An Intergenerational Theory of the Consumption Function

by

Simon C. Parker

A Thesis submitted in partial fulfilment
of the requirements for the degree of
Doctor of Philosophy

Economics

The University of Durham
1991

Abstract

This thesis presents a theory of the consumption function called the 'Inter-Generational Hypothesis' (IGH). The theory starts from the premise that individuals derive utility not only from their own consumption, but also from the welfare of their offspring. Individuals are supposed to maximise an intergenerationally altruistic utility function subject to a lifetime budget constraint and so derive their optimal consumption and bequest plans. From these plans, it is possible to construct an individual's consumption function. This contains earnings and inheritance terms, and is non-linear in earnings; this is consistently aggregated over all living individuals to yield the aggregate IGH consumption function. A feature of this function is the rich set of intergenerational information hypotheses it is able to encompass; there are also several implications with respect to earnings redistribution policy. The IGH function is estimated using 23 years of post-war UK data, and tested against rival consumption models, including Hall's (1978) REPIH. The principal finding is that the data do not appear to be consistent with either model in their pure form; however, they support a hybrid consumption function where a proportion of the population behave according to the altruistic IGH, and where the rest behave according to the 'selfish' REPIH. An additional finding, necessarily tentative given the imperfections of the distributional data, is a failure to detect significant non-linearity in the aggregate consumption function. This result casts doubt on the usefulness of policies designed to redistribute incomes in order to affect aggregate consumption.

Contents

Acknowledgments

I would like to thank my supervisor, Mr. John Ashworth, and all the staff at the Department of Economics at the University of Durham for their advice and encouragement. This thesis was written in two full time years and one part time year. It was purely self-financed, since no ESRC funding was received.

Important Note

Since this thesis was submitted, further development of the work contained herein has taken place. Whilst this does not alter the bulk of the theoretical and background material, the empirical results (section 5.2) have changed considerably, and Bevan's Hypothesis (section 3.3.3) has been generalised. The reader is referred to Parker (1992) for details.

Part I

Introduction

Chapter I

Introduction

This thesis presents a theory of the consumption function called the 'Inter-Generational Hypothesis' (IGH). The theory starts from the premise that individuals derive utility not only from their own consumption, but also from the welfare of their offspring. Individuals are supposed to maximise an intergenerationally altruistic utility function subject to a lifetime budget constraint and so derive their optimal consumption and bequest plans. From these plans, it is possible to derive an individual's consumption function; this can be aggregated over all living individuals to produce a tractable aggregate consumption function.

The last twenty years or so has seen the emergence and growth of intergenerational modelling in many areas of economic theory[1]. However, this growth in the application of intergenerational concepts to mainstream economic issues seems largely to have bypassed the empirical aggregate consumption function. Almost without exception, theories of the aggregate consumption function start from the postulate that individuals are selfish, maximising their own utility and not directly caring about the welfare of their children. We attempt to fill this gap by constructing a theory which allows for parental altruism to children at the outset. It is perhaps worth noting that before undertaking this endeavour, the author was motivated by curiosity as to what such a consumption function would look like, and how it would perform empirically. On the latter point, the principal questions were: would it be capable of explaining consumption behaviour substantially better than currently popular specifications? And would it be able to suggest that any empirical shortcomings of conventional consumption functions may be caused by their neglect of intergenerationally altruistic preferences?

[1] For example: in altruism and utility (eg Ray (1987), Hori and Kanaya 1989); resource distribution and inequality (eg Laitner (1979), Loury 1981); fiscal policy (Barro 1974, 1989); saving (eg Kotlikoff 1989); intergenerational justice (Arrow (1973), Dasgupta 1974); and bequest behaviour (Davies (1982), Bevan (1979), Laitner 1979).

A particular consumption function to which the latter question is addressed is that of Hall (1978). The literature relating to this consumption function is briefly reviewed in section 2.1 of chapter 2. Hall's function is derived from the assumption that households maximise a selfish utility function; the welfare of their children is not considered as a separate argument in their utility. Hall's theory has households choosing an optimal consumption stream over their lifetimes, in much the same manner as the Life-Cycle and Permanent Income hypotheses (Brumberg and Modigliani (1954), Modigliani and Ando (1963), and Friedman (1957), respectively). The principal difference is that Hall develops a version of this framework in which households face uncertainty and form rational expectations about the future.

Hall's model has been an extremely popular vehicle for applied and theoretical research into the consumption function. It forms an obvious point of comparison with an altruistic theory such as that developed in this thesis, because the utility function it is based on is 'selfish' rather than 'altruistic', the latter type of function admitting the welfare of others as an additional source of utility to the individual. An argument central to the thrust of this thesis is that consumption theories predicated on selfishness alone may give misleading indications of actual consumption behaviour if people are also motivated by altruism. This possibility is especially pertinent given (a) the weighty body of evidence which has now accumulated against both Modigliani's deterministic life-cycle model and Hall's stochastic version[2], and (b) growing evidence of the size and importance of intergenerational transfers[3]. It also seems intuitively plausible that preferences, at least for some people, are moulded by altruism, and altruism towards children in particular.

Altruistic preferences are in a sense a generalisation of selfish preferences. This is because altruistic utility functions invariably contain selfish as well as altruistic components, whereas selfish utility functions just embody selfishness. Altruists are presumed to have *some* concern about their own welfare: they are not *just* interested in the welfare of others. This might suggest that consumption functions built up from altruistic utility micro-foundations are bound to be generalisations

[2] For a listing of evidence against the former, see eg Kotlikoff (1989, p.27); for some evidence against the latter, see the references in chapter 2, section 2.1 of this thesis.

[3] See part B.4 of Appendix B.

of consumption functions predicated on selfishness. However, this does not follow if auxiliary assumptions used to construct consumption functions differ between selfish and altruistic theories. For example, it will become apparent in the thesis that the IGH function is *not* a generalisation of Hall's function, even though the former is built up from a more general utility function than the latter. The reason for the difference can be traced principally to a crucial asssumption about certainty: the IGH assumes that individuals possess perfect certainty about their futures, whereas Hall's model allows for uncertainty and unanticipatable stochastic shocks to the economy. Yet although the theories are not directly related to each other, they can be tested against each other. This is in fact a central topic of interest in the thesis, and tests of the two theories are conducted herein using UK aggregate time series data.

The IGH itself starts by considering an intergenerationally altruistic individual, who is taken to solve a dual optimisation problem at the start of his lifetime. This dual problem consists of an intergenerational consumption allocation problem to determine how much to bequeath; and a lifetime consumption allocation problem to determine how the remaining resources are to be consumed over the individual's lifetime. The IGH 'micro' consumption function is then constructed by substituting the solution of the former problem into the solution of the latter problem to give a non-linear function. Consistent aggregation both preserves the non-linearity and introduces a distributional term into the aggregate consumption function. The redistribution policy implications arising from the presence of an earnings distribution term in the consumption function are of interest in their own right, and so are discussed separately at some length (see chapter 4, section 4.1).

The consistency of the aggregation procedure suggests that the IGH may provide a suitable vehicle for rigorously testing the effect of the earnings distribution on aggregate consumption. This is an important testable prediction of the IGH. It will also be shown that the IGH is in principle capable of performing a second test: that of distinguishing empirically between three rival theories of intergenerational information.

Another theme which runs throughout this thesis is a belief in the importance of *testing* theories, rather than searching for post-hoc rationalisations of empirical phenomena. When sifting through the vast history of econometric results compiled on the consumption function this century, one is continually struck by the apparent willingness of the profession to tolerate the piling of new result upon result, without making any effort to interpret the findings of the past. Apart from engendering confusion and inconclusiveness, this practice calls the credibility of 'economics as a science' into dispute, as Mayer (1972) has noted. The situation has only become even more confused in recent years, with the increasing popularisation of sophisticated dynamic econometric modelling techniques. The cheapness of computer time superadded to continuing research effort in the field of consumer economics has swelled the compendium of econometric results yet further. This phenomenon has become increasingly serious with the rise of predominantly 'empirical' studies, and the decline of 'theoretical' work, at least in the UK. The divorce in these studies of empirics from rigorously constructed theory makes it hard to identify reasons for their breakdowns when these occur; hence yet more empirical studies are produced in an attempt to find them, and stacks of yet more econometric results are added to the thousands already in existence. This in turn generates yet more empirical studies, and so the depressing cycle goes on.

One possible way of cutting out of this vicious circle is to go back to theory and build up models from sound theoretical underpinnings. Building consumption equations from rigorous theoretical foundations allows us in principle to *test* theories and to identify weaknesses in them which may allow rectification. Not only does this add to our understanding – which may reduce our chances of following ultimately barren empirical avenues of research – but it also paves the way for explaining published results of other theories tested in the past. Thus the problem of trying to draw conclusions from an increasingly large and confusing literature may be tackled on two fronts: increased understanding of economic mechanisms, and greater knowledge about *why* previously published results were discovered.

It is not enough to prescribe or preach a methodology; it is incumbent on the proseltyser to translate his words into action. Thus the attempt in this thesis is to design a rigorous theory from first principles; to test as well as to estimate it; and to try to explain ('encompass') other models using the theory. Particular care is taken in the choice of a data set for the empirical work. Another point which is made by Mayer (1972), yet which has received surprisingly little attention in applied work, is the often cavalier way in which 'consumption' is measured for empirical purposes. Instead of recognising that optimising theories require a data series consistent with the notion of a *stock* yielding consumption services over time, much work in the applied arena has been content with utilising more readily available expenditure series. However, the latter is not congruent with the theoretical requirement of a *consumption* series. In our empirical work, we overcome this drawback by using a recently developed 'consumption' series; the empirical section discusses this issue further.

As mentioned earlier, what we now know as 'the' consumption literature consists of a huge and diverse array of studies. It is almost impossible to document it thoroughly. However, some discussion of the literature is required in a thesis such as this, for two important reasons. Firstly, description of the salient aspects of the literature is necessary as a sort of 'backdrop', in order to better understand the contribution of this thesis. Secondly, any theory which attempts to encompass others must be able to refer to the latter without ambiguity about their content. This necessitates the presentation of some summary material. We have decided to group all of this relevant material in chapter 2 of the thesis, which also describes the econometric technique of cointegration used later for the purpose of testing rival theories.

The IGH itself is presented in a chapter of its own (chapter 3). The IGH is a relatively large theory, which sustains and develops a line of logic for some 25 pages. In order to ease the burden on the reader, the chapter is broken down into four sections. For example, the optimisation problem is broken down into its life-cycle and intergenerational components in separate sections (3.2 and 3.3); they are only brought together in section 3.4. Considerable attention is paid in the chapter to the role and realism of the assumptions underpinning the theory – they are discussed as they are presented in turn. An advantage of explicitly listing and discussing the assumptions is the ability to go back to them in order to assess their possible responsibility for empirical failures of the theory based on them. As Darnell and Evans (1991) argue, the examination and modification of auxiliary assumptions represents a sound way of testing theory and improving it in the face of empirical adversity. Indeed, we adopt this very practice later in the thesis.

The purpose of chapter 4 is to illustrate that the IGH is not just a theory which suggests a testable time series equation, but that it is also rich in theoretical implications. One of the most interesting of these implications relates to redistribution policy – the idea that governments can redistribute earnings in order to hit a consumption target. The policy implications are set out in some detail, and numerical examples are provided. Yet it would be wrong to give the impression that policy is the only rationale for discussing implications of the theory. The chapter also deals with how the theory is related to various aspects of microeconomic behaviour. In particular, it is possible, at least in principle, to use empirical estimates of the IGH consumption function in order to make inferences about 'deep' structural parameters including intergenerational earnings mobility.

The empirical results themselves are presented in chapter 5; they are preceded by a relatively lengthy discussion about the data series used, their sources, construction, and limitations. A programme of testing the IGH and rival specifications is undertaken; as intimated earlier, encompassing is a major objective here. The results are interpreted both in this chapter, and further in chapter 6, which draws the thesis to a close by offering some final overall conclusions.

The precise layout of the thesis is as follows:

- **Chapter 2 (Background to the Theory).** Sections 2.1 and 2.2 contain brief descriptions of the two dominant approaches to consumption modelling at the present time: section 2.1 looks at the model of Hall (1978), and section 2.2 concentrates on the dynamic modelling approach of Hendry and others. Section 2.3 briefly chronicles some other recent work on the consumption function at the time of writing. Section 2.4 focusses on a previous attempt (by Boskin and Kotlikoff 1985) to model aggregate consumption using an altruism hypothesis. Finally, section 2.5 draws some conclusions about the work covered in this chapter.

- **Chapter 3 (The IGH).** Section 3.1 states and discusses the assumptions underpinning the analysis which follows. Sections 3.2 and 3.3 set out and solve both parts of the altruistic individual's dual optimisation problem: those of computing the optimal lifetime consumption path and the optimal bequest, respectively. Both of these solutions are used to derive the individual's optimal consumption function in section 3.4: this function is then consistently aggregated to yield the aggregate IGH consumption function.

- **Chapter 4 (Theoretical Implications of the IGH)**. Section 4.1 sets out the parameter restrictions suggested by the IGH, and considers some of the theoretical implications arising from the IGH consumption function, including the scope for government redistribution policy. Some important caveats are also mentioned. Section 4.2 discusses some additional theoretical implications of the three bequest functions examined in Section 3.3.

- **Chapter 5 (Estimation of the IGH Consumption Function)**. Section 5.1 describes the data sources, and Section 5.2 presents the econometric results.

- **Chapter 6 (Conclusions)**. This chapter draws some final conclusions from the theoretical and empirical work. Several promising avenues for future research are also suggested.

There are also four appendices, which are aimed at providing incidental or discursive material not central to the thrust of the IGH, but necessary as background information. These appendices may be omitted without in any way jeopardising the reader's grasp of the IGH and any of its implications.

Chapter II

Background to the Theory: Some Recent Research on the Consumption Function

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The purpose of this chapter is to describe the broad outline of recent work in the consumption literature which gives the basis for later developments. The immensity of the consumption literature means that only a limited discussion is possible; for more extensive overviews of the consumption literature, the reader is referred to the surveys by Fisher (1983), Hadjimatheou (1987) and Speight (1990).

The task of presenting recent work on the aggregate consumption function is simplified by the fact that most research since 1978 can be divided into two categories: 1) the 'Euler equation' model of Hall (1978); and 2) the dynamic modelling methodology commonly associated with Hendry (drawing from Davidson *et al* (1978) and hereafter labelled 'the Hendry approach'). The salient aspects of these two approaches are briefly discussed in Sections 2.1 and 2.2 respectively. Section 2.3 examines some more recent empirical work undertaken in the U. K., and Section 2.4 describes a previous attempt to model aggregate consumption using an altruism hypothesis. Section 2.5 draws some conclusions.

2.1 The Euler Equation Approach of Hall

In a landmark paper, Hall (1978) set out what is now known as the 'Euler equation' model of consumption. It is also known by other names, including REPIH (the Rational Expectations Permanent Income Hypothesis); RELCH (the Rational Expectations Life Cycle Hypothesis); and the 'random walk' model. This is one of the most important pieces of theoretical work in the consumption literature since the 1950's, the decade that saw the emergence of the Life Cycle Hypothesis (LCH) and the Permanent Income Hypothesis (PIH)[1]. Despite the sound micro-foundations of these two theories, theoretical and empirical objections to them have mounted steadily over the years (see eg Hadjimatheou 1987). Indeed, it was one of these criticisms, Lucas (1976), which prompted Hall to develop his Euler equation model.

The essence of Lucas's criticism is that econometric relationships such as consumption functions are unstable over time if their arguments contain expectational elements which are incorrectly modelled. Ideally, models should recognise that expectations are moulded by the policy regime currently in force, so that changes in policy regime alter expectations and so also behaviour. It can be shown that models which ignore this point may suffer from chronic structural instability.

Lucas's criticism applies to the LCH and PIH because these theories stress that future incomes as well as current incomes determine consumer behaviour. Under uncertainty, agents must form expectations about future incomes, expectations which must be modelled as fully rational (in the sense of using all available information) if Lucas's critique is to be overcome. The recognition of Lucas's argument is apparent in Hall's REPIH model.

Hall considered a representative household operating in an uncertain environment, and solved for its optimal consumption plan given a period-by-period budget

[1] These two great contributions crystallised the insight of the early theorists (such as Fisher (1907), Ramsey (1928) and Hicks (1939)), that people take consumption decisions with respect to total lifetime resources, not simply present income receipts in the manner envisioned by Keynes (1936). The LCH and PIH are neoclassical consumption theories, which derive consumption functions from the premise that rational households choose consumption plans which maximise their (selfish) utility subject to a lifetime budget constraint. According to these theories, bequests can only arise from selfish motives or as a result of imperfect information (see Appendix B).

constraint. Mathematically, the household seeks to maximise

$$E_t \sum_s \left(\frac{1}{1+\delta}\right)^s u(c_{t+s}) \tag{2.1}$$

subject to

$$\sum_s \left(\frac{1}{1+r}\right)^s (c_{t+s} - y_{t+s}) = A_t$$

where E_t is the expectations operator (conditional on all information at time t); δ and r are discount and interest rates respectively (both assumed constant); $u(\cdot)$ is the (strictly concave) instantaneous utility function; c_t is consumption; y_t is earnings; and A_t is physical assets. Hall solved the above problem to derive the following 'Euler equation':

$$E_t\, u'(c_{t+1}) = \left(\frac{1+\delta}{1+r}\right) u'(c_t)\,, \tag{2.2}$$

where $E_t\, u'(c_{t+1})$ is the household's expectation of period $t+1$ marginal utility of consumption formed in period t. Apart from studies which use instruments to try to estimate (2.1) directly (see below), most applied work follows Hall (1978), who postulated a quadratic utility function together with rational expectations. These assumptions yield the regression equation

$$C_t = \gamma C_{t-1} + \epsilon_t,$$

where γ is a constant, and ϵ_t is a random error term. When the discount and interest rates coincide, $\gamma = 1$, and consumption follows a random walk.

A feature of the above equation is that only last period's consumption should help to predict current consumption; no other regressors placed on the right-hand side of a consumption equation should be significant. This is a strong proposition, tests of which are called 'exclusion tests'. That the clear majority of such tests reject the proposition, by finding all sorts of additional variables to be significant determinants of consumption, is widely acknowledged to be a decisive empirical failing of the REPIH (Speight (1990) and Hall 1989). In the words of Hall: *"It is reasonably well established that the simple conclusion from the rational expectations* *permanent income model with constant expected real interest rates is inconsistent with the data: the rate of change of consumption can be predicted by past values of real income and past values of a number of financial variables"* (p.172). Two examples, one taken from the US and the other from a UK study, may be quoted to illustrate the failure of the REPIH by exclusion tests. The US example is taken directly from Hall's (1978) seminal study itself, in which lagged values of the per capita market value of corporate stock, s, are added to the right-hand side of a real per capita consumption equation. Hall reported the result

$$\underset{(8)}{C_t = -22} + \underset{(0.004)}{1.012C_{t-1}} + \underset{(0.051)}{0.223s_{t-1}} - \underset{(0.083)}{0.258s_{t-2}} + \underset{(0.083)}{0.167s_{t-3}} - \underset{(0.051)}{0.120s_{t-4}}$$

1948(I)-1977(I); R^2=0.999; se=14.4; DW=2.05.

In the above, standard errors are given in parentheses; se is the standard error of the regression; and DW is the Durbin-Watson statistic[2]. Clearly, lagged s values are significantly different from zero in this equation, so overturning the pure version of the REPIH. However, Hall rationalised these results by appealing to the idea that the lagged s values capture news-induced revisions to permanent income. This rationalisation can be sustained because the REPIH is based on the idea that last-period consumption embodies all systematic information about current consumption; if further information exists which is not picked up in C_{t-1}, then the REPIH may be modified by including variables in which the extra information inheres[3].

There are many other studies which report exclusion test failure for the REPIH, as the surveys of Hadjimatheou (1987), Hall (1989), and Speight (1990) confirm[4]. An early example from the UK is Daly and Hadjimatheou (1981), who added lagged and differenced values of income and assets, and also lagged values of consumption, to the right-hand side of the consumption autoregression. Three regressions were

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[2] Which is not actually valid in this regression since a lagged dependent variable is used – Durbin's h-statistic is relevant here.

[3] See also Nelson (1987) concerning this rationalisation.

[4] See also Davidson and Hendry (1981) using UK data, Cuddington (1982) for Canada, and Johnson (1983) for Australia.

reported in which the additional variables were found singly and jointly to be significant determinants of aggregate consumption. One example of these regressions is (with standard errors again in parentheses):

$$\underset{(0.1)}{C_t = 0.819C_{t-1}} + \underset{(0.1)}{0.219C_{t-2}} + \underset{(0.04)}{0.120Y_{t-1}} - \underset{(0.04)}{0.149Y_{t-2}}$$

1956(I)-1978(IV); R^2=0.995; se=1.986; F(3,87)=4.66.[5]

In addition to these results, Daly and Hadjimatheou examined the variation of the autoregression coefficient γ over different sample periods. Instability of this parameter casts further doubt on Hall's model.

Exclusion tests are only one way of testing the model. Two other approaches which use time series data and which have proven popular include 'excess sensitivity tests', and tests which estimate 'deep' (structural) parameters of the Euler equation (2.2). Excess sensitivity studies test an implication of the REPIH which suggests that anticipated changes in income are already captured by the autoregression process, and should therefore exert no influence on consumption over and above this. Investigations of this hypothesis typically decompose 'anticipated' and 'unanticipated' incomes from an autoregressive moving average representation of real income. Consumption is then regressed on both, and its sensitivity to the anticipated element is assessed. Studies typically find that, in contrast to REPIH, anticipated income plays a significant role in predicting consumption. Two early examples are Bilson (1980) and Flavin (1981), although Bilson uses questionable measures of income and consumption, and rather arbitrarily tacks on additional variables in the exclusion test regression. An interesting result in Flavin's paper is the marked divergence between observed consumption and that predicted by the REPIH.

The third species of time series REPIH test attempts to estimate directly parameters of the Euler equation (2.2) (ie without imposing restrictions on the form of the utility function $u(c_t)$). The pioneering paper here is Hansen and Singleton

[5] The F-statistic tests the significance of extra parameters, against the null that they are insignificantly different from zero. This F-statistic rejects the null.

(1982), which has since been followed by Mankiw, Rotemberg and Summers (1985) and Bean (1986), amongst others. Hansen and Singleton proposed a non-linear instrumental variables procedure, which is applied directly to the first order Euler condition. REPIH restrictions implicit in this condition are rejected by these authors. A similar result was obtained by Mankiw *et al*, who also tested whether the auxiliary REPIH assumption of consumption-leisure separability might be to blame for the negative results. Mankiw *et al*'s conclusion was that this did not appear to be the case, hence suggesting that the empirical problems with REPIH go somewhat deeper. Bean also produced evidence rejecting REPIH restrictions.

Tests of REPIH have also been undertaken using panel data, although these studies are not so numerous. Two examples here are Hall and Mishkin (1982) and Bernanke (1984). The idea behind Hall and Mishkin's study was to estimate the interest rate consistent with the consumption and income behaviour of members of the panel. A notable result was that unrealistic rates (for example, as high as 20%) were obtained, which suggests that problems with REPIH exist. A simple exclusion test finding lagged differenced income a significant determinant of differenced consumption confirmed suspicions about problems with the REPIH.

All of the evidence mentioned above tends to point to serious and pervasive problems with the REPIH. However, King (1985) has pointed out that tests of the REPIH are also tests of the modeller's choice of preference parameterisations. An example of the latter is the chosen form of the utility function. King's point is that it is possible that Hall's model is being rejected not because it is fundamentally wrong, but because it is being inappropriately specified. This point has been taken up by a large number of theorists, especially in the USA, and consequently several avenues of development have been actively explored. These include the modelling of information lags (Holden and Peel (1985), Wickens and Molana 1984), uncertain, variable and stochastic interest rates (Wickens and Molana (1984), Mankiw 1981); durability (Mankiw 1982); seasonality (Miron 1986); non-separability of preferences (Bernanke (1985), Mankiw *et al* (1985), Koenig 1990); transitory consumption (Holden and Peel 1985); and liquidity constraints[6]. However, the results of these studies tend to show that REPIH test failure cannot be easily blamed on weakness or inapplicability of auxiliary assumptions (see also Speight 1990, p.121).

[6] For an explanation and discussion about liquidity constraints, see Appendix A.

The overall conclusion must point towards problems which go to the heart of the REPIH.

One problem which has not been much investigated in empirical work is the limited nature of the REPIH utility function with respect to altruistic preferences. Hall specifies this as a (selfish) lifetime function, without any additional facility to render altruistic concern for descendants[7]. However, if individuals *are* motivated by altruism, then the Euler equation model will be fundamentally mis-specified, because Hall's entire analysis is built up from his initial utility function. No amount of tinkering with auxiliary assumptions can overcome this problem since it does not address the *fundamental cause* of the mis-specification. The performance of the altruistic theory developed in this thesis may therefore hold some additional interest for researchers working within the Euler equation framework.

[7] Even if utility in Hall's model is summed to an infinite horizon, the model cannot sustain a convincing intergenerational interpretation. Two parameters are required to weight future utilities, one for the individual and the other for his or her descendants. Hall only has one discount rate, and it is not feasible to suggest that this can perform a dual role. Setting the rate of time preference, δ, equal to zero is unrealistic in an infinite horizon context, since it implies that individuals place practically no weight at all on their own utility relative to the combined utility of all of their ancestors. And with $\delta > 0$, problems of intergenerational inconsistency arise in the sense of Kurz (1985). This is because discounting of descendants' utilities by an individual is unlikely to find favour with these same descendants – this engenders intergenerational conflict, and, in Kurz's terminology, an absence of a generationally consistent equilibrium. In any case, Hall's model under any interpretation cannot represent the distinctive aspect of intergenerational utility maximisation whereby mortal and partly selfish individuals form overlapping and interdependent dynastic chains.

2.2 The Hendry Modelling Methodology

In an influential paper, Davidson *et al* (1978) presented a 'data-based' strategy for modelling consumer expenditure. This paper – the forerunner of a whole series[8] – espoused an approach which has come to be associated with the name of David Hendry in particular: in the following, this will be referred to as 'the Hendry approach'.

The approach starts by positing that economic theory usually suggests long-run relationships between sets of key economic variables. However, it is assumed that data do not describe these relationships because consumers tend to be temporarily off their equilibrium paths: hence the data describes a disequilibrium manifestation of a presumed underlying pattern of behaviour. The novel feature of the Hendry approach is that the data themselves are used to estimate the precise form of the disequilibrium adjustment process. The econometrician is, at the outset, agnostic about both the variables of greatest explanatory importance and the lag structures which best describe the data. Initially, the econometrician must start with a relatively large[9] set of candidate explanatory variables in current and lagged form[10]. A regression run at this stage, and regressions run at all subsequent stages of the process, are used by the researcher in the Hendry approach to identify the variables which are statistically insignificant determinants of the dependent variable: these variables are then be dropped and re-estimation commences with the new set of explanatory variables. This process is called 'testing down'; and the entire search procedure is known as 'General to Specific modelling'. The final 'preferred' regression equation which emerges from this modelling exercise should ideally satisfy a list of criteria expounded in some detail by Hendry and Richard (1982): these include data coherency, parameter stability, consistency with the broad suppositions of underlying economic theory; and encompassment of previous studies and hence an explanation of their results[11].

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[8] eg Davidson and Hendry (1981), Hendry and von Ungern-Sternberg (1981), von Ungern-Sternberg (1981), Hendry and Richard (1982), Hendry (1983).

[9] Large relative to the number of observations, for example.

[10] This set may also include lagged values of the dependent variable.

[11] In practice, however, it is not always to be expected that preferred equations satisfy all of these criteria; see McAleer et al (1985).

Davidson *et al* used quarterly data from 1958–1970 to estimate the UK consumer expenditure (C_t) function. The following is their 'parsimonious' preferred equation:

$$\Delta_4 \ln C_t = 0.47\Delta_4 \ln Y_t - 0.21\Delta_1\Delta_4 \ln Y_t - 0.10 \ln \left(\frac{C}{Y}\right)_{t-4} + 0.01 D_t$$
$$- 0.13\Delta_4 \ln P_t - 0.28\Delta_1\Delta_4 \ln P_t$$

(OLS: 1958(I)–1970(IV); R^2 =0.77; s=0.0061; DW=1.80); where[12] Y_t is income; P_t is the implied consumption deflator; and D_t is a dummy variable for 1968(II). The above specification is an example of an Error Correction Model (ECM): this type of model allows agents to diverge from their 'long run' (equilibrium) plans in the short run, but has them continually adjusting towards their desired equilibrium paths. The term capturing the short run disequilibrium in the above is $\ln(C/Y)_{t-4}$; the presumed long run equilibrium (or steady state) relationship is $C_t = kY_t$, where k is a constant.

A somewhat arbitrary feature of the above equation is the presence of the inflation terms. The authors justify their inclusion on the basis of the improvement they make to empirical performance; they also appeal to the possibility of money illusion in the manner of Deaton (1977). However, this argument would tend to undermine the claims to a sound rationale which Davidson *et al* employed in constructing their ECM equation.

It is important to note that the ECM is not the only admissable disequilibrium specification. A popular alternative (eg Hendry and von Ungern-Sternberg (1981), von Ungern-Sternberg 1981) is the 'integral correction mechanism' (ICM), which assumes consumers wish to maintain a given asset-income ratio, so being continually prepared to devote some portion of their current income to maintain the real value of their asset holdings. The term capturing the short run disequilibrium in this case is $\ln(W/Y)$, where W is assets. The ICM together with the ECM ensures that consumers always move towards stock as well as flow equilibrium. The von-Ungern-Sternberg studies find that an ICM specification tends to be preferable to the ECM. This finding is supported by Carruth and Henley (1990) in a recent

[12] The standard errors for the regressors are, respectively, 0.04, 0.05, 0.02, 0.003. 0.07, and 0.15.

study: it is discussed briefly in the next section, where an example of an ICM is given.

Some additional work has been done on extending the ECM/ECM-ICM specifications, eg treating capital gains and losses more thoroughly (Pesaran and Evans 1984); stripping out non-household income from the measure of income used in the empirics (Borooah and Sharpe 1985); including proxies for constrained or uncertain income (Bean 1978); and allowing for distributional effects (Borooah and Sharpe 1986[13]). There has also been some work explicitly recognising potential observational equivalence between ECM and REPIH models. For example, Blinder and Deaton (1985) nested both models within a generalised specification, which also included a string of additional variables. In searching for the most acceptable consumption function, variables were decomposed into anticipated and unanticipated components in the manner of REPIH 'excess sensitivity' tests (see the previous section). Some support was found for REPIH relative to the ECM-ICM model, although these results are sensitive to the decomposition technique and so may best be regarded as tentative. Bean (1986) adopted a different encompassing technique, and found that both models were special cases without either being entirely satisfactory on their own (see also Davidson and Hendry 1981).

It is fair to say that, despite subsequent modifications of the sort mentioned above, and despite some empirical setbacks, the general Hendry approach has gained widespread support in this country, especially with forecasting bodies like the London Business School. The philosophy of data-based specification searches seems to have established itself quite widely, a trend which may come to be strengthened further with the advent of cointegration techniques (see the next section). However, acceptance of the Hendry approach has not been unanimous, on both methodological and empirical grounds. The empirical shortcomings of the approach are mentioned briefly in the next section, in the context of recent U. K. work on the consumption function. The rest of this section discusses some of the methodological objections which can be levelled at the general Hendry approach.

In a recent paper, Darnell (1989) launched a powerful attack on the methodology of the Hendry approach. A central point made by Darnell is that the Hendry

[13] Using annual data.

approach is 'verificationist' not 'falsificationist': that is, evidence is used not to test theories, but theories are used to account for empirical results. Apart from adding nothing to our understanding about which theories explain the world best[14], this approach is open to the charge of 'measurement without theory'. The role of theory according to the Hendry approach is either to provide a post-hoc rationalisation of results, or to suggest candidates for the opening list of explanatory variables. There is a substantial element of subjective judgement in the specification of the 'starting variables', something which tends to be overlooked by followers of the Hendry approach. This would not perhaps be so troublesome were it not for the fact that it is critical to the whole approach in that the starting point in general to specific searches tends to determine the finishing point (ie the preferred equation).

The concealment of judgment used to select the starting variables is but one example of hypotheses being 'present in disguise and not directly tested'. Other examples include the 'testing down' process, which also involves making judgements which are not fully explained; the choice of lag lengths on explanatory variables; and the chosen balance between unlagged explanatory variables and lags at the start, for any given number of degrees of freedom. Furthermore, Darnell criticises the use of lags on at least two separate grounds. Firstly, the inclusion of lags in the general-to-specific process may conceal the influence of certain explanatory variables whose importance becomes apparent later on in the modelling. Secondly, and perhaps more fundamentally, the entire hypothesis of consumer disequilibrium – which underpins the Hendry approach – is *assumed* but never directly tested. Even if the assumption *is* accurate, it is still not clear that the inclusion of lags into macroeconomic regression equations is a satisfactory way of modelling it. Far better would be a rigorous and above all testable economic theory of optimisation in a disequilibrium setting.

In this thesis, we follow Darnell's prescription of avoiding a data-based approach to consumption. We do this by constructing and testing a model based purely on economic theory. Following Darnell's methodology, our search for improvements to the model can be directed by sound theoretical considerations rather than *ad hoc* econometric experimentation.

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[14] The approach also cannot rigorously explain *why* particular equations work or break down.

2.3 Some Recent Work on the UK Consumption Function

It was argued in the previous section that the theoretical foundations of the Hendry methodology are so loose that sound empirical performance is the principal justification for the preferred equations it generates; a factor which is investigated below. At first, diagnostic tests run on error correction equations seemed to indicate data coherency both within and outside the sample period (see eg Davidson and Hendry 1981). However, a number of more recent studies have begun to cast doubt on the empirical success of the ECM approach. This section briefly considers a few of these studies.

The studies can be assorted into two main categories. The first type use standard econometric diagnostic statistics to test if the simple ECM model exhibits parameter stability and forecasts well outside its sample. The second type utilises recently developed cointegration techniques to test whether the 'equilibrium' manifestation of the Davidson *et al* model is supported by the data – ie whether consumption and income cointegrate. We first briefly discuss examples of the first type of studies, and then proceed to explain the ideas behind cointegration and two studies which use it. We spend some time explaining the econometrics of cointegration because, apart from being necessary for the understanding of the studies discussed in this section, it is utilised later in this thesis for testing the IGH.

Of the studies using standard diagnostic testing procedures, two important recent examples are Carruth and Henley (1990) and Currie, Holly and Scott (1989a). Carruth and Henley observed reasonable tracking behaviour for a standard ECM equation until the late 1980's, when the unpredicted and sustained decline in the U. K. personal savings ratio occurred. These authors also observed that the dramatic and prolonged divergence of consumption growth from income growth raised fundamental questions about the feasibility of an ECM which proposed transience of consumer disequilibrium. This doubt was borne out in tests of ECM equations which failed standard empirical stability tests based on data from these years. For example, a forecast $\chi^2(8)$ test statistic relating to the Davidson *et al* model was found by Carruth and Henley to take a value of 46.9, which lies considerably above the tabulated χ^2 statistic at 5% with 8 degrees of freedom. Carruth and

Henley first tested a Currie Holly and Scott (1989a) specification, which suggested that empirical failure of ECM equations might be caused by omitted variables, including the post-tax interest rate, the real value of the housing stock, and the age distribution[15]. Carruth and Henley found that the Currie Holly and Scott specification did not perform satisfactorily, and so searched elsewhere for potential sources of improvement. They ultimately recommended the use of a Hendry and von-Ungern Sternberg (1981) ICM model, where real net wealth (which includes housing wealth) rather than liquid assets appears in the integral correction term. The following is their favoured equation[16]:

$$\begin{aligned}\Delta c_t = {} & 0.0284 + 0.5772\Delta y_t - 0.2486\Delta^2 y_t - 0.2360(c-y)_{t-1} \\ & 0.0213(w-y)_{t-3} - 0.0298\Delta rr_t + 0.1339\Delta w_{t-2} + 0.0240\Delta d7301 \\ & - 0.0246\Delta d7901\end{aligned}$$

OLS 1969(II)-1985(IV); R^2=0.6833; DW=2.11; $\chi^2(12)$=10.80; $F_1(12,54)$=0.69; $F_2(7,40)$=0.10; H(16,37)=1.497.

In the above[17], lower case letters indicate logarithms; y is income adjusted for capital gains/losses; w is real wealth; rr is the real post-tax rate of interest; and the Δd terms are dummy variables. Of the diagnostic tests, χ^2 is a Chow forecast test; F_1 is a Chow parameter stability test; F_2 is a test for residual autocorrelation; and H is a test for heteroschedasticity. The latter three test statistics are all distributed as F. The performance of this specification is clearly excellent; a similarly good one was derived for unadjusted data. Carruth and Henley concluded that breakdowns in performance of Hendry-type equations may not therefore require completely new specifications: existing specifications are capable of explaining consumer expenditure, even in the singular years of the late 1980's.

Apart from Carruth and Henley and Currie Holly and Scott, other authors have attempted to rectify poor empirical performance within a standard dynamic modelling framework. Much of this work echoes Currie Holly and Scott's belief that

[15] where the latter factor is measured by the decline in the proportion of the population in the 45-64 age group.

[16] This is based on seasonally adjusted data; they also produced a regression using unadjusted data.

[17] standard errors of the coefficients are, in turn, 0.01, 0.09, 0.06, 0.09, 0.01, 0.01, 0.02, 0.01, 0.01.

omitted variables may be the cause of diagnostic test failure of existing Hendry-type equations. For example, Hendry *et al* (1989) have proposed adding further variables to the consumption function, including liquidity constraints, a measure of the increased fungibility of (especially illiquid) assets resulting from financial deregulation in the 1980's, and changes in expectations about permanent income in this decade as a consequence of sustained real growth and cuts in tax rates. And, as we shall shortly see, Drobny and Hall (1989) have followed the same route within a cointegration framework. It would probably be fair to conclude our discussion of these studies by noting that searching for omitted variables in this way is likely to continue to be a popular avenue for future research.

We turn now to the cointegration studies, of which Drobny and Hall (1989) and Molana (1991) are the two most prominent recent examples in the UK. Broadly speaking, the basic idea behind cointegration (Engle and Granger 1987) is that an equilibrium relationship between a set of variables means that these variables should track each other closely over time, with no tendency for them to diverge. It does not matter whether these variables are trended over time (ie nonstationary), or stationary; what matters is that the 'equilibrium error' of the regression of these variables is stationary. A stationary variable is said to be *integrated* of order zero, written I(0); if stationarity is only induced after first differencing a variable, then that variable is said to be I(1). This idea generalises to higher orders of integration.

The first task of the applied researcher in this framework is to examine the time series properties of the data. If the variables hypothesised to form the equilibrium relationship do not trend in the same way over time, then the idea of a stable relationship between them becomes hard to sustain. Hence the first task is to test whether all of the variables are integrated of the same order. If they are, a regression – called a *cointegrating regression* – is run on these variables, and the error term is tested for I(0). The rationale for this second task is that it is quite possible for variables to trend in the same way, but not to hold together over time. The I(0) test of the residuals tests whether the variables do hold together over time. If I(0) residuals are found, the variables in question are said to *cointegrate.*

The above two tests can be easily implemented in practice. The tests are structured as follows: for any variable X, the t-statistic of π_0 in the regression

$$\Delta X_t = \pi_0 X_{t-1} + u_t$$

is compared with critical values tabulated by Fuller (1976) (u_t is a stochastic error term). This statistic is called a *Dickey Fuller* (DF) statistic; if its absolute value exceeds the tabulated critical value, then the null hypothesis of X being nonstationary is rejected. This test is usually supplemented by the augmented regression

$$\Delta X_t = \pi_0 X_{t-1} + \sum_{j=1}^{p} \pi_i \Delta X_{t-j} + u_t \,,$$

where p lags of the first difference of X are added when the system displays marked serial correlation. The test statistic corresponding to this regression is called an *augmented Dicky-Fuller* statistic (ADF). The DF statistic is less powerful than the ADF when the lags in the latter are significant; however, it is more efficient when they are not. When conducting a unit root test, ie a test of the order of integration of a single variable, the tabulated values of the DF and the ADF statistic are identical. However, when investigating whether the error term of a cointegrating regression is I(0), the tabulated critical values differ, depending on the number of variables in the regression[18].

An example relating to the UK consumption function should help to fix these ideas. In a recent paper, Drobny and Hall (1989) used cointegration techniques to re-evaluate the performance of the favoured Davidson *et al* (1978) equation. The authors first demonstrated the failure by this equation of a standard forecasting test over the first five years of the 1980's; they then went on to show that the set of variables used by Davidson *et al* do not cointegrate over the period in question (1966(IV)-1985(IV)). ADF statistics for cointegrating regressions including log income, differenced log income, differenced log prices and a constant, failed to reach their critical values. In fact, excluding the constant term as in Davidson *et al* (1978), the ADF statistic was found never to exceed -2.06 – compared with a critical value of about -4.00 at 5% significance when four variables comprise the

[18] Other tests to investigate integration and cointegration also exist, including those of Phillips (1987) and Johansen (1988). These were not available to the author in a computer package at the time the thesis was submitted: see my note on page 6 and Parker (1992).

cointegrating regression[19]. Hence the regression residuals were not I(0), and so cointegration did not hold. Drobny and Hall took this to suggest that an important determinant of consumption had been omitted. A factor whose inclusion restored the property of cointegration to the equation was the income distribution[20] as proxied by a tax differential variable, called TAX. Their most successful regression, which also included wealth, W, and a dummy variable D for announced VAT changes, was

$$\ln C_t = 0.962 \ln Y_t + 0.067 \ln(W/Y)_t + 0.244TAX + 0.005D\,.$$

OLS: 1966(IV)-1985(IV); R^2 =0.977; CRDW=1.57; DF=-6.92; ADF=-3.35[21].

Since there are five variables in this regression, the Engle and Yoo (1987) tables give the 5% critical values for DF and ADF with a sample size of 100 as –4.58 and –4.36 respectively. Hence the DF statistic suggests cointegration, although the ADF does not. However, the CRDW statistic exceeds the critical value of 0.28 (also published in Engle and Yoo 1987), which supports the case that the above model exhibits cointegration. Finally, Drobny and Hall exploited a result known as Granger's 'Representation Theorem' (Engle and Granger 1987), which shows that a cointegrating regression always has a valid ECM representation. Drobny and Hall generated an ECM equation from their cointegrating regression, which exhibited parameter stability and fitted the data well.

Drobny and Hall (1989) is therefore another example of a study which follows the recent trend of identifying mis-specification of earlier ECM equations, and then rectifying the problem by suggesting a hitherto omitted variable. The power of the cointegration framework for conducting this sort of exercise derives from its ability to avoid complex dynamic lag structures of the type produced by the Hendry methodology, which may conceal omitted variables (see the discussion at then end of section 2.2, with reference to Darnell 1989). To be sure, it is possible to

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[19] See Engle and Yoo (1987).

[20] See Appendix C.

[21] CRDW is the cointegrating regression Durbin-Watson statistic, which is another cointegration test statistic. A CRDW significantly greater than zero implies stationary residuals; unfortunately, tabulated critical values are sparse. Standard errors are often not reported for cointegrating regressions because normal hypothesis testing is invalid with non-stationary series.

go on and use the residuals of a successful cointegrating vector in an ECM model, as the Granger Representation Theorem establishes; but the dynamic search which results has a valid equilibrium model underlying it, which is not necessarily present if the modeller constructs an ECM model without testing for cointegration first.

The second example of the application of cointegration techniques to the UK consumption function is a paper by Molana (1991). This study is of additional interest to us in this thesis, because it is one of the models against which the IGH is ultimately tested (albeit with a different functional form). Molana argued that the utility function may contain wealth, w, as an argument as well as consumption: that is, the utility function is $u(c, w)$ rather than just $u(c)$ of equation (2.1). The argument here is that wealth may be desired for its own sake, as well as for facilitating consumption purchases. Reasons for this may include the role of wealth in reducing anxiety about uncertain future labour income and liquidity constraints. In much the same way as Hall, Molana derived an Euler equation for his generalised model, in which consumption is no longer a random walk but accepts wealth as a determinant. Molana's model also difers from ECM equations since his model suggests wealth rather than income, which ECM equations tend to regard as the principal independent variable.

The problem for Molana was to show that consumption cointegrates with wealth, which would support his model, and not income, as in Davidson *et al*'s model. We have already seen how Drobny and Hall discovered that consumption and income do not cointegrate; this Molana confirmed using the (puzzlingly short) sample period 1966(IV)-1981(IV):

$$\ln C_t = 0.969 + 0.734 \ln Y_t$$

OLS: 1966(IV)-1981(IV); R^2 =0.860; CRDW=2.30; DF=-8.77; ADF=-2.17.

The ADF statistic suggests I(1) residuals and hence no cointegration. Interestingly, Molana also found initially that consumption and wealth do *not* cointegrate:

$$\ln C_t = -0.662 + 0.690 \ln W_t$$

OLS: 1966(IV)-1981(IV); R^2 =0.390; CRDW=0.83; DF=-3.42; ADF=-1.22.

That this result is so destructive to Molana's theory would seem to cast considerable doubt on it (a similarly negative result is also derived in chapter 5 of this thesis). However, Molana went on to search for other ways of testing for cointegration, until he found one, due to Campbell (1987), which supported his theory. An ADF statistic of -3.54 was produced which suggests cointegration between consumption and wealth. An acceptable ECM (disequilibrium) relationship based on this was also generated; hence Molana concluded that the evidence supported his model.

There are actually several awkward questions thrown up by Molana's empirics, including the surprisingly short sample period given the availability of nearly a decade more data at the time of the submission of his final typescript. Hence further tests of Molana's model need to be carried out to see if his results are sample specific. Furthermore, his choice of ignoring the negative result he obtained in the OLS cointegrating regression is somewhat disturbing. Johansen (1988) has recently developed a technique for examining multiple cointegrating vectors: it is possible to show that in a bivariate model such as Molana considers, the OLS cointegrating vector is unique: multiple cointegrating vectors will not occur. Yet this is precisely what Molana suggests is happening when he adopts another approach *designed* to maximise the absolute value of the ADF statistic. Hence the failure of Molana's OLS cointegrating regression is a damaging result, which requires explanation.

So what do all these recent empirical studies tell us about the UK consumption function? They are all fairly positive in outlook, insofar as they suggest new specifications which correct some empirical shortcoming, and which all seem to satisfy the diagnostic testing of their originators. However, it is hard not to be dismayed by the apparent fragility of many of these econometric specifications. The tendency of models to break down in the face of new phenomena not anticipated by their creators is no doubt largely responsible for the vogue of suggesting new, hitherto omitted, variables for inclusion in 'standard' consumption regressions. The fact that the 'standard' also varies over time is another worrying feature of modern applied work on the consumption function. Perhaps twenty years ago, the standard, or 'conventional' model would include one or more of (permanent) income, wealth, inflation, and assorted other terms, with only limited dynamics; following the seminal Davidson *et al* (1978) study, it became increasingly associated with dynamic ECM specifications. Now the ground seems to be shifting again, with growing interest in cointegration techniques. What this prolonged state of flux may be telling us is that we are still no nearer to possessing a definitive function relating consumption to a few important independent variables.

It is important to bear in mind, however, that the studies discussed in this section are almost entirely empirically based. This begs the question of the reliability of the data used in specification searches. However, only relatively little attention has been paid to this issue. A notable exception is Borooah and Sharpe (1985), who observed that whereas economic theory relates to the individual, published CSO data refers to the entire personal sector, which includes unincorporated businesses, and life assurance and pension funds as well as households. They noted that the behaviour of unincorporated companies, for instance, cannot be expected to match that of households. So there appears to be a risk of applying faulty data to (possibly correct) theory, with the consequent risk of mis-specification bias. However, this argument did not stand up to an investigation by Currie *et al* (1989b), who could only attribute a part of the decline in the personal sector's saving ratio to the activities of the non-household sector. And the effect of mismeasurement was found in the same paper to be of marginal importance. However, the problem of poor or questionable data sources will be one which will occupy much of our attention in the empirical work in this thesis.

Another point to note about the recent studies considered here is that they all relate to the case of the UK. Given the continuing emphasis on testing the REPIH in the USA, it is tempting to suggest an Atlantic divide in emphasis, with British research tending to focus on empirical structure, and American research tending to concentrate on a (particular) theoretical basis. The limitations of the REPIH have already been mentioned in section 2.1; these include the neglect by that model of altruistic motives. It therefore seems appropriate for us to turn our attention to an unique model which attempts to remedy that neglect.

2.4 The Consumption Model of Boskin and Kotlikoff

An attempt to model aggregate consumption using an altruism hypothesis has been undertaken by Boskin and Kotlikoff (1985)[22]. These authors' model of altruism is based on that of Bernheim and Bagwell (1985, 1988)[23], which postulates that ties of blood and marriage can bond seemingly disparate and unconnected altruists together in large groups or 'clans'. Boskin and Kotlikoff take this idea to its logical conclusion, by suggesting that an entire economy may be regarded as one utility-maximising clan, with a single objective function and budget constraint. The latter is described by the period-by-period evolution of the private sector's total net worth; the former is written as

$$U_t = E_t \sum_{\tau=0}^{\infty} \sum_{a=0}^{D} \alpha^{\tau} P_{t+\tau,a} \theta_a u(C_{t+\tau,a}) , \qquad (2.3)$$

where α is a discount factor; D is maximum longevity; P is the number of family members of age a ; θ_a are age-specific utility weights; and τ indexes generations. The government may be endogenised by adding the government budget constraint to that of the private sector.

Boskin and Kotlikoff's optimisation problem is essentially quite similar to that of Hall (1978) (compare (2.1) and (2.3)), with the obvious difference that what is taken as a family member by the latter is regarded as an entire economic group in the former. Retaining Hall's assumption of stochastic earnings and using a stochastic interest rate, the authors derive 1) a stochastic Euler equation for consumption; and 2) a dynamic equation for the economy's total net worth. These two equations describe the *evolution* of private sector consumption and assets over time, but they do not give rise to an *analytical representation of consumption as a function of net worth.* That is, Boskin and Kotlikoff cannot derive a tractable consumption function from their analysis. Instead, they have to use the technique of dynamic programming to solve for numerical values of consumption 'one by one' over time. They do this by exploiting the fact that a particular value of consumption implies a particular value of total net assets, and vice-versa; by picking a point

[22] See also Auerbach, Cai and Kotlikoff (1991), who adapt this model in order to simulate the US economy.

[23] See also Kotlikoff (1989).

in time far in the future, a particular configuration of consumption and assets can generate previous-period consumption and assets sequentially all the way back to the present day. In fact, Boskin and Kotlikoff's estimates of the former, denoted $\hat{C}_t$, provide the following means for testing their altruism hypothesis.

If Boskin and Kotlikoff's model is correct, then a regression of actual consumption values C_t on their estimates $\hat{C}_t$ should produce a unit coefficient on the latter and no intercept. Furthermore, variables proxying the possible effect of the age-income distribution should not be significant regressors, because the altruistic clans of Bernheim and Bagwell are hypothesised to react to any distributional changes by effecting offsetting transfers. Therefore, if $s_i \, (i = 1, ..., m)$ are the age-income shares of age groups i, and if the regression

$$E(C_t) = \beta_1 + \beta_2 \hat{C}_t + \sum_{i=1}^{m} \lambda_i \, s_{i,t} \tag{2.4}$$

is run, Boskin and Kotlikoff require $\beta_1 = 0$, $\beta_2 = 1$, and $\forall \lambda_i = 0$. However, when Boskin and Kotlikoff estimated (2.4) on annual US data (1968–1984), they found that all of these restrictions were violated.

Although this result would seem to reject the Boskin and Kotlikoff model, there might be other, less damaging, explanations. Boskin and Kotlikoff were quick to point out that it was the precise specification of their model rather than the hypothesis of altruism *per se* which could be rejected. For instance, their treatment of uncertainty could have been at fault: specifically, they assumed that current and future populations and age distributions are known with certainty. They also modelled earnings and the interest rate as a simple bivariate lagged structure with stochastic components: it is quite possible that this too could give misleading results. However, while these drawbacks may have had some role to play, there must be considerable doubt about the realism of the underlying Bernheim-Bagwell altruism hypothesis. The suggestion that agents are both altruistic and well-informed enough to form large and well-behaved clans was felt by Bernheim and Bagwell themselves to be so far-fetched that they used it as a critique of Barro's (1974) Debt Neutrality Proposition[24].

[24] Specifically, they asked: If agents can offset distributional changes by intergenerational altruism

To conclude, the Boskin and Kotlikoff model is an imaginative attempt to test the 'super altruism' hypothesis of Bernheim and Bagwell. However, this model evidently suffers from some important drawbacks. These include, principally, its failure to produce a tractable, 'closed form' consumption function and the fact that it is based on an unrealistic altruism hypothesis. As will be seen below, the IGH expounded in this thesis claims to be able to produce a tractable consumption function, derived from a widely accepted hypothesis of altruism, and based on a rigorous treatment of intergenerational preferences.

(as in Barro), can they not offset distributional changes intragenerationally as well via an extended type of altruism? The 'super neutrality' this would give rise to, as well as the type of altruism suggested, appear to be so intrinsically unrealistic, that this forms the core of their critique. See also Barro (1989) for a criticism of the clan hypothesis.

2.5 Conclusions

This chapter has summarised several recent developments in the consumption literature. A wide variety of studies have been discussed, ranging from the purely theoretical to the purely empirical. The purpose of the chapter was to provide a brief description of the current state of knowledge in the field, as well as to introduce concepts and models which will be referred to later in the thesis. Of the modelling techniques to be used later, cointegration is the principal example; Hall's pure REPIH and Molana's wealth-augmented REPIH are the most important studies to be considered and tested in the thesis.

Having provided a brief description of the current state-of-the-art, what overall conclusions (if any) can be drawn from it? Opinions here vary, but many researchers would surely concur with Speight who summed up a recent survey of the consumption literature thus:

"The resurgence of theoretical and empirical interest in consumption determination and the consumption function that followed the publication of Hall (1978) and Davidson *et al* (1978) continues. However the question of *the* theoretically appropriate *and* empirically valid representation of consumption behaviour is probably as unanswered as at any time in the history of thought on the issue. In short, the REPIH is strongly theoretically founded but lacks sufficient empirical support to be accepted without modification, while the ECM approach has a strong empirical foundation but a rather loosely-specified, non-commital theoretical basis. This contrast reflects a difference of purpose, the ECM approach being geared towards the tracking and forecasting of aggregate consumption behaviour, the REPIH seeking to 'explain' consumption determination theoretically, and appealing to empirical evidence for support." (1990, p.186)

The difference in purpose to which Speight alludes makes for a rather confusing and inchoate state of affairs. Judging by the content of the continuing output of research in the field, there seems to be a distinct lack of unity and direction. With a few notable exceptions, work tends to be grouped into a few broad, and almost mutually exclusive, categories corresponding roughly to the breakdown into the sub-sections of this survey chapter. Thus, one tends to find papers concerned with modifying Hall's REPIH (section 2.1); or applying data-based techniques to search for data-coherent empirical specifications (section 2.2); or applying cointegration techniques to examine the long-run properties of established models (section 2.3). There are also new theoretical models, although these are rarer – an example is the Boskin and Kotlikoff model discussed in section 2.4. The few exceptions to the current practice of specialising in one of these categories tend to heed Hendry and Richard's (1982) injunction to comparatively test models and search for new ones which *encompass* them (ie account for their past success or failure). Studies conducting nested tests of this sort between the REPIH and ECM models were briefly mentioned in section 2.2: they include Blinder and Deaton (1985) and Bean (1986). There is also a study by Muellbauer and Bover (1986) which tries to link these two models theoretically. However, it should be stressed that these studies are definitely exceptions to the general rule of proceeding to generate new results without trying to account for studies grounded in a different approach.

In the UK, the trend of producing purely empirical studies seems to be gaining strength. As noted in section 2.3, studies of this sort tend to augment Hendry-type equations with omitted variables. The author would like to make two comments here. Firstly, he notes the lack of unanimity about the perceived usefulness and robustness of specifications which include previously omitted variables. Secondly, he observes this overall empirical trend with no little disquiet. Several methodological objections to the Hendry approach have already been rehearsed in section 2.2: these include the replacement of falsificationism with verificationism; the secondary role of theory in the approach; and the presence of hidden, but potentially powerful, hypotheses. Of these, the author regards the subjugation of theory to empirics as the most invidious feature of the approach, and believes it to be responsible for its documented failure on its own (ie empirical) grounds (see section 2.3 for evidence). We believe its empirical failings arise from its choice of replacing *understanding* of economic processes, via theory, with mechanical econometric search routines. For example, Darnell (1989) argues that it is quite possible for econometricians using the Hendry approach to concoct a specification which passes whole batteries of diagnostic tests but which does not recognise a genuine breakdown in the underlying hypothesised relationship. This author believes that the best and most enlightening way forward is to return to theoretical basics, with an emphasis on the generation of testable, ie falsifiable, hypotheses. In presenting the IGH, this thesis aims to follow just this course.

What of the technique of cointegration, described in section 2.3? This technique provides, on one hand, an unusually powerful and efficient means of testing and falsifying theories. Later, in chapter 5 of the thesis, it is used for this very purpose. However, in common with Darnell and Evans (1991), the author believes that there is also a darker side to the technique, in that it can be used like other test procedures in the Hendry style to verify rather than falsify theories. Some authors are quite candid about their use of the technique in this manner, eg Hall *et al* (1990) in their modelling of UK monetary aggregates. Thus the procedure of searching for equation specifications with minimal reference to theory looks set to continue, at least in the UK.

Part II

The IGH: Theory

Chapter III

The IGH

3.1 Assumptions of the Model

The exposition of the IGH commences with a list of assumptions, given below. The assumptions, which are stated and evaluated one by one, are arranged in two groups. Members of the first group are commonly invoked by a variety of optimising models, especially those relating to consumer behaviour. Members of the second group are more specific to the IGH, although many of these have also been used in other consumption theories.

Group I: General preliminary assumptions

1. Individuals are rational utility maximisers. Although this assumption is fairly standard in most areas of economic theory, some economists regard it with scepticism. The (somewhat limited) evidence on its realism is mixed. Johnson *et al* (1987), for example, found that many of their human experimental subjects were unable to make coherent and consistent consumption planning decisions when they were placed in an hypothetical life-cycle environment. This suggestion of irrationality backs up earlier work by Diamond (1977), which suggested that elderly Americans persistently under-save for retirement, thereby exhibiting irrationality[1]. The rationality assumption is maintained in the IGH because of the current dearth of alternative assumptions which are as simple and convenient to work with. One such alternative worth mentioning is due to Thaler and Sheffrin (1981), who hypothesise that individuals have essentially dual personalities. That is, a 'higher order' self recognises the advantages of acting as a utility maximiser; but a 'lower order' self, more mindful of the benefits of immediate gratification, hijacks the ambitions of the higher order self. A self-control problem then ensues (see also Etzioni (1987), Schelling (1984) and Elster (1982), for example). However, as this

[1] However, Kotlikoff *et al* (1982) were able to overturn Diamond's findings empirically.

sort of modelling deserves a thesis in its own right, the standard assumption seems reasonable here.

2. Two-stage budgeting exists, whereby individuals allocate expenditures to a single 'composite good' in the first instance, and then to individual 'goods' or sub-groups of goods. At this second stage, sub-utility functions are weakly intertemporally separable. This assumption also tends to be invoked by most aggregate theories, not just the IGH. Abstraction from issues such as the composition of consumption and the relative prices of individual goods and services sacrifices detail for both greater simplicity and tractability.

3. Individuals' 'own' intertemporal utilities are strongly (additively) separable. A common functional form applies to all individuals of all generations: that is, tastes and preferences are homogenous and constant over time. The real subjective rate of time preference (which is positive) is denoted ρ and is also common to all individuals of all generations. This assumption is also fairly standard: it facilitates the use of a simple utility function from which (given suitable additional assumptions) optimal plans can be readily solved. The restriction of *constant* tastes simplifies the analysis considerably, though at what cost is uncertain; it does however seem intuitively likely that tastes do not remain fixed over lifetimes, but develop and evolve endogenously. Unfortunately, models of endogenously changing tastes[2] are as yet insufficiently developed to bear direct application to a theory such as the one developed below. A further interesting possibility in the context of this thesis is that altruism itself has changed over time. For example, Auerbach and Kotlikoff (1989) have suggested that the decline of the family in the US since the Second World War may have reduced peoples' tastes for bequests.

4. Leisure not defined by the consumption of goods and services is ignored. This assumption is intended to simplify the analysis of decision-making, admittedly at some cost to realism. A more general theory would recognise that rational individuals not only aim to allocate their resources optimally over their lifetimes, but that they also aim to allocate their time optimally between work and

[2] See, for example, Hammond (1976), Peleg and Yaari (1973).

leisure. Since resources are related to the amount of work undertaken, there will in practice be some interdependence between consumption and work decisions[3].

5. A single 'perfect' financial asset exists, which bears a single, constant, and non-negative real interest rate r, which applies for both borrowing and lending. The intragenerational capital market for this asset is perfect: there are no liquidity constraints over individuals' lifetimes. This assumption appears regularly in neoclassical consumption models. Concerning the constancy of the interest rate, there is some debate in the context of Hall's (1978) Euler equation model[4] about whether this assumption is a reasonable one. The evidence on this question (reviewed by Hall 1989) is mixed. However, most importantly, Blinder (1974) has demonstrated that extending a generalised life cycle optimisation model to include a time-variable interest rate does not substantially alter any of the results derived with a fixed interest rate. It may therefore be fair to conjecture that a constant interest rate will also do little damage to the IGH. Rather less satisfactory is the assumption of perfect intragenerational capital markets. Imperfections range from post-tax wedges between lending and borrowing rates[5], to the existence of liquidity constraints. The latter phenomenon in particular (which is explained and discussed more fully in Appendix A) has attracted a great deal of interest in the literature to date.

6. Labour income is exogenous: labour supply is constrained at some fixed level. Hence individuals make decisions about their consumption paths only; their earnings paths are fixed. This assumption conveniently removes any possible interdependence between consumption and work decisions (see Assumption 4).

Group II: Specific IGH assumptions

7. Individuals derive utility not only from their own consumption, but also from the utility of their offspring. However, they do not derive utility from the welfare of other friends or relations; and their utility is

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[3] For evidence against consumption-leisure separability, see eg Deaton and Muellbauer (1980, chapter 5) and Mankiw, Rotemberg and Summers (1985).

[4] See Section 2.1.

[5] Flemming (1973), Pissarides (1978), King (1986), Altig and Davis (1989).

also unaffected by government expenditure, or by the mere possession of wealth. The altruistic utility function assumed here belongs to the 'one-way non-paternalistic' class. This is the most common type of altruistic utility function in general usage. 'One-way' in this context means that parents derive satisfaction from the utility of their children, even though this regard is not reciprocated[6]. By 'non-paternalistic' is meant that the utility rather than the resources or consumption of others promotes happiness in the altruist. In defence of this specification of the utility function, it may be remarked that non-paternalistic functions seem intuitively more plausible than paternalistic functions (Ray 1987); there may also be problems of dynamic consistency with the latter (Goldman 1980). Furthermore, the one-way form has definite advantages of tractability over more complicated two-way[7] and multi-altruist[8] forms. It is also the case that the one-way parent-to-child form captures a particularly important class of transfer[9]; hence the particular one-way utility function used in the IGH may be a reasonably happy trade-off between tractability and a realistic representation of altruistic preferences. Another feature of the assumption is the non-appearance of government spending in individuals' utility functions. This is the rule rather than the exception in the consumption literature[10] and seems reasonable for our purposes. The treatment of *inter vivos* gifts to friends is a little more troublesome. *Inter vivos* gifts must be treated as a form of consumption, motivated by selfish impulses. This is perhaps not completely satisfactory in view of the evidence of Cox and Raines (1985), who claimed that this class of transfer is the predominant form of familial transfers among adults. However, there seems little alternative to this assumption without sacrificing the convenience of the one-way utility function; note also the counter-evidence of Japelli and Pagano (1989) disputing the importance of gifts *inter vivos.*

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6 The other obvious one-way function has children behaving altruistically towards their parents without reciprocity. If only one altruistic channel could be chosen, it would be likely that the child-parent channel would be rejected, for reasons associated with the *Selfish Gene* thesis of Dawkins (1989). According to this thesis, gene maximisation in many animals requires investment in children by parents rather than vice-versa (p.107). Note further that 'forced' child-parent transfers have been progressively reduced in most western countries by the simultaneous decline of the extended family and the rise of the Welfare State (so relieving many children of the burden of looking after their parents in old age).

7 eg Carmichael (1979), Burbidge (1983), Kimball (1987).

8 eg Bernheim and Bagwell (1988).

9 See eg Cox and Raines (1985).

10 But see eg Aschauer (1985), Djajic (1987) and Ihori (1990).

8. Perfect certainty applies over all individuals' lifetimes; intergenerational information is, however, imperfect. The hypothesised imperfection of the latter seems reasonable; however, the former is undeniably strong, and goes against the trend of much recent theoretical work on the consumption function (see section 2.1 of the previous chapter). However, an assumption of this sort is not uncommon in intergenerational models.

9. The unit of analysis is the individual, who unfailingly reproduces asexually by having one child. Children are non-earners and depend on parental consumption for survival until they reach adulthood. At this point they are assigned 'economic age' zero (henceforth, 'age' will be taken to mean 'economic age', unless otherwise indicated). All individuals die T years later. This hypothesised social structure also follows much intergenerational theory dealing with optimisation over several generations. It is obviously highly stylised, and vulnerable to the criticisms that it specifies the date of death as fixed and certain; and that it ignores important sources of heterogeneity in the population. Nevertheless, it does facilitate an analytical treatment of overlapping and finite-lived generations. Several versions of this assumption appear in the literature, which tend to retain the individual rather than the family as the logical unit of observation[11]. One variant (Bevan 1979, p.382) views families as consisting of one head who works; who has taken a spouse by perfectly assortative marriage[12]; and who has n composite children. This sort of married couple can effectively be treated as a composite single individual.

10. Bequests of (non-human) capital are made by the adult at an age σ; children inherit immediately at an age η. Bequests cannot be negative. An implication of the latter is that, given the absence of 'reverse' child-parent transfers implied by Assumption 7, intergenerational liquidity constraints may exist. That is, parents wishing to will debt to their children will be constrained from effecting such a 'transfer'. This seems reasonable in view of the fact that *"current estate law will not fully enforce liabilities in excess of assets in estates. One cannot 'will' bankruptcy to one's heirs. Nor does it seem there is any way a private individual can force liabilities on his children".* (Drazen 1978, p.5-6, footnote

[11] This has the advantage of obviating consideration of household dynamics.

[12] Which means individuals only marry those who receive the same inheritances as themselves.

1). Unfortunately, the assumption that all individuals come into their inheritance when they are of (economic) age η is less satisfactory. It conflicts with evidence assembled by Lansing and Sonquist (1969) from the 1963 Survey of Consumer Finances (USA), which showed that ages of inheritance displayed marked variation. This finding might not be too damaging in practice if η is regarded as an 'average age of inheritance'. The Swedish data of Blomquist (1979) suggest that the typical (human, not economic) age of inheritance is about 50.

11. All variables are measured in real terms and net of tax, where appropriate. Bevan (1979) has described this assumed role for taxation as 'emasculated': he also made the point that the taxation of inter-vivos gifts must be aligned with the taxation of inheritances such that individuals do not have a zero incentive to leave bequests. Note also that money illusion (Branson and Klevorick 1969) is ignored herein.

3.2 The Optimal Consumption Function of an Individual

3.2.1 The Intergenerational Utility Function

The one-way non-paternalistic utility function of Assumption 7 takes the following as its simplest form:

$$u_{\mathcal{G}} = u_{\mathcal{G}}(c_{\mathcal{G}}, u_{\mathcal{G}+1}), \tag{3.1}$$

where $u_{\mathcal{G}}$[13] and $c_{\mathcal{G}}$ is the total utility and total lifetime consumption respectively of a member of generation $\mathcal{G}$; and where $u_{\mathcal{G}+1}$ is the maximised total lifetime utility of the offspring.

Assuming separability, function (3.1) takes the form

$$u_{\mathcal{G}} = v(c_{\mathcal{G}}) + \lambda u_{\mathcal{G}+1} \qquad\qquad 0 < \lambda < 1, \tag{3.2}$$

where $v(c_{\mathcal{G}})$ is the personal 'utility from consumption' function, and where λ is the intergenerational weighting factor. Assumption 10 rules out negative bequests: hence a generation $\mathcal{G}$ individual must maximise (3.2) subject to his or her *lifetime* budget constraint. This is

$$w_{\mathcal{G}} = \xi k_{\mathcal{G}} + \int_0^T c_{\mathcal{G}}(s) e^{-rs}\, ds\,, \tag{3.3}$$

where w denotes lifetime wealth; s denotes (economic) age; and where ξk is the present value (at age zero) of a bequest k given at age σ; $\xi = e^{-r\sigma} > 0$. In words, equation (3.3) simply states, in present value terms, the fact that an individual's lifetime wealth is equal to the discounted sum of his outlays, which are of course bequests and lifetime consumption. The individual's lifetime wealth itself has two sources: inheritance and lifetime earnings: thus we have

$$w_{\mathcal{G}} = \psi k_{\mathcal{G}-1} + \tilde{y}_{\mathcal{G}}, \tag{3.4}$$

[13] The function $u_{\mathcal{G}}$ is strictly non-decreasing and concave ($u'_{\mathcal{G}}(\cdot) > 0$, $u''_{\mathcal{G}}(\cdot) < 0$); to rule out corner solutions, the conditions $u'_{\mathcal{G}}(0) = \infty$ and $u'_{\mathcal{G}}(\infty) = 0$ are imposed.

where $\tilde{y}_{\mathcal{G}}$ is lifetime earnings (ie the present value of the individual's flow of labour income over his lifetime); and where $\psi k_{\mathcal{G}-1}$ is the present value of the parental bequest inherited at age η; $\psi = e^{-r\eta} > 0$.

The separability of the utility function (3.2) allows optimal intra-generational consumption and optimal inter-generational bequest plans to be examined separately. We first analyse the former, assuming *for the time being* that the individual has determined his optimal breakdown between bequests and total consumption (the optimal breakdown itself is derived in section 3.3.).

3.2.2 The Optimal Consumption Stream of an Individual

Given the discount rate ρ, the total 'selfish' utility from consumption of an individual is

$$v(c) = \int_0^T v[c(s)]\, e^{-\rho s}\, ds \tag{3.5}$$

(the subscript $\mathcal{G}$ is temporarily supressed since there is no ambiguity about which generation is being considered). That is, total selfish utility is the discounted sum of utilities from consumption accruing over the individual's lifetime.

In the following, we need to give the general function $v[c(s)]$ a specific functional form. A popular choice in the consumption literature is the iso-elastic (see Blinder (1974, p.31-2), who discusses a rationale), which is written as

$$v[c(s)] = \frac{c(s)^{1-\beta}}{1-\beta} \qquad 0 < \beta \neq 1, \tag{3.6}$$

where β can be interpreted as the elasticity of the marginal utility of consumption. Now the optimal consumption stream (which is simply the pattern of consumption over the individual's lifetime) is given by the solution of the maximisation of (3.5) subject to (3.3) *given some optimal bequest k*. In order to solve this problem, we set up the following Lagrangean:

$$\Lambda[c(s)] = \int_0^T v[c(s)]\, e^{-\rho s}\, ds + \mu[w - \xi k - \int_0^T c(s)\, e^{-rs}\, ds],$$

where μ is the Lagrangean multiplier. The first order condition is

$$\frac{\partial \Lambda}{\partial c(s)} = v'[c(s)]\, e^{-\rho s} - \mu\, e^{-rs} = 0.$$

The solution to this equation, given the specific functional form (3.6), is

$$c(s) = \mu^{-1/\beta} e^{\frac{r-\rho}{\beta}s}. \tag{3.7}$$

'Initial' consumption at age zero, ie $c(0)$, is derived by putting $s = 0$ in (3.7): hence

$$c(0) = \mu^{-1/\beta},$$

so that (3.7) can be re-expressed as

$$c(s) = c(0) e^{\frac{r-\rho}{\beta}s}. \tag{3.8}$$

Equation (3.8) takes us some, but not all of the way towards our desired solution. The problem with (3.8) as it stands is that it does not tell us how initial consumption $c(0)$ is derived. This matter can be rectified by returning to the budget constraint, and substituting (3.8) into (3.3) to get

$$w = \xi k \,+\, c(0)\left[\int_0^T e^{-\left[\frac{r\beta+\rho-r}{\beta}\right]s}\, ds\right].$$

Hence if we define the constant

$$\begin{aligned}\phi &= \left(\int_0^T e^{-\left[\frac{r\beta+\rho-r}{\beta}\right]s}\, ds\right)^{-1} \\ &= \frac{(r\beta+\rho-r)/\beta}{1-\exp\{-(r\beta+\rho-r)/\beta\}}\end{aligned}$$

for notational ease[14], we have

$$c(0) = \phi.(w - \xi k). \tag{3.9}$$

Substituting (3.9) into (3.8) yields

$$c(s) = \phi.(w - \xi k)\, e^{\frac{r-\rho}{\beta}s}, \tag{3.10}$$

which is the final form of the individual's optimal consumption stream[15].

[14] By inspection, $\phi > 0$ unless by some fluke β is such that $r\beta + \rho - r = 0$. This latter case is degenerate since it makes ϕ undefined: it will be assumed not to apply in the following.

[15] This increases over the lifetime if $\rho < r$ and decreases if $\rho > r$.

However, note that equation (3.10) describes the optimal consumption stream *assuming* that the optimal value of k has been decided. The task of the next section is to derive the optimal value of k (strictly k_G) under three different hypotheses about intergenerational information.

Before turning to address this task, it is appropriate here to derive one more result which we will need in the next section. This is the maximal total utility $v(c)$ associated with the optimal consumption stream (3.10). This is found by substituting (3.10) into (3.6) and then into (3.5) to yield

$$\begin{aligned} v(c) &= \frac{1}{1-\beta}[\phi(w-\xi k)]^{1-\beta} \int_0^T e^{[\frac{\beta-1}{\beta}(r-\rho)-\rho]s}\, ds \\ &= L(w-\xi k)^{1-\beta}, \end{aligned} \tag{3.11}$$

where

$$L = \frac{\phi^{1-\beta}}{1-\beta} \int_0^T e^{[\frac{\beta-1}{\beta}(r-\rho)-\rho]s}\, ds$$

is a constant.

3.3 The Optimal Bequest Function of an Individual

3.3.1 Introduction

In the previous section, the first of the individual's two optimisation problems was solved: that of choosing the optimal stream of consumption over his lifetime. This section deals with the second problem, that of determining his optimal bequest.

The task here is in many ways more complex, because it involves modelling the behaviour of individuals who are concerned about what happens to their descendants, many generations into the future[16]. This poses several obvious difficulties, including tractability and sensitivity to assumptions about the stock of intergenerational information. The problem of tractability arises from the inter-relatedness of the actions of all generations in a dynasty. Specifically, in choosing a bequest, an individual of one generation affects the resources and so the bequests of individuals in the next generation; resources and beqeusts of the generation after that are likewise affected; and so the causal chain goes on to affect, ultimately, all future generations. If perfect integenerational information were assumed, then rational individuals would have to be modelled as taking every interdependency into account when solving their optimal bequest problem.

A scenario where intergenerational information is perfect is clearly neither very realistic nor conducive to tractability. For these reasons, we assume in the IGH that intergenerational information is imperfect and that agents form *expectations* about their descendants. Currently, the author is aware of only three well-developed hypotheses of imperfect intergenerational information and bequest behaviour in the literature. Each of these hypotheses differs with respect to both the way agents are presumed to form expectations about the future, and about the way lifetime earnings evolve over time. Although each hypothesis is capable of producing what is called a 'bequest function' (which is the relationship between desired bequests and

[16] Note, however, that this is only true if individuals *are* altruists. If they are not, then bequests will be motivated by other factors; for the sake of completeness, some alternative (mainly life cycle) theories of bequests are reviewed in Appendix B.

a number of explanatory variables[17]), the particular form of the bequest function differs between the hypotheses.

The bequest functions are the central objects of interest to us in this section. The fact that none of the information hypotheses giving rise to them are *a priori* more plausible than any other means that, for maximum generality, the various bequest functions must be nested inside a general structure, with parameter restrictions for each of the special cases being explicitly stated. As will be seen later in the thesis, the parameters of an estimated aggregate IGH consumption function can be interpreted in such a way as to make it possible to distinguish *empirically* between the three hypotheses.

Sub-sections 3.3.2, 3.3.3, and 3.3.4 contain an explanation of each hypothesis; sub-section 3.3.5 nests the three resulting bequest functions inside a generalised bequest function and outlines the special case parameter restrictions.

3.3.2 The Davies Hypothesis (DH) and Bequest Function

The first hypothesis is due to Davies (1982), who assumed that individuals believe their offspring will not desire bequests. The individual chooses $k_{\mathcal{G}}$ to maximise discounted expected intergenerational utility

$$\sum_{i=0}^{\infty} \lambda^i E\, v(c_{\mathcal{G}+i}),$$

where from equation (3.11), $v(c_{\mathcal{G}}) = L(w_{\mathcal{G}} - \xi k_{\mathcal{G}})^{1-\beta}$ (recall that L is a constant). Hence the individual seeks to maximise

$$\sum_{i=0}^{\infty} \lambda^i E\, \left(w_{\mathcal{G}+i} - \xi k_{\mathcal{G}+i}\right)^{1-\beta}, \tag{3.12}$$

assuming that the condition $k_{\mathcal{G}} \geq 0$ holds in practice. Since Davies assumes that individuals do not expect their offspring to leave bequests, $E(k_{\mathcal{G}+i})$ is $k_{\mathcal{G}}$ for $i = 0$

[17] Although this definition is the norm, Menchik (1979) has alternatively defined the mechanical relationship between bequests left by one generation and bequests received by the beneficiaries as a 'bequest function'. Also, what has been called the bequest function in this thesis has been termed the 'intergenerational savings function' by Pryor (1973).

and zero otherwise. Hence only the first two terms of the above sum are relevant. Substituting in (3.4), the expression (3.12) can therefore be re-written as

$$(w_G - \xi k_G)^{1-\beta} + \lambda(\psi k_G + \tilde{y}_{G+1})^{1-\beta}.$$

Differentiating the above expression with respect to k_G (for $\lambda > 0$) and setting to zero for a maximum yields the result

$$-\xi(1-\beta)(w_G - \xi k_G)^{-\beta} + \psi\lambda(1-\beta)(\psi k_G + \tilde{y}_{G+1})^{-\beta} = 0,$$

or

$$\xi(w_G - \xi k_G)^{-\beta} = \lambda\psi(\psi k_G + \tilde{y}_{G+1})^{-\beta}.$$

Re-arrangement yields the optimal bequest function

$$k_G = \beta_1 w_G - \beta_2 \tilde{y}_{G+1}\,, \tag{3.13}$$

where

$$\beta_1 = \xi^{-1/\beta}(\xi^{1-1/\beta} + \psi^{1-1/\beta}\lambda^{-1/\beta})^{-1},$$
$$\beta_2 = \beta_1(\xi/\lambda\psi)^{1/\beta},$$

are positive constants. Notice, incidentally, how the above bequest rule implies a breaking of the non-negative bequest constraint for poor individuals (low w_G) with high-earning offspring (high $\tilde{y}_{G+1}$).

We now proceed to convert (3.13) into a more manageable form for IGH purposes. A well-known earnings rule in the human capital literature (Becker 1967, 1981) hypothesises that log childrens' lifetime income is a linear function of log parental lifetime income. A variant of this rule, used by Davies in his simulations (though not his bequest function), uses lifetime *earnings* rather than income, in the following way:

$$\tilde{y}_{G+1} = h.\tilde{y}_G^{\gamma} \qquad\qquad h, \gamma > 0. \tag{3.14}$$

Here, γ is the (constant) elasticity of childrens' lifetime earnings with respect to parental lifetime earnings, and h is a constant[18,19].

Substituting (3.14) into (3.13) yields the optimal DH bequest function

$$k_{\mathcal{G}} = \beta_1 w_{\mathcal{G}} - \beta_2^* \tilde{y}_{\mathcal{G}}^{\gamma}, \tag{3.15}$$

where $\beta_2^* = \beta_2 h$. Thus the optimal bequest rule under the DH (though not derived by Davies) is non-linear, reflecting the non-linearity of the intergenerational earnings rule.

3.3.3 The Bevan Hypothesis (BH) and Bequest Function

Like the DH, the hypothesis due to Bevan (1979) utilises an intergenerational earnings rule, although it differs from the one used by Davies. Unlike the DH, individuals in Bevan's world accept that their offspring may desire to leave bequests; they also use the earnings rule explicitly in their optimisation exercise. Bevan's earnings rule (which, like the DH, allows regression to the mean) is

$$\tilde{y}_{\mathcal{G}+i} = \varphi^i \left\{ A^i \tilde{y}_{\mathcal{G}} + (1 - A^i) M_{\mathcal{G}} \right\} \qquad 0 \le A \le 1,$$

where φ is the intergenerational growth factor in earnings; A is the parameter determining the speed of regression to the mean; and $M_{\mathcal{G}+i} = \varphi^i M_{\mathcal{G}}$ is the perceived mean to which the regression proceeds. Ignoring the interest rate for simplicity, the individual maximises discounted expected intergenerational utility

$$\sum_{i=0}^{\infty} \lambda^i E \left(\frac{c_{\mathcal{G}+i}^{1-\beta}}{1-\beta} \right)$$

[18] The human capital rationale for Becker's specification is that family background effects and genetic factors link generations' income-generating capabilities. Whilst Becker's specification is probably better than (3.14) in capturing a family background effect, it is probably worse than (3.14) in portraying a genetic linkage. This is because childrens' genetic endowments are uncorrelated with parental choices about life cycle patterns of investment income accruals.

[19] We would normally expect to observe $0 < \gamma < 1$, which is consistent with regression of lifetime earnings to the mean; this is indeed the finding of studies which estimate (3.14) as a regression equation (see sub-section 4.2.1 of section 4.2).

with respect to lifetime consumption c_G, subject to the earnings rule and his budget constraint. The latter is, in present value terms,

$$c_{G+i} = k_{G+i-1} + \tilde{y}_{G+i} - k_{G+i}.$$

A broad similarity between Bevan's and Davies's set-up is that both conform to the general pattern of maximising the expected value of discounted intergenerational utility subject to an earnings rule and a budget constraint. Both also *assume* that optimal bequests are non-negative[20]. Apart from differing over the hypothesised type of expectations about descendants' bequest behaviour and the role of the earnings rule in the optimisation exercise, the two hypotheses also differ with respect to how they maximise intergenerational utility. It was seen in the previous sub-section that Davies maximised utility with respect to bequests themselves, solving optimal bequests from the first order maximisation condition. The earnings rule was then substituted into the solution to re-express the DH bequest function in terms of parental inheritance and lifetime earnings. In contrast, Bevan maximises utility with respect to *consumption*, substituting his earnings rule directly into the optimisation maximand. This explicit use of the earnings rule is apparent from Bevan's Lagrangean

$$\Lambda = \sum_{i=0}^{\infty} \left(\lambda^i \frac{c_{G+i}^{1-\beta}}{1-\beta} \right) + \mu \left[k_{G-1} + \sum_{i=0}^{\infty} \{A^i \tilde{y}_G + (1-A^i) M_G\} - \sum_{i=0}^{\infty} c_{G+i} \right].$$

When the solution from the above is substituted into the budget constraint, the following linear bequest function is derived (see Bevan (1979, p.385–6) for details):

$$k_G = v_1 k_{G-1} + v_2 \, \tilde{y}_G - v_3 M_G.$$

Here, the v's are positive constants. Assuming that lifetime earnings are expected to regress to the generation's actual lifetime earnings mean $\overline{y}_G$, the linear bequest function becomes

$$k_G = v_1 k_{G-1} + v_2 \, \tilde{y}_G - v_3 \overline{y}_G.$$

[20] Bevan went on to consider implications of the constraint binding, which included disruption of the formulation of infinite-horizon plans.

Unfortunately, this function cannot, as it stands, be used in the development of the IGH. This is because $\overline{y}_{\mathcal{G}}$ cannot be measured using aggregate time series data. Only a large series of longitudinal (panel) data could be detailed enough to isolate, at every point in time, a coherent 'generation' whose overall mean lifetime earnings could be calculated. In order to overcome this problem within a time series framework, the following strong simplifying assumption is invoked: that the average lifetime earnings of *all* generations are in a steady-state equilibrium. That is, we assume that $\tilde{y}_{\mathcal{G}} = \overline{y}_{\mathcal{G}} = g = constant \; : \forall \mathcal{G}$. Thus the BH bequest function becomes

$$k_{\mathcal{G}} = v_0 + v_1 k_{\mathcal{G}-1}, \tag{3.16}$$

where $v_0 = g(v_2 - v_3)$ is a constant, which may be less than, greater than, or equal to zero.

Arguably, the simplifying steady-state assumption removes much of the power of Bevan's bequest function. Indeed, the assumption weakens the BH function considerably in times of continuing economic growth. These matters are discussed further in section 4.2, sub-section 4.2.3, where the three information/bequest hypotheses are compared, contrasted, and critically evaluated. Meanwhile, we turn to consider the third hypothesis of intergenerational information and bequests.

3.3.4 The Laitner Hypothesis (LH) and Bequest Function

The third hypothesis, due to Laitner (1979), differs quite considerably from the others. The principal difference is that Laitner operates in a stochastic rather than in a deterministic world. According to Laitner's hypothesis, each individual's total lifetime earnings is an independent sampling from a given random variable, whose distribution remains unchanged over time. The distribution has a density function $p(\cdot)$ such that $p(\tilde{y})$ is continuous on an interval $[\tilde{y}_L, \tilde{y}_U]$; $p(\tilde{y}) = 0$ for $\tilde{y} \notin [\tilde{y}_L, \tilde{y}_U]$, and

$$\int_{[\tilde{y}_L, z]} p(\tilde{y})\, d\tilde{y} > 0 \qquad \forall z > \tilde{y}_L.$$

Importantly, even if parents subsequently observe their children to have a particular value of lifetime earnings, they are unable to revise planned bequests in the light of this evidence.

In his paper, Laitner was more interested in establishing various general properties of the bequest function arising from the above framework than in deriving a specific functional form for such a function. Working with an iso-elastic utility function, and restricting bequests to be non-negative, Laitner first proved the existence of an unique utility-maximising general bequest function $k_G(\cdot)$. This proof is complex and need not concern us here – the interested reader is referred to Laitner's paper for more details. Importantly from our perspective, Laitner then went on to show that $k_G(\cdot)$ is non-decreasing, continuous, *and in general a non-linear function of the lifetime wealth of the benefactor.* However, Laitner's analysis was presented purely in general notation: no specific form of the bequest function was suggested.

Since we require a specific functional form in order to derive the IGH consumption function, we suggest our own (admittedly rather *ad hoc*) specification as an approximation to the Laitner scenario. This is the following non-linear function:

$$k_G = \vartheta_0 + \vartheta_1 w_G + \vartheta_2 \tilde{y}_G^{\vartheta_3} \qquad \vartheta_0, \vartheta_1, \vartheta_2 \geq 0, \quad \vartheta_3 > 0. \tag{3.17}$$

This is not an ideal representation of Laitner's function $k_G(w_{G-1})$ because it includes only the lifetime earnings component of parental lifetime wealth in the non-linear term: the inheritance component is ignored. However, the former component is invariably much the greater of the two[21], which gives us reason to believe that the above approximation may not be too unsatisfactory.

3.3.5 The Generalised Bequest Function

The simplest bequest function which nests the DH, BH and LH functions (3.15), (3.16), and (3.17) as special cases of a generalised form is clearly

$$k_G = z_0 + z_1 k_{G-1} + z_2 \tilde{y}_G + z_3 \tilde{y}_G^{\varepsilon}. \tag{3.18}$$

[21] Blinder (1973, p.609); also, survey evidence suggests that the majority of individuals do not receive an inheritance of material wealth (Lansing and Sonquist (1969), Blomquist (1979), Menchik 1980). Furthermore, numerous other studies indicate that a sizeable proportion of the population have low or negative asset holdings (Diamond (1977), Diamond and Hausman (1983), Zeldes (1986, 1989), Hubbard and Judd 1986).

The Davies hypothesis suggests $z_0 = 0$; $z_1, z_2 > 0$; $z_3 < 0$; $\varepsilon > 0$; the Bevan hypothesis suggests $z_1 > 0$; $z_2 = z_3 = 0$; and the Laitner hypothesis suggests $z_0, z_1, z_2, z_3 \geq 0$; $\varepsilon > 0$. Some of these parameter restrictions are preserved throughout the construction of the aggregate IGH consumption function: these will be stated as the consumption function is developed. This takes place in the next section.

3.4 Derivation of the Aggregate IGH Consumption Function

In this section, the aggregate IGH consumption function is derived. This involves bringing together the bequest analysis of the previous section and the optimal lifetime consumption analysis of section 3.2. These analyses are brought together by substituting the generalised bequest function (3.18) and the lifetime wealth equation (3.4) into the consumption function (3.10), to yield

$$c_{\mathcal{G}}(s) = \phi[k_{\mathcal{G}-1}(\psi - \xi z_1) + \tilde{y}_{\mathcal{G}}(1 - \xi z_2) - \xi(z_0 + z_3\, \tilde{y}_{\mathcal{G}}^{\varepsilon})]\, e^{\frac{r-\rho}{\beta} s}. \tag{3.19}$$

This equation may be re-cast into a more convenient form by working with individuals' post-tax earnings at various (economic) ages, denoted $y_{\mathcal{G}}(s)$. Assume the following general earnings profile:

$$y_{\mathcal{G}}(s) = y_{\mathcal{G}}(0).f(s),$$

where $f(s)$ describes the age-earnings profile. The functional form of $f(\cdot)$ does not need to be specified in what follows: the principal and crucial restriction on $f(\cdot)$ is that it is independent of $y_{\mathcal{G}}(0)$.

Since $\tilde{y}_{\mathcal{G}}$ is the present value of net earnings flows, we have

$$\begin{aligned} \tilde{y}_{\mathcal{G}} &= y_{\mathcal{G}}(0) \int_0^T f(s)\, e^{-rs}\, ds \\ &= b.y_{\mathcal{G}}(s) f^{-1}(s), \end{aligned} \tag{3.20}$$

where

$$b = \int_0^T f(s)\, e^{-rs}\, ds$$

is a positive constant for all individuals of all generations.

Substituting (3.20) into (3.19) yields

$$c_{\mathcal{G}}(s) = \phi[k_{\mathcal{G}-1}(\psi - \xi z_1) + b.y_{\mathcal{G}}(s) f^{-1}(s)(1 - \xi z_2) - \xi(z_0 + z_3.b^{\varepsilon}\, y_{\mathcal{G}}(s)^{\varepsilon} f^{-\varepsilon}(s))]\, e^{\frac{r-\rho}{\beta} s} \tag{3.21}$$

as the consumption function for an age-s individual of generation $\mathcal{G}$. The forthcoming aggregation, which is performed over all living individuals irrespective of their age and the generation they belong to, now allows us to dispense with the generational and age indices.

Assume that the age distribution (denoted $a(s)$) and the size of the adult population (denoted N) are constant over time[22,23]. This is an important joint assumption because it simplifies the aggregation of (3.21) considerably. By ensuring that variations in s amongst individuals do not cause variations over time in the terms $f^{-1}(s)$, $f^{-\varepsilon}(s)$ and $e^{\frac{r-\rho}{\beta}s}$, aggregation over the population leaves the aggregates of these expressions as constants. Hence the only variables in (3.21) whose aggregation deserves further attention are inheritances $k_{\mathcal{G}-1}$ and earnings $y_{\mathcal{G}}(s)$. The inheritance term is straightforward to aggregate because it enters (3.21) linearly: defining I_t as the sum of bequests received, or to be received, by all those alive at t, we have $\sum_{\forall \mathcal{G} \in N_t} k_{\mathcal{G}-1} = I_t$. The same is true of the earnings term $y_{\mathcal{G}}(s)$ (but not $y_{\mathcal{G}}(s)^{\varepsilon}$), because it too enters in a linear fashion. Hence defining Y_t as aggregate earnings at t, we have $\sum_{\forall \mathcal{G} \in N_t} y_{\mathcal{G},t} = Y_t$. Unfortunately, however, $y_{\mathcal{G}}(s)^{\varepsilon}$ is less straightforward to aggregate because it enters (3.21) non-linearly. In order to aggregate this term, we must consider the frequency distribution of earnings, which we denote by $h(y_t;\theta_t)$. This distribution operates over the range H^*; θ_t is a vector of parameters at time t. Hence aggregate consumption is

$$C_t = \beta_0 + \beta_1 I_t + \beta_2 Y_t - \zeta \int_{H^*} y_t^{\varepsilon}.h(y_t;\theta_t)\, dy_t, \tag{3.22}$$

where

$$\beta_0 = -\phi\xi z_0 \int_N e^{\frac{r-\rho}{\beta}s} a(s)\, ds,$$

$$\beta_1 = \phi(\psi - \xi z_1) \int_N e^{\frac{r-\rho}{\beta}s} a(s)\, ds,$$

$$\beta_2 = \phi b(1 - \xi z_2) \int_N f^{-1}(s) e^{\frac{r-\rho}{\beta}s} a(s)\, ds,$$

$$\zeta = \phi\xi z_3 b^{\varepsilon} \int_N f^{-\varepsilon}(s) e^{\frac{r-\rho}{\beta}s} a(s)\, ds,$$

22 This might not be too restrictive an assumption in practice. Although several authors have recognised the potential role of the age distribution in explaining consumption (e.g. Duesenberry (1949), Friedman (1957), Modigliani and Ando (1963) and King 1985), the empirical evidence suggests that its impact is fairly weak (Hendershott and Peak (1984), Heien (1972), and Boskin (1978) - but see also Curry, Holly and Scott 1989a). Auerbach and Kotlikoff (1989) have also demonstrated how demographics are unable to explain post-war U.S. savings patterns.

23 Combined with the assumption that all individuals live to age T with certainty, this must mean that the age distribution is uniform over the range $[0, T]$.

are constants. We now briefly consider what these constants mean with respect to the DH, BH and LH.

We know from our earlier analysis that $\phi > 0$ (section 3.2, sub-section 3.2.2, footnote 14); we also know that ξ and ψ are greater than zero (sub-section 3.2.1), and that $b > 0$ (this section). All of the age integrals are also positive: therefore the signs of the β's and ζ in (3.22) depend on the values of the z's, which in turn depend on the bequest hypotheses of the previous section. Taking β_0 first, recall from sub-section 3.3.5 that $z_0 = 0$ according to the DH. From the equation of β_0, the DH therefore suggests $\beta_0 = 0$. In contrast, the BH places no restriction on β_0 (since it places no restriction on v_0 and z_0 - see sub-sections 3.3.3 and 3.3.5). The LH, by having $z_0 \geq 0$ from 3.3.5, suggests $\beta_0 \leq 0$. For β_1 and β_2, the DH imposes no restrictions, because although z_1 and z_2 are both restricted to be greater than zero (from 3.3.5), β_1 and β_2 multiply them by the 'sign ambiguous' terms $(\psi - \xi z_1)$ and $(1 - \xi z_2)$ respectively. The same indeterminacy also applies to the LH for these coefficients, for the same reason: this is also true for the BH with respect to β_1. However, the BH suggests that $z_2 = 0$ from 3.3.5, which from the equation of β_2, means that $\beta_2 > 0$. The restrictions on ζ implied by the hypotheses may be obtained from sub-section 3.3.5 in the same way. The DH has $z_3 < 0$ which implies $\zeta < 0$; the BH has $z_3 = 0$, which implies $\zeta = 0$; and the LH has $z_3 \geq 0$, which implies $\zeta \geq 0$.

We now proceed to evaluate the integral in (3.22). This requires, firstly, that the distribution $h(y_t; \theta_t)$ be given a specific form. There are a vast number of distributions in everyday use, of which there are several popular candidates for representing the income distribution. These include the beta, Singh-Maddala, lognormal, gamma, Weibull, Fisk, exponential, and several generalised forms[24]. The relative performance of the distributions are usually compared by fitting them to published income distribution data (usually in grouped form) and examining goodness-of-fit statistics. Two consistently good performers in empirical exercises of this type are the two parameter gamma (used by e.g. Salem and Mount 1974),

[24] See, for example, Thurow (1970), Metcalf (1972), Salem and Mount (1974), van Doorn (1975), Singh and Maddala (1976), Kloek and van Dijk (1978), McDonald (1978), McDonald and Ransom (1979), McDonald (1984), and Atoda *et al* (1988).

and the three parameter beta (see e.g. Thurow (1970), McDonald 1978)[25]. Both of these distributions have a functional form conducive to evaluating the integral in (3.22): we now proceed to discuss them further.

The two-parameter gamma distribution belongs to the Pearson Type III family and has a density function of the form

$$\frac{h(y;\mu,\alpha)}{N} = \frac{y^{\alpha-1}\, e^{-y/\mu}}{\mu^{\alpha}\,\Gamma(\alpha)} \qquad \alpha,\mu > 0; \quad y \geq 0,$$

where

$$\Gamma(\alpha) = \int_0^{\infty} \tau^{\alpha-1} e^{-\tau}\, d\tau$$

is the gamma function ($\Gamma(\alpha) = (\alpha - 1)!$). Recall that N is the population; note that in terms of our former general notation, $\theta = (\mu, \alpha)$ and $H^{\star} = (0, \infty)$. The parameter α can be interpreted as measuring the degree of inequality in the earnings distribution: 'equality' varies in direct proportion to α.

The beta distribution belongs to the Pearson Type I (or II) family and has a density function of the form

$$\frac{h(y;p,q,\omega)}{N} = \frac{1}{B(p,q)} \cdot \frac{y^{p-1}(\omega - y)^{q-1}}{\omega^{p+q-1}} \qquad p,q > 0; \qquad 0 \leq y \leq \omega,$$

where

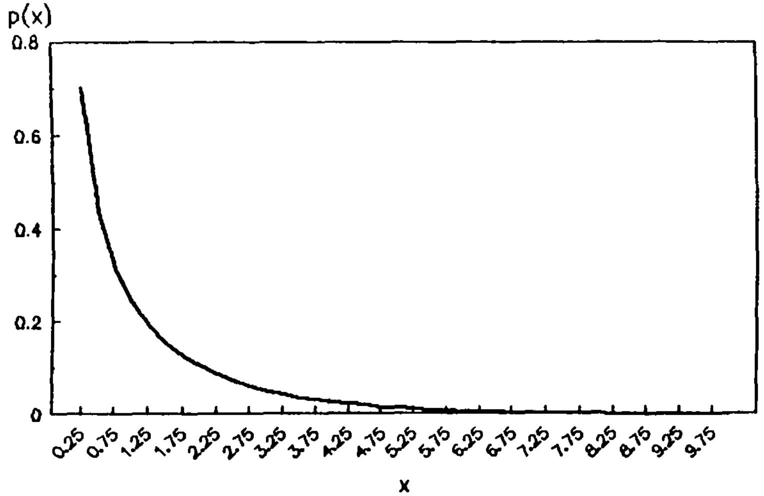
$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

is the beta function. Note that $\theta = (p, q, \omega)$ and $H^{*} = (0, \omega)$ for this distribution. Perfect income equality is approached as $p \to \infty$ **and** $q \to \infty$; perfect inequality is approached as $p \to 0$ and $q \to 0$.

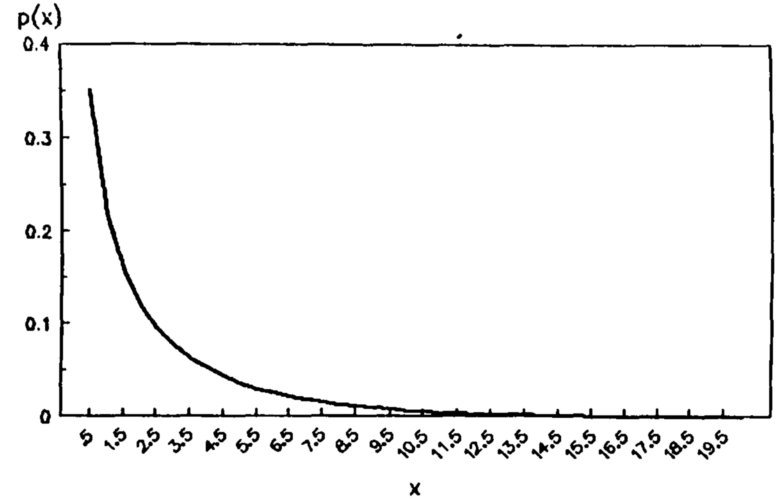
Both the gamma and the beta are flexible distributions, as the diagrams on the next two pages illustrate. Either distribution can be used to evaluate (3.22); however, only the gamma is developed and estimated in the text of this thesis. This is chiefly because the parameters of the gamma distribution, being fewer than those of the beta, prove easier in practice to estimate. This property was

[25] These distributions tend, not surprisingly, to be outperformed by generalised specifications, such as the generalised gamma (eg Atoda *et al* 1988) and the generalised beta (McDonald 1984).

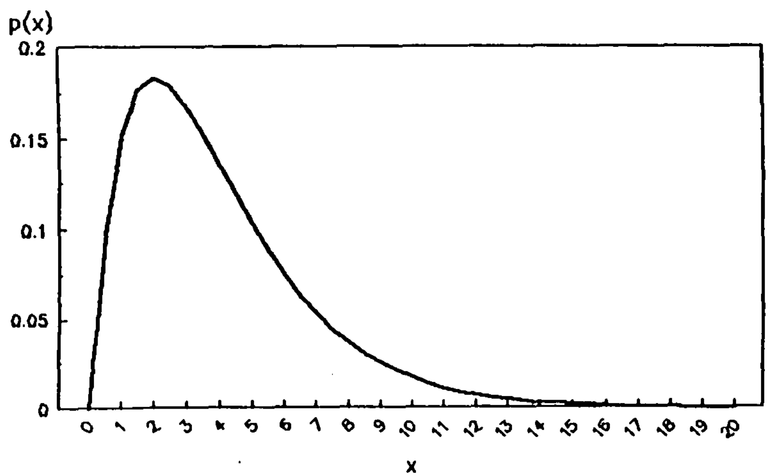
mu =2; alpha = 0.5 (Chi–square (1) distribution)



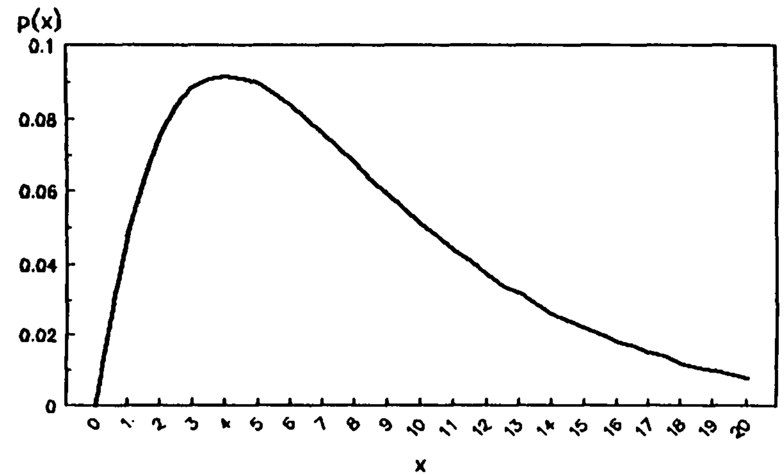
mu = 4; alpha = 0.5



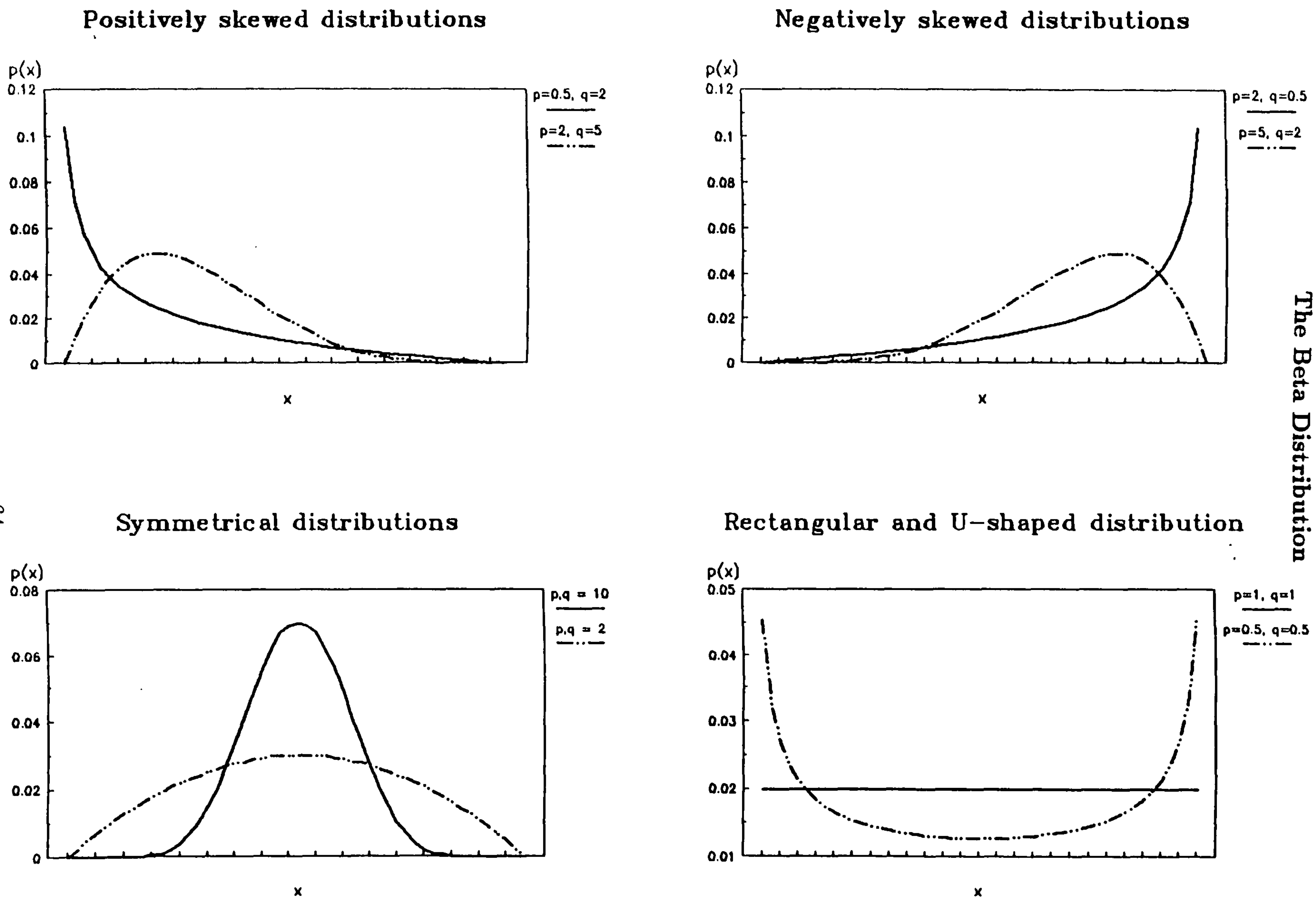
mu = 2; alpha =2 (Chi–square (4) distribution)



mu = 4; alpha = 2



The Gamma Distribution


Positively skewed distributions
p(x)
0.12
0.1
0.08
0.06
0.04
0.02
0
x
p=0.5, q=2
p=2, q=5
Negatively skewed distributions
p(x)
0.12
0.1
0.08
0.06
0.04
0.02
0
x
p=2, q=0.5
p=5, q=2
Symmetrical distributions
p(x)
0.08
0.06
0.04
0.02
0
x
p,q = 10
p,q = 2
Rectangular and U-shaped distribution
p(x)
0.05
0.04
0.03
0.02
0.01
x
p=1, q=1
p=0.5, q=0.5

confirmed by a series of experiments undertaken by the author using several years of British income distribution data. Furthermore, the parameters of the gamma also exhibited greater stability than the parameters of the beta. The parameters of both distributions can only be estimated by optimisation methods (such as grid search or gradient techniques); the gamma parameter estimates always converged quickly and were relatively robust to starting values. In contrast, the beta parameter estimates often took hours to converge, and sometimes did not converge at all. They were also invariably found to be sensitive to starting positions. Finally, and perhaps surprisingly given the greater apparent flexibility of the beta distribution, the differences between the beta and gamma goodness-of-fit statistics usually tended to be minimal.

Thus the gamma is the chosen distribution for this thesis. However, because of the possibility that, in the future, data from different countries and/or different time periods may suggest the beta instead of the gamma, it may also be of interest to derive the aggregate consumption function using the beta distribution. For the sake of completeness, this derivation is presented in Appendix D. Interestingly it shows that, with the exception of the structure of the distributional term, the aggregate consumption function is mathematically identical whichever distribution is used.

We now evaluate the integral in (3.22). Using the gamma density, this integral becomes

$$\frac{-\zeta N}{\mu_t^{\alpha_t}\,\Gamma(\alpha_t)} \int_0^\infty y_t^{\alpha_t+\varepsilon-1}\, e^{-y_t/\mu_t}\, dy_t\,; \tag{3.23}$$

the temporal subscripts on the earnings distribution parameters indicate how these parameters may be time-variable.

Euler's Second Integral establishes that

$$\begin{aligned} \frac{1}{\mu_t^{\alpha_t}\,\Gamma(\alpha_t)} \int_0^\infty y_t^{\alpha_t+\varepsilon-1}\, e^{-y_t/\mu_t}\, dy_t &= \frac{\mu_t^{\alpha_t+\varepsilon}\,\Gamma(\alpha_t+\varepsilon)}{\mu_t^{\alpha_t}\,\Gamma(\alpha_t)} \\ &= \mu_t^{\varepsilon}\,\frac{\Gamma(\alpha_t+\varepsilon)}{\Gamma(\alpha_t)}. \end{aligned} \tag{3.24}$$

Hence (3.23) becomes

$$-\zeta N\,\frac{\Gamma(\alpha_t+\varepsilon)}{\Gamma(\alpha_t)}\,\mu_t^{\varepsilon}. \tag{3.25}$$

However, average aggregate earnings is defined as

$$\overline{Y}_t = \frac{1}{N}\int_{H^*} y_t . h(y_t;\theta_t)\, dy_t$$
$$= \frac{1}{\mu_t^{\alpha_t}\,\Gamma(\alpha_t)}\int_0^\infty y_t^{\alpha_t}\, e^{-y_t/\mu_t}\, dy_t \tag{3.26}$$

using the gamma distribution. The right-hand side of equation (3.26) is clearly a special case of the left-hand side of (3.24) where $\varepsilon = 1$: hence (3.26) may be evaluated by putting $\varepsilon = 1$ into the right-hand side (result) of (3.24). This yields

$$\mu_t \frac{\Gamma(\alpha_t + 1)}{\Gamma(\alpha_t)} = \mu_t\, \alpha_t.$$

Therefore, $\overline{Y}_t = \mu_t\, \alpha_t$, or, since $\overline{Y}_t = Y_t/N$ (where Y_t is aggregate earnings), $N\mu_t\, \alpha_t = Y_t$. Hence

$$\mu_t^\varepsilon = \left(\frac{Y_t}{N\alpha_t}\right)^\varepsilon, \tag{3.27}$$

and so, substituting (3.27) into (3.25),

$$-\zeta N\, \frac{\Gamma(\alpha_t + \varepsilon)}{\Gamma(\alpha_t)}\, \mu_t^\varepsilon = -\zeta N^{1-\varepsilon} Y_t^\varepsilon \frac{\Gamma(\alpha_t + \varepsilon)}{\Gamma(\alpha_t)\alpha_t^\varepsilon}. \tag{3.28}$$

Now putting (3.28) into (3.22) yields the aggregate IGH consumption function

$$C_t = \beta_0 + \beta_1 I_t + \beta_2 Y_t + \beta_3 Y_t^\varepsilon\, \Psi(\alpha_t, \varepsilon) \tag{3.29}$$

where

$$\beta_3 = -\zeta\, N^{1-\varepsilon},$$

(constant) and where

$$\Psi(\alpha_t, \varepsilon) = \frac{\Gamma(\alpha_t + \varepsilon)}{\Gamma(\alpha_t)\, \alpha_t^\varepsilon} \tag{3.30}$$

is the Ψ function, which contains the time-variable shape parameter of the earnings distribution. As Appendix D demonstrates , the aggregate IGH consumption function derived using the beta instead of the gamma distribution is identical to (3.29), except that $\Psi(\alpha_t, \varepsilon)$ is replaced by $\Psi(p_t, q_t, \varepsilon)$, where

$$\Psi(p_t, q_t, \varepsilon) = \frac{\Gamma(p_t + q_t)\Gamma(p_t + \varepsilon)}{\Gamma(p_t)\Gamma(p_t + q_t + \varepsilon)}\left(\frac{p_t + q_t}{p_t}\right)^\varepsilon \tag{3.30'}$$

(recall that p and q are the beta distribution's shape parameters).

This completes the derivation of the IGH aggregate consumption function. All that remains now is to complete the guide to the IGH parameter restrictions implied by the bequest functions of section 3.3. The coefficients $\beta_0, \beta_1, \beta_2$ and ζ have already been discussed in the present section; we therefore only need to discuss β_3. However, β_3 merely changes the sign of ζ and re-scales it by a population factor, so it is a simple matter to derive the DH, BH, and LH restrictions for this parameter. The DH suggested $\zeta < 0$, which implies $\beta_3 > 0$. The BH had $\zeta = 0$, which implies $\beta_3 = 0$. And the LH had $\zeta \geq 0$, which implies $\beta_3 \leq 0$. Finally, note that ε takes the same restrictions as it did in sub-section 3.3.5 – aggregation has not changed this parameter at all. Thus the DH and LH both have $\varepsilon > 0$ (this parameter is not defined for the BH). Various implications of ε under each hypothesis are discussed in the next chapter, which also contains a table concisely summarising all of the parameter restrictions derived in this section.

Chapter IV

Theoretical Implications of the IGH

The purpose of this chapter is threefold: to state the parameter restrictions which can be used to distinguish between the three information hypotheses of Section 3.3; to describe the scope of government policy for altering aggregate consumption by redistributing incomes; and to discuss various implications of the three information hypotheses. At this stage, the discussion of the policy implications will be at a purely theoretical level. The empirical findings reported later in the thesis will add a practical dimension to this discussion (in addition to their role of testing the IGH theory).

Section 4.1 interprets the IGH consumption function parameters in terms of the information hypotheses of Section 3.3. This simply consists of a summary of parameter restrictions in tabular form. The section also considers how aggregate consumption is related to the distribution of income, and how governments can exploit this relationship in order to achieve some consumption target. It should be noted that the resulting policy implications only apply if the DH holds, because the DH is the only hypothesis which does not assume that individuals' incomes are in a steady state; income redistribution is inconsistent with all individuals' incomes being in a steady state. In this section, we concentrate exclusively on the consequences for consumption of income distribution: other aspects of redistribution policy (ethical, social and political) will not be considered. Governments will be assumed to be both able and willing to redistribute incomes[1] (by means of, say, its control over the tax-benefit system[2,3]).

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[1] See eg the *Effects of Taxes and Benefits on Household Income* articles published in *Economic Trends* for a demonstration of the redistributive power of governments in the UK (the most recent example, for 1988, appears in the March 1991 issue of *Economic Trends*). Note however that in a broader context, there is a danger that deployment of this power may be inhibited by self-serving and self-perpetuating ruling elites – see eg Bell (1974).

[2] Governments should in this case be assumed to be able to translate directly changes in income distribution parameters into changes in the tax-benefit system.

[3] The tax-unit data used to estimate the IGH consumption function (see Section 5.1) has the unforseen advantage of conforming to the standard unit of observation in tax-benefit work. This is

Redistribution implications of the IGH are important, but are not the sole topic of interest. Section 4.2 discusses various implications of the information and bequest hypotheses of section 3.3 with regard to inequality and Ricardian Equivalence. The realism of these hypotheses is also assessed, which gives us some idea of what empirical parameter restrictions to expect from the applied work of chapter 5.

principally because the benefit system still treats married couples and single people as single units; the abolition of joint taxation in April 1990, however, removed this method for the tax system.

4.1 Policy Implications of the IGH

Having derived the aggregate IGH consumption function in section 3.4, we now seek to interpret its parameters $\beta_1, \beta_2, \beta_3$ and ε. As stated in Section 3.3, the values taken by these parameters may also be used to distinguish between the three information/bequest hypotheses proposed by Davies, Bevan and Laitner. Furthermore, the parameters also suggest some precise implications for redistribution policy. We discuss each of these issues in turn.

Table 4.1 overleaf sets out a taxonomy of the IGH consumption function restrictions. This table summarises restrictions established during the construction of the IGH consumption function in section 3.4. It is apparent from the table that each bequest function is associated with an unique set of restrictions on the parameters of the IGH consumption function. Any applied work on the IGH consumption function should first estimate the parameters of the function (by non-linear regression analysis), and then compare them with the restrictions summarised in the table. In this way, the most appropriate information/bequest hypothesis may be empirically determined. A bequest hypothesis can only be accepted with confidence if *all* of the relevant restrictions are satisfied; if no hypotheses can be accepted, then either the IGH, the data-set, or the estimation process must be flawed.

It can be seen from Table 4.1 that a key parameter which distinguishes between the three information hypotheses is β_3. However, $\beta_3 = 0$ is consistent with both the LH and the BH[4]; and in practice, a value of β_3 insignificantly different from zero engenders problems of distinction between all three hypotheses. Hence testing decisively between the three hypotheses in practice may be less straightforward than Section 3.3 implies.

The presence of the Ψ-function in the aggregate IGH consumption function means that the distribution of earnings is a potential determinant of aggregate consumption. Distributional effects in general depend on the marginal propensity to consume out of earnings (henceforth mpc) being non-constant (Stoker 1986). A variable mpc exists when $\beta_3 \neq 0$ and $\varepsilon \neq 0, 1$; a constant mpc exists when $\beta_3 = 0$ or $\varepsilon = 0, 1$ (see Table 4.1).

[4] Although the fact that the LH bequest function is non-linear in general militates in favour of the BH in this instance.

TABLE 4.1
IGH PARAMETER RESTRICTIONS

Each of the below corresponds to a special case of the IGH consumption function

$$C_t = \beta_0 + \beta_1 I_t + \beta_2 Y_t + \beta_3 Y_t^{\varepsilon} \Psi(\alpha_t, \varepsilon)$$

THE DAVIES HYPOTHESIS (DH)

$\beta_0 = 0, \quad \beta_3 > 0, \quad \varepsilon > 0$		
$0 < \varepsilon < 1$	$\varepsilon = 1$	$\varepsilon > 1$
Decreasing mpc	Constant mpc	Increasing mpc

THE LAITNER HYPOTHESIS (LH)

$\beta_0 \leq 0, \quad \beta_3 \leq 0, \quad \varepsilon > 0$		
$0 < \varepsilon < 1$	$\varepsilon = 1$	$\varepsilon > 1$
Increasing mpc	Constant mpc	Decreasing mpc

THE BEVAN HYPOTHESIS (BH)

$\beta_2 > 0, \quad \beta_3 = 0$

The policy implications of the IGH consumption function are that, in the case where the mpc decreases with earnings[5], an equalisation of the earnings distribution tends to increase aggregate consumption; conversely, in the case where the mpc increases with earnings, an equalisation of the earnings distribution tends to reduce aggregate consumption; and in the case when the mpc is constant, aggregate consumption is independent of the distribution of earnings. These predictions can all be verified by a few simple calculations, which are not repeated here for brevity. The calculations simply involve choosing some ε, and then computing $\Psi(\alpha, \varepsilon)$ for several arbitrary values of α (in varying α, it is remembered that in the case of the gamma distribution, equality is approached as $\alpha \rightarrow \infty$, and inequality is approached as $\alpha \rightarrow 0$). The way that Ψ changes when α changes therefore tells us how changing inequality in the distribution affects aggregate consumption (since the latter is related directly to Ψ). These calculations are repeated for different values of ε corresponding to different mpc's.

The obvious policy suggestion arising out of the foregoing discussion is that the earnings distribution may constitute an additional policy instrument for demand management (Borooah and Sharpe 1986). That is, for any set of coefficient estimates for the IGH consumption function, a government could in principle vary redistribution programmes so as to hit a desired consumption target. Thus Table 4.1 may in principle allow a government to tailor its redistribution policy according to the world it finds itself in, ie according to the mpc (determined by β_3 and ε) and the prevailing intergenerational information structure (ie appropriate bequest hypothesis). The variability of Ψ with respect to α plays a central role in the effectiveness of such a policy, since it would avail the government nothing if, for example, Ψ (and hence C) were completely insensitive to any changes in the earnings distribution. Figures 4.1 and 4.2 overleaf illustrate the relationship between Ψ and the relevant shape parameter(s) of the earnings distribution for a range of values of ε: these figures relate to the gamma and beta distributions respectively. The figures show that redistribution policy becomes increasingly effective at changing Ψ as $\varepsilon \rightarrow \infty$. But the more reasonable assumption is that the mpc diminishes with resources, which suggests $\varepsilon < 1$ under the DH: from Figure 4.1a, this suggests that maximum sensitivity to Ψ is associated with a value of ε of just under a half (it

[5] Menchik and David (1983) and Diamond and Hausman (1983) have provided some evidence that mpc's decrease with resources.

will, however, be shown in the following that *overall* policy effectiveness varies in a slightly more complex way with ε).

A numerical example may best serve to show this and clarify the ideas so far. Suppose earnings inequality is measured by the Gini coefficient, G, which is initially 0.33; and that the government is interested in measuring the effects on aggregate consumption of amending the tax-benefit system. Suppose further that the government is considering four policy options, which have the following effects on inequality:

1. Decrease G (increase equality) by 10%
2. Decrease G by 50%
3. Increase G (decrease equality) by 10%
4. Increase G by 50%.

We invoke the *ceteris paribus* assumption that variables other than the shape of the earnings distribution, α, are unaffected by changes in this distribution. Thus we are interested only in changes in the term

$$\beta_3 Y^{\varepsilon} \Psi(\alpha, \varepsilon)$$

caused by changes in α. We assume in the following example that Y is at its 1979 value of £28625 m. (see section 5.1, chapter 5). For estimates of ε and β_3, we use the most reliable results from our empirical work in section 5.2. Although these parameters are not well-defined, since only limited non-linearity is detected in the IGH consumption function, we use them here for illustrative purposes. They are: $\varepsilon = 0.90$, and[6] β_3=44.190. From Table 4.1, we see that these results are consistent with the Davies Hypothesis and a diminishing marginal propensity to consume. We use our estimates to calculate $\beta_3.Y^{\varepsilon}$= £453310 m.

[6] Strictly speaking, 44.19 is β_3 weighted by a particular constant, but this need not concern us here.

The Ψ-function (Gamma Distribution)

Figure 4.1a
The Ψ-function for $\epsilon < 1$

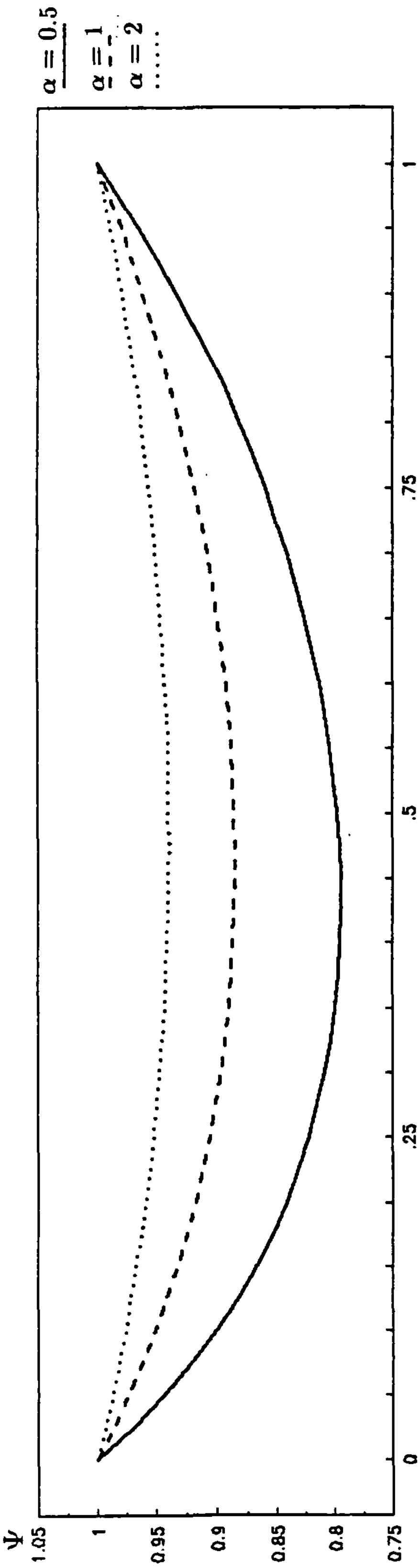


Figure 4.1b
The Ψ-function for $\epsilon > 1$

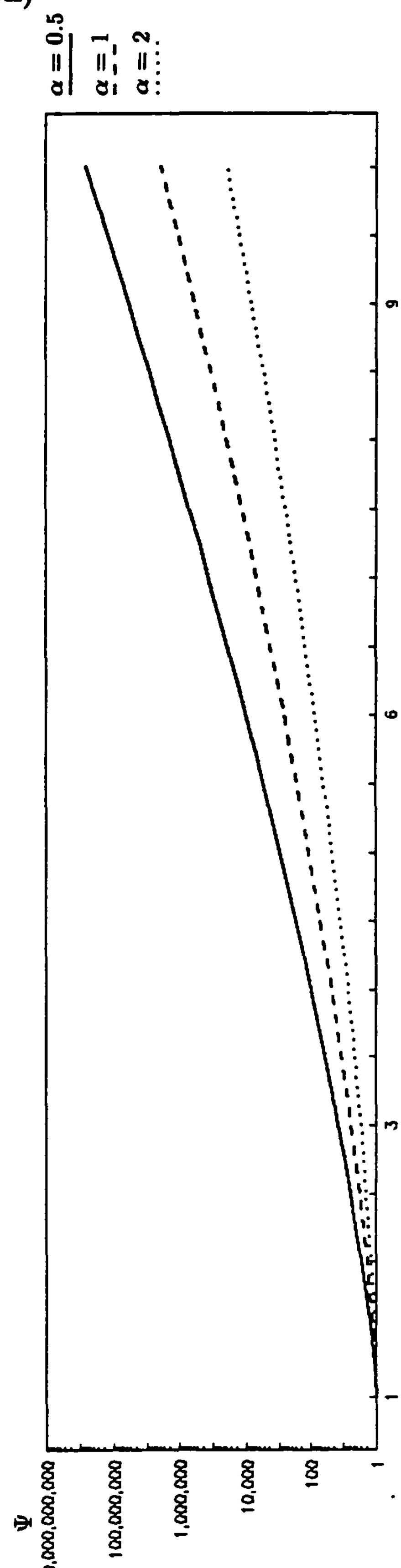


Figure 4.2a
The Ψ-function for $\epsilon < 1$

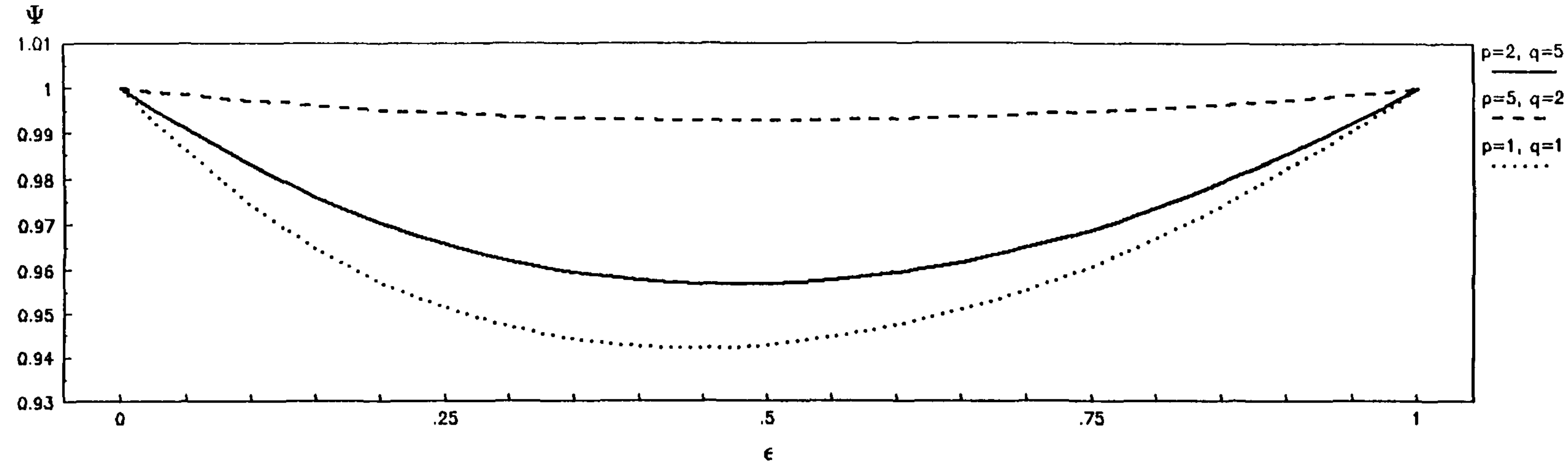
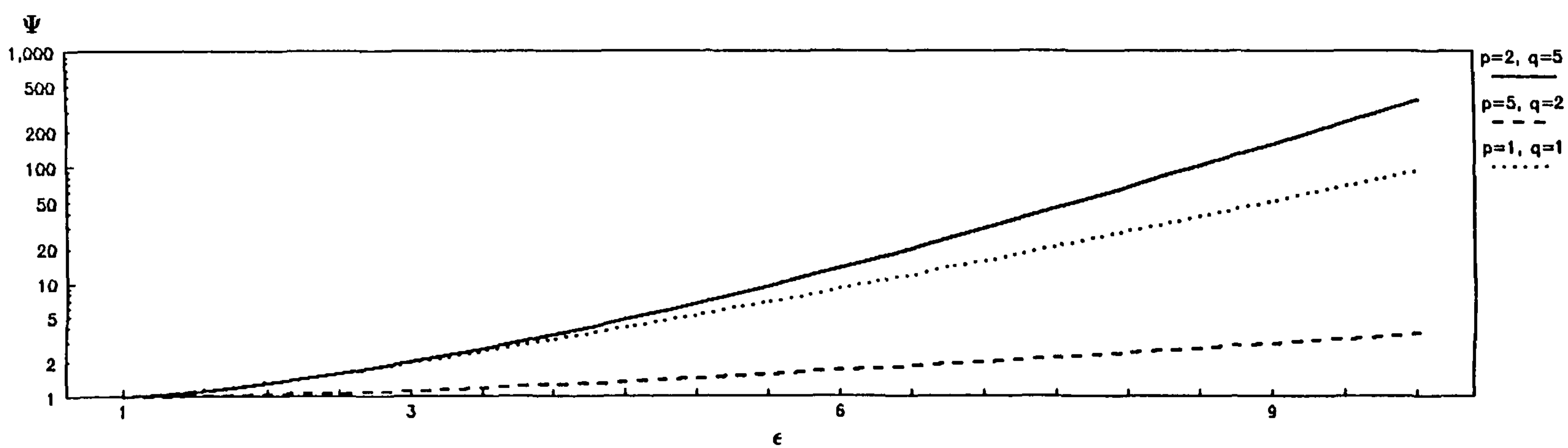


Figure 4.2b
The Ψ-function for $\epsilon > 1$



We clearly need a relationship between G and α in order to investigate changes in consumption brought about by changes in the Gini coefficient. Such a relationship exists: it is simply

$$G(\alpha) = \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)\sqrt{\pi}}.$$

This relationship enables us to translate unique Gini coefficients into unique α estimates and vice-versa – indeed, this technique is used to estimate some actual α_t values in section 5.1. Some iterative search technique is needed to do the transformation, since analytical solution of the relationship is impossible. Using a golden-section search routine, we found the α coefficients corresponding to the four options above to be 3.39, 11.44, 2.15, and 1.03, respectively. Substituting these into the Ψ function given $\varepsilon = 0.9$ gave the following Ψ-function values: 0.987, 0.996, 0.981, and 0.963, respectively (the Ψ-value corresponding to the initial situation is 0.984). This bears out the result of Figure 4.1a, that the Ψ function increases for given $\varepsilon < 1$ when equality increases. The initial value of the third term of the consumption function – before any policies are implemented – is $453310 \times \Psi(\alpha, \varepsilon) = 453310 \times 0.984 = 446057$.

Calculating the same quantity after the four policies have been implemented yields the following results:

1. C increases by £1 338 m.

2. C increases by £5 353 m.

3. C decreases by £1 338 m.

4. C decreases by £9 367 m.

An interesting feature of these numbers is that changes in inequality have quite powerful effects on overall consumption. For instance, policy 1 raises consumption by over £1.3 billion – roughly the revenue the government receives from inheritance tax. This supports Borooah and Sharpe's conjecture that redistribution policy may be a powerful extra tool of demand management.

Notice also that although policies are roughly symmetric for small changes, they are asymmetric for large ones. This reflects the slightly slewed dish shape of the Ψ function illustrated earlier. It means that, for large changes in redistribution policy, it is easier to reduce consumption by increasing inequality in this example than it is to raise consumption by reducing inequality. This observation is true irrespective of the parameterisation chosen. Of course, the other findings *are* sensitive to the given parameterisation, and so the poor determination of β_3 and ε urge caution in their interpretation. Furthermore, even if the above parameterisation was to be accepted as a realistic one, the effects on consumption suggested could be offset, possibly quite heavily, by second-round effects, especially those relating to discouraged labour supply. We take up this point below.

First, however, we examine the sensitivity of these results to the chosen parameterisation. Suppose ϵ was not 0.9, but was 0.5 – close to the point of maximal sensitivity of Ψ to changes in the earnings distribution (see Figures 4.1a and 4.2a). Then, initially $\beta_3.Y^{\varepsilon}=$ £7476 m. The initial Ψ value is now 0.954; those for policies 1 through 4 are now 0.964, 0.989, 0.944 and 0.889. This bears out the result that Ψ is more variable when $\varepsilon = 0.5$, as illustrated. Hence the leverage for redistribution policy, that is the change in consumption expressed as a proportion of the third term initially, is highest in this case. However, the effects of the policies under the new parameterisation are now:

1. C increases by £75 m.
2. C increases by £262 m.
3. C decreases by £75 m.
4. C decreases by £486 m.

Despite the greater variability of Ψ, the *overall* magnitude of the effect on consumption is lower, because the scale factor Y^{ε} is obviously greatly reduced when ε falls. This explains why the apparent effects of redistribution policy are greatest for high ε, even though the Ψ-function is less sensitive to policy changes in this case.

It has been mentioned above how the numerical values generated using these examples ought to be treated with some care. Apart from sensitivity to parameterisation, at least four caveats need to be mentioned. These apply also to the policy guide in general. The most important is that the policy guide only applies if the DH holds, since unlike the DH, the BH and LH are steady-state theories; income redistribution is inconsistent with all individuals' incomes being in a steady-state. The second recognises that the IGH consumption function may be mis-specified: if true, coefficient bias could result, so jeopardising consumption-based redistribution policies. The third recognises the point made by Bernheim and Bagwell (1985, 1988) that agents may be altruistically inter-connected. If true, this would mean a further reduction in the effectiveness of redistribution policy, because agents in the Bernheim and Bagwell scenario neutralise government policy to maintain desired consumption/bequest patterns within their 'clan'. The fourth caveat is that agents may adjust their behaviour in response to a redistribution programme, for example by reducing their labour supply if taxes on labour incomes are raised. This last caveat seems especially important, so we now proceed to discuss it further.

In the foregoing, it was implicitly assumed that agents passively accept any redistribution exercise that a government chooses to impose. This assumption was made in order to simplify the analysis of the effects of redistribution policy implied by the IGH consumption function. However, the assumption is not a particularly good one, since agents often possess sufficient power to alter their behaviour in response to government policy. In general, government policies which fail to allow for this eventuality are vulnerable to the Lucas Critique (Lucas 1976). We will briefly consider the relevance of the critique with respect to labour supply adjustments[7]; the effects of higher consumption (lower saving) on investment and thereby economic growth will be ignored in the following.

A redistribution exercise which involves higher taxation of those in work risks discouraging labour supply. If in response to a reduction of work incentives individuals substitute leisure for work, or switch their labour supply to alternative 'second choice' employment in reaction to the tax distortion, then it may be expected that total labour earnings and overall economic efficiency will be reduced. The 'equity-efficiency' tradeoff this implies will therefore be of direct practical interest to governments trying to choose a preferred redistribution programme. Although

[7] Adjustments may also be made, for example, by parents who invest less in the human capital of their children as a consequence of the reduced rate of return resulting from the redistribution programme (Becker and Tomes 1979). Long-term equalisation may therefore be impaired by redistribution policies.

the chosen point on the tradeoff is a political decision, the shape of the tradeoff is of considerable interest because it is this which determines the efficacy of following the 'policy guide' given in the foregoing. The greater the 'efficiency' which must be traded in for a given percentage gain in equality, the greater the 'second round' effects on aggregate income and its distribution. This will in turn affect aggregate consumption in a way not comprehended by the IGH, whose parameters will then be prone to instability, and whose predictions of aggregate consumption could cease to be accurate.

The IGH can give us no help in quantifying the equity-efficiency tradeoff[8]. We must therefore consult extraneous theory and evidence on the issue. The former has been copious from an historical perspective[9]; the latter has been scarcer. Some authors have stressed the tradeoff idea quite forcefully: eg Taubman (1978) and Gilder (1982), who asserted that redistribution is always at the expense of national income. However, others (eg Pahl (1984), Piachaud (1987) and Borooah 1988) have disputed the 'pessimistic' view that a tradeoff necessarily exists. Borooah, for example, cites several reasons why the existence of a tradeoff might not be clear-cut: a persuasive one relates to the 'efficiency wage' idea that rewarding low paid workers increases their efficiency and hence national income in the aggregate. The transmission of higher pay to greater efficiency may arise from better nutrition and health (especially in LDC's), or higher morale (more likely in DC's).

Some evidence on the issue of the equity-efficiency tradeoff does exist. Gevers and Rouyer (1979) proposed, in a study using US data for 1961 and 1970, that a 5.4% reduction in the Gini coefficient 'cost' only 1% of output. Moreover, in a study using cross-section data from 30 countries (DC's and LDC's), Lee and Koo (1988) found no statistical evidence of an 'equity-efficiency' tradeoff. However, in a recent paper Lambert (1990) used a simple model to show that the equity-efficiency tradeoff is approximately unity over a wide range of output levels: ie a 1% reduction in the Gini coefficient costs 1% of output. Thus the question of the precise magnitude of the tradeoff is still essentially unresolved. This means that it is very difficult to guage the true value of the policy implications outlined in this section.

[8] By Assumption 6 of section 3.1.

[9] See eg Breit (1974).

4.2 Implications of the Three Bequest Functions

This section spells out some of the implications of the three bequest functions explained in Section 3.3. Sub-section 4.2.1 is concerned with implications of the functions for inequality; sub-section 4.2.2 takes a look at implications for Ricardian Equivalence; and sub-section 4.2.3 contains a brief discussion about the various merits and demerits of the functions.

4.2.1 Implications for Inequality

As stated in sub-section 3.3.2 of chapter 3, the earnings rule used in the Davies Hypothesis is a variant of a widely-used intergenerational earnings function. The implications of the rule for inequality depend on the value taken by the intergenerational earnings parameter ε, formerly written as γ (this symbol is replaced by ε because it is the latter which is estimated in empirical work not the former; also, we want to use γ in later work to refer to Hall's (1978) autoregression parameter without ambiguity). If ε lies between zero and unity, lifetime earnings will tend to regress to the mean (ie inequality of lifetime earnings will tend to decrease over time); if ε is greater than unity, however, inequality will tend to increase over time. With regression to the mean being the likelier case in practice (Becker 1974), the issue then becomes the *speed* of regression to the mean. The closer to zero is ε, the quicker this will be.

Estimates of ε in the literature tend to relate to earnings functions where childrens' lifetime earnings are a function of parental lifetime *incomes* (not earnings as in Section 3.3)[10]. Perhaps the best recent estimate of ε in this case is that of Behrman and Taubman (1990), who were able to exploit the longitudinal aspect of their data to average out transitory influences on earnings and incomes. Averaging over ten years yielded sets of earnings and incomes with more of a permanent, or 'lifetime', character than single years. Without controlling for gender, race, or other factors, these authors reported $\hat{\varepsilon} = 0.8$. This is a considerably higher estimate than previous studies have generated[11], though as Behrman and Taubman

[10] An exception to this is Atkinson et al (1978).

[11] Eg using a number of cross-section samples drawn from five countries, Becker and Tomes (1986) found a median estimate of ε of 0.17. This implies a relatively fast regression to the mean. However,

observed, it is largely due to the use of more 'permanent' measures of earnings and incomes.

What are the effects of the DH bequest rule on inequality? It was seen in sub-section 3.3.2 how optimal bequests varied negatively with childrens' earnings: that is, the DH bequest rule is 'compensatory'[12]. This feature of the bequest rule is equalising, and offsets, to some extent, the disequalising reaction by rich families of giving greater bequests in order to thwart regression to the mean. The impact of bequests is therefore ambiguous in the DH. Not surprisingly, the same intuition can be applied to the BH; in a series of simulations, Bevan (1979) has produced some results which confirm this.

Evidence on the implications of Bevan's earnings rule for lifetime earnings inequality is limited. Bevan was only able to estimate the speed of regression to the mean, A, using *socioeconomic* data. Assuming that ordering by class can be taken as a rough proxy for orderings by lifetime earnings, Bevan (1979, p.387–388) used maximum likelihood methods to estimate A using two data-sets: the Oxford Mobility Survey (covering England and Wales) and data from Brittain (1977, Table 3.11), relating to Cleveland, Ohio, USA. The estimates of A were 0.46 and 0.53 respectively. Bevan found his intergenerational earnings rule fitted the data quite well for the Ohio sample but not for the Oxford survey. Hence the evidence supporting Bevan's rule is inconclusive.

As far as the Laitner hypothesis is concerned, the variance of lifetime earnings is fixed by assumption: inequality measured at any one particular time may fluctuate due to stochastic variation, but the 'expected' degree of inequality will not. Evidence of even only limited intergenerational regression to the mean therefore casts considerable doubt on the LH. However, where the LH does throw up an interesting result is in its (implied) role for bequests. Laitner (1979) rigorously demonstrated that in an economy where goods are not perishable, where money exists, and where earnings are not heritable, bequests will either increase or leave unchanged the degree of equality of the (stationary) distribution of consumption.

Davies (1982) had earlier used a value of ε of $\sqrt{1/2}$ based on a supposition due to Griliches (1979).

[12] The reader should not confuse this term with its specific use by Tomes (1981), who referred to parents compensating poorly endowed members of their offspring by leaving them larger bequests.

A similar point about the potential equalising role of bequests has also been made by other authors, including Stiglitz (1978) and Bevan and Stiglitz (1979). However, the rationale for this equalising role under the Laitner hypothesis is especially appealing intuitively. Consider the following scenario, where an individual is well endowed with resources, and so (according to the LH bequest function) leaves a large bequest: if his child is poor, this bequest clearly performs an equalising role. If his child is rich, its large inheritance makes it even richer; but the increasing non-linear form of the LH bequest function ensures that a proportionately even larger bequest is left than the parent's, thereby mitigating consumption inequality. The latter process therefore equalises consumption even in spite of a string of 'lucky' (high earning) generations: and when the string is broken with an 'unlucky' (low earning) generation, the latter's lifetime wealth is brought up to the mean by the large accumulated inheritance received courtesy of its (rich) ancestors.

4.2.2 Implications for Ricardian Equivalence

The recent evolution of thought on Ricardian Equivalence can be traced back to a seminal paper by Barro (1974). This paper, which has since generated a considerable amount of theoretical and empirical work[13], used a model of intergenerational altruism to demonstrate that government debt issue[14] has no real effects in an economy when people have operative bequest motives. This proposition (also known as the Debt Neutrality Proposition) is simple and intuitively appealing: any future tax increases required to finance the debt issue will be anticipated by the beneficiaries, whose behaviour will adjust accordingly. When the tax increase is levied on their offspring, an altruistic individual with an operative bequest motive will simply adjust his or her bequest to maintain the real value of its impact. In short, debt issue does not enlarge individuals' possibility sets; and people will always try to choose the same optimal outcome from these opportunity sets.

Do the bequest rules possess the properties of Ricardian Equivalence? The DH bequest rule does, at least in its 'primary' form (equation 3.13, Section 3.3) before the earnings rule is specified. This is easily demonstrated, assuming $r = 0$

[13] Two recent surveys include Bernheim (1987) and Barro (1989).

[14] Or social security transfers: since the principle is the same, we can talk about either.

for simplicity[15] so that $\xi = \psi = 1$. Then

$$\Delta k_G = \left(\frac{1}{1+\lambda^{-1/\beta}}\right)\Delta w_G - \left(\frac{\lambda^{-1/\beta}}{1+\lambda^{-1/\beta}}\right)\Delta \tilde{y}_{G+1}.$$

But the inheritance k_{G-1} is fixed, so that $\Delta w_G = \Delta \tilde{y}_G$. However, a new debt issue increases $\tilde{y}_G$ by the same amount that $\tilde{y}_{G+1}$ decreases, so we also have $\Delta \tilde{y}_G = -\Delta \tilde{y}_{G+1}$. Therefore

$$\begin{aligned}\Delta k_G &= \left(\frac{1+\lambda^{-1/\beta}}{1+\lambda^{-1/\beta}}\right)\Delta \tilde{y}_G \\ &= \Delta \tilde{y}_G.\end{aligned}$$

This demonstrates Equivalence: individuals increase bequests 'one-for-one' with the debt issue to maintain the real value of the bequest to their offspring.

The earnings rule (3.14), by which childrens' earnings are correlated with those of their parents, destroys 'one for one' Equivalence in an interesting manner. Debt issue increases individuals' earnings, thereby increasing their childrens' earnings automatically via the earnings rule. To see how this may overcome one for one neutralisation, consider the case where intergenerational earnings are linked by a family background effect. The payment of social security to the parents of a child may improve the family background, and thus also the eventual lifetime earnings, of the child. One for one neutralisation is destroyed because bequest adjustments by parents will compensate for only a fraction of the increase in the child's earnings: the 'one for one' (linear) correspondence between the generations' lifetime earnings is replaced by a new (non-linear) relationship.

Neither the BH nor the LH bequest rules can be seen to exhibit Equivalence. In the case of the LH, this follows directly from the informational limitations of the hypothesis. Information about future tax hikes which is required for the Equivalence property to hold are absent from the Laitner world where no more precise information exists about future generations than a probability distribution of lifetime earnings.

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[15] The result also holds for $r > 0$, but the interpretation is complicated by the need to adjust variables in order to represent them in present value terms.

4.2.3 The Three Hypotheses Assessed

Which of the three information hypotheses offers the best description of reality? There does not appear to be a clear *a priori* answer to this question. One reason for this is the absence of a well-established body of empirical work on intergenerational information sets. Another is the paucity of use of these hypotheses in the literature. Nevertheless, it is possible to make a few tentative guesses about the relative merits and demerits of the hypotheses at this stage, before estimating the IGH consumption function and thereby testing their relative performances.

Arguably a major drawback of the LH is the way it rules out specific parental information about offspring. This does not seem realistic because in view of the contribution to utility that such information may account for, it is to be expected that parents would have a powerful incentive to acquire it. Moreover, if parents are living with their offspring during the latter's childhood (the norm), the costs of gathering this information are negligible or zero. Parents should be able to generate a good estimate of a child's earnings abilities from a young age.

It is in fact a restriction on the stochastic distribution of lifetime earnings which raises a strong objection to the LH bequest function. This is the 'zero growth' restriction: ie the assumed fixity of the distribution from which individual samplings are 'drawn'. Since this thesis's version of the BH also assumes zero growth (though Bevan's original paper did not), it too is vulnerable to this objection. Given the fact that economic growth tends to reduce bequests *ceteris paribus* (Bevan and Stiglitz 1979), the LH and BH bequest functions would be expected to over-predict bequests if their parameters were known.

In a practical sense, the author believes that the zero growth restriction is so strong that it alone is able to suggest the DH, which does not impose it, as the most realistic *a priori* bequest function. The DH also has the advantage over the other hypotheses of allowing use of the redistribution policy guide set out in the previous section. However, the information hypothesis underpinning the DH bequest function is not beyond reproach. For although it may be more realistic than the Laitner information hypothesis, it does not compare particularly favourably with aspects of the BH. The BH has (what is arguably) the advantage of being consistently 'rational': the intergenerational earnings rule is known and used explicitly by the optimising individual, even though the rule may turn out to be 'wrong'. In contrast the DH is slightly less 'rational', in the sense that individuals are assumed to mistakenly believe that their offspring will not leave a bequest; furthermore, individuals do not believe, understand, or know about the earnings rule sufficiently well for them to use it in predicting future lifetime earnings and bequests. On one hand this might be more realistic than the Bevan scenario (agents cannot look further ahead than their own and their childrens' lifetimes); but on the other hand, it might not. After all, the earnings rule should impart *some* information to rational altruists; and the expectation of parents that childrens' bequests will be zero whilst their own are non-zero does not seem consistent.

To conclude, it would seem that of the three information *hypotheses*, the LH is the least realistic, with the DH lagging or leading the BH according to how much intergenerational information individuals actually use. The author believes, however, that out of the three bequest *functions*, the Davies function is the most realistic. This is because it is the only one which does not impose zero economic growth.

Part III

The IGH: Estimation and Conclusions

Chapter V

Estimation of the IGH Consumption Function

5.1 The Data

This section provides a summary of the raw data sources and data transformations used to create a data-set for estimating the IGH consumption function. Sub-section 5.1.1 deals with the real aggregate consumption series; sub-section 5.1.2 describes the construction of a nominal net earnings series with a distributional breakdown; and sub-section 5.1.3 discusses the inheritance series.

5.1.1 Real Aggregate Consumption

The real aggregate consumption series was taken directly from a paper by Patterson (1989)[1]. This annual series covers the period 1964-1986: more recent observations have not yet been constructed. Patterson's series relates specifically to the (estimated) *consumption* of goods and services rather than to *consumer expenditure* – an important distinction, and one which ensures that the theoretically correct dependent variable is used in the econometric work. Patterson also derived a price deflator series derived on a consumption rather than an expenditure basis; this was later used to deflate the nominal earnings and (the relevant stretches of) the two nominal inheritance series.

Although the superiority of a consumption over an expenditure series has been apparent for a long time (see eg Friedman (1957), Mayer (1972)), much applied work has utilised consumer expenditure data instead. This is not unreasonable in view of the large proportion of expenditure taken up by non-durables; but the use of expenditure data in applied work surely owes more to the lack of availability of a suitable aggregate consumption time series. Therefore Patterson's series is an especially welcome contribution to the applied consumption area.

[1] The author is grateful to Dr. Patterson for permitting him to use these data.

Patterson used a disaggregated CSO data-set to construct his (Divisia) consumption index. Quantitatively, it seems to be fairly similar to the expenditure series, although it is slightly smoother, indicating a slightly smaller fall in personal savings in the 1980's than implied by the CSO data.

5.1.2 Nominal Net Earnings with a Distributional Breakdown

Two principal sources of data were used to construct a time series of nominal net earnings with a distributional breakdown. These were: aggregated Central Statistical Office (CSO) data, whose distributional breakdowns are based on the Family Expenditure Survey (FES); and the Inland Revenue's *Survey of Personal Incomes* (SPI). We will first describe how an estimated aggregate net earnings series was constructed, and then we will discuss the pedigree of the distributional series.

Aggregate nominal net earnings were defined as the sum of incomes from employment and self-employment, less a certain proportion of income taxes. This proportion was defined as the ratio of incomes from employment and self-employment to the difference between total personal income and current government transfers. This definition of aggregate net earnings is similar to Muellbauer's (1983, p.41-42), which multiplied the sum of incomes from employment and self-employment by the 'retention ratio': the principal difference is that Muellbauer regards national insurance contributions as a form of taxation, whereas we argue that they should be treated as a form of saving out of income. This is because employee national insurance contributions are simply earnings which are 'forced' into savings: these savings are of course realised later in the life cycle (ie in retirement). The estimates of aggregate real net earnings, Y_t, are given in the last column of Table 5.1 (which appears at the end of this sub-section).

The distributional data were harder to generate and involved using data of doubtful quality. The central problem here was that both sources of available raw data - the FES and the SPI - suffer from serious drawbacks. The problems with the FES stem chiefly from the survey nature of the data: they include

- the under-recording of top and self-employment incomes - see eg Atkinson and Micklewright (1983), who cite an unpublished CSO study which suggests that the top 1% of taxable income recipients are under-represented in the FES by about 30%; and that self-employment income is under-recorded by about 25%;

- the collected data relate to incomes rather than earnings;
- compilation at the household level, where a household is not a family but any group of people living under the same roof: this unit is clearly inconsistent with the IGH theory;
- the short time span of the Gini series, which dates back only as far as 1975: furthermore, the CSO has stopped reporting Gini coefficients for unequivalised households after 1987;
- unreliability, arising from an incomplete and biased response rate. In comparing the FES with the SPI and New Earnings Survey, Rendall and Wolf (1983) conclude: *"Although the [FES] results may be sufficiently reliable for the purpose it serves, ie, in establishing weights for the RPI, it is questionable whether really meaningful income distribution results can be obtained".* (p.175);
- numerous changes in definition over time.

Despite these shortcomings, especially the fact that total income rather than earnings is measured, it would seem that FES distributional data is preferable to that of the SPI. This is chiefly because the SPI only contains data relating only to taxpayers' incomes: non-taxpayers (who amount to about one quarter of the tax unit population[2]) are ignored. The reason for this is that the tax office does not need to know the incomes of those whose personal allowances exceed their incomes. This property biases estimates of overall distribution considerably. For example, it is widely known that income inequality increased between 1979 and 1986: estimates of the Gini coefficient based on FES data bear this trend out, being 0.33 in 1979 and 0.36 in 1986 (qv later). Yet the implied Gini coefficients computed from SPI data using the gamma distribution (see Table 5.1) were 0.35 in 1979, 0.18 in 1982, and 0.16 in 1986! Such a dramatic apparent increase in equality can be attributed to the large increase in unemployment arising from the

[2] Under the system of joint taxation (which was replaced by independent taxation in April 1990), single people and married couples were both treated as one unit for tax purposes - hence the term 'tax unit'. Of course, the IGH theory is framed in terms of individual decision making, not tax-unit decision making; however, these units may be viewed as being suitably close.

1979-1981 recession - this removed much of the lower tail of the taxpayers' earnings distribution. Other problems with the SPI include:

- the data is compiled on a financial rather than a calendar year basis (unlike the consumption series used in the IGH);

- changes in coverage and composition have occurred over time, a recent example being the inclusion in 'income', from 1985 onwards, of estimates of employees' superannuation contributions.

- no data were published in 1980 and 1981, due to a Civil Service strike.

However, the SPI does have some compensating advantages: these include

- a detailed breakdown of earnings by earnings class, stretching back to the early 1960's;

- the ability to generate fairly accurate estimates of net earned income, for all earnings classes, relying on the minimum of assumptions. For example, it is relatively straightforward to allocate tax liabilities on earned income to each class. Also, the distribution of 'non-investment' income is broken down into useful component categories. This enables the researcher, for example, to remove pensions from 'earned income', which in an intertemporal optimising model should be regarded as realised savings not earnings. The SPI also correctly treats national insurance contributions as earnings (see the second paragraph of this sub-section)[3];

- the reliability of the data, coming as it does directly from tax offices.

On balance, however, the drawbacks of the SPI data were felt to greatly outweigh the advantages. It is worth noting that the shortcomings of the SPI are not readily amenable to correction by adjusting the raw data series. Filling in the 'missing' tax unit population would not be straightforward, since this would entail estimating distributions of personal tax allowances (which vary with individual circumstances). Another drawback is that the unit of account is the tax unit not the individual (see footnote 2); however, sufficient data does not exist to effect

[3] Blinder and Deaton (1985) also excluded these contributions, though it is not clear from their paper why they did so.

a ready transformation from the tax unit to the individual level. It would also not have been straightforward to adjust for the financial year compilation of the data: especially problematical would have been the application of this procedure to the distributional parameters estimated from the data. We therefore used the SPI data when we had no better data: this is for the period 1964–1974.

FES estimates of the Gini coefficient for disposable income are used as raw data for 1975-1986: estimates for alternate years starting in 1975 can be found in Table 3 on page 118 of the Redistribution of Income article in May 1990's *Economic Trends*. Interpolation is used for the missing years. Recall from the structure of the IGH consumption function (3.29) that estimation work requires estimates of the shape parameter of the gamma distribution of earnings, α, for each year. Since the implied Gini coefficient is related to α by the formula

$$G(\alpha) = \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)\sqrt{\pi}},$$

a simple iterative search can be used to generate estimates of α from observed values of G. These estimates and their corresponding Gini values for 1975–1986 are as follows: $\hat{\alpha}$: 2.849, 2.948, 3.053, 2.849, 2.662, 2.662, 2.662, 2.662, 2.662, 2.492, 2.337, 2.194; $G(\hat{\alpha})$: 0.32, 0.315, 0.31, 0.32, 0.33, 0.33, 0.33, 0.33, 0.33, 0.34, 0.35, 0.36.

For the years 1964-1974, the raw SPI data were transformed by first, creating two vectors for each year, one giving the average net earnings of each earnings class, and the other listing the proportions of the earnings population in each class. Distributional breakdowns of net earnings were estimated by allocating tax payments to each distributional class (in any year) in proportion to the class's ratio of earned to total income in that year. The assumption implicit in this procedure is that earned and investment income are taxed equally, an assumption which has only strictly been true in the UK since 1984 (setting aside the distributional effects of the composite rate tax, abolished in April 1991). However, this assumption seems to be a fairly good one: adjustments for years prior to 1984, to reflect the relatively higher taxation of investment income, were not deemed to be worthwhile. A few examples using the pre-1973 system with earned income and small income reliefs bore this out – the amendments made to the 'proportions' defined above are very small, and are themselves subject to caveats. Furthermore, coping with the investment income surcharge system which applied between 1973 and 1984 requires making assumptions about income ranges which are not judged to be secure enough to merit confidence in the improved accuracy they might provide. Finally, the estimated tax payed on earned income was then deducted from each class of pre-tax earned income to obtain the estimated distribution of post-tax earnings.

The above procedure produced the required two vectors for each year. Gamma distributions were then fitted to these data for each year, and the parameters of the distributions were estimated. The fitting of the gamma distribution involved minimising the sum of squared residuals between the observed number of members for each earnings class, and the 'predicted' number of members arising from the functional form of the gamma distribution for some estimate of its two parameters (α, μ). The Hooke and Jeeves (1961) pattern search algorithm was used to search the parameter space for the $(\hat{\alpha}, \hat{\mu})$ which minimised the sum of squared residuals[4]. The Hooke and Jeeves method was chosen because of its robustness with respect to parameter 'starting positions'; it also has the desirable feature that it tends to successfully follow ridges in the parameter space[5]. The Hooke and Jeeves estimates always converged; furthermore, checking by perturbing the solutions seemed to suggest that these were located at global rather than local minima.

The results of fitting the post-tax SPI earnings distribution for the years 1964-1986 appear in Table 5.1. For each year's distribution, the following information is presented: estimates of the shape (α) and scale (μ) parameters; the associated (implied) Gini coefficient $G(\alpha)$; the residual sum of squares of the fit (rss); and the number of observations (n) used to estimate the two parameters. Comparing the α and $G(\alpha)$ values derived from the FES (above) with those presented in Table 5.1 (on page 95) illustrates the enormous differences between the two data-sets – and their poor quality and unreliability. They almost seem to describe two

[4] The Hooke and Jeeves algorithm was programmed by the author in the high-level programming language APL using APL★PLUS PC on a Commodore personal computer; programs are available from the author on request.

[5] See eg Goldfeld and Quandt (1972, pp.27–38). A more regularly used technique to fit the gamma distribution is the Newton-Raphson algorithm, which is used to solve the maximum likelihood equations. This technique was tried but rejected due to excess sensitivity to starting positions.

completely different earnings distributions. Hence the α estimates must be treated with considerable caution.

TABLE 5.1
THE SPI (GAMMA) EARNINGS DISTRIBUTION DATA ESTIMATES
AND CSO AGGREGATE EARNINGS SERIES

	$\hat{\alpha}$	$\hat{\mu}$	$G(\hat{\alpha})$	rss	n	Y (£m)
1964	0.592	105.478	0.603	0.020	28	19625
1965	0.739	51.223	0.559	0.017	25	20400
1966	0.798	41.055	0.544	0.017	25	20871
1967	0.862	32.954	0.529	0.018	25	21181
1968	0.921	27.594	0.516	0.019	25	21471
1969	0.947	24.417	0.511	0.022	24	21767
1970	0.636	59.989	0.589	0.018	24	22907
1971	0.831	31.431	0.536	0.021	24	23018
1972	0.526	20.311	0.627	0.032	17	25315
1973	0.843	12.160	0.533	0.041	17	27977
1974	1.266	7.143	0.455	0.050	17	27875
1975	3.648	1.433	0.285	0.049	17	27684
1976	1.255	7.247	0.457	0.016	17	27771
1977	1.702	5.053	0.402	0.014	17	26679
1978	2.232	3.683	0.357	0.010	17	28180
1979	2.338	3.991	0.350	0.006	17	28675
1980	-	-	-	-	-	28269
1981	-	-	-	-	-	27787
1982	9.519	1.015	0.181	0.004	17	27005
1983	10.176	1.009	0.175	0.005	17	27846
1984	11.705	0.896	0.163	0.007	16	28467
1985	12.792	0.884	0.156	0.014	16	28859
1986	12.509	0.969	0.158	0.022	16	29739

Notes. Distributional data for 1980 and 1981 are unavailable due to a Civil Service strike.

5.1.3 The Inheritance Data

This series proved the most difficult to construct. Two measures of inheritance were used for I_t, the first being a direct but cumbersome method of adding up past inheritances. This method utilises many tenuous assumptions and so the measure based on it is at best of highly questionable accuracy. The second, and we believe more reliable, measure is a total net wealth proxy for I_t. However, there are problems with this measure too, and we discuss these together with the data sources.

We start by describing the direct 'adding up' measure. This measure utilised Inland Revenue inheritance data combined with a battery of simplifying assumptions. Recall from section 3.4 that I_t is the sum of inheritances received *or to be received* by every living adult at time t. This therefore includes adults (of economic age less than η) who are still to come into their bequest, yet who form intergenerational consumption plans taking their future inheritances into account. An immediate practical problem becomes apparent if $\eta > 0$, since data on inheritances received in future periods ($t+1$, $t+2$, ...) will be needed to construct I_t for any t. Such data obviously does not exist, and so we are forced to adopt a 'second best' strategy for estimating this measure.

Two approaches for estimating an approximate series stand out. One involves collecting survey data on intentions to bequeath, factoring the results up to the macro level, and then adding them to historical macro inheritance data. Although this approach is closest to the spirit of the IGH, this author is unaware of any UK survey of this type, and so a second approach is followed. This entails assuming that $\eta = 0$, ie that individuals inherit when they reach adulthood. Recourse to this assumption obviates the need to consult future inheritance data: all that is needed is data on current aggregate inheritance and a vector of inheritance aggregates stretching back $T-1$ periods into the past (recall that T is adult longevity). Past inheritance data is required because adults alive at t include all those reaching adulthood from period $t-T+1$ up to t. The 'adding up' estimate of I_t is just the sum of all these inheritances.

An immediate problem with the assumption of $\eta = 0$ is that it is flatly contradicted by Blomquist's (1979) evidence that people tend to inherit at around human age 50. It also implies unrealistic childbirth-timing decisions: if individuals are of economic age zero when they inherit, then with a childhood of 20 years and a human lifespan of 70 years, (all) parents are aged 50 when they give birth. Hence the assumption is deeply flawed. However, ignoring this for the moment, consider the case when parents bequeath before they die, ie $\sigma < T$. Then appropriate inheritance data will look at *inter vivos* transfers only. That is, estate data will be inappropriate. However, the scope for measurement error with *inter vivos* bequest transfer data is quite substantial, and no sufficiently detailed or reliable series currently exists in the UK. Therefore we make an additional simplifying assumption, that $\sigma = T$, so that bequests are terminal and estate data is appropriate. The reliability and length of the inheritance series in the UK makes this assumption particularly convenient. The Inland Revenue has collected estate data since the 1894 Finance Act, and it is readily available in Inland Revenue Commissioner's Reports and *Inland Revenue Statistics.* Taking $T = 51$[6], and noting that the sample starts in 1964, inheritance data was needed for as far back as 1914. Total inheritance in a year was defined as the total net capital value of estates passing at death in that year[7]; the values for 1964-1986 were deflated using Patterson's series, and earlier data were deflated by the old 'Cost of Living' index suitably adjusted to splice with Patterson's series. Nominal inheritance data were interpolated for the years 1914-1919.

The resulting measure of I_t is therefore an attempt at direct estimation, but it suffers from obvious shortcomings. The assumptions used to construct it – including people inheriting when they reach adulthood and the non-existence of *inter-vivos* transfers – are almost certainly seriously wrong. Consequently, the constructed series is probably of marginal usefulness. We therefore considered a second possible measure of I_t which made no such unrealistic assumptions. An appropriate proxy was felt to be total net wealth, since it contains both bequests made in the past, and the wherewithall to provide for bequests to be made in the future, as required. Therefore, net wealth is likely to move in the same direction as

[6] If 'childhood lasts for 15-20 years, T=51 implies a lifespan of about 66-71, which seems quite reasonable.

[7] Incomplete coverage of the raw data should be noted: until the 104^{th} Commissioner's Report, estates whose value was less than the then exemption limit were not analysed. Other problems include: prior to 1974-75 the data relate to Great Britain not the United Kingdom; and the fact that data are in financial not calendar year terms.

the stock of inheritances received or be received. The recent increase in popularity of *inter vivos* transfers in order to minimise tax loss will not distort this measure because the wealth out of which these transfers are made, or will be made (in the case of living adults waiting for their bequest), is fixed regardless. However, the direct 'adding up' measure will fail to register this trend, and so will be biased downwards, an especially important point to bear in mind when data from recent years is being used (see Figure 5.2 in section 5.2).

Total net wealth estimates are readily available from the personal sector balance sheets published in the Blue Book and *Financial Statistics*: these are annual and relate to stocks held at calendar year end. Apart from questions of accuracy which must beset any official estimate of wealth[8], there is an important reason to treat the series with caution. This relates to the closeness of wealth as a proxy for the 'true' total inheritance, I_t. Specifically, total wealth includes life-cycle savings, which should ideally be separated from wealth accumulated by inheritance. Of course, the published series do not do this.

[8] As testimony to this, the wealth series is constantly being revised.

5.2 Empirical Results

This section reports empirical results arising from the estimation and testing of the IGH consumption function

$$C_t = \beta_0 + \beta_1 I_t + \beta_2 Y_t + \beta_3 Y_t^{\varepsilon} \Psi(\alpha_t, \varepsilon). \tag{3.29}$$

After estimation of its parameters is undertaken, the function is compared to two rival specifications, the Euler equation model of Hall (1978) and the wealth-augmented Euler equation model of Molana (1991)[9]. The object of this exercise is to determine which model (if any) is consistent with the UK data described in the previous section.

The time series of aggregate consumption and labour earnings are graphed for the 1964–1986 sample in Figure 5.1. The two time series designed to measure I_t are graphed in Figure 5.2: INHDIV3 and WDIV3 are the 'direct' and proxy measures described in the previous section, divided by a factor of three for ease of graphical presentation. It is apparent from Figure 5.1 that there is little obvious non-linearity in the relationship between consumption and earnings. However, earnings do not seem to track consumption very closely over this sample. From Figure 5.2, it would appear that wealth rather than the direct measure of I_t tracks consumption the closest. This is as we would expect, given the discussion about the measures contained in sub-section 5.1.3. An implication of this is that we should expect the empirical work to favour the wealth over the direct measure in terms of successful econometric performance. These points should be borne in mind as the econometric results of this chapter are presented.

The fact that (3.29) is non-linear raises several immediate problems. What is the most appropriate estimation technique? What are the properties of the chosen estimator, including small sample behaviour? How sensitive to starting positions is the chosen estimator? The commonest approach to dealing with a function like (3.29) is to assume an identically and independently distributed error term, and to choose the non-linear least squares (NLLS) estimator. As its name suggests, the NLLS estimator attempts to find the coefficient estimates which are consistent

[9] See sections 2.1 and 2.3 respectively for more details about these models.

with minimisation of the sum of squared residuals (ssr) of the model. The word 'attempts' is used because unlike OLS, there is no neat analytical formula with NLLS for calculating the optimum coefficient estimates. Minimisation of the sum of squared residuals involves using some iterative, or direct search, routine which keeps adjusting coefficient estimates until it is no longer possible to reduce the ssr[10]. One of the perils of non-linear estimation is that there is never any guarantee that global, rather than local, optima are reached by a search routine. Related to this is common problem that the 'solution' to a search program can often be highly dependent on the starting values of the search. Lack of convergence of the ssr in applied work often suggests some oddity of the function being estimated.

The NLLS estimator, while probably the best available, does not possess particularly encouraging small sample properties. Unlike the OLS estimator, which is always unbiased (but only asymptotically efficient), the NLLS estimator is only asymptotically unbiased; and test statistics with known distributions for diagnostic checking are limited in number[11]. These are important caveats which any applied researcher undertaking non-linear estimation should be aware of.

We turn now to the sample period used for estimation of (3.29). Recall from the previous chapter that the source of raw data for the distribution parameter α changed in 1975. So pronounced was the effect of this change on α (see sub-section 5.1.2), that we thought it advisable to consider four separate sample periods. The first is 1964–1974, for which the SPI (taxpayer) data was utilised. The second is 1975–1986, based on the FES data source. The third sample combines both sub-samples, despite the discontinuity; and the fourth uses just SPI data for the longest available period 1964–1986, with 1980 and 1981 omitted. Corresponding to each of these samples are the econometric results presented in columns 1 to 4 respectively of Table 5.2 and 6 to 9 of Table 5.3. Table 5.2 describes results using the direct inheritance measure; Table 5.3 presents results using the wealth measure instead. Column 5 of Table 5.2 and column 10 of Table 5.3 are regressions which do not use any of the distributional data. The idea here was to try to detect non-linearity whilst avoiding the distortions caused by the error-prone distributional data. Results for all 10 regressions were generated using TSP version 4.1B.

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10 In actual fact, algorithms invariably reduce the ssr to within some given accuracy scalar.

11 See eg Amemiya (1983).

Figure 5.1
Consumption and Aggregate Earnings

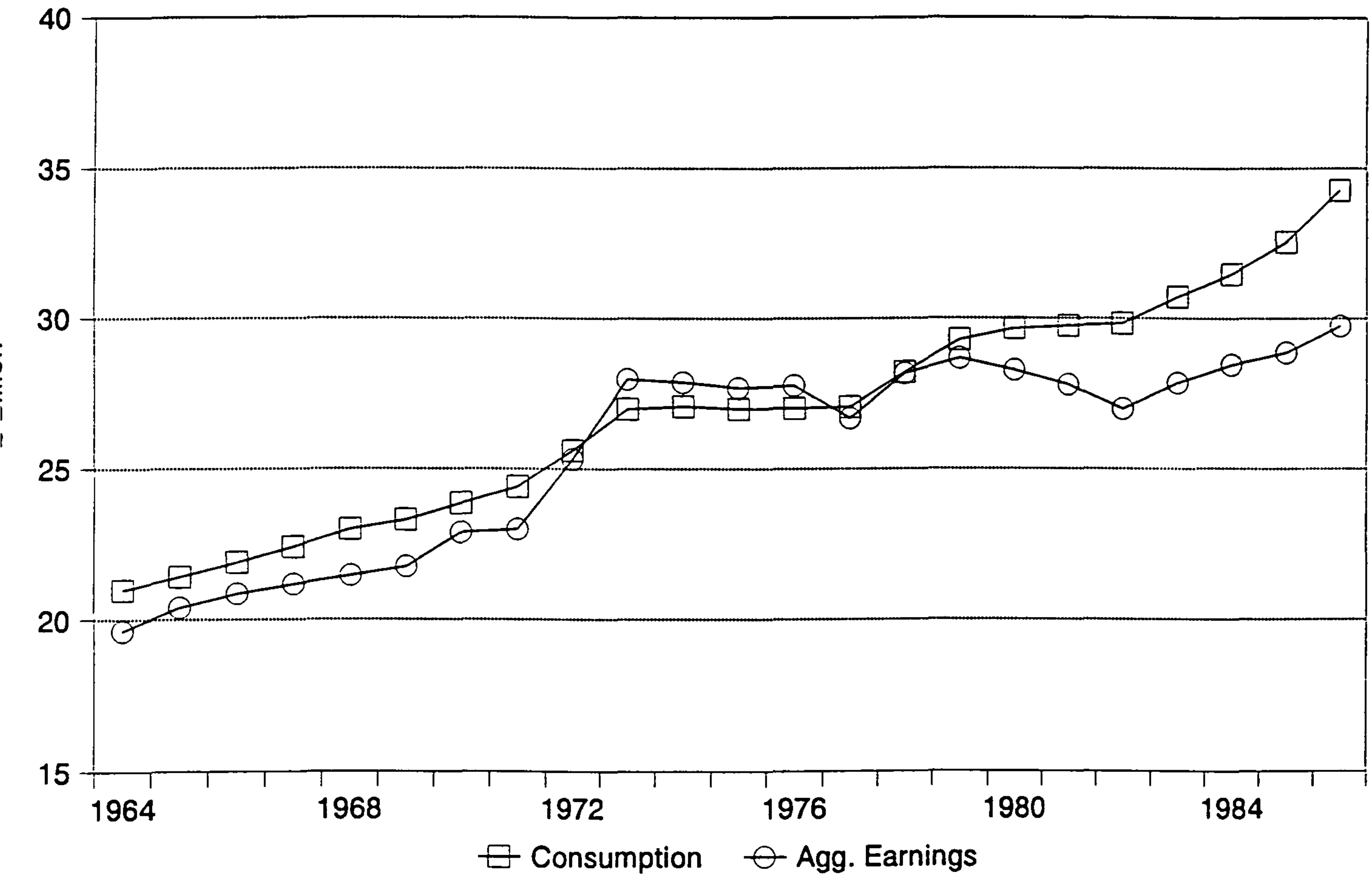
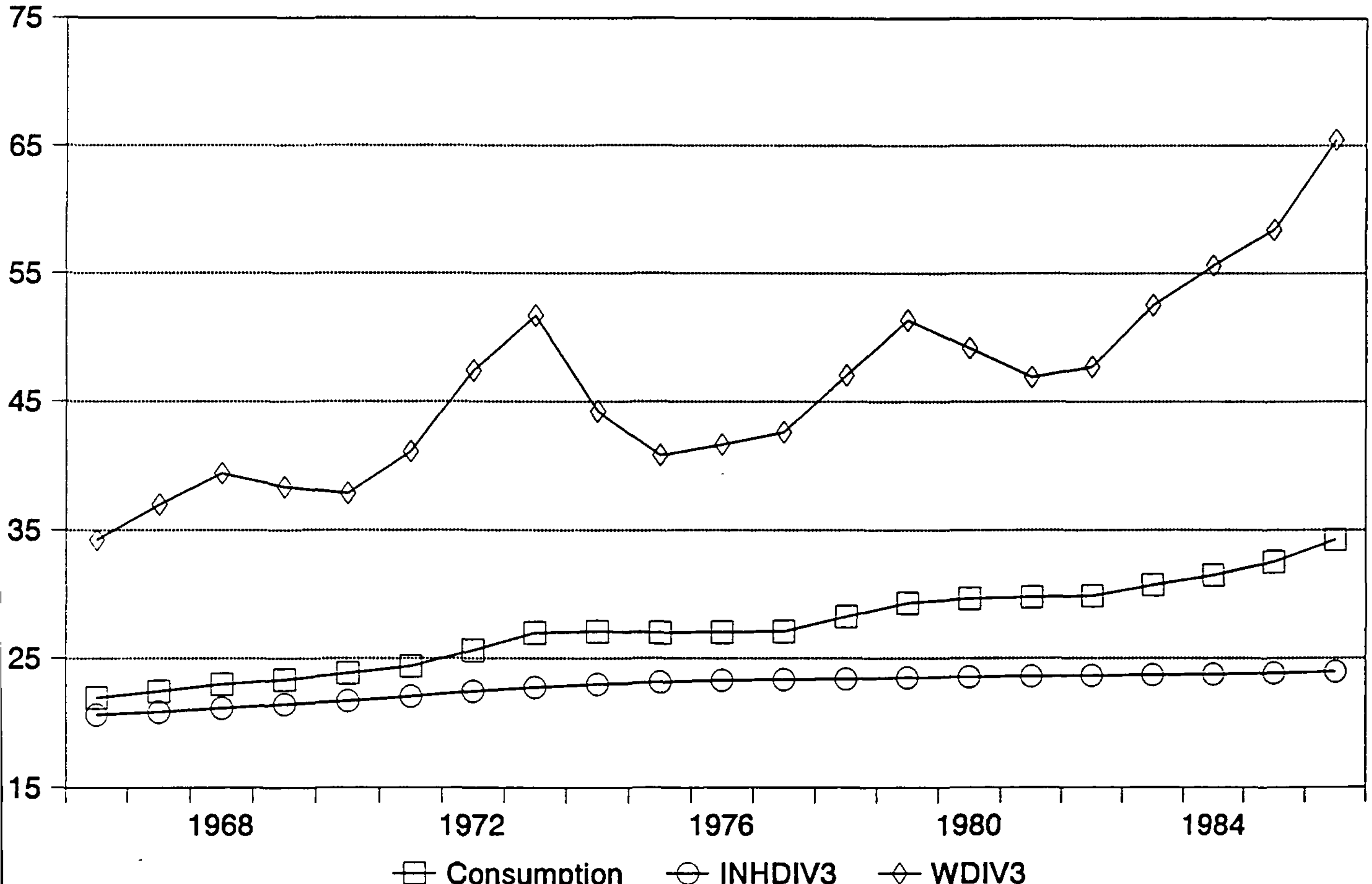


Figure 5.2
Consumption, INHDIV3, WDIV3



The results contained in the tables are uniformly disappointing for the IGH, although the validity of the results is highly questionable given the likelihood of small sample biases of the NLLS estimator. This caveat is especially important for the sub-samples, ie the first two columns of each table. There is very little difference between the tables, and so the following comments should suffice to summarise the essential features of both. The non-linear term is insignificantly different from zero in all of these regressions: the highest t-statistic on the non-linear term is around 0.7. The poor Durbin-Watson statistics indicate pervasive, and in some cases, severe mis-specification problems. Hence these regressions are probably worthless for practical policy purposes. They are also useless for inferring bequest hypothesis applicability and speed of 'regression to the mean' (see chapter 4).

The limited variation in the FES α estimates probably accounts for why both regressions (2) and (5) (and (7) and (10)) indicate zero non-linearity, where the latter holds α perfectly constant in order to investigate whether any residual non-linearity exists. These results bear out the implication of Figure 5.1, that the relationship between consumption and aggregate earnings is linear in this sample.

TABLE 5.2

THE COEFFICIENT ESTIMATES OF EQUATION (3.29)

('Direct' Inheritance measure)

Dependent Variable: C_t

	(1) 1964–1974 (SPI)	(2) 1975–1986 (FES)	(3) 1964–1986 (FES & SPI)	(4) 1964–1986† (SPI)	(5) 1964–1986†* (none)
β_0	-12706.950	-167576.400	-41040.530	-41544.840	-36193.10
	(2513.46)	(13635.16)	(16094.07)	(76051.68)	(13704.88)
β_1	0.459	2.507	1.045	1.040	0.909
	(0.06)	(0.23)	(0.41)	(1.87)	(0.34)
β_2	0.344	0.716	1.473	1.316	0.056
	(0.31)	(0.21)	(1.84)	(7.21)	(0.37)
β_3	-0.007	0	-3.202	-2.264	0
	(0.19)	(0)	(4.57)	(15.24)	(0)
ε	1.180	-	0.933	0.939	-
	(1.86)		(0.07)	(0.37)	
–LL	66.60	87.11	195.12	209.34	195.52
se	139.63	450.10	1322.30	5917.88	1345.32
DW	1.16	1.43	0.25	0.01	0.19

† Years 1980 and 1981 omitted.

* Treating the α_t as a constant.

Notes. Method of estimation: NLLS. Estimated standard errors are given in parentheses beneath coefficients; LL is the log-likelihood function; and se is the standard error of the regression.

TABLE 5.3
THE COEFFICIENT ESTIMATES OF EQUATION (3.29)
(Wealth measure)

Dependent Variable: C_t

	(6) 1966–1974 (SPI)	(7) 1975–1986 (FES)	(8) 1966–1986 (FES & SPI)	(9) 1966–1986† (SPI)	(10) 1966–1986†* (none)
β_0	7131.206	26124.610	3269.949	3328.873	7346.50
	(2968.03)	(9013.61)	(2062.58)	(2015.16)	(3823150)
β_1	0.016	0.117	0.090	0.089	0.016
	(0.02)	(0.01)	(0.01)	(0.02)	(0.04)
β_2	0.115	-0.495	0.825	1.902	-0.545
	(0.84)	(0.38)	(1.32)	(1.24)	(47908)
β_3	2.426	0	-0.019	-1.295	2.126
	(5.07)	(0)	(0.26)	(1.82)	(34862)
ε	0.854	-	1.282	1.010	0.946
	(0.15)		(0.97)	(0.06)	(2201)
–LL	63.14	90.35	168.05	154.26	184.92
se	404.07	589.76	828.35	946.33	1849.85
DW	0.90	1.16	1.00	1.50	0.08

† Years 1980 and 1981 omitted.

* Treating the α_t as a constant.

Notes. Method of estimation: NLLS.

Whilst treating the NLLS results with great caution, the fact that we cannot identify any significant non-linearity in these regressions prompts us to ask: how does a linear version of the IGH perform (ie with the non-linear term excluded)? Specifically, since at least some of the variables in the function seem to be trended, do the variables C, I, and Y cointegrate? If the linear special case of the IGH is data-coherent, this would support the BH rather than the non-linear form of the DH. The principles of cointegration were briefly described in Section 2.3 of chapter 2 earlier, when some recent empirical work on the UK consumption function was discussed. As stated there, the first task when testing for cointegration is to establish the time series properties of the data. The tests of the principal variables for I(0) and I(1) characteristics are reported in Table 5.4 below. Just the ADF(p) statistic is reported, since it is more powerful than the DF: p, the number of lags, is determined by the data. Clearly, p=0 corresponds to the DF statistic.

TABLE 5.4
TESTS FOR STATIONARITY

	I(0)		I(1)	
	ADF(p)	p	ADF(p)	p
C	0.83	2	-3.07*	3
W	0.64	2	-3.79*	1
Y	-1.40	0	-3.63*	0
I_D	-3.45*	1		

Notes:

Annual data 1964–1986 for all variables, except W, for which the sample is 1966–1986.

Critical values are –1.95; an asterisk indicates stationarity after differencing d times.

Here, I_D denotes the 'direct' measure of I, and W wealth (the alternative measure of I). Asterisks indicate that the 'alternative hypothesis' of I(d) is accepted by the data. Hence consumption, income, and the wealth measure all appear to be I(1) variables, whilst the inheritance measure appears to be I(0). The intuitive meaning of this can be understood by referring back to Figures 5.1 and 5.2, where all variables except I_D were seen to trend definitely upwards over time, though with a fairly constant rate of change.

The implication of all this is that only Y and W may potentially cointegrate with C: it is impossible for I_D to be a member of the cointegrating vector. This is not altogether surprising given our discussion in sub-section 5.1.3 of section 5.1: I_D suffers from serious measurement problems, including omission of lifetime inheritance transfers. Since such transfers have become increasingly popular in recent years, this may well explain why the I_D measure displays I(0) rather than I(1) characteristics (we would expect the true series to be I(1), like wealth – see the bequest hypotheses of section 3.3). On the basis of this reasoning, we interpret the above result as evidence that I_D is too poor a measure to warrant further empirical use. Thus we do not utilise I_t any further in this thesis: only the alternative measure W will be used hereafter.

We now proceed to test whether C, Y and W cointegrate, as the IGH under the Bevan Hypothesis (BH) predicts. Table 5.5 on page 109 contains the results of the cointegration exercise. Three regressions have been run, all based on the linear version of the IGH function

$$C_t = \beta_0 + \beta_1 W_t + \beta_2 Y_t \,.$$

Column (11) in the table is for the completely unrestricted case; (12) reports the special case where β_2 is constrained to zero – this is a test in levels of Molana's (1991) model (originally formulated in logs). Finally, regression (13) is for the special case where β_1 is constrained to zero – this is a test of the assumed long run relationship underpinning the ECM.

Critical values for N-variable cointegrating regressions for a sample size of n=25 are not published; however, using those of Engle and Yoo (1987) for n=50 give the following 5% critical values. For regression (11), tabulated DF is -4.11; ADF(4) is -3.75; and tabulated CRDW is unavailable. For regressions (12) and (13), tabulated DF is -3.67; ADF(4) is -3.29; and CRDW is either 0.78 or 1.03, depending on the degree of underlying serial correlation. Hence the results in Table 5.5 indicate that neither wealth nor income, nor both together, cointegrate with consumption (there is a possibility in (11), however, that an undetected cointegrating vector may exist; but we do not test for this here). On the basis of this evidence we reject the IGH, Molana's generalisation of Hall's REPIH in levels, and the ECM approach which suggests a stable 'long run' relationship between consumption and income. The latter two results are not very surprising: similar results have been obtained by other authors, as shown in section 2.3 of chapter 2. However, the failure of the pure IGH, is a new and, we believe, important result deserving further investigation.

Our route of investigation involves going back to one of the more tenuous IGH auxiliary assumptions, and modifying it. However, before we follow this line of enquiry, we first find it interesting to assess the performance of the pure REPIH of Hall. We therefore estimated the simple consumption autoregression $C_t = \gamma.C_{t-1} + u_t$, where γ is a constant and u is an error term. If the REPIH were correct, the autoregression would fit the data well, be well specified, and fail to admit other regressors on the right-hand side as signficant explanatory variables. Regression (14) in Table 5.6 reports the results using our data-set.

Column (14) of Table 5.6 shows quite clearly that the pure REPIH is rejected by the data. There is marked serial correlation and heteroschedasticity in the residuals, as evidenced by failures of the h, LM and H statistics at 5% significance. Furthermore, a quick glance at the other regressions presented in this table shows that the REPIH fails simple exclusion tests.

Koskela and Viren (1984) have observed that serial correlation may be caused in REPIH regressions by breakdowns in the auxiliary assumptions, eg imperfect labour market information and constraints, real interest rate variability, and non-separable consumption-leisure preferences. However, given extensive evidence about the limited responsibility of these assumptions for REPIH test failure (see section 2.1), it is possible that there is a different reason for this failure. We devote the remainder of this chapter to considering two candidates. Both hypothesise that population heterogeneity exists; the difference between them relates to the type of presumed heterogeneity.

TABLE 5.5

TESTS FOR COINTEGRATION

Dependent Variable: C_t

	(11)	(12)	(13)
β_0	2095.7	8566.8	250.273
	(2027.6)	(1833.6)	(2448.0)
β_1	0.083	0.136	
	(0.02)	(0.01)	
β_2	0.527		1.039
	(0.12)		(0.09)
R^2	0.93	0.85	0.85
se	992.11	1364.50	1475.60
BJ(2)	7.68*	0.76	0.33
CRDW	0.73	0.63	0.20
ADF(p)	-2.85	-2.94	0.07
p	1	1	0

Notes. Method of estimation: OLS. Annual data 1966–1986 except regression (13), which is 1964–1986. BJ(·) is the Jarque-Bera statistic (distributed as a χ^2 variate) which tests normality of the residuals; an asterisk indicates non-normality. To give a rough idea of the contribution of coefficients to the regressions, standard errors are given in parentheses; however, t-ratios are not distributed as a Student's t, and so conventional hypothesis testing cannot be undertaken. CRDW is the cointegrating Durbin-Watson statistic, another test of stationary residuals.

TABLE 5.6

REPIH AND HYBRID REGRESSIONS

	(14)	(15)	(16)	(17)	(18)
$\beta_0^\dagger$			659.864	10383.500	
			(368.36)	(10908.80)	
$\gamma^\dagger$	1.023	0.996	0.780	0.778	0.778
	(0.004)	(0.08)	(0.03)	(0.03)	(0.03)
$\beta_1^\dagger$			0.039	0.038	0.038
			(0.003)	(0.004)	(0.004)
$\beta_2^\dagger$		0.028	0.016	-4.356	22.091
		(0.08)	(0.03)	(4.90)	(22.50)
ζ_1				0.394	-4.331
				(0.44)	(4.45)
ζ_2					0.213
					(0.22)
R^2	0.982	0.982	0.998	0.998	0.998
se	482.00	492.42	177.71	178.79	178.62
h	2.17*	2.50*	0.32	0.26	0.25
LM(1)	3.69*	4.33*	0.02	0.04	0.03
RESET(1)	0.24	0.55	1.48	0.50	0.47
BJ(2)	0.90	0.69	5.84	5.17	5.14
H	5.63*	6.82*	0.14	0.12	0.12

† For regressions (14) and (15), $\beta_0^\dagger$ is β_0; $\gamma^\dagger$ is γ; and $\beta_2^\dagger$ is β_2 – all as before. For regressions (16), (17) and (18), $\beta_0^\dagger$, $\gamma^\dagger$, $\beta_1^\dagger$ and $\beta_2^\dagger$ are functions of their respective non-† coefficients: these are defined later in the text.

Notes. Method of estimation: OLS. Annual data 1966–1986 except regressions (14) and (15), which are 1964–1986. h is Durbin's serial correlation statistic for regressions with a lagged dependent variable; LM(1) is the Lagrange Multiplier statistic for first order serial correlation (distributed as $\chi^2(1)$); RESET is Ramsey's test of functional form (distributed as $\chi^2(1)$); BJ is the Jarque and Bera test for normality of the residuals (distributed as $\chi^2(2)$); and H is an F-test for heteroschedasticity. An asterisk indicates that the regression fails the referenced diagnostic test.

In a popular modification of the REPIH, it is suggested that the population consists of the following two groups. In the first are people who obey the REPIH, and in the second, a minority who are 'Keynesian', or 'liquidity constrained', consumers. The latter do not separate their consumption and income streams in the usual life-cycle manner; instead, they passively consume their current income in each period. One reason for such behaviour may be the presence of liquidity constraints, which bind income and consumption paths together (Hall (1978), Flavin (1981), Hayashi (1982), Summers (1982), DeLong and Summers (1986), Campbell and Mankiw (1987, 1989); see also Appendix A). Hence the second group may be thought of as liquidity constrained. Modifying the REPIH to account for this form of population heterogeneity is quite straightforward: the usual representation is

$$C_t = \lambda_1 C_{t-1} + \lambda_2 Y_t + u_t \,,$$

where λ_2 measures the proportion of the population who are liquidity constrained. As stated in Appendix A, US evidence suggests that this proportion is at most 20%. Estimation of λ_2 under this liquidity constraint interpretation should involve use of an instrumental variables (IV) procedure, because we expect the error term u to be related to Y under this interpretation. If OLS is used instead, λ_2 will be a biased estimator of the proportion liquidity constrained. But in this thesis we are not interested in examining the liquidity constraint rationale behind the above equation: that is, we do not want to estimate λ_2 for these purposes. What we *are* interested in is the econometric performance of the augmented specification, in particular, whether such a model is well-specified (data coherent). Hence we use OLS to investigate this and therefore stress that the λ_2 coefficient should *not* be interpreted as a 'proportion liquidity constrained' parameter.

The results of estimating the above equation are presented as regression (15) in Table 5.6. The coefficients λ_1 and λ_2 correspond to $\gamma^\dagger$ and $\beta_2^\dagger$ in the table. A reasonable fit notwithstanding, the equation exhibits severe mis-specification problems: autocorrelation and heteroschedasticity. Hence we conclude that the REPIH-Keynesian heterogeneity rationale does not explain the data.

We can however suggest an alternative source of heterogeneity which relates directly to the IGH. This involves returning to the IGH auxiliary assumption about homogenous preferences, and modifying it to make it more realistic. Specifically, we suggest the following modification to assumptions 3 and 7 of section 3.1:

> *The population can broadly be divided into two groups, the first of which follow Hall's REPIH, and the second of which follows the IGH.*

This hypothesis is based on suggestions by some authors, including Kurz (1984, 1985) and Boskin and Lau (1984), that the majority of the population (say q) behave according to the REPIH, whilst the remainder, $1 - q$, behave intergenerationally. Kurz (1985) suggests that richer, dynastically-orientated, consumers are more likely to be intergenerational planners, while more modestly endowed individuals (the bottom 70% of the income distribution) are probably less obviously altruistic towards their offspring[12]. Another convenient fact which supports the implementation of a hybrid REPIH-IGH specification concerns the role of assumed uncertainty about the future in these theories. The IGH posits certainty about the future whereas the REPIH does not: this corresponds to the breakdown of the population into rich IGH consumers, who are likely to be more certain about their futures and who therefore find it easier to intergenerationally plan; and poorer REPIH consumers, who are likely to face greater uncertainty about the future and who may therefore be less able (and perhaps willing) to intergenerationally plan. In other words, the breakdown into richer and poorer sectors matches the certainty-uncertainty assumptions of the models relating to these sectors.

With a fraction q behaving according to the REPIH, and $(1 - q)$ behaving according to the IGH under the BH, we have the following component equations:

$$qC_t = q\gamma C_{t-1}$$
$$(1-q)C_t = (1-q)(\beta_0 + \beta_1 W_t + \beta_2 Y_t).$$

Hence aggregate consumption will be a simple linear regression of current consumption on lagged consumption, current income and the wealth measure, and a constant term. Notice that we expect smaller coefficients in this regression than we have been used to seeing hitherto, because the usual coefficients have now been

[12] Interestingly in this respect, Wolff (1990,p.151) has found from US cross-section data that families below the poverty line have hump-shaped asset holdings (which suggests the LCH), while those above it have continually increasing asset holdings as they age (which suggests the IGH).

multiplied by fractions (of the population). In particular, we expect the coefficient on lagged consumption to be considerably below unity, in contrast to the straightforward REPIH autoregression. The greater the degree of intergenerational altruism in the population, the lower q, and hence the lower the coefficient on lagged consumption. Notice also how this aggregate consumption regression encompasses the REPIH-Keynesian (liquidity constrained) hybrid model we considered a short while ago.

The estimates of this regression are listed as column (16) of Table 5.6. In this column, $\beta_0^\dagger = (1-q)\beta_0$; $\gamma^\dagger = q\gamma$; $\beta_1^\dagger = (1-q)\beta_1$; and $\beta_2^\dagger = (1-q)\beta_2$. This is a data-coherent model since every diagnostic test is passed[13]. Also, coefficient values are indeed lower than they have been hitherto. Interestingly, $\gamma^\dagger$ (the REPIH autocorrelation coefficient multiplied by q) is 0.78 – which is consistent with an independent conjecture of Kurz (1985) if we accept that the interest rate, r, and the discount rate, δ, should be very nearly equal under REPIH. The reason for this is that Hall's autoregression parameter γ is unity if $r = \delta$: hence we have q=0.78 (ie only about 22% of the population are intergenerational planners). Kurz conjectured that the top 30% of the income distribution belong to this category – and the UK figure is likely to be lower than this (Britain has proportionately fewer millionaires)[14]. An important feature of (16) is that neither the pure REPIH nor the pure IGH can be accepted as parsimonious special cases in this regression, since exclusion tests based on W (for REPIH) and C_{t-1} (for IGH) fail. Hence the data would seem to support a hybrid REPIH-IGH specification (under the Bevan Hypothesis), as hypothesised.

It is worth noting before we go on that exclusion test failure has formerly been rationalised by Hall (1978) (and others, including Nelson 1987) by appealing to the role of 'news', as proxied by the significant additional regressors in an exclusion test equation. The idea here is that the additional variables represent news whose effect on permanent income valuations was not apparent at $t-1$ and hence whose

[13] Although the test statistic for normality of the residuals is close to its critical value – but see later. The DF and ADF statistics for the residuals are –4.38, which suggests stationarity. However, we cannot be certain of this, because no tabulated critical values for cointegrating regressions with lagged dependent variables exist at the current time.

[14] These conclusions should however be tempered by realisation of the fact that the OLS estimator suffers from small sample bias in the presence of a lagged dependent variable.

effect on C_t could not be picked up by C_{t-1}. Hence the REPIH can justify the addition of such variables to the autoregression. Does this interpretation square with the results in (16)? The answer is almost certainly not, because the coefficient on C_{t-1} is too low in regression (16) (significantly lower than unity: $t^*=6.58$) to be consistent with this story. We expect Hall's γ to be close to unity in theory (and empirics bear this out), because (a) $\gamma = (1+r)/(1+\delta)$ and (b) consumers are only in equilibrium when their marginal rate of substitution of present for future consumption $(1+\delta)$ equals the relative price of future to current consumption $(1+r)$. That is, equilibrium implies $\gamma = 1$; so a value of γ=0.78 is plainly inconsistent with this theoretical condition. Furthermore, no plausible reason exists as to why making allowance for 'news' could produce such a dramatically low γ value anyway. We are therefore left with the conclusion that the only cause for the dramatic change in the observed constant is multiplication of Hall's γ by the fraction q. The latter, we have argued, takes on a very realistic value under the hybrid REPIH-IGH specification.

Before we conclude this section, we make one last effort to test for non-linearity in the consumption function. We do this because the small sample limitations of the NLLS estimator could be obscuring a 'true' non-linear functional form. The idea here is to construct a linear approximation to the non-linear term $\beta_3 Y^{\varepsilon}$ so that OLS estimation (which is unbiased in small samples) can be used[15]. In order to use an approximation, we effectively have to ignore the distributional parameter α (or assume it constant) as we did in regressions (5) and (10). We choose Taylor's expansion, which expands the function of interest around a given point. Since NLLS estimates of ε earlier suggested that ε is in the region of unity, we decided to expand the function around one. Taylor's expansion of a function $f(\varepsilon + \sigma)$ is

$$f(\varepsilon + \sigma) = f(\varepsilon) + \sigma f'(\varepsilon) + \frac{\sigma^2}{2} f''(\varepsilon) + \dots,$$

[15] I am grateful to Jonathon Rougier for this suggestion.

where in our case, $f(\varepsilon) = Y^{\varepsilon}$. Hence

$$f'(\varepsilon) = Y^{\varepsilon} \ln Y$$
$$f''(\varepsilon) = \ln Y (Y^{\varepsilon} \ln Y)$$
$$\dots$$

Expanding the function around one (ie putting $\varepsilon = 1$) yields

$$f(1+\sigma) = Y + \sigma(Y \ln Y) + \frac{\sigma^2}{2}\left(Y \ln^2 Y\right) + \dots,$$

ie $\varepsilon = 1 + \sigma$. Hence considering just the first three terms, Taylor's approximation to

$$C_t = (1-q)\beta_0 + q\gamma C_{t-1} + (1-q)[\beta_1 W_t + \beta_2 Y_t + \beta_3 Y_t^{\varepsilon}]$$

is

$$\begin{aligned} C_t = {} & (1-q)\beta_0 + q\gamma C_{t-1} + (1-q)\beta_1 W_t + (1-q)(\beta_2 + \beta_3) Y_t \\ & + (1-q)\beta_3 \sigma (Y_t \ln Y_t) + (1-q)\beta_3 \frac{\sigma^2}{2} (Y_t \ln^2 Y_t), \end{aligned}$$

which may be re-written as

$$C_t = \beta_0^{\dagger} + \gamma^{\dagger} C_{t-1} + \beta_1^{\dagger} W_t + \beta_2^{\dagger} Y_t + \zeta_1 (Y_t \ln Y_t) + \zeta_2 (Y_t \ln^2 Y_t), \qquad (5.1)$$

where

$$\begin{aligned} \beta_0^{\dagger} &= (1-q)\beta_0 \\ \gamma^{\dagger} &= q\gamma \\ \beta_1^{\dagger} &= (1-q)\beta_1 \\ \beta_2^{\dagger} &= (1-q)(\beta_2 + \beta_3) \\ \zeta_1 &= (1-q)\beta_3(\varepsilon - 1) \\ \zeta_2 &= (1-q)\beta_3 (\varepsilon - 1)^2/2 . \end{aligned}$$

We first estimated a version of (5.1) where only the first two terms of the Taylor expansion, Y and $Y_t \ln Y_t$, were added to regression (16). The results, which are given as regression (17) in Table 5.6, are encouraging. The t-statistic of ζ_1 is now 0.90, suggesting that slightly more non-linearity exists than is implied by Tables

5.2 and 5.3. Since β_3 is positive in this scenario (see below) and ε is less than unity, reference to Table 4.1 in section 4.1 tells us that this result is consistent with the Davies Hypothesis and a diminishing marginal propensity to consume. As mentioned in chapter 4, this is what we expected.

More important than this result, however, is the fact that the model specification improves relative to (16), with greater apparent normality of the residuals. Regression (18) in the same table includes the third term of the Taylor's expansion as well as the first two. Since there is a non-linear restriction on the ζ_1 and ζ_2 coefficients, a restricted least squares estimator should ideally be used here. However, since that would remove the desirable property of small sample unbiasedness which OLS enjoys, we decided to estimate the full version of (18) without imposing the restriction (we test for it afterwards). In any case, the purpose of running this regression is primarily to check on the accuracy of the two-term approximation. The fact that the difference between (17) and (18) is negligible suggests that the two-term approximation is a good one.

Parameters are very stable between (17) and (18) as expected, except for $\beta_2^\dagger$. The difference here primarily reflects the exclusion of a constant term in (18), which was removed in order to avoid collinearity. It also reflects the variability of β_3, since $\beta_2^\dagger$ is a function of this, as defined above.

In summary, the results seem to support the hybrid REPIH-IGH formulation, which is consistent with a population mix where most people are 'selfish' REPIH consumers and a rich minority are intergenerationally altruistic IGH consumers. There also seems to be a small element of non-linearity present, although a joint variable deletion test for ζ_1 and ζ_2 produced a likelihood ratio statistic of only 4.69 – which is lower than the critical value $\chi^2(2)_{5\%} = 5.99$ ([16]although not at 10% significance: $\chi^2(10)_{10\%} = 4.605$).

We conclude the discussion of the results by briefly testing the validity of two assumptions underpinning regressions (17) and (18). For these regressions, which use the Taylor expansion, we want to check (a) the choice of expanding around

[16] The F-version of this test produced a test statistic of only 2.00, which is lower than the critical value $F(2,16)_{5\%}$=3.63.

$\varepsilon = 1$, and (b) the non-linear restriction between ζ_1 and ζ_2. To check (a), note from regression (18) in Table 5.6 that we have

$$(1-q)(\beta_2+\beta_3) = 22.091$$

$$(1-q)\beta_3(\varepsilon-1) = -4.3306$$

$$(1-q)\beta_3(\varepsilon-1)^2/2 = 0.2127\,.$$

There are three equations in three unknowns: since these equations are linear, there is exact identification. Solving the last two yields $\hat{\varepsilon} = 0.902$, which is fairly close to unity as required. The fact that we chose a point of expansion close to the true point explains why our second-order approximation (17) was so accurate: the closer the chosen point is to the 'true' point, the fewer terms in the expansion are needed to attain any given level of accuracy.

To check (b), we solve the equations above for $(1-q)\beta_3 = 44.190$. This gives a 'theoretical value' of the third equation above of 0.2122 – which compares with the 'actual' ζ_2 value of 0.2127. Therefore, the restriction between the two ζ coefficients holds almost exactly, as required.

Although the results of the hybrid REPIH-IGH specification seem promising, we believe that more work is needed before we can advocate this hypothesis with the confidence we would like. It should not be forgotten that we have been using proxies for I_t – and the wealth proxy, while probably better than our constructed Inland Revenue measure, is not perfect. In particular, the usefulness of this proxy is weakened by its wrongful inclusion of lifetime saving. We conclude by making a plea for better data generally (including that relating to the earnings distribution), and further testing of the hybrid REPIH-IGH model. It is open to question whether any better data can be found in the UK at the present time.

Chapter VI

Conclusions

This thesis presents a new theory of the consumption function called the IGH. This theory is built up from the hypothesis that individuals are altruistic utility maximisers. The foundations of the IGH were shown in chapter 3 to possess two distinct strands: the choice by individuals of the optimum bequest to be left to their offspring; and the allocation over their own (mortal) lifetimes of the remaining resources. When these two strands are brought together, it was demonstrated that a tractable consumption function at the individual level may be derived, although the bequest decision will in general embody elements of non-linearity, the latter property carrying over into the consumption function. Consistent aggregation of this function over all living agents also preserves the non-linearity of the consumption relationship and furthermore introduces a new distributional term into the consumption function. The greater part of chapter 4 was devoted to examining some implications for redistribution policy arising from this feature of the IGH: some comments on the limitations of this policy were also mentioned. Despite potentially serious limitations, UK data were used, in chapter 5, to estimate the aggregate IGH consumption function. The results did not support the IGH in its basic form; but nor did it support rival specifications, including those of Hall's (1978) REPIH; Molana's (1991) wealth-augmented REPIH; and a hybrid REPIH-Keynesian (liquidity constrained) model. We then proceeded to test whether an auxiliary assumption underpinning the IGH was at fault. Specifically, we examined whether relaxing the assumption that everyone is intergenerationally altruistic was able to produce a modified model consistent with the data. The conclusion reached in the previous chapter was that a modified model where a proportion of the population behave according to the 'selfish' Hall theory, and where the remainder behave according to the intergenerationally altruistic IGH, was supported by the data. The model passed all of its diagnostic tests and produced realistic parameter values.

Interestingly, the data could not identify a significant role for the non-linear distributional term of the IGH function. A visual inspection of the time series of consumption and income suggests that this is a genuine description of the aggregate consumption characteristics of the UK economy, rather than any intrinsic failing in the earnings distribution data series. However, it is acknowledged that the shortcomings of the latter are serious and that better data are needed. Echoing the findings of van Doorn (1975), also in the case of the UK, the current data suggest that there is practically no scope for using earnings redistribution policy as a demand management tool in the UK at the present time (see Borooah and Sharpe 1986). The failure to establish a non-zero coefficient on the non-linear term in the IGH consumption function implies acceptance of Bevan's hypothesis about bequest formation, in which there is no role for distributional factors to affect aggregate consumption.

Of course, the above results were derived using a particular data-set, and there is no reason to suppose that the experience of other countries will replicate that reported for the UK. The principal reason for this – one emphasised in the chapter describing the empirical work – is the rather poor quality of some of the data used to estimate and test the IGH. The earnings distribution parameter estimates and the inheritance series are especially prone to error. However, in view of the coherent results attained using the wealth proxy for the latter, it would seem that the empirical results are not worthless. Indeed, the author would argue that the results are sufficiently well-founded for them to form the basis of future research.

However, a problem with the results which were derived is the smallness of the data set. The desire to use only theory-consistent consumption data – ie that relating to the flow of consumption services rather than consumer expenditure – meant that the sample ran from 1984–1986 only. The availability of only 23 observations for each series was especially serious when estimating equations using NLLS: as stated in section 5.2, this estimator is only *asymptotically* unbiased and efficient. Hence the failure to detect significant non-linearity in the NLLS equations could conceivably have been caused by bias in the estimates. However, further examination of the data using OLS in conjunction with Taylor's expansion, confirmed the conclusions following from the NLLS results.

We stress that despite the avowed data problems, we do not believe that the empirical results are useless. Caution only is urged. A danger with pointing out sample-based problems is that creators of fundamentally mis-specified models can take refuge in data-insufficiency 'explanations' of poor performance instead of tackling the real shortcomings of the models themselves. To prove that we are aware of this danger, we now proceed to consider some potentially serious theoretical problems with the IGH. Each suggested problem is discussed in turn, together with our thoughts about their likely individual importance. It will ultimately be seen that the problem we claim to be the most serious is consistent with the interpretation of the chapter 5 results that envisages a population split between those who follow the IGH, and those who follow the REPIH.

- **Binding intragenerational (lifetime) liquidity constraints.** Appendix A explains how the existence of binding liquidity constraints can cause actual consumption to diverge from desired consumption plans over individuals' lifetimes. This explanation has been used to account for the empirical shortcomings of a number of consumption functions in the past, especially that of the lifetime model of Hall[1] – does it also apply to the IGH? Apart from the fact that the precise importance of liquidity constraints is open to question (see Appendix A), there are at least two reasons to suppose that they would play less of a role in the IGH than in, say, the LCH. One reason is that individuals may be more interested in saving for bequests over their lifetimes than in borrowing (Deaton 1989). Hence they will be less susceptible to liquidity constraints which restrict consumption rather than saving. Another reason is that intergenerational transfers of wealth can be expected to at least partially offset the occurrence of binding constraints. For example, someone inheriting when young may be able to use their newly-found wealth to maintain actual consumption at the same level as desired consumption. Therefore, we doubt whether the IGH's neglect of this type of constraint will greatly jeopardise the IGH specification.

- **Binding intergenerational (dynastic) liquidity constraints.** It is assumed in the IGH that bequests will always be non-negative. This constraint

[1] See chapter 2, section 2.1.

is *assumed* rather than *imposed* in section 3.3 in order to simplify the consumer's optimisation problem. How accurate is this assumption likely to be? As Bevan and Stiglitz (1979) have pointed out, the assumption of positive bequests will only be correct if, in the absence of a bequest, children would be worse off than the parent. Of course, technical progress can be expected to reduce the frequency of cases where this applies; and in model simulations, Bevan (1979) and Bevan and Stiglitz found that negative bequests would be widespread if they were permissible. Although these points do not apply to the Laitner hypothesis of bequest behaviour (because precise information about childrens' lifetime earnings is unavailable), they do apply to the Bevan and Davies hypotheses. It is easy to see in these latter hypotheses how those with low lifetime wealth but high-earning children would want to leave negative bequests (see section 3.3). At higher levels of lifetime wealth for any fixed lifetime wealth of the offspring, bequests become positive and increase monotonically.

But offsetting the tendency of technical progress to increase the likelihood of individuals wishing to leave a negative bequest, is the effect of inheritance playing a smaller role in individuals' overall lifetime wealth as the population grows. The idea here is that the lifetime wealth of children can remain smaller than that of parents if smaller inheritances offset larger incomes arising from technical progress. Hence positive bequests may still be made, despite technical progress. Yet despite the importance of total inheritance as a component of aggregate lifetime wealth (Menchik 1980), the majority of individuals do not seem to receive an inheritance of material wealth; furthermore, bequests seem to be predominantly the preserve of the rich (Lansing and Sonquist (1969), Blomquist (1979), Menchik 1980). Unfortunately, the possibility that individuals are at bequest corner solutions may have serious implications for the IGH consumption function: continuity could be destroyed, and IGH parameters could be susceptible to mis-specification bias. Parameter bias may arise, for example, if the incidence of corner solutions is negatively related to the size of lifetime resources – which seems a reasonable proposition.

- **The neglect of other forms of transfers**. The IGH assumes that all individuals are altruistic, and it restricts manifestations of altruism to offspring only. Could this be an important cause of IGH mis-specification bias? Evidence on the existence of intergenerational altruism is mixed. For example, Hurd (1986) found that parents decumulated assets at the same rate as they aged, regardless of whether they had children or not. This contradicts the hypothesis of the IGH, which suggests that parents with children would decumulate assets slower, in order to leave a bequest. Furthermore, Cox and Raines (1985) have cited evidence emphasising the importance of *inter vivos* transfers between unrelated adults; Kurz (1985) independently found that at least 40% of intergenerational transfers went outside the extended family. However, other studies have supported the notion of intergenerational planning. For example, Friedman and Spivak (1986) have discovered that people with more children buy fewer annuities (although this result could be explained simply by poorer, less well-informed members of the population being more fertile). In addition, casual anecdotal evidence supports the notion that rich families are more concerned about passing wealth on to their heirs. Hence the evidence about the veracity of the intergenerational altruism assumption is inconclusive.

This should not of itself cast doubt on the hybrid REPIH-IGH specification, however, which is consistent with some individuals being selfish and others altruists. Therefore this model should be immune to the above criticisms. However, transfers of material wealth are only form of admissible transfer. As Davies (1982, p.476, footnote 4) has pointed out, intergenerational models invariably focus either on material transfers between generations (ie bequests) or on non-material transfers (eg human capital, family values). The IGH has been true to this pattern, modelling bequests but omitting a rigorous treatment of human capital. It cannot be doubted that the neglect of human capital is a serious shortcoming of the IGH: investment in human capital may constitute the only transfers made by the poor. The argument here is that parents will only augment human capital investment with material transfers when the rate of return on the former (which starts out very high) falls to the point where bequest-making becomes optimal. Although some elements of human capital and family background factors enter the IGH via the lifetime earnings 'rules' of the Davies and Bevan bequest functions, the treatment of human capital here is clearly superficial. The impact of this on the empirical performance of the IGH consumption function is hard to evaluate. As in the case of binding dynastic liquidity constraints, the inclusion of human capital investment decisions may induce a discontinuity in the consumption function. A continuous model such as the IGH might then be vulnerable to some mis-specification bias. Furthermore, since the IGH component of a hybrid REPIH-IGH specification would then be vulnerable to bias, the whole specification would be similarly afflicted. However, the evidence presented in the previous chapter detected no such mis-specification problems with the hybrid formulation. This casts doubt on the force of the above objections.

- **Omission of the distribution of aggregate inheritance**. A case can be made for both the DH and the LH bequest functions to contain a non-linear inheritance term. Concerning the DH, sub-section 3.3.2 of section 3.3 points out that lifetime earnings may be related to parental lifetime *incomes* (not earnings) in the intergenerational earnings rule, if family background effects are strong. Income may consist in part of the return on wealth, and wealth may include inherited wealth: hence there is a possible role of inheritance in the non-linear intergenerational earnings rule. The case for a non-linear inheritance term in the LH bequest function is stronger. Recall from sub-section 3.3.4 of section 3.3 that the bequest function (3.17) was only an approximation to Laitner's general form $k_G(w_{G-1})$. The approximation took $k_G(w_{G-1})$ to be a non-linear function of parental lifetime earnings, not parental lifetime earnings *and* inheritance (as it should have been).

In both cases, a non-linear inheritance term was omitted on the grounds of tractability. Given that consistent aggregation would have introduced a non-linear inheritance term into the aggregate IGH consumption function in just the same way as it introduced a non-linear earnings term, how serious is the omission of such a term? As far as the DH is concerned, the relation between childrens' and parents' earnings in the earnings rule was justified in sub-section 3.3.2 as a valid alternative to a relationship between childrens' earnings and parental incomes. The idea here was that an earnings link better conveyed the idea of genetic ability transmission than an earnings-income link. Therefore the omission of a non-linear inheritance term may not be very serious in the case of the DH. The same may be true for the LH in view of the fact that inherited wealth is a small component of lifetime wealth for most people – see the evidence referred to in sub-section 3.3.4. To conclude, we doubt whether the inclusion of a non-linear inheritance term would constitute much of an improvement, especially when the measurement problems encountered with the variable I_t are considered (see sub-section 5.1.3).

- **Irrationality**. If individuals are unable to even *subconsciously* formulate lifetime and intergenerational plans, then the IGH will not describe real-world consumption behaviour. It is important to distinguish irrationality, which implies the mis-use or non-use of information, from uncertainty and inadequate information. If it were known that all the necessary information for consumption optimisation existed with certainty, and if it were also observed that actual consumption was poorly predicted by the optimising model, then it would be sensible to start questioning the rationality predicate. However, when perfect lifetime information does not exist with certainty, it becomes impossible to assess the accuracy of the rationality predicate from the IGH results alone. Clues can only be found in studies which have tested rationality explicitly. As the discussion of Assumption 1 in section 3.1 indicates, these studies do not refute rationality, but nor do they provide overwhelming support for it; the question therefore remains open. However, we do not believe that altering the rationality assumption is a particularly promising route to follow. Any problems attributed to the IGH from this source must also be attributed to its theoretical rivals, eg the REPIH.

- **Imperfect information and uncertainty**. The assumptions of perfect lifetime information and certainty, possibly more than any other, do not seem to accord well with reality. And yet these assumptions are crucial ingredients of the IGH, which, if removed, open up all sorts of alternative patterns of inter- and intra-generational resource allocation. The implications of this for the IGH consumption function are serious indeed: the less people are able to plan ahead in their lifetimes, the less likely it is that they will behave in accordance with the IGH. In extreme cases, the IGH will become an inappropriate model: it will be mis-specified and unable to track actual consumption decisions with any degree of accuracy. In this sense, the stochastic selfish REPIH, which explicitly models uncertainty, enjoys an important advantage over the IGH. This accords with the final conclusions of chapter 5, that the population is split between IGH and REPIH consumers. Arguably those most prone to uncertainty about their future are those with the fewest resources, who are least able, and willing, to intergenerationally plan. These consumers could behave according to the REPIH. Conversely, those who are wealthy are likely to be more secure, altruistic, and better able to plan far into the future: these are potential IGH consumers.

The foregoing comments are especially important given our belief that the greatest theoretical problem with the IGH as it currently stands is its certainty assumption. The possibility exists that a generalisation of the IGH to deal with uncertainty might remove any traces of REPIH-like behaviour – assuming, of course, that *everybody* is intrinsically intergenerationally altruistic, not just the rich. If some people are not intergenerationally altruistic under any circumstances, then a generalisation of this sort may not effect a dramatic improvement in the IGH: in other words, we may always need a combination of the REPIH and the IGH. Such people may include the childless and the unflinchingly misanthropic. Is the existence of people like this sufficient to frustrate any attempt at successfully generalising the IGH to take account of uncertainty? We doubt it, given our belief in the intrinsic strength of cross-generational altruism in human nature; but we are also under no illusions about the modelling difficulties involved in analysing intergenerational planning under uncertainty. Not the least of these difficulties is the specification of an expectations-generating process. A risk here is that the particular process chosen could mistakenly mis-specify the model, which in all other respects could be correct. Other extensions of the IGH, while doubtless desirable in themselves, might also face formidable modelling obstacles. Such extensions relate not only to the above-mentioned weak-spots of the IGH (many of which may, as we pointed out, pose no great threat to the theory); but also to a host of other assumptions. These include variable and dynamic tastes; variable mortality probabilities; a different specification of the instantaneous utility function; and demographic effects. The latter may be especially important in economies undergoing rapid population change, although the IGH lends itself *in principle* to an extension to cope with this: $f(s)$ and an age distribution functional form may be developed so as to be amenable to consistent aggregation (see eg Murphy and Welch (1990) for an examination of age-earnings profiles). Investigations into the feasibility of these extensions constitute interesting topics for future research.

However, instead of concentrating on theoretical extensions of the IGH, we believe that future research would be better directed at effecting data improvements. The top priorities here include generating inheritance estimates more in line with the IGH theory, and collecting consistent, reliable earnings distribution estimates. We regard the former as the most urgent task. This might be addressed by constructing survey data on bequest intentions, or by improving the applicability of the specific wealth measure used to proxy I_t. Unfortunately, however, the prospects for improvements in the data do not look very promising in the UK at the present time. Perhaps our research suggestion could be more successfully prosecuted in the immediate future by utilising data from overseas.

Part IV

Appendices

Appendix A

Lifetime Liquidity Constraints

The purpose of this appendix is to describe and highlight some of the salient points about liquidity constraints, especially as they relate to consumer behaviour. The material here is not intended to be an exhaustive or even complete survey of the enormous literature on these constraints: for a more thorough treatment, the reader is referred to, for example, Hayashi (1987).

When people talk about a 'liquidity constraint', they usually refer to a situation where an individual is unable to borrow all (or even any) of what they wish to in the credit market. A common reason for individuals being denied access to loans is imperfect information combined with an adverse selection problem: lenders cannot always be sure about the trustworthiness of the potential borrower, even if the borrower genuinely can finance a desired consumption plan out of future earnings. Consequently, some borrowers will be restricted to loans of a certain size (and possibly at higher rates of interest than average, to reflect the lender's perceived increased exposure to risk), while others will be refused a loan of any size. Individuals restricted access to credit in this way are referred to as 'liquidity constrained'.

In the following, the type of liquidity constraints considered will be of the 'lifetime' variety. This term is used to refer to individuals encountering some obstacle to borrowing based on command over future resources received *during their lifetimes.* In contrast, intergenerational liquidity constraints refer to restricted funds which the individual could be eligible to borrow against the resources of future generations. The existence of intergenerational liquidity constraints seems well established in the legal systems of most countries (see the discussion of Assumption 10 in section 3.1, chapter 3). However, the scale and prevalence of lifetime liquidity constraints is rather more a matter of dispute: we will shortly review some evidence on this issue. The issue is especially important in the way it relates to consumption theory.

The implications of liquidity constraints for neoclassical consumption theories are rather serious. These theories usually rely on an assumption of perfect capital markets to permit rational individuals to separate their optimal consumption paths from the timing of their command over resources. The existence of liquidity constraints therefore jeopardises this result when individuals' desired consumption paths exceed their current command over resources. In the extreme case where the individual is refused access to any credit, actual consumption will follow a discontinuous time path, being the minimum of desired consumption and current resources. In any case, smooth neoclassical consumption functions premised on perfect capital markets are susceptible to mis-specification bias, because factors inhibiting unconstrained lending are omitted from the list of explanatory variables and may disrupt the functional form of the consumption function. Hence the latter could break down altogether, especially if constraints vary with changes in financial regimes[1].

There is evidence about the importance of liquidity constraints, but, because of the difficulty of observing liquidity constrained households, much of it is indirect. An example of this is the finding of 'excess sensitivity' of consumption to current income. As Hall (1978) and Flavin (1981) have suggested, excess sensitivity may be a natural consequence of liquidity constraints, which serve to bind income and optimal consumption paths closer together. This would suggest that current income Y_t is a good predictor of C_t, in contrast to Hall's REPIH model which postulates that only lagged consumption is a relevant explanatory variable (see section 2.1). A measure of the excess sensitivity may be provided by dividing the population into two groups: (a) a minority of liquidity constrained 'Keynesian' consumers who always consume their current period income, and (b) a majority for whom the Hall (1978) REPIH equation holds. Hence the value of the parameter λ_2 in the equation

$$C_t = \lambda_1 C_{t-1} + \lambda_2 Y_t + \epsilon_t$$

is a measure of the degree of liquidity constrainedness in the economy. Evidence from studies of the above type suggests that at most 20% of US households are liq-

[1] There are other implications of liquidity constraints, for human capital theory, labour markets and social insurance/tax policy (to name a few). See eg Becker and Tomes (1986), Baily (1974) and Hubbard and Judd (1986). Liquidity constraints may increase the stabilising properties of tax cuts, and may also violate Ricardian equivalence (Bernheim (1987), Yotsuzuka 1987).

uidity constrained (see, for example, the surveys of Hayashi (1987), Zeldes (1989), and Wilcox 1989).

However, evidence from the above equation needs to be treated with some caution. Excess sensitivity could arise because of poor forecasting by consumers, or inconsistent aggregation, or even the separability of consumption and leisure in utility functions. A more telling objection, however, is that it is surely naive to model liquidity constraints as an 'all or nothing' phenomenon: the timing and severity of constraints are likely to vary across lifetimes, and ideally this needs to be modelled rigorously. Following Zeldes (1989), individuals' behaviour itself may be affected by the presence of liquidity constraints, so that the fraction of people who are liquidity constrained is an *endogenous* variable, which may vary over time[2]. Furthermore, the parameters of the above equation are likely to exhibit instability in view of the finding of Fissel and Jappelli (1990) that the proportion of individuals in the two assumed groups varies significantly over time. This casts doubts on the accuracy of the estimates of liquidity constrainedness in the economy derived from these studies.

It should be noted that the theoretical case for liquidity constraints has not gone unchallenged. Cox (1990), for example, has made the point that groups of individuals may be able to overcome each others' liquidity constraints either by making altruistically-motivated loans, or by exploiting their informational edge over banks and lending to the credit-worthy. The informational edge is presumed to derive from (informal) knowledge about the personal characteristics of the credit-worthy, which professional lenders do not have. However, the strength of Cox's argument is impaired by the possibility that individuals in the lowest income classes (who are arguably the prone to liquidity constraints) will be linked only to similarly constrained members of their own class. And evidence from Jappelli and Pagano (1989) has shown how altruistically motivated *inter vivos* transfers are not large enough to overcome borrowing constraints[3].

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[2] Zeldes has also raised the possibility that individuals may reduce consumption when future labour incomes are uncertain because of the risk of future liquidity constraints.

[3] Using cross-section data from Italy and the USA, they found that the proportion of households receiving transfers from relatives or friends was 9.8% in the US (in 1983) and 4% in Italy (in 1988).

There is also a body of evidence disputing the importance of liquidity constraints. A glance at the time series of consumption and income is sufficient to confirm the noticeably greater smoothness of the former than the latter. This suggests that capital markets are operating well enough to allow consumption smoothing to take place. In a careful panel study, Altonji and Siow (1986) found that the 'perfect capital market' assumption was quite well supported. Caution in assessing the importance of liquidity constraints is therefore needed on both theoretical and empirical grounds.

Appendix B

The Alternative View of Bequests as Non-Altruistic

B.1 Introduction

In contrast to the IGH, the Life Cycle Hypothesis (LCH) is a non-altruistic (selfish) theory of human behaviour. According to the LCH, individuals or households are concerned with maximising utility only over their own lifetimes: they are not interested in the utility or consumption accruing to others, such as descendants and other close relatives. Consequently, evidence of widespread and important intergenerational transfers has induced LCH theorists to search for essentially non-altruistic explanations of bequests, explanations which are consistent with the selfish premise of the LCH. This appendix provides a short survey of these explanations, together with a critical discussion. The survey is meant to be illustrative, not exhaustive.

The LCH explanations of bequest behaviour have been grouped into two categories for convenience. These are: bequests as planned selfish transfers (Part B.2) and bequests as unplanned 'accidental' transfers (Part B.3). Additionally, as a point of interest, some implications about the role of bequests in national savings are also briefly discussed (Part B.4).

B.2 Bequests as Planned Selfish Transfers

Early versions of the LCH[1] ignored bequests in their development of a consumption function. This was done in order to simplify their analysis of 'selfish' optimising consumers. Later generalisations of the LCH which explicitly modelled bequests[2] essentially continued in this tradition: bequests were regarded merely as 'terminal' consumption, which generated utility to the benefactor in the form of a 'warm glow' effect. The basic life cycle premise of the selfish consumer was left intact.

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[1] Brumberg and Modigliani (1954), Modigliani and Ando (1963).

[2] Eg Yaari (1964), Atkinson (1971), Blinder (1974).

Since the publication of these studies, several other 'selfish' models of bequests have been suggested. One of the most popular (Bernheim *et al* 1985) postulates bequests arising from game-theoretic interactions between selfish individuals. So, for example, parents may use bequests as a device to induce children to behave in a desired way[3]. This might take the form of dutiful and obedient offspring being 'rewarded' by inclusion into their parents' will; less compliant offspring could be bequeathed less or even excluded altogether (the 'King Lear' effect). The central point of the Bernheim *et al* thesis is that parents who only care about their own utility may still find it optimal to leave bequests. However, a problem with this thesis is that if children form part of a market for services, wages rather than bequests may be the appropriate means of payment (Barro 1989).

A similar story to Bernheim *et al*'s has been suggested by Kotlikoff and Spivak (1981). According to these authors, parents and children may enter into implicit risk-sharing arrangements, under which parents trade bequests for a guarantee that they will be looked after by their children for the remainder of their lifetimes (which are of uncertain duration). Kotlikoff *et al* (1986, 1987) have demonstrated that this could be an important bequest motive in practice.

It is also possible that selfish consumers reach a point of 'consumer satiation', where they are unable to consume the value of all the resources they acquire. Bequests are 'left over', and are non-altruistic.

B.3 Bequests as Unplanned 'Accidental' Transfers

Another possibility is that bequests are accidental (Abel 1985). The idea here is that risk-averse individuals save against uncertain future health expenditures and an uncertain date of death, but die before all their savings can be realised[4]. The accidental bequest hypothesis clearly hinges on the existence of imperfect annuities markets or imperfect rental markets for housing and other durable assets[5]. For if

[3] Becker has called this the 'enforcement theory of giving'.

[4] Some authors have cited this as an explanation for the LCH's well-known over-prediction of asset-decumulation by the elderly - see eg Davies (1981), King and Dicks-Mireaux (1982) and King (1985).

[5] Bevan and Stiglitz (1979).

annuity markets were well developed, risk averse individuals would be expected to use them, so dispensing with the need to accumulate precautionary savings.

In favour of the accidental bequest hypothesis, Kotlikoff, Shoven and Spivak (1986,1987) and Davies (1981) have illustrated that imperfect insurance markets can account for a potentially large share of intergenerational transfers. However, the demand for actuarially fair annuities appears in practice to be weak[6]. Under one interpretation (eg Barro 1989), this supports the altruism hypothesis, since a choice not to annuitise implies that people prefer to bequeath instead. However, a rival interpretation due to Bernheim *et al* (1985) suggests that this militates *against* the altruism hypothesis, since according to these authors, this hypothesis should actually predict annuitisation![7]. It is therefore difficult to reach any clear-cut conclusions about the role of altruism from the fact that annuitisation is a fairly rare occurrence.

However, it would seem that the accidental bequests hypothesis suffers quite a setback from the finding of limited annuitisation[8]. It is certainly hard to agree with LCH theorists who assert that the demand for annuities is weak because annuity markets are poorly developed. A number of substitutes for annuities do seem to exist: eg unemployment and disability insurance; there is also a case to be made that public pensions compensate for the any shortfall in the supply of indexed annuities (see Diamond 1977). And although adverse selection may restrict the supply of annuities relative to the demand for them, families can effectively *self*-insure by forming an implicit self-insurance market to spread risk (Kotlikoff and Spivak 1981). The fact that there is evidence of under-annuitisation even after the possibility of family annuitisation points to a weakness with the accidental bequest hypothesis[9]. Surely if the utility gains from annuitisation were great enough,

[6] Bernheim *et al* (1985), Friedman and Wahrshawsky (1985a).

[7] Their argument is that rational benefactors should prefer lifetime gifts to terminal bequests, for reasons of minimising tax liabilities; minimising liquidity constrainedness of young beneficiaries; and allowing young beneficiaries to annuitise early.

[8] See Friedman and Wahrshawsky (1985b), for example, who concluded from their life-cycle model of saving and portfolio behaviour with empirically observed annuity prices that an intentional bequest motive must be operative to explain the observed low degree of participation in annuity markets.

[9] Note, however, Kotlikoff's (1988) point that the possibility of large future medical expenses may deter people from committing all of their assets to the relatively illiquid form of annuities. This seems a more satisfactory explanation of limited annuitisation than others commonly advanced, which include the suppositions that individuals are unable to understand annuities, or are irrationally

widespread annuity markets would have developed by now (Kurz 1985).

B.4 The Role of Bequests in Capital Accumulation

An interesting footnote to the above discussion is the quantitative importance of intergenerational transfers. Given the relative weakness of some of the LCH 'explanations' of bequests, it is probably the case that the greater are bequests as a component of total savings, the greater is the inaccuracy of the LCH. Not surprisingly then, there has been a fierce debate in the literature about the magnitude of bequests, whose two main protagonists are Laurence J. Kotlikoff and Franco Modigliani.

The debate began in earnest with a paper by Kotlikoff and Summers (1981), which claimed that intergenerational transfers accounted for up to 80% of total US capital formation. This figure, which seemed very high, was challenged by Modigliani (1985), whose own estimate was closer to 20%. The debate continued in the Spring 1988 issue of the *Journal of Economic Perspectives*, with both Kotlikoff and Modigliani maintaining their positions with little apparent possibility of resolution in sight.

The very next year, the same journal published an article by Kessler and Masson (1989) which attempted to explain the large disparity between the estimates of Kotlikoff and Modigliani. Kessler and Masson pointed out that the disparity could be traced to different definitions of what should be treated as an intergenerational transfer, and to different computations of inherited wealth resulting from a given transfer. Since the two disputants both took up extreme positions in both cases, it seemed likely to Kessler and Masson that the 'true' contribution of intergenerational transfers to total savings would lie somewhere inbetween these two positions. The authors were able to back up this point with a range of evidence from France and Canada. An estimate by Davies and St-Hilaire (1987), for example, put the share of savings due to inheritance at 42% – an intermediate value between Modigliani and Kotlikoff, though somewhat closer to the former than the latter.

afraid of 'betting' on a long life.

Appendix C

The Aggregate Consumption-Income Distribution Literature

C.1 Introduction

This appendix discusses the relationship between aggregate consumption and the distribution of income. Since this is a question which has been accorded some importance in the literature, and on which this thesis claims to make a contribution, the previous literature relating to this issue is examined in some depth.

It is a matter of surprise to some researchers - for example Blinder (1975) and Borooah and Sharpe (1986) - that there is such a paucity of work on the aggregate consumption-income distribution relationship. Indeed, this author has been able to identify only a few[1] pieces of research in this area. All of them fail, for reasons outlined below, to treat the aggregate consumption-income distribution relationship in a satisfactorily rigorous way.

Before the state of the literature is summarised, some necessary groundwork about the distribution of income is introduced. This has been kept deliberately short for the sake of brevity - a more detailed discussion of aspects of income distributions are contained in such standard texts as Pen (1971), Atkinson (1975), Sen (1973), and Cowell (1977).

There are broadly two ways in which an income distribution can be numerically described. The first (adopted by the IGH) assigns a functional form to the observed density (or frequency) function of the distribution; the second condenses all of the information encapsulated by the former into a single number or index. Such numbers or indices are meant to vary in a well-defined way with some interesting feature of the distribution - typically, implied income inequality.

There is a small but growing literature on the econometrics of fitting functional forms to observed distributions: see section 3.4. Suffice it to say here that most

[1] Two additional studies which were known of, but unavailable at the time of writing, were Kesenne (1980) and Tahir (1981). Borooah (1991) is a partial survey of a handful of the more recent contributions to this literature.

functional forms in general usage are flexible enough to convey at least the broad outlines of most real-world distributions, if not their exact shapes. The shape of an income distribution is almost always characterised by a high degree of positive skew, meaning that many people are bunched in the low income tail, and relatively few are located in the high income tail. Two moments of especial interest to researchers are the mean of the distribution, and the variance (which is a measure of the "spread" of incomes).

In contrast to the functional approach to income distribution, the usage of summary measures involves compressing the information of the former into a much simpler form. The chief advantage of summary measures is their ease of handling and interpretation. Whereas a change in an income distribution can alter the parameters of a functional form in a less than straightforward manner, such a change causes a simple and unambiguous shift in a summary measure. The principal summary measures include the Gini coefficient, transformations of the variance, Dalton's (1920) measure, Atkinson's (1970) welfare-based measure, Pareto's α, and Paukert's (1973) measures. Quantile shares form another set of measures. What the measures all try to capture is the inequality of incomes. Their mathematical forms are therefore crucial in determining just how a distributional change is picked up and translated into a change in inequality. A discussion of the mathematical forms themselves is beyond the scope of this study – the interested reader is therefore referred to eg Nyård and Sandström (1981) for more details.

As Metcalf (1972,p.9-10) has pointed out, inequality measures can be misleading indicators of income inequality. Also, while they are undoubtedly useful in certain applications, it will be argued in the following that they are inappropriate for analysing the aggregate consumption-income distribution relationship. If a model of this relationship is not to be *ad hoc* and biased, the whole distribution must be considered, not some artificially condensed version of it.

It is now appropriate to evaluate the various efforts in the literature to account for the impact of the income distribution on aggregate consumption. Much of the work takes Keynes's Absolute Income Hypothesis (AIH) $C_t = a + bY_t$ as a starting point of a working model. It will therefore be useful to mention what Keynes had to say about the importance of income distribution on consumption.

It is clear that in the simple linear AIH consumption function, no role can be ascribed to the *distribution* of income in determining aggregate consumption. Distributional effects are synonymous with non-linearity, not linearity (see eg Stoker 1986). Keynes did, however, acknowledge the importance of income distribution; and the idea that a strong relationship between the two variables exists at all can be traced back directly to his *General Theory* (1936). Therein, Keynes stated (p.90-91) that *"the amount that a community spends on consumption depends partly on ... the principles on which income is divided between them"* (members of the population). The reason why the distribution of income may have an effect on aggregate consumption relates to Keynes's supposition that an individual's savings ratio will vary positively with the individual's disposable income. This supposition is equivalent to the statement that $c_t' > 0$; $c_t'' < 0$, where c_t' is an individual's consumer expenditure differentiated with respect to his or her personal disposable income – the *marginal propensity to consume* (mpc). With a variable mpc, a redistribution from rich to poor individuals will result in the rich reducing their consumption, but not by as much as the poor will increase theirs. Hence Keynes's propositions attribute to the distribution of incomes a direct role in explaining consumption. Specifically, if income were "progressively" redistributed from rich to poor, then consumption in the aggregate would increase. This mechanism has been the (often unstated) rationale behind the studies claiming that the distribution of income affects aggregate consumption.

Since Keynes, efforts to explore the relationship have manifested themselves in four broad approaches. These are now discussed in turn.

C.2 The First Approach: Augmenting with Summary Measures

The first approach arbitrarily augments some "standard" (usually AIH) aggregate consumption function with a summary measure G of the type mentioned above. A typical AIH specification of this sort is

$$C_t = a + b_0 Y_{dt} + b_1 G_t,$$

where Y_{dt} is personal disposable income. If the coefficient b_1 turns out to be significantly different from zero, then it is inferred that income inequality (and hence the distribution of income) is an important determinant of aggregate consumption. A recent example of the first approach is the study of Khan (1987). Khan estimated the two equations

$$\frac{C^*}{Y^*} = \alpha_0 + \alpha_1.G \qquad (C.1)$$

$$\frac{C^*}{Y^*} = \alpha_0 + \alpha_1.G + \alpha_2.Y^* \qquad (C.2)$$

on cross-section data from twenty developing countries for each of the years 1975-1979. Because of data limitations, C^* was defined as real per capita consumption and Y^* was defined as real per capita (rather than disposable) income. Averaging over 1975-1979, Khan reported the following results using Ordinary Least Squares estimation:

$$\underset{}{\frac{C^*}{Y^*}} = \underset{(9.46)}{0.97} - \underset{(1.67)}{0.37\,G} \qquad \overline{R}^2 = 0.12 \qquad (C.1')$$

$$\frac{C^*}{Y^*} = \underset{(10.02)}{1.01} - \underset{(2.11)}{0.44\,G} - \underset{(1.91)}{0.00004\,Y^*} \qquad \overline{R}^2 = 0.30, \qquad (C.2')$$

where t-ratios are given in parentheses. Evidently, the Gini coefficient is a significant regressor in these equations (though only at 10% significance in $(C.1')$); it is also signed in accordance with Keynes's propositions.

Khan's paper is typical of the studies comprising the first approach. Other studies (which account for the majority of studies in the literature) include Staehle (1937), Polak (1939), Ferber (1953), Metcalf (1972), Blinder (1975), Della Valle and Oguchi (1976), Cramer (1976), Musgrove (1980) and three papers associated with Drobny and Hall's (1989) recent *Economic Journal* article. The summary measures used by these authors actually vary quite widely. For example, Staehle related consumption to his own index for the concentration of labour income; Polak used Pareto's α on U.S. data; and Blinder related the average propensity to consume to five variance terms and the Gini coefficient (in current period and lagged form, also on U.S. data). Metcalf regressed consumption on several variables, including the upper and lower quantile tails of the income distribution relative to the mean. Della, Valle and Oguchi supplemented the Gini coefficient with Paukert's (1973) inequality measures in their regressions on the average propensity to consume (international data); and Musgrove supplemented the Gini coefficient with a term designed to capture the asymmetry of the Lorenz curve (also international data). Finally, Drobny and Hall (1989), Hall (1991) and Cuthbertson and Barlow (1991) are three papers which use a variable, originally suggested by Drobny and Hall (1989), which measures tax rate differentials (see section 2.3 of chapter 2).

It hardly needs to be stated that studies employing the first approach are very *ad hoc*. This must inevitably cast doubts on the value of the results derived from them. Perhaps not surprisingly, given the degree of *ad hoc*-ery involved, the results themselves tell conflicting stories. Staehle; Cramer; Khan; Della Valle and Oguchi; and the three papers associated with Drobny and Hall's method all concluded that the distribution of income significantly affects aggregate consumption, and that an increase in inequality will reduce aggregate consumption. Polak, Blinder and Musgrove, however, have concluded that distributional effects are either weak or non-existant[2]; and Blinder and Metcalf found evidence of a positive relationship between inequality and consumption.

C.3 The Second Approach: Empirical Studies

The second approach looks purely at the empirical aspect of the aggregate consumption- income distribution relationship. The papers of Lubell (1947) and Borooah and Sharpe (1986) exemplify this approach. In neither of these papers is the theoretical rationale behind this relationship discussed. Borooah and Sharpe estimated a form of the Davidson *et al* (1978) consumption function (see section 2.2) for each of five quintiles on their U.K. data set of 1963-1982. They then reported that use of Zellner's (1962) seemingly unrelated regression technique indicated the existence of significant coefficient differences. This result implies that aggregation bias will be a serious problem in consumption studies which ignore the impact of the income distribution (see part C.5 of this appendix). Interestingly, Borooah and Sharpe went on to use an aggregated form of the Davidson function

[2] Though Musgrove found distributional effects to be significant when a sub-group of developed countries were examined separately.

to forecast that a simulated equalisation of incomes would increase aggregate consumption in the future. However, extrapolating results up to seventeen years in the future and assuming that the U.K. growth rate is constant until the year 2000 is arguably not the most reliable way of measuring the impact of the income distribution on aggregate consumption. Furthermore, the results are model-specific in the sense that the consumption function chosen at the outset determines to a large extent the precise predictions about the effects of hypothetical redistributions. For example, a model with unstable parameters would be of little use for predictive work of this type. It should be noted in this context that instability has been observed in Davidson *et al*-type equations (see section 2.3 of chapter 2). The model-specificity of the Borooah and Sharpe results did not receive sufficient attention from the authors.

C.4 The Third Approach: Analysis by Income Class

The third approach splits the income distribution into classes, making each income class a seperate regressor in a consumption function. If differences between estimated coefficients are significant, it is inferred that the income distribution affects consumption. As we have just seen, Borooah and Sharpe's (1986) paper utilises this approach; so do the recent papers by Moulaert and de Canniere (1987) and Bunting (1991). In some ways, this third approach could be regarded as a direct descendant of the work of Kaldor (1956,1966) and Pasinetti (1962), which dwelt on the importance of the *composition* of income[3]. Moulaert and de Canniere's paper exemplifies the third approach. These authors sub-divided a year of Belgian cross-section household data into ten income-earning classes (Y_j), ranged from lowest to highest. They first estimated the (arbitrary) non-linear equation[4]

$$C_j = a + b.Y_j + c.Y_j^2$$

and found that $\hat{c}$ was negative ($= -1.59 \times 10^7$) and statistically significant ($|t| = 34.295$). This result, which suggests that the marginal propensity to consume out

[3] Several authors have attempted to test the hypothesis that the composition of income is a determinant of aggregate consumption. For a survey, see eg Hadjimatheou (1987), chapter 9.

[4] The same equation estimated by Husby (1971), although Husby estimated it for all income classes.

of income decreases as income rises, is of course the basic Keynesian premise for income redistribution.

Next, Moulaert and de Canniere split the income distribution into classes. Rather arbitrarily, they regrouped the two lowest classes into one sub-group; all other groups constituted the second sub-group. The AIH equation was then specified and estimated for both sub-groups. The parameters representing the marginal propensities to consume of the two groups were found to be different[5]: this difference turned out to be significant.

An immediate objection to the third approach is that the choice of income class regrouping is a matter of purely personal judgement (as is the choice of the consumption function used). Thus the approach, like the one described in the previous section, is *ad hoc*. The process of condensing the entire income distribution into a small number of classes is unsatisfactory, because important inter-group differences and changes can be ignored or mis-represented. Ideally, a researcher should look at the entire distribution, without splitting up and manipulating it.

Finally, the Fourth Approach involves consistently aggregating over all individuals in order to derive an aggregate consumption function. This is the ideal theoretical approach and, since it is adopted by the IGH, it will now be examined in some detail. Part C.5 of this appendix clarifies what is meant by the term 'consistent aggregation'; and Part C.6 considers the study of van Doorn (1975), who has attempted to use this approach.

C.5 On Consistent Aggregation

Ideally, a theory of consumer expenditure should start at the micro level, explaining how much a rational individual will spend out of, say, his or her personal disposable income at any point in time. With c_{jt} and y_{jt} denoting the consumption and personal income respectively of an individual j at time t, consider the model

$$c_{jt} = f_j(y_{jt}), \qquad (C.3)$$

[5] These were: 0.741 for the first group; 0.509 for the second group; and 0.541 for the combined groups.

where f_j is some particular function pertaining to individual j. Now "consistent aggregation" exists when aggregate consumption C_t is obtained by summing the individual consumer expenditures c_{jt} directly over all individuals. That is, consistent aggregation means that

$$C_t = \sum_j f_j(y_{jt}). \tag{C.4}$$

With consistent aggregation, any differences in the way that individuals spend out of income will be reflected without distortion by equation (C.4). That is, the income distribution comes through into the aggregate consumption function without any bias or distortion. Terms capturing an "income distribution" or "income inequality" effect – such as were employed by the First Approach described above – are irrelevant in the context of a consistently aggregated consumption function. In contrast, aggregation bias will occur with inconsistent aggregation, which arises when aggregate consumption is obtained by summing over individual incomes, even though the "micro" function f_j does not permit this. That is, inconsistent aggregation occurs when f_j is non-linear and the aggregation used is described by

$$\begin{aligned} C_t &= f_j(\sum_j y_{jt}) \qquad (C.5) \\ &= f_j(Y_t), \end{aligned}$$

where Y_t is total disposable income. Although (C.4) and (C.5) can sometimes be the same ((C.5) is a special case of (C.4)), in general they will not be. They will only be the same if (a) the f_j are identical for all individuals j, and (b) if the f_j are linear functions. In a practical context, they will also be observationally equivalent if the distribution of income is constant over time.

Crucial though the aggregation problem is, few studies in the consumption-income distribution literature pay sufficient attention to it. We now turn to one study which has attempted to address the problem directly.

C.6 The Fourth Approach: van Doorn (1975) and the IGH

Van Doorn's paper enjoys the advantage of representing the income distribution by a function, as opposed to a summary measure of the sort discussed in Part

C.2. Van Doorn's use of a distribution function is therefore a significant advance. He started with the logarithmised Keynesian AIH micro consumption equation

$$c_i = e^{\alpha}.y_i^{\beta}.z_i^{\gamma}, \tag{C.6}$$

where α, β, γ are constants, and where c_i, y_i and z_i are the consumption, income and size respectively, of a household i. The above equation is non-linear, suggesting a decreasing marginal propensity to consume as income rises when $\beta < 1$. Van Doorn tried to aggregate his micro consumption function in the following way. By taking logarithms of (C.6), summing over all n households and dividing by n, the following equation is derived:

$$\left\{\frac{1}{n}\sum_{i=1}^{n}\ln c_i\right\} = \alpha + \beta\left\{\frac{1}{n}\sum_{i=1}^{n}\ln y_i\right\} + \gamma\left\{\frac{1}{n}\sum_{i=1}^{n}\ln z_i\right\}. \tag{C.7}$$

Now a convenient property of a lognormally distributed variable is that the logarithm of its geometric mean value can be expressed as a linear combination of its arithmetic mean and variance. Since the values in curly brackets in (C.7) are logarithms of geometric mean values, the above property can be exploited to yield

$$\ln\overline{c} - \frac{1}{2}\sigma_c^2 = \alpha + \beta\left(\ln\overline{y} - \frac{1}{2}\sigma_y^2\right) + \gamma\left(\ln\overline{z} - \frac{1}{2}\sigma_z^2\right).$$

This assumes, of course, that household size, and household income, and household consumption are all lognormally distributed. Assuming further that the variance of consumption, σ_c^2, is directly proportional to the variance of household size σ_z^2, with proportionality parameter γ, van Doorn obtained the final consumption function

$$\ln\overline{c} = \alpha + \beta\left(\ln\overline{y} - \frac{1}{2}\sigma_y^2\right) + \gamma\ln\overline{z}, \tag{C.8}$$

where the effect of income distribution is picked up by the variance of income σ_y^2. Van Doorn contrasted (C.8) with the special 'no distribution effects' case where $\sigma_y^2 = 0$. Using 1970-1 UK cross-section data, regressions were run on both specifications. However, van Doorn was able to detect only a weak (statistically insignificant) negative relationship between consumer spending and income inequality.

There are several obvious problems with van Doorn's model. To start with, no rationale was given for the rather unusual 'micro' consumption function chosen[6]. In addition, van Doorn's choice of representing the income distribution by the lognormal distribution can be criticised at both a theoretical (Aitchison and Brown (1957), Metcalf 1972) and an empirical[7] level. And van Doorn's assumptions that (a) household size, and household income, and household consumption are all lognormally distributed; and (b) $\sigma_c^2 = \gamma\sigma_y^2$ are very strong and lacking any justification. It would seem that the only rationale for them – and the lognormal and logarithmised AIH specifications – is mathematical expedience: ie, they make consistent aggregation possible. Hopefully, the IGH consumption presented in the main text of the thesis indicates a slightly less restrictive way of consistently aggregating a micro consumption function.

C.7 Conclusion

To conclude, the majority of the work to date on the aggregate consumption-income distribution relationship suffers from important drawbacks. These include: the use of *ad hoc* consumption functions; the use of inequality measures instead of income distributions; and a failure to aggregate consistently. The literature seems to have tolerated a remarkable degree of *ad hoc*-ery for a remarkably long time.

Part of the reason for this is the popularity of what we have termed the 'First Approach' to modelling the consumption-income distribution relationship. However, this survey should have demonstrated that a better approach to modelling exists, one which offers greater hope for the accurate estimation of the consumption-income distribution relationship. This is the Fourth Approach of van Doorn (1975) and the IGH (see section 3.4 of chapter 3).

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[6] And, rather surprisingly, the mechanism by which income distribution was supposed to affect aggregate consumption (ie diminishing mpc's) was not explicitly mentioned.

[7] Aitchison and Brown (1957,p.116) report that the lognormal distribution consistently underestimates the number of members in the lowest and highest income classes. Metcalf (1972,p.14) has also criticised it for over-correcting for positive skew in income distribution data, and forcing a symmetric treatment of movements in the two tails of the distribution. Additionally, see Salem and Mount (1974), McDonald and Ransom (1979), McDonald (1984) and Atoda et al (1988) for evidence of the poor performance of the lognormal in describing real world income distributions.

Appendix D

Derivation of the IGH Consumption Function Using the Beta Distribution

Using the beta density stated in section 3.4 of chapter 3, the integral in (3.22) becomes

$$\frac{-\zeta N}{B(p_t, q_t)\,\omega_t^{p_t+q_t-1}} \int_0^{\omega_t} y_t^{p_t+\varepsilon-1}(\omega_t - y_t)^{q_t-1}\,dy_t. \tag{3.23'}$$

By Euler's First Integral,

$$\begin{aligned}\frac{1}{B(p_t, q_t)\,\omega_t^{p_t+q_t-1}} \int_0^{\omega_t} y_t^{p_t+\varepsilon-1}(\omega_t - y_t)^{q_t-1}\,dy_t &= \frac{B(p_t+\varepsilon, q_t)\,\omega_t^{p_t+\varepsilon+q_t-1}}{B(p_t, q_t)\,\omega_t^{p_t+q_t-1}} \\ &= \omega_t^{\varepsilon}\frac{\Gamma(p_t+q_t)\Gamma(p_t+\varepsilon)}{\Gamma(p_t)\Gamma(p_t+\varepsilon+q_t)}.\end{aligned} \tag{3.24'}$$

Hence (3.23′) becomes

$$-\zeta N\left\{\frac{\Gamma(p_t+q_t)\Gamma(p_t+\varepsilon)}{\Gamma(p_t)\Gamma(p_t+\varepsilon+q_t)}\right\}\omega_t^{\varepsilon}. \tag{3.25'}$$

However, average aggregate earnings is defined as

$$\begin{aligned}\overline{Y}_t &= \frac{1}{N}\int_{H^*} y_t.h(y_t;\theta_t)\,dy_t \\ &= \frac{1}{B(p_t, q_t)\,\omega_t^{p_t+q_t-1}} \int_0^{\omega_t} y_t^{p_t}(\omega_t - y_t)^{q_t-1}\,dy_t\end{aligned} \tag{3.26'}$$

using the beta distribution. The right-hand side of equation (3.26′) is clearly a special case of the left-hand side of (3.24′) where $\varepsilon = 1$: hence (3.26′) may be evaluated by putting $\varepsilon = 1$ into the right-hand side (result) of (3.24′). This yields

$$\omega_t\frac{B(p_t+1, q_t)}{B(p_t, q_t)} = \omega_t\left(\frac{p_t}{p_t+q_t}\right).$$

Therefore, $\overline{Y}_t = \omega_t p_t/p_t + q_t$, or, since $\overline{Y}_t = Y_t/N$, we have $N\omega_t p_t/p_t + q_t = Y_t$. Hence

$$\omega_t^{\varepsilon} = \left(\frac{Y_t(p_t+q_t)}{Np_t}\right)^{\varepsilon}, \tag{3.27'}$$

and so, substituting (3.27′) into (3.25′),

$$-\zeta N\left\{\frac{\Gamma(p_t+q_t)\Gamma(p_t+\varepsilon)}{\Gamma(p_t)\Gamma(p_t+\varepsilon+q_t)}\right\}\omega_t^\varepsilon = -\zeta N^{1-\varepsilon}Y_t^\varepsilon\frac{\Gamma(p_t+q_t)\Gamma(p_t+\varepsilon)}{\Gamma(p_t)\Gamma(p_t+q_t+\varepsilon)}\left(\frac{p_t+q_t}{p_t}\right)^\varepsilon . \tag{3.28′}$$

Now putting (3.28′) into equation (3.22) (see the text of section 3.4) yields the aggregate IGH consumption function

$$C_t = \beta_0 + \beta_1 I_t + \beta_2 Y_t + \beta_3 Y_t^\varepsilon\,\Psi(p_t, q_t, \varepsilon) \tag{3.29′}$$

where the β_3 is defined in the text of section 3.4, and where

$$\Psi(p_t, q_t, \varepsilon) = \frac{\Gamma(p_t+q_t)\Gamma(p_t+\varepsilon)}{\Gamma(p_t)\Gamma(p_t+q_t+\varepsilon)}\left(\frac{p_t+q_t}{p_t}\right)^\varepsilon . \tag{3.30′}$$

Clearly the only difference between using the beta instead of the gamma distribution to aggregate the consumption function is the structure of the Ψ function.

Part V

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