Report No. 63
May, 1953.

THE COLLEGE OF AERONAUTICS

CRANFIELD

The Strain Energy Analysis of Swept Boxes with Ribs Normal to the Spars *

- by -

D. Howe, D.C.Ae.

---oOo---

SUMMARY

The root constraint problem associated with uniform rectangular swept boxes, having ribs normal to the spars is considered. A strain energy method using self-equilibrating internal end load systems is used.

Solutions are presented for the cases of a single cell box having either all ribs rigid, or the root rib flexible. In addition, a consideration is made of second order effects combined with a flexible root rib. The case of a box having two equal cells, with all ribs rigid and a built in root is investigated, and the method of dealing with special root connections in this case is indicated. The effect of the flexibility of the root rib in the two cell box is also considered.

In all cases, the boxes are analysed for loading by a torsion couple, and a normal force applied on the centre-line at the tip.

The basic theory apertaining to this method of solution, together with the equivalent unswept solutions, are given in appendices.

BHF

---

* Work carried out under the auspices of a 1951 Clayton Research Fellowship, awarded by the Institute of Mechanical Engineers.

LIST OF CONTENTS

NOTATION

The following notation is used throughout this report:-

Oxyz System of orthogonal axes, based on rib and spar directions.

A_i (i=1,2,3,R) Boom area.

$\sum A = A_2 + 2A_1$ Sum of boom areas on one side of plane of symmetry.

a Typical length of shear panel, in particular rib pitch.

a_o Rib pitch in first bay of swept boxes.

a_{ij} Strain Energy Coefficients, defined in Appendix 1 Eqs.(1), (2) and (3).

a_{kk}, a_{kE}, $a_{k\ell}$ Particular Values of a_{ij}, defined in Appendix 1.

B^b Coefficient used to define shear flow, given by Appendix 2 Eq. (19).

$$B^b = \frac{2c}{t} \Bigg/ \left(\frac{3b}{t'} + \frac{2c}{t}\right)$$

b Total depth of spar web.

C_1, C_2, C Constants used in solution of strain energy equations, and defined in Appendix 1 Eqs. (6), (8) and (9).

c Width of skin panel in direction of y axis.

d Typical length of shear panel.

E Young's Modulus of Elasticity.

G Shear Modulus

k Particular rib station, used in derivation of formulae.

L_i Torsion Couple (applied at rib i).

ℓ Length of box along the front spar.

n Number of ribs in box, root defined as 0.

P_i (i=1-6,R) End Load in booms.

S_i (i=1-6,w,R) Shear flow.

T_k^i (i=a,b,c) End Load systems used as unknowns in strain energy calculations.

/ t ....

t	Skin thickness
t'	Spar web thickness
t_R	Rib web thickness
U	Strain Energy
W	Loading in Z direction, applied on x axis at tip
x	Specifically used to denote section between ribs k, $k + 1$, ($x = 0$ at rib k)
Z	Normal load on box
α	Coefficient used in calculating C. Appendix 1, Eq. (5), $\alpha = \cosh \phi$
θ	Complement of angle of sweepback
ϕ	Function used in definition of α, Appendix 1, Eq. (5)

/ 1. ....

1. INTRODUCTION

The solution of the root problem of a swept wing having ribs normal to the spars, using the fundamental method involves much algebra, and it would seem that a strain energy method is preferable. It is known that one of the more satisfactory ways of applying a strain energy method to the solution of a structure, is to use the method of self equilibrating end load systems, suggested by Ebner and Köller(1) and developed by Hemp(2). This method has been applied to a single cell swept box with rigid ribs, by Hemp in some unpublished work.

Amongst the advantages of this method are that it provides a solution which is relatively simple to apply, and that it can be readily modified to take into consideration the effects of flexible ribs and second order quantities, such as shear lag. Different root conditions can be solved without the necessity of evolving a completely new theory. It must be mentioned that one disadvantage of the method is that it can only consider discrete panels carrying pure shear, and hence only average shears can be calculated. When attempts are made to overcome this, the calculations become lengthy and most of the advantages are lost.

In addition to the solution given by Hemp, which is included for completeness, this work gives solutions when shear lag and a flexible root rib are considered. The case of a two cell box with a fixed root is also dealt with, and the method of solving the problem when special root conditions apertain is indicated. There is evidence that the flexibility of the first rib joining the mainspar root to the front spar is of importance, and the modifications to the solution for the two cell box with fixed root to cover this case are given.

All the solutions are given for loading by a torsion couple, and a normal force applied on the centreline of the box, at the tip.

Appendices contain the basic theory, procedure for solving the strain energy equations, and the solutions for the equivalent unswept boxes.

2. SINGLE CELL SWEPT BOX

2.1 All Ribs Rigid

This solution has been given by W.S. Hemp, but is included here for completeness, and because the results are needed for the other cases to be considered.

The geometry of the box structure is shown in Fig. 7. It is basically similar to the unswept case, Appendix 2, § 1, except for the root configuration. The first bay is triangular in planform, and the hypotenuse of the triangle, which forms the root, is built in. The first rib pitch is a_o.

The assumptions made are the same as those for the unswept case, the triangular skin being treated by the method of Appendix 3

Internal Load Systems

As for the unswept case, the basic internal load system is the doubly antisymmetrical form.

Consideration of the spar and rib boom equilibrium in Bay 0 gives:-

$$S_2 = S_1 - \frac{T_o}{a_o} \qquad S_R = - S_1 \qquad S_2 = S_R$$

$$\therefore S_1 + S_2 = 0$$

$$\therefore S_1 = - S_2 = - S_4 = \frac{T_o}{2a_o} \qquad \text{.. (1)}$$

The distribution in bay k ($k \neq 0$) will be as for the unswept case App.2 Eqs. (1) and (2).

Calculation of Strain Energy Coefficients

$$\left.\begin{aligned} a_{oo} &= \tfrac{1}{2}\, a_{oo} \\ a_{o1} &= \tfrac{1}{2}\, a_{o1} \end{aligned}\right\} \text{ for unswept case as there is only half the structure contributing to the coefficients.}$$

/ .. ..

$\therefore$ Using App. 2, Eqs (7),(8) and (9):-

$$\left.\begin{aligned} a_{oo} &= \frac{2a_o}{3EA} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t}\right) \\ a_{o1} &= \frac{a_o}{3EA} - \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t}\right) \end{aligned}\right\} \quad .. (2)$$

There will be half the unswept contribution to a_{11} from bay 0, and a full contribution from bay 1.

$$\therefore a_{11} = \frac{4}{3EA}\left(\frac{a_o}{2} + a\right) + \frac{1}{2G}\left(\frac{1}{2a_o} + \frac{1}{a}\right)\left(\frac{b}{t'} + \frac{c}{t}\right) \quad .. (3)$$

All the other coefficients will be as for the unswept case.

External Load Systems

(1) Bending by Z wise force applied on x axis at tip $Z = -W$

$$\text{Statically Correct Solution:-}\quad \left.\begin{aligned} S_1 &= -S_3 = \frac{W}{2b} \\ P &= \frac{W}{2b}(\ell - ak) \end{aligned}\right\} \quad .. (4)$$

where P is the end load in the upper spar booms at rib k.

Strain Energy Coefficients

End Load System T_o.

There will be a contribution from Bay 0 only

$$\text{End Load at section x in Bay 0} = \frac{W}{2b}(\ell - x) + T_o\frac{(a_o - x)}{a_o} + T_1 \cdot \frac{x}{a_o}$$

$$\text{Shear in Web} = \frac{W}{2b} + \frac{T_o}{2a_o} - \frac{T_1}{2a_o}$$

$$\therefore a_{oE} = \frac{Wa_o\ell}{2bEA}\left(1 - \frac{a_o}{3\ell}\right) + \frac{W}{4Gt'} \quad .. (5)$$

the contributions being from two booms and one web.

End Load System T_1

In this case there will be contributions from both bay 0 and bay 1. The bay 1 contribution will be zero, due to the internal loads being doubly antisymmetric, and the external loads singly antisymmetric about the Oxy plane

$$\text{From Bay 0;-}\quad a_{1E} = \frac{Wa_o\ell}{2bEA}\left(1 - \frac{2a_o}{3\ell}\right) - \frac{W}{4Gt'} \quad .. (6)$$

End Load System T_k $(k \neq 0, 1)$

All contributions will cancel

$$\therefore a_{kE} = 0 \quad (k \neq 0, 1) \quad .. (7)$$

/ (2) .....

(2) Loading by Torsion Couple L_i applied at i^{th} rib

Statically Correct Solution:- $-\dfrac{L_i}{2bc} = S_1 = S_2 = S_3 = S_4$ (8)

Strain Energy Coefficients

End Load System T_o

$a_{oE} = \frac{1}{2} a_{oE}$ for the unswept case, as there is only half the structure

$$\therefore \text{From App.2 Eq.(10):-}\quad a_{oE} = \frac{L_i}{4Gbc}\left(\frac{c}{t} - \frac{b}{t'}\right) \quad .. (9)$$

End Load System T_1

From Bay 0:- Contribution $= -\dfrac{L_i}{4Gbc}\left(\dfrac{c}{t} - \dfrac{b}{t'}\right)$

From Bay 1:- Contribution $= \dfrac{L_i}{2Gbc}\left(\dfrac{c}{t} - \dfrac{b}{t'}\right)$

$$\therefore a_{1E} = \frac{L_i}{4Gbc}\left(\frac{c}{t} - \frac{b}{t'}\right) \quad .. (10)$$

End Load System T_k $(k \neq 0, 1)$

This will be similar to the unswept case

$$\left.\begin{aligned} \therefore a_{kE} &= 0 \quad (k \neq 0, 1, i) \\ a_{iE} &= -\frac{L_i}{2Gbc}\left(\frac{c}{t} - \frac{b}{t'}\right) \end{aligned}\right\} \quad .. (11)$$

2.2 Flexible Root Rib

Apart from the fact that the flexibility of Rib 1 is considered, the details of this box are the same as those for §2.1

Internal Load Systems

The doubly antisymmetric system will still apply. From App.2, Eqs. (1), (2), and (3) we have:-

Shear in rib webs due to T_k: $S_{R_{k+1}} = S_{R_{k-1}} = \dfrac{T_k}{2a}$

$$S_{R_k} = -\frac{T_k}{a}$$

$$\therefore \text{Shear in web of rib 1} = S_{R_1} = -\frac{T_1}{2a_o} + \frac{T_o}{2a_o} + \frac{T_2}{2a} - \frac{T_1}{2a} \quad (12)$$

/ Modification ....

Modification of Strain Energy Coefficients

$$\left.\begin{aligned}
&\text{Contribution to } a_{oo} = \frac{bc}{4a_o^2 Gt_R} \qquad \text{where } t_R \text{ is rib web thickness}\\
&\text{Contribution to } a_{o1} = -\frac{bc}{4a_o Gt_R}\left(\frac{1}{a}+\frac{1}{a_o}\right)\\
&\text{Contribution to } a_{11} = \frac{bc}{4Gt_R}\left(\frac{1}{a_o^2}+\frac{2}{a_o a}+\frac{1}{a^2}\right)\\
&\text{Contribution to } a_{12} = -\frac{bc}{4aGt_R}\left(\frac{1}{a}+\frac{1}{a_o}\right)\\
&\text{Contribution to } a_{22} = \frac{bc}{4a^2 Gt_R}\\
&\text{Contribution to } a_{o2} = \frac{bc}{4a_o . a . Gt_R}
\end{aligned}\right\} \quad .. \; (13)$$

The values of the coefficients now become:-

Using Eq.(2) & (3):-

$$\left.\begin{aligned}
a_{oo} &= \frac{2a_o}{3EA} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t} + \frac{bc}{a_o t_R}\right)\\
a_{o1} &= \frac{a_o}{3EA} - \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t} + \frac{bc}{t_R}\left[\frac{1}{a} + \frac{1}{a_o}\right]\right)\\
a_{11} &= \frac{4}{3EA}\left(\frac{a_o}{2} + a\right) + \frac{1}{2G}\left(\frac{1}{2a_o} + \frac{1}{a}\right)\left(\frac{b}{t'} + \frac{c}{t}\right) + \frac{bc}{4Gt_R}\left(\frac{1}{a^2} + \frac{2}{a_o a} + \frac{1}{a_o^2}\right)\\
a_{12} &= \frac{2a}{3EA} - \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{c}{t} + \frac{bc}{2t_R}\left[\frac{1}{a} + \frac{1}{a_o}\right]\right)\\
a_{22} &= \frac{8a}{3EA} + \frac{1}{Ga}\left(\frac{b}{t'} + \frac{c}{t} + \frac{bc}{4at_R}\right)\\
a_{o2} &= \frac{bc}{4a_o . a . Gt_R}
\end{aligned}\right\} \quad (14)$$

There is no external load in the ribs, hence the external coefficients are unchanged. All the other coefficients will be identical to those calculated in §2.1

/ 3. ....

3. TWO CELL SWEPT BOX

3.1 Fixed Root Condition

Fig.8 shows the structure of this box. The general arrangement is similar to the two cell unswept box considered in App.2 §2, but there are two triangular skins at the root, which are assumed to be built in. The same assumptions are made as for the unswept case, and the triangular skins are dealt with by the method of Appendix 3.

Internal Load Systems

All the three load systems of Fig.2 are possible. Consideration of the spar and rib boom equilibrium conditions, shows that the load distributions are the same in the root bays, as for the unswept case, App.2, Eqs. (13),(16),(17),(20),(22) and (25).

Calculation of Strain Energy Coefficients

End Load System a

Only the front spar booms and skins will contribute to $a_{oo}^{(a)}$. From App.2, Eq (26):-

$$a_{oo}^{(a)} = \frac{2a_o}{3EA_1} + \frac{c}{Gta_o} \qquad .. (15)$$

In the case of $a_{o1}^{(a)}$ there will be contributions from the same components as above

$$\therefore a_{o1}^{(a)} = \frac{a_o}{3EA_1} - \frac{c}{Gta_o} \qquad .. (16)$$

Both bays 0, and 1, will contribute to $a_{11}^{(a)}$

Front spar booms give:- $4a_o/3EA_1$

Main spar booms give:- $8a_o/3EA_2$

Bay 0 Skins:- c/Gta_o Bay 1 skins:- $3c/Gta_o$

$$\therefore a_{11}^{(a)} = \frac{4a_o}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{4c}{Gta_o} \qquad .. (17)$$

/ The front .....

The front and mainspar booms and skins will contribute to $a_{12}^{(a)}$.

Booms contribute:- $\frac{a_o}{3E}\left(\frac{1}{A_1}+\frac{4}{A_2}\right)$

and skins:- $-3c/Gta_o$

$$\therefore a_{12}^{(a)} = \frac{a_o}{3E}\left(\frac{1}{A_1}+\frac{4}{A_2}\right) - \frac{3c}{Gta_o} \quad .. (18)$$

Bay 2 will contribute fully to $a_{22}^{(a)}$, but the bay 1 effects will not be complete. Bay 2 gives:- $\frac{4a}{3E}\left(\frac{1}{A_1}+\frac{2}{A_2}\right)+\frac{4c}{Gta}$

Bay 1, spar booms contribute:- $\frac{2a_o}{3E}\left(\frac{1}{A_1}+\frac{4}{A_2}\right)$

Skins give:- $3c/Gta_o$

$$\therefore a_{22}^{(a)} = \frac{2}{3E}\left\{\frac{(a_o+2a)}{A_1}+\frac{4(a+a_o)}{A_2}\right\}+\frac{c}{Gt}\left(\frac{3}{a_o}+\frac{4}{a}\right) \quad .. (19)$$

In all other cases, $k, \ell > 2$, App.2 Eq (26) will apply.

End Load System b.

The front booms, web and skins will contribute to $a_{oo}^{(b)}$. Using App.2 Eq.(27),

Booms give $2a_o/3EA_1$

Skins give $\frac{c}{Ga_o t}(1-B^b)^2$, and web $\frac{a_o b}{Gt'}\left(\frac{B^b}{a_o}\right)^2 = \frac{b(B^b)^2}{Gt'a_o}$

$$\therefore a_{oo}^{(b)} = \frac{2a_o}{3EA_1}+\frac{1}{Ga_o}\left\{\frac{b}{t'}(B^b)^2+\frac{c}{t}(1-B^b)^2\right\} \quad .. (20)$$

The shear contributions to $a_{o1}^{(b)}$ will be equal to that for $a_{oo}^{(b)}$, but opposite in sign.

Shears give:- $-\frac{1}{Ga_o}\left\{\frac{b}{t'}(B^b)^2+\frac{c}{t}(1-B^b)^2\right\}$

and booms give $a_o/3EA_1$

$$\therefore a_{o1}^{(b)} = \frac{a_o}{3EA_1}-\frac{1}{Ga_o}\left\{\frac{b}{t'}(B^b)^2+\frac{c}{t}(1-B^b)^2\right\} \quad .. (21)$$

For $a_{11}^{(b)}$; Bay 0 gives:- $\frac{2a_o}{3EA_1} + \frac{1}{Ga_o}\left\{\frac{b}{t'}(B^b)^2 + \frac{c}{t}(1 - B^b)^2\right\}$

Bay 1 gives:- $\frac{2a_o}{3E}\left(\frac{1}{A_1}+\frac{4}{A_2}\right) + \frac{1}{Ga_o}\left\{5\frac{b}{t'}(B^b)^2 + \frac{3c}{t}(1-B^b)^2\right\}$

$$\therefore a_{11}^{(b)} = \frac{4a_o}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{2}{Ga_o}\left\{3\frac{b}{t'}(B^b)^2 + \frac{2c}{t}(1 - B^b)^2\right\} \quad .. (22)$$

In the case of $a_{12}^{(b)}$, the skin effects are equal to those from Bay 1 for $a_{11}^{(b)}$, but opposite in sign.

$$\therefore a_{12}^{(b)} = \frac{a_o}{3E}\left(\frac{1}{A_1} + \frac{4}{A_2}\right) - \frac{1}{Ga_o}\left\{5\frac{b}{t'}(B^b)^2 + \frac{3c}{t}(1 - B^b)^2\right\} \quad .. (23)$$

The contribution to $a_{22}^{(b)}$ from Bay 2 will be complete:-

$$\frac{4a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{2}{Ga}\left\{3\frac{b}{t'}(B^b)^2 + \frac{2c}{t}(1 - B^b)^2\right\}$$

Bay 1 gives: $\frac{2a_o}{3E}\left(\frac{1}{A_1} + \frac{4}{A_2}\right) + \frac{1}{Ga_o}\left\{5\frac{b}{t'}(B^b)^2 + \frac{3c}{t}(1 - B^b)^2\right\}$

$$\therefore a_{22}^{(b)} = \frac{2}{3E}\left\{\frac{(2a+a_o)}{A_1} + \frac{4}{A_2}(a+a_o)\right\} + \frac{b(B^b)^2}{Gt'}\left(\frac{6}{a} + \frac{5}{a_o}\right) + \frac{c(1-B^b)^2}{Gt}\left(\frac{4}{a}+\frac{3}{a_o}\right) \quad (24)$$

For coefficients where $k, \ell > 2$, App.2 Eq. (27) will hold.

End Load System c.

The front booms, web and skins contribute to $a_{oo}^{(c)}$. Using App.2 Eq.(28):-

$$a_{oo}^{(c)} = \frac{2a_o}{3EA_1} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t}\right) \quad .. (25)$$

The shear contribution to $a_{o1}^{(c)}$ will be equal and opposite to that for $a_{oo}^{(c)}$

$$\therefore a_{o1}^{(c)} = \frac{a_o}{3EA_1} - \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t}\right) \quad .. (26)$$

/ The bay ......

The bay 0 contribution to $a_{11}^{(c)}$ is the same as that for $a_{oo}^{(c)}$:-

$$\frac{2a_o}{3EA_1} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{c}{t}\right)$$

Bay 1 contribution:- $\frac{2a_o}{3EA_1} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{3c}{t}\right)$

$$\therefore \; a_{11}^{(c)} = \frac{4a_o}{3EA_1} + \frac{1}{2Ga_o}\left(\frac{b}{t'} + \frac{2c}{t}\right) \qquad .. (27)$$

The shear contribution to $a_{12}^{(c)}$ will be equal and opposite to that from Bay 1, for $a_{11}^{(c)}$

$$\therefore \; a_{12}^{(c)} = \frac{a_o}{3EA_1} - \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{3c}{t}\right) \qquad .. (28)$$

$a_{22}^{(c)}$ has a full contribution from bay 2:- $\frac{4a}{3EA_1} + \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right)$

Bay 1 contributes $\frac{2a_o}{3EA_1} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{2c}{t}\right)$

$$\therefore \; a_{22}^{(c)} = \frac{2(a_o + a)}{3EA_1} + \frac{1}{4G}\left\{\frac{b}{t'}\left(\frac{1}{a_o} + \frac{2}{a}\right) + \frac{c}{t}\left(\frac{3}{a_o} + \frac{4}{a}\right)\right\} \qquad (29)$$

For $k, \ell > 2$, $a_{kk}^{(c)}$ and $a_{k\ell}^{(c)}$ will be as App.2 Eq. (28).

<u>External Load Systems</u>

1) <u>Bending by Z-wise force applied on x axis at tip Z = - W</u>

Statically Correct Solution:-

This is assumed to be the same as for the unswept case.

$$S_1 = - S_4 = \frac{WA_1}{b\sum A} \qquad\qquad S_w = \frac{WA_2}{b\sum A}$$

where $\sum A = 2A_1 + A_2$

End load in front and rear upper booms at rib k:-

$$P_1 = P_3 = \frac{WA_1}{b\sum A}(\ell - a\,k)$$

End load in mainspar upper boom at rib k:-

$$P_2 = \frac{WA_2}{b\sum A}(\ell - a\,k)$$

(30)

/ Calculation .....

Calculation of Strain Energy Coefficients

End Load System a

All the contributions will cancel about the Oxy plane, due to the nature of the internal loading system.

$$\therefore \; a_{kE}^{(a)} = 0 \qquad .. (31)$$

End Load System b

The front booms and webs will give the only contributions to $a_{oE}^{(b)}$

Booms give:- $\frac{2}{EA_1}\int_o^{a_o} -\frac{WA_1}{b\Sigma A}(\ell - x)\frac{(a_o - x)}{a_o}dx = -\frac{Wa_o\ell}{bE\Sigma A}\left(1 - \frac{a_o}{3\ell}\right)$

Webs give :- $\frac{a_o b}{Gt'} \cdot \frac{WA_1}{b\Sigma A} \cdot -\frac{B^b}{a_o} = -\frac{WA_1 B^b}{Gt'\Sigma A}$

$$\therefore \; a_{oE}^{(b)} = -\frac{Wa_o\ell}{bE\Sigma A}\left(1 - \frac{a_o}{3\ell}\right) - \frac{WA_1 B^b}{Gt'\Sigma A} \qquad .. (32)$$

There will be contributions to $a_{1E}^{(b)}$ from bays 0, and 1

Bay 0, front booms give:- $\frac{2}{EA_1}\int_o^{a_o} -\frac{WA_1}{b\Sigma A}(\ell - x)\cdot\frac{x}{a_o}dx = -\frac{Wa_o\ell}{bE\Sigma A}\left(1 - \frac{2a_o}{3\ell}\right)$

Front Web:- $\frac{WA_1 B^b}{Gt'\Sigma A}$

Bay 1:- Front booms give $\frac{2}{EA_1}\int_o^{a_o} -\frac{WA_1}{b\Sigma A}(\ell - a_o - x)\frac{(a_o - x)}{a_o}dx = -\frac{Wa_o}{bE\Sigma A}\left(\ell - \frac{4a_o}{3}\right)$

Main booms give $\frac{2}{EA_2}\int_o^{a_o} \frac{2WA_2}{b\Sigma A}(\ell - a_o - x)\frac{(a_o - x)}{a_o}dx = \frac{2Wa_o}{bE\Sigma A}\left(\ell - \frac{4a_o}{3}\right)$

Webs:- $-\frac{WA_1 B^b}{Gt'\Sigma A} + \frac{2B^b}{a_o}\cdot\frac{WA_2}{b\Sigma A}\frac{a_o b}{Gt'} = \frac{WB^b}{Gt'\Sigma A}(2A_2 - A_1)$

$$\therefore \; a_{1E}^{(b)} = -\frac{2Wa_o^2}{3bE\Sigma A} + \frac{2WB^b A_2}{Gt'\Sigma A} \qquad .. (33)$$

/ The boom .....

The boom contributions to $a_{2E}^{(b)}$ in bay 2, will cancel one another, and the front and main web shears will cancel from bay 1 to bay 2.

Bay 2:- Rear Web gives $-\frac{WA_1 B^b}{Gt'\Sigma A}$

Bay 1:- Front booms:- $\frac{2}{EA_1}\int_0^{a_o} -\frac{x}{a_o}\frac{WA_1}{b\Sigma A}(\ell - a_o - x)dx = -\frac{Wa_o}{bE\Sigma A}\left(\ell - \frac{5a_o}{3}\right)$

Main booms:- $\frac{2}{EA_2}\int_0^{a_o} \frac{2x}{a_o}\frac{WA_2}{b\Sigma A}(\ell - a_o - x)dx = \frac{2Wa_o}{bE\Sigma A}\left(\ell - \frac{5a_o}{3}\right)$

$$\therefore a_{2E}^{(b)} = \frac{Wa_o}{bE\Sigma A}\left(\ell - \frac{5a_o}{3}\right) - \frac{WA_1 B^b}{Gt'\Sigma A} \quad .. (34)$$

For $a_{kE}^{(b)}$, $k > 2$ App.2 Eq.(31) will apply.

End Load System c

The front spar yields the only contribution to $a_{oE}^{(c)}$

Front booms give:- $\frac{Wa_o \ell}{bE\Sigma A}\left(1 - \frac{a_o}{3\ell}\right)$ and web $\frac{a_o b}{Gt'}\cdot\frac{1}{2a_o}\cdot\frac{WA_1}{b\Sigma A} = \frac{WA_1}{2Gt'\Sigma A}$

$$a_{oE}^{(c)} = \frac{Wa_o \ell}{bE\Sigma A}\left(1 - \frac{a_o}{3\ell}\right) + \frac{WA_1}{2Gt'\Sigma A} \quad .. (35)$$

In the case of $a_{1E}^{(c)}$ also, only the front spar will contribute, the webs cancelling from bay to bay

Bay 0, booms give $\frac{Wa_o \ell}{bE\Sigma A}\left(1 - \frac{2a_o}{\ell}\right)$

Bay 1, booms give $\frac{Wa_o \ell}{bE\Sigma A}\left(1 - \frac{4a_o}{3\ell}\right)$

$$\therefore a_{1E}^{(c)} = \frac{2Wa_o \ell}{bE\Sigma A}\left(1 - \frac{2a_o}{\ell}\right) \quad .. (36)$$

There will be no contribution to $a_{2E}^{(c)}$ from bay 2.

Bay 1, front spar booms give $\frac{Wa_o}{bE\Sigma A}\left(\ell - \frac{5a_o}{3}\right)$, and web $\frac{-WA_1}{2Gt'\Sigma A}$

$$\therefore a_{2E}^{(c)} = \frac{Wa_o}{bE\Sigma A}\left(\ell - \frac{5a_o}{3}\right) - \frac{WA_1}{2Gt'\Sigma A} \quad .. (37)$$

For $a_{kE}^{(c)}$, $k > 2$ App. 2 Eq. (32) applies.

/ 2) ....

2) <u>Loading by Torsion Couple L_i, applied at the i^{th} rib</u>

Statically Correct Solution:-

$$-\frac{L_i}{4bc} = S_1 = S_2 = S_3 = S_4 = S_5 = S_6 \quad .. (38)$$

End Load System a

All contributions will cancel about the Oxy plane

$$\therefore a_{kE}^{(a)} = 0 \quad .. (39)$$

End Load System b

The web and skins contribute to $a_{oE}^{(b)}$

Front Web gives:- $\frac{a_o b}{Gt'} \cdot \frac{-L_i}{4bc} \cdot \frac{-B^b}{a_o} = \frac{L_i B^b}{4Gbc} \cdot \frac{b}{t'}$

Skins give:- $\frac{ca_o}{Gt} \cdot \frac{-L_i}{4bc} \frac{(1 - B^b)}{a_o} = -\frac{L_i(1 - B^b)}{4Gbc} \cdot \frac{c}{t}$

$$\therefore a_{oE}^{(b)} = \frac{L_i}{4Gbc}\left\{B^b \cdot \frac{b}{t'} - (1 - B^b) \cdot \frac{c}{t}\right\} \quad .. (40)$$

The web contributions to $a_{1E}^{(b)}$ will cancel.

Bay 0 skins give $\frac{L_i}{4Gbc}(1 - B^b)\frac{c}{t}$

Bay 1 skins give $\frac{2ca}{Gt} \cdot \frac{-L_i}{4bc} \frac{(1-B^b)}{a_o} - \frac{L_i ca_r}{4bcGt} \cdot \frac{-(1-B^b)}{a_o} = \frac{-L_i(1-B^b)}{4Gbc} \frac{c}{t}$

$$\therefore a_{1E}^{(b)} = 0 \quad .. (41)$$

All the contributions from bay 2 to $a_{2E}^{(b)}$ will cancel about the x axis.

Bay 1 skins give $\frac{L_i}{4Gbc}(1 - B^b)\frac{c}{t}$, and webs $-\frac{L_i B^b}{4Gbc} \cdot \frac{b}{t'}$

$$\therefore a_{2E}^{(b)} = \frac{L_i}{4Gbc}\left\{(1 - B^b)\frac{c}{t} - B^b \cdot \frac{b}{t'}\right\} \quad .. (42)$$

/ End Load .....

End Load System c

The web and skins contribute to $a_{oE}^{(c)}$

$$\text{Webs} \quad -\frac{L_i}{4Gbc} \cdot \frac{b}{2t'} \quad \text{and skins} \quad \frac{L_i}{4Gbc} \cdot \frac{c}{2t}$$

$$\therefore a_{oE}^{(c)} = \frac{L_i}{8Gbc}\left(\frac{c}{t} - \frac{b}{t'}\right) \quad .. (43)$$

Only the skins contribute to $a_{1E}^{(c)}$ as the webs cancel from bay to bay.

$$\therefore a_{1E}^{(c)} = \frac{L_i}{4Gbt} \quad .. (44)$$

The only contribution to $a_{2E}^{(c)}$ will be from the rear web, and a "triangular half" skin from bay 2, as the rest will cancel with bay 1.

$$\therefore a_{2E}^{(c)} = \frac{L_i}{8Gbc}\left(\frac{c}{t} - \frac{b}{t'}\right) \quad .. (45)$$

3.2 <u>Effect of a Flexible Root Rib</u>

There is some evidence that the degree of flexibility of the half rib joining the mainspar to frontspar at station 1, is of importance, and the modifications to the strain energy coefficients are given in this section.

End Load System b.

The load distribution is given in App.2 Eq. (21)

$$\left.\begin{aligned}
&\text{Contribution to } a_{oo}^{(b)} = \frac{2(1-2B^b)^2 c^3}{3EA_R a_o^2} + \frac{bc(B^b)^2}{Gt_R a_o^2} \\
&\qquad = \text{Contribution to } a_{22}^{(b)} \text{ and } a_{o2}^{(b)} \\
&\text{Contribution to } a_{o1}^{(b)} = \frac{-4(1-2B^b)^2 c^3}{3EA_R a_o^2} - \frac{2bc(B^b)^2}{Gt_R a_o^2} \\
&\qquad = \text{Contribution to } a_{12}^{(b)} \\
&\text{Contribution to } a_{11}^{(b)} = \frac{8(1-2B^b)^2 c^3}{3EA_R a_o^2} + \frac{4bc(B^b)^2}{Gt_R a_o^2}
\end{aligned}\right\} \quad .. (46)$$

/ End Load ....

End Load System c

This load distribution is given in App.2 Eq. (25).

$$\left.\begin{aligned}
\text{Contribution to } a_{oo}^{(c)} &= \frac{bc}{4Gt_R a_o^2} \\
&= \text{Contributions to } a_{o2}^{(c)} \text{ and } a_{22}^{(c)} \\
\text{Contribution to } a_{o1}^{(c)} &= \frac{-bc}{2Gt_R a_o^2} \\
&= \text{Contribution to } a_{12}^{(c)} \\
\text{Contribution to } a_{11}^{(c)} &= \frac{bc}{Gt_R a_o^2}
\end{aligned}\right\} \quad .. (47)$$

There will be no contribution to the coefficients due to the statically correct solution. The contributions of Eqs. (46) and (47) must be added to their respective coefficients as given in §3.1.

3.3 Special Root Conditions

The effect of special root conditions is to reduce the number of redundancies at the section corresponding to the special connection. For example, consider the case of a two cell box, similar to that discussed in §3.1, except that only the mainspar is built in at the root, the front and rear spars being arranged to transfer shear, but not end loads, to their supports.

At station 0, the only internal end load system will be similar to that considered for T_o in the single cell swept box §2.1, and it will be statically determinate in that it must be equal and opposite to the front spar boom loads given by the statically correct solution at rib 0.

/ The load ......

The load system T_1, will also give only one possible system, which will act in the front and mainspars, and be similar to the load system T^c. At rib 2, the load system must fulfil certain equilibrium conditions. There must be equilibrium of the system on the inboard side of rib 2, although there can be no end load corresponding to the rear spar booms. Also there must be no discontinuity of end load across rib 2. The only end load system which will meet these requirements is one similar to that of T_1. This will give no internal loads in the rear spar booms, and an additional statically determinate system must be added to cancel the statically correct loads in the rear spar booms at station 2.

The distribution of these loads into the individual structural components, and the calculation of the strain energy coefficients, are made as given in § 3.1.

/4. .....

4. SINGLE CELL SWEPT BOX - SECOND ORDER EFFECTS

The structure of this box is shown in Fig.8. Except for the omission of the centre web and the half rib joining it to the front spar at the root, it is similar to the two cell box. The centre booms are retained and the root is built in. The flexibility of rib 1, which in this case is complete, is considered. All the other assumptions are identical to those of the previous cases, Appendix 3 indicating the method used for dealing with the triangular skin panels.

Internal Load Distribution

All the three load systems are possible, and the load distributions resulting from them will be similar to those for the unswept case. App. 2 §3.

Calculation of Strain Energy Coefficients

The coefficients due to the internal load systems (a) and (b) will be the same for internal loads only.

End Load Systems a and b.

The contributions to $a_{oo}^{(a)}$ will be from the front booms, with part contributions from the centre booms, front and rear skins and rib booms.

Centre booms give $\frac{2}{EA_2}\int_{a_{o/2}}^{a_o}\left(\frac{2x}{a_o}\right)^2 dx = \frac{a_o}{3EA_2}$

The front and rear skins will give $\frac{2ca_o}{Gt}\left(\frac{1}{a_o^2}\right) = \frac{2c}{Gta_o}$

The booms of rib 1 give $\frac{4}{EA_R}\int_0^c\left(\frac{y}{c}\cdot\frac{c}{a_o}\right)^2 dy = \frac{4c^3}{3EA_Ra_o^2}$

$$\therefore a_{oo}^{(a)} = a_{oo}^{(b)} = \frac{a_o}{3E}\left(\frac{2}{A_1}+\frac{1}{A_2}\right) + \frac{2c}{Gta_o} + \frac{4c^3}{3EA_Ra_o^2} \quad \text{.. (48)}$$

In the case of $a_{o1}^{(a)}$ there will be similar contributions.

Centre booms give $2a_o/3EA_2$ and skins $-2c/Gta_o$

Rib booms give $\frac{4}{EA_R}\int_0^c -\left(\frac{c}{a_o}\cdot\frac{y^2}{c^2}\left\{\frac{c}{a_o}+\frac{c}{a}\right\}\right) dy = -\frac{4c^3}{3EA_Ra_o}\left(\frac{1}{a}+\frac{1}{a_o}\right)$

$$\therefore a_{o1}^{(a)} = a_{o1}^{(b)} = \frac{a_o}{3E}\left(\frac{1}{A_1}+\frac{2}{A_2}\right) - \frac{2c}{Gta_o} - \frac{4c^3}{3EA_Ra_o}\left(\frac{1}{a}+\frac{1}{a_o}\right) \quad \text{(49)}$$

There will be the normal unswept contribution to $a_{11}^{(a)}$ from bay 1, in addition to the boom and reduced skin contribution from bay 0.

From bay 1 $\quad \frac{4a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{4c}{Gta}$

Bay 0, centre booms give $\frac{2}{EA_2}\int_{a_o/2}^{a_o}\left(\frac{2x}{a_o}\right)^2 dx = \frac{7a_o}{3EA_2}$

Skins give $2c/Gta_o$ and front spar booms $2a_o/3EA_1$

Rib booms give: $\frac{4}{EA_R}\int_0^c\left\{\left(\frac{c}{a}+\frac{c}{a_o}\right)\cdot\frac{y}{c}\right\}^2 dy = \frac{4c^3}{3EA_R}\left(\frac{1}{a^2} + \frac{2}{a_o a} + \frac{1}{a_o^2}\right)$

$$\therefore a_{11}^{(a)} = a_{11}^{(b)} = \frac{2}{3EA_1}(2a+a_o) + \frac{1}{3EA_2}(8a+7a_o) + \frac{2c}{Gt}\left(\frac{2}{a}+\frac{1}{a_o}\right) + \frac{4c^3}{3EA_R}\left(\frac{1}{a^2}+\frac{2}{a_o a}+\frac{1}{a_o^2}\right) \quad (50)$$

There will be contributions from rib 1 to $a_{12}^{(a)}$, $a_{22}^{(a)}$ and $a_{o2}^{(a)}$, which will otherwise be as for the unswept solution.

Rib Boom contribution to $a_{12}^{(a)} = \frac{4}{EA_R}\int_0^c -\left(\frac{c}{a}\cdot\frac{y^2}{c^2}\left\{\frac{c}{a}+\frac{c}{a_o}\right\}\right) dy = -\frac{4c^3}{3EA_R}\left(\frac{1}{a^2}+\frac{1}{a_o a}\right)$

$$\therefore a_{12}^{(a)} = a_{12}^{(b)} = \frac{2a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) - \frac{4c}{Gta} - \frac{4c^3}{3EA_R a}\left(\frac{1}{a} + \frac{1}{a_o}\right) \quad .. \ (51)$$

Rib boom contribution to $a_{22}^{(a)}$ is $\frac{4}{EA_R}\int_0^c\left(\frac{c}{a}\cdot\frac{y}{c}\right)^2 dy = \frac{4c^3}{3EA_R a^2}$

$$\therefore a_{22}^{(a)} = a_{22}^{(b)} = \frac{8a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{8c}{Gta} + \frac{4c^3}{3EA_R a^2} \quad .. \ (52)$$

Rib boom contribution to $a_{o2}^{(a)} = \frac{4}{EA_R}\int_0^c\left(\frac{c}{a}\cdot\frac{y}{c}\cdot\frac{c}{a_o}\cdot\frac{y}{c}\right) dy$

$$\therefore a_{o2}^{(a)} = a_{o2}^{(b)} = \frac{4c^3}{3EA_R a_o a} \quad .. \ (53)$$

For the other bays, $k, \ell > 2$, the unswept values, App. 2 Eqs. (46), (47) will apply.

/ End ......

End Load System c

The contribution to $a_{oo}^{(c)}$ will be due to the front spar, and rib 1 web, together with part contributions from the skins.

Rib web contribution $= \frac{2cb}{Gt_R}\left(\frac{1}{2a_o}\right)^2 = \frac{bc}{2Gt_R a_o^2}$

$$\therefore\ a_{oo}^{(c)} = \frac{2a_o}{3EA_1} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{2c}{t}\right) + \frac{bc}{2Gt_R a_o^2} \qquad ..\ (54)$$

The rib web contribution to $a_{o1}^{(c)}$ is $\frac{2cb}{Gt_R}\left(\frac{1}{2a_o}\right)\left\{-\left\{\frac{1}{2a} + \frac{1}{2a_o}\right\}\right\} =$

$$-\frac{bc}{2Gt_R a_o}\left(\frac{1}{a} + \frac{1}{a_o}\right)$$

$$\therefore\ a_{o1}^{(c)} = \frac{a_o}{3EA_1} - \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{2c}{t}\right) - \frac{bc}{2Gt_R a_o}\left(\frac{1}{a} + \frac{1}{a_o}\right) \qquad ..\ (55)$$

In addition to the effect due to the rib web, there will be a normal unswept contribution to $a_{11}^{(c)}$ from bay 1, and a partial contribution from bay 0. Bay 1: $\frac{4a}{3EA_1} + \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right)$

Bay 0:- $\frac{2a_o}{3EA_1} + \frac{1}{4Ga_o}\left(\frac{b}{t'} + \frac{2c}{t}\right)$

Rib Web:- $\frac{2bc}{Gt_R}\left(\frac{1}{2a} + \frac{1}{2a_o}\right)^2 = \frac{bc}{Gt_R}\left(\frac{1}{a^2} + \frac{2}{a_o a} + \frac{1}{a_o^2}\right)$

$$\therefore\ a_{11}^{(c)} = \frac{2}{3EA_1}(2a+a_o) + \frac{1}{4G}\left\{\frac{b}{t'}\left(\frac{2}{a}+\frac{1}{a_o}\right) + \frac{2c}{t}\left(\frac{2}{a}+\frac{1}{a_o}\right)\right\} + \frac{bc}{2Gt_R}\left(\frac{1}{a^2} + \frac{2}{a_o a} + \frac{1}{a_o^2}\right) \qquad (56)$$

There will be rib web contributions to $a_{12}^{(c)}$, $a_{22}^{(c)}$ and $a_{o2}^{(c)}$ which will otherwise be similar to the unswept case.

Rib 1 effect on $a_{12}^{(c)} = \frac{2cb}{Gt_R}\left\{\frac{1}{2a}\left(-\frac{1}{2a} - \frac{1}{2a_o}\right)\right\} = \frac{-bc}{2Gt_R a}\left(\frac{1}{a} + \frac{1}{a_o}\right)$

$$\therefore\ a_{12}^{(c)} = \frac{2a}{3EA_1} - \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right) - \frac{bc}{2Gt_R a}\left(\frac{1}{a} + \frac{1}{a_o}\right) \qquad ..\ (57)$$

/ Contribution .....

Contribution from rib 1 to $a_{22}^{(c)} = \frac{2bc}{Gt_R}\left(\frac{1}{2a}\right)^2 = \frac{bc}{2Gt_R a^2}$

$$\therefore a_{22}^{(c)} = \frac{8a}{3EA_1} + \frac{1}{Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right) + \frac{bc}{2Gt_R a^2} \quad .. (58)$$

Contribution from rib 1 to $a_{o2}^{(c)} = \frac{2bc}{Gt_R}\left(\frac{1}{2a} \cdot \frac{1}{2a_o}\right)$

$$\therefore a_{o2}^{(c)} = \frac{bc}{2Gt_R a_o a} \quad .. (59)$$

For the other bays, k, ℓ 2, the unswept values given by App.2 Eq. (48) will apply.

External Load Systems

1) Bending by Z wise force applied on x axis at tip $Z = -W$

Statically Correct Solution

This is assumed to be the same as for the unswept case App. 2 Eq. (49):-

$$\left.\begin{aligned} S_1 &= -S_4 = \frac{W}{2b} \qquad P_1 = P_3 = \frac{WA_1}{b\Sigma A}(\ell - ak) \\ P_2 &= \frac{WA_2}{b\Sigma A}(\ell - ak) \\ S_2 &= S_6 = -S_3 = -S_5 = \frac{W}{b}\left(\frac{1}{2} - \frac{A_1}{\Sigma A}\right) \end{aligned}\right\} (60)$$

where $\Sigma A = 2A_1 + A_2$

Calculation of Strain Energy Coefficients

End Load System a

The doubly symmetric nature of the internal system, together with the fact that the statically correct solution is antisymmetric about the Oxy plane only, implies that all contributions will cancel about the plan of symmetry $\therefore a_{kE}^{(a)} = 0$

End Load System b

There will be contributions to $a_{oE}^{(b)}$ from the front and rear skins and front and centre booms.

The skins contribute: $\frac{1}{a_o} \cdot \frac{2a_o c}{Gt} \cdot \frac{W}{b}\left(\frac{1}{2} - \frac{A_1}{\Sigma A}\right) = \frac{2Wc}{Gtb}\left(\frac{1}{2} - \frac{A_1}{\Sigma A}\right)$

/ Main ......

Main booms contribute: $\frac{2}{EA_2}\int_{a_o/2}^{a_o} \frac{2(a_o - x)}{a_o}\,\frac{WA_2}{b\Sigma A}(\ell - x)dx = \frac{Wa_o\ell}{bE\Sigma A}(\frac{1}{2} - \frac{a_o}{3\ell})$

Front booms give: $-\frac{2}{EA_1}\int_{o}^{a_o} \frac{(a_o - x)}{a_o} \cdot \frac{WA_1}{b\Sigma A}(\ell - x)dx = -\frac{Wa_o\ell}{bE\Sigma A}(1 - \frac{a_o}{3\ell})$

$$\therefore a_{oE}^{(b)} = \frac{2Wc}{Gtb}(\frac{1}{2} - \frac{A_1}{\Sigma A}) - \frac{Wa_o\ell}{2bE\Sigma A} \quad .. (61)$$

Both bays 0 and 1 will contribute to $a_{1E}^{(b)}$. In bay 1, the booms will cancel one another, leaving only a skin contribution.

Bay 1: $\frac{4Wc}{Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\}$

The components contributing from Bay 0, will be the same as for $a_{oE}^{(b)}$.

Skins: $-\frac{2Wc}{Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\}$

Front booms: $-\frac{2}{EA_1}\int_{o}^{a_o} \frac{x}{a_o}(\ell - x) \cdot \frac{WA_1}{b\Sigma A} \cdot dx = -\frac{Wa_o\ell}{bE\Sigma A}(1 - \frac{2a_o}{3\ell})$

Centre booms: $\frac{2}{EA_2}\int_{a_o/2}^{a_o} \frac{2x}{a_o}(\ell - x)\,\frac{WA_2}{b\Sigma A}\,dx = \frac{Wa_o\ell}{bE\Sigma A}(\frac{3}{2} - \frac{7a_o}{6\ell})$

$$\therefore a_{1E}^{(b)} = \frac{2Wc}{Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\} + \frac{Wa_o\ell}{2bE\Sigma A}(1 - \frac{a_o}{\ell}) \quad .. (62)$$

$a_{kE}^{(b)}$, $k > 1$ will be as for the unswept case, App.2 Eq. (51).

End Load System c

The front booms, web, and skins will contribute to $a_{oE}^{(c)}$

Front booms: $\frac{2}{EA_1}\int_{o}^{a_o} \frac{(a_o - x)}{a_o} \cdot \frac{WA_1}{bE\Sigma A}(\ell - x)dx = \frac{Wa_o\ell}{bE\Sigma A}(1 - \frac{a_o}{\ell})$

Front Web: $\frac{a_o b}{Gt'} \cdot \frac{W}{2b} \cdot \frac{1}{2a_o} = \frac{W}{4Gt'}$

Front Skins: $\frac{W}{b}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\} \cdot \frac{-1}{2a_o} \cdot \frac{a_o c}{Gt} \cdot \frac{3}{2} = -\frac{3Wc}{4Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\}$

Rear Skins: $-\frac{W}{b}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\} \cdot \frac{-1}{2a_o} \cdot \frac{a_o c}{Gt} \cdot \frac{1}{2} = \frac{Wc}{4Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\}$

$$\therefore a_{oE}^{(c)} = \frac{Wa_o\ell}{bE\Sigma A}(1 - \frac{a_o}{\ell}) - \frac{Wc}{2Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\} + \frac{W}{4Gt'} \quad .. (63)$$

In the case of $a_{1E}^{(c)}$, the contributions from bay 1, will be self cancelling due to the nature of the load systems. The bay 0 contribution will be from the front spar and skins.

$$\text{Front booms} = \frac{Wa_o \ell}{bE\sum A}\left(1 - \frac{2a_o}{\ell}\right)$$

$$\text{Front Spar} = -\frac{W}{4Gt'}$$

$$\text{Front and Rear Skins} = \frac{Wc}{2Gtb}\left\{\frac{1}{2} - \frac{A_1}{\sum A}\right\}$$

$$\therefore\ a_{1E}^{(c)} = \frac{Wa_o \ell}{bE\sum A}\left(1 - \frac{2a_o}{\ell}\right) + \frac{Wc}{2Gtb}\left\{\frac{1}{2} - \frac{A_1}{\sum A}\right\} - \frac{W}{4Gt'} \quad \text{.. (64)}$$

$a_{kE}^{(c)}$, $k > 1$ will be as for the unswept case App.2 Eqs.(52),(53).

2) <u>Loading by Torsion Couple L_i applied at i^{th} rib</u>

Statically Correct Solution:

$$-\frac{L_i}{4bc} = S_1 = S_2 = S_3 = S_4 = S_5 = S_6 \quad \text{.. (65)}$$

<u>Calculation of Strain Energy Coefficients</u>

End Load System a

All coefficients due to this system will be zero.

$$\therefore\ a_{kE}^{(a)} = 0$$

End Load System b

The contributions will be due to the skins only.

$$\text{Front Skins: } \frac{3}{2}\frac{ca_o}{Gt}\cdot\frac{1}{a_o}\cdot -\frac{L_i}{4bc} = -\frac{3L_i c}{8bc.Gt}$$

$$\text{Rear Skins: } -\frac{1}{2}\cdot -\frac{L_i}{4bc}\cdot\frac{a_o c}{Gt}\cdot -\frac{1}{a_o} = \frac{L_i}{8Gbc}\frac{c}{t}$$

$$\therefore\ a_{oE}^{(c)} = -\frac{L_i}{4Gbt} \quad \text{.. (66)}$$

/ In the .....

In the case of $a_{1E}^{(b)}$ all the bay 1 contributions will be self cancelling. Bay 0 effects will be similar to $a_{oE}^{(b)}$ but opposite in sign.

$$\therefore \; a_{1E}^{(b)} = \frac{L_i}{4Gbt} \qquad .. (67)$$

$a_{kE}^{(b)}$, $k > 1$, will be as the unswept solution App.2 Eq. (55).

End Load System c

Both webs and skins will contribute to $a_{oE}^{(c)}$

Front Web; $\quad - \frac{L_i}{4bc} \cdot \frac{a_o b}{Gt'} \cdot \frac{1}{2a_o} = - \frac{L_i}{8Gbc} \cdot \frac{b}{t'}$

Skins: $\quad \frac{L_i}{4bc} \cdot \frac{2a_o c}{Gt'} \cdot \frac{1}{2a_o} = \frac{L_i}{4Gbc} \cdot \frac{c}{t}$

$$\therefore \; a_{oE}^{(c)} = \frac{L_i}{8Gbc} \left(\frac{2c}{t} - \frac{b}{t'} \right) \qquad .. (68)$$

The bay 0 contribution to $a_{1E}^{(c)}$ will be equal to that for $a_{oE}^{(c)}$ but opposite in sign, whilst the bay 1 effect will be twice as much but of the same sign

$$\therefore \; a_{1E}^{(c)} = \frac{L_i}{8Gbc} \left(\frac{2c}{t} - \frac{b}{t'} \right) \qquad .. (69)$$

For $a_{kE}^{(c)}$, $k > 1$, the unswept values apply, App.2 Eq. (56).

REFERENCES

No.	Author	Title
1	Ebner H. and Köller H.	Stress distribution in reinforced cylindrical shells A.R.C. Report No. 3470 Dec. 1937
2	Hemp, W.S.	On the analysis of statically indeterminate structures. R & M. No. 2396 May 1947

/ Appendix 1 .....

APPENDIX 1

Basic Theory

In this method, the unknown quantity is the unit of a self equilibrating internal end load system. The end load system is assumed to act only in the bays adjacent to the rib at which it is applied, and to fall linearly to zero over the rib pitch. Fig.1 shows the typical form of an end load system. This latter assumption results in the minimum of overlap of the effects due to the unknowns, and a resulting simplification to the final equations.

One or more of these systems are applied to the structure at each rib station. The number of separate systems at each rib station is determined by the number of spanwise booms, m, and is equal to (m - 3). The three systems used for the solutions considered here are shown in Fig. 2.

The externally applied loads are distributed in the structure in a statically correct solution. As the internal loads are in self equilibrium, the overall equilibrium conditions are fulfilled. The actual stress distribution is that due to the most general combination of the statically correct solution, and the internal systems.

The numerical values of the units of the self equilibrating internal end load systems are determined by using the theorem of minimum strain energy.

In general, strain energy, $U = \frac{1}{2}\sum_{i}\sum_{j} a_{ij} T_i T_j + \sum_{i} a_{iE} T_i +$ constant (1)

where T_i, T_j are the unknown units of the internal load systems, a_{ij} are coefficients dependent upon on the geometry and elastic properties of the structure, and a_{iE} are coefficients dependent upon the applied load as well as the structural properties.

Using Castiliano's Theorem of Minimum Strain Energy, and differentiating the strain energy with respect to the end load system at rib k, for the case where the 'overlap' of the end load systems is restricted to one bay:-

$$a_{k-1,k}\, T_{k-1} + a_{kk}\, T_k + a_{k,k+1}\, T_{k+1} + a_{kE} = 0 \qquad .. (2)$$

/ Particular .....

Particular Cases for calculating the coefficients, a_{ij}, are:-

(1) Members subjected to end loads:-

$$(a_{ij}) \cdot T_i T_j = \int_0 \frac{T_i T_j}{EA} dx \qquad \cdot\cdot (3)$$

(2) Members subjected to shear (panels):-

$$(a_{ij}) \cdot T_i T_j = S_i S_j \cdot \frac{ad}{Gt}$$

where, S_i, S_j are the shear flows resulting from T_i, T_j respectively.

In practice, the coefficient a_{kE} may be zero for parts of the structure not adjacent to discontinuities, and Eq.(2) becomes:-

$$a_{k-1,k} T_{k-1} + a_{kk} T_k + a_{k,k+1} T_{k+1} = 0 \qquad \cdot\cdot (4)$$

Also $a_{k-1,k} = a_{k,k+1}$ if the structure is uniform.
Use is made of these occurrences in simplifying the solution of the equations.

$$\text{Write:-} \quad \frac{a_{kk}}{a_{k,k-1}} = -2\alpha \qquad \alpha = \cosh \phi \qquad \cdot\cdot (5)$$

The solution of the Homogeneous Equations (4) is:-

$$T_k = C_1 \cosh K\phi + C_2 \sinh K\phi \qquad \cdot\cdot (6)$$

A further assumption is that the internal load system falls to zero at the extremity of the structure.

$$\text{Hence} \quad T_n = 0 \qquad \cdot\cdot (7)$$

and from Eq. (6):-

$$\frac{C_1}{C_2} = -\tanh n\phi \qquad \cdot\cdot (8)$$

$$\therefore C_1 = C \sinh n\phi \;; \quad C_2 = -C \cosh n\phi$$

$$\text{and} \quad T_k = C \sinh (n-k)\phi \qquad \cdot\cdot (9)$$

/ Substitution ....

Substitution From Eq. (9) into the equation for the internal load system at the discontinuity, which takes the form of Eq. (2), enable the value of C to be found, and hence the numerical values of T_k.

Procedure for Solution

1) Idealise the structure to conform with that dealt with in the solution given.

2) Distribute the applied loads in the manner of the statically correct solution used in the calculation of a_{kE}.

3) Calculate the strain energy coefficients, a_{kk}, $a_{k\ell}$, a_{kE}

4) Form the elasticity equations of the type of Eqs. (2),(4).

5) Solve the equations as shown above.

6) Calculate the load distributions due to the internal load systems.

7) Superimpose these results upon the statically correct solution in the most general way possible.

This will give the corrected distribution.

/ Appendix 2 .....

APPENDIX 2

Unswept Box Theory

1. SINGLE CELL

The geometry is shown in Fig.3. The box is of uniform rectangular section, referred to the axes Oxyz. The skins $z = \pm b/2$ and the spar webs $y = \pm c/2$, are assumed to carry only shear loads, any end load carrying capacity that they possess being integrated with that of the spar booms. The total effective area of each spar boom is A. The ribs are rigid and placed at a pitch a. The box is built in at the root which corresponds to rib O.

Each of the shear panels is assumed to be subjected to a constant shear flow, this implying a linear variation in end load between the ribs.

Internal Load Systems

In this case, only the doubly antisymmetric load system T^c of Fig.2 is possible. The shear notation is shown in Fig.5.

Consideration of the equilibrium of internal loads at the spar and ribs booms, due to T_k, in the bays adjacent to rib k, yields:-

At rib (k+1):- $S_{2_k} - S_{1_k} = -\frac{T_k}{a}$ $\qquad S_{R_{k+1}} + S_{2_k} = 0$

$$S_{R_{k+1}} - S_{1_k} = 0$$

$$\left.\begin{aligned} \therefore\ S_{R_{k+1}} &= +\frac{T_k}{2a} \\ S_{1_k} = S_{3_k} = -S_{2_k} = -S_{4_k} &= \frac{T_k}{2a} \end{aligned}\right\} \quad .. (1)$$

At rib (k-1):- $S_{2_{k-1}} - S_{R_{k-1}} = 0$ $\qquad S_{1_{k-1}} + S_{R_{k-1}} = 0$

$$S_{2_{k-1}} - S_{R_{k-1}} = 0$$

$$\left.\begin{aligned} \therefore\ S_{R_{k-1}} &= +\frac{T_k}{2a} \\ S_{1_{k-1}} = S_{3_{k-1}} = -S_{2_{k-1}} = -S_{4_{k-1}} &= -\frac{T_k}{2a} \end{aligned}\right\} \quad .. (2)$$

At rib k:- $-S_{2_{k-1}} + S_{2_k} - S_{R_k} = 0$

$$\therefore\ S_{R_k} = -\frac{T_k}{a} \qquad .. (3)$$

There are no rib boom end loads

External Load Systems

1) Bending by Z wise force on x axis at tip $\quad Z = -W$

Statically correct solution:-
$$\left.\begin{aligned} S_1 &= -S_3 = \frac{W}{2b} \\ P &= \frac{W}{2b}(\ell - ak) \end{aligned}\right\} \quad .. (4)$$

where P is the end load in the upper spar booms at rib k.

2) Loading by Torsion Couple, L_i, applied at $(i)^{th}$ rib

Statically correct solution:-
$$-\frac{L_i}{2bc} = S_1 = S_2 = S_3 = S_4 \quad .. (5)$$

Calculation of Strain Energy Coefficients

At a section in the k^{th} bay defined by x from rib k, the end load the front spar top boom is:-

$$\frac{W}{2b}(\ell - ak + x) + T_k \cdot \frac{x}{a} + T_{k+1}\frac{(a-x)}{a}$$

In the $(k-1)^{th}$ bay:- $\frac{W}{2b}\left(\ell - a(k-1) + x\right) + T_k\frac{(a-x)}{a} + T_{k-1} \cdot \frac{x}{a}$

where in this case, x is from rib (k-1).

Shear Flows in k^{th} bay:-

$$S_1 = \frac{T_k}{2a} - \frac{T_{k+1}}{2a} + \frac{W}{2b} - \frac{L_i}{2bc} \qquad S_2 = -\frac{T_k}{2a} + \frac{T_{k+1}}{2a} - \frac{L_i}{2bc}$$

$$S_3 = \frac{T_k}{2a} - \frac{T_{k+1}}{2a} - \frac{W}{2b} - \frac{L_i}{2bc} \qquad S_4 = -\frac{T_k}{2a} + \frac{T_{k+1}}{2a} - \frac{L_i}{2bc}$$

In $(k-1)^{th}$ bay:-

$$S_1 = -\frac{T_k}{2a} + \frac{T_{k-1}}{2a} + \frac{W}{2b} - \frac{L_i}{2bc} \qquad S_2 = \frac{T_k}{2a} - \frac{T_{k-1}}{2a} - \frac{L_i}{2bc}$$

$$S_3 = -\frac{T_k}{2a} + \frac{T_{k-1}}{2a} - \frac{W}{2b} - \frac{L_i}{2bc} \qquad S_4 = \frac{T_k}{2a} - \frac{T_{k-1}}{2a} - \frac{L_i}{2bc} \quad .. (6)$$

The ribs are rigid.

Then
$$a_{kk} \cdot T_k^2 = \frac{8}{EA}\int_0^a T_k^2 \cdot \frac{x^2}{a^2}\,dx + \frac{T_k^2}{4a^2} \cdot \frac{4}{G}\left(\frac{ba}{t'} + \frac{ca}{t}\right)$$

(Contribution from 4 booms, 2 webs, and 2 skins in each of bays k, k-1)

$$\therefore a_{kk} = \frac{8a}{3EA} + \frac{1}{Ga}\left(\frac{b}{t'} + \frac{c}{t}\right) \quad .. (7)$$

/ $a_{k\ell}$ ....

$$a_{k\ell} \cdot T_k \cdot T_\ell = \frac{4}{EA}\int_0^a T_k \cdot T_\ell \cdot \frac{x}{a}\frac{(a-x)}{a}dx - \frac{T_k T_\ell}{4a^2} \cdot \frac{2}{G}\left(\frac{ba}{t'} + \frac{ca}{t}\right)$$

for $\ell = k+1, k-1$

(Contributions from 4 booms, 2 skins and 2 webs in bay k)

$$\left.\begin{aligned} \therefore a_{k\ell} &= \frac{2a}{3EA} - \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{c}{t}\right) \qquad & \ell &= k-1, k+1 \\ a_{k\ell} &= 0 & \ell &\neq k-1, k+1 \end{aligned}\right\} \quad .. (8)$$

At the root there is a contribution from bay 0 only

$$\therefore a_{oo} = \tfrac{1}{2} a_{kk} \quad .. (9)$$

The a_{kE} due to the Z wise force are zero, as the contributions from the front and rear spars cancel. This implies that for this case $T_k = 0$ i.e. the solution assumed is correct.

Due to the torsion couple L_i, in bay k:-

$$a_{kE} \cdot T_k = \frac{2T_k}{G \cdot 2a} \cdot \frac{L_i}{2bc}\left(-\frac{ba}{t'} + \frac{ca}{t}\right) \qquad (k < i)$$

$$a_{kE} = \frac{L_i}{2bc \cdot Ga}\left(\frac{c}{t} - \frac{b}{t'}\right)$$

There will be an equal and opposite contribution from bay k-1.

$$\left.\begin{aligned} \therefore a_{kE} &= 0 \quad (k \neq o, i) \\ a_{oE} &= -a_{iE} = \frac{L_i}{2Gbc}\left(\frac{c}{t} - \frac{b}{t'}\right) \end{aligned}\right\} \quad .. (10)$$

This allows for warping constraint effects.

2. TWO CELL

The structure of the two cell box is shown in Fig.4. It is similar to the single cell box, apart from the additional centre spar. The same assumptions are made.

Internal Load Systems

There are three possible internal load systems, as shown in Fig.2. All three are considered, although in practice, for the external loads which give rise to stresses which are antisymmetric about the Oxy plane, the doubly symmetric system, T^a, is zero.

Using the notation of Fig.5 for the resultant shears across the section to be zero:-

$$\left.\begin{aligned} S_i &= S_1 - \frac{\sum_{o}^{j=i-1} P_j}{a} & i &= 2,6 \\ S_i &= S_1 - \frac{\sum_{o}^{j=i-1} P_j}{a} + S_w & i &= 3,4,5 \end{aligned}\right\} \quad .. (11)$$

For zero resultant moment on section:-

$$S_1 = S_4 + \tfrac{1}{2}(S_2 + S_3 + S_5 + S_6) \quad .. (12)$$

End Load System a

$$\left.\begin{aligned} P_1 &= P_3 = P_4 = P_6 = T^a \\ P_2 &= P_5 = -2T^a \end{aligned}\right\} \quad .. (13)$$

From Eqs. (11), (12), (13):-

$$S_1 = -S_4 = -\frac{S_w}{2}$$

$$\left.\begin{aligned} S_2 &= -\frac{S_w}{2} - \frac{T^a}{a} & S_3 &= \frac{S_w}{2} + \frac{T^a}{a} \\ S_5 &= \frac{S_w}{2} - \frac{T^a}{a} & S_6 &= -\frac{S_w}{2} + \frac{T^a}{a} \end{aligned}\right\} \quad .. (14)$$

/ The strain ...

The strain energy of the shear flows is:-

$$U = \frac{a}{2G}\left[\frac{b}{t'}\left(S_w^2 + \frac{S_w^2}{4} + \frac{S_w^2}{4}\right) + \frac{c}{t}\left\{\left(\frac{S_w}{2} + \frac{T^a}{a}\right)^2 + \left(\frac{S_w}{2} + \frac{T^a}{a}\right)^2 + 2\left(\frac{S_w}{2} - \frac{T^a}{a}\right)\right\}\right]$$

$$\therefore \quad \frac{\partial U}{\partial S_w} = \frac{\partial}{\partial S_w}\left[S_w^2\left(\frac{3b}{2t'} + \frac{c}{t}\right)\right]$$

For minimum energy $\frac{\partial U}{\partial S_w} = 0$

i.e. $S_w\ (\frac{3b}{t'} + \frac{c}{t}) = 0$

$$\therefore S_w = 0 \qquad .. (15)$$

From Eqs. (14),(15):-

$$\left.\begin{aligned} S_1 &= S_4 = S_w = 0 \\ S_2 &= S_5 = -S_3 = -S_6 = -\frac{T^a}{a} \end{aligned}\right\} \qquad .. (16)$$

End Load System b

$$\left.\begin{aligned} P_1 &= P_3 = -P_4 = -P_6 = -T^b \\ P_2 &= -P_5 = 2T^b \end{aligned}\right\} \qquad .. (17)$$

From Eqs. (11),(12),(17):-

$$\left.\begin{aligned} S_1 &= -S_4 = -\frac{S_w}{2} \\ S_2 &= S_6 = -S_3 = -S_5 = -\left(\frac{S_w}{2} - \frac{T^b}{a}\right) \end{aligned}\right\} \qquad .. (18)$$

Strain Energy: $U = \frac{a}{2G}\left[\frac{b}{t'}\left(\frac{S_w^2}{4} + S_w^2 + \frac{S_w^2}{4}\right) + \frac{c}{t}\left\{4\left(\frac{S_w}{2} - \frac{T^b}{a}\right)^2\right\}\right]$

$$\therefore \frac{\partial}{\partial S_w}\left[S_w^2\left(\frac{3b}{2t'} + \frac{c}{t}\right) - 4\left(S_w\frac{T^b}{a} \cdot \frac{c}{t} - \left(\frac{T^b}{a}\right)^2\frac{c}{t}\right)\right] = 0$$

$$\therefore S_w = \frac{T^b}{a} \cdot \frac{4c}{t}\Big/\left(\frac{3b}{t'} + \frac{2c}{t}\right)$$

or writing $B^b = \frac{2c}{t}\Big/\left(\frac{3b}{t'} + \frac{2c}{t}\right)$ .. (19)

From Eqs. (18) and (19):-

$$\left.\begin{aligned} S_1 &= -S_4 = S_{w/2} = -\frac{T^b}{a}(1 - B^b) \\ S_2 &= S_6 = -S_3 = -S_5 = \frac{T^b}{a}(1-B^b) \end{aligned}\right\} \qquad (20)$$

/ Consideration .....

Consideration of the equilibrium of the rib booms gives:-

$$\left.\begin{aligned} S_{R_{k-1}} &= S_{R_{k+1}} = -\frac{S_{R_k}}{2} = -\frac{T_k^b B^b}{a} \\ P_{R_{k-1}} &= P_{R_{k+1}} = -\frac{P_{R_k}}{2} = \frac{T_k^b c}{a}(1 - 2B^b) \end{aligned}\right\} \quad \cdot\cdot (21)$$

End Load System c

$$\left.\begin{aligned} P_1 &= P_4 = -P_3 = -P_6 = T^c \\ P_2 &= P_5 = 0 \end{aligned}\right\} \quad \cdot\cdot (22)$$

From Eqs. (11), (12), (22):-

$$\left.\begin{aligned} S_1 &= -\frac{S_w}{2} + \frac{T^c}{2a} & S_4 &= \frac{S_w}{2} + \frac{T^c}{2a} \\ S_2 &= S_6 = -\left(\frac{S_w}{2} + \frac{T^c}{2a}\right) & S_3 &= S_5 = \frac{S_w}{2} - \frac{T^c}{2a} \end{aligned}\right\} \quad \cdot\cdot (23)$$

Strain Energy:- $U = \frac{a}{2G}\left[\frac{b}{t'}\left\{\left(\frac{T^c}{2a} - \frac{S_w}{2}\right)^2 + S_w^2 + \left(\frac{T_c}{2a} + \frac{S_w}{2}\right)^2\right\} + \frac{c}{t}\left\{2\left(\frac{S_w}{2} + \frac{T^c}{2a}\right)^2 + 2\left(\frac{S_w}{2} - \frac{T^c}{2a}\right)^2\right\}\right]$

$$\therefore \frac{\partial}{\partial S_w}\left[\frac{b}{t'}\left\{\frac{3}{2}S_w^2 + \left(\frac{T^c}{2a}\right)^2\right\} + \frac{c}{t}\left\{S_w^2 + 4\left(\frac{T^c}{2a}\right)^2\right\}\right] = 0$$

$$\therefore S_w\left\{\frac{3b}{t'} + \frac{2c}{t}\right\} = 0$$

$$\therefore S_w = 0 \quad \cdot\cdot (24)$$

From Eqs. (23) and (24):- $S_1 = S_4 = T^c/2a$

$$S_2 = S_3 = S_5 = S_6 = -\frac{T^c}{2a}$$

Consideration of the rib boom equilibrium gives:-

$$S_{R_{k-1}} = S_{R_{k+1}} = -\frac{S_{R_k}}{2} = \frac{T_k^c}{2a} \quad \cdot\cdot (25)$$

/ Calculation .....

Calculation of Strain Energy Coefficients

End Load System a

$$a_{kk}^{(a)} = \frac{8a}{3EA_1} + \frac{16a}{3EA_2} + \frac{8c}{Gta}$$

where the contributions are from the front and rear spar booms, mainspar booms, and skins respectively.

$$\left.\begin{aligned}
\therefore\ a_{kk}^{(a)} &= \frac{8a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{8c}{Gta} \\
a_{oo}^{(a)} &= \frac{1}{2}\, a_{kk}^{(a)} \\
a_{k\ell}^{(a)} &= 0 \qquad (\ell \neq k-1,\ k,\ k+1) \\
a_{k\ell}^{(a)} &= \frac{2a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) - \frac{4c}{Gta}, \quad \ell = k-1,\ k+1
\end{aligned}\right\} \quad .. \ (26)$$

where the contributions are as for $a_{kk}^{(a)}$

End Load System b

The boom contributions to $a_{kk}^{(b)}$ are as for $a_{kk}^{(a)}$

$$\text{Shear contribution} = 2\left[\left(\frac{B^b}{a}\right)^2 \cdot \frac{2ab}{Gt'} + \left(\frac{2B^b}{a}\right)^2 \cdot \frac{ab}{Gt'} + \left(\frac{1}{a}\left\{B^b - 1\right\}\right)^2 \cdot \frac{2ac}{Gt} + \left(\frac{1}{a}\left\{1 - B^b\right\}\right)^2 \frac{2ac}{Gt}\right]$$

$$\left.\begin{aligned}
\therefore\ a_{kk}^{(b)} &= \frac{8a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{4}{Ga}\left(3\left\{B^b\right\}^2 \frac{b}{t'} + \frac{2c}{t}\left\{1 - B^b\right\}^2\right) \\
a_{oo}^{(b)} &= \frac{1}{2}\, a_{kk}^{(b)} \\
a_{k\ell}^{(b)} &= 0 \qquad (\ell \neq k-1,\ k,\ k+1)
\end{aligned}\right\} \ (27)$$

The boom contribution to $a_{k\ell}^{(b)}$ $(\ell = k-1,\ k+1)$ is as for $a_{k\ell}^{(a)}$.
The shear is $(-\frac{1}{2})$ the contribution to $a_{kk}^{(b)}$

$$\therefore\ a_{k\ell}^{(b)} = \frac{2a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) - \frac{2}{Ga}\left(3\left\{B^b\right\}^2 \cdot \frac{b}{t'} + \frac{2c}{t}\left\{1 - B^b\right\}^2\right) \qquad \ell = k-1,\ k+1$$

/ End Load ....

End Load System c

There is no end load in the mainspar booms, and the contributions to $a_{kk}^{(c)}$ from the front and rear spar booms will be as for the other end load cases.

$$\therefore a_{kk}^{(c)} = \frac{8}{3}\frac{a}{EA_1} + \frac{1}{Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right)$$

the contributions being from the front and rear spar booms, front and rearspar webs, and skins respectively.

$$a_{oo}^{(c)} = \frac{1}{2}a_{kk}^{(c)}$$

$$a_{k\ell}^{(c)} = 0 \qquad (\ell \neq k-1,\ k,\ k+1) \qquad (28)$$

The end load contribution to $a_{k\ell}^{(c)}$ is the same as that for the front and rear spars to $a_{k\ell}^{(a)}$, ($\ell = k+1,\ k-1$). The skin contribution is $(-\frac{1}{2})$ that to $a_{kk}^{(c)}$

$$\therefore a_{k\ell}^{(c)} = \frac{2a}{3EA_1} - \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right) \qquad \ell = k-1,\ k+1$$

<u>External Load Systems</u>

1) <u>Bending by Z wise force applied on x axis at tip $Z = -W$</u>

Statically correct solution:- $S_1 = -S_3 = \frac{WA_1}{b\sum A}$

$$S_W = \frac{WA_2}{b\sum A}$$

where $\sum A = A_2 + 2A_1$

At rib k:-

End Load in upper front and rear spar booms:- $\frac{WA_1}{b\sum A}(\ell - ak)$ (29)

End Load in upper mainspar booms:- $\frac{WA_2}{b\sum A}\ (\ell - ak)$

<u>Strain Energy Coefficients</u>

End Load System a

The internal loads are symmetric, and the external loads antisymmetric about the Oxy plane, hence the contributions from members on either side of the plane will cancel.

$$\therefore a_{kE}^{(a)} = 0 \qquad (30)$$

/ End Load .....

End Load System b

Due to the choice of the external load distribution, Eq.(29), and the nature of the internal loads, the end load contribution to $a_{kE}^{(b)}$ is zero. The mainspar boom effect is cancelled by that of the front and rear spar booms. There is no shear in skins from the external loads, and hence no contribution from the skins.

In bay k:- Contribution from front web $= - \frac{WA_1 B^b}{Gt'\Sigma A}$

$=$ Contribution from rear web

Contribution from centre web $= \frac{2WA_2 B^b}{Gt'\Sigma A}$

$$\therefore a_{kE}^{(b)} = - \frac{2WB^b}{Gt'\Sigma A} (A_1 - A_2)$$

There will be an equal and opposite contribution from bay k-1.

$$\therefore a_{kE}^{(b)} = 0 \quad (k \neq 0, n)$$

$$a_{oE}^{(b)} = - a_{nE}^{(b)} = - \frac{2WB^b}{Gt'\Sigma A} (A_1 - A_2) \quad .. (31)$$

End Load System c

In this case the contribution from the internal and external loads will be zero, as the former is doubly antisymmetric, and the latter singly symmetric. Cancellation will take place on either side of the x axis.

$$\therefore a_{kE}^{(c)} = 0 \quad .. (32)$$

2) <u>Loading by Torsion Couple L_i, applied at i^{th} rib</u>

Statically correct solution:-

$$- \frac{L_i}{4bc} = S_1 = S_2 = S_3 = S_4 = S_5 = S_6 \quad .. (33)$$

<u>Strain Energy Coefficients</u>

End Load System a

The statically correct solution gives a doubly antisymmetric load distribution, and as the internal load system is doubly symmetric, there is no resultant contribution.

$$\therefore a_{kE}^{(a)} = 0 \quad .. (34)$$

/ End Load .....

End Load System b

In this case the internal load system is singly antisymmetric about the Oxy plane, and contributions will cancel about the x axis.

$$\therefore a_{kE}^{(b)} = 0 \qquad .. (35)$$

End Load System c

There is no external load in the booms, and therefore no contribution from this source.

In bay k:- the contribution from the webs is:-

$$-\frac{L_i}{4bc} \cdot \frac{b}{Gt'}$$

and the contribution from the skins:- $+\frac{L_i}{4bc} \cdot \frac{2c}{Gt}$

There will be an equal and opposite contribution from bay (k-1)

$$\therefore a_{kE}^{(c)} = 0 \quad (k \neq 0, i)$$

$$a_{oE}^{(c)} = -a_{iE}^{(c)} = \frac{L_i}{4Gbc}\left(\frac{2c}{t} - \frac{b}{t'}\right) \qquad .. (36)$$

3. <u>SINGLE CELL - SECOND ORDER EFFECTS</u>

The structure of this box is similar to that of the two cell box, Fig.4, except that there is no centre web. The centre booms are retained, and the assumptions are the same as for the single cell case.

<u>Internal Load Distribution</u>

The three systems of Fig.2 are possible.

End Load System a

$$\left.\begin{aligned} P_1 = P_3 = P_4 = P_6 &= T^a \\ P_2 = P_5 &= -2T^a \end{aligned}\right\} \qquad .. (37)$$

/ S_W ....

S_w is zero, as there is no centre web, hence from Eq. (16):-

$$\left. \begin{array}{l} S_1 = S_4 = 0 \\ S_2 = S_5 = - S_3 = - S_6 = - \frac{T_k^a}{a} \end{array} \right\} \quad .. (38)$$

As there is no spar web shear, there can be no rib web shear. Consideration of the equilibrium at the rib booms gives:-

$$\left. \begin{array}{l} \frac{P_{R_{k+1}}}{c} = \frac{P_{R_{k-1}}}{c} = - \frac{T_k^a}{a} \\ \frac{P_{R_k}}{c} = \frac{2T_k^a}{a} \end{array} \right\} \quad .. (39)$$

for both upper and lower surfaces, where P_R is the end load in the rib boom at $y = 0$, falling to zero at $y = \pm c$.

End Load System b

$$\left. \begin{array}{l} P_1 = P_3 = - P_4 = - P_6 = - T^b \\ P_2 = - P_5 = 2T^b \end{array} \right\} \quad .. (40)$$

Using Eqs. (20), (21), and writing $S_w = 0$

$$\left. \begin{array}{l} S_1 = S_4 = 0 \\ S_2 = S_6 = - S_3 = - S_5 = \frac{T_k^b}{a} \end{array} \right\} \quad .. (41)$$

Again there is no rib web shear.
Rib boom loads on top surface:-

$$\left. \begin{array}{l} \frac{P_{R_{k+1}}}{c} = \frac{P_{R_{k-1}}}{c} = \frac{T_k^b}{a} \\ \frac{P_{R_k}}{c} = - \frac{2T_k^b}{a} \end{array} \right\} \quad .. (42)$$

These will be opposite in sign on the lower surface.

/ End Load ......

End Load System c

$$\left.\begin{aligned} P_1 = P_4 = - P_3 = - P_6 = T^c \\ P_2 = P_5 = 0 \end{aligned}\right\} \quad .. (43)$$

From Eq. (25):-

$$\left.\begin{aligned} S_1 = S_4 = \frac{T_k^c}{2a} \\ S_2 = S_3 = S_5 = S_6 = - \frac{T_k^c}{2a} \end{aligned}\right\} \quad .. (44)$$

In this case there is a rib web shear, but no rib boom load:-

$$\left.\begin{aligned} S_{R_{k-1}} = S_{R_{k+1}} = \frac{T_k}{2a} \\ S_{R_k} = - \frac{T_k}{a} \end{aligned}\right\} \quad .. (45)$$

<u>Calculation of Strain Energy Coefficients</u>

The ribs are assumed to be rigid.

End Load System a

The load distribution is identical to the two cell case.

Hence:-

$$\left.\begin{aligned} a_{kk}^{(a)} &= \frac{8a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) + \frac{8c}{Gta} \\ a_{oo}^{(a)} &= \frac{1}{2}\, a_{kk}^{(a)} \\ a_{k\ell}^{(a)} &= 0 \qquad (\ell \neq k-1,\ k,\ k+1) \\ a_{k\ell}^{(a)} &= \frac{2a}{3E}\left(\frac{1}{A_1} + \frac{2}{A_2}\right) - \frac{4c}{Gta} \qquad (\ell = k-1, k+1) \end{aligned}\right\} \quad .. (46)$$

End Load System b

The distribution is similar to the two cell case with B^b zero. The coefficients are then identical to those obtained for load system a.

$$\therefore \quad \left.\begin{aligned} a_{kk}^{(b)} = a_{kk}^{(a)} \\ a_{k\ell}^{(b)} = a_{k\ell}^{(a)} \end{aligned}\right\} \quad .. (47)$$

/ End Load .....

End Load System c

This is again identical to the two cell case.

$$\left.\begin{aligned}
\therefore a_{kk}^{(c)} &= \frac{8a}{3EA_1} + \frac{1}{Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right) \\
a_{oo}^{(c)} &= \frac{1}{2}\, a_{kk}^{(c)} \\
a_{k\ell}^{(c)} &= 0 \qquad (\ell \neq k-1,\ k,\ k+1) \\
a_{k\ell}^{(c)} &= \frac{2a}{3EA_1} - \frac{1}{2Ga}\left(\frac{b}{t'} + \frac{2c}{t}\right) \qquad (\ell = k-1, k+1)
\end{aligned}\right\} \quad .. \ (48)$$

External Load Systems

1) Loading by Z wise force on x axis at tip $\quad Z = -W$

Statically Correct Solution:- $S_1 = -S_4 = \frac{W}{2b}$

The end loads assumed to be distributed to give constant stress across the section.

i.e.:- Load in upper booms at rib k:-

$$P_1 = P_3 = \frac{WA_1}{b\sum A}(\ell - ak)$$

$$P_2 = \frac{WA_2}{b\sum A}\quad(\ell - ak) \qquad (49)$$

where $\sum A = A_2 + 2A_1$

There will also be skin shears:-

$$S_2 = S_6 = -S_5 = -S_3 = \frac{W}{b}\left(\frac{1}{2} - \frac{A_1}{\sum A}\right)$$

Strain Energy Coefficients

End Load System a

In this case the internal loads are doubly symmetric, and the external loads antisymmetric about the Oxy plane. Therefore all contributions will cancel about the Oxy plane.

$$\therefore a_{kE}^{(a)} = 0 \qquad .. \ (50)$$

/ End Load .....

End Load System b

There are no web shears from the internal loads, and the boom contributions will cancel due to the choice of the statically correct solution.

In bay k:- Skin shears:-

$$S_2 = S_6 = -S_3 = -S_5 = \frac{T^b}{a} + \frac{W}{b}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\}$$

$$\therefore a_{kE}^{(b)} = \frac{4Wc}{Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\}$$

There will be an equal and opposite contribution from bay (k-1).

$$\left.\begin{aligned} \therefore a_{kE}^{(b)} &= 0 \qquad (k \neq o, n) \\ a_{oE}^{(b)} &= -a_{nE}^{(b)} = \frac{4Wc}{Gtb}\left\{\frac{1}{2} - \frac{A_1}{\Sigma A}\right\} \end{aligned}\right\} \quad .. (51)$$

This allows for shear lag effect.

End Load System c

As the internal loads are doubly antisymmetric, whilst the external loads are antisymmetric about the Oxy plane only, the contributions will cancel, on either side of the x axis.

$$\therefore a_{kE}^{(c)} = 0 \qquad .. (52)$$

2) <u>Loading by Torsion Couple L_i, applied at i^{th} rib</u>

Statically Correct Solution:-

$$\frac{-L_i}{4bc} = S_1 = S_2 = S_3 = S_4 = S_5 = S_6 \qquad .. (53)$$

<u>Strain Energy Coefficients</u>

End Load System a

The statically correct solution yields loads which are doubly antisymmetric, the internal loads being doubly symmetric.

$$\therefore a_{kE}^{(a)} = 0 \qquad .. (54)$$

End Load System b.

Here the internal loads are singly symmetric about the Oxy plane, and contributions will cancel about x axis.

$$\therefore a_{kE}^{(b)} = 0 \qquad .. (55)$$

End Load System c

The load distributions are the same as for the two cell case. Hence:-

$$a_{kE}^{(c)} = 0 \quad (k \neq o, i)$$

$$a_{oE}^{(c)} = - a_{iE}^{(c)} = \frac{L_i}{4Gbc} \left(\frac{2c}{t} - \frac{b}{t'}\right) \qquad .. (56)$$

APPENDIX 3

Treatment of Triangular Skins

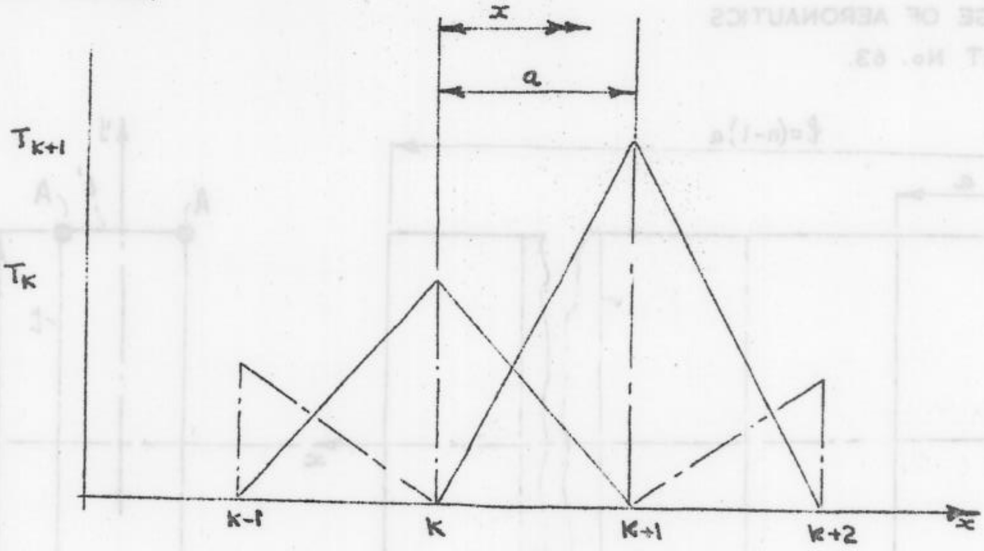
The forces on the triangular skin are shown in Fig. 6. A uniform shear flow, S, is assumed to act along the two perpendicular sides. This assumption corresponds to that made for the rectangular panels previously considered. Equilibrium is maintained by a shear flow, S', along the hypotenuse, and a system of uniformly distributed normal forces, p'.

Resolution of forces normal to, and parallel to the hypotenuse gives:-

$$-2Sc \cos \theta = \sum p' = P' \qquad .. (57)$$

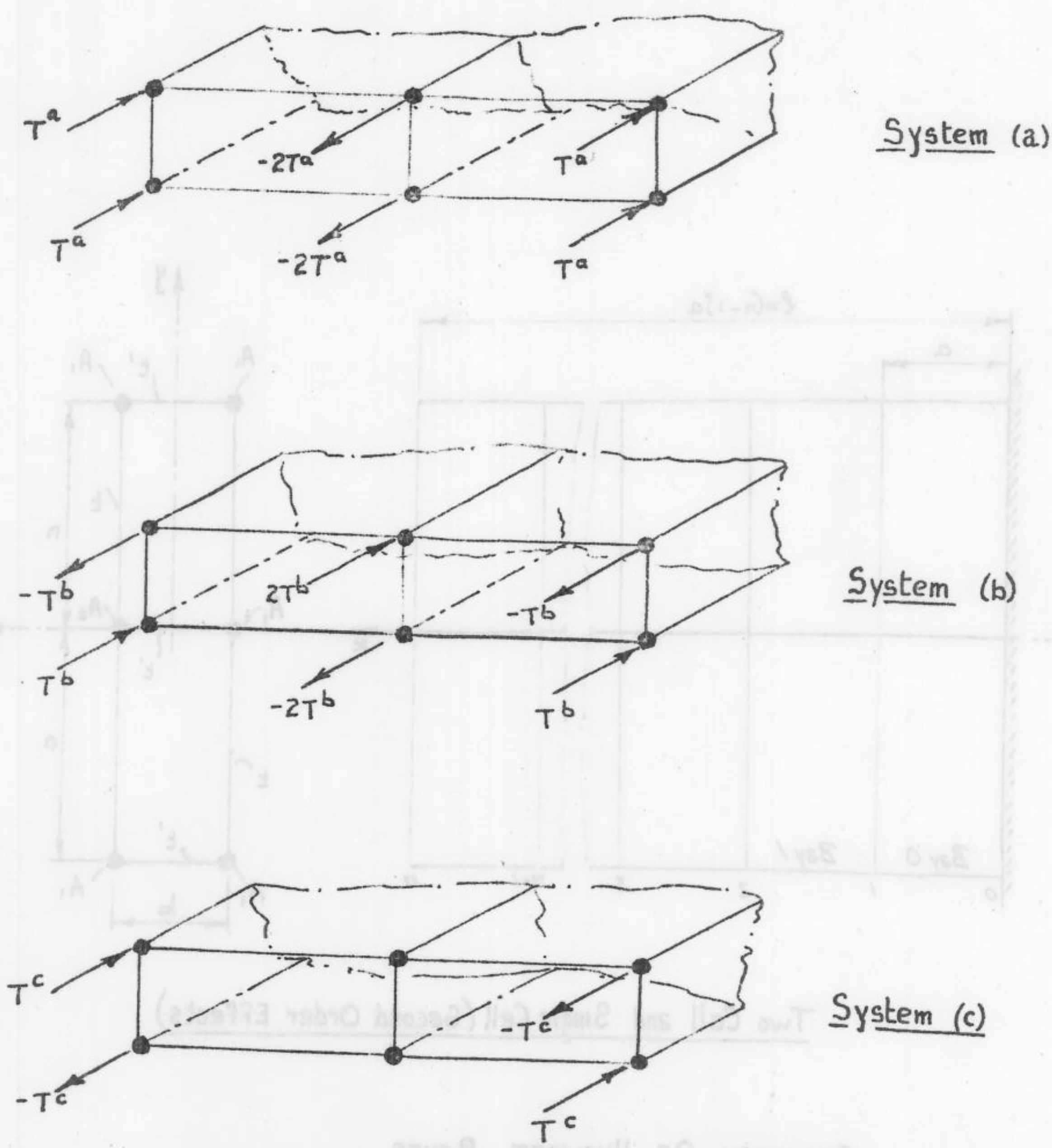
Where the root is built in, P' will be reacted along the built in edge. For the case where the hypotenuse is not built in, P' will be reacted at the spars.

COLLEGE OF AERONAUTICS
REPORT No. 63.



END LOAD SYSTEMS.

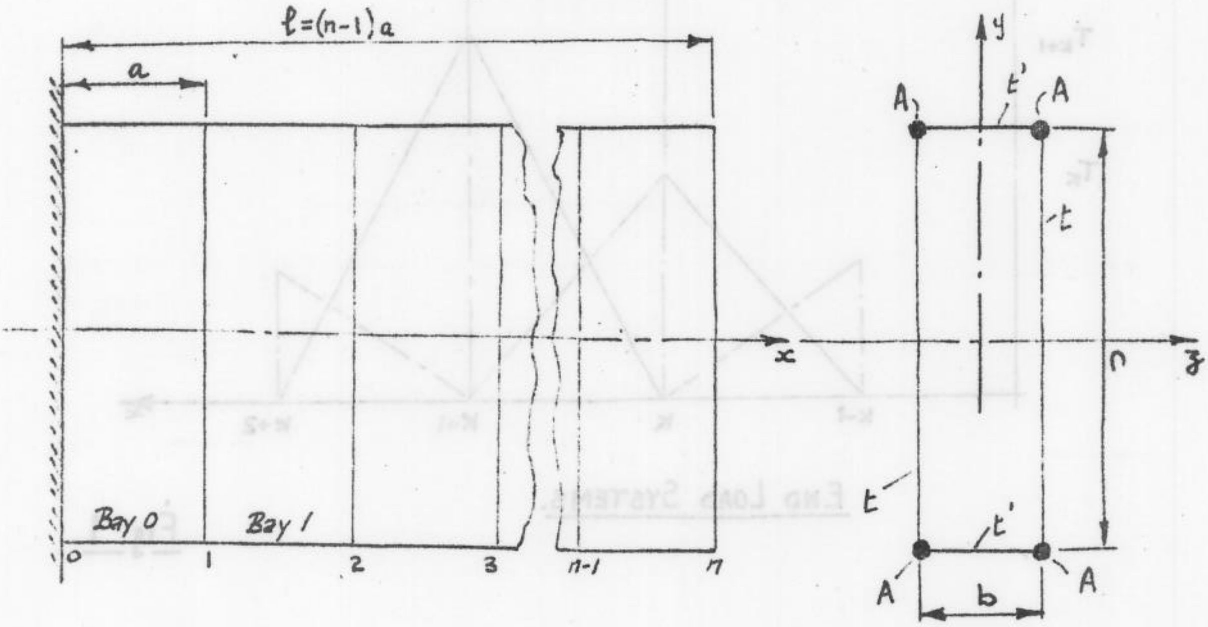
Fig. 1.



INTERNAL END SYSTEMS

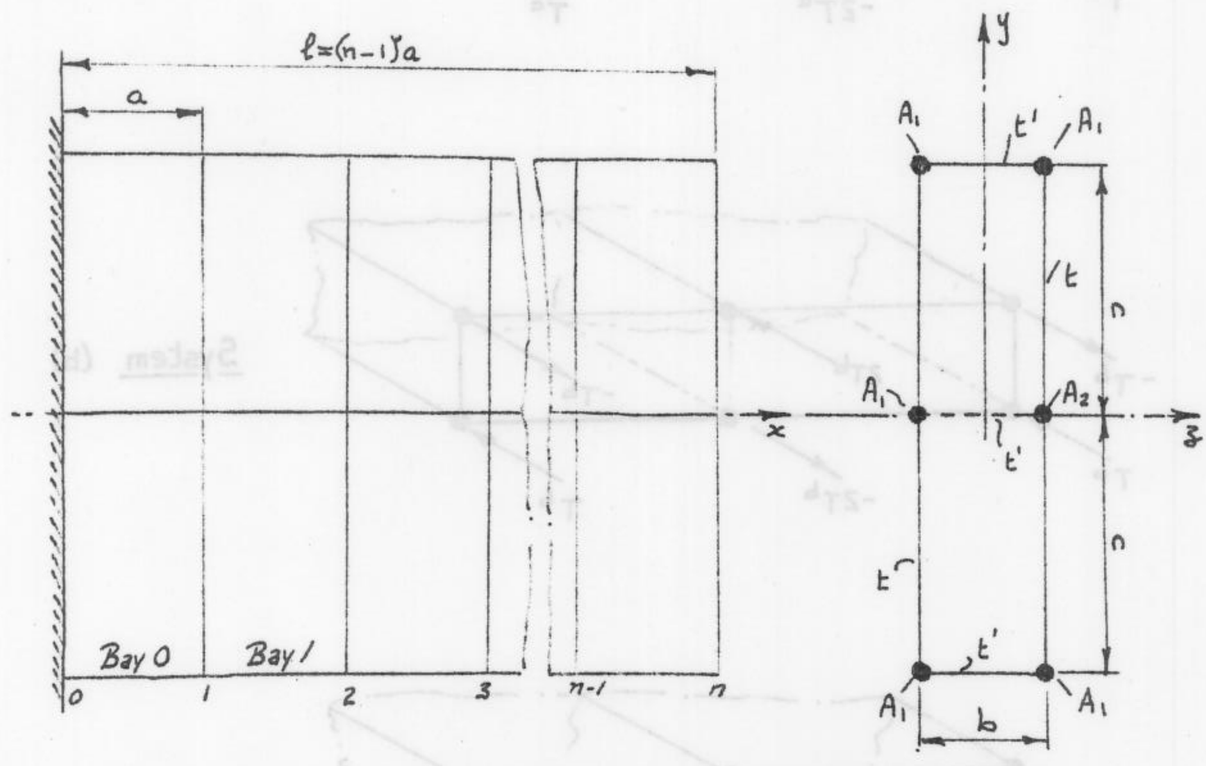
Fig. 2

COLLEGE OF AERONAUTICS
REPORT No. 63.



Single Cell

Fig. 3



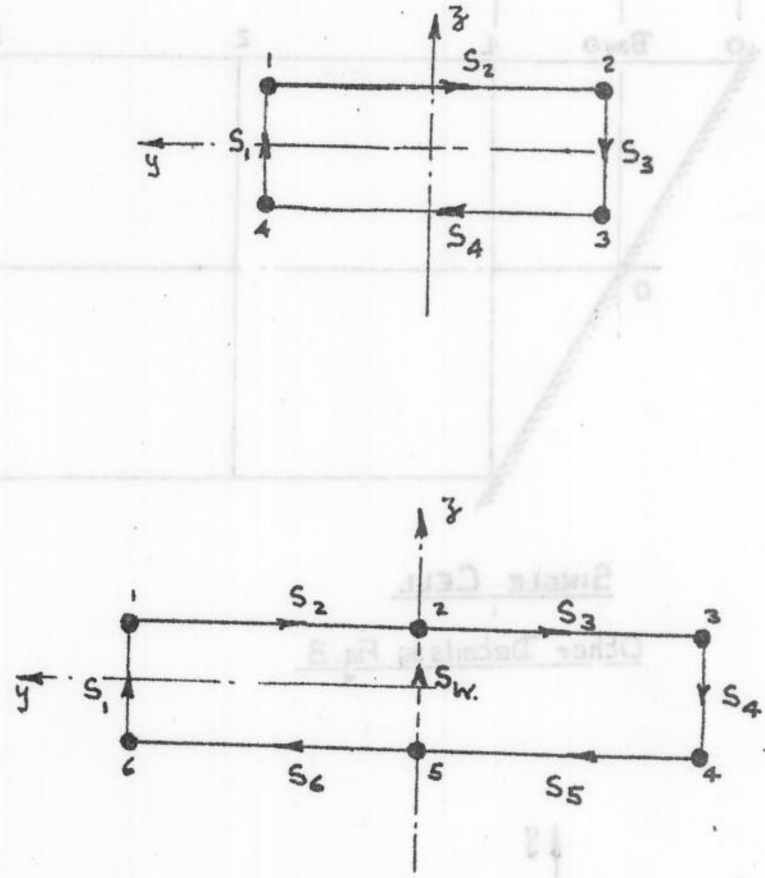
Two Cell and Single Cell (Second Order Effects)

GEOMETRY OF UNSWEPT BOXES

Fig. 4.

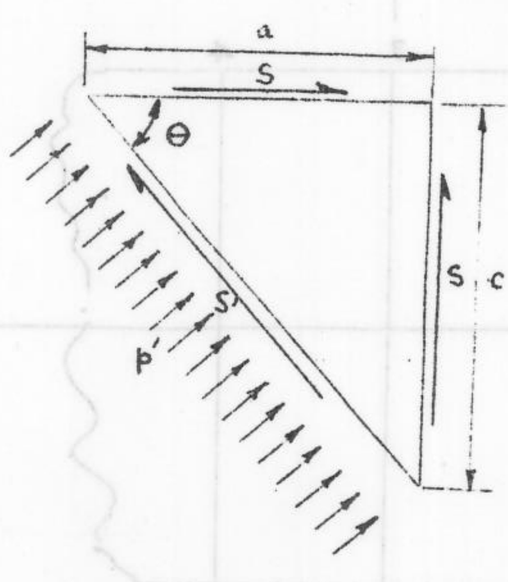
Single Cell

Two Cell and Single Cell (Second Order Effects).



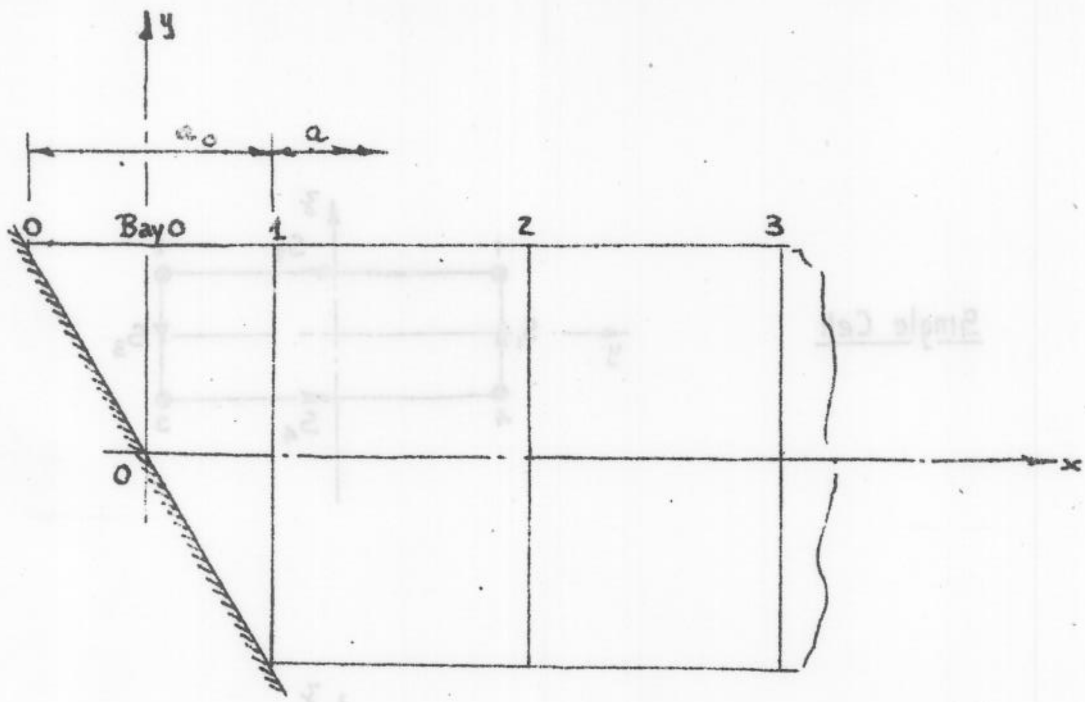
SHEAR NOTATION

Fig. 5.



TRIANGULAR SKIN

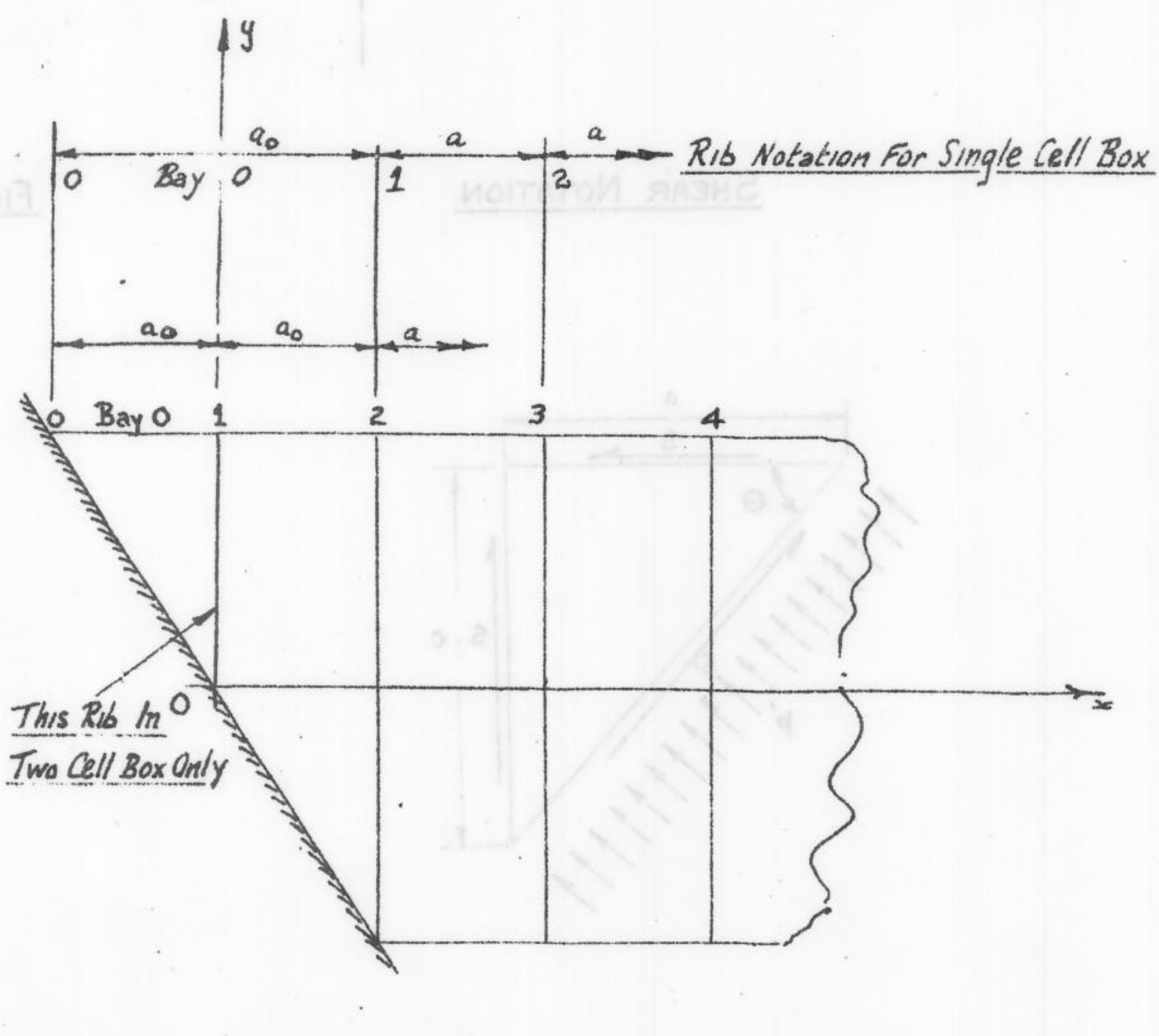
Fig. 6



SINGLE CELL

Other Details as Fig. 3

Fig. 7



TWO CELL AND SINGLE CELL (Second Order Effects).
Other Details as Fig. 4.

GEOMETRY OF SWEPT BOXES - Ribs Normal To Spars

Fig. 8