dataset	Architecture	Algorithm and Iterations		Classification Accuracy (%) Training Validation Testing	
Random	2:15:2	Backpropagation 48	97.25	97.62	98.37
Stratified	2:8:2	Conjugate gradient 55	98.25	98.75	96.62

Training								
Class	$\mathbf{1}$	$\overline{2}$	3	$\overline{4}$	Total	User's $(\%)$		
1	398	40	$\overline{2}$	θ	440	90.45		
$\overline{2}$	$\overline{2}$	360	Ω	Ω	362	99.44		
3	$\boldsymbol{0}$	θ	398	Ω	398	100		
$\overline{4}$	$\overline{0}$	Ω	$\mathbf{0}$	400	400	100		
Total	400	400	400	400	1600			
Producer's (%)	99.50	90.00	99.50	100				
			Validation					
Class	$\mathbf{1}$	$\overline{2}$	3	$\overline{4}$	Total	User's $(\%)$		
$\mathbf{1}$	490	9	$\mathbf{0}$	θ	499	98.19		
$\overline{2}$	10	91	θ	θ	101	90.09		
3	θ	$\mathbf{0}$	100	Ω	100	100		
$\overline{4}$	$\overline{0}$	Ω	$\mathbf{0}$	100	100	100		
Total	500	100	100	100	800			
Producer's (%)	99.00	91.00	100	100				
			Testing					
Class	$\mathbf{1}$	$\overline{2}$	3	$\overline{4}$	Total	User's $(\%)$		
1	493	6	Ω	θ	499	98.79		
$\overline{2}$	7	94	Ω	$\overline{0}$	101	93.06		
3	θ	θ	100	Ω	100	100		
$\overline{4}$	θ	θ	$\boldsymbol{0}$	100	100	100		
Total	500	100	100	100	800			
Producer's (%)	98.60	94.00	100	100				

Table 7. Confusion matrices for the network selected using a validation set formed by random sampling.

Training (a)								
Class	$\mathbf{1}$	$\overline{2}$	3	$\overline{\mathbf{4}}$	Total	User's $(\%)$		
1	386	14	Ω	θ	400	96.50		
$\overline{2}$	14	386	Ω	θ	400	96.50		
3	θ	$\mathbf{0}$	400	$\overline{0}$	400	100		
4	θ	$\overline{0}$	θ	400	400	100		
Total	400	400	400	400	1600			
Producer's (%)	96.50	96.50	100	100				
		(b)	Validation					
Class	$\mathbf{1}$	$\overline{2}$	3	4	Total	User's $(\%)$		
$\mathbf{1}$	195	5	θ	θ	200	96.50		
$\overline{2}$	5	195	Ω	0	200	96.50		
3	θ	θ	200	$\overline{0}$	200	100		
4	Ω	θ	$\mathbf{0}$	200	200	100		
Total	200	200	200	200	800			
Producer's (%)	97.50	97.50	100	100				
		(c)	Testing					
Class	$\mathbf{1}$	$\overline{2}$	3	$\boldsymbol{4}$	Total	User's $(\%)$		
1	473	$\mathbf{0}$	θ	$\overline{0}$	473	100		
2	27	100	Ω	θ	127	78.74		
3	θ	$\overline{0}$	100	$\overline{0}$	100	100		
4	θ	$\overline{0}$	θ	100	100	100		
Total	500	100	100	100	800			
Producer's (%)	94.60	100	100	100				

Table 8. Confusion matrices for the network selected using a validation set formed by stratified sampling.

Table 9. Cross-tabulation of labelling from classifiers using random (columns) and stratified (rows) validation sets for the simulated data.

	Correct	Incorrect	Total
Correct	767		773
Incorrect			
Total	787		

Table 10. Key characteristics of the networks selected for the analyses of the remotely-sensed data.

The selected neural networks were able to classify the data accurately. In each case, however, it was evident that classification accuracy was slightly, no more than 1.51%, less accurate when the stratified, rather than random, validation dataset had been used in network selection. Although small, these differences can still be significant. Indeed, a key result was that the accuracy of the classification of the testing dataset, which indicates the accuracy of the land cover map obtainable, was higher when the validation set formed using random (overall accuracy = 97.87%), rather than stratified, sampling (overall accuracy = 96.36%). Although small, a test of the significance of the difference in overall accuracy, using a one-sided McNemar test to recognize the directional nature of the test and the use of

the same testing set, revealed it to be statistically significant at the 95% level of confidence (Table [13\)](#page-11-2). Specifically, given the seven discordant cases observed (Table [13\)](#page-11-2), Equation (4) yields $z = 1.889$.

The difference in accuracy between the classifications of the testing sets by the two selected neural networks (Tables [11](#page-11-0) and [12\)](#page-11-1) was attributable mostly to seven cases of sugar beet being commissioned into the potato class when the validation set formed using a stratified sample was used. As a result of these errors the producer's accuracy for the sugar beet class declined from 99.00% when the validation sample acquired with a random sample was used to 93.00% when the validation sample had been generated with a stratified sample. The user's accuracy for the potato class also differed for the classifications of the testing set obtained when using the validation set defined with random and stratified sampling, the accuracies being 96.00% and 77.42%, respectively.

Given that the testing set had been generated using a simple random sample design the variation in the number of cases per-class reflected the relative abundance of the classes in the region to be mapped. Critically, the size of the sample of cases for the potato class was approximately one quarter of that for the sugar beet class. As hypothesized, the overall accuracy of the testing set decreased when the stratified rather than random sample was used in validation because cases of an abundant class (sugar beet) were commissioned by a relatively rare class (potato). Thus, even in a situation such as that encountered here, in which the classes are very highly separable, the sample design used in the formation of the validation dataset can have a statistically significant effect on the overall accuracy of the final land cover map, as reflected in the accuracy of the classifications of the testing set (Tables [11](#page-11-0) and [12\)](#page-11-1).

Class	S	W	B	C	P	G	Total	User's $(\%)$
S	99	Ω		0	0	0	100	99.00
W	0	96	0	0		0	97	98.97
B	0	າ	52	Ω	0	0	54	96.30
	0	0	0	34		0	35	97.14
P		0	0	0	24	0	25	96.00
G	Ω		0	Ω	0	18	19	94.74
Total	100	99	53	34	26	18	330	
Producer's $(\%)$	99.00	96.97	98.11	100	92.31	100		

Table 11. Confusion matrix for the testing set from the network selected using a validation set formed by random sampling.

Table 12. Confusion matrix for the testing set from the network selected using a validation set formed by stratified sampling.

Class	S	W	B	C	P	G	Total	User's $(\%)$
S	93	Ω		0	Ω	0	94	98.94
W	0	97	0	0		0	98	98.98
B	0	2	52	0	0	0	54	96.30
	0	0	0	34		0	35	97.14
D	7	0	0	0	24	0	31	77.42
G	0	0	0	0	0	18	18	100
Total	100	99	53	34	26	18	330	
Producer's $(\%)$	93.00	97.98	98.11	100	92.31	100		

Table 13. Cross-tabulation of labelling from classifiers using random (columns) and stratified (rows) validation sets for the remotely sensed data.

Finally, it was also evident that the networks selected for the analyses of the remotely-sensed data differed, most notably in terms of architecture (Table [10\)](#page-10-2). When the validation set constructed with simple random sampling had been used, the data acquired in wavebands 1 and 8 were deemed unnecessary and, hence, only nine input units used. In addition this latter network also had fewer hidden units than the network selected when the validation set had been formed with a stratified sample. Overall, the network formed with the use of the validation sample acquired by random

sampling was smaller and less complex than that selected when the validation set formed with a stratified sample was used. As such the network might be expected to be less likely to over-train and have a higher ability to generalize than that selected from the use of the validation sample formed via stratified sampling. Slightly different trends were observed for the simulated dataset. Here, however, it should be noted that a large number of networks of very different size, but very similar performance in terms of ability to classify the data, were generated, limiting the ability to comment on the issue; note, for example, that some candidate networks that yielded the same accuracy for the classification of the testing set after use of the random validation set had the smallest number of hidden units.

The results show that the design of the validation sample has a significant effect on classification by a neural network. Thus, the sample design used to form the validation set should be considered carefully when using neural networks. If, for example, there are constraints that limit design possibilities, it may be possible to make simple adaptations to standard practice. For example, if a stratified sample must be used for the validation sample, then this feature of the dataset should be accounted for in the assessment of the accuracy of the classification of the validation set. Thus, rather than use a standard confusion matrix, as indicated in Table [5,](#page-8-0) the elements of the matrix could be converted from raw counts to proportions via $p_{ij} = W_i \frac{n_{ij}}{n_i}$ $\frac{n_l}{n_i}$ where W_i is the proportion of the area mapped as class *i* [\[18\]](#page-13-14). The use of this approach, for example, shows that the estimated accuracy with which the validation set formed by stratified sampling of the simulated data was slightly less (98.12%) than the naïve assessment of the matrix (98.75%) and may indicate that the network it is associated with is, therefore, less attractive as a candidate for the task at hand than it first appears from the naïve assessment. Alternatively, if the focus of the application is on a subset of the classes it may be sensible to weight errors differentially or focus on only on the classes of interest rather than use overall accuracy. Critically, consideration needs to be given to the design of the validation sample in classification by a neural network. It should also be noted that this is only one small part of a set of broader validation issues that should be considered in the use of neural networks [\[47\]](#page-15-3).

4. Conclusions

Feedforward neural networks are often constructed with the aid of a validation dataset. The latter data are typically used to indicate the accuracy of the neural network on a dataset independent of that used in its training phase. Commonly, the optimal network is selected on the basis of the overall accuracy with which the validation dataset is classified based on the analysis of a raw confusion matrix. A problem with this approach is that all classification errors are typically treated equally and the magnitude of the overall accuracy can be distorted by the sample design used to form the validation sample. Here, it was shown that the use of a stratified, rather than random, sample of cases as a validation set resulted in a statistically significant reduction of the accuracy with which an independent test set was classified. This difference in accuracy arose because of the commission of cases of an abundant class by a relatively rare class. Moreover, these results were obtained for analyses of a datasets in which the classes were very highly separable within the feature space of both the simulated and the remotely-sensed datasets used. Assuming that the desired aim is the production of a map with high overall accuracy, simple ways to address issues connected with the design of the validation dataset are to the use of a validation sample acquired by simple random sampling or to ensure the accuracy of its classification is based on proportions calculated on the basis of class abundance rather than basic raw counts.

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