TITLE

An Asset and Liability Management Model Incorporating Uncertainty

A thesis submitted for the degree of Doctor of Philosophy

By

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ABSTRACT

Asset and Liability Management (ALM) is a well-established method, which enables companies to match future liabilities with future cash flow streams of assets. The first stage is to develop a deterministic model with forecast cash flow streams. In reality this can lead to results that are often volatile to deviations of future cash flows from their predicted values.

There are two main stages to this problem. Firstly, there is the issue of representing the future uncertainties. To this end we have developed a scenario generator that forecasts alternative realizations of future cash flows streams of different assets using alternative scenarios about a financial Index and the Capital Asset Pricing Model (CAPM). Considering this with the deterministic model leads to the creation of ALM models which incorporate uncertainty.

Having represented the uncertainty, we use an optimisation model to generate the current decisions concerning acquisition and disposal of assets. This model is a two stage stochastic programming model that aims to achieve targeted cash flows for each future year. Risk is represented in the form of assigning shares to different risk groups. In this thesis we describe our models of randomness and how they are captured in the two-stage stochastic programming model. We compare our model to a mean-variance representation. Both models are simulated through time. Backtesting is used to investigate the quality of both approaches.

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Chapter 1: INTRODUCTION

1.1 Investment Characteristics

In our epoch a phenomenon that characterises a vast majority of individuals, businesses and organisations around the world is to invest considerable amounts of their savings. By investing in shares, bonds, mutual funds or other financial derivative products makes them feel comfortable and enthusiastic that they could earn more on their investment rather than put the money in a bank and wait patiently for it to grow from the interest rates. They also like to manage any potential liabilities that they might have in the near or distant future.

Most investment decisions share three key characteristics in varying degrees. 1) The investment is partially or completely irreversible. The initial cost of investment is at least partially sunk [134]. For example capital controls may make it impossible for foreign (or domestic) investors to sell assets and reallocate their funds and investments in new workers thus, investment could be partly irreversible because of high costs of living, training and firing. 2) There is uncertainty over the future rewards from the investment. 3) The timing of the investment remains unknown. These characteristics interact to determine the optimal decisions of investors.

A way to make a 'safe' investment incorporating the above three characteristics and others such as risk is to develop an integrated system or model.

1.2 Risks and Investment

Investors who wish to allocate their assets and manage their liabilities must develop a strategic risk management system. There is a plethora of financial risks with the most important being market, credit and currency risk.

In the stock market, market risk is associated with the movements in the market index of portfolio returns [135]. According to the Capital Asset Pricing Model (CAPM), [22], all securities must be priced so that their expected returns at equilibrium are a linear combination of the risk-free return and the market index portfolio return. The weight of the latter for a particular security is the security beta (β), which indicates the relative marginal variation of the returns of that security with respect to the market portfolio. The CAPM is a single-factor model of security returns i.e. it assumes a single source of risk: the market. In more general models one encounters several independent risk factors. Each risk factor has a risk premium associated with it and, at equilibrium, security returns are determined as the sum over all factors of the total value of that factor in the security. This hypothesis is termed as Arbitrage Pricing Theory (APT), [4]. Credit risk covers the up or down grading of a borrower's credit worthiness. These changes are caused by changing prospects on the issuer's ability to meet all future obligations. Currency risk is the risk caused by exchange-rate fluctuations. Investors who own portfolios in foreign currency denominated securities will lose when exchange rates depreciate and gain when they appreciate.

1.3 Modelling Investment Strategies

Optimisation models are required to incorporate risk and uncertainty. If it is assumed that an investor's preference can be represented by some utility function over the mean and variance of the portfolio's returns, thus favouring portfolios with higher means and lower variances. The optimal portfolios for this investor are those that achieve the highest expected return for a given level of risk. Such portfolios are called mean-variance portfolios and are analysed in detail in a later chapter of this thesis. Mean-variance models have the characteristic that they take a decision at a time t and not beyond that. They are often called static models.

Having introduced the single-period CAPM and APT models and the static mean-variance, a broader modelling concept is the one of stochastic programming. In cases where uncertainty prevails at all the stages of the planning horizon and corrective action (recourse) would be possible between periods t and $t+1$, then stochastic programming models become appropriate.

Asset and Liability Management (ALM) modelling is successfully employed by using stochastic programming. ALM modelling incorporates the uncertainty in a unique manner, which is the generation of scenarios.

1.4 Thesis Outline

Chapter 1 gives a general introduction to investment characteristics and how these can be employed into modelling systems. It introduces the concepts of capturing and then modelling uncertainty. Two topics that this thesis covers in a novel way.

Chapter 2 discusses the single-period models in asset pricing. The mean-variance model and its formulation is analysed. The advantage of such models is that investors could build optimal portfolios under conditions of uncertainty by using statistical measurements of expectation and variance of return. The drawback is that if there are too many shares then the covariance matrix generation consumes large computational time. The mean variance framework has been one of the most popular models in empirical validations and comparisons. The CAPM and APT single-period factor models are also analysed and compared. The empirical validations of such models proved to be of enormous interest as their conclusions support their use and usefulness in the finance industry.

Chapter 3 introduces the concept of representing uncertainty. Forecasting procedures like stochastic processes (Geometric Brownian motion) and econometric techniques are discussed. The authors own scenario generator that incorporates uncertainty is analysed in detail. The generator produces consistent share and Index scenarios by using the CAPM.

In Chapter 4, optimisation models that incorporate the uncertainty are introduced. The Quadratic programming (QP) models are first analysed. The Wait-and-See, Expected-Value and Here-and-Now approaches are presented together with their inter-relationship and bounds. Two-stage and multi-stage stochastic linear models are given special treatment because the concept is used for the computational study of this thesis.

In Chapter 5, the ALM modelling is given special attention, as it is the main topic of this thesis. The author's new ALM study is presented together with its contributions to this field.

Chapter 6 presents the computational study, the findings and its results. This study also introduces a comparison of the ALM model with a Quadratic model, run with exactly the same data. The S&P 100 share Index is as well compared with the two models.

Finally, Chapter 7, concludes the research by pointing out the key findings and future directions.

Chapter 2: SINGLE-PERIOD RANDOM CASH FLOWS

2.1 Introduction

Single-period investment models were the starting point in asset pricing. Mean-Variance single-period models are characterized by the fact that the covariance term has to be evaluated. The drawback is that if there are too many shares to be evaluated then the covariance matrix generation takes large computational time. Other pricing models include the Capital Asset Pricing Model (CAPM), which is a special case of the Arbitrage Pricing Theory (APT) and is one of the most tested and widely used models. This chapter describes the Mean-Variance, CAPM and APT models explaining the theory behind these concepts. Empirical tests that have been performed by various academics and professionals in the finance and economics field are also discussed. Other pricing models within the CAPM concept are also analysed.

The background of the mean-variance framework is introduced together with a simple Markowian outline formulation of the model. The Markowitz concept provides the milestone for single-period investment theory and together with the CAPM and the APT gave the initiative to develop completely new and innovative investing techniques.

2.2 The Mean-Variance Model

2.2.1 Background

In June 1952 a leading academic journal published an article entitled 'Portfolio Selection'. Its author was Harry Markowitz [45], (although the full exposition was available several years later in 1959), [46], an unknown graduate student at that time. The topic that Markowitz chose for research was classified by many journals as too dicey and speculative. His objective was to use the notion of risk to construct portfolios[1] for investors who 'consider expected return a desirable thing and variance of return an undesirable thing [47]. Variance is a statistical

1 The word has a Latin root. Portare which means to carry and foglio which means leaf or sheet. So, the actual meaning of Portfolio is a collection of paper assets.

measurement of how widely the returns on an asset vary around their average. The greater the variance or standard deviation around the average, the less the average return will indicate about what the outcome is likely to be. One other key insight in Markowitz's methodology is that of diversification. He declares 'diversification, is both observed and sensible; a rule of behaviour which does not imply the superiority of diversification must be rejected both as a hypothesis and as a maxim'. With diversification one can combine a group of risky securities with high-expected returns into a relatively low-risk portfolio, so long as the covariances among the returns are minimized.

The investors' problem after Markowitz's research can be separated into three levels [48]. First, is that investors must obtain estimates (which are not simply point estimates) of future outcomes for individual securities. This is the task of security analysis. Attention should be made for the risk of each security and its relation to other securities. Using a suitable set of such estimates the investor needs to identify the set of efficient portfolios. The definition of an efficient portfolio is if and only if it offers a higher overall expected return than any other portfolio with comparable risk. This second task is the portfolio analysis. To perform this task Markowitz designed a specific algorithm (formally known as quadratic programming). The final task that investors face is the portfolio selection. Having identified the efficient portfolios it is a question of which one to choose. This depends on the individual investor and how he/she prefers to treat risk with regard to expected returns.

2.2.2 The Markowitz Model

The Markowitz concept provides the milestone for single-period investment theory. The problem explicitly addresses the trade-off between expected rate of return and variance of the rate of return in a portfolio [28]. Markowitz showed that investors could build optimal portfolios under conditions of uncertainty by using statistical measurements of expectation and variance of return. These portfolios are identified and the efficient set can be found by solving a quadratic problem.

Assuming that there are n assets whose expected rates of return or mean are $\bar{r}_1, \bar{r}_2, ..., \bar{r}_n$ and the covariances are $\sigma_{i,j}$, for $i, j = 1,2,...,n$. A portfolio is defined with weights w_i, $i = 1,2,...,n$, that sum to 1. In order to find a minimum-variance portfolio, the portfolio return is fixed at some arbitrary value $\bar{r}$. The mathematical formulation of the Markowitz problem is as follows (the following is only an overview for the understanding of the mean-variance concept and extensive discussions and formulations are found in Chapter 4).

Minimize $\frac{1}{2}\sum_{i,j=1}^{n} w_i w_j \sigma_{i,j}$

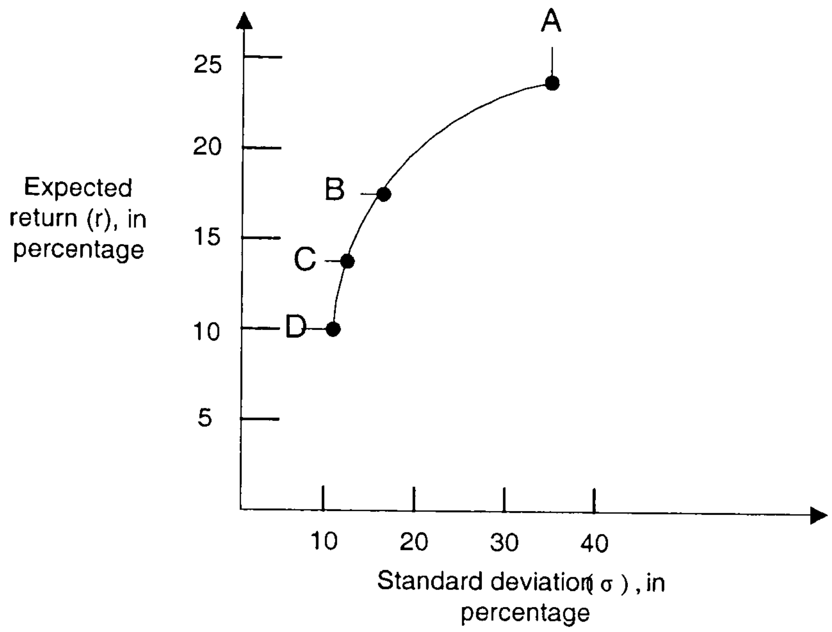
Subject to $\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r}$

$$\sum_{i=1}^{n} w_i = 1 \quad w_i \geq 0, \quad i = 1,...,n$$

The fraction $\frac{1}{2}$ in front of the variance is for ease of expressing the partial derivative equations.

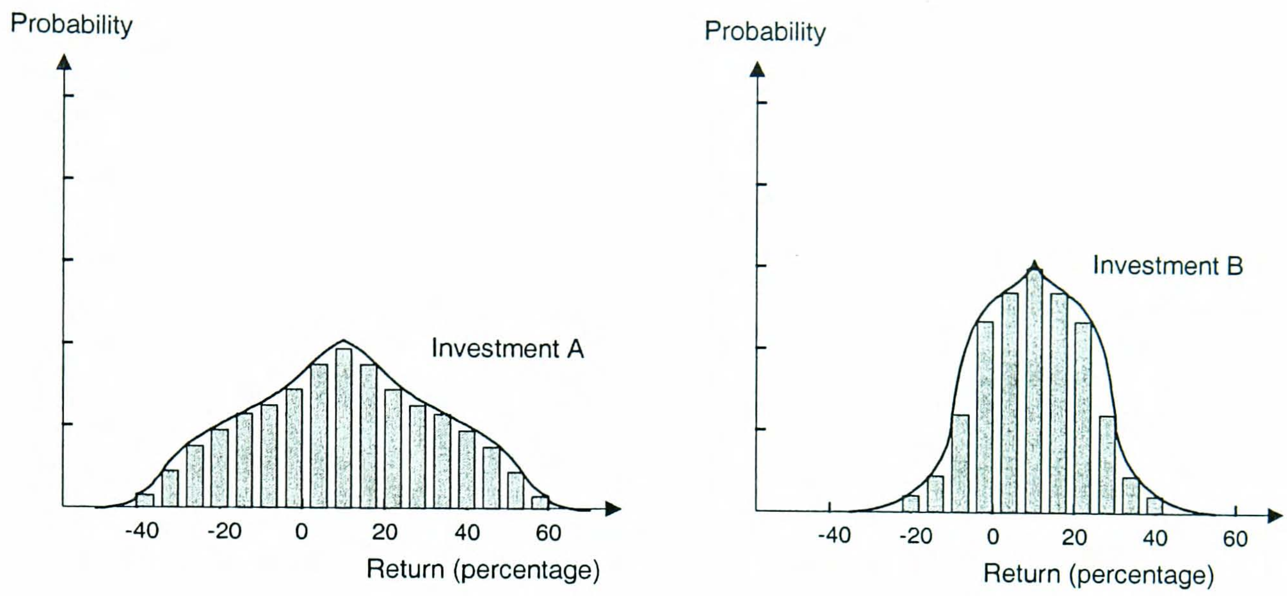
By varying the level of return a graph can illustrate the answer to this problem (graph 2-1):

Graph 2-1: Efficient Frontier (Pareto Analysis)



Graph 2-1, represents a plot in standard deviation-return terms whose efficient set forms the efficient frontier of an investment. An investor would want to increase the expected return hence to go up, and to reduce risk hence to go to the left of the line. Points A, B, C, and D are supposed to be portfolios and Markowitz calls them efficient portfolios as they offer the highest-expected returns for a given standard deviation. The line has the characteristic of offering the same mean rate of return but different standard deviations or variances. Point D is also known as the minimum-variance point (MVP). As investors prefer portfolios with the smallest standard deviation (leftmost point on the line), they seek to minimize risk (measured by standard deviation), so they are said to be risk averse. Graph 2-2, below, shows two different investments of portfolios which both have an expected return of 10%. Investment A, has a greater spread of possible returns. The spread is measured by the standard deviation, which in this case is 15% for A and 7.5% for B. Investors would prefer B to A for two reasons. Firstly, it has a lower standard deviation and secondly, it has a greater spread of possible returns, meaning it is more risky.

Graph 2-2: Two possible investments or portfolios



2.3 Empirical Validation of the Mean-Variance model

Mean-Variance portfolio analysis by Markowitz's research [45], [49], [46], provided the first quantitative treatment of the tradeoff between profit and risk. This issue though raised many questions and was the cause of extensive research, tests and validations. Efficient portfolios have the property of using a Von Neumann–Morgenstern [50] utility function, which maximizes the expected utility of the return on an investment. This concept was widely used by other researchers in the mean-variance field. Work in utility functions which are justified by their relationship to the corresponding risk premiums within the mean-variance framework have been made by Tobin in 1958 [51], Pratt in 1964 [52], Lintner in 1970 [53], Arrow in 1971 [54], Rubinstein in 1973 [55], Duncan in 1977 [56], Kira and Ziemba in 1980 [57], Ross in 1981 [58], Hubermann and Ross in 1983 [59], Pratt and Zeckhauser in 1987 [60], Li and Ziemba in 1989 and 1993 [61],[62]. Markowitz [46], discussed the advantages and disadvantages of replacing the variance by alternative risk measures. These considerations and the framework of stochastic dominance were exploited by Levy in 1992 [63], Levy and Wiener in 1998 [64]. Asymmetric risk measures like expectation of loss and semi variance were tackled by Markowitz, Todd, Xu and Yamane in 1993 [69], King in 1993 [68], Ogryczak and Ruszczynski in 1999 [71]. Bawa and Lindenberg in 1977 [65]. Konno in 1990 [66], used a piecewise linear risk function. Konno and Yamazaki in 1991 [67] and Zenios and Kang in 1993 [70], dealt with the mean-absolute deviation as a risk measure. Rockafellar and Uryasev in 2000 [72] and Uryasev in 2000 [73] introduced the Conditional Value-at-Risk or CVAR as their risk measure.

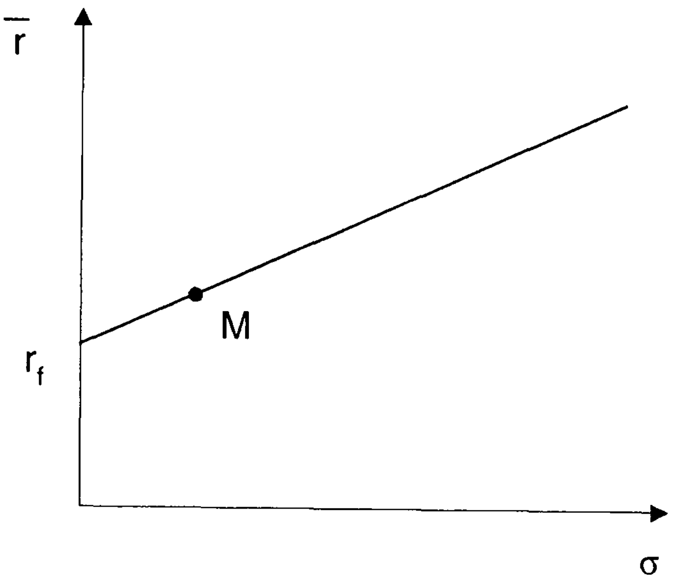
Many researchers used different approaches within the mean-variance framework to alternatively represent risk. The Markowitz problem though has two main difficulties that made it impossible to be established as the only technical model. The first drawback is that the computational time to generate the covariance matrix is very large. The other disadvantage is that Quadratic Programming is more difficult to solve than linear programs. Furthermore, Markowitz's model neither utilizes transactions costs nor any taxes. For these reasons people started to look into alternative models to compensate the risk issue.

2.4 Market Equilibrium

A simple definition of the Market Equilibrium is as follows. The return of an asset depends on two factors, the assets initial price and its final price. Suppose that investors decide to invest in the market in order to synthesize their portfolios. Asset prices, which have large demand, tend to increase while those with low demand tend to decrease. The change in asset prices affects the estimates of asset returns resulting in investors reconstructing their portfolios. This process is repeated until demand has an absolute match with supply. The process at this stage is then said to be in equilibrium. The theory of equilibrium is usually applied to assets that are repeatedly traded over time, such as the stock market.

The Capital Market Line (CML), shows the relation between the expected rate of return and the risk of return for asset portfolios or efficient assets (graph 2-3). It is also referred to as a pricing line. This line states that as risk increases, then the corresponding expected rate of return also increases.

Graph 2-3: The Capital Market Line (CML)



In mathematical terms, the CML is translated as:

$$\bar{r} = r_f + \frac{\bar{r}_M - r_f}{\sigma_M}\sigma \qquad (2.1)$$

where $\bar{r}_M$ and σ_M are the expected value and the standard deviation of the market rate of return. $\bar{r}$ and σ are the expected value and the standard deviation of the rate of return of an arbitrary efficient asset and r_f is the risk free asset

The slope of the CML is: $K = \frac{\bar{r}_M - r_f}{\sigma_M}$. This value is also called the price of risk.

2.4.1 Overview of the CAPM

The CML does not show how the expected rate of return of an individual security is related to its individual risk. However, this relation is explained by the Capital Asset pricing Model. CAPM was initially developed by Sharpe in 1963, 1964 [21], [22], and Treynor in 1961 [23]. Future developments were made by Lintner in 1965b, 1969 [24], [25], Mossin in 1966 [26], and Black in 1972 [27]. It is a model of capital market equilibrium which attempts to measure and price risk. The CAPM is a factor model and relates the expected return of an asset to its systematic risk. The required expected rate of return on any asset (r_i) equals the risk free rate of return (r_f) plus a risk premium. Its mathematical formulation is:

$$r_i = r_f + \beta_i * (r_M - r_f) \quad (2.2)$$

where $\beta_i = \frac{\sigma_{iM}}{\sigma^2_M}$ or $\beta_i = \frac{\text{cov}(r_i, r_M)}{\text{var}(r_M)}$

r_i is the expected return of the asset i

r_f is the interest rate

r_M is the expected rate of return of the Market index

β_i is the beta value of asset i

$\sigma_{i,M}$ is the covariance of asset i with the Market index

σ^2_M is the variance of the Market index

The risk premium can be thought of as the extra compensation, above the risk free rate, that the investors require for investing in the market portfolio. It is the product of the quantity of risk with the price of risk. The price of risk is the difference between the expected rate of return on the market portfolio and the risk free rate. The market Index could be referred to as the FTSE 100 or the S&P 100 etc. The quantity of risk usually called beta is a number that measures the degree to which the expected return on an asset moves with the expected return on the market. An asset with a high beta is one that is sensitive to moves in the overall market and tends to move in the same direction. On the other hand, if beta is zero, then that asset has no tendency to move with the market. The variance (σ^2) is the degree of possible deviation from the mean. Covariance of two or more variables is their mutual dependence. Suppose that two random variables x_1, x_2, have the property $\sigma_{12} = 0$, then they are said to be uncorrelated (one variable gives no information about the other). While if $\sigma_{12} > 0$ then the two variables are said to be positively correlated (if one variable is above its mean then the other is likely to be above its mean) and in the case of $\sigma_{12} < 0$ the variables are said to be negatively correlated (if one variable is below its mean then the other is likely to be above its mean).

The CAPM is a pricing model but the (standard) equation 2.2, contains only expected rates of return. So, if one wants to price a particular asset with a price P and a payoff Q, 2.2 becomes:

$$P = \frac{Q}{1 + r_f + \beta(\bar{r}_M - r_f)} \qquad (2.3)$$

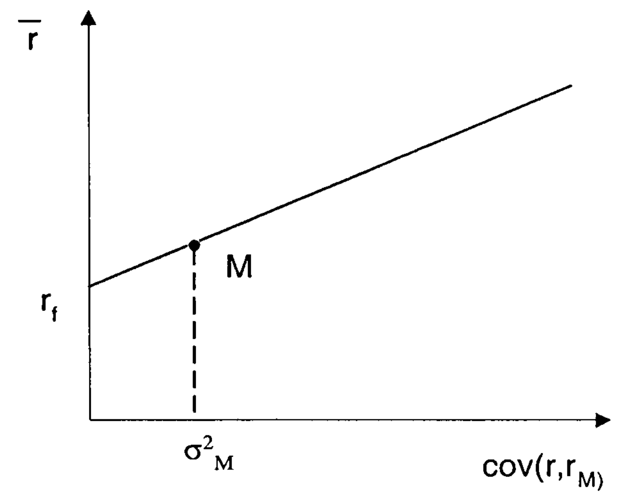
2.4.2 The Security Market Line

By regarding the CAPM formula (2.2) as a linear relationship it can be graphically expressed in Graphs 2-4, 2-5. The relationship is termed the Security Market Line (SML). Graph 2-4, expresses the linear variation of $\bar{r}$, in covariance form with $\mathrm{cov}(r, r_M)$, (horizontal axis). In this case the market portfolio corresponds to point σ_M^2 on the horizontal axis. In graph 2-5, the linear relationship is expressed in beta form, where beta is the horizontal axis. The market portfolio corresponds to point $\beta = 1$.

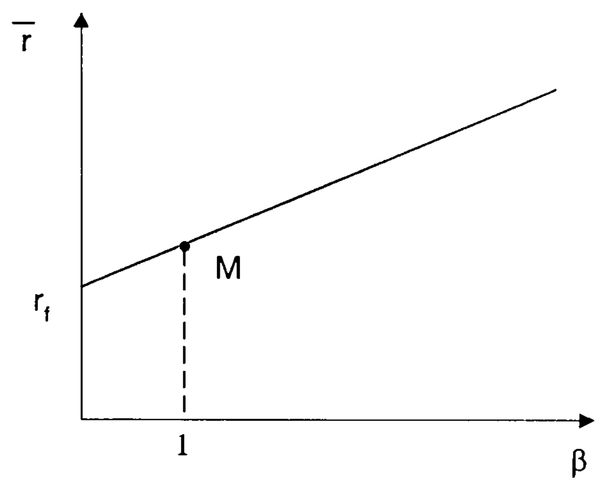
The SML expresses the risk-reward asset structure according to the CAPM: the risk of a particular asset is a function of its covariance with the market or a function of its beta [28].

Graph 2-4: The expected rate of return increases linearly as the covariance of the market increases

Graph 2.4



Graph 2.5



Graph 2-5: SML: The expected rate of return increases linearly as increases

In order to understand why beta plays the dominant risk role in the CAPM formula 2.2 can be written as:

$$r_i = r_f + \beta_i\left(r_M - r_f\right) + \varepsilon_i \qquad (2.4)$$

The expected value of the above formula according to the CAPM implies $E(\varepsilon_i) = 0$. Furthermore, by using the beta definition $\mathrm{cov}(\varepsilon_i, \sigma_M) = 0$. Formula 2.4 can be expressed by using these two last statements:

Derived from: $\mathrm{var}(r_i) = \sigma_i^{\,2} = \mathrm{var}\left(r_f + \beta_i\left(r_M - r_f\right) + \varepsilon_i\right)$

$$= 0 + \beta_i^{\,2}\sigma_M^{\,2} + \beta_i^{\,2}0 + \mathrm{var}(\varepsilon_i)$$

$$= \beta_i^{\,2}\sigma_M^{\,2} + \mathrm{var}(\varepsilon_i) \qquad (2.5)$$

The first part of the right hand side of 2.5, $\beta_i^2 \sigma_M^2$, is termed as 'systematic risk' and it has two characteristics: a) it is associated with the market and it cannot be reduced by diversification because every nonzero beta asset contains this risk and b) the second part, $\mathrm{var}(\varepsilon_i)$, is termed as 'nonsystematic', 'idiosyncratic' or 'specific risk'. This type of risk has exactly the opposite effect from the systematic risk. It is uncorrelated with the market and can be reduced by diversification. It must be mentioned that systematic risk (measured by beta), is important as it directly combines the systematic risk of other assets.

2.5 Assumptions of the CAPM

The CAPM is based on equilibrium rates of return, which make it unrealistic and it is developed in a hypothetical world. For this case the following assumptions have been made from investors to support this model [29]:

1. There are no market imperfections like taxes, regulations or restrictions on short selling.

2. Investors are price takers and have homogeneous expectations about asset returns that have a joint normal distribution.

3. Asset markets are frictionless and information is costless and available to all investors.

4. There exists a risk-free asset such that investors may borrow or lend unlimited amounts at a risk-free rate.

5. Investors are risk-averse individuals who maximize the expected utility of their end-of-period wealth.

6. The quantities of assets are fixed, marketable and perfectly divisible.

2.6 Empirical Tests and Validity of CAPM

The initial testing of the CAPM concentrated on the properties of the SML. Meaning that if the market portfolio is mean-variance efficient, then there will be a linear positive relationship between beta and the expected returns. Black, Jensen and Scholes in 1972 [30], concentrated on the SML. Their results are in favour of the CAPM. Fama and MacBeth's in 1973 [18], research, also concentrated on the properties of SML. They attempted to predict the future rates of return of portfolios on the basis of risk variables estimated in previous periods. Again the results were highly supportive of the CAPM. In 1977 Roll [31], wrote an extensive working paper in which he criticized Black, Jensen, Scholes and Fama, MacBeth's research claiming that their tests were tautological. He stated that it was not improbable to obtain results like theirs no matter how stocks were priced in relation to risk in the real world. He also claimed that since the only real prediction of the CAPM is that the market portfolio is efficient, this is the prediction that should be tested. Others include Miller and Scholes in 1972 [32], Blume and Friend in 1973 [33], Blume and Husick in 1973 [34], who found that economic factors other than the beta play an important role in the model. Basu in 1977 [35], found that low price/earnings portfolios have higher rates of return than the CAPM could explain. Litzenberger and Ramaswamy in 1979 [36], concluded that the market requires higher rates of return on equities with high dividend yields. Gibbons in 1982 [37] and Stambaugh in 1982 [38], explained how returns are positively related to beta and the relationship appears to be linear. Also, unsystematic risk is not priced in the market. Overall their results support the CAPM. Reinganum in 1981 [39], showed that size and earnings per share are important along with beta. He concluded that beta is not the only variable that explains the expected return. Keim in 1983 and 1985, [40]-[41], came with the conclusion of seasonality in stock returns the so-called January effect. Shanken in 1985 [42], put forward a variety of tests whether the market portfolio is mean-variance efficient Fama and French in 1992 [43], examined the relationship between the expected rates of return and other financial and accounting variables. This was the first study investigating these issues. They found that beta had no explanatory power either on its own or when it is included with other accounting variables like leverage. Their main conclusion was that beta coefficient was dead. Antoniou, Garrett and Priestley in 1995 [44], found that the more general multifactor APT model provides a more accurate description of the risk return relationship than the CAPM. This accuracy can be improved further by including the market portfolio in the APT specification. Therefore, the market portfolio has a role to play in pricing risky assets, but within a multifactor framework.

2.7 Arbitrage Pricing Theory (APT)

Asset pricing models are the link between literature theories and the behaviour of stock markets. They are used in pricing the individual risky assets or even within a portfolio, which provides a thorough understanding of business cycles, calculate the corresponding discount rates and finally take the appropriate investment decisions.

The Arbitrage Pricing Theory, or APT, was first formulated by Merton [1], and then by Ross[2]. APT offers a testable alternative to the Capital Asset Pricing Model (CAPM) and it does not request which portfolios are efficient. It assumes that each stock's return depends partly on a number of macroeconomic factors and 'noise' (events that are unique to that particular organization). The return is assumed to obey the relationship in formula (2.6):

$$\text{Return} = a + b_1(r_{\text{factor 1}}) + b_2(r_{\text{factor 2}}) + b_3(r_{\text{factor 3}}) + \ldots\ldots + \text{noise} \quad (2.6)$$

The theory does not state what the factors are and this has been a subject of empirical tests. Some stocks though could be sensitive to a particular factor for example Texaco would be more sensitive to an oil factor than Pepsi. Thus, if factor 1 represents changes in oil prices, b_1 will be a higher value for Texaco than for Pepsi.

2.7.1 Risk and APT

For any stock there are two main sources of risk (Brealy, Myers [3]). First is the risk, which is associated with the spread of the macroeconomic factors, which cannot be eliminated by diversification (the spread of a portfolio into different investments in order to minimize the risk exposure). Second is the risk uniquely associated with the organisation. Although diversification does indeed eliminate unique risk (see empirical evidence), the expected risk premium on a stock is a affected by factor or macroeconomic risk.

APT argues that the stock's expected risk premium should depend on the expected risk premium associated with each factor and the stock's sensitivity to each of the factors (b_1, b_2, b_3, etc.). Thus, the formula is (2.7):

Expected risk premium on investment= $r - r_f$ (2.7)

$$= b_1(r_{factor\,1} - r_f) + b_2(r_{factor\,2} - r_f) + \ldots\ldots$$

Quoting from Brealy, Myers [3], formula (2.7), states the following: "If you plug in a value of zero for each of the b's in the formula, the expected risk premium is zero. A diversified portfolio that is constructed to have zero sensitivity to each macroeconomic factor is essentially risk-free and therefore must be priced to offer the risk-free rate of interest. If the portfolio offered a higher return, investors could make a risk-free (or 'arbitrage') profit by borrowing to buy the portfolio. If it offered a lower return, one could make an arbitrage profit by running the strategy in reverse. A diversified portfolio that is constructed to have exposure to, say, factor 1, will offer a risk premium, which will vary in direct proportion to the portfolio's sensitivity to that factor".

APT applies to well-diversified portfolios where the unique risk has been diversified. However, if this theory applies for all diversified portfolios then it should apply to the issue of the individual stocks, which must give an expected return proportionate with its contribution to portfolio risk. As far as the APT is concerned, this contribution relies upon the issue of the stock's sensitivity in return to unexpected changes in the macroeconomic factors.

2.8 Empirical Evidence on APT

The empirical tests of APT involve the determination of a) systematic or nondiversifiable factors that explain asset returns and b) whether risk premiums are associated with the factor betas (pricing of factors).

Researchers exploit the issue that the APT does not say anything about the nature of the pervasive factors or their number and thus two main approaches have been used in an attempt to determine their nature and number. The first approach is Factor Analysis and the second is Principal Component Analysis. These are both statistical methods to extract factors from historical returns. In particular, the second approach uses a set of Economic and Financial factors to represent pervasive sources of risk.

2.8.1 Factor Analysis

Factor Analysis is a natural method of extracting factors:

$$R_t = \Omega + \psi$$

where Ω is systematic risk and ψ is unsystematic risk. Factor Analysis takes Ω, the variance-covariance, and extracts common correlations amongst the assets and stops when for example 95% of the covariance of returns has been explained by the factors.

Roll and Ross in 1980 [4], performed the first test of the APT using factor analysis. After they constructed portfolios of stocks, they found 4-5 systematic risk factors. Chen in 1983 [5], by using this analysis showed that APT outperforms the CAPM. Dhrymes, Friend and Gultekin in 1984 [6], found that if one increases the number of assets in a portfolio then the number of factors extracted from it, is also increased. Dhrymes, Friend, Gultekin and Gultekin in 1985 also found [7], that if the time period under investigation changes so does the number of factors.

Overall, results from factor analysis studies are questionable as firstly the technique has certain statistical limitations and secondly the number of factors is indeterminate. The number of factors may vary according to the sample size and the number of time series observations. Furthermore, one has to raise the issue of whether the different versions of the technique are compatible with each other in order to produce consistent results. Dhrymes and associates in 1984 [8] and 1985 [9] did exactly that. They questioned the validity, performance and empirical results obtained from the factor analysis technique.

2.8.2 *Principal Component Analysis*

The alternative approach that researchers use is principal component analysis. This method also extracts factors, from the covariation of returns, as described by Chamberlain and Rothschild in 1983 [10]. Trzcinka in 1986 [11], Connor and Korajczyk in 1988 [12], Brown in 1989 [13], and finally Shukla and Trzcinka in 1990 [14], found that up to 5 pervasive factors could well be present. This is consistent with the dominance of a market factor in stock returns, as researched by King in 1966 [15]. Connor and Korajczyk in 1988 [16], and Brown in 1989 [13], detected one dominant factor, which is the equally weighted market index.

Neither factor analysis nor the principal component analysis techniques are adequate to determine the number of factors and testing for pricing.

2.8.3 *Economic and Financial Factors*

An alternative approach to test the APT is to pre select economic variables as factors based on the economic theory and intuition to determine the degree to which the set of factors explains the cross-sectional variation in returns according to the APT's pricing relation. Any factor that either affects the future cash flows of the organization or the discount rate is a candidate as a pervasive source of systematic risk. Chen, Roll and Ross (CRR) in 1986 [17], by using the Fama and MacBeth [18] procedure, identified four sources of systematic risk: unexpected inflation, unexpected industrial production, change in expected inflation and changes in the term structure of interest rates. Similar results to CRR have been published by Burmeister and Wall in 1986 [19] and Berry, Burmeister and McElroy in 1988 [20].

Tests and research such as these on APT provide much more empirical meaning to the models and usefulness in investment practice.

2.9 CAPM v. APT

The CAPM and the APT are two theoretical models that enable investors to price risky assets. The appropriate measure of risk in the CAPM sceptic is the covariance of returns between the risky asset and the market portfolio of all the assets. On the other hand the APT model is more general indeed as a plethora of factors may explain asset returns. For each individual factor within the APT framework, the appropriate measure of risk is the sensitivity of asset returns to changes in that individual factor. The empirical tests for the APT have shown that asset returns can be explained by approximately four factors and have relaxed the concept of the variance of an asset return as one of them. The market portfolio plays an important role in the CAPM, as it does not in the APT. Both the CAPM and the APT can be applied to cost of capital and capital budgeting problems. Since the CAPM has not perfectly been empirical validated and tested, its main implication has been defended and that is the fact that the beta or systematic risk, remains a valid measure of risk.

2.10 Other pricing models

Two very important pricing models within the CAPM framework are described in this section. The Consumption-Based CAPM (CCAPM) and the Multibeta CAPM (ICAPM). These two alternative models are an extension of the original CAPM with more assumptions and a different concept on asset pricing.

The Consumption-based CAPM was developed by Rubinstein in 1976 [74], Breeden and Litzenberger in 1978 [75] and Breeden in 1979 [76]. The sceptic of CCAPM is that the covariance of an asset with aggregate consumption growth is a better measure of systematic risk, rather than the covariance with the return on a market index. The assumptions of the CCAPM are:

1. There are no taxes or transaction costs in the capital market.

2. Investors seek to maximize a lifetime utility consumption function that increases in a marginally decreasing rate, with higher levels of real consumption.

3. Multiperiod horizon.

Chen, Roll and Ross have done empirical tests on the CCAPM in 1983 [77], where they tested whether innovations in macroeconomic variables are risks that are rewarded in the stock market.

Trying to create a more realistic asset pricing model with more realistic assumptions and more amenable empirical testing, Merton in 1973 [1], came with the Multibeta CAPM or ICAPM. He relaxed the assumption that the investment opportunity set and the riskless rate are constant over time. The point, which makes this model more consistent with reality, is that, the investment opportunity changes stochastically over time. Merton showed that investors' portfolio decisions would be in accordance with a three-fund separation theorem. The three funds are:

1. the riskless asset.

2. the market portfolio.

3. a portfolio whose returns are perfectly negatively correlated with changes in the investment opportunity.

This theorem states that given a set of n risky assets and the riskless asset investors would be indifferent between choosing portfolios from among the original $(n+1)$ assets or from three portfolios ('mutual funds') constructed from these assets.

Chapter 3: MODELLING OF UNCERTAINTY

3.1 Introduction

The common problem among short and long-term investors is how to represent uncertainty. Investors who wish to achieve goals and meet future obligations need to establish the market expectations, which means that they have to establish their beliefs for the major asset categories such as stocks, bonds, currency, etc., in different regions of the world. Some investors express their expectations in terms of distributions of the return or interest rates for the different asset classes. The challenge is first to convert these expectations into an asset allocation format [78], which possibly a stochastic programming model can handle and second to construct an optimisation model which gives the optimal asset mix given this input.

The first part, the scenario generation, is in fact the most challenging. Scenario generation is the construction of possible future asset outcomes. The second part is discussed in Chapter 4. There is a need for accurate and efficient algorithms in scenario generation as in many cases when there are many asset classes the scenario generation can become a bottleneck process: the input data procedure could be computer-time consuming. Stochastic processes are widely used for scenario generation. It is critical that the decision-makers can express the market expectations in a way that they find most convenient and move a step forward to convert these expectations to model the inputs in a consistent manner.

This chapter gives an insight of how to model uncertainty and how it is possible to generate scenarios. Scenarios can be generated by using econometric techniques, time series or stochastic processes (wiener processes and the Geometric Brownian Motion). Major scenario systems, which are used in the market, for long term strategic asset liability planning are discussed and analysed.

The authors own scenario generator system is fully analysed, discussed and numerical examples and illustrations are given.

3.2 Stochastic Processes

Any variable whose value changes over time in an uncertain way is said to follow a stochastic process. There are two types of stochastic processes [79], the discrete-time and the continuous-time. The discrete-time process is defined as the value of a variable that can change only at certain fixed points in time. In a continuous-time stochastic process changes can take place at any time. For stock prices a continuous-time stochastic process is considered. A stochastic process can be described as a variable that evolves over time in a random way. A descriptive example would be the temperature in Athens. It usually rises from early morning at 6 o'clock until 4 o'clock in the afternoon and then drops for the rest of the time. Another example in the financial field would be the variation of a stock price. It fluctuates randomly providing a positive return in some time periods and negative returns in others. Both these examples are continuous-time stochastic processes in the sense that the time period is a continuous variable.

Stock prices are usually assumed to follow a particular type of stochastic process, which is the Markov process. This process has the characteristic that only the present value of a variable is relevant for predicting the future value. The past history of the variable and the way that the present has emerged from the past are irrelevant.

3.2.1 Wiener Process - Geometric Brownian Motion

Robert Brown [80] was a Scottish botanist and during his career he studied the way that pollen was transmitted in plants. He observed that pollen could be broken down into particles. These particles did not come to rest even in still water and 'danced' in a seemingly haphazard and very irregular way N.H. Bingham [81]. This discovery gave the initial 'push' to scientists to further apply it to other fields like mathematics and physics. In the field of physics, the Brownian motion can be defined as the motion of a particle that is subject to a large number of small molecular shocks. After Brown, the mathematician Norbert Wiener, constructed a stochastic process which contains all the properties for a rigorous mathematical model of Brownian motion:

i. Continuous Paths, so the process can model the displacement through time, of particles that move continuously, just like Brownian particles do.

ii. No Drift, or zero mean displacement.

iii. Mean Square displacement proportional to time or variance.

iv. Normally Distributed increments (variance that increases linearly over the time interval [81]).

A Wiener process obeys the Markov rule, (mentioned above), which creates the probability distribution of future values of the process, based only on its current value.

The stochastic process of Brownian motion is now called a 'Wiener process'. It has the same foundations and roots of Brownian Motion but with sophisticated terms and relations added.

In order to understand a Wiener process, consider a small interval of time Δt and let the change in process ζ during Δt be $\Delta\zeta$ then:

Lemma 1: the relationship of $\Delta\zeta$ with Δt is:

$\Delta\zeta = \varepsilon\sqrt{\Delta t}$ where ε is a random variable taken from a normal distribution

with mean of zero and standard deviation of 1.

Lemma 2: $\Delta\zeta$'s values for any two different short intervals of time Δt are completely independent

From Lemma 1, $\Delta\zeta$ has a normal distribution with:

mean of $\Delta\zeta = 0$

standard deviation of $\Delta\zeta = \sqrt{\Delta t}$

variance of $\Delta\zeta = \Delta t$

Lemma 2, defines that ζ follows a Markov process.

This model is applied to value stocks and shows how to predict their future price in the following way:

Let S be the stock price and the expected drift rate (average drift per unit of time) in S be μS where μ is a constant parameter. So, in a short time interval of time Δt, the expected increase in S is $\mu S \Delta t$.

If the variance of the model is always zero then:

$$\frac{dS}{S} = \mu dt$$

In practice, the stock price has volatility. If σ^2 is defined as the variance rate of the proportional change in the stock price, then that means that $\sigma^2 \Delta t$ is the variance of the proportional change in the stock price at time Δt and $\sigma^2 S^2 \Delta t$ is the variance of the actual change in the stock price

$$\frac{dS}{S} = \mu dt + \sigma d\zeta$$

S during Δt and leads to the following model:

The equation above is known as *Geometric Brownian motion.*

The variance σ is referred to as the stock price volatility.

The discrete-time version of the model is:

$$\frac{\Delta S}{S} = \mu \Delta t + \sigma \varepsilon \sqrt{\Delta t}$$

or

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t} \qquad (5.1)$$

where ΔS is the change in the stock price S

Δt is a small time interval

ε is a random variable from a standardized normal distribution (a normal distribution with a mean of zero and standard deviation of 1).

The term $\mu\Delta t$ is the expected value of the return and the term $\sigma\varepsilon\sqrt{\Delta t}$ is the stochastic

$$\frac{\Delta S}{S} \approx \phi(\mu\Delta t, \sigma\sqrt{\Delta t}) \qquad (6.1)$$

component of the return. Equation (5.1) can be expressed as follows:

where $\varphi(\mu, \sigma)$ denotes a normal distribution with mean μ and standard deviation σ.

3.2.2 Monte Carlo Simulation

A Monte Carlo simulation of a stochastic process is a procedure for sampling random outcomes for the process. An example of how the simulation works according to the share price process is as follows:

The S&P 100 share Index in 2000 had an expected return of 13% per annum with a standard deviation or volatility on return of 21% per annum. Considering changes in the share price at intervals of 3.65 days leads to $\mu = 0.13$, $\sigma = 0.21$ and $\Delta t = 0.01$. Equation (5.1) then becomes:

$$\Delta S = 0.13 * 0.01S + 0.21\sqrt{0.01}S\varepsilon$$

or

$$\Delta S = 0.0013S + 0.021S\varepsilon \qquad (7.1)$$

A path for the Index can be simulated by repeatedly sampling for ε from $\varphi(0,1)$ and substituting into equation (7.1). The initial Index price was 434.47 and for the first period, ε is sampled as 0.46. From equation (7.1), the change in the Index is:

$$\Delta S = 0.0013*434.47+0.021*434.47*0.46 = 4.756$$

At the beginning of the second time period the Index price is therefore 4.756 + 434.47 = 439.226. Table 3-1 shows a particular set of outcomes, which have actually been used for the current research.

Table 3-1: Simulation of the S&P 100 Share Index with μ = 0.13, σ = 0.21 and Δt = 0.01 years.

S&P 100 at Start Period	*Random Sample, for ε*	*Change in Index Price During Each Period*
434.47	0.46	4.761
439.231	0.323	3.554
442.786	1.183	11.584
454.370	0.697	7.250
461.620	-0.049	0.116
461.737	-0.766	-6.830
454.907	1.237	12.412
467.319	0.923	9.667
476.987	0.954	10.184
487.171	-1.648	-16.232
470.939	-0.689	-6.210
464.728	0.996	10.327

The above is one possible pattern of the S&P 100 Index movement, which means that different random samples would lead to different Index paths. There are 12 different time intervals and obviously by repeatedly simulating movements for the Index, its complete distribution at the end of this time interval could be obtained.

The Geometric Brownian Motion together with the Monte Carlo Simulation could be extended to generate scenarios for shares and Indexes. It is a straightforward method and it is used by many academics and researchers to model uncertainty. Nevertheless, it is not the only method used. The next section is dedicated to other econometric techniques and methods that are used to model uncertainty.

3.3 Econometric Techniques

Econometric modelling and in particular time series analysis, in both its theoretical and empirical aspects, has been for many years an integral part of study of financial markets, with empirical research beginning with the papers by Working [82], Cowles [83], [84] and Cowles and Jones [85].

Working focused on stock prices and commodities and insisted on the fact that they resemble accumulations of purely random changes. Cowles concentrated on the ability of market analysts and financial services to predict future price changes. He concluded that there was little evidence that they could. Cowles and Jones reported evidence of positive correlation between successive price changes.

Stochastic processes (analysed above) belong to the Univariate linear stochastic model sector. The Autoregressive (AR) and Moving Average (MA) processes also belong to this sector. Their combination is also widely used and it is known as the ARMA process [86]. Their integrated formulation is known as the autoregressive-integrated-moving average or ARIMA [87],[88]. These kinds of models are statistical models, which use time series for forecasting. It should be emphasized that stock prices are not the only financial time series [89],[90],[91], of interest as there are financial markets other than those for stocks, most notably for bonds and foreign currency, but there also exist the various futures and commodity markets.

A different group of models is that of the Univariate non-linear stochastic models. These models are capable of modelling higher conditional moments, such as the autoregressive conditionally heteroskedastic (ARCH) model introduced by Engle [92]. The popularity of ARCH models can be seen in an extensive survey made by Bollerslev et al. [93]. Further issues in ARCH models can be found in Milhoj's [94] and Weiss's research [95]. A practical difficulty with ARCH models is that very often estimations lead to the violation of the non-negativity constraints. To obtain more flexibility, a further extension to the generalized ARCH or GARCH has been proposed by Bollerslev [96],[97].

Another group of models is the regression models. Vector autoregressive or VAR process is one of the widely used methodologies. In VAR models, each member of a group of random variables is expressed as a linear function of past values of itself or past values of the other members of the group. VAR is based on the work of Hansen [98], White [99] and White et al. [100].

3.4 Scenario Generation

Having introduced in the previous chapter the concept of the CAPM and in this chapter the framework of the stochastic processes and in particular the Wiener process and the Geometric Brownian Motion, the author's own scenario generator is now analysed in detail.

3.4.1 Generating Scenarios for the S&P 100 Index

The theory of stochastic processes and in particular the Geometric Brownian motion with drift is utilized to generate forecasts for the S&P 100 U.S. stock market Index. By using formula 5.1, (the discrete-time version of the Brownian motion),

$$\Delta S = \mu S \Delta t + \sigma S \varepsilon \sqrt{\Delta t}$$

with μ, mean, (drift) and σ, standard deviation, were calculated by taking monthly historical data. These two variables were calculated by considering 5 years of historical values. Δt was calculated in monthly terms (0.0833). ε, was simulated as mentioned in 3.2.2, (sampling repeatedly from $\varphi(0,1)$). This was achieved by using MS Excels' random number generation option. The routine of continuously repeating and generating the Index scenarios was achieved by using the Monte Carlo simulation procedure. The start date was August 1997 and the close date was June 1998. A complete year or 12 months were considered to model the uncertainty annually because it would give an insight of the performance of the Index. If half a year or 6 months were considered then the study would be inadequate as this would not capture any effects as the turn-of-the-year, Christmas period, or any other dates which are crucial for shares in general and Indexes (rise or drop) in particular for this case. From the 12 months, six values were considered which represent the year as a whole meaning that forecasted values of every other month are taken. 1997 and 1998, were the initial dates for forecasting but for back testing reasons and further research the scenarios framework was expanded to model a decade. That means the actual start date was January 1988 and the end date was January 1998. This particular decade was crucial as the majority of the worlds Indexes, including the S&P 100, faced unexpected falls and rises. Falls were reported in the recession of the US economy in 1991 and from 1992 it began its current upswing. Below is a graphical representation of the S&P 100 Index actual values and sample scenarios using the Geometric Brownian motion.

Graph 3-1: S&P 100 and 29 sample scenarios. The bold dark blue line is the actual value of the Index.

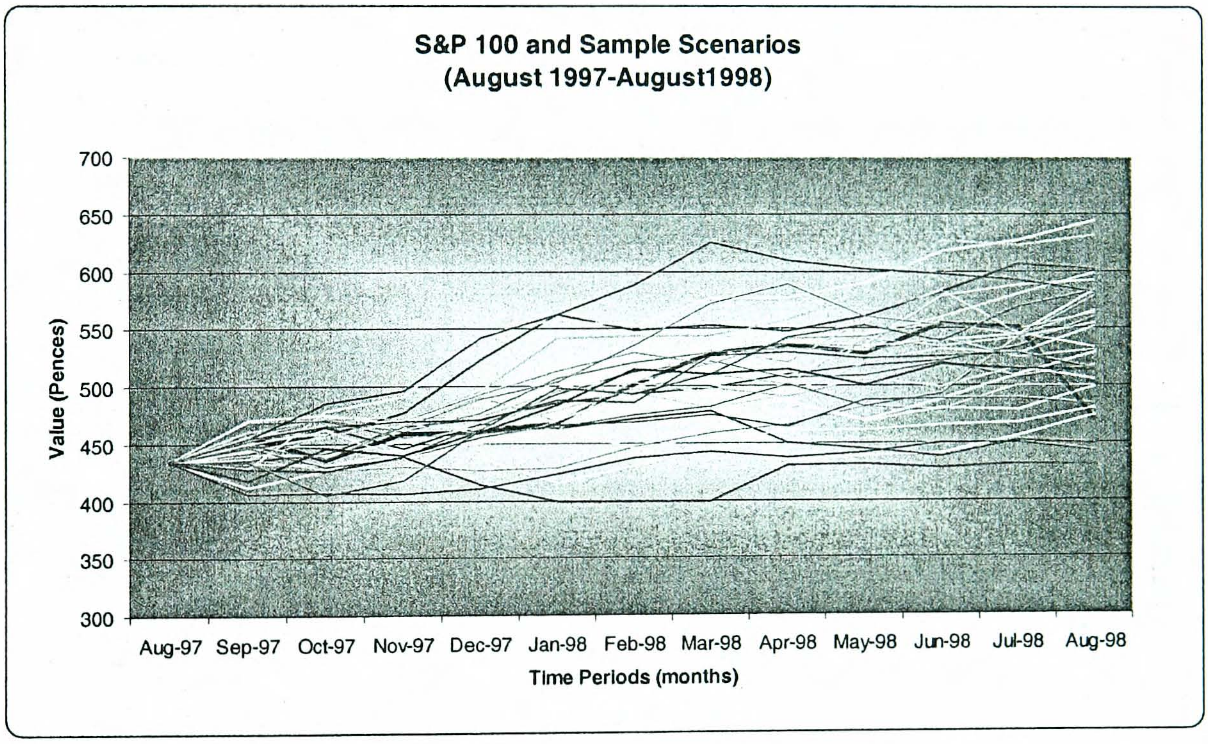


Table 3-2, illustrates the above graph in numbers. A total of 92 scenarios were generated using the Geometric Brownian Motion. A random sample of 29 is illustrated below to show that 'good' and 'bad' scenarios are taken into consideration.

Table 3-2: The Actual S&P 100 Value and 29 random scenarios (August 1997-August 1998)

Actual	**S1**	**S2**	**S3**	**S4**	**S5**	**S6**	**S7**	**S8**	**S9**	**S10**
434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47
456.91	437.14	412.01	453.52	442.71	451.08	450.15	407.35	437.98	444.28	441.07
436.71	442.28	426.87	466.56	466.11	458.68	447.78	408.06	427.75	458.62	441.02
459.1	466.04	449.04	468.15	446.12	477.55	440.55	406.85	430.58	470.63	452.5
459.94	472.23	438.40	491.64	468.24	523.76	466.67	411.51	426.77	473.34	465.83
468.79	485.81	414.98	469.56	463.14	563.28	493.46	423.78	429.92	504.13	469.58
501.93	493.59	430.04	504.32	471.78	549.24	512.48	437.50	447.24	503.52	496.60
528.91	498.95	446.87	520.70	478.65	553.85	522.38	443.70	458.93	494.48	523.54
536.48	511.08	434.89	537.07	450.96	548.24	507.87	437.76	467.31	484.57	508.33
529.04	518.07	438.97	535.37	445.60	561.06	509.72	442.27	483.11	468.85	534.97
554.56	522.29	440.71	578.10	439.08	582.80	521.57	450.93	478.73	481.18	539.15
550.93	534.05	456.43	587.43	452.78	606.41	506.97	450.68	500.23	478.35	545.14
470.91	559.73	473.92	592.15	449.48	603.20	517.18	442.88	500.43	499.03	583.25

S11	**S12**	**S13**	**S14**	**S15**	**S16**	**S17**	**S18**	**S19**	**S20**	**S21**
434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47
471.46	446.67	433.6	455.72	428.95	447.2	458.53	461.73	450.37	457.09	460.36
475.14	431.32	431.23	473.71	479.01	431	458.15	458.74	462.84	465.96	472.7
467.64	444.27	438.53	473.37	491.32	437.91	470.93	468.76	437.74	464.97	461.09
487.76	450.11	453.26	474.4	545.42	455.74	493.09	459.52	451.4	463.92	477.1
498.84	469.71	461.7	497.22	558.57	466.38	514.28	497.04	460.08	480.1	507.27
507.71	494.62	510.65	503.40	552.28	468.6	529.96	498.58	446.69	497.94	521.11
501.97	481.62	530.35	532.12	577.2	475.33	519.3	510.31	453.75	498.95	529.76
486.89	492.18	543.72	521.57	576.76	464.71	541.25	502	460	490.97	533.58
470.56	520.51	545.2	557.73	589.47	488.51	543.55	541.34	461.24	506.78	530.95
488.04	532.08	560.04	583.83	617.8	489.34	554.72	534.75	466.08	498.75	542.1
509.10	527.43	580.62	545.57	627.1	489.68	544.32	541.86	468.03	516.36	524.92
529.1	551.6	598.12	533.55	643.85	483.58	565	554.72	480.25	505.06	530.68

S22	**S23**	**S24**	**S25**	**S26**	**S27**	**S28**	**S29**

434.47	434.47	434.47	434.47	434.47	434.47	434.47	434.47
469.49	437.66	454.97	449.251	418.3709	427.9516	435.1159	454.9761
470.59	456.73	486.29	438.3115	446.3717	426.4532	405.9488	466.2791
456	472.65	496.89	465.3031	439.6765	439.3611	418.5757	470.4291
459.16	496.13	542.78	489.5465	460.598	414.823	462.8513	470.6443
465.14	541.68	563.21	504.1471	483.5223	399.9989	498.7826	488.5362
475.46	543.88	588.54	535.9118	514.777	399.3077	493.8499	486.5266
482.39	543.78	626.37	572.6673	510.9444	399.8403	510.4631	526.9451
502.41	557.47	609.7	590.2671	514.9492	430.7154	542.7251	529.799
482.4	579.32	603.08	560.9796	501.4235	431.6838	553.5898	524.2425
494.1	627.15	597.94	553.9517	519.6388	428.2306	540.0053	526.5941
541.6	622.94	593.21	549.0797	515.2277	431.9846	569.3858	529.3267
578.53	631.89	582.29	562.6066	513.7069	430.9596	583.6892	514.8408

The scenario generator (Figure 3-1) so far consists of the geometric Brownian motion used together with a Monte Carlo simulation in Visual Basic code (Excel). This technique reproduces the Index scenarios

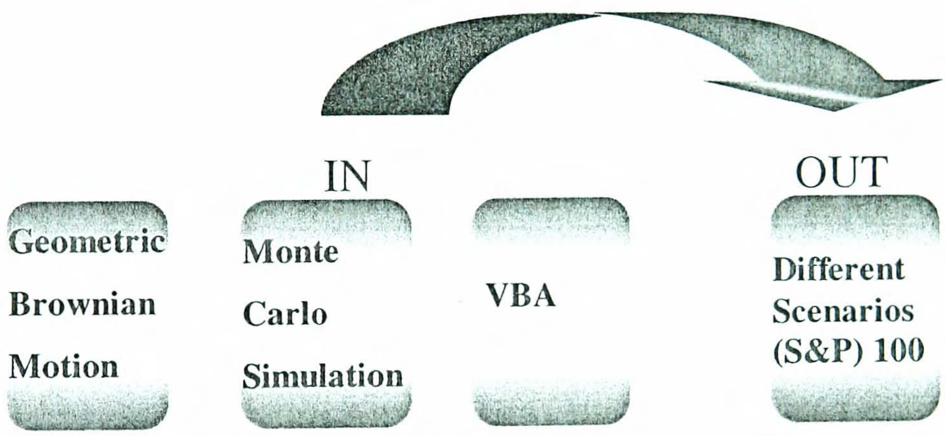
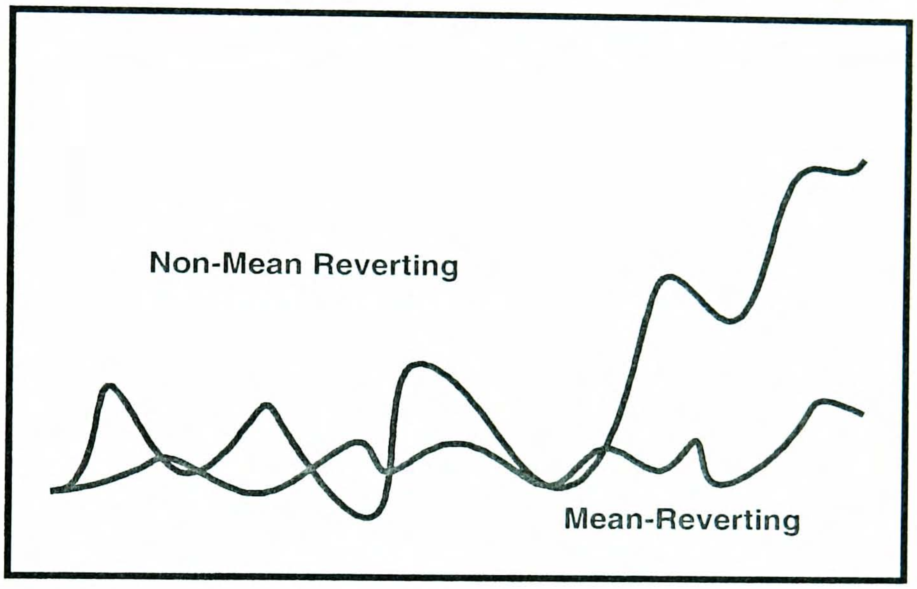


Figure 3-1: The scenario Generator for the Index

The difficulty raised at this stage is generating individual share prices in the S&P 100. For shares and Index consistency, an econometric or another type of model must be utilized. The main purpose of scenario generation is to generate scenarios for different asset classes. At this stage the shares asset class is nonexistent and completely inconsistent. The CAPM model provides a means for mapping the Index scenarios into prices for individual shares.

3.4.2 Interest Rates Scenarios

Interest rates are characterized by a process named mean reversion. Mean reversion is a tendency for certain random variables to return over time to a long-run average level. For example, interest rates and implied volatilities tend to be mean reverting. Exchange rates and stock prices tend to be non-mean reverting. It is not possible to ascertain if a variable is mean reverting by looking at its performance over any short period of time. This is because a mean reverting tendency often reveals itself over long horizons. Graph 3-2 provides an intuitive illustration of the difference between mean reverting and non-reverting behaviour.



Graph 3-2: Mean and Non-Reverting processes

In the literature there are many interest rate models and the majority of them deal with fixed income securities. Well-known models have been developed by Cox, Ingersoll and Ross or CIR [117], who explain in detail the term structure (relationship among the yields on default-free securities that differ only in their term to maturity) of interest rates. Vasicek in 1977 [118], derived a general form of the term structure of interest rates. Hull and White made an extension of the Vasicek model in 1990 [119]. Ho and Lee in 1986 [120], presented an interest rate model in the form of a binomial tree of bond prices with several parameters. Giles in 2000 [121], constructed a rather simple model on U.K. interest rate and inflation forecasting. This model uses upward and downward interest rate targets.

The model used in this research is the Black, Derman and Toy's model [122]. This model of interest rates can be used to value any interest-rate-sensitive security. It is applied to a Treasury zero-coupon bond option - a long-term coupon-bearing instrument issued by the government to finance its debt. Coupon is the interest rate on a fixed income security, determined upon issuance, and expressed as a percentage of its face value. Zero-coupon bond is the one, which pays no coupons, is sold at a deep discount to its face value or the nominal amount assigned by the issuer and matures at its face value. The model has three key features:

1. Its fundamental variable is the short rate - the annualised one-period interest rate.

2. The model takes as inputs the yield curve - a curve that shows the relationship between yields (the annual rate of return on an investment, expressed as a percentage. For bonds and notes, it is the coupon rate divided by the market price) and maturity dates for a set of similar bonds, usually Treasuries, at a given point in time - and the volatility curve (array of yield volatilities).

3. The model varies an array of means and an array of volatilities for the future short rate to match the inputs. As the future volatility changes, the future mean reversion changes.

The term structure of interest rates is quoted in yields. Today's annual yield, y, of say the N-year zero-coupon Treasury in terms of its price, S, is given by the y that satisfies:

$$S = \frac{100}{(1+y)^N}$$

where S is the expected price of a security S say one year from now

100 is the maturity price of S in one year from now

y is the annual yield

N is the duration in years of the security S

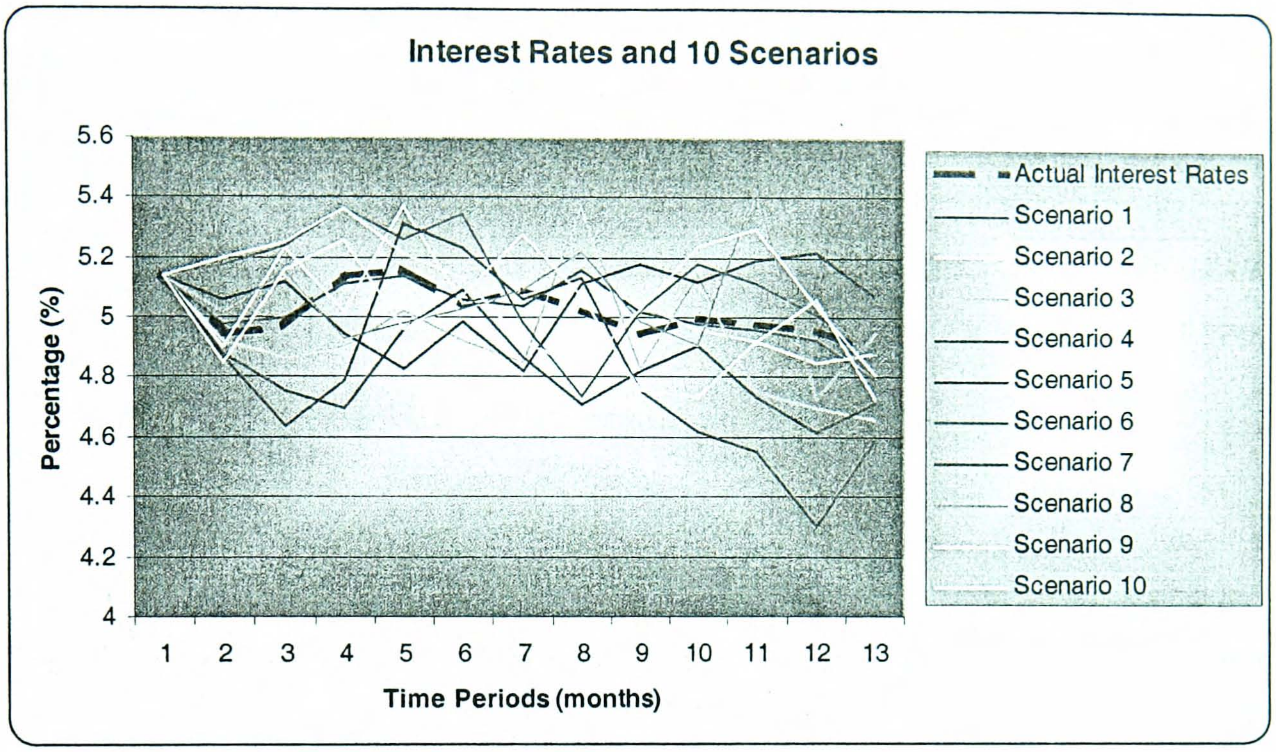
The yields y_u (yield that can move up) and y_d (yield that can move down) one year from now corresponding to the security prices S_u (price that can move up) and S_d (price that can move down) are given. Suppose that $a = S_{u,d}$ and $b = y_{u,d}$ then:

$$a = \frac{100}{(1+b)^{N-1}}$$

This is the methodology behind the BDT's model of forecasting short rate interest rates. This methodology has been followed to produce a scenario for the risk-free asset of the CAPM. Table 3-3 shows the actual interest rates and the scenario, which was generated using the above method. Graph 3-3 illustrates the actual interest rates and an average sample path of 10 different interest rates scenarios.

Table 3-3: Scenario for Interest Rates

Date	*Actual Interest Rates in %*	*Scenario*
Aug-97	5.14	5.14
Sep-97	4.95	4.98
Oct-97	4.97	5
Nov-97	5.14	5.11
Dec-97	5.16	5.14
Jan-98	5.04	5.06
Feb-98	5.09	5.04
Mar-98	5.03	5.08
Apr-98	4.95	5.02
May-98	5	4.98
Jun-98	4.98	4.96
Jul-98	4.96	4.93
Aug-98	4.9	4.8



Graph 3-3: The path of 10 random sample scenarios and the actual interest rates

The result of the above graph indicates that the BDT model is a good methodology for interest rates prediction. Although it cannot closely follow the actual path it captures the trend.

3.4.3 The CAPM and scenario consistency

CAPM is a factor model, which consists of the risk free asset or the interest rate, the market or index return and the beta of the share, which acts as a risk measurement. The formula, (analysed in Chapter 2), is the following:

$$r_i = r_f + \beta_i * (r_M - r_f)$$

So far, r_M is the only variable known after its scenarios were generated. The risk free asset, r_f, must be computed. A mean reverting model was used for this purpose, which was developed by Black, Derman and Toy (known as BDT model and described in the previous section). Only one scenario was generated for the risk-free asset (interest rate), as it does not affect the models' performance as much as the Markets' expected rate of return and the beta values of the stocks.

The beta values of each share were calculated by applying: $\beta_i = \dfrac{\operatorname{cov}(r_i, r_M)}{\operatorname{var}(r_M)}$. Betas were calculated by using the above formula in a MS Excel spreadsheet. New betas were produced in order to correspond with each of the ten years of this study. Initially the betas were held constant throughout the period of forecasting. However, this led to some large errors on model validation. The model was improved by dynamically updating the betas using all the time series of return data including forecast data upto the current forecast period. This technique provided better forecasts. Table 3-4 lists some companies, (used in this study), and their corresponding beta (β), estimated at a particular date.

Table 3-4: Beta (β) values in August 1997 (S&P100)

Ticker Symbol	*Company Name*	*Beta (β)*
GD	General Dynamics	0.32
AA	Alcoa Inc.	1.22
GM	General Motors	1.29
KO	Coca Cola Co.	0.47
PEP	PepsiCo Inc.	1.05
DOW	Dow Chemicals	0.74
IFF	International Flav/Frag	0.82
HRS	Harris Corp.	0.76
NT	Nortel Networks	1.69
HWP	Hewlett-Packard	1.28
GE	General Electric	1.16
HON	Honeywell International	0.95
ROK	Rockwell International	0.5
RTN.B	Raytheon Co.	0.43
TEK	Tektronix Inc.	1.99
TXN	Texas Instruments	1.45
DIS	Disney	0.86
BNI	Burlington Northern Santa Fe Corp.	0.6
AEP	American Electric Power	-0.28
SO	Southern Co.	-0.2

In order to capture more than half the number of the shares from the S&P 100, 59 of them were randomly chosen. At this stage, consistency has been achieved with the Market scenarios, betas and interest rates by using the CAPM. For example, Index' scenario number 1 (out of 92) would be consistent for all the 59 shares given the corresponding interest rates and betas corresponding to each share. Below, there are graphs accompanied from tables that illustrate different scenarios and share prices.

Table 3-5: General Electric and sample scenarios

	Actual	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5	Scenario6	Scenario7
Aug' 97	67.75	67.75	67.75	67.75	67.75	67.75	67.75	67.75
Sep '97	66.25	75.70318	70.99984	58.862	66.68418	67.33467	67.71149	68.38802
Oct '97	69.06	75.72486	68.44384	61.82594	72.69923	60.03564	69.66608	70.50154
Nov '97	69.13	72.59936	65.71153	68.35516	73.97001	62.44283	76.84444	74.92141
Dec '97	73.44	66.73664	65.62363	71.25196	79.1234	73.22658	69.85037	71.42941
Jan '98	75.31	69.10188	68.95585	76.53766	80.22869	71.61428	76.36466	65.67096
Feb '98	76.69	67.342	71.82071	79.71198	79.71654	67.05198	74.84854	66.83919
Mar '98	76.38	69.71205	82.58994	83.42707	89.75717	68.7398	76.3968	78.98939
Apr '98	87.38	73.53862	74.20515	78.20754	87.79111	70.49246	72.05452	84.16159
May '98	83.31	63.69011	71.01845	83.54529	86.75475	75.78886	75.6588	80.3205
June '98	84.75	62.05427	78.2463	80.96582	87.29929	74.49363	82.58904	79.84845
July '98	90.94	62.17758	75.83271	84.04162	80.7171	84.49642	82.52282	92.86677
Aug '98	86.75	67.11947	68.39776	78.4575	88.88304	84.25671	83.07063	83.76516

Graph 3-3: General Electric

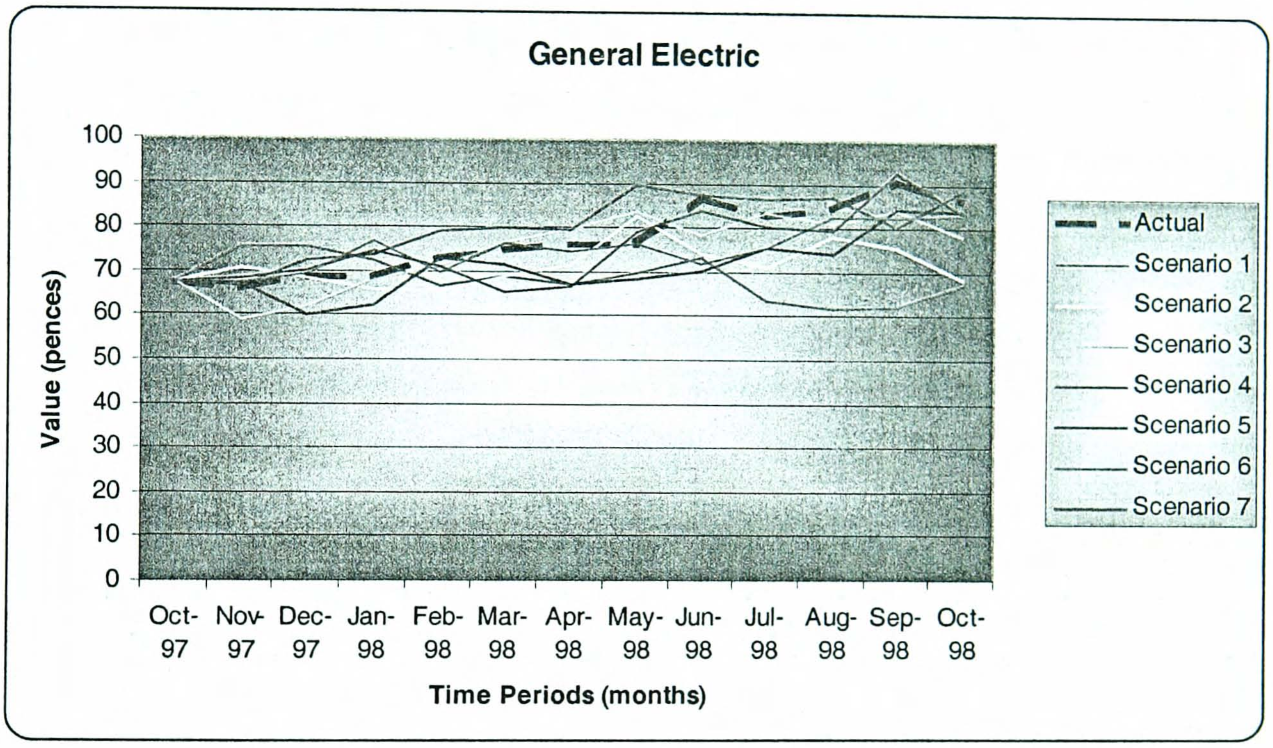


Table 3-6: Disney and sample scenarios

	Actual	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5	Scenario6	Scenario7
Aug' 97	26.64	26.64	26.64	26.64	26.64	26.64	26.64	26.64
Sep '97	26.06	27.6122	22.3313	25.73456	26.01757	26.18151	26.47586	29.43013
Oct '97	28.12	26.36213	23.45403	28.16304	22.75548	26.87534	27.2372	30.72676
Nov '97	28.37	25.2046	26.20081	28.71513	23.77099	29.94646	29.1337	35.38375
Dec '97	31.33	25.04502	27.30228	30.78937	28.19797	26.78536	27.48989	34.78608
Jan '98	32.93	26.16045	29.2253	30.90667	27.18258	29.23739	24.71759	35.63346
Feb '98	36.1	27.15465	30.33365	30.44217	25.04996	28.36222	25.00722	34.48787
Mar '98	35.06	31.54597	31.77063	34.55699	25.64253	28.89234	29.93217	31.17208
Apr '98	36.98	27.67245	29.23906	33.35809	26.0978	26.77316	31.7878	35.03359
May '98	42.22	26.10187	31.177	32.61418	28.02631	28.00764	29.88887	34.43535
June '98	38.23	28.83278	29.86294	32.58024	27.27253	30.62259	29.45582	38.66585
July '98	35.48	27.59987	30.86047	29.58103	31.08915	30.33114	34.51535	41.47384
Aug '98	33.56	24.18974	28.12648	32.44644	30.52297	30.09676	30.25219	43.21003

Graph 3-4: Disney

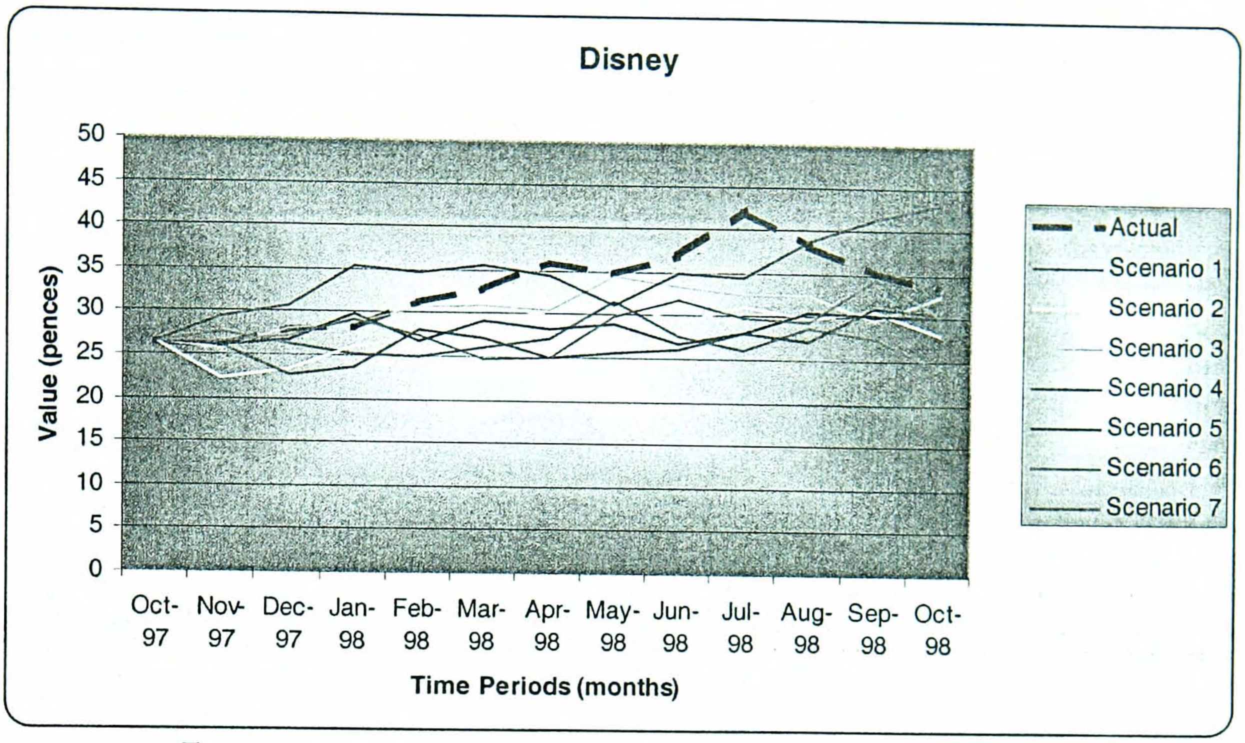


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Table 3-7: Ralston Purina and sample scenarios

	Actual	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5	Scenario6	Scenario7
Aug' 97	29.06	29.06	29.06	29.06	29.06	29.06	29.06	29.06
Sep '97	29.83	29.98542	24.5127	24.94953	25.61767	31.51534	28.89842	30.99018
Oct '97	29.87	28.69218	25.68059	25.17793	27.64027	32.57043	25.16854	33.2319
Nov '97	28.88	27.53162	28.57032	24.12368	28.58059	34.09814	29.09065	35.12698
Dec '97	31.12	27.36667	29.71205	21.63248	30.35433	31.95666	24.00682	35.64619
Jan '98	29.93	28.48007	31.65189	20.68593	33.94603	30.9971	24.99666	34.61613
Feb '98	30.35	29.48747	32.76909	19.73407	34.09034	32.36972	25.79168	34.43422
Mar '98	31.86	34.02312	34.24984	19.15161	32.4445	33.47256	27.28696	34.55961
Apr '98	34.21	30.0087	31.61131	21.33408	32.21453	31.81846	28.89435	38.86654
May '98	35.58	28.35938	33.56884	20.03588	28.03608	33.49622	30.52913	33.18528
June '98	36.67	31.15728	32.19992	21.63899	24.63396	28.1897	25.90695	32.16861
July '98	38.33	29.8637	33.19297	23.78611	24.47397	31.80569	25.35503	32.91379
Aug '98	30.69	26.28485	30.31769	23.96116	24.13976	31.4983	24.85578	36.84563

Graph 3-5: Ralston Purina

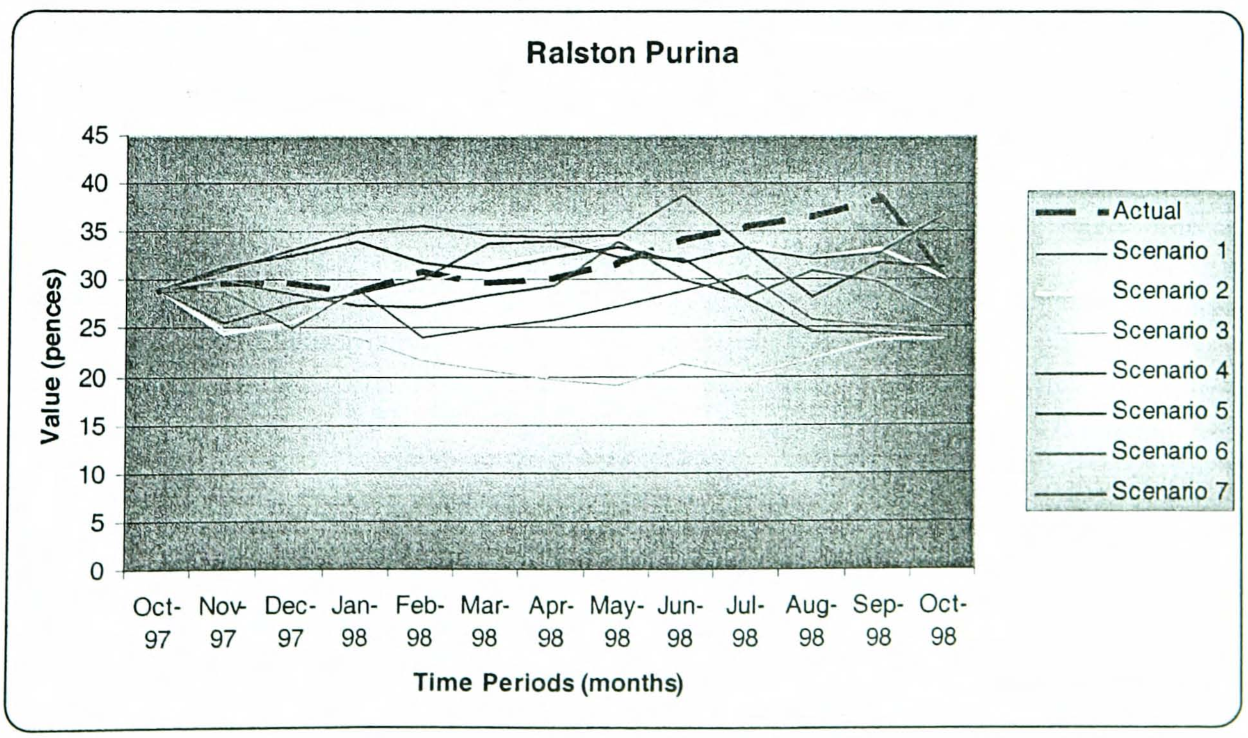


Table 3-8: Colgate and sample scenarios

	Actual	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5	Scenario6	Scenario7
Aug' 97	36.69	36.69	36.69	36.69	36.69	36.69	36.69	36.69
Sep '97	33	34.1102	34.64235	29.65541	33.7984	37.46885	33.87003	36.74664
Oct '97	35.5	32.7006	39.35161	34.4054	35.69091	38.97674	29.23637	39.78076
Nov '97	32.81	34.19698	43.55343	34.62971	37.46642	40.80777	34.07867	42.08811
Dec '97	36.03	33.34481	45.15406	37.2606	40.82298	38.13717	27.69276	42.89047
Jan '98	37.44	39.32383	44.61196	45.53985	45.49872	37.36154	29.27896	42.07097
Feb '98	38.81	39.80452	44.46207	48.17673	52.79048	39.43538	30.50626	42.13618
Mar '98	39.59	41.72468	44.04769	44.37064	52.98359	40.97998	32.49544	42.39001
Apr '98	43.87	46.77935	48.16803	41.53742	48.93548	39.21357	34.92885	48.59865
May '98	44.56	41.10299	57.24307	45.66824	55.58943	41.82009	37.39697	41.30009
June '98	44.75	38.91863	61.21145	46.92685	61.7009	34.90911	31.50285	40.22347
July '98	44.12	39.57769	54.46428	40.76555	68.06575	40.07259	31.02995	41.56795
Aug '98	44.87	42.62431	51.69939	41.15223	63.18371	40.33635	30.89534	47.68785

Graph 3-6: Colgate

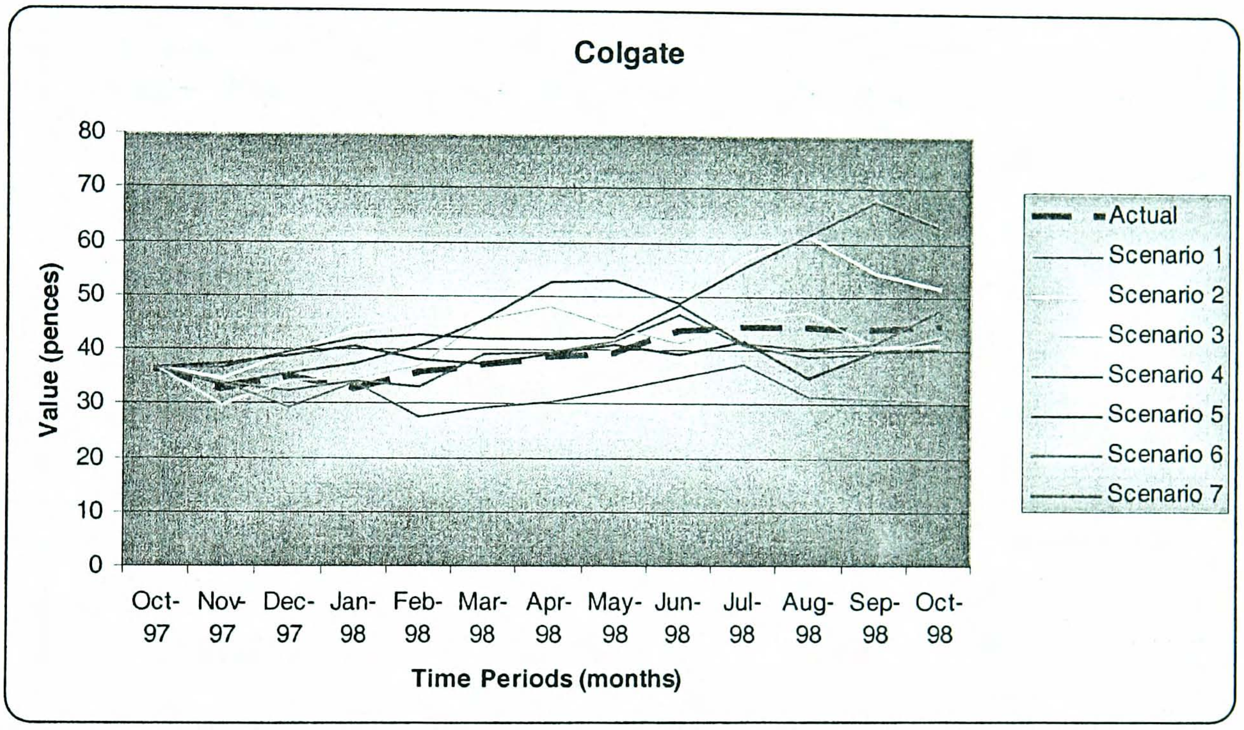
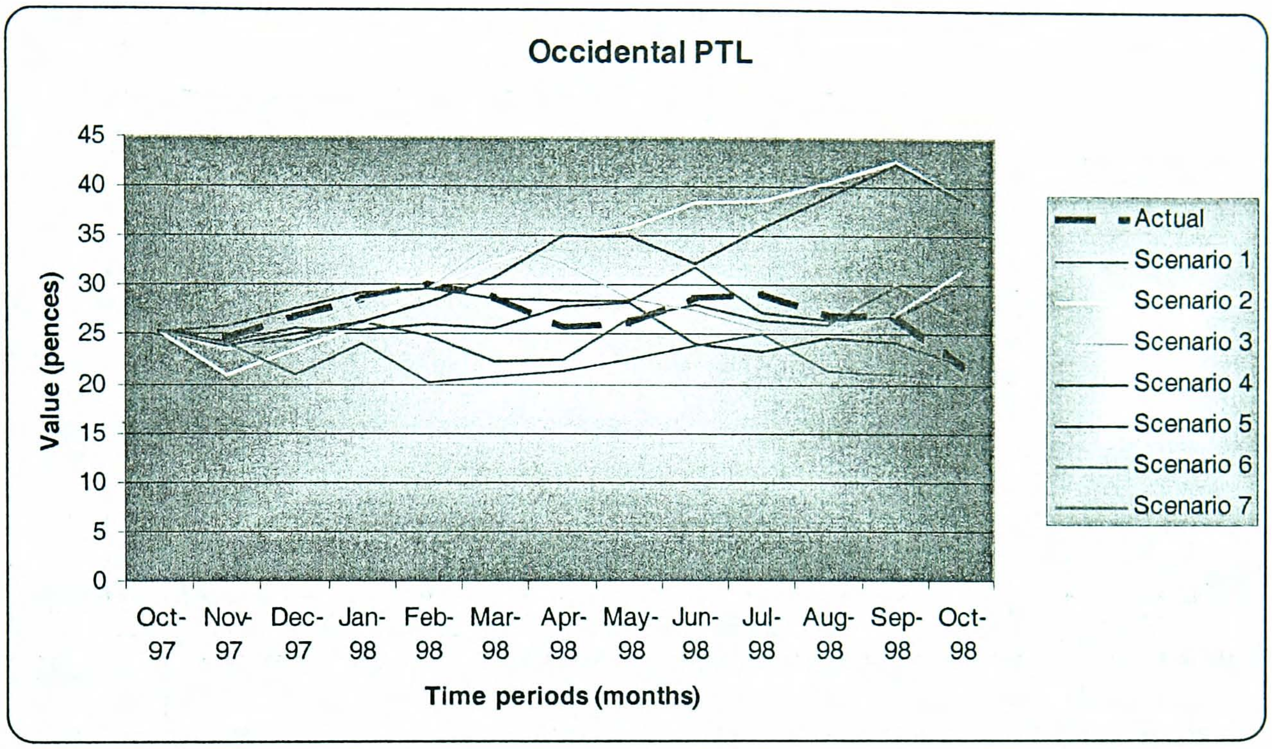


Table 3-9: Occidental PTL

	Actual	Scenario1	Scenario2	Scenario3	Scenario4	Scenario5	Scenario6	Scenario7
Aug' 97	25.31	25.31	25.31	25.31	25.31	25.31	25.31	25.31
Sep '97	24.63	23.95434	20.88004	23.42091	24.30995	23.98725	24.03173	25.8183
Oct '97	26.81	24.56829	23.55737	25.22082	25.61498	25.08989	20.96497	27.62282
Nov '97	28.75	26.21727	25.89117	28.54142	25.5471	26.3318	24.18992	29.19539
Dec '97	30.25	24.81797	29.70923	29.66955	26.04666	28.36349	20.02426	29.5943
Jan '98	28.81	22.36533	32.54243	34.52258	25.67762	30.87953	20.7683	28.65871
Feb '98	25.88	22.54976	34.65985	32.31837	27.97446	35.07253	21.37551	28.45509
Mar '98	26.25	26.67772	35.88842	28.70787	28.32687	35.10217	22.57803	28.53974
Apr '98	28.81	28.10754	38.45628	27.42974	23.99714	32.2488	23.81798	31.94631
May '98	29.31	26.4468	38.71721	25.48108	23.21394	35.85659	25.08206	27.29002
June '98	27	26.01888	40.33812	25.37996	24.65563	39.09969	21.30895	26.41667
July '98	27.13	30.10269	42.54128	27.08128	24.19952	42.3569	20.81709	26.95889
Aug '98	21.69	26.43045	38.78835	31.44912	22.17396	38.77828	20.32127	29.99546

Graph 3-5: Occidental PTL



The above graphs and tables show how the CAPM is applied to generate scenarios. These results can also be validated by applying confidence intervals analysis. **APPENDIX I** presents all the shares utilised in this study.

3.4.4 Confidence Intervals Analysis

There are numerous statistical procedures, like hypothesis testing for example, to investigate the quality of scenario generation, but confidence interval analysis seems a more informative approach.

An interval estimate of an unknown population parameter is a random interval constructed so that it has a given probability of including the parameter.

A confidence interval is a numerical range, from a lower to an upper bound, within which the true value lies with a stated probability, usually 95%.

The width of the confidence interval gives an idea about how uncertain we are about the estimates. A very wide interval may indicate that more data should be collected before anything very definite can be said about the parameter.

The confidence interval analysis is used in all shares for all scenarios. The formula is as follows:

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

where $\bar{x}$ is the mean of a random sample

$\pm$ 1.96 is the upper/lower bound for a 95% confidence interval

$\frac{\sigma}{\sqrt{n}}$ is the sample standard deviation

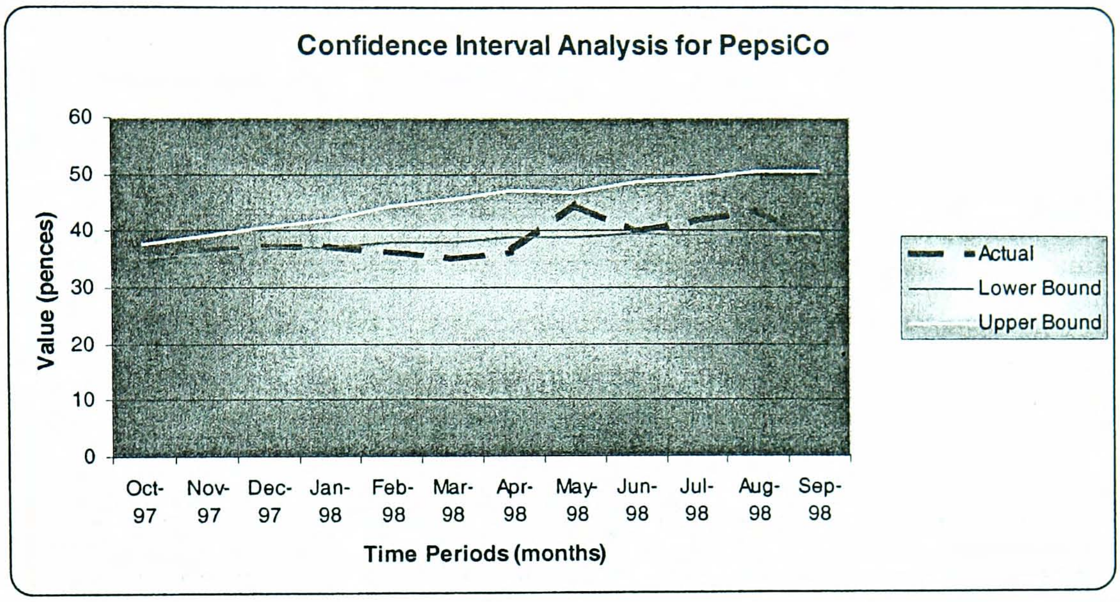
Consider Disneys' share during the period of 05/08/1997 to 05/08/1998. A 95% confidence interval for the price of this share is calculated by following the procedure for all the twelve time periods to capture the whole year. Table 3-10 illustrates the results of a confidence interval analysis for PepsiCo.

Table 3-10: Confidence Interval for PepsiCo

Period	*Lower Interval*	*Upper Interval*	*Actual Value*
Aug '97	35.10995	37.6011	34.6
Sep '97	36.06558	39.26703	36.79
Oct '97	37.02315	40.65516	37.38
Nov '97	36.93741	41.83441	37.31
Dec '97	37.96136	44.28019	36.5
Jan '98	38.03817	45.66248	35.25
Feb '98	39.01481	46.99884	36.56
Mar '98	38.77457	46.75268	44.69
Apr '98	39.57247	48.63038	40.19
May '98	39.20802	49.43144	42
June '98	39.33723	50.42536	43.69
Aug '98	39.35569	50.4836	37.75

Graph 3-9 shows PepsiCo's share actual path together with a confidence interval analysis of 95%. Confidence Interval Analysis has been carried out for all 59 shares.

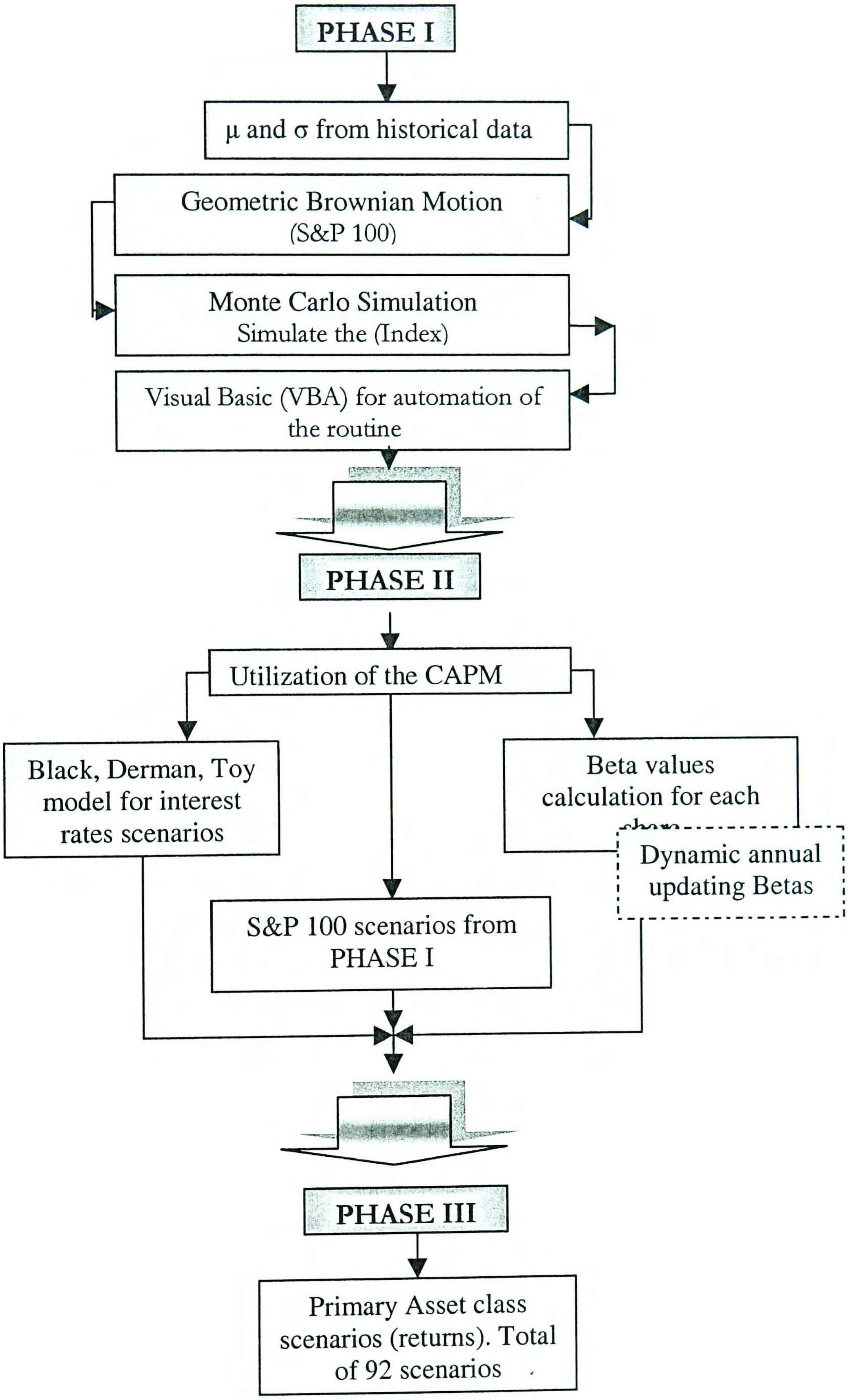
Graph 3-9: Confidence Interval Analysis for PepsiCo.



The confidence interval technique has been carried out successfully not only for the shares but also for the Index's scenarios.

Figure 3-2, shows schematically the structure adopted for the scenario generator system and the links between each variable.

Figure 3-2: Scenario Generator System



3.5 Scenario Generators

The most important issue in financial modelling is the data. The number and form of the scenarios are the most crucial factors in the size and complexity of any financial model. There are a number of scenario generators constructed by leading academics and institutions. Their careful selection is crucial for a successful application. The most well known scenario generators are referenced in this section, as they are state-of-the-art systems.

Carino, Ziemba and Myers [101] and Carino and Ziemba [102], have constructed one of the leading scenario generators for the Russell (financial institution) -Yasuda Kasai (insurance organization). This scenario generator uses econometric techniques to produce 256 different scenarios for 7 asset classes (stocks, bonds etc.). The strongest assumption of the scenarios is that they are independent across periods, which substantially reduces the amount of data. The generator creates scenario outcomes from the input (historical) data using several adjustments. In a most recent study Geyer, Herold, Kontriner and Ziemba [108], developed a scenario generator for a pension fund for the Bank of Vienna. The generator uses means, standard deviations and correlations from historical data for equities and bonds. The total number of scenarios was 960.

Another robust scenario generator was developed by Mulvey and Thorlacius [103], Mulvey et al. [104], for the Towers Perrin Tillinghast, the worlds largest actuarial consulting companies. The generator uses econometric techniques to generate scenarios for interest rates, inflation and real yields. There are also currencies and stock returns. The innovation of this generator is that it is calibrated in more than 17 countries throughout America, Europe and Asia. The number of scenarios generated was 500.

Dempster et al. [105], have been involved in scenario generation systems. The first is the Swiss Re/Falcon Asset Model. The system uses a non-linear autoregressive AR econometric technique for forecasting. For each country (as it is an international application), both economic and financial variables were projected. The second Consigli and Dempster [106], is the CALM-

WATSON model for a pension fund, where up to 2688 scenarios were generated for 5 asset classes and 5 funds. Again the system used economic factors like interest rates for evaluating the scenarios.

Zenios [107], has developed a system for fixed income securities. This system uses interest rate contingencies. A similar methodology with this research was done by Consiglio and Zenios [116]. They use the Monte Carlo simulation procedure, calibrated using historical observations of volatility and correlations, to generate jointly scenarios of interest rates and exchange rates. 512 scenarios were produced for tracking International Indices.

Other scenario generator systems have been developed by Yakoubov et al. [109], where they utilized fundamental macroeconomic factors. They considered price and salary inflation for U.K. bonds, overseas and U.K. equities. Kouwenberg [110], generated scenarios for liabilities and the economic scenarios were generated using a VAR model. Mulvey et al. [111], proposed well-known estimation techniques critical to scenario generation. Dupacova [114] and Dupacova et al. [115], concentrated on the representation of the data and underlined the issue of probability distributions and their manipulation.

Chapter 4: INCORPORATING UNCERTAINTY

4.1 Introduction

Having developed a methodology for capturing uncertainty, this chapter focuses on incorporating it into optimisation models. The simplest form is a single period model. The most common and widely used are the quadratic programs or QP. They incorporate uncertainty by estimating return and variance. A more complex formulation is that of multiple time period models.

This chapter gives an insight into single period quadratic or mean-variance models, and then the framework of multiple time period models for tackling uncertainty (wait-and-see, expected value and here-and-now) and their relationships are discussed. The here-and-now approach is the basic concept behind the stochastic linear programs, which are analysed by considering the 2-stage and multi-stage methodologies. Furthermore, the end effects in multi-stage stochastic programs and their implications are debated. Extensions of stochastic programming into Asset and Liability Management (ALM) are exploited and finally a computational financial study is introduced.

Optimisation problems are made up of three basic ingredients [123]: An **objective function,** which needs to be minimized or maximized. For instance, in a manufacturing process, the profit might need to be maximized or the cost minimized. Almost all optimisation problems have an objective function. A set of **unknowns** or **variables,** which affect the value of the objective function. In the manufacturing problem, the variables might include the amounts of different resources used or the time spent on each activity. Variables are essential because their non-existence, cannot define the objective function and the problem constraints. Sets of **constraints** that allow the unknowns to take on certain values. For the manufacturing problem, it does not make sense to spend a negative amount of time on any activity, so all the 'time' variables are constrained to be non-negative. Most problems require constraints, however, the field of unconstrained optimisation does not need any constraints at all. Within the finance field general optimisation techniques for portfolio selection problems can be found in [137].

4.2 Quadratic Models

Harry Markowitz [45], introduced the theory of efficient portfolios where for a given risk they yield the highest expected return. This set of portfolios is known as the efficient set and can be identified by solving a quadratic program. Chapter 2 gives an extensive discussion about the Markowitz's mean-variance model. When plotted in a risk-return graph the efficient set forms the efficient frontier. In order to compute the variance/covariance-a measure of the degree of possible deviations from the mean (average value obtained by regarding the probabilities as frequencies)- a quadratic function that measures risk needs to be computed. To minimize the variance the quadratic function should also be minimized. Quadratic programming utilizes a quadratic objective but the constraints are linear equalities or inequalities.

Consider the Quadratic program (QP):

Minimize $$\frac{1}{2}\sum_{i,j=1}^{n} w_i w_j \sigma_{i,j}$$

Subject to $$\sum_{i=1}^{n} w_i \bar{r}_i = \bar{r} \qquad \text{(C1)}$$

$$\sum_{i=1}^{n} w_i = 1 \qquad \text{(C2)}$$

$$w_i \geq 0 \qquad i = 1,\ldots,n$$

where $\bar{r}_i$ is the expected return of asset i and $\bar{r}$ is the desired level of the portfolio's return. w_i are the weights of the portfolio $i = 1,2,\ldots,n$ and $\sigma_{i,j}$ are the covariances for stocks $i,j = 1,2,\ldots,n$. The objective function is to minimize the covariance. Constraint (C1), specifies the expected return on the portfolio and (C2) satisfies the fact that all the portfolio money is invested in different assets. By indicating a level of expected return for the portfolio the QP computes the corresponding minimum variance. By considering different levels of return, and the corresponding portfolio variance then the results can be plotted graphically (minimum variance versus specified return). This curve represents the efficient frontier. The characteristic of these portfolios is that they have the highest return for a given level of risk and vice versa. Some additional restrictions-constraints can be added to the quadratic model such as cardinality, buy-in threshold and roundlots. The first restricts the number of companies allowed in a portfolio. A buy-in threshold constraint defines the minimum level (threshold) that a share can be purchased. The last constraint restricts the investor to make transactions in multiples of these roundlots.

Special computer packages are available in the market for solving the QP's, but small sized problems can be solved easily in spreadsheet programs. In the financial sector there are powerful frame stations that can solve QP's with hundreds or even thousand of assets. Researchers have tried to find techniques to reduce computational times. A well known academic, Professor Pardalos, published [136] a paper where he utilises parallel algorithms for Quadratic programs in order to cut down the computational time.

Many users of mean-variance optimisation models obtain poor results because of errors in forecasting asset returns and this is a major drawback. Often this is due to the common practice of naively extrapolating historical returns and correlations into the future and producing portfolios that are optimal, based on past data. Setting investment allocations this way produces poor future performance. By minimizing the variance one actually makes allowance for the uncertainty and does not effectively target it. Another disadvantage in quadratic programming is that if there are too many assets the covariance matrix becomes difficult to compute. Models that use more than just one time period and efficiently manage uncertainty are discussed in the next section.

4.3 Planning Models Incorporating Uncertainty

Consider a portfolio optimisation problem. In a deterministic approach the underlying assumption is that the returns of the equities traded in the market are known parameters [124]. The optimal solution of this problem would contain as many shares as possible of the equity with the highest assumed return. This means that if one asset seems superior to another, the latter will never, ever appear in any recommended portfolio. The returns of a portfolio selected by a deterministic model may be significantly different from those expected. It is uncommon for investors to attempt to implement the solution of a deterministic portfolio optimisation problem. To overcome this problem diversification constraints are added to the model. Diversification improves risk-adjusted returns. It can provide some hedge against the volatility of the individual assets. Diversification is best achieved by owning different asset classes rather than trying to pick the 'best' securities in a given market or sector. This is because even in the unlikely event that one truly possesses that skill, when that particular market torpedoes, so will his/her portfolio. Due to the lack of satisfactory solution in these kinds of problems different techniques can be utilized to compensate this disadvantage. Post-Optimal analysis on the solution such as sensitivity analysis [125], in the sense that if the solution of the problem is sensitive to a particular parameter then its value is revised, and scenario analysis is carried out (discussed in the previous chapter). Scenario analysis within a model captures the uncertainty at a particular time period or periods and can lead to Stochastic Programming (SP). Stochastic models tackle the disadvantages of deterministic models directly. The assumption of the uncertain parameters of the model to be known is relaxed by assuming that their distributions are known [126], [127].

To gain insight one first solves all the individual scenarios using the wait-and-see technique. Next, one looks for feasibility by solving the expected value problem and finally one makes a decision about the uncertainty before carrying out the here-and-now technique. In the following three sub-sections, the different approaches to the stochastic problem are discussed.

Let (Ω, F, P) denote a (discrete) probability space where $\omega \in \Omega$ denotes realizations of the uncertain parameters and $P(\xi(\omega))$ the corresponding probability. The realizations of A, b, c for a given ω as:

$$(A, b, c)_\omega = \xi_\omega \quad \text{or} \quad \xi(\omega)$$

4.3.1 The Wait-and-See Approach

The Wait-and-See (WS) approach [124] assumes that one can wait until the uncertainty is resolved at the end of the planning horizon and an outcome $\omega \in \Omega$ can be monitored, before any optimal decision x can be made. This approach assumes perfect information about the future, which is unrealistic and usually such a solution, is not implementable and it is known as the 'passive approach'. Wait-and-See models are often used to analyse the probability distribution of the objective value, and it can be said that it consists of a family of Linear Programming models, each associated with an individual scenario in the event tree. The problem can be formulated as follows:

$$Z(\omega) = \min c(\omega) x$$

$$\text{subject to} \quad x \in F^\omega$$

The expected value of the Wait-and-See solutions is defined as:

$$Z_{ws} = E[Z(\omega)] = \sum_{\omega \in \Omega} Z(\omega) p(\omega)$$

Figure 4-1 shows a generic scenario tree for the wait-and-see (WS) approach:

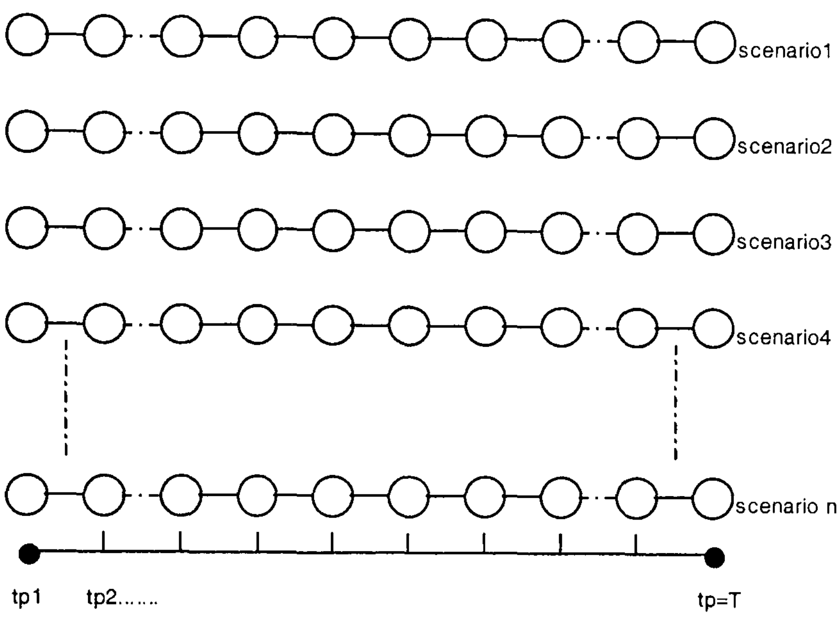


Figure 4-1: Scenario tree for WS

4.3.2 The Expected-Value Approach

In the expected-value (EV) approach the stochastic parameters are replaced by their expected values [124]. This EV model is a linear program, as the uncertainty is dealt with before it is introduced into the underlying linear optimisation model. The EV problem is formulated and solved to gain some insight into the decision problem. By letting the constraints:

$$F^{\omega} = \{x | Ax = b, x \geq 0 \} \quad \text{for} \quad (A,b,c)_{\omega} \quad \text{or} \quad \xi(\omega)$$

the expected value problem is be reconsidered where:

$$(\overline{A},\overline{b},\overline{c}) = \overline{\xi} = E[\xi(\omega)\ ] = \sum_{\omega \in \Omega} p(\omega)\xi(\omega)$$

then the optimisation problem is defined as:

$$Z_{ev} = \min \overline{c}x$$

$$\text{subject to} \quad x \in \overline{F} \equiv \{x | \overline{A}x = \overline{b}\ \}$$

where:

- $z = cx$ represents the objective function

- x is a (n x 1) vector of unknown decisions, which are discovered by solving the problem, hence it is called the decision variable

- c is a (1 x n) vector of known data and represents the coefficients of the objective function

- A is a (m x n) vector of known data and represents the technical coefficients

- b is a (m x 1) vector of the known data representing the right hand side values

- $Ax = b$ represent the constraints

Letting x^*_{ev} be the optimum solution to the above expected value problem then this solution can be evaluated for all possible scenarios $\omega \in \Omega$, the corresponding objective function values can be determined and the expectation of the expected value can be computed as follows:

$$Z_{eev} = E[c(\omega)x^*_{ev}]$$

If however, an ω exists such that: $x^*_{ev} \notin F^{\omega}$, that is, x^*_{ev} is not be feasible for some realizations of the random parameters then in this particular case $Z_{eev} \rightarrow +\infty$ is set. Figure 4-2 shows a generic scenario visualization for the EV approach.

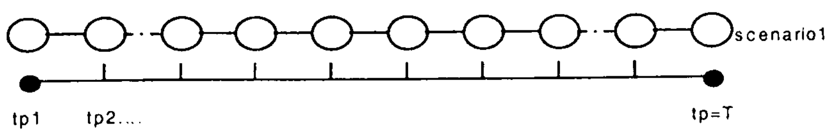


Figure 4-2: Generic scenario visualization for the EV approach

4.3.3 *The Here-and-Now Approach*

This underlines the true stochastic optimisation problem. A decision x has to be made 'here and now', before observing an outcome from Ω [124]. The decision maker takes his decision at the present point in time without waiting to find out the actual values of the random parameters at a future point in time. The value x is chosen such that the expected

costs $E[c,x]$ assume a minimum:

$$Z_{hn} = \min E[cx]$$

$$\text{subject to} \qquad x \in F$$

$$F = \bigcap_{\omega \in \Omega} F^{\omega}$$

The optimal objective function value Z_{hn} denotes the minimum expected costs of the stochastic optimisation problem. It is noted that x has to be feasible for all scenarios-realizations $\omega \in \Omega$. The optimal solution $x^* \in F$ hedges against all possible contingencies $\omega \in \Omega$ that may occur in the future.

4.3.4 *Inter Relationship and Bounds of the Approaches*

The relationship between the three different approaches for a minimization problem is as follows [124]:

$$z_{ws} \leq z_{hn} \leq z_{eev}$$

The difference $z_{hn} - z_{ws}$ is defined as the expected value of perfect information or EVPI. It measures the maximum amount a decision maker is willing to pay in return for complete and accurate information about all the future scenarios. A small EVPI indicates that better forecasts will not lead to much improvement. A large EVPI indicates that the representation of uncertainty needs more refinement.

A different measure is the value of the stochastic solution known as VSS:

$$VSS = z_{eev} - z_{hn}$$

VSS measures how much better the solution of the stochastic optimisation problem is in relation to the expected solution of the expected value approach. A small VSS indicates that there is not much benefit in solving the here-and-now problem. The larger that measure becomes the more important it is to solve the stochastic optimisation program.

EVPI and VSS can be bounded as follows [124]:

$$0 \leq EVPI \leq z_{hn} - z_{ev} \leq z_{eev} - z_{ev} \quad \text{and}$$

$$0 \leq VSS \leq z_{eev} - z_{ev}$$

4.4 Stages of Stochastic Linear Models

Stochastic programs are mathematical programs where some of the data incorporated into the objective or constraints is uncertain [128]. Uncertainty is usually characterized by a probability distribution on the parameters. Although the uncertainty is rigorously defined, in practice it can range in detail from a few scenarios (possible outcomes of the data) to specific and precise joint probability distributions. The outcomes are generally described in terms of elements w of a set W. W can be, for example, the set of possible demands over the next few months.

When some of the data is random, then solutions and the optimal objective value to the optimisation problem are themselves random. A distribution of optimal decisions is generally unimplementanble. Ideally, we would like one decision and one optimal objective value.

In the following two subsections the 2-stage and multi stage stochastic programming models are analysed and their formulations are discussed.

4.4.1 *Two-Stage Stochastic Models*

The fundamental idea behind stochastic linear programming is the concept of recourse. Recourse is the ability to take corrective action after a random event has taken place. A simple example is the two-stage recourse.

- A variable x is chosen in order to control the situation at this very time period
- Some random incident happens lets assume during the night
- At the next time period a recourse action y is taken in order to correct what the random incident may have caused.

In two-stage stochastic programming formulations one decomposes the model into two main parts. The first par involves taking a decision x that is feasible for all scenarios $\omega \in \Omega$ and has minimum expected costs. That is the first stage decision. The second stage decision $y(\omega)$ compensates for and adapts to different scenarios ω. The decision x is made only with the knowledge of the distribution $(\omega, p(\omega))$ of the random parameters, while the second stage decision is made after the realisation of the random parameters. The sequence of events is the following:

First Stage Decision → Observation → Recourse Decision-Second Stage Decision

Two-stage linear programs with recourse form an important class of models, which incorporate uncertainty within an optimisation model. It is obvious that under uncertainty, the circumstances lead to the fact that not all information for the future is available and parameters like demand and prices are modelled by random variables. Under this situation the activities that cannot be postponed for the future should be planned here-and-now. Decisions for remaining activities are postponed until further or better information is available. The main issues of the two-stage stochastic program (SP) are classified as:

- identifying the first stage decision variables and the related deterministic parameters
- identifying the recourse variables and related random parameters

The two-stage SP is formulated as follows [124]:

$$\min z = cx + E_{\omega} Q(x,\omega)$$

subject to $$Ax = b$$

$$x \geq 0,$$

where:

$$Q(x,\omega) = \min f(\omega)y(\omega) \qquad \text{(a)}$$

$$\text{subject to } D(\omega)y(\omega) = d(\omega) + B(\omega)x$$

$$y(\omega) \geq 0.$$

$$\omega \in \Omega$$

The matrix A and the vector b are known with certainty. The function $Q(x,\omega)$, referred to as the recourse function, is in turn defined by the linear program (a). the technology matrix $D(\omega)$, also known as the recourse matrix, the right-hand side $d(\omega)$, the inter-stage linking matrix $B(\omega)$ and the objective function coefficients $f(\omega)$ of this linear program are random. For a given realization ω, the corresponding recourse action $y(\omega)$ is obtained by solving the problem set out in (a).

The difficult part in these problems is to make a decision during the first stage that satisfies all scenarios $\omega \in \Omega$ and minimises the expected costs for the second stage. In these kinds of stochastic problems the first time period decision x, is independent of which second time period scenario actually occurs. These are called the non-anticipativity constraints.

By considering Ω being discrete and finite with $\Omega = \{1,\ldots,K\}$ defining an index set, the deterministic equivalent algebraic formulation of the problem is as follows:

$$\min z = cx + p_1 q_1 y_1 + p_2 q_2 y_2 + \ldots + p_K q_K y_K$$

subject to

$$
\begin{aligned}
&Ax && && = b_1 \\
&T_1 x + W_1 y_1 && && = b_2 \\
&T_2 x && + W_2 y_2 && = b_3 \\
&\cdots\cdots\cdots && && \\
&T_K x && && + W_K y_K = b_K \\
&x \geq 0 \quad y_1, && y_2, \quad \ldots, && y_K \geq 0
\end{aligned}
$$

where:

- $K \in \Omega$ represents the different values each scenario can take
- p_K is the probability of occurrence for each different state of the world

This representation clearly shows that for each scenario ω the second stage decision $y(\omega)$ is the solution of a linear program.

Figure 4-3 illustrates a generic scenario tree for the two-stage here-and-now SP

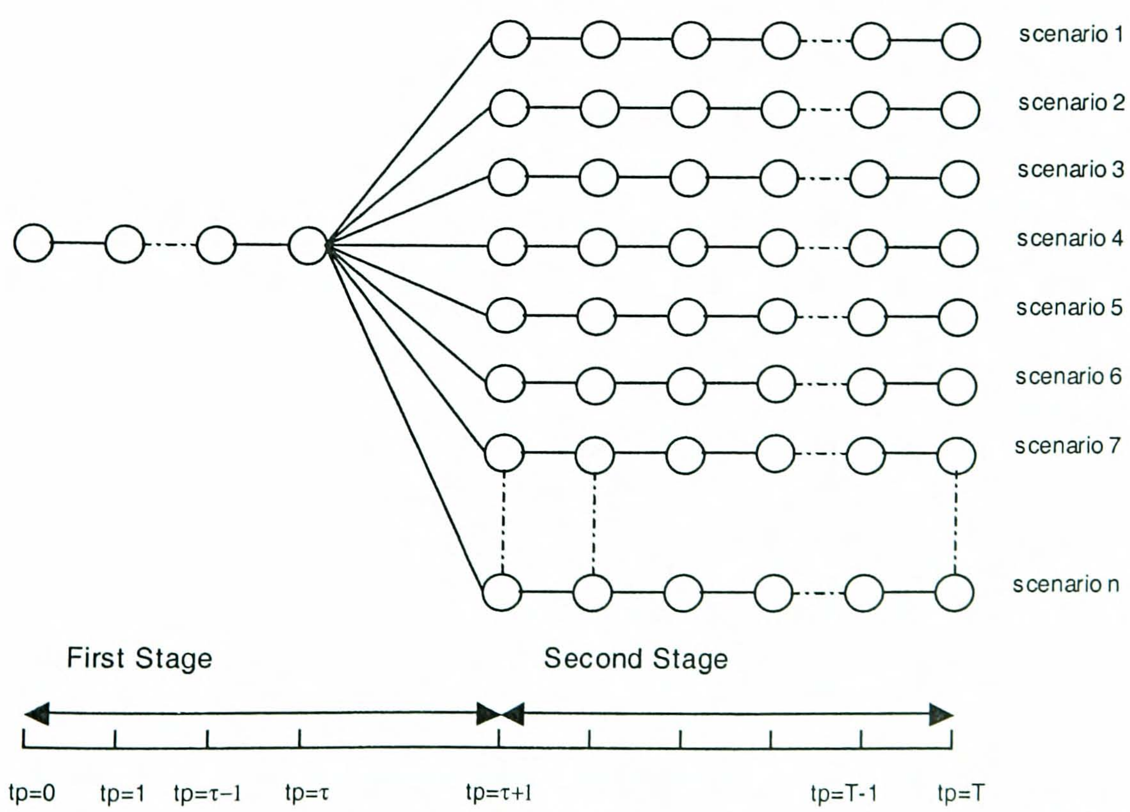
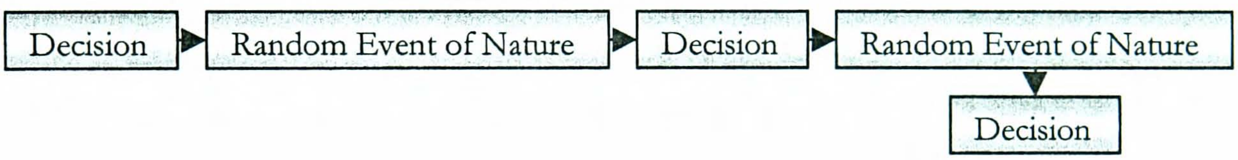


Figure 4-3: Generic scenario tree for the two-stage here-and-now SP

4.4.2 *Multi-Stage Stochastic Models*

Multi-stage stochastic programs are an extension to the above two-stage SP with recourse but with more stages. A planning horizon is considered and is divided into a number of discrete time periods of equal lengths for points $t = 1,...,T$. The first time period's decisions are known with certainty. The second stage decisions are made after the result in the second time period has been observed but without knowing anything about time period 3 and so on. The main objective is to minimize the expected costs of all decisions taken. The sequence of events and decisions is shown below:



The multi-stage models formulation is as follows:

$$\min_{x1}\{c_1 x_1 + E_{\xi_2}\left[\min_{x2} c_2 x_2 + E_{\xi_3|\xi_2}\left(\min_{x3} c_3 x_3 + \ldots + E_{\xi_T|\xi_{T-1}|\cdots|\xi_2} \min c_T x_T \ \right)\right]\}$$

subject to

$$\begin{aligned}
&A_{11}x_1 && = b_1 \\
&A_{21}x_1 + A_{22}x_2 && = b_2 \\
&A_{31}x_1 + A_{32}x_2 + A_{33}x_3 && = b_3 \\
&\qquad\qquad\ddots && \vdots \\
&A_{T1}x_1 + A_{T2}x_2 + A_{T3}x_3 + \ldots \qquad + A_{TT}x_T && = b_T \\
&\qquad l_t \le x_t \le u_t, \qquad t = 1,\ldots,T
\end{aligned}$$

where:

- $t \in (1, \ldots, T)$ represents the planning horizon

- $E_{\xi_2}\left[\min_{x2} c_2 x_2 + E_{\xi 3|\xi 2}\left(\min_{x3} c_3 x_3 + \ldots + E_{\xi_T|\xi_{T-1}|\ldots|\xi_2} \min c_T x_T \;\;\right)\right]$, is the stochastic part of the objective function

- The vectors and matrices: $\xi_t = (b_t, c_t, A_{t1}, \ldots, A_{tT}) \qquad \forall t \in [2, \ldots, T]$, are random parameters on a probability space (Ω, F, P)

The objective function is a sequence of nested optimisation problems corresponding to different stages. At time period 1 the user has to pick out a decision whose outcome completely relies on the future realizations of the corresponding multi-stage stochastic program. The solution of this program is referred to as the here-and-now. After that, for each realization of the history ξ_t of the data process up to time t, a recourse problem is considered in which decisions are allowed to be a function of observed realization (x_{t-1}, ξ_t) only. The multi stage stochastic programming problems capture the concept of the uncertainty through the different stages. This is why it is more realistic to make corrections (recourses) at different time periods when new stages of uncertainty take place. Figure 4-4 represents a generic form of scenario tree that corresponds to the multi-stage here-and-now SP.

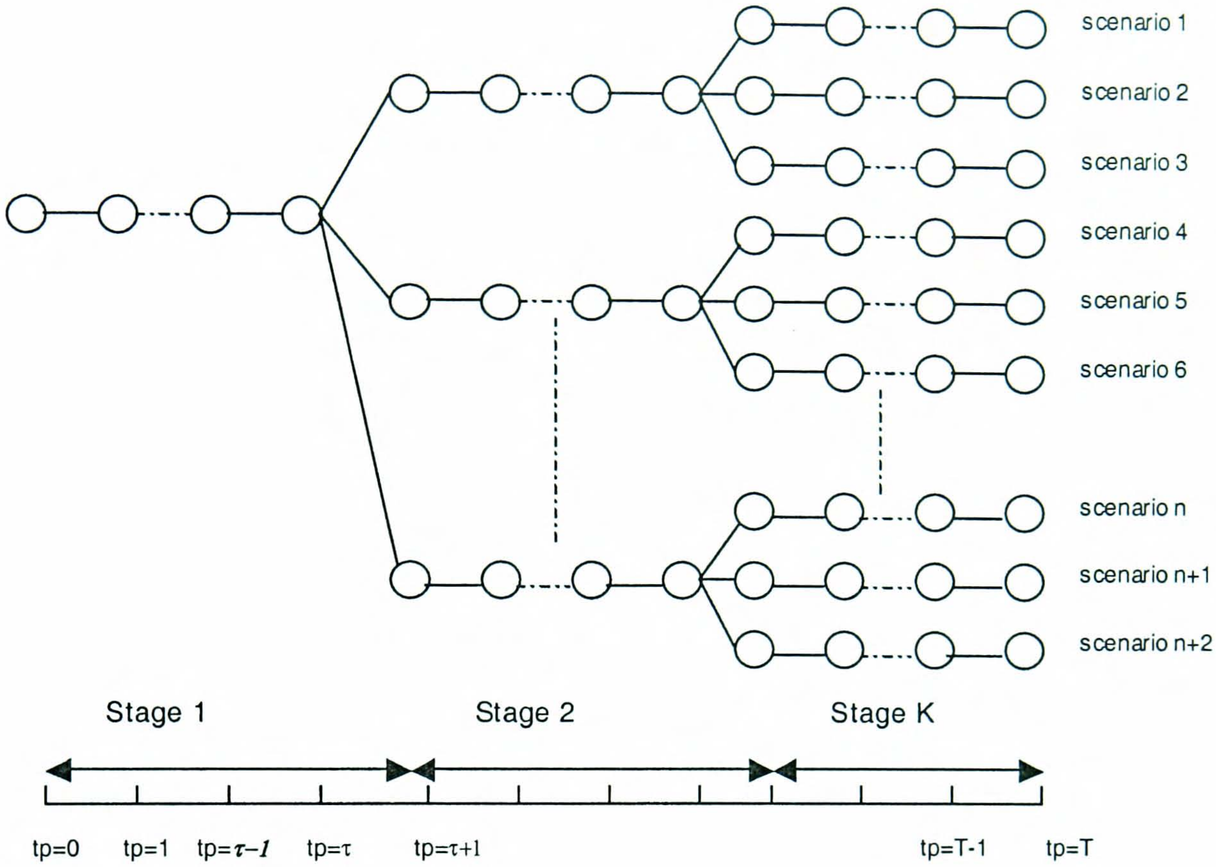


Figure 4-1: A generic scenario tree illustration for the multi-stage here-and-now SP

4.4.3 *End-Effects in Multi-Stage Models*

The end-effects period is designed to reduce the effects of ending the model at a finite horizon, while in reality the firm expects to operate indefinitely beyond the horizon. It assumes that continuity of the model is vital, as realistically any institution will continue to function after the planning horizon of the model. Carino and Ziemba [101], [102], developed a multi-stage financial planning stochastic model for Russell-Yasuda (RY). In their model they utilize but not implement the end-effects methodology. Ziemba argues that for financial models, end effects are less important and do not appear in the subsequent models but only in their general study. The methodology considers the base problem up to period T (defined by the user), and then a steady-state terminal value $d_T x_T$ is added to the objective function. This term and the constraints associated with it create an extra time period in the base problem. Thus, the problem has $T+1$ periods. Grinold [129], [130], [131] studied various approaches for adding the steady-state period and wrote the basic papers for deterministic models. He used this methodology for an energy-planning model. The most widely used technique is the dual equilibrium. The idea behind this technique is to assume that the dual variables of the T stage problem increase period by period in proportion to the assumed discount factor. End-effects are important when one makes factories or other capital goods - with financial models, one usually can get away without end-effects in financial ALM models and use the extra modelling space for a bigger model in the earlier periods. The technique of the end-effects and its formulation appears in **APPENDIX III**. Stochastic Programming models are not only used within the financial or the energy-production field. An example of different industries where SP is applicable is illustrated in table 4-1.

ENERGY	*ENGINEERING*	*FINANCE*	*OTHER*
Energy Production	Machine Scheduling	Portfolio Planning	Crew Scheduling
Energy Distribution	Production Planning	Asset and Liability Management	Farm Planning
Electrical Generation Capacity Planning	Supply Chain Management	Financial Variables Forecasting	Blending
Demand Forecasting	Inventory Management	Strategic Investment Decisions	Traffic Management
Energy Costs Estimation	Demand Forecasting		Depot Location

Table 4-1: Applications of Stochastic Programming Models

Chapter 5: ALM MODELS – A REVIEW AND NEW DIRECTIONS

5.1 Introduction

One of the most challenging and at the same time difficult problems that investors and organizations face, is that of the management of their assets and the commitment to cover their future liabilities. In order to achieve desirable returns the assets must be invested over a period of time subject to certain constraints that the institution or individual might have. Requirements like taxes, policies, legal and regulatory issues, transactions costs and obviously their liability commitments [133]. The main problem faced is that most of them fail to diversify successfully across time or markets and in particular fail to match their liability streams. Many investors and organizations face the dilemma of [132]:

Buy and you'll be sorry,
Sell and you'll regret,
Hold and you'll worry,
Do nothing and you'll fret

Asset and Liability Management (ALM) modelling is a field of great practical importance and high complexity thus resulting in a challenging research area that provides users with organized and diversified systems to help manage their financial commitments in an increasingly complex and difficult financial world. Such models, force diversification and therefore help minimize the possibility of financial disasters or any other embarrassing situations, while at the same time providing adequate advice in ordinary circumstances balancing the investors situation.

5.2 Foundations of ALM

ALM refers to those who wish to achieve goals and meet future obligations by utilising optimal investment policies. Application areas of ALM do not only include insurance companies, pension plans or banks but also wealthy and ordinary individuals. Because these investors possess future goals and liabilities they must make investment decisions while considering the use of their funds, in other words, investing for a purpose. The key issue is to develop an optimisation model for supporting the decision-making concerning the allocation of assets and managing the liabilities over several time periods in such a way that the liabilities are met and the goals of the company are achieved. Stochastic programming optimisation models are used to evaluate long-term investment strategies. These types of problems-models have an inherently stochastic nature. The roots of stochasticity include besides the assets (shares, bonds, etc.) the liabilities, which could be any future commitments. Consequently adequate models for ALM should account for the stochastic nature of the problem.

The concepts of 'achieving goals' and 'meeting liabilities', are certainly two very important issues but to be successfully implemented and achieved they need to be researched hierarchically assessing the individual's or institutions:

i. Specific goals
ii. Investment objectives
iii. Investment knowledge
iv. Risks
v. Volatility tolerance

Specific goals of an individual can be an early retirement, college education for their children or buying a vacation home. For institutions it can be to provide retirement benefits to participants in a particular scheme or fund the charitable pursuits of an endowment fund. Even when such goals are expressed in such a manner they lack specificity. A characteristic example is what does retirement really mean. Will it be at the age of 50, 55 or 60? What lifestyle does the retiree have in mind? From an institutions perspective, goals generally involve a target return on capital and the ability to provide prompt payments to their customers arising out of contractual obligations. Cash flows are also a goal that has to be specified for the day-to-day progress of any institution. Thus, management's role is to be proactive in reacting to changes in the environment.

Once the goals are clearly specified, the next step is to develop the investment objectives. These objectives are also often more ambitious than can realistically be achieved. Pension fund companies often realise too late that their clients wish to modify their lifestyles a few years prior to retirement.

Investment management is the secret for long-term success and depends on the individual's or institution's understanding of how their portfolios are structured and the manner in which they will behave. ALM can include as many home or foreign asset classes (shares, bonds) as the user wishes. This provides a well-diversified portfolio.

ALM models incorporate uncertainty as discussed in chapter 4. In ALM one is not only interested in maximising wealth but also to limit the exposures to possible losses. These exposures can be defined as risk. There are two types of risks: intrinsic and contextual as Ziemba [133], characteristically states. Intrinsic risk refers to uncertainty surrounding a single security-share where its price fluctuates accordingly to the market. This type of risk cannot be easily eliminated through diversification strategies by asset-only investors. On the other hand, risks that are unrelated to market movements, the so-called non-systematic risks, can be mitigated through diversification across asset classes but also among elements such as interest rates and liabilities. For example, currency risk can be eliminated by diversifying through assets in different countries of the world.

Volatility tolerance is an important issue. The issue here is the amount of volatility that an institution or an investor can tolerate. If the incremental return expected from common stocks is not sufficient to compensate for the volatility of returns, then the volatility tolerance is appropriately lowered [132]. Often investment strategies look for volatility tolerances signs based on the investors or institutions situation. As an example, an individual who is working for one employer for a lifetime and his hobby is golf, may be more volatile-averse than for someone who changes jobs relatively often in advancing his career and likes to rock climb at the weekends.

Good ALM modelling requires an amount of research in these five different areas described above. This will set a foundation in determining the exact goals that someone would like to achieve and meet the future liability obligations. Having achieved this the ALM model can perform the optimisation subject to the individual's constraints and give realistic solutions to the problem.

Figure 5-1, illustrates how the planning and design of asset-liability portfolios could possibly be performed. Any asset portfolio requires its constraints, which are an input by the user. These could include limitations in certain asset trades. Asset portfolio's data should fit the capital markets expectations. This can be achieved by generating scenarios that capture future expectations. The time horizon of the investment should be clear according to the user's needs. The cash flow testing is performed to decide the investment strategy that should be followed in association with the credit availability by the organisation or individual. If there is enough cash on reserve then the strategy is given the green light to go forward.

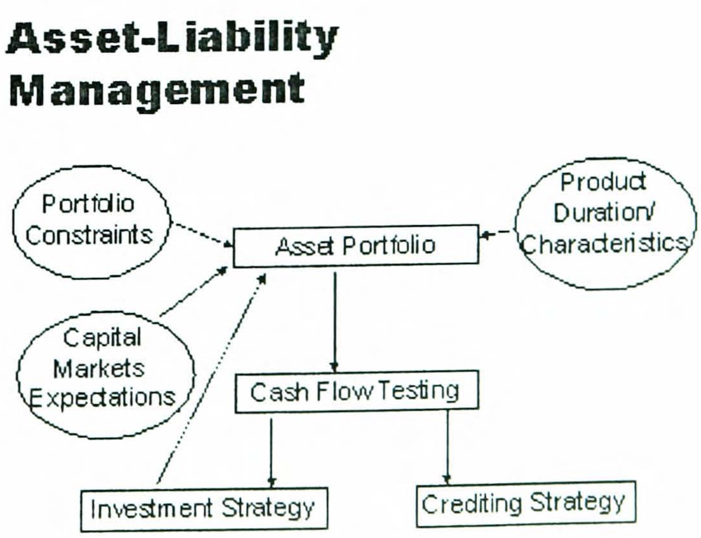


Figure 5-1: Asset and Liability Management hierarchy

5.3 A Review of ALM Models

Asset and Liability Management is a methodology that attracts a big majority of financial organisations, private investors and academics. It is the challenge of meeting these goals that would make them gain competitiveness over their rivals. Stochastic models are sophisticated integrated systems that incorporate uncertainty, institutional constraints and policies. In the literature there is a considerable number of such systems mainly developed by academics to be used by financial institutions. One of the most well known is that of Carino and Ziemba's 'Russell-Yasuda' model [101], [102]. The model assists the firm in deciding how available funds should be allocated among potential investments to provide returns to cover liabilities and provide for long-term growth of the firm's wealth. There are several asset classes such as bonds and shares. The model uses a piecewise linear convex cost function in the objective function of the multi-stage stochastic model to minimize the shortfalls occurred in every time period for every target not met. The main focus is in the liability side. Liability balances change over time as scheduled payments are made, as reserves are set aside for existing obligations, or as new obligations are taken on. Klaasen [112], [113], considers a problem of an investor who faces a sequence of liability payments in the future, and wants to construct a portfolio of securities that allows him to meet these liabilities under a variety of plausible scenarios. From all feasible portfolios he wants to choose the one that optimises a given criteria such as minimum cost. He uses a multi-stage stochastic linear model, which explicitly includes portfolio rebalancing when new data information becomes available. Time aggregation techniques are used. This technique is performed in state n at time t in an event tree by replacing the transitions to the successors of its successors. Aggregation eliminates some trading dates in an event tree as Klaasen characteristically argues and this affects dividends and riskless one-period securities. Time aggregation is performed when one chooses to do so in parts of the event tree, where one thinks or knows that they are less critical to an optimal solution.

Mulvey et. al. [103], [104], use an asset and liability management system (CAP:Link) for Towers Perrin-Tillinghast. The system simulates asset and liability decisions across a long-term planning horizon of 5-15 years. It simulates asset policy with liability decisions so that the company's wealth or pension plan surplus is maximized, while maintaining a safe level of operations. There are three main features that make this model unique: a set of structural economic factors (such as interest rates) for driving both assets returns and liability movements, a set of policy rules that underlie the decision making processes and a full actuarial analysis of pension design and cash flow liabilities for each economic scenario. A multi-time period stochastic programming model is utilized and 500 scenarios are developed. Asset and Liability Management under uncertainty for fixed income securities are discussed in Zenios's paper [107]. The problem was the mismatch of assets and liabilities that exposed some organizations to substantial interest rate risk. An organization faced the problem of maturing liabilities while its assets had a long remaining time to mature. Zenios uses a multi-stage optimisation stochastic model where the first-stage decision deals with the purchase of a portfolio of fixed-income securities. The uncertain future is the level of interest rates and the cash flows that will be obtained from the portfolio. The second stage decision deals with borrowing decisions when the fixed-income cash flows lag the target liabilities. Scenarios of interest rates are based on Monte Carlo simulation of the term structure. He also includes decisions to rebalance the portfolio at some future time period(s), by purchasing or selling securities. The objective (terminal wealth), is computed by accumulating the total surplus net of any outstanding debt at the end of the planning horizon and liquidating any securities that remain in the portfolio.

Dempster [129], has developed a multi-period asset and liability stochastic model the so called CALM model. This model is designed for pension fund management and it uses 5 asset classes and five funds. The objective is to maximize terminal wealth at the end of the horizon. The model utilizes 10 stages with five investment opportunities (bank deposit, fixed rate securities, index-linked securities, share index, real estate). The liabilities in this case are borrowings with a penalty rate when the target is not achieved. The future uncertainty affects price processes, interest rates and pension payments. Up to 2688 scenarios were generated with equal probability. Professor AD Wilkie, developed the Wilkie Model [130],[131], which is a stochastic asset model that models the random behaviour of various economic series (including price and wage inflation, short and long interest rates, share yields and dividends, and exchange rates) over time. Wilkie's model is based on actual data from the U.K. for the period 1924-1991 and is formulated as a set of simultaneous autoregressive equations of up to the third order in recursive form, all dependent on an underlying inflation process. It generates data paths for annual returns in the U.K. market for ordinary shares, fixed-interest irredeemable bonds, bank deposits, index-linked securities and real estate together with predictions of annual pension payments and an estimated reassurance-to-close representing future payment liabilities discounted to the horizon. These are also used for calculating the required random coefficients for the CALM model. The Wilkie model is frequently used in asset liability modelling work to help assess financial risk for pension funds, insurance companies and charities.

One of the latest studies in ALM is that of Ziemba et.al. [108]. It is a financial planning ALM model developed for Austrian pension funds. The model uses a multiperiod stochastic linear programming framework with a flexible number of time periods of varying length. Various forecasting models yield inputs that provide the generation and aggregation of multiperiod discrete probability scenarios for random return and other model parameters. The correlation across asset classes, of bonds, stocks, cash and other financial instruments, are scenario dependent using multiple covariance matrices that correspond to differing market conditions. Austrian pension law and policy considerations are modelled as constraints in the optimisation. A concave risk averse preference function is to maximize the expected present value of terminal wealth at the specified horizon net of expected convex (piecewise linear) penalty costs for wealth and benchmark targets in each decision period. This model's characteristics have similarities with the study that is performed by the author of the present thesis. His new ALM study and the contribution within the ALM field is described in the following section.

5.4 A New ALM Study

After researching the financial literature ALM is a most interesting field for further developments. It is a complex environment as it involves data analysis, conceptualising specific needs, avoiding exposure to high risks while meeting certain goals and liabilities.

The contribution of this study to the Asset and Liability Management-ALM field can be separated into several areas. Scenario generation using geometric Brownian motion with drift is utilised to generate scenarios for the S&P 100 share Index. The Monte Carlo simulation is used to repeat the routine of generating automatically the Index's scenarios. Having done that the Capital Asset Pricing model is used to achieve the consistency within the Index scenarios and shares. Interest rates scenarios are generated using the Black, Derman and Toy model. Numerical values of the mean, standard deviation and beta values are calibrated using a 60 months period of historical data. Shares, Index and interest rates scenarios are validated by using a 95% confidence interval analysis. Having achieved the modelling of uncertainty (by fitting past data to forecast their future movements), a measurement of risk had to be modelled. This was done by grouping the 59 shares used for this study into risk groups. These risk groups were developed by taking the S&P 100 as a benchmark. Its mean and standard deviation was measured and accordingly with those of the shares they were grouped into 5 risk groups in line with the Index's performance. The optimisation part of the study involved the construction of a 2-stage stochastic programming model. The optimisation runs the simulation (scenarios) by identifying decision strategies that best fit the proposed objective that is the maximisation of the portfolio's return for a given target at a specific time period. The risk parameter of the model is the risk groups that control the shares standard deviation and allocate them in the portfolio. Transactions costs are considered in every trade that occurs. Liabilities are considered in the form of a loan in the situation when the target-return is not achieved. No interest is assumed for the outstanding liabilities. The above ALM integrated modelling system is run and dynamically rebalanced automatically through the construction of Visual Basic macro commands.

Chapter 6: A COMPUTATIONAL STUDY

6.1 Introduction

This chapter presents analytically the computational study. The framework implemented for this study is discussed and a schematic representation is given in the next section. The Quadratic Programming model and the 2-stage Stochastic Programming model's algebraic forms are then presented. The results are shown in the form of tables. The purpose for this is to examine the risk profiles performance of the two models. The strategy is to assume that no investments at the beginning of the planning period exist so an initial portfolio for both the QP and SP is first solved. Then, the models are updated with new data and rebalanced every two months for a period of 10 years. The Quadratic programming model's results with the corresponding risk levels are presented followed by the Stochastic Programming model's results. A comparison of the QP and SP is then presented and discussions concerning the two techniques are given. The Index S&P 100 and the risk free asset are also compared with the QP and SP.

The investigation is concentrated on the results of the first run of the scenario generator.

6.2 A Framework for Computational Experiments

The framework that was developed to enable the current study to be performed successfully is described in this section.

The need of Stochastic Programming in this particular study was essential. Asset prices are largely volatile and uncertain especially when one is trying to make forecasts. Similarly, all the liabilities have a large degree of variability. Furthermore, issues like volatile market periods i.e. interest rates rise/cut etc., can be represented only by implementing an SP. This is due to the fact that by generating scenarios, which are able to reflect stock market anomalies investors can gain access to assets that would never be selected by using other types of models such as the QP.

6.2.1 Historical Data

The framework for the historical data is the following: all historical data were gathered in MS Excel. The expected monthly rate of return was estimated from the mean series of past monthly returns. The covariance between two assets was estimated on the basis of 'sample covariance' over the same series of monthly returns. The disadvantage here is that when estimating the true rate of return, sample estimates of stock returns can be unreliable and unstable. For that reason many organisations, investors and academics in order to decrease the sampling error produce sample estimates based on lengthy history of past returns.

The historical set used in this study range over a period of 60 months (January 1983-January 1988). This period though was dynamically changed to reflect the updating (rebalancing) of the data from August 1988-June 1997. The whole procedure was done automatically by using codes-macros in Visual Basic (MS Excel).

6.2.2 Models procedure

A Quadratic Programming model and a 2-stage Stochastic Programming model were utilised using the mathematical programming language MPL. FortQP and FortMP respectively were the solvers utilised. The solvers have been developed by the mathematical research group at Brunel University. A ten-year investment period (August 1988-June 1998) is considered. The problem is to determine an investment strategy for each of the 10 years of the investment period. But as this does not reflect the realistic institutions or investors strategies the rebalancing technique was adopted.

The assumption is that there are no investments at the beginning of the planning period. The two models are solved to generate the initial optimum hedged portfolios using historical statistics as described above. This historical set was used to compute the QP's mean-variance and as an input to the SP's scenario generator. In every two monthly time periods after the initial portfolio was developed, the two models with their parameters (historical sets, mean variance, scenarios set), were updated and re-balanced (adjust) from the initial portfolio weights to form a new optimal QP and SP's portfolios. The re-balancing process is carried out until the end of the planning horizon November 1997. Input and output data are stored using MS Excel. An integrated VBA routine is used to call MPL to adjust the model according to the changed parameters and also to generate and update all the data used. Once the data are read in Excel and the model is generated it is sent to the solver. As soon as it is solved, a solution file is generated which is read back by the spreadsheet.

Below, is a schematic representation of the framework of the study. In the next chapter the algebraic formulation of the SP and QP models used for the study are analysed and the results obtained presented.

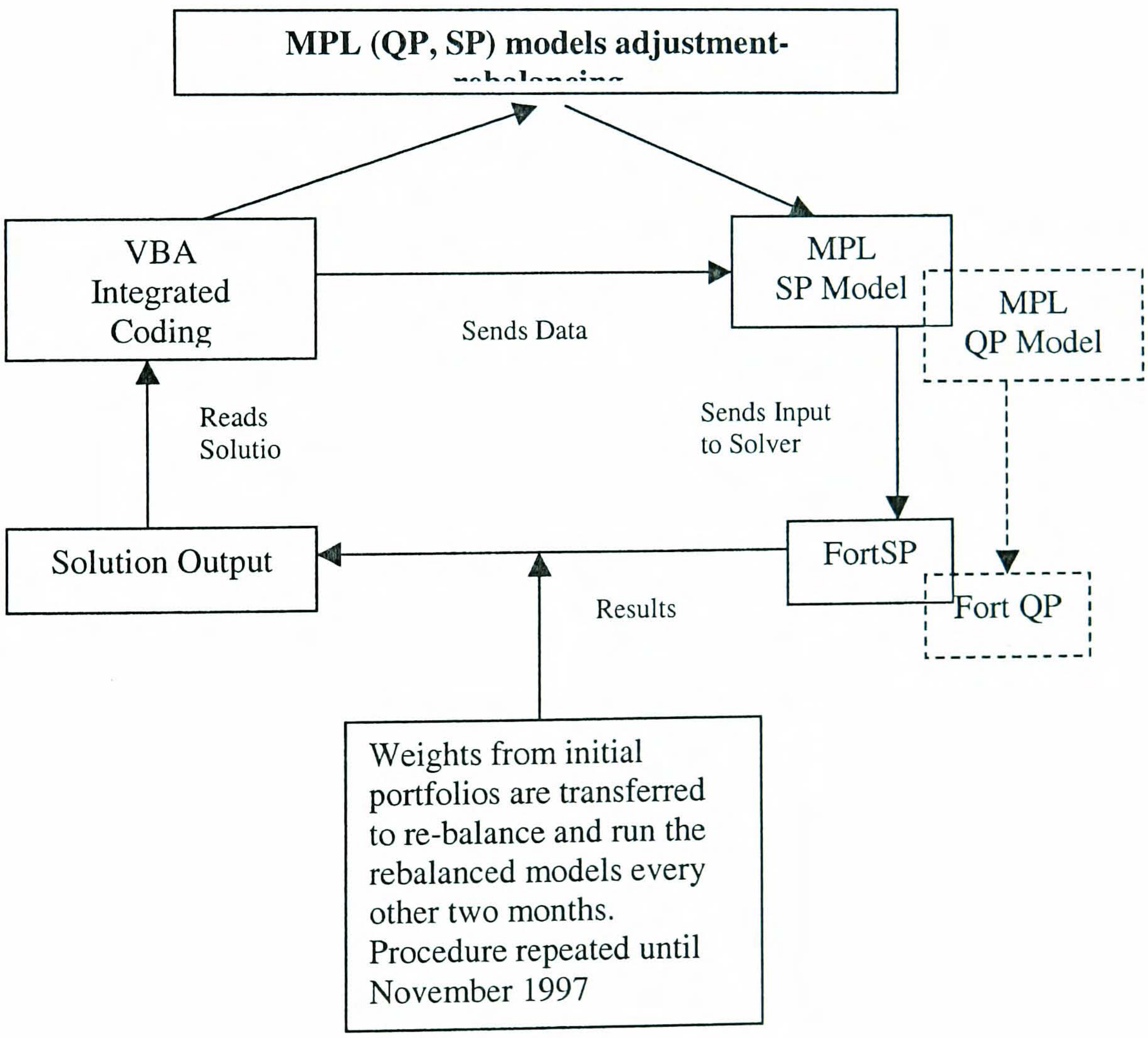


Figure 6-1: Framework of the study

6.3 The QP's Algebraic Form Utilised for the Study

Introduced in Chapter 2, the mean-variance framework also known as Quadratic Programming is widely used for comparison purposes and validations. Before presenting the SP model's framework developed by the author purely for this study, it is essential to show the QP's formulation as its parameters and results are used for comparing it with the SP.

Indices:

$i = 1 \ldots 59, (n)$, denotes the shares

$\zeta = 1 \ldots Z$, denotes the number of risk groups that the shares are classified into. In this case there are 5 different risk groups.

Model Coefficients:

$TC = 0.025$, denotes the transactions costs for every trade

$\mathrm{Re}\,turn_i$, denotes the returns of the shares estimated from historical data

Wo_i, denotes the original weights of the model -used for the second run of the model

$\mathrm{cov}_{i,j}$, denotes the covariance matrix of the shares

$lambda$, denotes the numerical value of the risk. In this case it varies and it is split into 5 different values: 0, 0.25, 0.5, 0.75 and 1. For the underlying study, lamda values are simply distinguished by 5 different risk profiles: min, medium-low, medium, medium-high and max.

$gamma = 1 - lambda$

ρ_ζ denotes the threshold level for each risk group in the portfolio

Models Decision Variables:

W_i, denotes the weights of asset i in the model

B_i, denotes the amount of asset i bought

S_i, denotes the amount of asset i sold

R, denotes the return of the portfolio

Objective Function:

$$\min gamma * \sum_{i=1}^{59} \sum_{j=1}^{59} W_i * \text{cov}_{i,j} * W_j - lambda * R$$

It can be seen from the objective function of the model that by varying the lamda, one can get different returns of the portfolio.

Weights Constraints:

C1: $\sum_{i=1}^{59} W_i = 1,$

C2: $W_i = B_i$, used in the first run of the model $\forall i$

C3: $W_i = Wo_i + B_i - S_i \quad \forall i$

Transactions Constraints:

C4: $R = \sum_{i=1}^{59} W_i * \text{Re}turn_i - \sum_{i=1}^{59} B_i * TC - \sum_{i=1}^{59} S_i * TC$

This constraint incorporates the transactions costs in the buying and selling of shares.

Risk Constraints Group

C5: $\sum_{i \in g_\zeta} W_i \geq \rho_\zeta * \sum_{i=1}^{n} W_i \ , \ \zeta = 1 \ldots Z$

The risk group constraint is analysed in the Stochastic Model's algebraic formulation

Bounds:

$S_i \leq Wo_i$

The bounds constraint is important as it actually stops short selling.

The bounds constraints conclude the description of the QP's algebraic formulation. In the following section the analysis of the 2-stage Stochastic Program model is introduced.

6.4 The 2-stage Stochastic Programming Model

This section describes a 2-stage stochastic programming model, for a financial portfolio optimisation. The problem is to determine an investment strategy for the next 12 months (12 time periods). The goal is to maximize wealth after a period of 12 months. There are three time periods when the portfolio can be rebalanced. Rebalancing occurs at months 2, 6 and 12.

To structure the portfolio, 59 shares that are traded in the S&P 100 American stock market Index were randomly chosen and 92 scenarios of alternative returns were generated using the technique described in chapter 3. The models notation and algebraic form are analysed and discussed below.

Indices:

tp	$= 1 \ldots 3$,	denotes the time periods
i	$= 1 \ldots 59\ (n)$	denotes the shares
s	$= 1 \ldots 92$,	denotes the scenarios
ζ	$= 1 \ldots Z$,	denotes the number of risk groups that the shares are classified

Model coefficients:

P_s	$= 1/92$,	denotes the probability of scenarios occurrence
TC	$= 0.025$,	denotes the transactions costs for every trade
$r_{i,s,tp}$		denotes the values (returns) of the stocks for a given scenario and time period

Wo_i denotes the initial weights of i

$target_{tp}$ denotes the required target return for each time period

L_{tp} denotes the liability at time period tp

$lambda$ denotes the investors risk aversion $(0 \leq lambda \leq 1)$

ρ_ς denotes the threshold level for each risk group in the portfolio

Models Decision Variables

$B_{i,s,tp}$ denotes the Amount Bought of asset i,s at tp

$S_{i,s,tp}$ denotes the Amount Sold of asset i,s at tp

$W_{i,s,tp}$ denotes the Weights of asset i,s at tp

dev_{tp} denotes the deviation from the $target$ at tp

$sslack_{s,tp}$ denotes the shortfall of s at tp

$return_{s,tp}$ denotes the portfolio's return on each time period

$portfolioreturn_{tp}$ denotes the total return of the portfolio

Objective function:

$$Max \sum_{tp=1}^{3} portfolioreturn_{tp} * lambda - (1 - lambda) * \sum_{tp=1}^{3} dev_{tp}$$

The objective function of the model maximizes the portfolio's return in all three time periods but subtracts potential deviations from it. The 'multiplier' $lambda$, indicates the risk aversion of the user. If $lambda = 1$ then the model is transformed to a maximum risky one, which is an indication that the user is interested only in maximising the portfolio's return and he is not interested in the scenarios deviation from the target.

The objective function can be satisfied subject to the following constraints

Weights Balance Constraints Group:

C1: $\sum_{i=1}^{59} W_{i,s,tp} = 1 \qquad \forall_{s,tp}$

C2: $W_{i,s,tp=1} = Wo_i + B_{i,s,tp=1} - S_{i,s,tp=1} \qquad \forall_{i,s}$

C3: $W_{i,s,tp} = W_{i,s,tp-1} + B_{i,s,tp} - S_{i,s,tp} \qquad tp = 2,\ldots,T \ , \forall_{i,s}$

The weights constraints group is very important as it is balancing the weights throughout the modelling procedure. Constraint 1, (C1), makes the summation of all scenarios and time periods of the shares weights equal to the unity. C2 and C3 ensure that the new weights depend on what it is sold and bought.

Utility Constraint Group:

C4: $return_{s,tp} = \sum_{i=1}^{59} r_{i,s,tp} * W_{i,s,tp} - \sum_{i=1}^{59} B_{i,s,tp} * TC - \sum_{i=1}^{59} S_{i,s,tp} * TC \qquad \forall_{s,tp}$

C5: $portfolioreturn_{tp} = \sum_{s=1}^{92} return_{s,tp} * P_s \qquad \forall_{tp}$

Constraint C4 ensures that the return reflects any losses incurred for trading. Hence, the two terms of transactions costs. C5 denotes the total return of the portfolio.

Target-Return Constraints Group:

C6: $dev_{tp} = \sum_{s=1}^{92} sslack_{s,tp} * P_s \qquad \forall_{tp}$

C7: $return_{s,tp} - L_{tp} + sslack_{s,tp} \geq t\arg et_{s,tp} \qquad \forall_{s,tp}$

C6, is the constraint that according to the target return of the model, performs a type of a downside risk aversion contribution. It looks at the deviation from a given target (expected downside risk). The $sslack$, is an actual measure of how much the model deviates from the target.

Constraint C7 contributes to the target-return concept of the model. It is the inequality between the $return$ plus the shortfalls of a given $target$ for all scenarios and time periods. This constraint measures the under performance from the target by considering the shortfalls $(sslack)$. The liabilities (L) are subtracted on the left hand side of the equality. It must be mentioned that in the comparison of the two models the liabilities are ignored. The Stochastic ALM programming model is run with the liabilities and is compared to the one without them.

Non-Anticipativity Constraints Group $(tp = 1)$:

C8: $W_{i,s=1,tp=1} = W_{i,s',tp=1}$ $\quad s' = 2,\ldots,S$

C9: $B_{i,s=1,tp=1} = B_{i,s',tp=1}$ $\quad s' = 2,\ldots,S$

The non-anticipativity constraints are applied to two-stage and multi-stage stochastic programs only. In this case they are applied for the weights and the amount bought. This set of constraints ensures that at the 1st time period all scenarios make the same decision.

Risk Constraints Group

C10: $$\sum_{i \in g_\zeta} W_{i,s,tp} \geq \rho_\zeta * \sum_{i=1}^{n} W_{i,s,tp} \;,\; tp = 1...T, s = 1...S, \zeta = 1...Z$$

The set of constraints from C10, are the risk groups constraints. RiskGrades™ introduced by RiskMetrics (2000) represents a new measure of volatility. It is a risk indicator based on the volatility of the returns relative to the major market indices. Typically, RiskGrades are measured in a scale from 0 to 1000. A value of 100 corresponds to an average RiskGrade, while a value of zero should correspond to cash, and a value greater than 100 corresponds to riskier than the market assets. RiskGrades can vary over time, and, thus, can help investors to dynamically monitor their exposure to the market risk.

Based on the RiskGrades methodology we introduce the *RiskGroups* constraint, which works as follows. First, the risk grades of the asset universe, i=1..n, are computed. This can be relative to the major financial indices, similar to the RiskGrades approach, or relative to any other index individually, similar to what it is followed in this study. The assets are then sorted and classified in Z different risk groups, g_ζ, relative to their RiskGrade value. Let $g_\zeta \subseteq n$ for ζ=1...Z, and $\bigcup_\zeta g_\zeta = n$, with $g_j \cap g_\zeta \neq 0$, $\forall(j,\zeta)$, then the RiskGroup constraint can be incorporated in the model in the form of a threshold constraint, that bounds each risk group to constitute a pre-specified ρ % of the portfolio.

ζ is split into 5 different risk groups. According to the market Index, the S&P 100 in this case, the volatility of the shares and the Index's were computed and then the shares were separated into the 5 groups. These groups control the risk of the portfolio. By having the five different risk groups the volatility of the portfolio is kept levelled according to the Index. A pre-specified ρ % threshold has been allocated and is 0.2. It has been kept constant so as shares can equally be selected from the lower to the highest risk group.

The investor's perspective to risk is employed by using five different risk profiles: 'Low' risk profile corresponds to the utilization of low risk and is achieved in both QP and SP by using the multiplier '$lambda$'. For the 'Low' risk profile $lambda$ equals 0. The other risk profiles are 'Medium-Low', 'Medium', 'Medium-High' and 'Max' with $lambda$ to be equal to 0.25, 0.5, 0.75 and 1 respectively. By utilising this multiplier one can eliminate the aggressiveness of the model. By using $lambda$ =0 (or Low risk profile), the model goes for the lowest return possible. On the other hand by setting $lambda$ =1 (or Max risk profile), the model has the maximum risk possible hence the highest return.

In **APPENDIX II** the MPL formulation of the QP and SP is illustrated.

6.5 SP versus QP Results

This section provides the results of the two models. A comparison between the QP and the SP is performed. In the tables below, the results present the portfolio's returns (in percentage) of each model together with the standard deviation recorded for the period of August 1987 to July 1989. The first table illustrates the QP and SP's results with the risk groups constraints and the second table without them.

Portfolio returns (%) with Risk Groups constraints for QP and SP													
Risk Profiles	Low		Med-Low		Med		Med-High		Max		S&P 100	Long Int.Rates	Short Int.Rates
	Ret	STDV	Ret	STDV	Ret	STDV	Ret	STDV	Ret	STDV			
					QP								
Aug '97	-3.66	2.08	1.87	3.86	2.41	4.16	2.67	5.67	4.84	6.54	**0.9**	**1.06**	**0.83**
Oct '97	-3.55	2.09	-2.67	3.77	-1.19	4	1.21	4.99	1.43	6.36	**0.51**	**1**	**0.86**
Dec '97	0.52	2.1	1.19	3.8	1.3	4.08	2.87	4.98	5.82	6.26	**5.31**	**0.98**	**0.85**
Feb '98	9.45	3.79	12.9	4.24	12.97	4.32	11.03	5.08	13.3	6.42	**9.12**	**0.99**	**0.83**
Apr '98	-7.18	3.49	-4.55	4.09	-3.82	4.19	-3.7	4.79	-2.57	6.96	**6.88**	**0.95**	**0.83**
Jun '98	-8.32	3.63	-8.23	4.64	-8.1	4.86	-5.31	5.05	1.43	6.02	**3.37**	**0.92**	**0.82**
					SP								
Aug '97	4.66	3.73	6.38	4.56	8.93	5.33	11.18	6.98	12.73	7.23			
Oct '97	0.13	2.56	1.44	4.44	2.91	4.87	6.13	6.52	10.51	7.14			
Dec '97	0.41	2.69	0.85	3.81	1.27	4.47	4.42	5.75	6.76	6.88			
Feb '98	4.84	3.94	10.082	4.8	10.93	5.95	10.94	6.87	11.009	7.19			
Apr '98	0.23	2.5	1	4.1	1.85	4.73	2.6	5.42	2.65	6.45			
Jun '98	-1.55	2.94	0.24	4.08	2.09	4.84	3.34	5.66	6.1	6.74			

Table 6-2: Results of QP and SP without the risk groups constraints

Portfolio returns (%) with no Risk Groups constraints for QP and SP													
Risk Profiles	Low		Med-Low		Medium		Med-High		Max		S&P 100	Long Int.Rates	Short Int.Rates
	Ret	STDV	Ret	STDV	Ret	STDV	Ret	STDV	Ret	STDV			
					QP								
Aug '97	-3.78	3.66	-1.21	5.12	0.52	5.19	1.37	7.08	2.86	7.98	**0.9**	**1.06**	**0.83**
Oct '97	-3.21	3.52	-2.46	5.28	-1.27	5.45	0.55	6.87	1.38	7.92	**0.51**	**1**	**0.86**
Dec '97	0.83	3.7	1.17	5.04	1.47	5.33	2.31	7.32	6.25	8.51	**5.31**	**0.98**	**0.85**
Feb '98	10.04	4.23	10.15	5.84	10.88	6.06	10.92	7.87	11.24	8.87	**9.12**	**0.99**	**0.83**
Apr '98	-6.27	4.04	-5.64	5.31	-3.29	5.72	-3.22	7.55	-2.12	8.14	**6.88**	**0.95**	**0.83**
Jun '98	-9.44	4.18	-8.36	5.64	-7.21	5.87	-5.37	7.65	-3.19	8.31	**3.37**	**0.92**	**0.82**
					SP								
Aug '97	4.71	4.67	5.71	5.41	6.33	6.22	10.57	7.66	8.31	8.41			
Oct '97	0.1	3.99	1.27	5.16	2.88	6.14	6.15	7.39	10.49	8.56			
Dec '97	0.56	4.22	0.81	5.07	1.27	5.89	5.11	7.27	6.76	8.39			
Feb '98	4.58	4.52	10.44	5.76	10.67	6.31	10.82	7.83	10.97	8.68			
Apr '98	0.17	4.17	1.32	5.24	1.53	5.77	2.54	7	2.74	8.07			
Jun '98	-1.67	4.41	0.17	5.02	2.18	6.07	3.44	7.07	5.82	8.31			

Below is a table illustrating how the shares are split into the five different risk groups. A technique analysed in the SP's algebraic formulation.

Table 6-3: Risk Groups (August 1987 – June 1989)

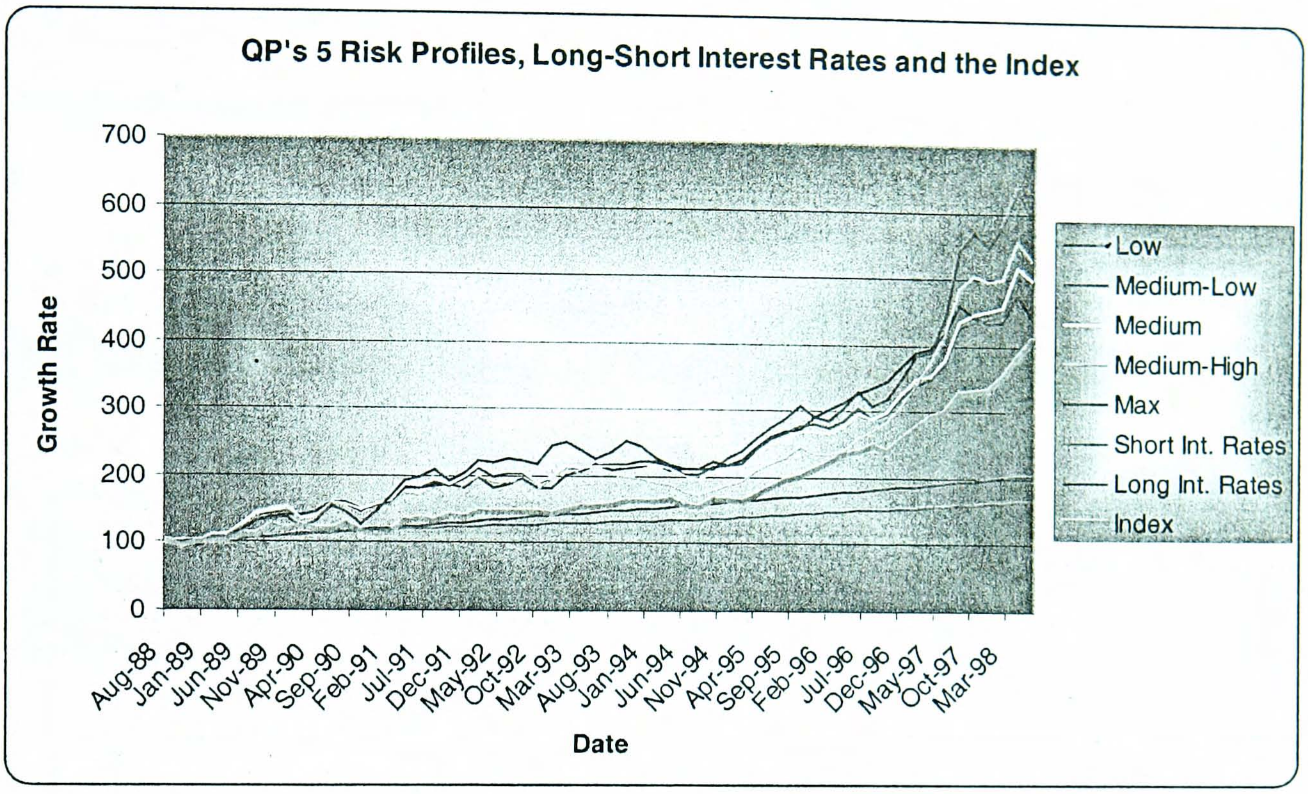
Low		Medium-Low		Medium		Medium-High		High	
Share	Volatility	Share	Volatility	Share	Volatility	Share	Volatility	Share	Volatility
51	87.15256	19	141.146	7	161.5975	12	179.3295	52	222.5819
50	102.4869	30	144.7707	18	163.201	17	182.84	16	227.3929
23	111.7686	13	151.225	6	163.4331	2	187.6225	31	227.9387
42	122.2563	57	152.5281	35	163.8707	56	195.0529	59	232.6694
28	124.2831	34	153.6292	58	163.9077	53	203.0629	48	237.9279
20	125.0309	37	155.8044	3	171.0793	8	207.7655	32	239.0175
24	126.0978	21	157.6729	11	171.1204	15	210.1896	55	241.2076
4	128.4727	41	158.2573	36	171.3694	46	214.7432	43	250.8758
5	129.5509	27	160.8636	49	172.1181	10	217.0404	40	269.1823
25	132.2664	39	161.0268	1	172.2439	44	219.4965	47	273.5209
14	140.4589	9	161.095	33	174.2912	54	219.5056	22	285.0503
		26	161.2262	45	178.6979	38	220.4419	29	376.1866

In the following table the allocation of the shares that correspond to table 6-3 is presented. August 1987 is considered to show how shares are allocated-split when the risk groups are utilised and when the risk groups are not utilised (Basic QP, SP).

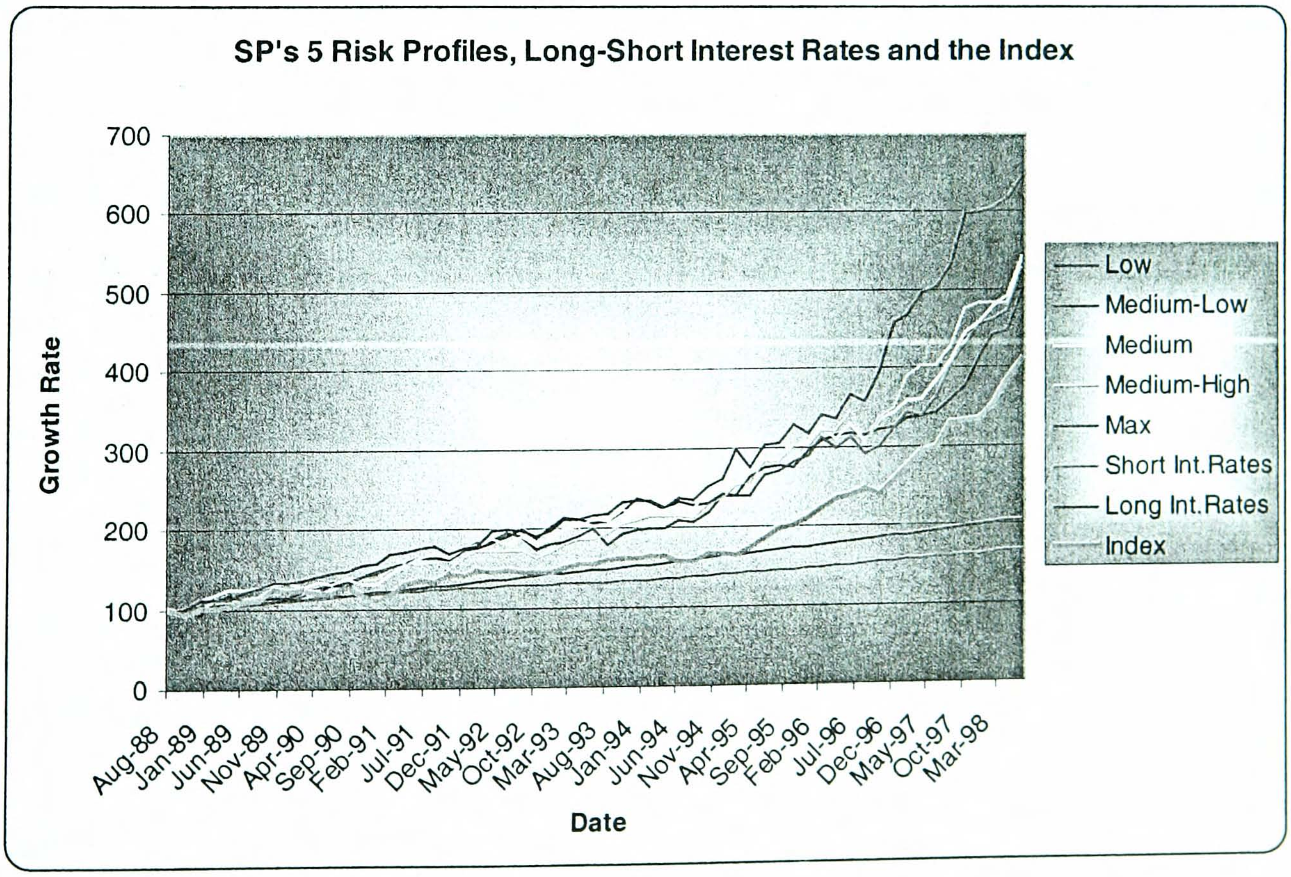
Table 6-4: August 1987: The allocation of shares in the Portfolio and Risk Groups

August 1987 - Number of shares allocated in Portfolio							*Basic QP*	*Basic SP*
Risk Groups	*Low*	*Med-Low*	*Medium*	*Med-High*	*Max*	*Total*		
Risk Profiles			**QP**					
Low	7	5	4	2	1	19	10	
Med-Low	4	3	2	1	1	11	8	
Medium	4	2	1	1	1	9	5	
Med-High	2	2	2	1	1	8	4	
Max	1	1	1	1	1	5	1	
			SP					
Low	4	4	4	2	1	15		9
Med-Low	4	3	3	1	1	13		7
Medium	3	3	3	1	1	11		4
Med-High	3	2	2	2	1	10		4
Max	1	1	2	2	1	7		1

The two graphs following show the QP and SP with the Index and the interest rates growth rates for the planning horizon of ten years.



Graph 6-1: QP, Interest rates and the Index



Graph 6-2: SP, Interest rates and the Index

Below is a representation from an investor's point of view. In the case that he/she invested £100 at the beginning of the planning horizon and wants to see the wealth by comparing the two models the Index and the Interest rates. BQP and BSP stands for 'Basic' that is without the risk constraints group in the models formulations.

Table 6-5: Wealth after 10 years of £100 invested

	QP	*SP*	*BQP*	*BSP*	*Index*	*Short*	*Long*
Low	306.4	390.1	301.6	378.1	**323.89**	**69.3**	**107.8**
Med-Low	349.1	444.3	337.7	436.8			
Med	358.7	474.5	354.4	468.6			
Med-High	402.9	497.3	405.9	499.5			
Max	500.2	548	509.5	512.9			

From the above results it is clear that the SP outperforms the QP. The SP has a fraction higher volatility than the QP but for the reason that it utilises scenarios makes it more coherent and reliable for forecasting. The risk groups play an important role in controlling the risk. It can be seen that it spreads out the allocation of shares resulting in not only choosing shares from all the risk groups but also creating a less volatile environment. In the QP model there is a large number of shares in the portfolio in the low risk profile but as we move up the risk profile scale they get fewer. This is because the QP goes for shares with high returns whose number is small. On the contrary the SP although in the low risk profile it has less shares in the portfolio as it moves to different risk profiles it does pick more shares than the QP and spreads the volatility. One more reason is that it has a greater choice of shares to choose from the scenarios and that lies into the reliability of the generator.

Without the risk groups both models go for less shares. Towards the high risk profiles shares in the portfolio drop dramatically with unfortunate results to the volatility being at its peak as there is no risk control. The portfolio's return in the Medium-High and Max risk profiles is greater as is not what an investor would prefer despite the higher returns. He/she prefers less return but controllable risk in all cases.

6.6 Liabilities

This section illustrates the results of the stochastic programming model run with the same set of scenarios but in two different 'versions'. The first version is without liabilities and the second is when liabilities are utilized in the model's framework. The results are presented in percentage returns.

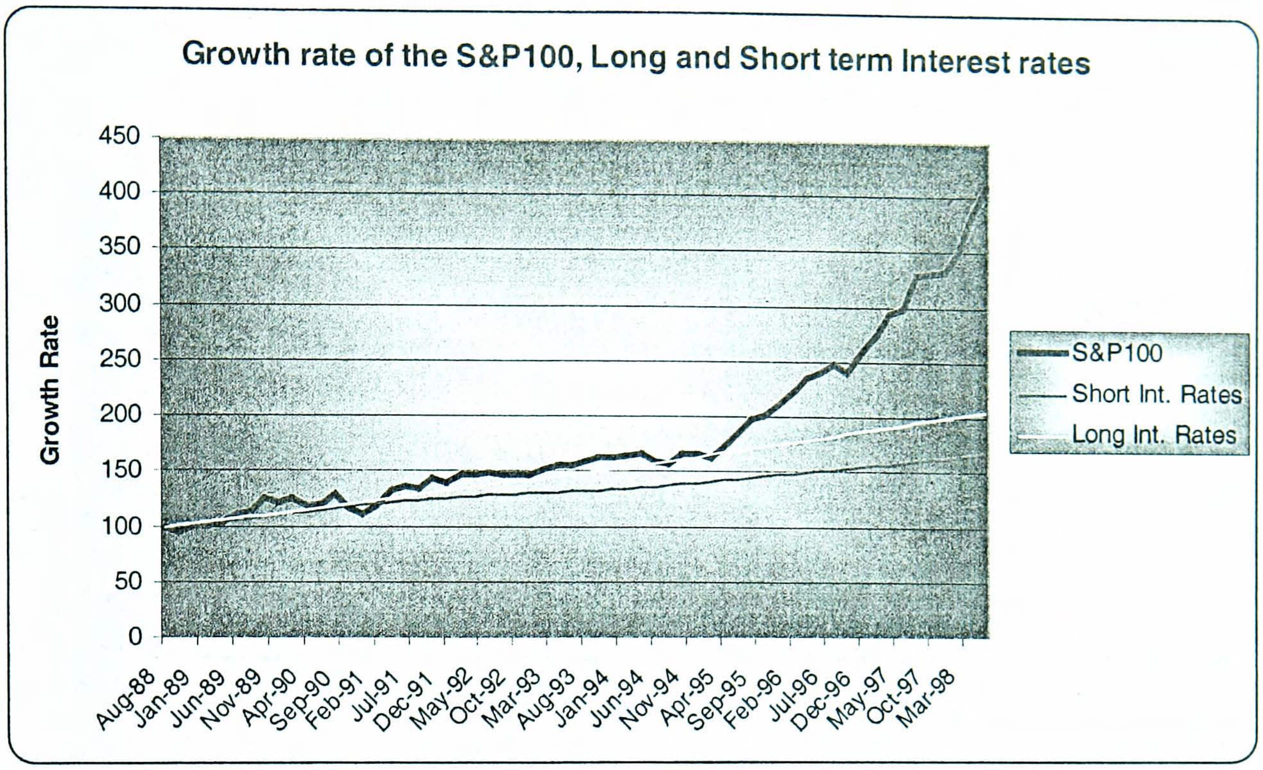
Table 6-6: Liabilities of SP

Dates	*Aug '97*	*Oct '97*	*Dec '97*	*Feb '98*	*Apr '98*	*Jun '98*
	SP model returns (%) without liabilities					
Risk Profiles						
Low	4.66	0.13	0.41	4.84	0.23	-1.55
Med-Low	6.38	1.44	0.85	10.082	1	0.24
Medium	8.93	2.91	1.27	10.93	1.85	2.09
Med-High	11.18	6.13	4.42	10.94	2.6	3.34
Max	12.73	10.51	6.76	11.009	2.65	6.1
	SP model returns (%) with liabilities					
Low	2.66	-1.87	-1.59	2.84	-1.77	-3.55
Med-Low	4.38	-0.56	-1.15	8.082	-1	-1.76
Medium	6.93	0.91	-0.73	8.93	-0.15	0.09
Med-High	9.18	4.13	2.42	8.94	0.6	1.34
Max	10.73	8.51	4.76	9.009	0.65	4.1

The liabilities are assumed to be constant throughout the planning horizon of this study. Because of the reason that the data of the SP model are returns it is assumed that liabilities are 2% or have a return of 0.02 for a period of twelve months. From the above graph it is clear that the performance of the SP has been reduced due to the liabilities.

6.7 S&P 100 Share Index, Long and Short term Interest Rates

The use of the S&P 100 stock Index in this study is twofold. First, it is used as a benchmark to create the corresponding risk groups for the shares. Second, it is used for performance comparison purposes with the QP and SP. The graph below illustrates the growth rate of the Index, long and short-term interest rates for the underlying study period.



Graph 6-1: Growth rates of Index, Short and Long Interest Rates

The next table shows the Index, Short and Long term interest rates returns in percentage.

When the value of the market rises and the Interest rates drop then the two models do rise as well. They both make profits but with the following pattern. The models pick the largest amount of shares when the Low risk profile is utilised but as far as the risk groups is concerned, the majority of shares are distributed within the Low, Medium-Low and Medium risk groups. However, when the Index drops and Interest rates rise then shares take a rather aggressive swing on picking shares from the Medium-High and Max risk groups as they try to compensate the Index's losses and make profits and finally outperform its profile. This can be seen clearly in Graphs 6-1 and 6-2.

The interest rates do not fluctuate as much as the Index's and obviously the models paths. Short Interest rates are less volatile than Long Interest rates. They generally follow the market's pattern but in some cases as seen in Graph 6-3 they move the opposite way. When they do drop though shares 'prefer' the higher risk groups just like when the Index drops.

Table 6-7: Returns (%) of the S&P 100, Long and Short Interest Rates

	S&P 100						Short Term Interest Rates						Long Term Interest Rates					
	Aug	Oct	Dec	Feb	Apr	Jun	Aug	Oct	Dec	Feb	Apr	Jun	Aug	Oct	Dec	Feb	Apr	Jun
1988	-5.16	6.48	-0.1	3.85	6.22	1.3	1.23	1.35	1.42	1.44	1.36	1.32	1.48	1.5	1.5	1.51	1.38	1.35
1989	10.95	-2.81	3.57	-5.12	0.56	8.13	1.27	1.27	1.29	1.30	1.29	1.24	1.33	1.32	1.42	1.46	1.41	1.48
1990	-9.82	-5.4	7.11	12.12	2.1	-0.65	1.2	1.12	0.99	0.94	0.93	0.89	1.48	1.37	1.34	1.37	1.41	1.36
1991	5.77	-1.44	4.74	-0.16	1.18	-1.46	0.83	0.68	0.64	0.63	0.61	0.52	1.32	1.28	1.31	1.33	1.31	1.23
1992	0.14	-0.44	3.66	2.77	-0.13	2.36	0.48	0.54	0.49	0.48	0.51	0.5	1.26	1.24	1.18	1.14	1.14	1.05
1993	2.77	-0.03	0.29	1	-4.3	-1.16	0.5	0.51	0.54	0.61	0.69	0.75	0.99	1.04	1.08	1.21	1.23	1.25
1994	6.62	0.07	-2.09	6.29	7.03	6.21	0.83	0.93	0.96	0.94	0.91	0.9	1.32	1.31	1.27	1.23	1.1	1.14
1995	2.41	4.78	5.4	5.31	2.17	2.7	0.88	0.86	0.81	0.83	0.85	0.84	1.06	1.01	1.04	1.13	1.18	1.14
1996	-2.76	8.11	5.77	6.74	2.23	9.6	0.83	0.82	0.84	0.86	0.82	0.86	1.14	1.09	1.12	1.18	1.13	1.1
1997	0.9	0.51	5.31	9.12	6.88	3.37	0.83	0.86	0.85	0.83	0.83	0.82	1.06	1	0.98	0.99	0.95	0.92
1998	-5.16	6.48	-0.1	3.85	6.22	1.3	1.23	1.35	1.42	1.44	1.36	1.32	1.48	1.5	1.5	1.51	1.38	1.35

Chapter 7: CONCLUSIONS - FUTURE DIRECTIONS

This thesis is concerned with Asset and Liability Management (ALM) modelling by using stochastic programming.

The research extends the sceptic of asset pricing models and focuses on how to model the uncertainty. Processes like the 'Wiener' or the Geometric Brownian Motion are discussed. Simulation techniques like the Monte Carlo are analysed. That kind of simulation procedure is one of the most widely used throughout the financial and banking sectors, as it is a straightforward method. Furthermore, econometric techniques that represent the uncertainty are introduced. Econometric models capture the behaviour of macroeconomic and microeconomic variables. These models are used when one wants to incorporate factors like interest rates, inflation and others within an econometric framework-model in order to gain market consistency. Scenario generation, which is a sophisticated mathematical procedure of capturing the uncertainty is thoroughly discussed. The author's own scenario generator is analysed and random shares scenario samples are illustrated. Scenarios were also generated for the interest rates by utilizing an existing interest model. Capturing the uncertainty by generating scenarios is a very important aspect of the ALM framework as it is a mirror towards the optimal decisions. The scenario generator developed in this thesis is novel. Its contribution is:

1. The Brownian motion stochastic process is used to generate the market scenarios by using the Monte Carlo simulation. The integration of the market and interest rates scenarios comes together in the CAPM formula after the 'betas' correlation is found. The same historical set was used to find the 'betas' and generate the market scenarios. The same market forecast is linked in every time period of the planning horizon with the shares 'betas' and interest rates. This process helps to get consistent scenarios between shares, market, interest rates and 'betas'.

2. Data (scenarios) are generated such as they are already in a database format and directly linked with the optimisation model.

3. Confidence intervals analysis is utilized for validation and robustness of scenarios.

Controlling the risk is an important issue in financial modelling. In this study this is performed in a novel way. The risk groups are employed to control the risk and allocate shares in the portfolio based on the volatility of the returns relative to the market Index. By introducing the risk groups into the models shares in the portfolio do not have large risk measures like standard deviation. A stochastic programming ALM model was used to encapsulate the above concepts and get optimum investment decisions. A mean-variance model has also been used to compare and test its performance versus the stochastic for over a period of ten years for shares that belong to the S&P 100 share Index. Different investor's risk perceptions are employed to accomplish this within a 'forward rolling' framework, which back-tests the performance of the portfolios.

The stochastic programming framework outperforms and exhibits better results than the mean-variance model. This confirms that for the current study the use of stochastic programming in financial planning is superior to other types of models such as the mean-variance in this case. When the two different model methodologies are compared with the S&P 100 share Index they both outperform the Index.

Future considerations should include research of other risk measures like the CVAR methodology that could possibly be employed into the model. Furthermore, the utilization of econometric models to research how factors like inflation, Consumer Price Index and interest rates are correlated and put into the scenario generator. This would lead into the flexibility of the generator to create scenarios for different countries. Another topic that is going to be researched and considered in the future is the liabilities. The author will consider generating liabilities stochastically. This is more realistic in real life applications where one does not know his/hers future payment obligations with certainty.

APPENDIX I

1. GEN.DYNAMICS	21. HOMESTAKE MNG.	41. XEROX
2. ALCOA	22. BLACK & DECKER	42. MCDONALDS
3. GENERAL MOTORS	23. BRISTOL MYERS SQUIBB	43. MAY DEPT.STORES
4. COCA COLA	24. JOHNSON & JOHNSON	44. K MART
5. PEPSICO	25. MERCK	45. SEARS ROEBUCK
6. DOW CHEMICALS	26. BAXTER INTL.	46. WAL MART STORES
7. INTL.FLAV.& FRAG.	27. COLGATE-PALM.	47. LIMITED
8. HARRIS	28. PROCTER & GAMBLE	48. DELTA AIR LINES
9. NORTEL NETWORKS (NYS)	29. BRUNSWICK	49. BURLINGTON NTHN.
10. HEWLETT-PACKARD	30. MINNESOTA MNG.& MNFG.	50. AMER.ELEC.PWR.
11. GEN.ELEC.	31. UNITED TECHNOLOGIES	51. SOUTHERN
12. HONEYWELL	32. HALLIBURTON	52. WILLIAMS COS.
13. ROCKWELL INTL.NEW	33. SCHLUMBERGER	53. BANK ONE
14. RAYTHEON 'B'	34. OCCIDENTAL PTL.	54. US BANCORP DEL.
15. TEKTRONIX	35. BOISE CASCADE	55. BANK OF AMERICA
16. TEXAS INSTS.	36. INTL.PAPER	56. AMER.EXPRESS
17. DISNEY (WALT)	37. WEYERHAEUSER	57. AMER.GENERAL
18. CAMPBELL SOUP	38. AVON PRODUCTS	58. AMER.INTL.GP.
19. HEINZ HJ	39. EASTMAN KODAK	59. MERRILL LYNCH
20. RALSTON PURINA	40. POLAROID	

APPENDIX II

Quadratic Programming - Initial - model's MPL formulation:

```
TITLE
Initial_mean_variance;

INDEX

i = DATABASE(returns,rowid);
j = DATABASE(returns,colid);


DATA

Return[i]:= DATABASE(returns,"returns");
Cov[i,j]:= DATABASE(covariance,"cov");
TC:= EXCELRANGE("Markowitz results.xls",transaction_costs);
lamda:= EXCELRANGE("Markowitz results.xls",risk_profile);
gamma:= 1 - lamda;
tar_return:= EXCELRANGE("Markowitz results.xls",tar_return);
relationA[i]:=EXCELLIST("Markowitz results.xls",relationIA);
relationB[i]:=EXCELLIST("Markowitz results.xls",relationIB);
relationC[i]:=EXCELLIST("Markowitz results.xls",relationIC);
relationD[i]:=EXCELLIST("Markowitz results.xls",relationID);
relationE[i]:=EXCELLIST("Markowitz results.xls",relationIE);

DECISION

W[i] EXPORT TO EXCELRANGE ("Markowitz results.xls","weight");
B[i];
S[i];
R;
holdings;
risk_one_holdings;
risk_two_holdings;
risk_three_holdings;
risk_four_holdings;
risk_five_holdings;

MODEL

Min portfolio=gamma * sum(i,j: W[i]*Cov[i,j]*W[i:=j]) - lamda* R;

SUBJECT TO

Weight:Sum(i:W[i])= 1;

Transactions:R=Sum(i:W[i]*Return[i])-sum(i:TC*B[i])-sum(i:TC*S[i]);
```

```
!RISK GROUPS

all_holdings:Sum(i:W[i])=holdings;

group_one_holdings:SUM(i:W[i] where (relationA[i]=1))=risk_one_holdings;

group_two_holdings:SUM(i:W[i] where (relationB[i]=1))=risk_two_holdings;

group_three_holdings:SUM(i:W[i] where (relationC[i]=1))=risk_three_holdings;

group_four_holdings:SUM(i:W[i] where (relationD[i]=1))=risk_four_holdings;

group_five_holdings:SUM(i:W[i] where (relationE[i]=1))=risk_five_holdings;

b1:0.2*holdings<=risk_one_holdings;

b2:0.2*holdings<=risk_two_holdings;

b3:0.2*holdings<=risk_three_holdings;

b4:0.2*holdings<=risk_four_holdings;

b5:0.2*holdings<=risk_five_holdings;

End
```

Quadratic Programming - Rolling - model's MPL formulation:

```
Title
Quadratic_R1;

INDEX

i = DATABASE(returns,rowid);
j = DATABASE(returns,colid);

DATA

Return[i]:= DATABASE(returnsR1,"returns");
Cov[i,j]:= DATABASE(covarianceR1,"cov");
Wo[i]:= DATABASE(wI,"weights");
TC:= EXCELRANGE("Markowitz results.xls",transaction_costs);
lamda:= EXCELRANGE("Markowitz results.xls",risk_profile);
gamma:= 1 - lamda;
tar_return:= EXCELRANGE("Markowitz results.xls",tar_return);
relationA[i]:=EXCELLIST("Markowitz results.xls",relation1A);
relationB[i]:=EXCELLIST("Markowitz results.xls",relation1B);
relationC[i]:=EXCELLIST("Markowitz results.xls",relation1C);
relationD[i]:=EXCELLIST("Markowitz results.xls",relation1D);
relationE[i]:=EXCELLIST("Markowitz results.xls",relation1E);

DECISION

W[i]EXPORT TO EXCELRANGE ("Markowitz results.xls","weightR1");
B[i];
S[i];
R;
holdings;
risk_one_holdings;
risk_two_holdings;
risk_three_holdings;
risk_four_holdings;
risk_five_holdings;

MODEL

Min gamma * sum(i,j: W[i]*Cov[i,j]*W[i:=j]) - lamda * R;

SUBJECT TO

Weight:Sum(i:W[i]) = 1;

Transactions:R = Sum(i:W[i]*Return[i])-sum(i:TC*B[i])-sum(i:TC*S[i]);

ExportWeights[i]:  W[i]=Wo[i]+B[i]-S[i];
```

RISK GROUPS

```
all_holdings:Sum(i:W[i])=holdings;

group_one_holdings:SUM(i:W[i] where (relationA[i]=1))=risk_one_holdings;

group_two_holdings:SUM(i:W[i] where (relationB[i]=1))=risk_two_holdings;

group_three_holdings:SUM(i:W[i] where (relationC[i]=1))=risk_three_holdings;

group_four_holdings:SUM(i:W[i] where (relationD[i]=1))=risk_four_holdings;

group_five_holdings:SUM(i:W[i] where (relationE[i]=1))=risk_five_holdings;

b1:0.2*holdings<=risk_one_holdings;

b2:0.2*holdings<=risk_two_holdings;

b3:0.2*holdings<=risk_three_holdings;

b4:0.2*holdings<=risk_four_holdings;

b5:0.2*holdings<=risk_five_holdings;

Bounds

S[i]<=Wo[i];

END
```

Stochastic Programming model's MPL formulation:

```
TITLE

Model_Stochastic;

DATA

lamda=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",lamda);
A=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",assets);
S=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",scenarios);
T=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",timeperiods);
gamma = 1-lamda;
relationA[i]:=EXCELLIST("Markowitz results.xls",relation1A);
relationB[i]:=EXCELLIST("Markowitz results.xls",relation1B);
relationC[i]:=EXCELLIST("Markowitz results.xls",relation1C);
relationD[i]:=EXCELLIST("Markowitz results.xls",relation1D);
relationE[i]:=EXCELLIST("Markowitz results.xls",relation1E);

INDEX

shares=1..A;
scenarios=1..S;
tp=1..T;

DATA

r[tp,shares,scenarios]=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",Generic_Return);
Wo[shares]=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",Generic_Holdings);
TC=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",Transaction_Costs);
P=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",probability);
target[tp>1]=EXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",Generic_Risk);
L[tp>1]=(0.05,0.05);

DECISION VARIABLES

W[tp,shares,scenarios];
Buy[tp,shares,scenarios];
Sell[tp,shares,scenarios];
sslack[tp,scenarios];
surplus[tp,scenarios];
dev[tp];
return[tp,scenarios];
portfolioreturn[tp];
firststagehold[tp=1,shares]EXPORTTOEXCELRANGE("C:\WINNT\Profiles\Administrator\Desktop\George-Inputs.xls",Results);
```

```
holdings;
risk_one_holdings;
risk_two_holdings;
risk_three_holdings;
risk_four_holdings;
risk_five_holdings;

MODEL

Max  value = lamda * Sum(tp:portfolioreturn) - Sum(tp:gamma*dev);

SUBJECT TO

Weightstotal[scenarios,tp]:Sum(shares:W[tp,shares,scenarios])=1;

Weightsbalance[tp=1,shares,scenarios]:W[tp=1,shares,scenarios]=Wo[shares]+Buy[tp=1,shares,
scenarios]-Sell[tp=1,shares,scenarios];

Weightsbalance[tp>1,shares,scenarios]:W[tp>1,shares,scenarios]=W[shares,scenarios,tp-
1]+Buy[tp>1,shares,scenarios]-Sell[tp>1,shares,scenarios];

Util[scenarios,tp]:Sum(shares:r*W)-sum(shares:TC*Buy)-sum(shares:TC*Sell)=return;

Totalwealth[tp]:Sum(scenarios:P*return[scenarios,tp])=portfolioreturn[tp];

Deviation[scenarios,tp]:return-L+sslack=target+surplus;

Totaldev[tp]:Sum(scenarios:P*sslack[scenarios,tp])=dev[tp];

firstholdings[tp=1,shares]:SUM(scenarios:P*W)=firststagehold;

na1[shares,scenarios>1,tp=1]:W[scenarios]=W[scenarios-1];

na2[shares,scenarios>1,tp=1]:Buy[scenarios]=Buy[scenarios-1];

RISK GROUPS

all_holdings:Sum(i:W[i])=holdings;

group_one_holdings:SUM(i:W[i] where (relationA[i]=1))=risk_one_holdings;

group_two_holdings:SUM(i:W[i] where (relationB[i]=1))=risk_two_holdings;

group_three_holdings:SUM(i:W[i] where (relationC[i]=1))=risk_three_holdings;

group_four_holdings:SUM(i:W[i] where (relationD[i]=1))=risk_four_holdings;

group_five_holdings:SUM(i:W[i] where (relationE[i]=1))=risk_five_holdings;

b1:0.2*holdings<=risk_one_holdings;

b2:0.2*holdings<=risk_two_holdings;

b3:0.2*holdings<=risk_three_holdings;
```

```
b4:0.2*holdings<=risk_four_holdings;

b5:0.2*holdings<=risk_five_holdings;

Bounds

Sell[tp=1,shares,scenarios]<=Wo;

END
```

APPENDIX III

This section illustrates the end-effects methodology in an energy planning model.

```
TITLE

End_Effects;

DATA

T=2;
alpha=0.9;
C[Plant]=(29  ,38);
K[Plant]=(0.5 , 1);
A[Plant]=( 1 , 1);
b[t]=(100,105,110.25);
M=10000;

INDEX

Plant= ("AA","BB");
t= 0..T;

DECISION

x[Plant,t];
z[t];
g[t];

MODEL

Min  SUM(t=0,Plant:  C*x)+SUM(t=1,Plant:  alpha*C*x)+SUM(t=2,Plant:  alpha*alpha*C*x)+
SUM(t: M*z);

SUBJECT TO

link[t=0]: SUM(Plant: A*x[t]) +z[t] - g[t] = b[t];

link[t=1]: SUM(Plant: K*x[t-1]) + SUM(Plant: A*x[t])  +z[t] - g[t] = b[t];

link[t=T]: SUM(Plant: K*x[t-1]) + SUM(Plant: A*x[t]+K*alpha*x[t])  +z[t] - g[t] = b[t];

BOUNDS

x[t]<=50;

END
```

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