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On Some Aspects of the Noise Propagation from Supersonic Aircraft

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G.M. Lilley, M.Sc., D.I.C., A.F.R.Ae.S.,
R. Westley, B.Sc., D.C.Ae.,
A.H. Yates, B.Sc., B.Sc. (Eng.), A.F.R.Ae.S.,
J.R. Busing, B.E., B.Sc., D.C.Ae.

of the Department of Aerodynamics

SUMMARY

The noise problem associated with an aircraft flying at supersonic speeds is shown to depend primarily on the shock wave pattern formed by the aircraft. The noise intensity received by a ground observer from a supersonic aircraft flying at high as well as low altitudes, is shown to be high although it is of a transient nature.

A study of the shock wave patterns around an aircraft in accelerated and retarded flight is shown to lead to an explanation of the one or more booms, of short duration, heard by ground observers after an aircraft has dived at supersonic speeds.

The shock wave patterns associated with an aircraft flying in accelerated or retarded flight at transonic speeds are shown in certain cases to be very different from the corresponding patterns observed in steady flight. The significance of these results, with reference to problems of flight at supersonic speeds is briefly discussed.

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§2. Notation

a	speed of sound
a_o	speed of sound in undisturbed air
c	chord of aerofoil, critical speed in water $(=\sqrt{gh})$
d	distance apart of shock waves
f	acceleration
g	acceleration due to gravity
h	depth of water
ℓ	body length
ℓ, m, n	direction cosines
M	Mach number
M_o	freestream Mach number
n	frequency
p	pressure
r, θ	polar coordinates
R	radius of pulse wave
t	time
v	wind velocity
V	velocity
x, y, z	Cartesian coordinates
$\underset{\sim}{x}$	position of observer
$\underset{\sim}{y}$	position of acoustic source
$2y_m$	aerofoil maximum thickness
α	angle between the forward Mach cone and the acceleration vector
β	angle of sound ray to ground normal
γ	$(\pi/2 - \mu)$, ratio of specific heats
δ	fineness ratio of body
θ	direction of flight path to ground normal
λ	wave length
μ	Mach angle
ξ	angle of shock wave
ξ, η, ζ	Cartesian coordinates of flight path
ρ	density, radius of curvature of flight path
τ	retarded time
ϕ	total angle of sound ray to ground normal.

§3. Introduction

The aerodynamic noise emitted by high speed aircraft has attracted increasing attention in recent years; it is already a serious nuisance and it is threatening to become worse.

It is known that the major noise, created by an aircraft travelling at subsonic speeds, is due to the jet and/or the propeller. In these cases most of the annoyance to the general public arises during running up on the ground, the take-off and landing. On the other hand the noise level produced at the ground from a jet aircraft flying at high altitudes is less than the average noise level of a busy city street. It is interesting to note that the noise level on the ground will in general be very much greater for a single high speed aircraft flying at low altitude than for a large formation of high speed aircraft flying at high altitude, although in the latter case the time duration of noise will be extended.

When an aircraft is flying at supersonic speeds a shock wave pattern is formed around the aircraft. The shock waves, which are formed as a result of the aircraft speed being greater than the speed of propagation of the pressure disturbances created by the aircraft during its flight, envelope the sound waves created by the aircraft and its jet in the direction of motion. Hence a ground observer must hear the shock waves produced by an advancing aircraft before the noise from the jet. The sensation of noise caused by the shock waves, even when they are weak, is very much greater than that produced by other sources, although the time duration of the noise is in most cases very small. Examples of the sensation of noise caused by shock waves are provided by bullets, shells, missiles[24] and aircraft (at present) in dives. In the latter case, when the supersonic part of the flight occurs at high altitudes, one, two and sometimes more 'bangs' are heard by ground observers. Although the pressure changes received near the ground have not, as yet, been strong enough to cause damage to the human ears or to property there is understandably interest in the likelihood of these 'bangs' being stronger when emitted from faster and low flying aeroplanes and missiles. Even if stronger 'bangs' prove unlikely to reach the ground, owing to atmospheric refraction, the possible effects on other aircraft flying in the vicinity of a supersonic aeroplane must be assessed.

It will be shown below that the explanation of the 'bangs' and related problems with reference to the noise from supersonic aircraft, lies in an understanding of the shock wave

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pattern around the aircraft in the cases of uniform, accelerated and retarded flight. It is then possible for the relative importance of the height, speed and size of the aeroplane to be assessed and for the strength of the 'bang' to be estimated.

It is emphasised that, owing to the limited amount of data available with regard to shock wave patterns around bodies in unsteady flight, some of the conclusions stated in this report must be regarded as qualitative only and may need slight modification when more accurate data are obtained.

§4. Response of the ears to transient pressure disturbances

It is pertinent to consider briefly the response of the ears of a ground observer to transient pressure disturbances as a preliminary to a discussion on the noise emission from high speed bodies in flight. It is well known that the human ear is a highly selective frequency analyser, sound locator and an indicator of the loudness, pitch and the timbre of sounds. (See reference 3 and the references quoted therein). Thus an incident pressure disturbance, having a complex wave form is resolved by the listener's ear into its Fourier components and the sensation is that of a fundamental sound and a series of overtones whose intensities are greater or smaller than that of the fundamental. In the case of continuous noise, whose spectrum is not 'white' (i.e. continuous and uniform as a function of frequency) but peaks over a discrete band of frequency, the listener will, as a result of the Fourier analysis, sense the high intensity noise above the background, provided that the difference intensities and the bandwidth are within definite ranges. It has also been observed[4] that only two successive cycles are sufficient for a listener to determine the pitch of a pure note, at least up to a frequency of about 1000 cycles per second. In general, however, the duration of the note must exceed 0.01 seconds for accurate pitch determination to be made. In addition for sounds of shorter duration, although the ear cannot detect the pitch, it continues to respond for about 0.01 seconds, and this response is commonly called the aftersound. Thus under normal conditions, two following sounds of short duration will not be distinguished one from the other, unless their time interval is greater than 0.01 seconds.

There is an upper limit of frequency of sound that is audible to the human ear and this limit is about 20 - 25kc. But the ear will register a single pulse of short duration provided that the intensity is sufficiently great. In all such cases,

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due to the aftersound, the duration appears to be about 0.01 seconds. For example an electric spark of 1μ sec. duration is heard as a crisp 'chirp', whereas an intermittent spark triggered at the rate of 1000 per sound is heard as a prolonged 'chirp' of about 1000 c.p.s. pitch.

There appears to be very little correlated experimental evidence between the aural sensation and a high fidelity microphone for transient disturbances such as shock waves. Records obtained from projectiles fired in air at supersonic speeds and at low altitudes indicate that the shock pattern associated with the projectile passes in less than 0.01 seconds, whereas the explosive (or gun) wave is of longer duration. From what has been stated above it can be seen that the ears will respond to the passage of shock waves, and will identify their amplitude and direction.

§5. Doppler effect

When a body is in motion relative to an observer, the observed pitch is different from the pitch of sounds emitted from the body or the flow field around the body. If the sound radiators, which are moving with the velocity $\underset{\sim}{M}\, a_o$, where $\underset{\sim}{M}$ is the Mach number and a_o is the speed of sound in undisturbed air, send out vibrations of frequency n at the point $\underset{\sim}{y}$, then the observed pitch[5] at $\underset{\sim}{x}$ is

$$\frac{n}{1 - \dfrac{\underset{\sim}{M} \cdot (\underset{\sim}{x} - \underset{\sim}{y})}{|\underset{\sim}{x} - \underset{\sim}{y}|}}$$

The frequency is increased for sound emitted forwards and decreased for sound emitted backwards. The Doppler effect applies equally well to each band of frequencies in the spectrum of noise emitted by a sound radiator.

It has often been stated that for a body moving towards an observer at rest, with unit Mach number, the apparent frequency of the emitted sound, as a result of Doppler effect, is infinite, and therefore outside the audible range. The <u>incorrect</u> conclusion is then drawn that at the instant the aircraft passes the observer the aircraft will not be heard. This remark is then, more often than not, followed by the <u>correct</u> statement that the noise, heard by the observer as the aircraft recedes, is at frequencies equal to half that of the sound emitted. This paradox only arises when we neglect (a) the sources of sound that are

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associated with the body in motion, and are not themselves travelling at the same speed as the body, and (b) the shock waves around the body, which arise from the pressure field of the body and its propagation, and which are readily converted by the ear into noise.

§6. The noise emission from bodies at subsonic speeds

The sound emitted from a body travelling through the atmosphere may be classified under the headings external and internal. In the latter class we include the noise from engines, compressors, turbines and pumps etc., whereas in the former class we include the noise due to propellers, piston engine exhausts[6], jets[5], boundary layers, wakes, tip vortices and to the pressure field associated with the body motion. An analysis of the noise received by a ground observer shows that the internal noise accounts for only a small fraction of the total noise. In the case of high speed aircraft, propulsion is usually by means of a jet and the noise emitted from the jet generally dominates the remainder of the noise, at least at subsonic flight speeds below the critical Mach number. We will therefore only consider in detail this type of noise, in addition to that associated with the body pressure field.

(i) Jet noise

The noise emitted from a jet arises from the turbulent motion[5] in the mixing region between the main flow near the jet centre, and the air at rest outside. The noise, which is strongly directional, is a function of the jet speed[7], temperature and diameter[8]. The intensity falls off approximately as the inverse square of the distance from the jet. For jets below choking the emission is stronger in the downstream field (relative to the jet) and weaker in the upstream field. For jets above choking the noise field is modified by the interaction of turbulence with the stationary shock waves[9] (relative to the jet exit) in the jet and in certain ranges of frequency and pressure ratio the upstream radiation may be greater than the downstream.[10]

(ii) Pressure field due to body motion

If a body starts from rest and is gradually accelerated until it reaches a uniform subsonic speed then at each instant each part of the body sends out pulse waves of small amplitude, which move with sonic speed. We sometimes loosely, if graphically, refer to these waves as warning the air ahead that the body is approaching.

/(See figure 1) ...

(See figure 1). Since we are at present considering subsonic body speeds, these waves will travel both upstream and downstream relative to the body. The complete system of waves is directional and a simple argument shows that their amplitude and wave length will be respectively functions of the body volume and its length in the direction of motion. (On the rather crude assumption that the body may be replaced by a source-sink combination.[1])

The system of waves can be easily seen by dragging a two-dimensional body through shallow water at low speeds. It can be shown that the elevation and depression of the weak surface waves (apart from the capillary waves of small wave length) correspond to the longitudinal wave motion produced by movements of a similar body through air. The analogy between shallow water waves and pulse waves in air follows from the fact that both have a speed which is independent of wavelength.

An approximate calculation[1] shows that in accelerated flight at high subsonic speeds the amplitude of the pressure disturbances at a large distance from the moving body is not negligible in terms of noise intensity. This problem, which relates to the calculation of the pressure history at a fixed point due to the motion of a body, needs further investigation.

§7. The noise emission from bodies at supersonic speeds

The sources of noise associated with a body travelling at supersonic speed are similar to those at subsonic speeds except that, as stated above, the pressure disturbances created by the body during its motion form a system of shock waves around the body. In the case of a jet propelled body the main noise, at a large distance from the body, will be a combination of jet, wake and boundary layer noise and the noise due to shock waves.

(i) Pulse waves, Mach waves and Shock waves

When a body is flying at steady supersonic speeds the pulse waves, which are emitted continuously by the body during its flight, and which are moving at sonic speed relative to the local flow, can no longer propagate upstream since their speed is less than that of the body. The pulse waves which are emitted from an infinitely small body form envelopes and the surfaces generated are called Mach waves (see figure 2). We refer to a Mach wave across which there is an infinitesimal increase (decrease) in pressure as a compression (expansion) wave. By definition, the velocity normal to a Mach wave is sonic. The

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angle the Mach wave makes with the direction of the local flow, relative to the Mach wave, is called the Mach angle. The infinitesimal disturbance created at each point along the flight path is propagated along the surface of a cone (the forward Mach cone) whose semi-apex angle is equal to the complement of the Mach angle. The pulse waves, emitted by a point source, are propagated with sonic velocity relative to the undisturbed flow ahead of the source, because, owing to its size, the induced velocity due to the motion of the source is zero. In the case of a body of finite size the velocity of the pulse waves is still sonic relative to the flow around the body, but owing to the finite induced velocity due to the body motion, the velocity and angle of forward propagation, relative to the undisturbed flow are increased and decreased respectively. (See figure 3). The induced flow over the body cannot, however, be generated by Mach waves alone, since sudden finite changes in flow direction and finite pressure increases are called for. Consequently, we find ahead or springing from the body, bow and tail waves of finite amplitude called shock waves. Just as Mach waves may be regarded as envelopes of pulse waves so shock waves may loosely be regarded as envelopes of compression Mach waves. Shock waves propagate normal to themselves with supersonic speed relative to the undisturbed flow. The bow and tail shock waves around a two-dimensional body will be straight only when the velocity of the body is uniform in a straight line, the flow behind the shock is supersonic relative to the body, and when no expansion waves (such as the fan of waves POQ in fig. 3) interact with the shock waves. In the latter case the expansion waves reduce the strength of the shock wave and its velocity of propagation. The shock wave is, crudely, the mechanism by which undisturbed air ahead of a body, moving at supersonic speeds, is rapidly accelerated in the direction of motion of the body, in order that smooth flow may occur over the body. The thickness of a shock wave in air is very small and for many practical purposes it may be assumed that it is of zero thickness.

For a complex body such as an aeroplane shock waves will be generated at the leading and trailing edges of the wings and tail surfaces and at the nose and tail of the body. Additional shock waves will in general also be present around intakes, fairings etc. Since the main problem under consideration is that of the noise received by a ground observer from a body moving at supersonic speeds not too close to the ground, it is the shock wave configuration in the distant field which is of direct interest and not that corresponding to the vicinity of the aircraft. Simple reasoning will show that, in general, at distances from the aircraft,

/large ...

large compared with its dimensions, the complex shock pattern will be mainly determined by the stronger shocks especially if they are partly plane. Thus, for an aircraft, the shock pattern will be represented approximately by two main shocks, although at any finite distance from the aircraft we cannot assume that the remaining shocks have coalesced with the main shocks, and that their local strength is less than that of the main shocks. A diagram showing the interaction of shock waves from tandem aerofoils is given in figure 4. For simplicity, however, it is legitimate in the approximate treatment below to consider only the shock waves that develop around a two-dimensional aerofoil or an axisymmetric body of revolution.

In order to determine the noise produced by shock waves in motion, we must first calculate the pressure rise across them at a large distance from the body. This involves assessing the amount by which shock waves decay with distance. The processes of decay are

(a) by the action of viscous and thermal effects
(b) by the interaction of expansion and shock waves
(c) by the spreading of free spherical shock waves away from their source.

In most practical problems the effects (a) can be safely neglected. Figure 5a shows how the expansion waves interact with the shock waves and gradually reduce their strength to zero at an infinite distance from the aerofoil. The typical pressure signature is shown in figure 5b, and is representative of that obtained by calculation both for aerofoils and bodies of revolution in uniform straight line motion.

If an aircraft or projectile flies past a ground observer at constant supersonic speed, the observer can hear nothing until the bow wave reaches him. The saw tooth (N-wave) form of the pressure disturbance due to the bow and tail waves will be observed as one or two cracks or booms depending on the body length, speed, altitude and the ground terrain. When small projectiles are fired at low altitudes the time interval between the bow wave, the tail wave and the ground reflection waves (see figure 6) will be less than 0.01 seconds and hence two or more discrete sounds cannot then be detected by the ears.

(ii) Jet, wake and boundary layer noise

The noise emission from jets, wakes and boundary layers will have a similar character to that created at subsonic body speeds. However one main difference is that sound waves

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propagated out from a jet or wake cannot penetrate into the region between the bow and tail waves, apart from the small percentage of the total output which is transmitted forward through the subsonic part of the boundary layer. Hence disturbances, giving rise to noise, will exist in the region between the bow and tail waves, although the main noise, apart from that due to the moving shock waves, will exist to the rear of the tail wave. A common example of the wake noise, following that due to shock waves, is that experienced from a shell or bullet. This gives rise to the prolonged hissing noise, the pitch of which decreases on passing, as a result of Doppler effect.

(iii) General conclusions

It has been shown qualitatively how the noise from bodies moving at subsonic speeds differs from that at supersonic speeds. The main difference lies in the discontinuities in pressure associated with the formation of shock waves around the body. The remarks relating to the ear response show that in general two shock waves cannot be heard separately unless their time interval is greater than 0.01 seconds.

One important conclusion is, that in order to calculate the noise intensity from aircraft travelling at supersonic speeds we must know the shock wave pattern around the aircraft. This is known reasonably well, apart from certain minor gaps, for bodies travelling at uniform speeds but the development of shock waves in accelerated and retarded flight has as yet received little attention.

§8. Mach wave patterns formed by a moving source

Before considering the changes that occur in the shock pattern around a body when it is accelerated or retarded, we will first discuss the corresponding changes in the Mach wave pattern formed by a moving source. In section (i) the physics of some simple illustrative patterns will be discussed, in section (ii) the theory will be outlined.

(i) Examples of some simple patterns

It is assumed that the source moves through a uniform fluid which is initially at rest, and that the spherical pulse waves, emitted by the source during its motion, travel through the fluid with constant sonic speed. Figures 1 and 2, which have been referred to above, show part of the complete set of waves at a given time instant. At subsonic speeds the crowding

/together ...

together of the pulse waves ahead of the source is clearly seen, whereas at steady supersonic speed the conical envelope at the forward part of the pulse waves is visible. Figures 7,8,9,10 and 11 show that in unsteady supersonic motion the envelopes form Mach waves of concave and convex curvature divided by cusps.[11], [12] Obviously a property of the Mach wave is that it moves everywhere normal to itself at the speed of sound. Hence a Mach wave, whose curvature is concave to the direction of motion, will focus at a point ahead, which must be its centre of curvature. Beyond this point, however, it will be seen that the Mach wave becomes a wave whose curvature is convex to the direction of motion. This helps to explain why a cusp and a rear wave of convex curvature are formed when a source accelerates in a straight line through the speed of sound, (see figure 7). Thus the Mach waves formed by the accelerated source generate a closed loop in any longitudinal plane passing through the axis. When the source is now retarded to subsonic speeds the Mach waves become convex forwards and tend to move ahead of the source (see figure 8) since their speed of propagation is greater than that of the source. Both the front and rear waves of the closed loop then eventually move ahead of the source. The distance apart of the waves, of the loop, along the axis (straight line motion) remains constant but the height of the loop increases. The cone angle of the loop at a large distance from the position at which the highest Mach number, M_{max}, of the source is reached, is equal to

$$2\left(90^{\circ} - \sin^{-1}\left(\frac{1}{M_{max}}\right)\right) .$$

When a source is accelerated after steady motion at a supersonic speed has been attained, two cusps are formed in the forward Mach wave (see figure 9). The inner cusp is generated along the forward Mach cone, corresponding to the initial Mach number, with its apex at the point of initial acceleration. The outer cusp spreads outwards. Its path depends upon the history of the accelerated motion.

(ii) <u>The equation of the wave envelope for an arbitrarily moving source</u>

Although the complete wave envelope may be geometrically constructed using the principle of expanding spherical waves, the drawback to this method is that it becomes rather indeterminate in the region of the cusp points unless an excessively large number of circles is drawn. To overcome the difficulty of locating cusps, two simple expressions will be introduced to locate the wave

/envelope ...

envelope and the cusp lines.

Consider a source which has moved along the line BA (figure 13) and has reached the point A at the present time t=0. Spherical pulse waves will have been emitted along its path at each past instant. These waves will be expanding outwards away from their points of formation with the speed of sound (a). The whole system of pulse waves will be viewed at the present time. Consider the spherical wave which was emitted at the time $t = -\tau$ as the source passed through the point C whose Cartesian coordinates are (ξ, η, ζ). Let a point D, on the sphere, have coordinates (x, y, z).

The equation of the sphere is.-

$$R^2 = (x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2$$

where $R = a\tau$,

or $$a^2\tau^2 - (x - \xi)^2 - (y - \eta)^2 - (z - \zeta)^2 = 0 \quad \text{.....8.1}$$

If we differentiate equation 8.1 with respect to τ, keeping x,y,z constant, we get

$$a^2\tau + (x - \xi)\dot{\xi} + (y - \eta)\dot{\eta} + (z - \zeta)\dot{\zeta} = 0 \quad \text{.....8.2}$$

and on differentiating again with respect to τ, we get

$$a^2 + (x - \xi)\ddot{\xi} + (y - \eta)\ddot{\eta} + (z - \zeta)\ddot{\zeta} - \dot{\xi}^2 - \dot{\eta}^2 - \dot{\zeta}^2 = 0 \quad \text{.....8.3}$$

where $\dot{\xi} = \frac{d\xi}{d\tau}$ etc.

Let the velocity of the source at the point (ξ, η, ζ) be V, having the direction cosines (ℓ, m, n).

Therefore

$$\left.\begin{aligned} \dot{\xi} &= -V\ell \\ \dot{\eta} &= -Vm \\ \dot{\zeta} &= -Vn \end{aligned}\right\} \quad \text{.....8.4}$$

and

$$\dot{\xi}^2 + \dot{\eta}^2 + \dot{\zeta}^2 = V^2. \quad \text{.....8.5}$$

since $\dot{\xi} = \frac{d\xi}{d\tau} = -\frac{d\xi}{dt}$ etc.

Thus equations 8.1, 8.2, and 8.3 become

$$a^2\tau^2 - (x - \xi)^2 - (y - \eta)^2 - (z - \zeta)^2 = 0 \quad \text{.....8.1a}$$

$$a\,R\left[1 - \frac{V}{a}\left\{l\frac{(x-\xi)}{R} + m\frac{(y-\eta)}{R} + n\frac{(z-\zeta)}{R}\right\}\right] = 0. \qquad \ldots\ldots\ldots\ldots 8.2a$$

$$R\left[\frac{a^2-V^2}{R} + \left\{\frac{\ddot{\xi}(x-\xi)}{R} + \frac{\ddot{\eta}(y-\eta)}{R} + \frac{\ddot{\zeta}(z-\zeta)}{R}\right\}\right] = 0. \qquad \ldots\ldots\ldots\ldots 8.3a$$

A point (x,y,z) which satisfies equation 8.1a will lie on the spherical pulse wave. If it also satisfies equation 8.2a then the point will lie on the wave envelope. The third expression 8.3a is the condition that the wave envelope forms a cusp. Thus if the point D satisfies 8.1a it lies on the spherical wave, if it satisfies 8.1a and 8.2a it lies on the wave envelope and if it satisfies 8.1a , 8.2a and 8.3a then it lies at a cusp point on the envelope.

Let γ equal the angle the tangent to the source path at the point (ξ,η,ζ) makes with the radius vector, R, drawn to the point where the spherical wave touches the envelope. Then γ is the complement of the Mach angle, and from 8.2a,

$$\frac{a}{V} = \cos\gamma$$

or

$$\gamma = \cos^{-1}\frac{1}{M} \qquad \ldots\ldots\ldots\ldots 8.6$$

where M is the instantaneous Mach number at the point C. Thus, as the source passes through the point (ξ,η,ζ) a circular element of the wave envelope is emitted. This circular element will travel away from the point of emission with the speed of sound, and will trace out a cone of semi vertex angle equal to $\cos^{-1}\frac{1}{M}$, and whose axis is the direction of motion of the source as it passed through the point (ξ,η,ζ).

Let the magnitude of the acceleration of the source at (ξ,η,ζ) equal $f = \sqrt{\ddot{\xi}^2 + \ddot{\eta}^2 + \ddot{\zeta}^2}$. Then, if the radius vector from (ξ,η,ζ) to the wave envelope make an angle α with the direction of acceleration, condition 8.3a reduces to.-

$$R\left(\frac{a^2 - V^2}{R} + f\cos\alpha\right) = 0$$

or

$$R = \frac{a^2(M^2 - 1)}{f\cos\alpha} \qquad \ldots\ldots\ldots\ldots 8.7$$

For motion in a straight line equation 8.7 can be written

$$R = \frac{a(M^2 - 1)M}{\dot{M}} \qquad \ldots\ldots\ldots\ldots 8.7a$$

where $\dot{M} = \frac{1}{a}\frac{dV}{dt}$.

/A cusp ...

A cusp will form on the envelope when condition 8.7 is satisfied. If $f \cos \alpha$ is positive when the source passes through the point (ξ, η, ζ) then a cusp is formed after an interval of time

$$\frac{a(M^2 - 1)}{f \cos \alpha} \qquad \text{.....8.8}$$

The complete wave envelope is formed by constructing all the circular elements corresponding to each point on the path for which the motion was supersonic. The envelope will consist of concave and convex sheets which are separated by cusp lines.

The formation of cusp points may perhaps be more easily understood if one considers the curvature of a wave element at its instant of formation. The element will be initially concave outwards with radius of curvature,

$$\frac{a^2(M^2 - 1)}{f \cos \alpha} .$$

That is, the element will come to a focus at its centre of curvature, and then diverge in the form of a convex element. Figure 14 illustrates the formation of a cusp from a concave wave envelope, EF. The wave envelope which is formed at some later instant may be constructed by producing all the normals to the wave front for a constant distance such that $B_1C_1 = B_2C_2 = B_3C_3 = GG_1 = EE_1$. It will be seen that the envelope between F_1 and G_1 is still concave since none of the normals intersect, the wave element at G is focussed at its centre of curvature G_1 and the remaining part of the envelope G_1E_1 is convex.

Figure 12 illustrates the method of constructing the wave envelope for the arbitrary motion of a source without resorting to spherical pulse waves. Let the instantaneous forward Mach cones, having the semi-apex angles $\gamma_1, \gamma_2, \gamma_3 \ldots$, be drawn from the corresponding points $A_1, A_2, A_3 \ldots$ on the flight path. The slant side of each cone A_1B_1, A_2B_2, $A_3B_3 \ldots$ will be of length $a\tau_1$, $a\tau_2$, $a\tau_3 \ldots$ where $\tau_1, \tau_2, \tau_3 \ldots$ are the times taken for the source to travel from $A_1, A_2, A_3 \ldots$ respectively to its present position D. The surface drawn touching the base circles of all cones will give the instantaneous Mach wave when the source is at D. The cusps can be located by determining which points satisfy the relation

$$\tau = \frac{a(M^2 - 1)}{f \cos \alpha} .$$

Figures 15 and 16 show a three dimensional sectional model of the wave envelope formed by a source moving in a circle at a constant Mach number of 2. (See also figure 10).

If, in the motion illustrated in figures 10, 15 and 16, ρ

/is the ...

is the radius of curvature in the circular path, the radius of the pulse wave at the cusp in the plane of the motion is, by equation 8.7,

$$R = \rho \cos \mu \qquad \text{.....8.9}$$

and the radius of the circle generated by the cusp is ρ/M.

Similarly in the motion illustrated in figure 11, if the equation of the equiangular spiral in polar coordinates is

$$r = r_o \exp\left(-\frac{\theta}{\sqrt{M^2-1}}\right) \qquad \text{.....8.10}$$

where M is constant, the radius of the pulse waves at the cusp in the plane of motion is

$$R = r . \qquad \text{.....8.11}$$

Hence, the cusp is stationary with respect to the moving source, since the source reaches the centre or origin at the same time as the inner wave. If the source completes only one revolution the internal Mach line traces out a circle.

§9. The shock wave formation around isolated two-dimensional and axisymmetric bodies of revolution

In this paragraph the changes that occur in the system of shock waves around a body in steady, accelerated or retarded flight will be discussed and compared.

(i) Steady motion

Observations of the flow around bodies at steady high subsonic speeds above the critical Mach number show that a small nearly normal shock wave is formed near the surface downstream of the sonic point. With increase in Mach number this shock wave spreads to a greater distance from the surface and at the same time moves back towards the trailing edge. The latter wave is of finite length, since the region of supersonic flow relative to the body is finite also. The shape of the tail wave is in general either bifurcated, with its rear member nearly normal to the surface or wake, or it is nearly normal to the surface itself. The exact shape depends upon the freestream Mach number, the shape of the body, and the interaction with the boundary layer.[13][22] The extent of the tail waves increases as the steady body speed tends to a Mach number of unity.

/At steady ...

At steady supersonic speeds a bow wave is formed ahead of the body. In becomes attached to the nose of the body, if its nose is sharp, above a certain Mach number, which is a function of the nose angle. Between the bow and tail waves there exist expansion waves which are formed either continuously over the surface of a curved body or in the form of a fan of waves at the shoulders of a straight sided body, (see figure 5). The expansion waves are the means by which the flow, relative to the body, is accelerated between the bow and tail waves. The interaction of the expansion waves with the bow and tail waves reduces the latter's strength at a distance from the body. It has been shown by Lighthill[14] and others[15,16] that the shock wave pattern around two-dimensional aerofoils in steady motion at supersonic speeds can be represented by parabolae at large distances.

As the Mach number is reduced the above process is reversed, the bow wave becomes detached at a Mach number depending on the nose angle. The distance between the bow wave and the nose increases as the Mach number approaches unity from above and when the Mach number equals unity the separation distance is infinite.

The pressure change across the bow wave of a two-dimensional aerofoil at large distances is,[15] (see figure 5),

$$\frac{\Delta p}{p_o} = \frac{2\gamma}{\sqrt{\gamma+1}}\sqrt{\frac{y_m}{y}}\left[1 + \frac{3M_o^2 - 4}{4(M_o^2-1)}\sqrt{\frac{(\gamma+1)y_m}{y}} \ldots\right] \qquad \ldots\ldots\ldots\ldots 9.1$$

where M_o is the Mach number of the aerofoil, $2y_m$ is its maximum thickness and y is the distance normal to the aerofoil. The pressure falls between the bow and tail shock waves and increases across the tail wave by a value similar to that given in equation 9.1 above.

The distance apart of the shock waves at distance y from the aerofoil of chord c is

$$d = \frac{c\,M_o^2}{(M_o^2-1)}\sqrt{(\gamma+1)\frac{y_m}{c}\cdot\frac{y}{c}} + \ldots \qquad \ldots\ldots\ldots\ldots 9.2$$

The corresponding calculations by Whitham,[17,18] for a parabolic shaped body of revolution of fineness ratio δ and length ℓ, show that across the bow wave

$$\frac{\Delta p}{p_o} \sim 0.53 \quad (M_o^2-1)^{1/8} \quad \frac{\delta}{(y/\ell)^{3/4}} \qquad \ldots\ldots\ldots\ldots 9.3$$

and a similar result holds for the tail wave. The distance apart of the shock waves is

$$\frac{d}{\ell} \sim 1.82 \quad \frac{M_o^2\,\delta\,(y/\ell)^{1/4}}{(M_o^2 - 1)^{3/8}} \qquad \ldots\ldots\ldots\ldots 9.4$$

/The ...

The formulae above all apply to the case of zero body incidence and where the shock waves close to the body are not too strong. The effect of incidence is roughly to increase the strength of part of the shock front by an amount similar to that obtained with a body of increased thickness.

No attempt has been made to present formulae for the pressure changes across shock waves in subsonic flow. It is known that the strength of these waves is not negligible and it will be shown in the next sub-section that they play a very important part in determining the shock formation in accelerated flight through the speed of sound.

(ii) Accelerated and retarded motion

When a body+ is uniformly accelerated up to high subsonic speeds, below the critical Mach number, the flow pattern around the body at any instant will not be markedly different from that at the corresponding steady speed. The critical Mach number will however be a function of the acceleration but its change from the steady value may not be significant. At flight speeds above the critical the body shock wave begins to develop, but since it cannot be formed instantaneously its form and position will be a little different from that at the corresponding steady speed. Let us consider the growth of this shock wave when an aerofoil is accelerated slowly through the critical Mach number. Just after the local speed of the flow relative to the aerofoil, outside the boundary layer, reaches sonic, a region of supersonic flow develops of finite extent. At its forward end, the expansion waves accelerate the flow, whereas near its rearward end the compression waves, which retard the flow, coalesce and form a weak shock wave approximately normal to the aerofoil surface. As the speed increases so the region of supersonic flow expands and the normal shock, which is increasing in height, moves backwards approximately normal to itself. If boundary conditions are favourable, for instance if boundary layer separation occurs, the disturbances generated by the separation will coalesce to form a second shock wave in front of the first. The formation of the two shock waves will probably be similar to the formation of Mach waves around an accelerated source (see figure 7). The result will be a bifurcated wave, whose rear member is the original normal shock. It can easily be shown that the strength of the rear member will be less than the front member of the tail wave.

/When the ...

+ In what follows the words body and aerofoil will be used to describe the same moving object.

When the subsonic speed is sufficiently high the shock formation will reach the trailing edge (see figure 17). It should be remarked here that since the acceleration has been assumed small, the shock wave pattern at each speed should quickly develop to its corresponding steady state configuration. It can therefore be assumed that, since the steady state shock configuration is fairly well known so the accelerated shock pattern will also be known at high subsonic speeds. Thus so far no new ideas need be introduced when we consider the modification to the shock pattern consequent to an acceleration in the subsonic regime.

This is not the case when we consider the effect of a retardation back to a subsonic speed below the critical. The tail shock waves will move slowly forward towards the nose and will not vanish as soon as the body speed drops below the critical Mach number.+ The shock waves will tend to coalesce into a single wave which moves ahead of the nose. The distance of separation will be a function of the future history of the body. The wave, or waves, will be weak and will be propagated with the speed of sound. They must, however, represent pressure waves of large amplitude and would be heard by an observer together with the remainder of the noise emitted by the body. Since these waves are of finite length and are approximately normal to the flight path when they leave the aerofoil, their effect, relative to the ground, will be very localised.

When the acceleration of the aerofoil is continued so that its speed just reaches the speed of sound, a bow wave is formed, which, unlike the tail wave, remains approximately normal to the flight path, near the body, at least over a finite range of supersonic speeds. The bow wave commences to form ahead of the body and its distance from the body is a function of the acceleration, the time history of the motion, and the body geometry.

/At steady ...

+ It should be noted that experiments in wind tunnels at steady or unsteady speeds will not show these effects. This is because the instantaneous shock formation around the wind tunnel model, during an increase or decrease in speed, is the result of disturbances created at times immediately prior to the given instant. Thus disturbances forming shock waves around the model during the time of accelerated flow cannot themselves form shock waves during the time of retarded flow, since by then they will have moved far downstream. When the freestream Mach number falls below the critical the shock waves disappear.

At steady sonic speed the bow wave will be at an infinite distance ahead of the nose, but in this case the motion is implicitly assumed to have been going on for an infinite time previously. We must expect therefore that for all practical flight cases the bow wave is at a finite distance from the nose. The bow wave is not formed instantaneously. Thus for a body initially at rest the bow wave can strictly speaking only reach its steady state configuration after an infinite time. It should also be noted that no proof has yet been advanced as to whether the bow shock wave, produced by the motion of an accelerated bluff body, occurs when the speed is just sonic or slightly less than sonic. It is known[15] that in one-dimensional flow shock waves occur ahead of bodies at subsonic speeds if they are accelerating. In general as a body accelerates above the speed of sound the bow wave, as stated above, remains approximately normal to the flight path, near the body, and it will become attached to the body at a sufficiently high supersonic Mach number depending on the nose shape. Away from the body axis, the interaction of the expansion fan with the shock wave reduces its inclination relative to the flight path. It must be emphasised that the field of flow associated with a body of finite size particularly ahead of it, inhibits the formation of a concave bow wave having a cusp similar to that shown in fig. 7. If an acceleration occurs after a body has reached a steady supersonic speed, the cusps which are formed in the picture of Mach waves (see figure 9) are not reproduced with shock waves. A simple analysis of the flow in the loop behind the front wave will show why shock waves, with expansion waves interacting with them, cannot generate this pattern. The experiments in shallow water, discussed below, show that in all the observed continuous motions so far, the bow wave has remained continuous also.

Let us now consider what changes occur in the tail wave pattern as the accelerated body exceeds the speed of sound. It has been shown above that when the body reaches sonic speed the front member of the tail wave, in general, is inclined backwards, relative to the direction of motion, near the body and becomes more normal at a distance. Also, in general, a rear nearly normal wave exists in the wake of the front member of the tail wave, and is attached to it at its outer extremity. As the speed of the body increases so the distance apart of the two parts of the tail wave increases also, since the front member is attached to the body, and the rear member is moving normal to itself at approximately the speed of sound. The suggested shock formation for a body accelerated from subsonic to supersonic speeds is a bow wave ahead of or attached to the nose, and separated from the front tail wave by a system of expansion waves, and a rear transverse

/wave ...

wave joined to the front tail wave at its outer extemity. (See figure 17).

When the body is retarded (see figure 18) and its speed falls below the critical Mach number[+] the bow wave will now be far ahead of the nose since it detaches itself from the body when the speed is a little above sonic.[++] The front tail wave will move slowly forward towards the nose and in doing so becomes approximately normal to the surface. Its shape changes and its curvature becomes convex in the direction of motion. The rear tail wave moves towards the aerofoil, overtakes it and finally, it also moves ahead of the body. The suggested shock formation ahead of a body, retarded from supersonic motion of finite duration to subsonic, is a bow wave separated from the front tail wave by a system of expansion waves and trailing further behind a rear transverse wave joined to the front tail wave at its outer extremity. If these three shock waves were formed around a body whose maximum speed only just exceeded the speed of sound, the waves will all be approximately plane and a simple calculation[(15)] will then show that the rear tail wave must eventually overtake the front tail wave, whilst the bow wave must move faster than the front tail wave due to the expansion waves between them. Thus at a distance from the body we must expect to find either three or two shock waves. For higher maximum body speeds a similar result is to be expected.

Thus for a simple body or aerofoil the formation of shocks far ahead of the region of accelerated and retarded motion, in which the motion of the body just exceeded the speed of sound, is not unlike the simple Mach patterns for a source executing a similar motion. The distance apart of the 'two' waves can be crudely calculated from the time history of the motion and in particular from the positions on the flight path at which sonic speed and the critical Mach number was reached during the acceleration and retardation. A similar answer will be obtained if it is assumed that the distance between the bow and front tail waves, increases according to $\sqrt{t}$ [(15)] where t is the time

/after ...

+ It appears that the critical Mach number in steady motion will be different from that in accelerated or retarded motion. In the latter cases the instantaneous flow pattern is a function of the previous time history of the body.

++ The bow wave will commence to move away from the nose of the body at a low supersonic Mach number which is a function of the body shape.

after these waves detached themselves from the body. This result will only apply to plane or nearly plane waves and will therefore be approximate only in a range of body speeds close to sonic.

The results of the shallow water experiments discussed below, support the above qualitative argument. However, the possibility cannot be excluded that the arguments will require modification in the light of more detailed experiment and a rigorous analytic treatment.

An experimental investigation of this problem could only be done conveniently by performing a series of free flight tests under fixed conditions of acceleration and retardation. The results obtained from projectiles fired from guns are not suitable, since in this case the bow wave is formed first at the instant the projectile penetrates the gun wave. The bow wave is therefore of greater extent than the tail wave as is shown in figures 19 and 20.[19] The rear tail wave in figure 20 bears a striking resemblance to the rear tail waves obtained behind moving bodies in shallow water.

Very little can be stated about the magnitude of pressure across the shock waves far distant from the accelerated and retarded body. For straight flight paths the pressure change is probably not very different from that calculated, at a similar distance, for a body moving at a steady speed corresponding to the maximum speed during the manœuvre. Alternatively we can assume that the pressure excess is inversely proportional to the distance from the source or origination of the whock wave. This result is correct for the expansion outwards of a weak spherical shock wave when interaction with expansion waves is neglected. For curved flight paths the approximate theory of Warren[1] could be used.

§10. Experiments in a hydraulic analogy channel

A series of simple experiments have been performed on bodies moving in shallow water in order to establish the shock pattern around bodies in accelerated and retarded motion. It is well known[20] that the surface gravity waves propagated by moving bodies in shallow water are analogous to the infinitesimal disturbances caused by the same two-dimensional body moving through a compressible gas. In water the wave velocity is a function of the wave length but if a depth of $h = 0.25$ inches is chosen[21] for ordinary tap water, the wave velocity, c, will be independent of the wavelength, λ, except for the small capillary waves. Thus, when $h/\lambda \ll 1$

$$c = \sqrt{gh} \qquad \text{.....10.1}$$

It has been shown that the analogy applies only to a perfect gas whose ratio of specific heats, $\gamma = 2$. The analogy is not exact between shock waves propagated in a gas and bores or hydraulic jumps+ formed in water.(21) However, if the analogy is restricted to values of M 1.5 the differences can be neglected. It will be shown in the tests described below that large numbers of capillary waves form upstream of the bores. Their presence is unavoidable although their effect on the formation of the bores is almost negligible.

Since the strength of the bores, during an acceleration of a body to supercritical speeds++ and after a retardation to subcritical speeds, are very small the reproduction of observed formations of bores by photographic methods is extremely difficult. The height of the bore corresponding to a pressure discontinuity of 10 lb. per sq.ft. in air is about 0.001 inches. The apparatus used was not designed for this investigation and great accuracy cannot be claimed for the results obtained.

(i) Apparatus

The tests were performed in a tank 5ft. long by 3ft. wide having a glass bottom (see figure 21). The model was attached to a carriage mounted on rails fixed over the tank. The carriage was either moved by hand or its acceleration and deceleration were controlled by springs, fixed at one end to earth and to the carriage at the other end. In the case of method (b) below, the spark was operated by means of a metal strip, placed on the side of the tank, coming in contact with a microswitch fixed to the carriage. Ordinary tap water was used to a depth of 0.25 inches. Photographic records were taken as follows.-

(a) The glass bottom was covered on the inside with a flat metal plate. Glancing light was obtained from three 250 watt photo-flood lamps. A 35mm camera was held either directly above the model or at an acute angle to the water surface.

(b) The metal plate was removed and the water surface was illuminated by a short duration spark placed 4ft. below the tank bottom. The exposures were made direct on Kodak bromide WSG 1.S paper placed 0.25 inches above the water surface. For these

/tests ...

+ In hydraulics the term hydraulic jump is applied only to a stationary discontinuity in water surface and a moving discontinuity is known as a bore.

++ In this section apart from subsection (iii) the terms subcritical and supercritical will be used to denote body speeds less than or greater than $\sqrt{gh}$.

tests a grid of wires was suspended below the tank bottom from the moving carriage.

(c) The surface of the water was completely covered with aluminium particles to give a strong reflection from the surface. The exposures were taken with a 1/4 plate camera.

The following models were used

(1) 0.5in. chord and 0.18in. maximum thickness double wedge section

(2) 3.0in. chord and 1.0in. maximum thickness

The results obtained by use of the three methods (a), (b) and (c) above were very similar. Best results were, however, obtained with method (c), and it is for this reason that this series alone will be discussed below.

(ii) Experimental results

Method (c)

Typical results are shown in figures 22 and 23. They show the growth of the bow wave and the front tail wave as well as the rear tail wave during an acceleration and retardation from subcritical to supercritical, and back to subcritical speeds. Figure 24 shows the modified wave pattern when an aerofoil is accelerated to supercritical, retarded to critical, accelerated back to supercritical and finally retarded to subcritical speeds. Figure 25 shows the wave pattern around tandem aerofoils during an acceleration and retardation from subcritical to supercritical, and back to subcritical speeds. Figure 26 shows the wave pattern around an aerofoil moving in a circle. Figure 27 shows diagrammatically the changes in the water level associated with the accelerated motion and should assist in an appreciation of the wave patterns detailed in the above photographs.

(iii) Discussion of results

Detailed comments on the results are unnecessary since they confirm qualitatively the broad conclusions stated in the earlier parts of the paper. The main results+ are as follows.-

(a) Above the critical Mach number the front tail wave is formed and at approximately sonic speed the rear tail wave appears,

/connected ...

+ The results will be discussed in terms of the analogous motions of the aerofoils through air. Thus the terms subsonic, critical and supersonic Mach numbers have their usual meanings. Since all the results refer to accelerated and/or retarded motion of an aerofoil the words accelerated and retarded will only be used where it is essential to avoid confusion.

connected to it at its outer extremity.

(b) The rear tail wave is very weak except near its point of attachment to the front tail wave.

(c) As the speed increases the rear tail wave moves backwards relative to the aerofoil.

(d) When the speed falls below sonic, the bow wave becomes detached and moves off ahead of the nose. The front tail wave changes shape and becomes more normal close to the aerofoil.

(e) The front tail wave moves slowly over the aerofoil surface and detaches itself from the nose when the speed falls below the critical Mach number. It follows in the wake of the bow wave.

(f) The rear tail wave overtakes the aerofoil, passes over it, and follows in the wake of the front tail wave. It overtakes the latter in finite time which depends on the time history of the motion in the high subsonic and supersonic parts of the flight path. There are always one, two or more shock waves ahead of the nose of the aerofoil when it returns to subsonic speeds below the critical.

(g) When an aerofoil is accelerated to supersonic, retarded to subsonic, accelerated to supersonic and then retarded to subsonic, the formation of shock waves ahead of the nose depends entirely on the time history of the motion and in particular on the speed reached during the first retardation. Two, three or four shock waves will result although some of the waves will coalesce after a finite time has elapsed.

(h) The shock patterns around tandem aerofoils depend on their distance apart and their respective dimensions.

§11. Discussion on the noise received by ground observers when the time of supersonic flight is finite

(i) Uniform atmosphere

It has been shown in paragraph 7 above, that the noise received by a ground observer from a body travelling at supersonic speeds for a finite time is mainly a function of the shock waves around the body at each instant during the manoeuvre, together with the noise associated with the jet and boundary layers etc.

/In order ...

In order to see clearly the nature, intensity and duration of the noise let us consider a specific example. A jet propelled wing of chord length 30ft., having a critical Mach number of 0.9, is in a vertical dive. It accelerates through a Mach number of unity at 35,000ft., attains a maximum Mach number of 1.05 at 30,000ft., and decelerates through a Mach number of unity at 25,000ft. The atmosphere is assumed to be uniform throughout and its density and speed of sound are 0.001 slugs per ft.^{3} and 1,000ft. per sec. respectively. This example was considered in reference (1) and a similar example was considered in reference (2). In the first instance let us consider the noise received by an observer positioned on the ground directly below the aircraft. Since the aircraft speed is subsonic, below the critical, from 60,000ft. down to 36,000ft. the main noise emitted in this region is from the jet. If we assume that the sound radiators emitting noise upstream of the jet are at rest relative to the aircraft, and that the jet noise is independent of the aircraft speed but is proportional to the square of the distance from the aircraft, a simple calculation shows that the first noise arriving at the ground comes 60 secs. after the aircraft commenced its dive. The noise level is, say, 74.5 decibels or 0.001 lb. per sq.ft. r.m.s. pressure. The noise level remains approximately constant for a further 140.4 secs. (see figure 28), during which time the aircraft has reached a height of 37,000ft. and a Mach number of 0.8.

Between 36,000ft. and 35,000ft. shock waves begin to form over the rear of the wing. The noise from the jet that is emitted forwards can no longer escape past the nose of the body and must pile up behind the tail shock wave. This shock wave continues to expand and forms its rear member at about sonic body speed, that is when the aircraft has reached 35,000ft. The bow wave now forms and increases in dimensions as the aircraft falls a further 10,000ft. During this same period the rear part of the tail wave falls behind the aircraft and continues to encompass the sound waves produced by the jet between 60,000ft. and 35,000ft. which did not escape in front of the aircraft, as well as the noise emitted between 35,000ft. and 25,000ft. which was either propagated in the downstream direction from the region between the two members of the tail wave, or was created in the jet downstream of the rear tail wave. The remainder of the jet noise emitted between 35,000ft. and 25,000ft. will be confined to the region between the front and rear members of the tail wave. At 25,000ft. the bow shock wave separates from the nose of the wing and reaches the ground approximately 200.8 secs. after the commencement of the manoeuvre. Figure 28 also shows that some of the noise emitted at 36,500ft. arrives at the same instant.

/However ...

However the intensity of the latter noise is small compared with 2 lb. per sq.ft. which is the estimated increase in pressure across the bow wave near the ground according to reference (1). The expansion waves will now meet the ground and will tend to reduce the excess pressure to a suction. However, in this same period the reflection of the bow wave from the ground, and adjacent buildings, will pass the observer (i.e. about 0.01 secs. after the passage of the bow wave). Thus the bow wave and its reflections will give rise to a one prolonged crack or boom.

Let us now follow the path of the front tail wave. It cannot leave the aircraft until the speed has dropped to the critical Mach number of 0.9 which occurs at 19,000ft. Its rear wave has lagged behind and although it was 230ft. behind the nose at 25,000ft. it catches up with the front tail wave at about 20,000ft. Hence when the tail wave separates from the nose it is 230ft. behind the bow wave. The tail wave reaches the ground 0.23 secs. after the bow wave and is also heard as a prolonged crack or boom. The increased distance between the bow and tail waves due to the spreading effect of the expansion waves will cause the delay time, between the arrival of the two shock waves at the ground, to increase to about 0.24 secs. The rise in pressure across the tail shock cancels out the suction produced by the expansion waves. The mean pressure aft of the tail wave, neglecting the reflection effects, is approximately ambient atmospheric pressure.

Part of the jet noise created between 25,000ft. and 35,000ft., in that order, will be heard immediately afterwards. This will be followed by the remainder of the jet noise created between 25,000ft. and 35,000ft. and the 'subsonic noise' emitted between 35,000ft. and 60,000ft. The jet noise spectrum will be modified by Doppler effect and the jet noise will appear to the observer, first as a high pitched shriek followed by a low pitched roar.

<u>In this example, therefore, two bangs or booms are heard, each of about 0.02 seconds duration, separated by about 0.25 seconds.</u>

If in a similar example the Mach number fell to 0.9 at 24,000ft., instead of 9,000ft. the front tail wave would leave the aircraft only 50ft. behind the bow wave and the rear tail wave would still be 180ft. further behind the front tail wave. Hence the time interval between the bow and front tail waves striking the ground would be 0.05 secs. and the rear tail wave would follow about 0.20 secs. later. The first and second booms would be of considerably greater amplitude than the third.

/If we ...

If we consider typical examples relating to the flight of missiles at low altitudes we can show that the time delay between the bow and tail waves may be little greater than 0.03 seconds and the rear wave arrives about 0.20 secs. later. In this case the time delay between the bow and tail waves, allowing for the effects of reflection, reverberation and the aftersound, is insufficient for two separate booms to be heard. An observer will experience one large boom followed 0.20 secs. later by one weak boom.

We have considered above, the sequence of the booms and jet noise which is heard by an observer stationed directly under the aircraft. Let us now consider the noise heard by an observer who is in line with the normal to the freestream Mach wave at the position along the flight path at which the aircraft reached its maximum speed. He will hear both the bow and front tail waves only, since the rear tail wave will be either very close to or will have overtaken the front tail wave. Hence either one or two booms will be heard depending on whether the time delay between the two shock waves is less than or greater than 0.02 secs. approximately. The sequence and number of booms depends critically on the flight path and the position of the observer.

When the maximum speed of the aircraft exceeds the Mach number corresponding to attached shock waves at the nose of the body, wings etc., the simplified picture above will not now hold. The number of shock waves which are propagated forwards when the aircraft speed falls below the critical Mach number may exceed three. The number depends on the aircraft geometry and the time history of the manoeuvre. In order to determine whether an observer will hear booms or not, it will probably be sufficiently accurate, for many practical purposes, to construct the complete Mach wave pattern (see figure 12) at each instant along the flight path and beyond, around a moving point source which replaces the aircraft. In this simplified approach the criterion that an observer will hear a boom, is that the component of the speed of the source along the instantaneous line joining the source to the observer must be sonic.[1] + This criterion clearly does not strictly apply to the body of finite size even though the shock waves far distant from the body are weak.

/(ii) ...

+ By definition, this line, along which the component of the source speed is sonic, is normal to a Mach wave.

(ii) Non-uniform atmosphere

When shock waves are propagated in a non-uniform atmosphere they suffer refraction due to variations in wind velocity and temperature with altitude. If we assume that far distant from the body the shock waves are travelling at the speed of sound normal to themselves, then a simple calculation[4] shows that the condition for the waves originating at height, h_1 say, to be refracted parallel to the ground at the height h_2 is (see figure 29)

$$\frac{a_1 \sin \beta_1 + v_1}{a_2 \sin \beta_2 + v_2} = \sin^2 \phi_1 \qquad \text{.....11.1}$$

where β is the angle the sound ray, (i.e. the normal to the Mach wave), makes with the ground normal; ϕ is the angle the sound ray makes with the ground normal when the wind velocity is included; a is the speed of sound; v is the horizontal component of the wind velocity. Suffix '1' refers to conditions at a given altitude h_1 and suffix '2' to the altitude h_2 at which the rays are parallel to the ground.

If θ is the angle the flight path makes with the ground normal when the body speed is U_1, then if $v = 0$ and μ is the Mach angle,

$$\beta = \phi = \mu - \frac{\pi}{2} + \theta \qquad \text{.....11.2}$$

and equation 11.1 reduces to

$$U_1 = \frac{a_2 \sin \beta_1}{\sin \mu_1} \qquad \text{.....11.3}$$

Thus when θ equals $\frac{\pi}{2}$ equation 11.3 shows that the waves will be refracted back at height h_2 where the speed of sound, a_2, is equal to the speed of the body, U_1, at height h_1.

If an aircraft+ is in a shallow dive at $\theta = 75^\circ$ to the ground normal at $h = 40{,}000$ft., and the Mach number at this instant is 1.1, equation 11.3 shows that refraction of the rays upwards will commence at 25,000ft. Whether or not the remaining waves will reach the ground depends on the spatial extent of the wave which in turn is critically related to the time history of the aircraft motion.

(iii) Steady flight at supersonic speeds

Although at first sight this problem should afford a simple solution, regarding the time sequence and the intensity of

/noise ...

+ In this example the aircraft is represented as a point source. The atmosphere is assumed to be the Standard Atmosphere.

noise produced by the shock waves passing a ground observer, it still requires a detailed knowledge of the shock pattern around the complete aircraft. Once this is known the solution follows from equations 9.1 to 9.4 inclusive. It is worth noting (see figure 30) that for a given aircraft at a given height the pressure increase across the shock waves does not increase rapidly with increase in Mach number. When the flight path is parallel to the ground (i.e. $\theta = \frac{\pi}{2}$) equation 11.3 shows that the shock wave pattern will not penetrate below altitudes at which the local speed of sound is greater than the forward speed of the aircraft.

It can be shown from equation 11.1, for bodies moving at uniform supersonic speed and at constant altitude, that it is only for forward speeds at low supersonic Mach numbers that the shock waves are refracted back to higher altitudes. Thus for flight just above sonic speed at high altitude the shock waves will not be transmitted to ground level. The noise heard by a ground observer would then be limited to that of the jet etc.

§12. Future problems related to the noise from bodies flying at supersonic speeds

In this section some of the more important problems are discussed which will in all probability require consideration before frequent supersonic flights at both low and high altitudes could be readily tolerated over towns and cities.

(i) Ground level effects

It has been shown in the discussions above, that for an aircraft flying at supersonic speeds the shock waves, which are formed around the aircraft, will be transmitted, in general, to ground level. The intensity of these shock waves increases greatly, not as might have been expected from increase in speed, but from a decrease in the altitude of flight. The time interval for the passage of these shock waves will in general be small, but it must not be overlooked that the pain and/or physical distress to persons exposed to shock waves of frequent occurrence may not be tolerable. It is stated in reference 23 that a high noise level of 140 decibels above 0.0002 dyne per cm^2 causes physical pain whereas a noise level of 160 db causes mechanical damage to the inner ear.+ These values of noise level correspond to pressures about 4.0 and 40lb per sq.ft. r.m.s. respectively. It can be

/shown ...

+ Nothing has been said relative to the damage of property by shock waves. It is felt, however, that this problem has been discussed much more fully in papers relating to the damage of buildings exposed to high explosive blast. In this connection it is worth noting that 160 decibels noise level corresponds to a pressure about 40 lb. per sq.ft. r.m.s.

shown from figure 30, however, that provided aircraft in straight and level flight at supersonic speeds operate at altitudes above 5000ft. these values of pressure excess are unlikely to be exceeded. If a limit of 120 db above 0.0002 dyne/cm^2 is set then aircraft must operate at altitudes above 20,000ft. at a Mach number of 2.0 and above 22,000ft. at a Mach number of 3.0. There still remains the problem of assessing the allowable height above which aircraft manoeuvres, such as diving, must be restricted in order to ensure that the shock waves reaching ground level have a pressure excess below the limiting values above. It has been stated in section 9 that the pressure change across the shock waves emitted by an aircraft in a vertical dive, during which supersonic speeds were reached for a finite time, can be calculated approximately as follows. The pressure change at h feet below the point on the flight path at which the maximum supersonic speed was reached, is assumed equal to the similar pressure change at a normal distance of h feet from the same body moving at a steady Mach number equal to that of the maximum Mach number during the dive. It immediately follows that the same conditions for noise abatement apply to aircraft both in steady and diving flight. Thus if a limit of 120 db above 0.0002 dyne per cm^2 is set, the tentative conclusion is that aircraft must fly supersonically at altitudes in excess of 20,000ft.

(ii) Flying problems

In this subsection we will consider some of the more obvious problems which must arise from the flight of aircraft at supersonic speeds.

We have shown above that the tail shock waves move towards the nose of a body as it decelerates below the critical Mach number. The pressure changes across these shock waves will, in general, be very much greater than 40 lb. per sq.ft. and it is therefore pertinent to ask the question as to whether or not the pilot, crew, and passengers will experience a very loud explosive bang as these shock waves travel over the body. The probable answer is that the passage of the shock wave will be heard but it seems likely that the increase in noise level and the time duration will be decreased and increased respectively as compared with booms heard at ground level. This is inferred from the fact that the pressure rise across the shock wave suffers appreciable diffusion in the subsonic viscous boundary layer adjacent the body surface.

A similar problem relates to the noise experienced inside the cabin of a low speed aircraft as an aircraft moving at supersonic speeds flies past. The separation distances of the two aircraft are assumed to be less than 4 miles. (If the distance

/is greater ...

is greater the problem can be discussed on similar lines to that described in subsection (i) above). Although in such cases some diffusion of the pressure rise across the shock waves, which strike the low speed aircraft, still occurs in the viscous boundary layer adjacent to the surface of the low speed aircraft, the amount of diffusion will probably be insufficient to suppress the boom. Here again we are forced to the conclusion that to avoid very unpleasant or even dangerous effects supersonic flight of aircraft may have to be limited to the region outside a radius of at least four miles from either the nearest aircraft or ground level.

Although it is outside the scope of this paper to discuss the problems of stability and control of aircraft in accelerated and retarded flight at subsonic and supersonic speeds consequent on the movement and/or passage of shock waves across the wings or body there can be little doubt that these effects are important. These problems are made more difficult since the theory has yet to be completed and relevant experiments in the wind tunnel are difficult to perform. The best approach is probably that of free flight experiments under conditions of fixed acceleration and retardation. A point that may need consideration is the possible effects on the control characteristics of a low speed aircraft as shock waves, from passing supersonic aircraft, travel over their control surfaces. It is clear that the advent of supersonic flight will bring with it a number of problems and possibly hazards of a kind that we have not hitherto had to consider seriously.

§13. Acknowledgement

The authors wish to thank many of their colleagues for help and criticism relating to the preparation of this paper. The subject has attracted wide and considerable interest, and it must be acknowledged that many of the ideas mentioned in this paper have arisen out of discussions both inside and outside the College of Aeronautics.

§14. Conclusions

(1) It has been shown that the noise problem associated with an aircraft flying at supersonic speeds depends primarily on the shock wave pattern formed by the aircraft.

(2) When a body accelerates to supersonic speeds three main shock waves are formed, viz. the bow wave, the front tail wave and

/the rear ...

the rear tail wave. The existence of the rear tail wave and related phenomena has been established from a theoretical discussion of the Mach waves generated by an accelerated and retarded source and experimentally by the use of towed model in a hydraulic analogy tank. The shock waves are each of finite dimensions which depend on the time history of the motion. It is noted that the steady shock wave pattern is strictly speaking established only after the steady motion has persisted for an infinite time.

(3) When the body now decelerates to subsonic speeds below the critical Mach number it is shown that (i) the bow wave moves away from the nose of the body when the speed has fallen to a low supersonic Mach number, (ii) the front tail wave changes shape and moves slowly towards the nose of the body and separates when the speed has fallen below the critical Mach number, (iii) the rear tail wave moves towards the tail of the body, eventually travels over the body and follows in the wake of the bow and front tail waves. Under favourable conditions the rear tail wave coalesces with the front tail wave before the speed of the body has fallen below the critical Mach number.

(4) An analysis of the time sequence and intensity of these moving shock waves, which are formed say by an aircraft diving from a high altitude when the Mach number exceeds unity for a limited period, shows that they must be the cause of the booms experienced recently by ground observers.

(5) The effects of these shock waves, formed by bodies in both steady and unsteady flight, on people on the ground and near other flying aircraft are analysed. It is shown that provided aircraft flying at supersonic speeds up to Mach number of 3, restrict their flights to altitudes greater than 22,000 ft., and are at least 4 miles from the nearest aircraft, the noise level experienced by observers will be less than 120 decibels (ref. 0.0002 dynes per cm^2).

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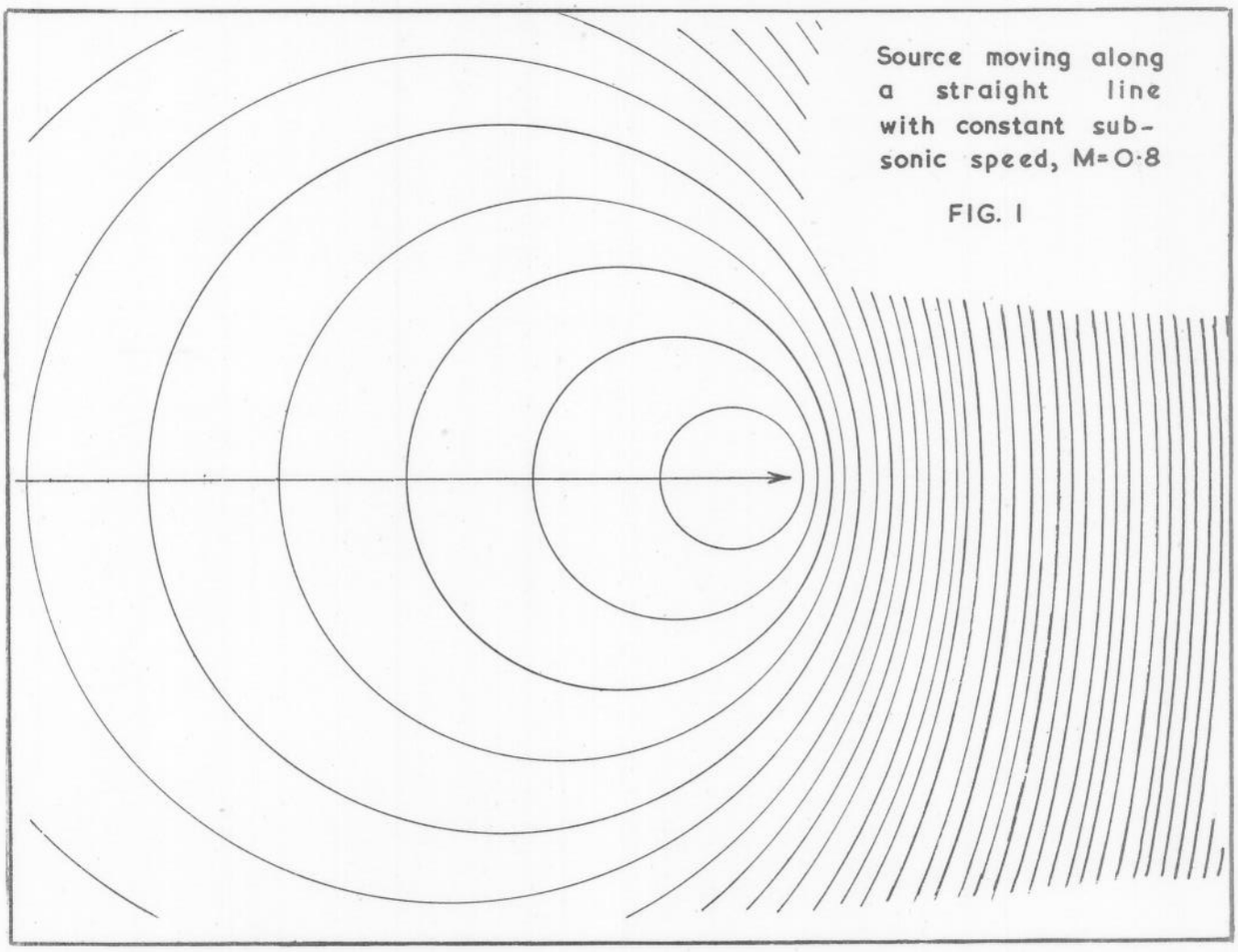
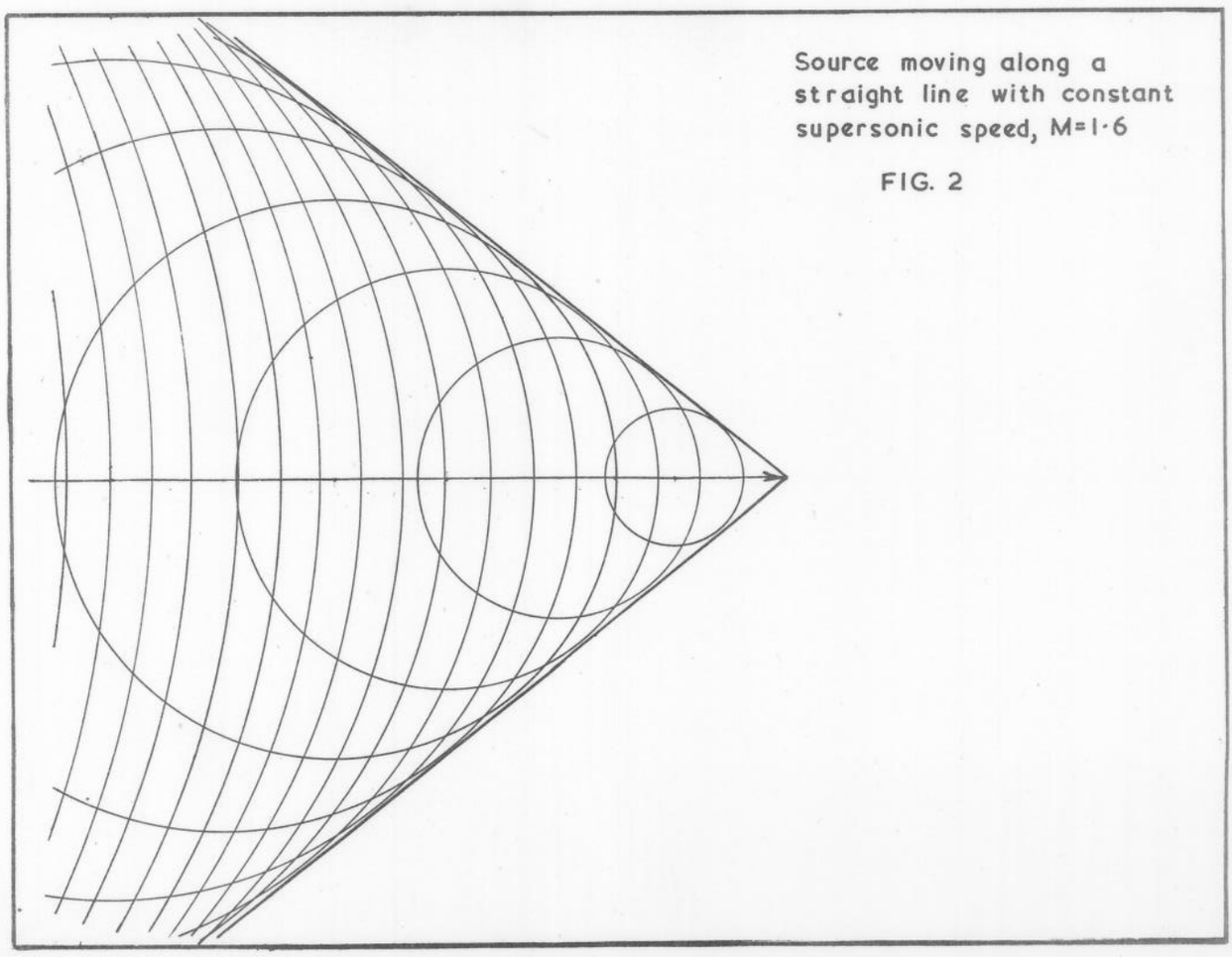


Source moving along a straight line with constant sub-sonic speed, M=0·8

FIG. 1



Source moving along a straight line with constant supersonic speed, M=1·6

FIG. 2

FIG. 3.

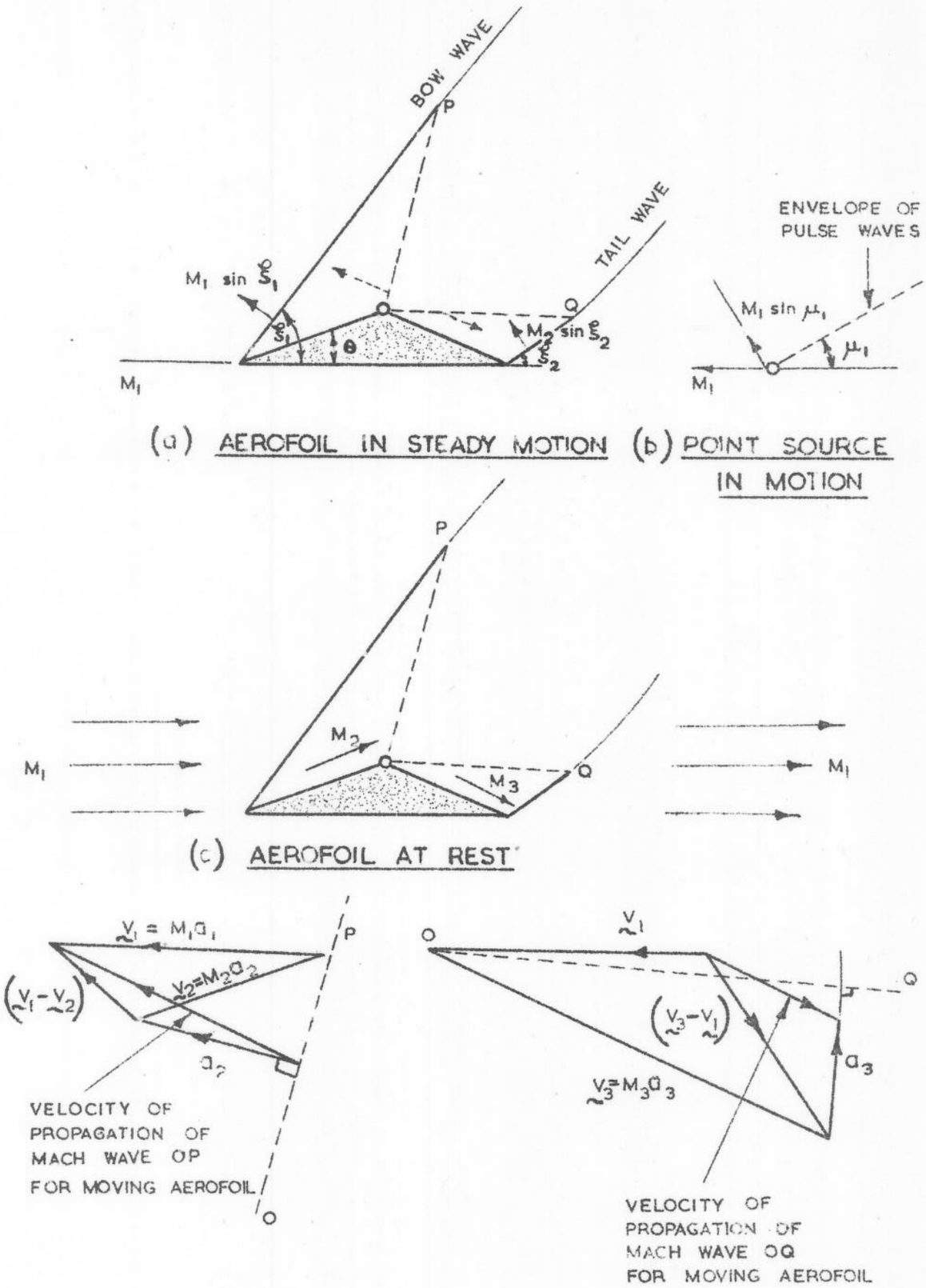
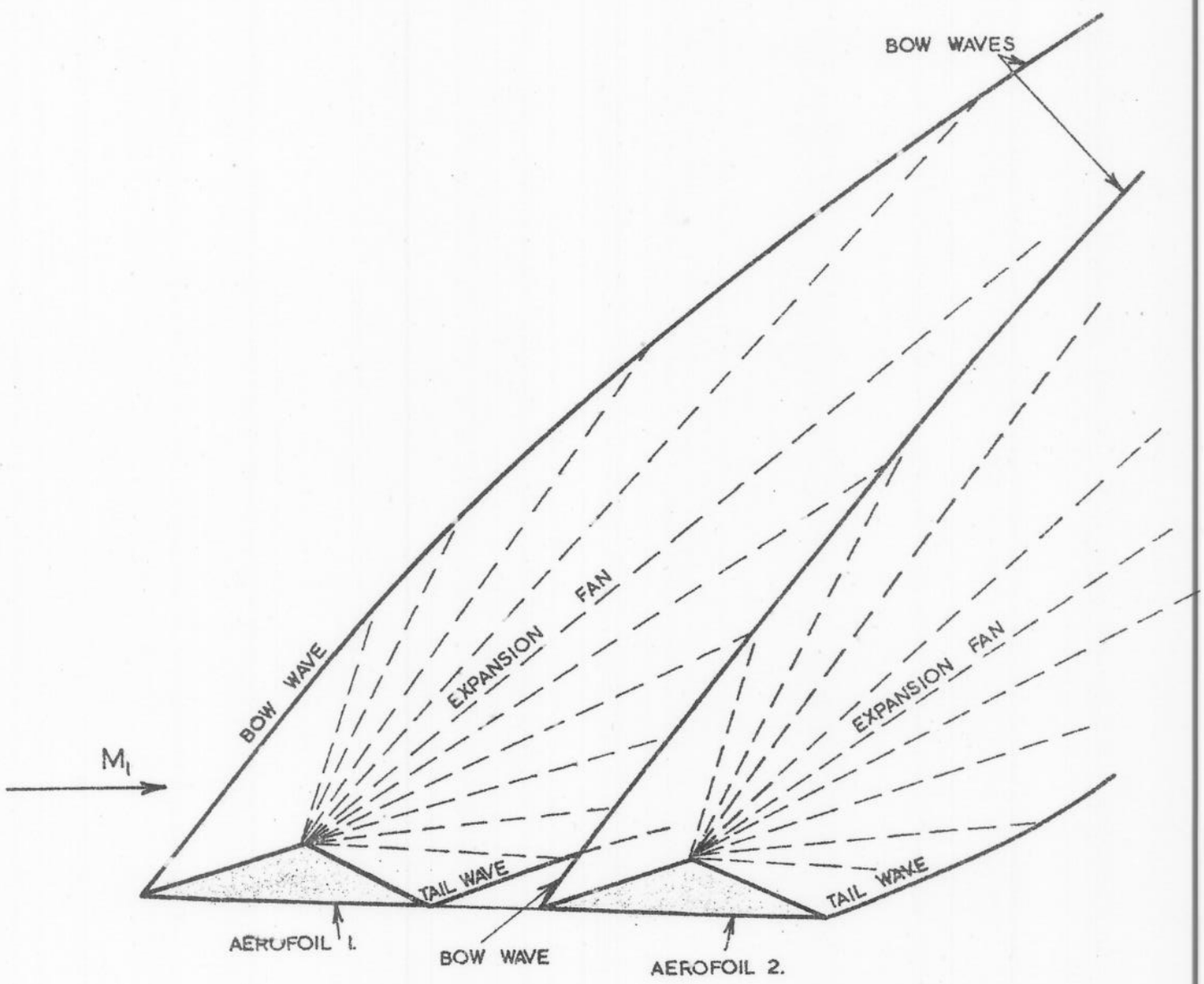


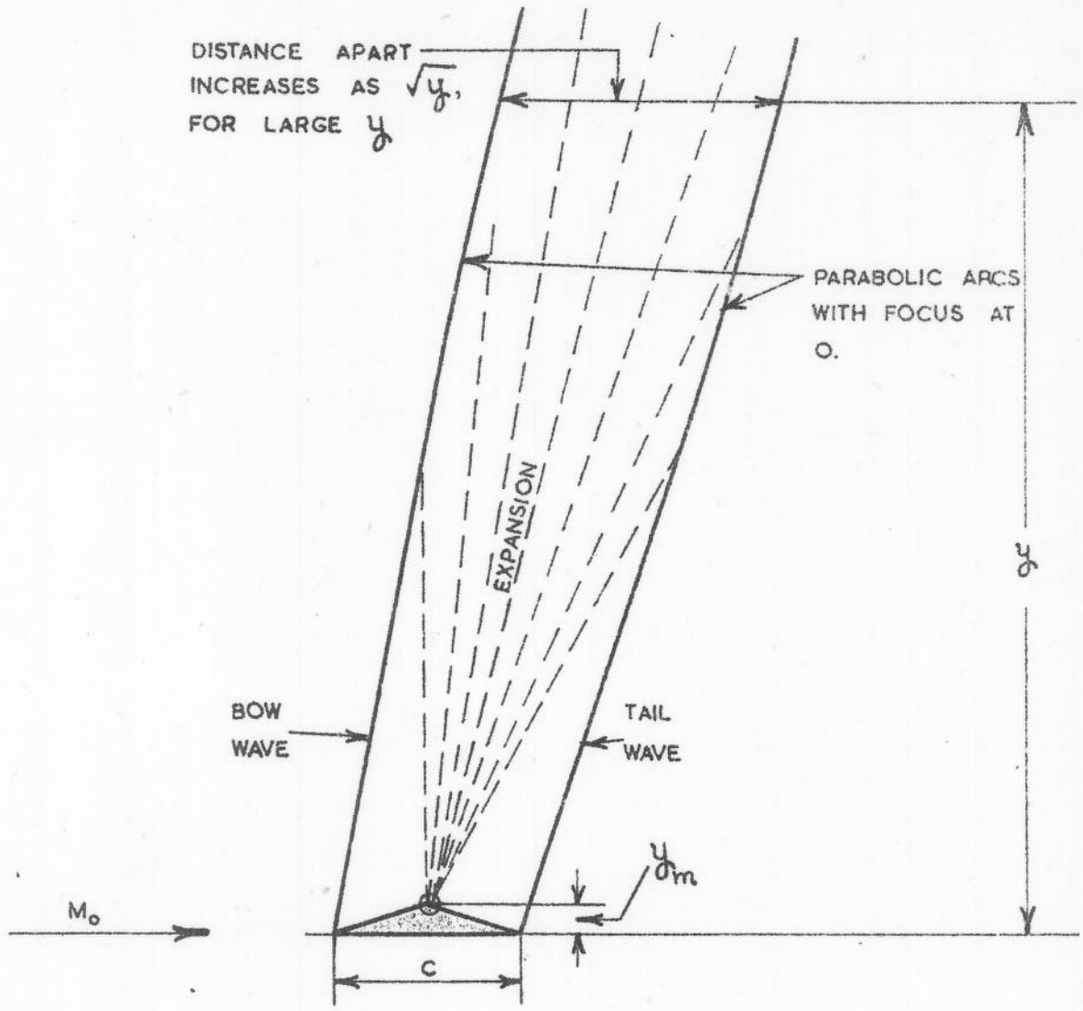
DIAGRAM SHOWING THE PROPAGATION OF SHOCK WAVES AND MACH WAVES FOR A TWO-DIMENSIONAL AEROFOIL IN STEADY MOTION.

FIG. 4.

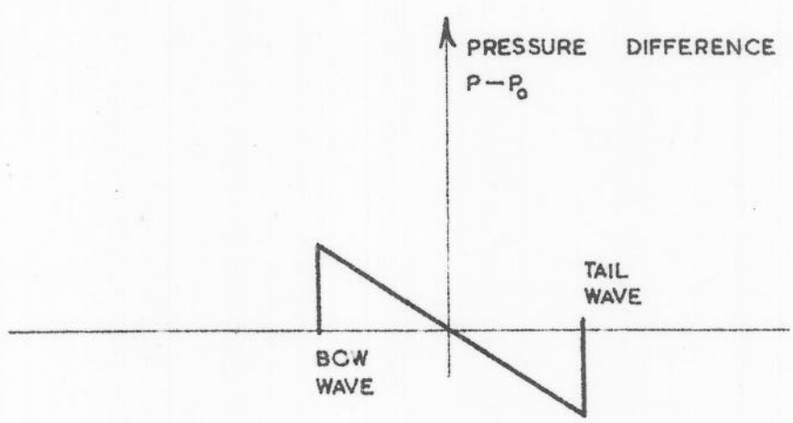


APPROXIMATE FORMATION OF SHOCK WAVES AROUND TWO ADJACENT AEROFOILS.

FIG. 5.



a. SHOCK WAVE AROUND A WEDGE SHAPED AEROFOIL AT STEADY SUPERSONIC SPEEDS.



b. PRESSURE SIGNATURE OF SHOCK WAVES AT A LARGE DISTANCE FROM A TWO-DIMENSIONAL AEROFOIL MOVING AT STEADY SUPERSONIC SPEEDS.

DIAGRAM SHOWING THE FORMATION OF SHOCK WAVES AROUND AN AEROFOIL IN STEADY MOTION.

FIG. 6.

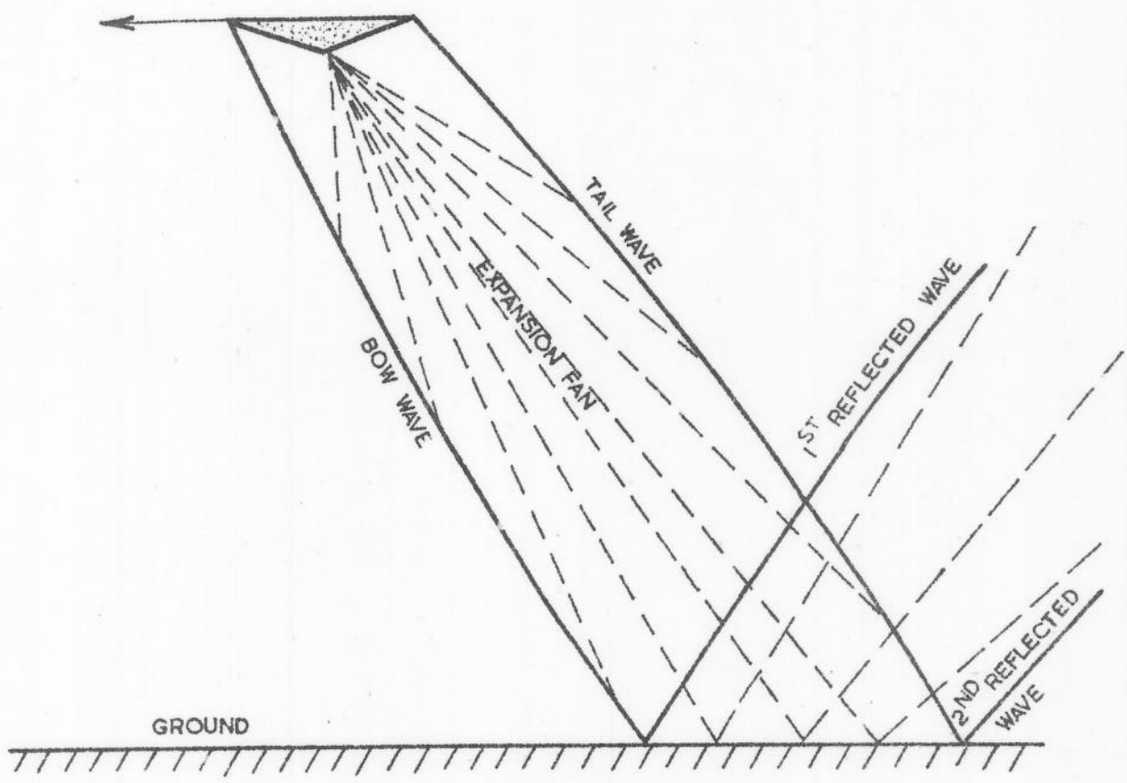
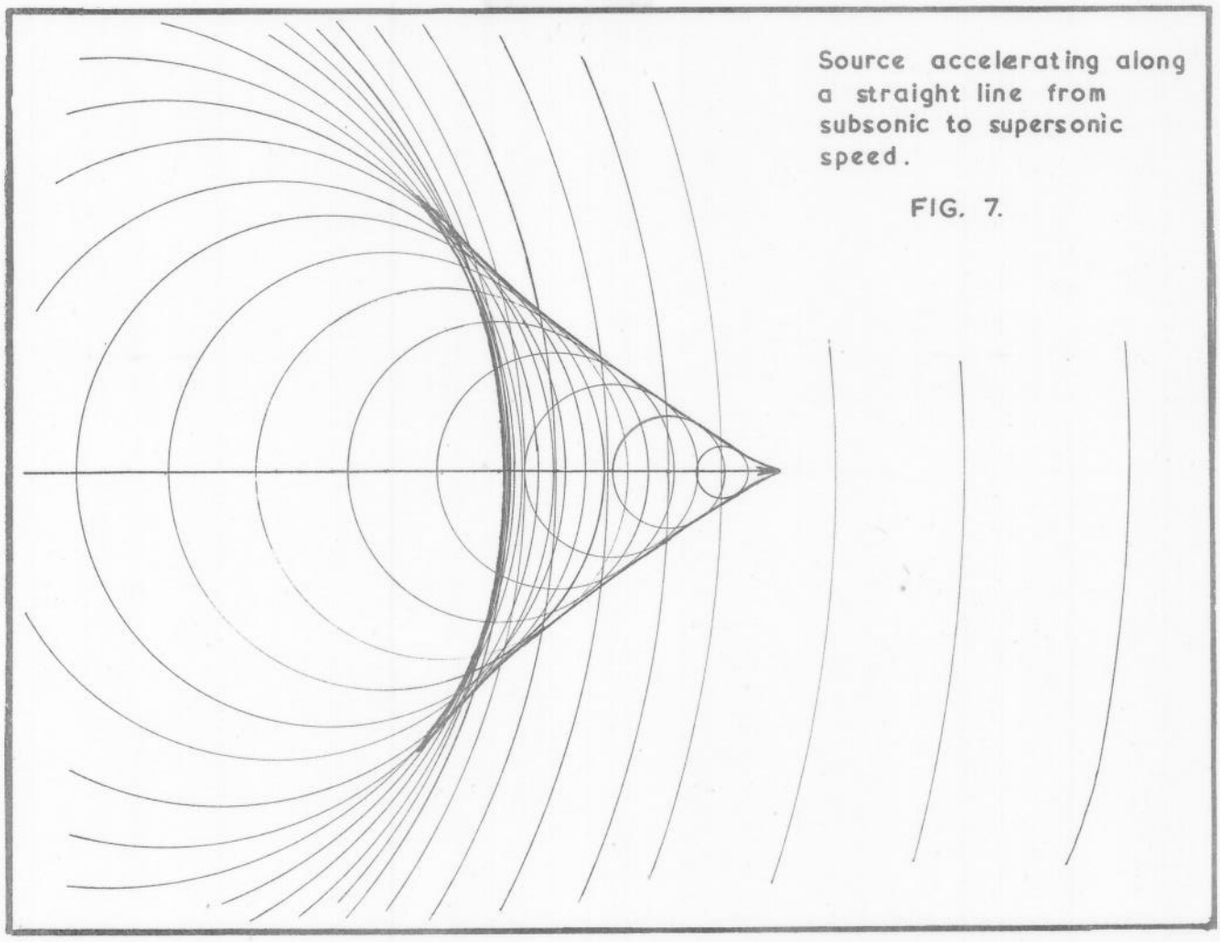
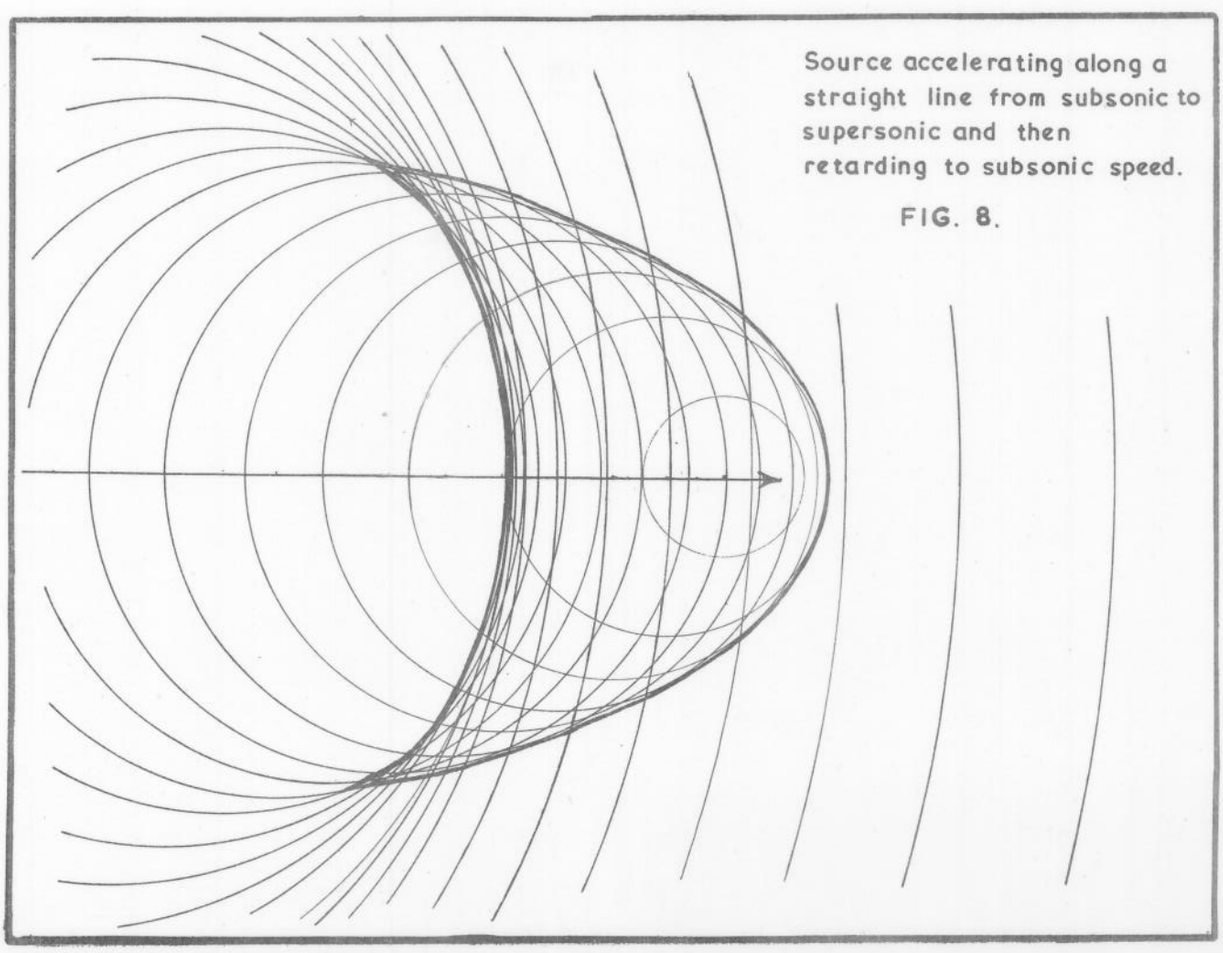


DIAGRAM SHOWING THE FORMATION OF REFLECTED SHOCK WAVES FROM A SUPERSONIC AEROFOIL TRAVELLING PARALLEL TO THE GROUND.



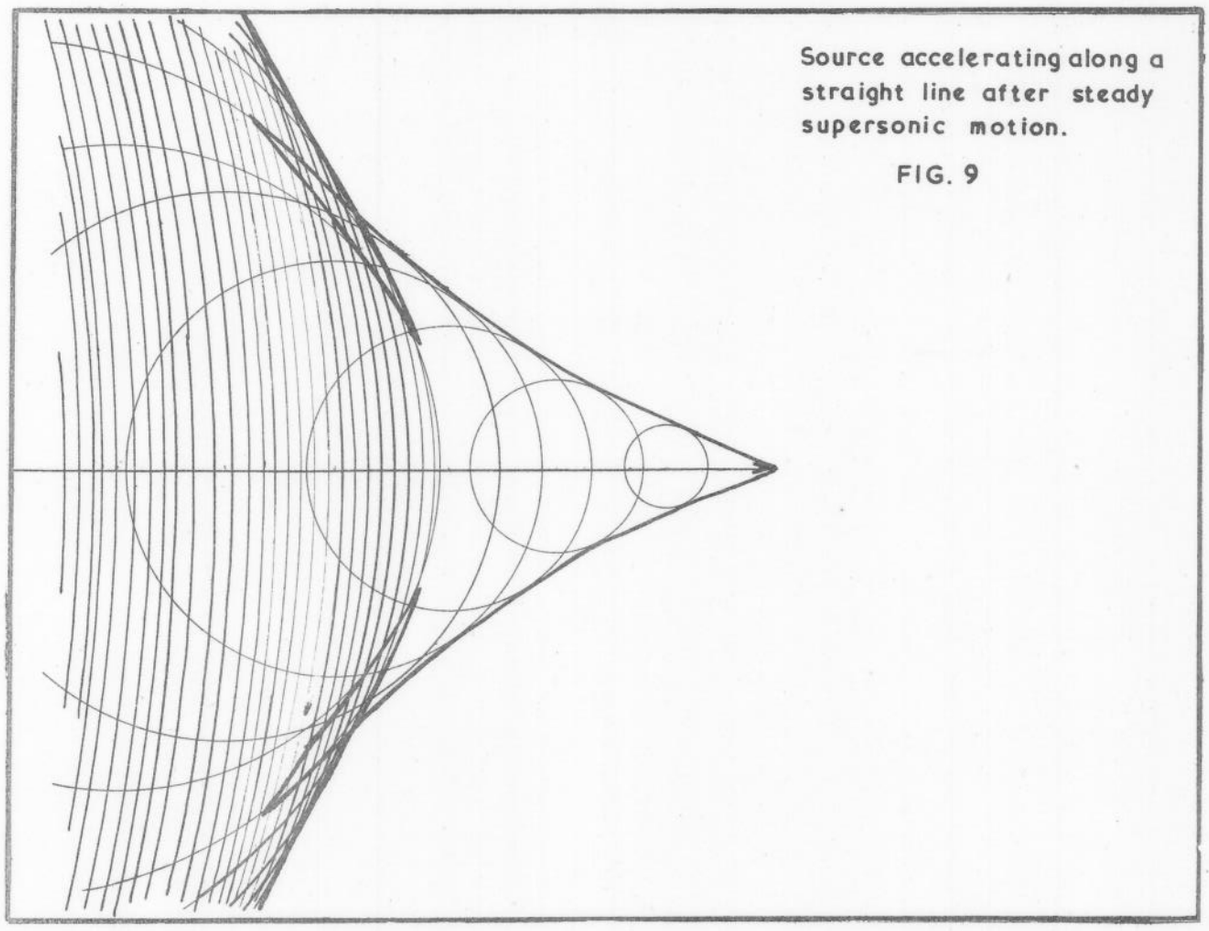
Source accelerating along a straight line from subsonic to supersonic speed.

FIG. 7.



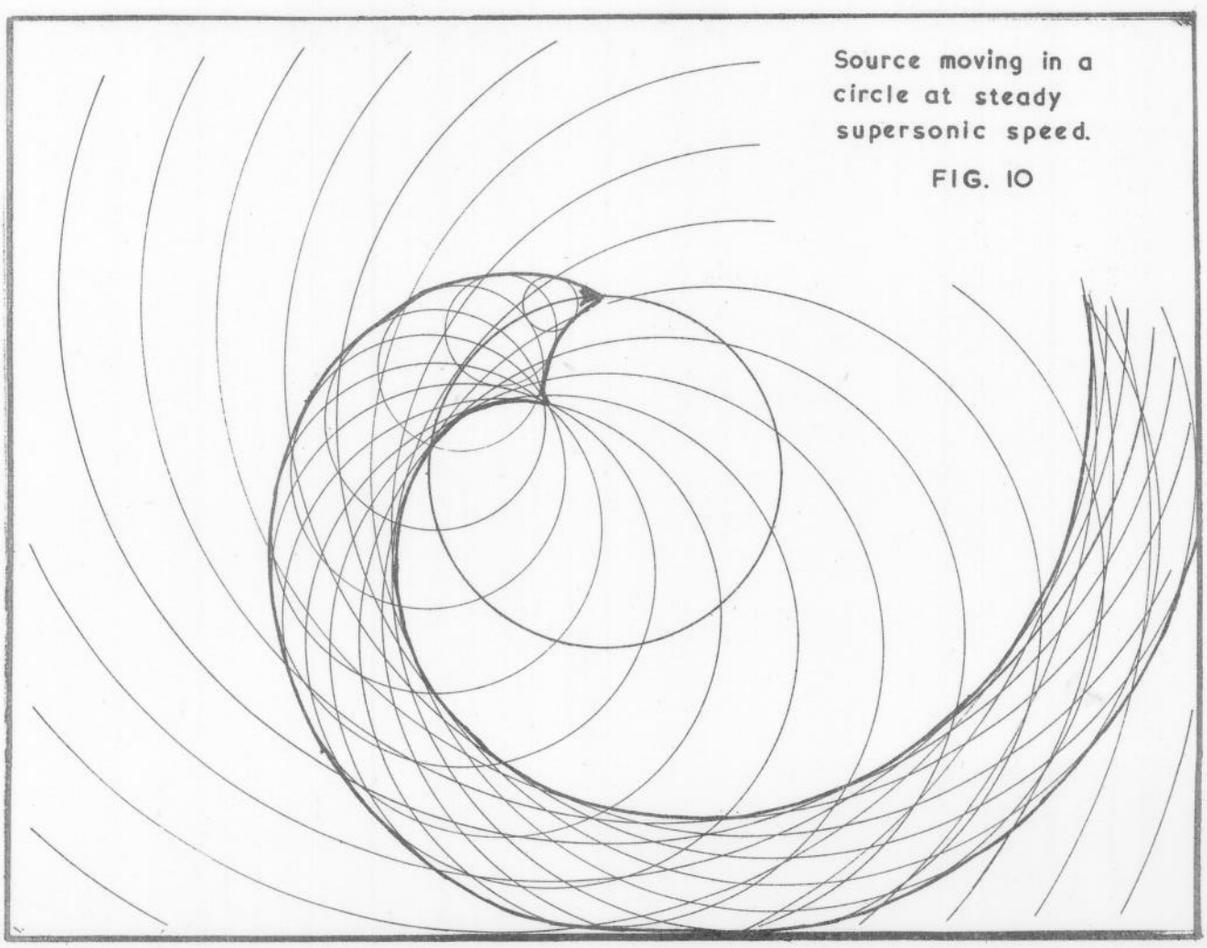
Source accelerating along a straight line from subsonic to supersonic and then retarding to subsonic speed.

FIG. 8.



Source accelerating along a straight line after steady supersonic motion.

FIG. 9



Source moving in a circle at steady supersonic speed.

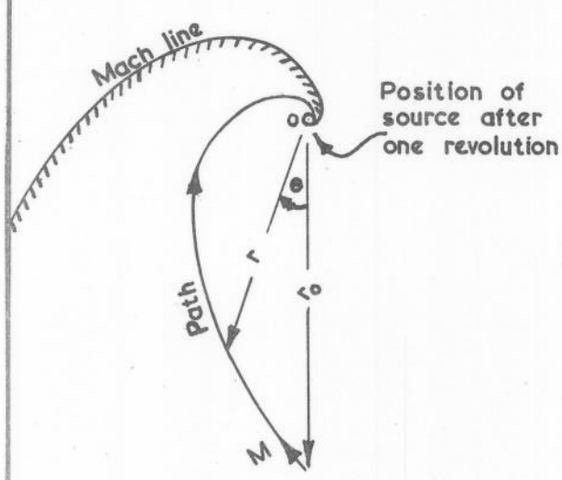
FIG. 10

The whole of the inner mach line reaches O at the same instant provided

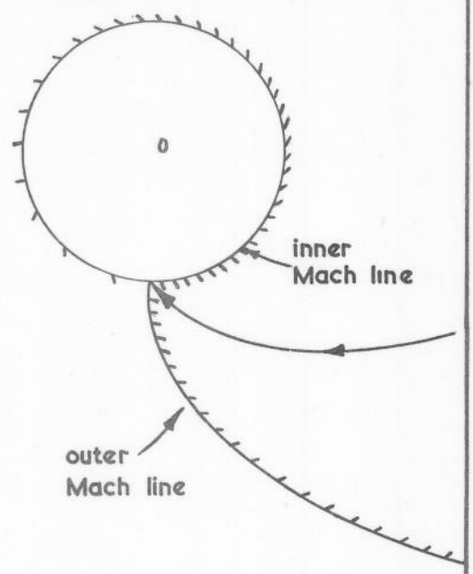
$$r = r_0 e^{-\frac{\theta}{\sqrt{M^2-1}}}$$

Source moving at constant speed along an equiangular spiral $M=\sqrt{2}$

FIG. 11.

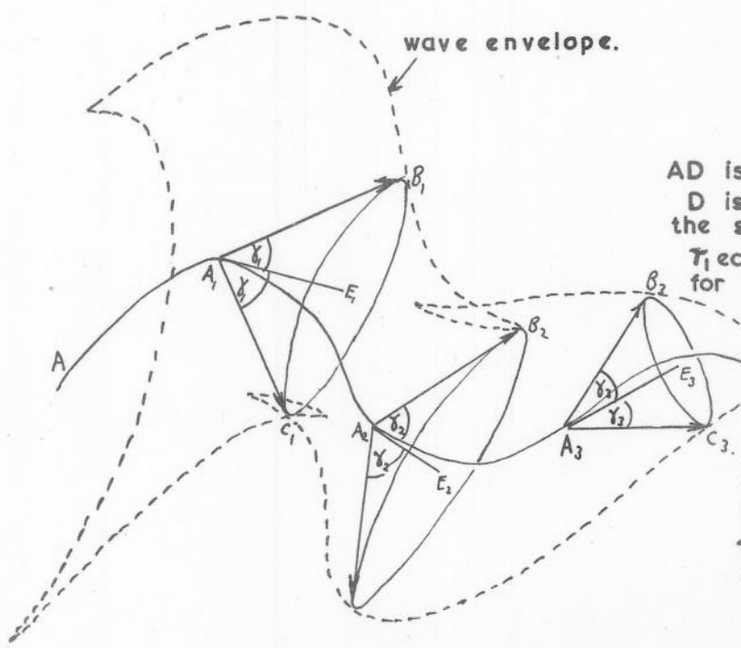


Enlarged view at O.



Construction of wave envelope for arbitrary motion.

FIG. 12.



KEY

AD is the flight path

D is the present position of the source.

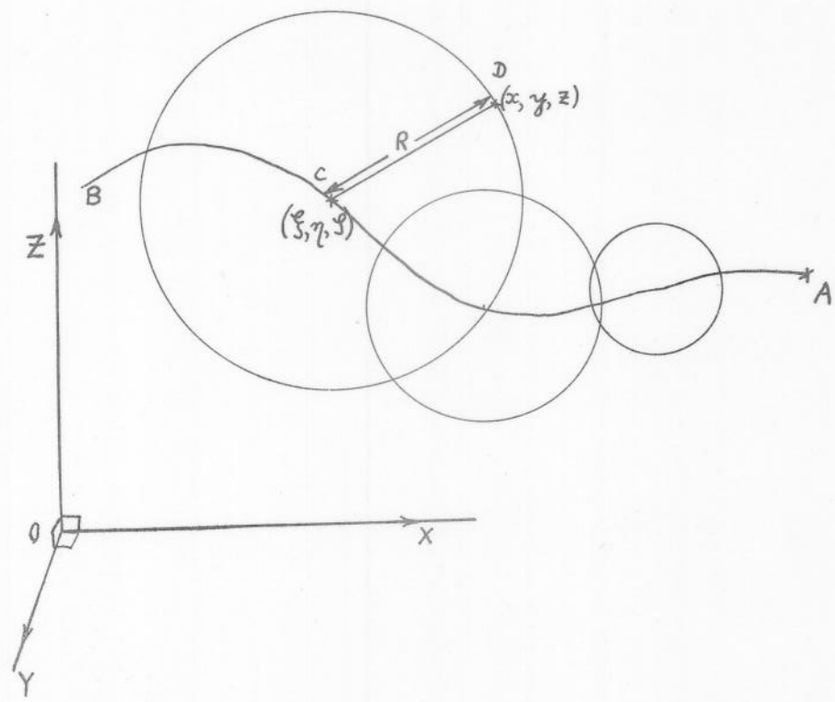
τ_1 ect. is the time taken for for the source to travel the distance AD.

The cone $A_1B_1C_1$ has its axis A_1E_1 along the flight path at A_1 and its semi vertex angle is $\gamma = \frac{\pi}{2}$ - Mach angle at A_1

The length $A_1B_1 = A_1C_1 = \tau_1 a$ where a is the constant speed of sound.

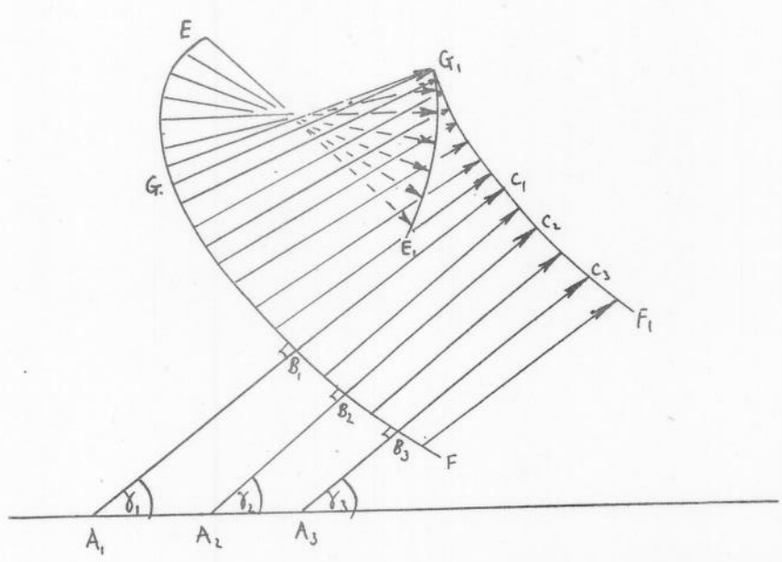
Coordinate system for a moving source.

FIG. 13.



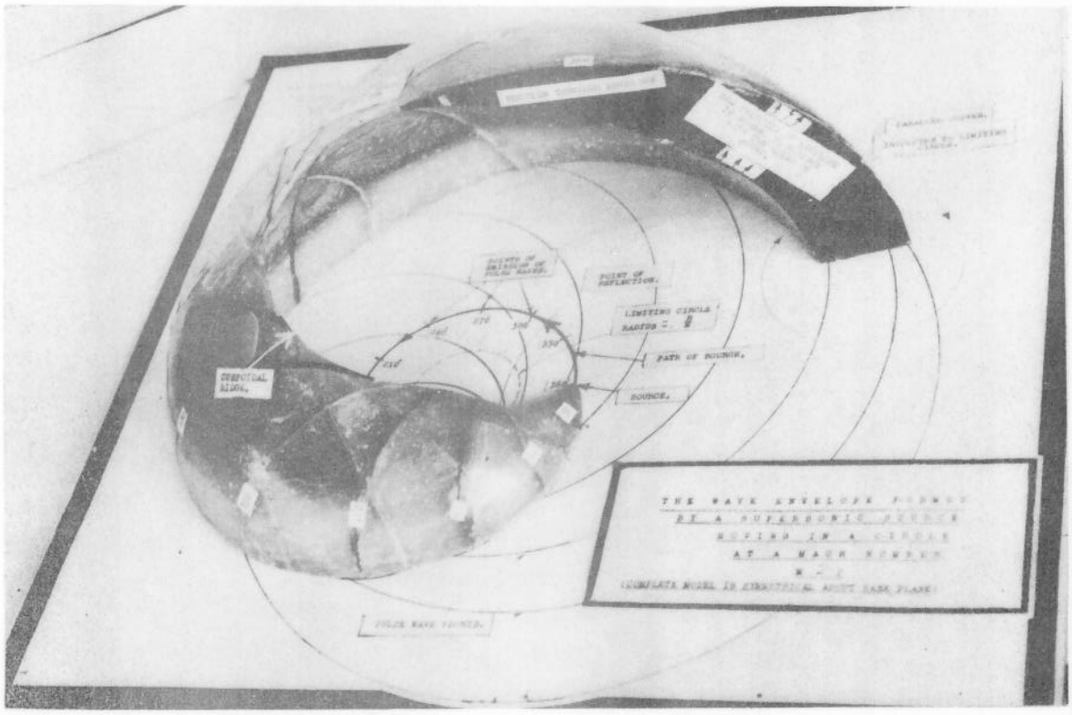
Formation of a cusp by a concave wave.

FIG. 14.



SECTIONAL MODEL OF THE WAVE ENVELOPE FORMED BY A SOURCE MOVING IN A CIRCLE AT M=2

FIG. 15.



SECTIONAL MODEL OF THE WAVE ENVELOPE FORMED BY A SOURCE MOVING IN A CIRCLE AT M=2

FIG. 16.

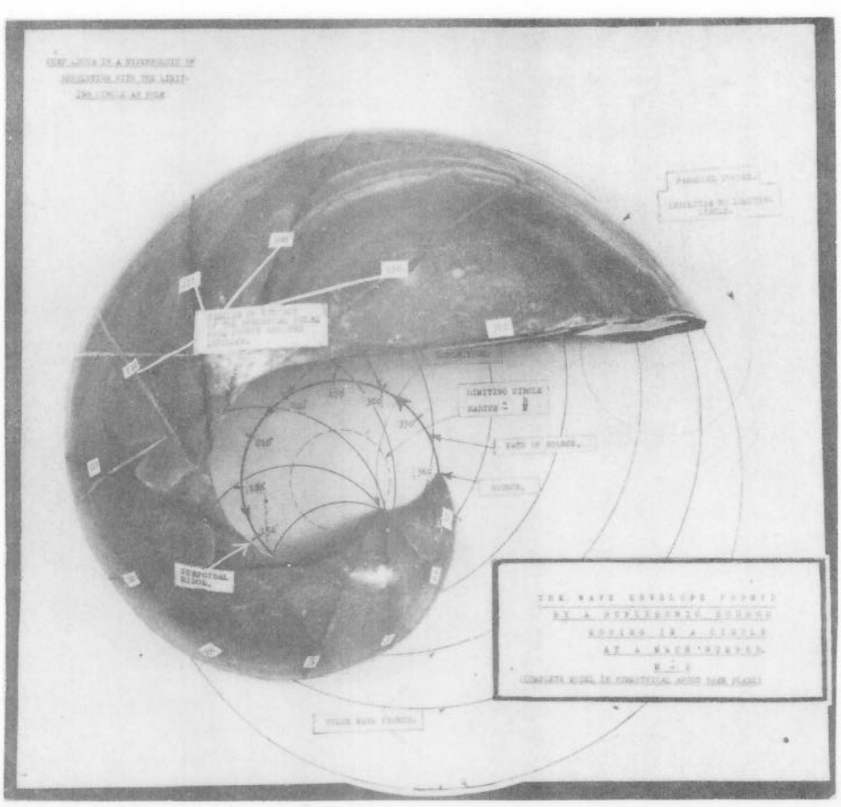
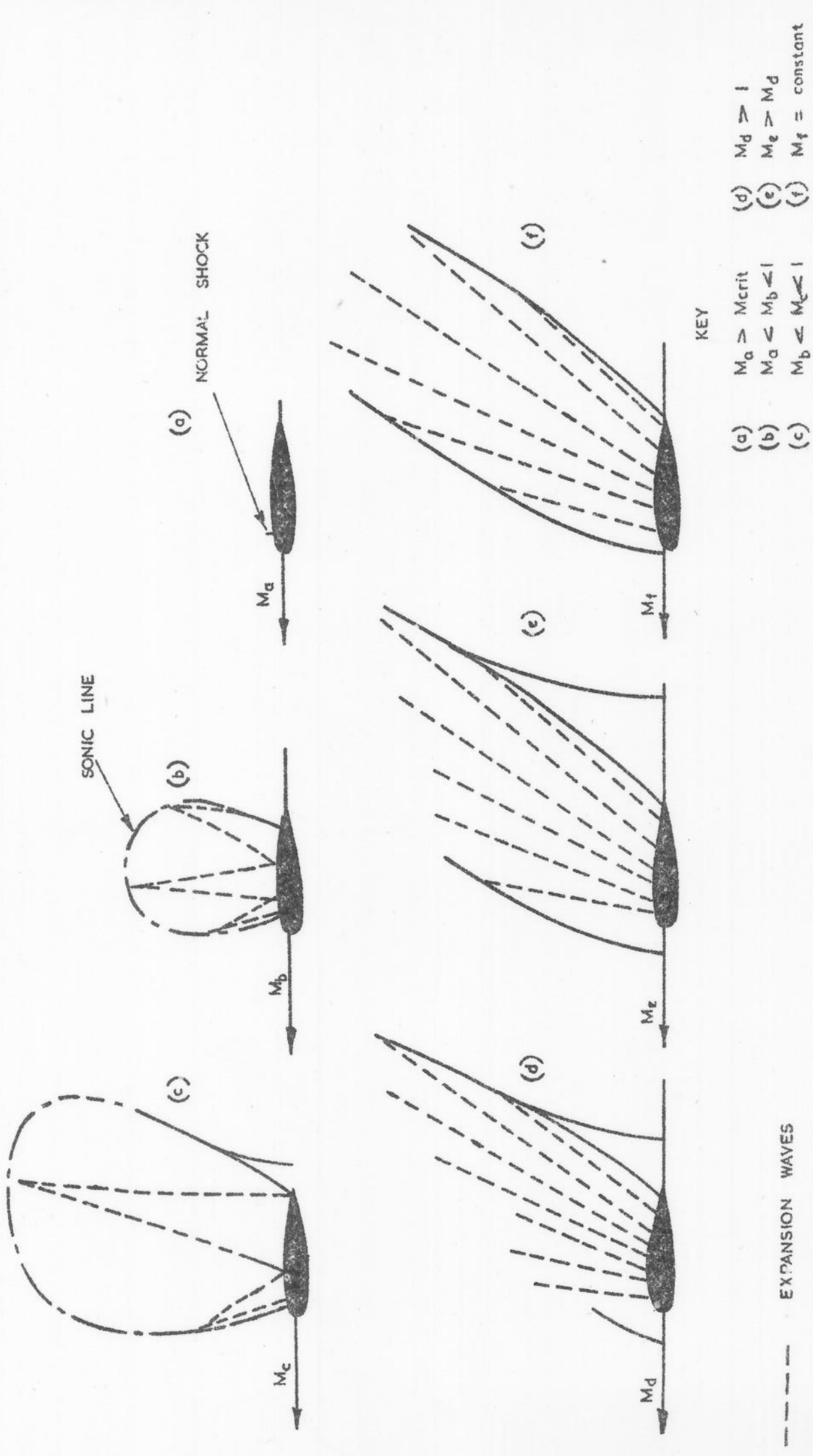
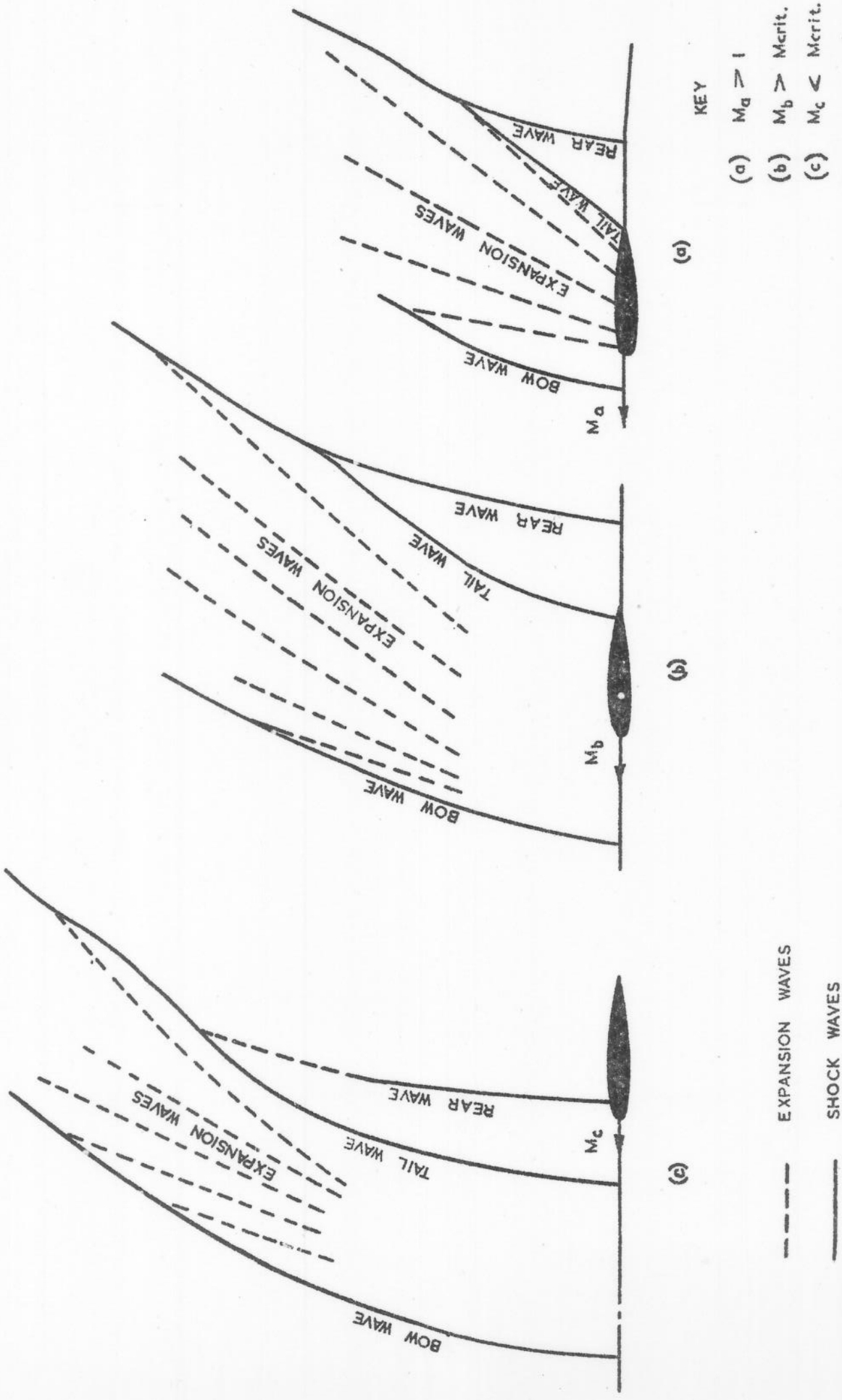


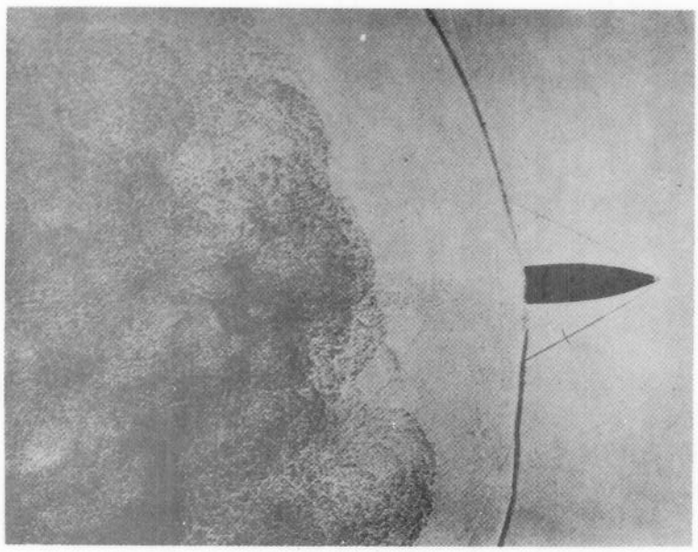
FIG. 17.



THE APPROXIMATE FORMATION OF SHOCK WAVES AROUND AN AEROFOIL AT VARIOUS STAGES DURING AN ACCELERATION FROM SUBSONIC TO SUPERSONIC SPEEDS



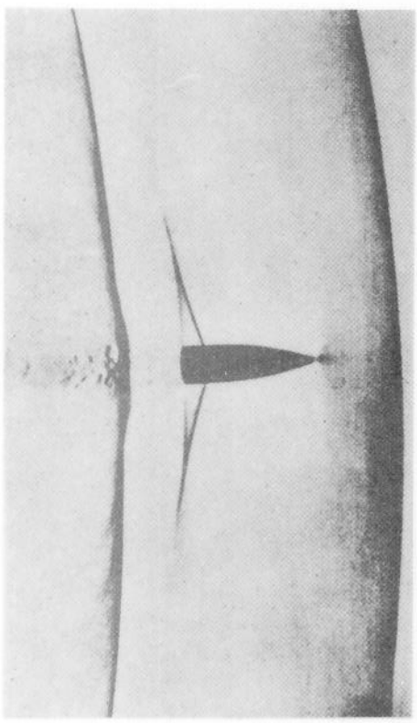
THE APPROXIMATE FORMATION OF SHOCK WAVES AROUND AN AEROFOIL AT VARIOUS STAGES DURING A RETARDATION FROM SUPERSONIC TO SUBSONIC SPEEDS.



THE COMMENCEMENT OF THE BOW WAVE FROM A POINTED BULLET.

(Taken from ref. 19)

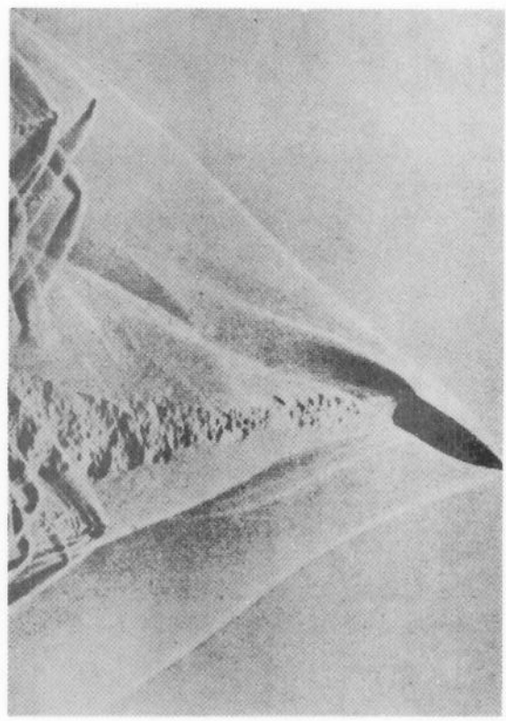
FIG. 19.



A

THE BOW AND TAIL WAVES FROM A BULLET AT M=1·0

(Taken from ref 19)

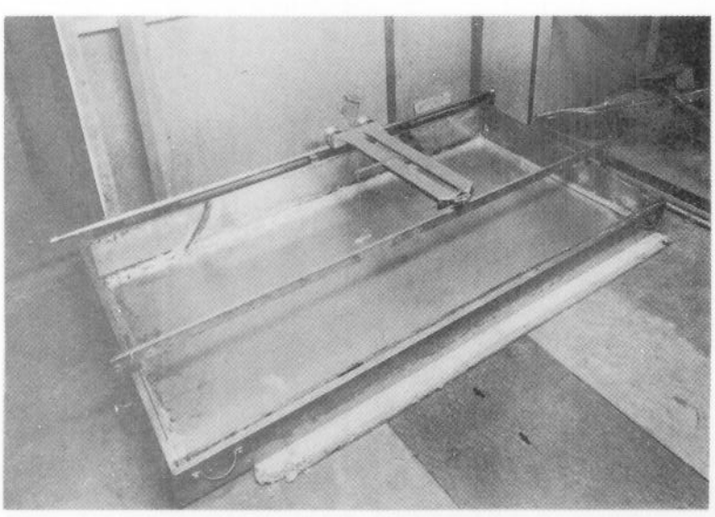


B

CURVED SHOCK WAVES PRODUCED BY A BULLET WHICH HAS GRAZED A WOODEN BOARD

(Taken from ref 19)

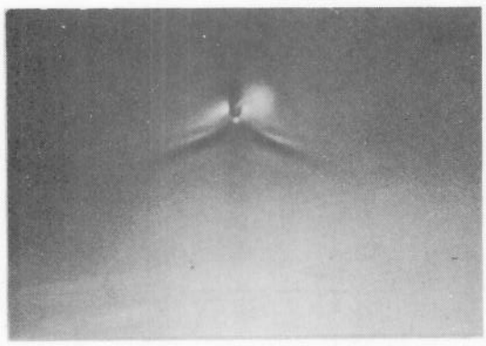
FIG. 20.

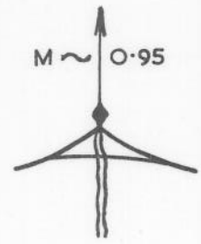


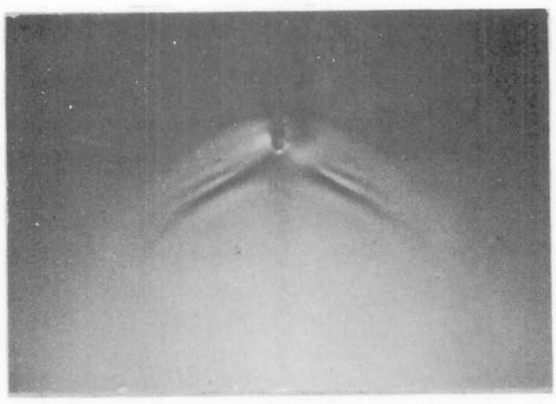
ARRANGEMENT OF HYDRAULIC ANALOGY TANK

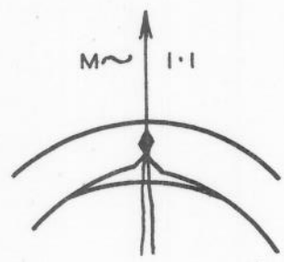
FIG. 21.

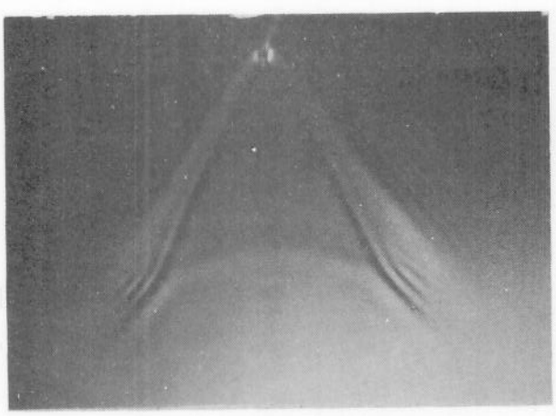
FIG. 22.

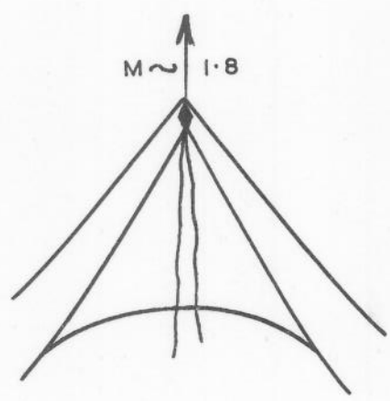


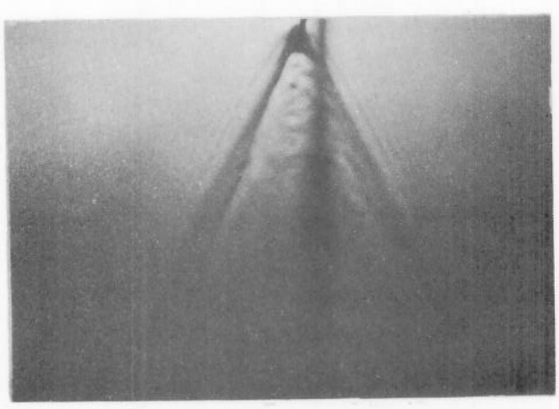


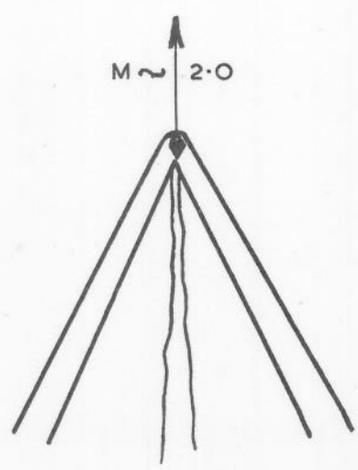




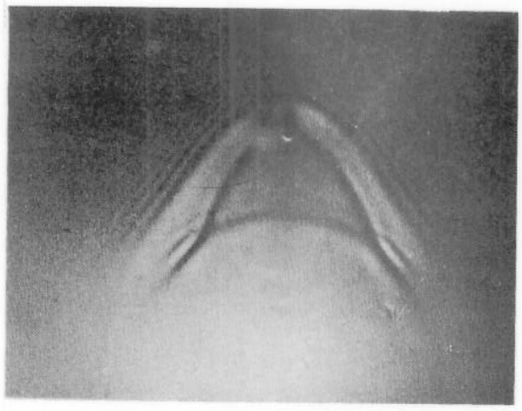


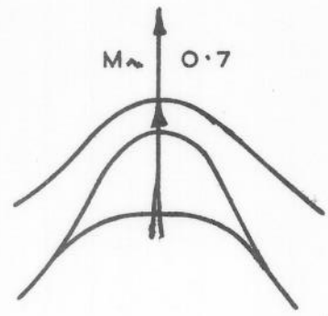


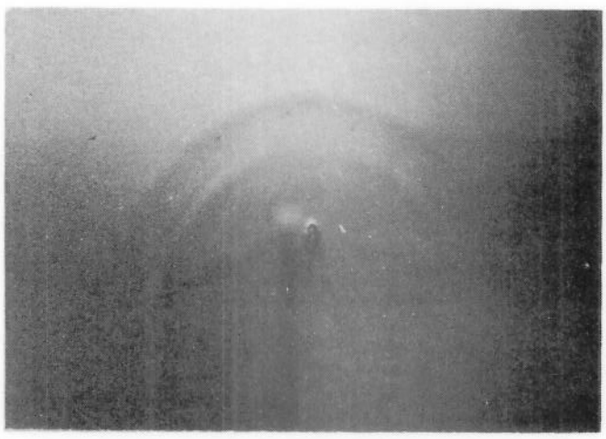


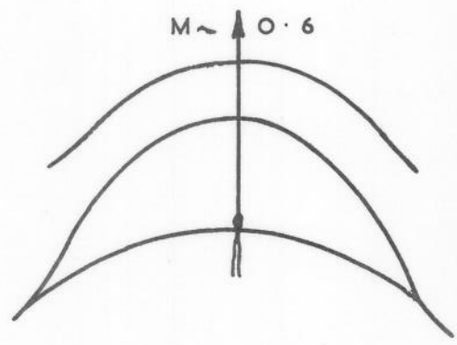


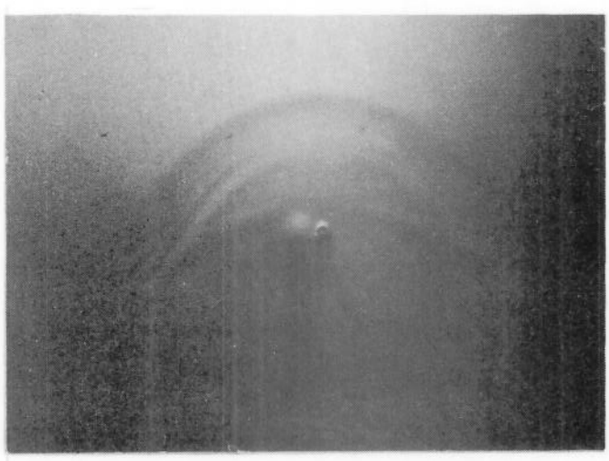
ACCELERATED AEROFOIL FROM SUB—TO SUPERCRITICAL SPEEDS
AEROFOIL DOUBLE WEDGE 0·5IN CHORD x 0·18IN THICK.











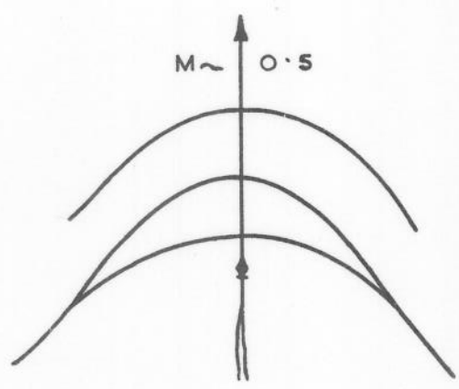
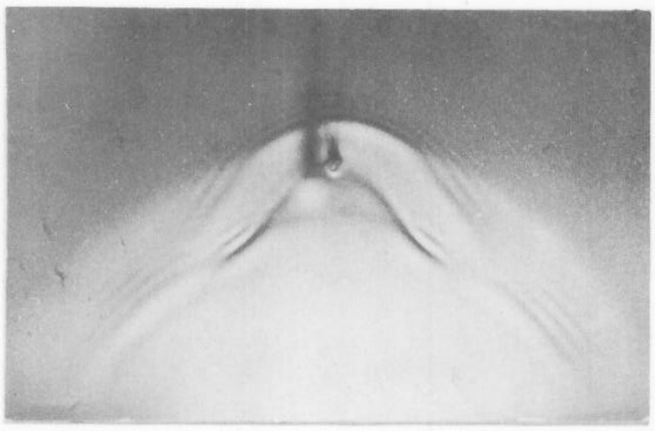


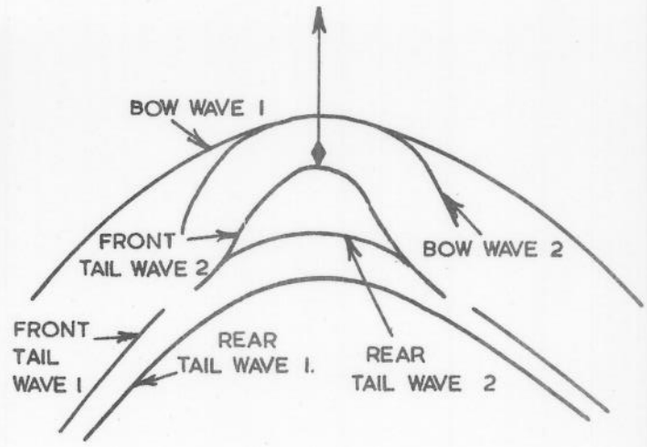
FIG. 23.

RETARDED AEROFOIL FROM SUPER-TO SUB CRITICAL SPEEDS

AEROFOIL DOUBLE WEDGE 0·5 in CHORD X 0·18 in THICK

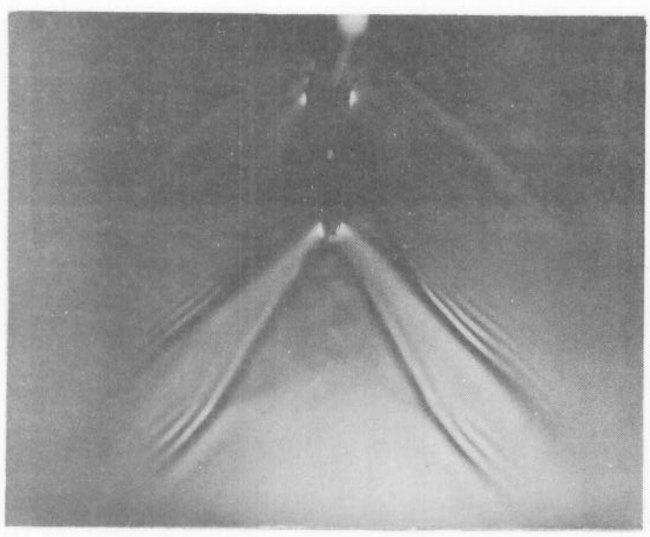
FIG. 24.

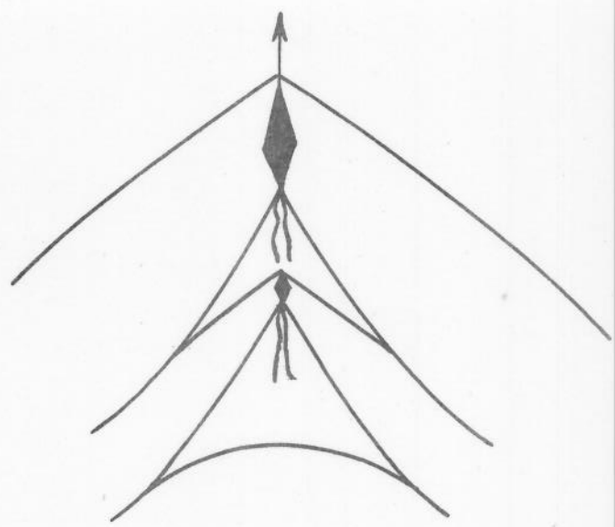




DOUBLE ACCELERATION AND RETARDATION FROM SUB TO SUPERCRITICAL AND BACK TO SUBCRITICAL SPEEDS.

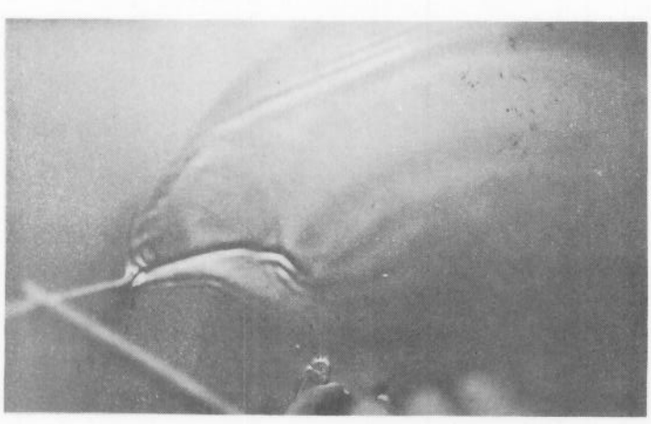
FIG. 25.





ACCELERATION AND RETARDATION OF TANDEM AEROFOILS.

FIG. 26.



AEROFOIL MOVING IN A CIRCLE.

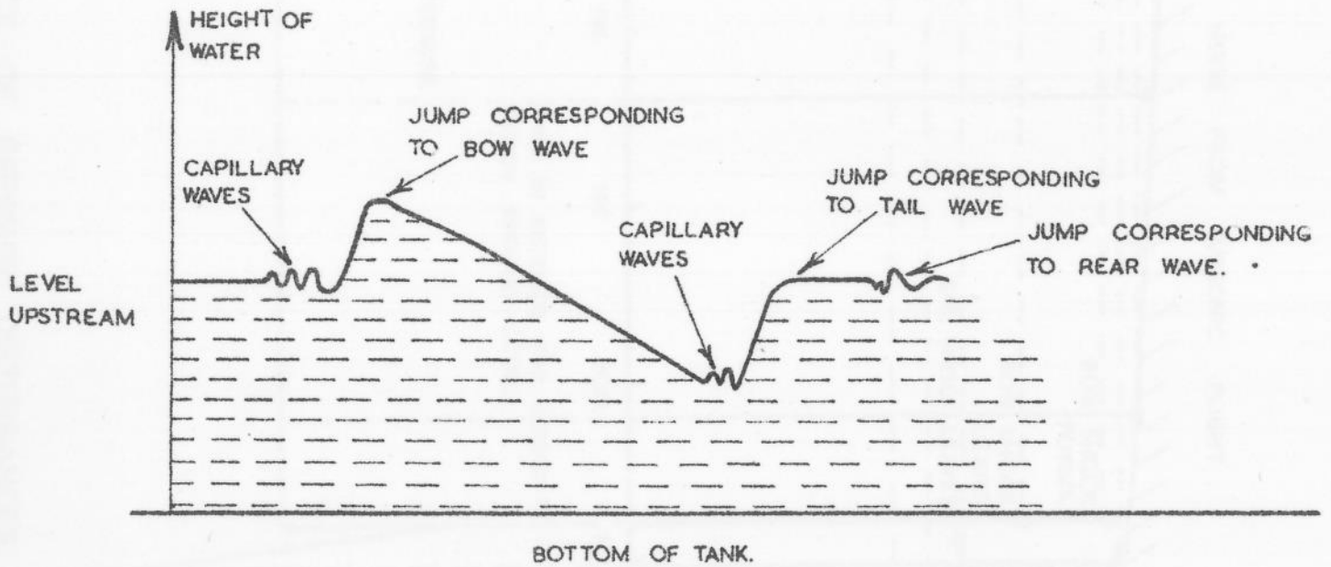
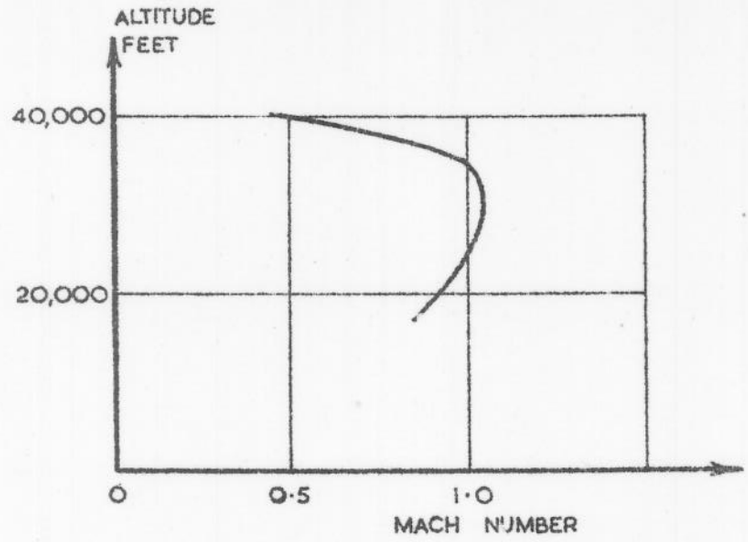
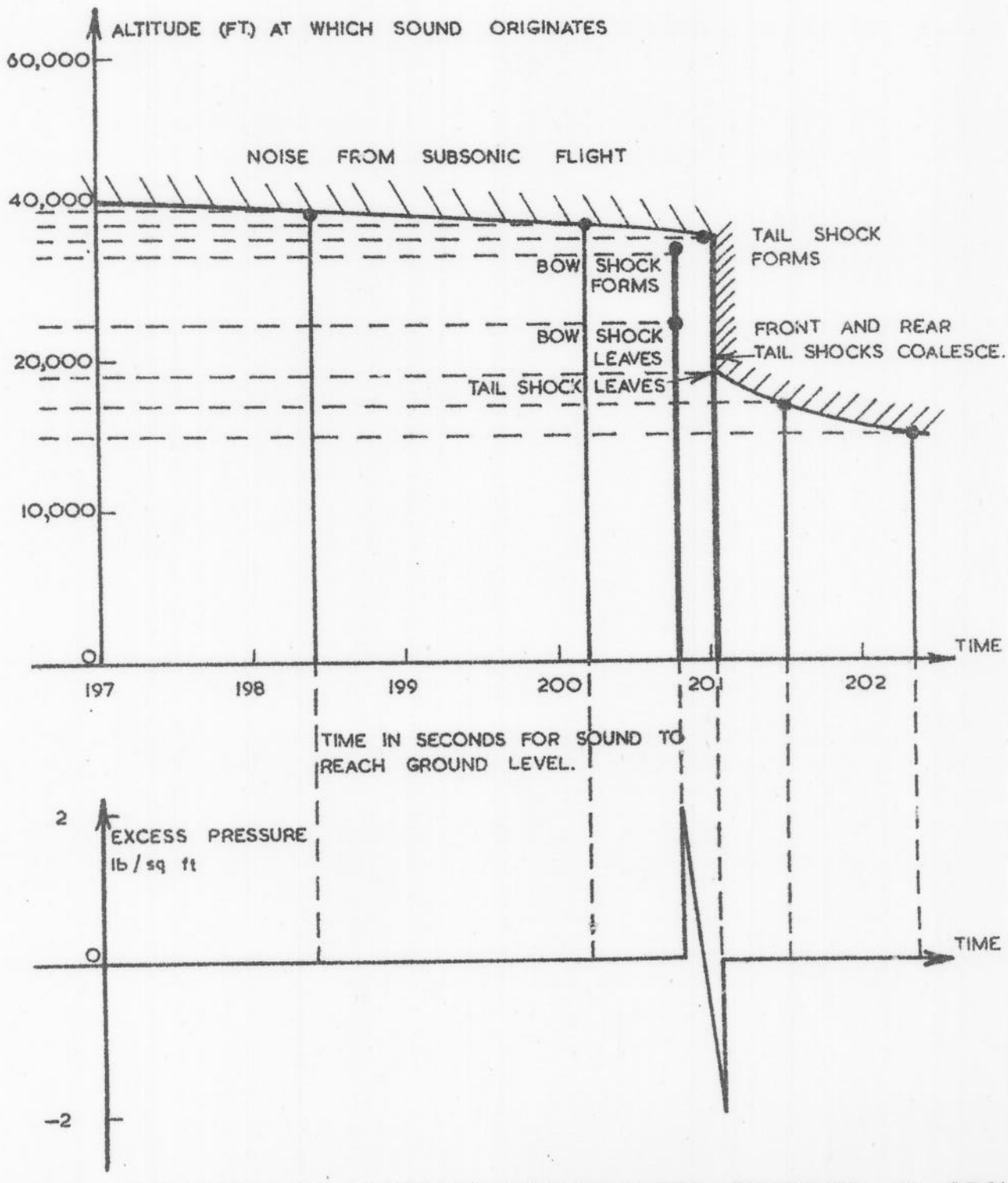


DIAGRAM OF THE CHANGES IN WATER HEIGHT ASSOCIATED WITH THE ACCELERATED MOTION OF A TWO-DIMENSIONAL AEROFOIL.

FIG. 28.





THE TIME SEQUENCE OF PRESSURE DISTURBANCES OBSERVED AT GROUND LEVEL FROM AN AIRCRAFT IN A HIGH SPEED DIVE.

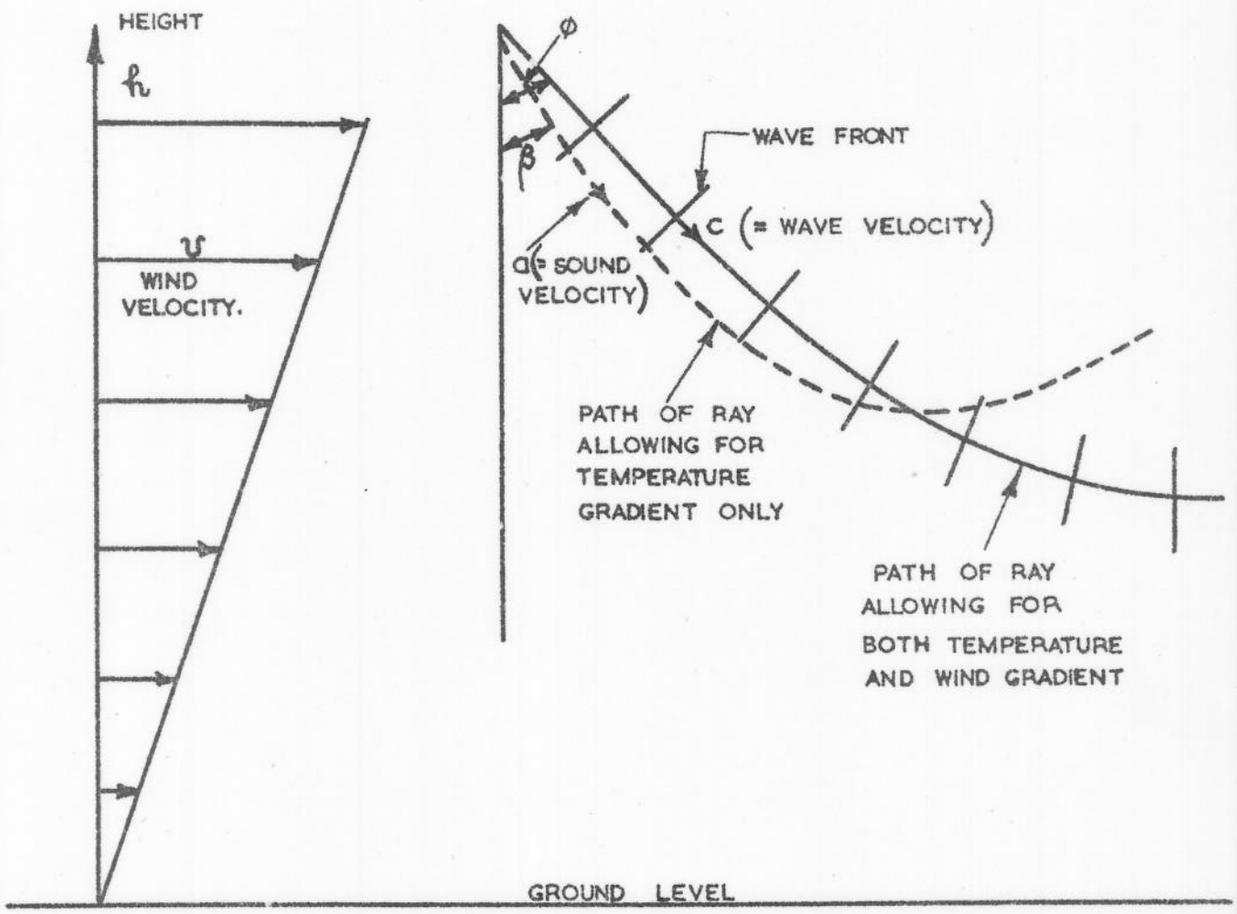
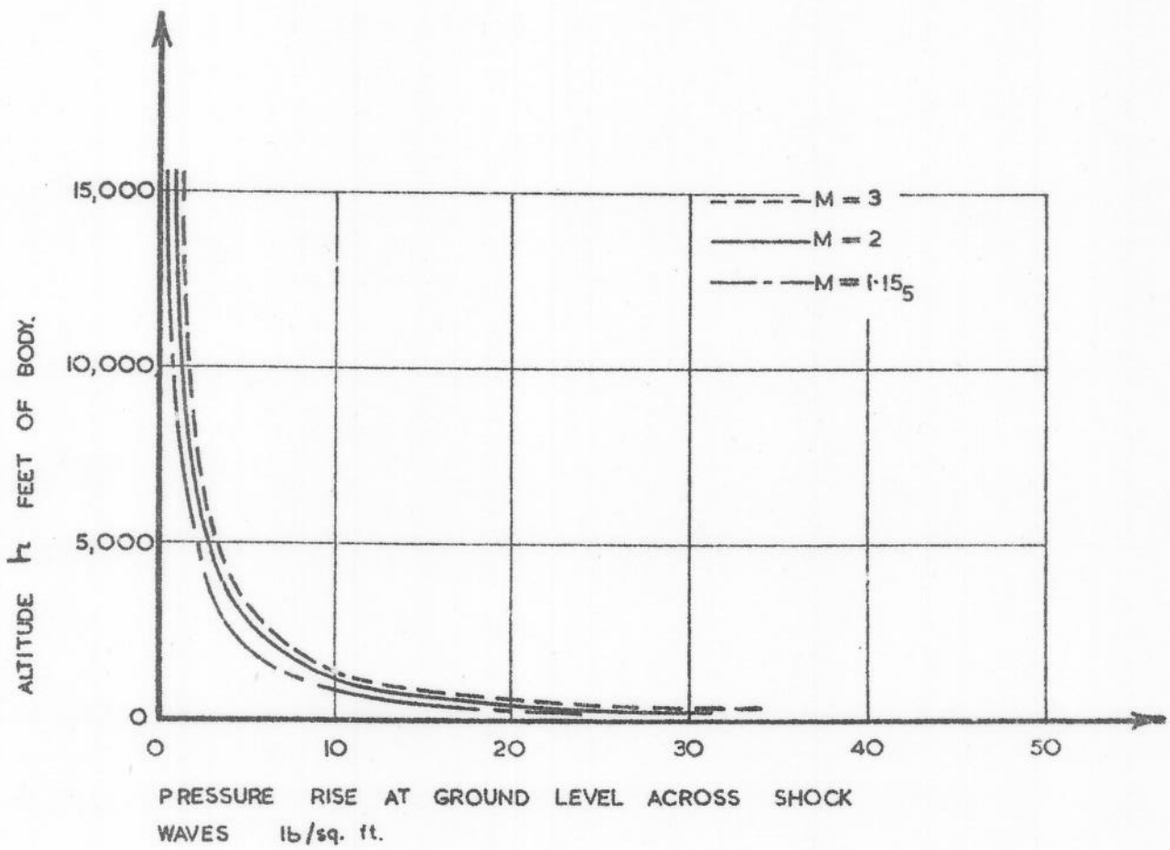


DIAGRAM SHOWING THE REFRACTION OF SOUND WAVES DUE TO ATMOSPHERIC WIND AND TEMPERATURE GRADIENTS.

FIG. 30.



THE APPROXIMATE PRESSURE RISE AT GROUND LEVEL ACROSS THE BOW WAVE OF A BODY OF REVOLUTION MOVING IN STEADY FLIGHT AT AN ALTITUDE h (FT.) WITH MACH NUMBER (M)

$$(\ell = 30\text{ft} \qquad \delta = \frac{d}{\ell} = 0{\cdot}1)$$