Ampatzidis, Theofanis and Chronopoulos, Dimitrios (2017) Parametric study of control of frequency banded behaviour of periodic pressurised composite structures. In: 24th International Congress on Sound and Vibration, 23-27 July 2017, London, UK.

PARAMETRIC STUDY OF CONTROL OF FREQUENCY BANDED BEHAVIOUR OF PERIODIC PRESSURISED COMPOSITE STRUCTURES

Theofanis Ampatzidis and Dimitrios Chronopoulos
Institute for Aerospace Technology & The Composites Group, University of Nottingham, NG7 2RD, UK
email: Theofanis.Ampatzidis@nottingham.ac.uk

Periodic structures are very common in engineering, such as airplane fuselages and train rails. This periodicity has been observed to be the cause of banded frequency response after mechanical excitation. This response can be engineered so that noise and vibrations to be isolated or even annihilated. In addition to this, further methods of inducing band-gaps without weight penalty are of interest among the researchers. In this paper a parametric survey was conducted examining the impact of the core geometry and the pressure in the core cells on the suppression of the vibrations. An infinite composite sandwich beam with hollow and pressurised core cells as periodic band gap inducing factors was examined. The periodic theory was used to predict the effect of pressured core cells periodicity on wave propagation and band gaps generation. Three low order finite elements (FE) models were used in this survey, which consisted of a small section of the simple sandwich beam with homogeneous core, with hollow core and with pressurised hollow core

Keywords: band gap, wave finite elements, vibrations, pressurised composite structures

1. Introduction

Periodic structures consist of infinite assembly of identical elements, usually called cells, joined in an identical manner. These structures, also called *banded* structures, have been subject of research for more than a century. Floquet [1] was the first to publish research on periodic structures, where he studied 1D Mathieu's equation. His work was followed by Rayleigh [2], who arrived at a form of Floquet's theorem. In this century, Mead firstly introduced Wave Finite Elements (WFE) Method in [3] which is based on Brillouin's periodicity theory (PT) [4] and Floquet's and Bloch's theorems. In [5] his work on wave propagation in periodic structures was reviewed. The WFE has recently found applications in predicting the vibroacoustic and dynamic performance of composite panels and shells [6, 7, 8, 9, 10, 11, 12] , with pressurized shells [13, 14] and complex periodic structures [15, 16, 17, 18] having been investigated. The variability of acoustic transmission through layered structures [19, 20], as well as wave steering effects in anisotropic composites [21] have been modelled through the same methodology.

Periodic structures exhibit *band-gaps*, where wave propagation is significantly attenuated. Due to this attenuation and their potentials to passively damp vibration, numerous researches have been published examining periodic structures' banded frequency response. Some of the most important work are Ruzzene's et al. [22, 23] and Hussein's et al. [24, 25]. Ruzzene et al. focused on the control of wave propagation and banded behaviour, firstly in sandwich composite beams with periodic auxetic core [22] and then in 2D sandwich plate with periodic honeycomb [23]. In both works it was proved that banded behaviour can be controlled by changing parameters such as the length ratio of the periodic cells of the core. Hussein et al. derived dispersion relations for periodic materials and examined the analysis [24] and design [25] of them. Based on these works, Liu et al. [26] produced a research focusing on the wave motion and banded response of four different types of periodicity in 1D beams. In addition to this work, Wu et al. [27] examined the banded behaviour of sandwiches with corrugated core, focusing on the geometry of the core and Chen et al. [28] examined the wave propagation in sandwich with periodic core. In the latter work, two different materials periodically forming the core of the sandwich were examined. WFE method has, also, been used to examine the banded behaviour of a periodic beam in [29].

In this work the periodic theory used in [30] has been adopted to examine the banded behaviour of infinite composite sandwich beam with hollow core as band gap inducing factor. Additionally, pressure was examined as a method to actively control banded behaviour of the structure. The paper is organised as follows: in Sec.2 the methodology used to get the banded behaviour of the examined structures is described. In Sec.3 the wave dispersion characteristics of each case are sought using the methodology described in the previous section. Numerical results are presented and all the cases are compared with each other commenting on the effect of hollow core length and pressure on the banded behaviour of the beam. In Sec. 4 conclusions and thoughts on the results of the presented work are drawn.

2. Methodology

2.1 Description of the method

The periodic theory adopted on 1D in current work is the one used in [30]. A general structure with 1D periodicity was considered. A periodic shell can be extracted from the structure and modeled using a FE model with degrees of freedom **q** (see Fig. 1). Steady-state harmonic vibration of frequency ω is considered in what follows and all response quantities are represented by complex amplitudes so that

$$\mathbf{q}(t) = \mathrm{Re}\{\mathbf{q}e^{i\omega t}\} \tag{1}$$

In 1D the degrees of freedom **q** of the cell can be partitioned into left (*L*), interior (*I*) and right(*R*) degrees of freedom. According to Floquet's theorem, the equation that relates the displacements on the two edges of the section is [3]:

$$\mathbf{q}_R = \lambda\mathbf{q}_L, \quad \mathbf{f}_R = -\lambda\mathbf{f}_L, \tag{2}$$

where $\lambda = e^{-ikL_x}$, with L_x being the periodic element's lenght, k being the wavenumber and $\varepsilon_x = kL_x$ being the 'phase constant'.
The complete vector of local degrees of freedom for 1D can be ordered so that

$$\mathbf{q} = \begin{bmatrix}\mathbf{q}_I^T & \mathbf{q}_L^T & \mathbf{q}_R^T\end{bmatrix}^T \tag{3}$$

The undamped equation of motion for the cell is given by

$$\left[\mathbf{K} - \omega^2\mathbf{M}\right]\mathbf{q} = \mathbf{f} \tag{4}$$

where **K** and **M** are the stiffness and mass matrices, respectively, **f** is the vector of the nodal forces. In order to write the propagation relation in Eq. (1) in matrix form, we consider matrix **R**, which

$$\mathbf{R} = \begin{bmatrix}\mathbf{I} & 0 \\ 0 & \mathbf{I} \\ 0 & \mathbf{I}e^{-i\varepsilon_x}\end{bmatrix} \tag{5}$$

This way we get

$$\mathbf{q} = \mathbf{R}\mathbf{q}', \quad \text{where} \quad \mathbf{q}' = [\mathbf{q}_I \mathbf{q}_L]^T \tag{6}$$

The resulting homogenous quation in the reduced set of coordinates is then given by

$$\left[\mathbf{K}' - \omega^2 \mathbf{M}'\right] \mathbf{q}' = \mathbf{f} \tag{7}$$

where

$$\mathbf{K}' = \mathbf{R}^H(\varepsilon_x)\mathbf{K}\mathbf{R}(\varepsilon_x), \quad \mathbf{M}' = \mathbf{R}^H(\varepsilon_x)\mathbf{M}\mathbf{R}(\varepsilon_x) \tag{8}$$

and where $\mathbf{R}^H$ denotes the complex conjugate, or else called Hermitian, transpose of $\mathbf{R}$. When a particular set of phase constants ε_x are specified then we get a standard eigenvalue problem. The eigenvalues Ω^2 indicate the frequencies at which a wave can propagate in the structure when a given phase is specified between the edges of the cell.

2.2 Stress stiffening

As in this work a scenario of pre-stressed structure was examined, pre-stress stiffness matrix $\mathbf{K}_s$ had to be calculated. Considering that a static analysis had been solved, the updated stiffness matrix was calculated $\mathbf{K}$ [31]:

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_s \tag{9}$$

where $\mathbf{K}_0$ the original element stiffness matrix and:

$$\mathbf{K}_s = \iiint \mathbf{G}^T \tau \mathbf{G} \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \tag{10}$$

where $\mathbf{G}$ is a matrix of shape function derivatives and τ is a matrix of the current Cauchy (true) stresses σ in the global Cartesian system.

The updated matrix $\mathbf{K}$ was then used in the periodic theory described in the previous subsection to get the wavenumbers and eigenvectors of the pre-stressed structure.

3. Numerical Results

In this work the flexural wave of an infinite composite sandwich beam was examined, as shown in Fig. 1. The mechanical characteristics of each material used in the models are listed in Table 1, where E_i is the modulus of elasticity in direction i, v_{ij} is the Poisson's ratio for i and j being the directions of extension and contraction, respectively, ρ is the density and G_{ij} is the shear modulus of elasticity in direction j on the plane whose normal is in direction i. In Fig.2 z axis is depicted. ANSYS 14.0 was used during the FE modelling. Linear 8-node ANSYS SOLID45 solid element was chosen for the segment's meshing, which comprises a 3D displacement field and three degrees of freedom per node (translations in the x, y, and z directions) [31].

All three models had the same core ($h_c = 10mm$) and skin thickness ($h_s = 1mm$) and four different ratios were tested ($Ratio = (hollow\ core\ length)/(cell\ length) = L_h/L_x$), along with homogenous core one ($Ratio = 0$) . The sandwich cell was $L_x = 16cm$ long and each element was $L_e = 1cm$. The beam's width was $2cm$.

3.1 Results

In Fig. 4 the wavenumbers of all the ratios without any pressure asked are depicted. As it was expected ([22], [26] and [28]), banded behaviour is noticed on the graphs of the periodically hollow core beams. This can be explained by the hollow core part of the beam which acts as source of

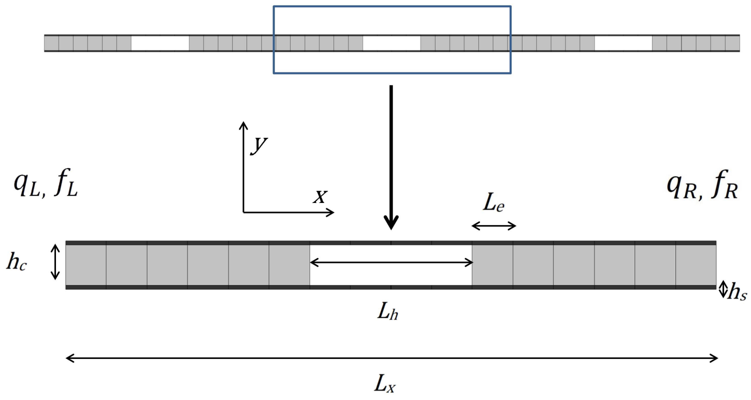

Figure 1: Infinite beam with periodic cell.

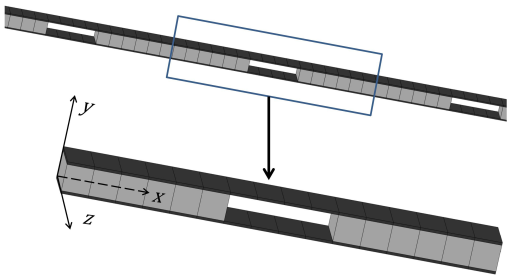

Figure 2: Geometry of infinite beam.

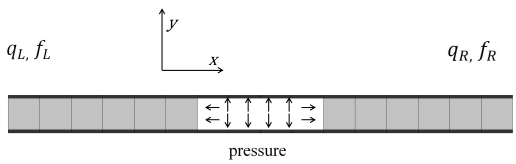

Figure 3: Pressure asked in the hollow part of the core.

Table 1: Material properties

Material I	Material II
$\rho = 1870kg/m^3$	$\rho = 110kg/m^3$
$E_x = 60e9Pa$	$E_x = 145e6Pa$
$E_y = 40e9Pa$	$E_y = 145e6Pa$
$E_z = 60e9Pa$	$E_z = 145e6Pa$
$\nu_{xy} = 0.4$	$\nu_{xy} = 0.45$
$\nu_{yz} = 0.4$	$\nu_{yz} = 0.45$
$\nu_{xz} = 0.25$	$\nu_{xz} = 0.45$
$G_{xy} = 1.2e9Pa$	$G_{xy} = 50e6Pa$
$G_{yz} = 1.2e9Pa$	$G_{yz} = 50e6Pa$
$G_{xz} = 3.6e9Pa$	$G_{xz} = 50e6Pa$

impedance mismatch which is responsible for the creation of band gaps. For the same reason it can be seen that the band gaps frequencies change significantly as the $Ratio$ changes, since the source of the impedance mismatch alters characteristics. It worths noting that the third band-gap of $Ratio = 1/4$ is significantly smaller than the other cases examined.

In Table 2 the results of the pressurised beams are presented. It should be noted that a structural integrity check was done using FE analysis for every examined situation so that to be sure that the beam can withstand the stress of the pressure asked on its skins. For this reason $1MPa$ pressure was not examined for $Ratio = 1/2$ since it led to very high stress values. Going through the results it can be seen that pressure affects the band gap frequency in most of the cases but it does not offer any significant control in the specific examined structure. Nevertheless, it was confirmed that pressure has effect on wave propagation (as it was shown in [14]), and on banded behaviour in this case, which

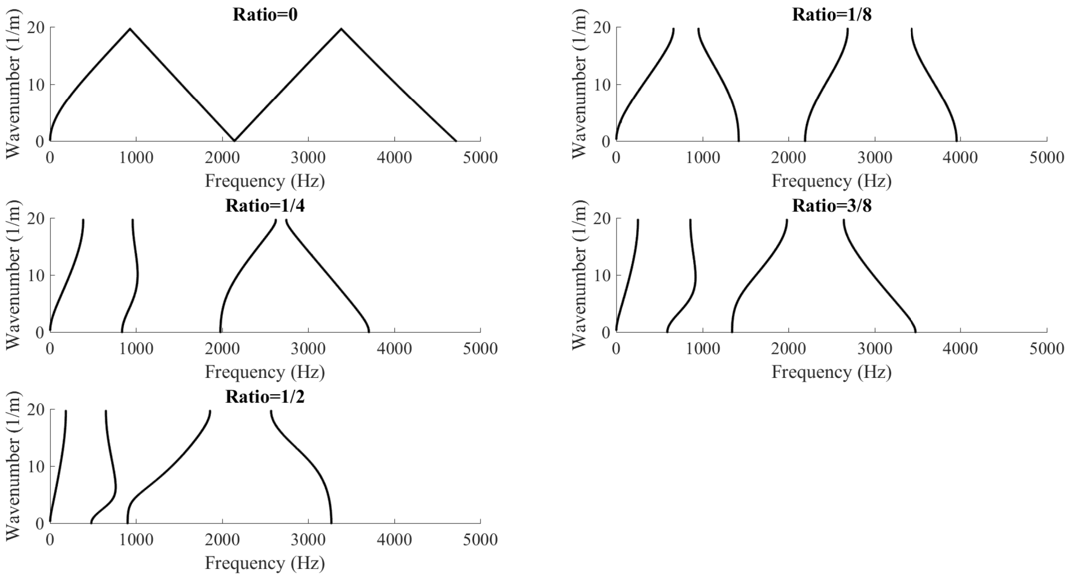

Figure 4: Graphs of the flexural waves of the different ratios $Ratio$ with no pressure asked on the skins.

might lead to practical applications in future research. Additionally, further research might lead to stiffer and more durable materials which will increase the pressure capacity of the structure and hence potential band-gap control.

Table 2: Band gaps frequencies, in Hz

	1^{st}	2^{nd}	3^{rd}
Ratio=1/8			
no pressure	$664.2 - 955.1$	$1422 - 2192$	$2687 - 3433$
$p = 10kPa$	$664.2 - 955.1$	$1422 - 2192$	$2687 - 3433$
$p = 100kPa$	$664.4 - 955.2$	$1422 - 2192$	$2687 - 3433$
$p = 1MPa$	$665.6 - 956.2$	$1426 - 2194$	$2690 - 3436$
Ratio=1/4			
no pressure	$385.2 - 837$	$1018 - 1980$	$2624 - 2746$
$p = 10kPa$	$385.2 - 837.2$	$1018 - 1980$	$2624 - 2746$
$p = 100kPa$	$385.6 - 838.3$	$1018 - 1981$	$2624 - 2748$
$p = 1MPa$	$388.5 - 849.7$	$1022 - 1987$	$2628 - 2759$
Ratio=3/8			
no pressure	$251.5 - 593.1$	$916.2 - 1344$	$1981 - 2642$
$p = 10kPa$	$251.6 - 593.1$	$920.5 - 1344$	$1981 - 2642$
$p = 100kPa$	$252.3 - 596.1$	$921.2 - 1346$	$1982 - 2643$
$p = 1MPa$	$259 - 621.9$	$927.7 - 1358$	$1995 - 2652$
Ratio=1/2			
no pressure	$183.3 - 479.2$	$762.5 - 900.9$	$1859 - 2569$
$p = 10kPa$	$184.8 - 481.9$	$763.8 - 902.6$	$1860 - 2572$
$p = 100kPa$	$197.6 - 505.6$	$775.9 - 917.8$	$1871 - 2593$
$p = 1MPa$	NA	NA	NA

4. Conclusion and further work

In this work a parametric study of frequency banded behaviour of and infinite periodic pressurised composite sandwich beam was examined. Both the length of the hollow part of the core and the pressure asked on its skins were considered as the parameters in the analyses. It was proven that the length of the hollow core plays significant role in the wave propagation and hence the frequencies of the band gaps. On the other hand, the pressurised beams did not have notable different banded behaviour concerning the flexural vibration that was the one examined in this work. Nevertheless, the behaviour pressurised beams exhibited allows the space for promising potentials as a method to actively control banded behaviour of structures and further research on the specific case is due.

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