Adaptive Motion Synthesis and Motor Invariant Theory

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Yet to be decided

This work is dedicated to my loving parents, who are always ready to sacrifice everything for me.

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Abstract

Generating natural-looking motion for virtual characters is a challenging research topic. It becomes even harder when adapting synthesized motion to interact with the environment. Current methods are tedious to use, computationally expensive and fail to capture natural looking features. These difficulties seem to suggest that artificial control techniques are inferior to their natural counterparts.

Recent advances in biology research point to a new motor control principle: utilizing the natural dynamics. The interaction of body and environment forms some patterns, which work as primary elements for the motion repertoire: Motion Primitives. These elements serve as templates, tweaked by the neural system to satisfy environmental constraints or motion purposes. Complex motions are synthesized by connecting motion primitives together, just like connecting alphabets to form sentences.

Based on such ideas, this thesis proposes a new dynamic motion synthesis method. A key contribution is the insight into dynamic reason behind motion primitives: template motions are stable and energy efficient. When synthesizing motions from templates, valuable properties like stability and efficiency should be perfectly preserved. The mathematical formalization of this idea is the *Motor Invariant Theory* and the preserved properties are *motor invariant*

In the process of conceptualization, new mathematical tools are introduced to the research topic. The Invariant Theory, especially mathematical concepts of equivalence and symmetry, plays a crucial role. Motion adaptation is mathematically modelled as topological conjugacy: a transformation which maintains the topology and results in an analogous system.

The *Neural Oscillator* and *Symmetry Preserving Transformations* are proposed for their computational efficiency. Even without reference motion data, this approach produces natural looking motion in real-time. Also the new motor invariant theory might shed light on the long time perception problem in biological research.

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Nomenclature

Roman Symbols

q	Generalized Coordinates
$\dot{q}$	Generalized Velocity
$\mathbf{x}$	State Variable
u	Control Input
g	Gravity
Q	Configuration Space or Configuration Manifold
TQ	Tangent Bundle of Q
M	State Space Manifold
TM	Tangent Bundle Manifold
$\mathcal{A}$	Attractor
$\mathcal{B}(\mathcal{A})$	Basin of Attraction of $\mathcal{A}$
$\simeq$	Topology Conjungacy
S	Neural Oscillator
u_{i}	Input Signal to the Neural Oscillator

u_{o} Output of the Neural Oscillator

h_{i} Input Coefficient of the Neural Oscillator

h_{o} Output Coefficient of the Neural Oscillator

G A Lie Group

g_a an element in Lie Group G with parameter a

$I(x)$ Invariant Function of x

ε The parameter of a lie group element

Acronyms

CMS Character Motion Synthesis

PD Proportional Derivative

LC Limit Cycle

CPG Central Pattern Generator

EPH Equilibrium Point Hypothesis

UMH Uncontrolled Manifold Hypothesis

CPG Central Pattern Generator

MoIT Motor Invariant Theory

DOF Degree of Freedom

BoA Basin of Attraction

Chapter 1

INTRODUCTION

1.1 The Challenge

Character Motion Synthesis (**CMS**) research aims at generating motion for virtual characters. It is a topic of significant value in terms of theory and application. Besides major applications in the media industry, where both computer games and animation films depend heavily upon character motion for storytelling, current research also has applications in user interface design, psychology, sport and medicine.

The challenge of **CMS** is not to make characters move, but to make them lifelike. Underlying this challenge is the marvellous human ability of motion perception. In real life, people's motion is very similar, yet individuals vary considerably. From the varieties in motion details, humans can infer mental states, health conditions or the surrounding environment. Human motion perception has some very peculiar properties. When watching a film with computer generated characters, some awkward artefacts are spotted instantly even though they are physically feasible, while many physically impossible motions are accepted as realistic and entertaining.

Nowadays in industry, high quality motions are mainly generated manually. Very often, characters are complex and contain a large number of joints, making animation tedious work. To make it worse, reusing motion animation is also difficult and prone to artefacts. Therefore high level animation tools are badly needed.

Real life motions interact extensively with the environment. Currently, the most important research endeavour is the physics based approach. Besides the addition of the dynamic interactive responses, it is expected that the elimination of artefacts that violates physics will make motions more natural looking. However, there is a key problem in applying this method for **CMS**: dynamics of biological systems are much more complex than artificial systems; attempts to dynamically simulate biological system face prohibitive computational costs and modelling difficulties. In fact, this problem has already been identified by biological researchers.

Motor Control and Motion Perception are close related. Difficulties in **CMS** reflect the inferiority of artificial control method. The peculiarity of motion perception and control suggests biological systems may adopt a very different principle. To keep motions natural looking, it is worthwhile to synthesize motion following the biological motor control principle. This thesis is founded on biological research findings.

1.2 Agile Animals

Although animals have fascinated us for thousands of years, we still do not fully understand how they move. Animals are very different from artificial machines and such comparisons may reflect the biological motor control principle.

- **Degrees of freedom (DOFs)**. From a mechanical perspective, animals have many more **DOF**s than their artificial counterparts. An artificial ship can be approximated by a simple rigid body; whereas the flexible spine of a fish is made up of tens of **DOF**s.

  In principle, the extra **DOF**s allows for more variations in adapting the environment. However, for the control system, too many extra **DOF**s become a disaster because of the computational burden. For a human to take one step, the neural system controls more than 600 muscles. Even with nowadays computers, solving this problem directly would cost thousands of hours(Anderson and Pandy, 2001).

- **Versatility** Most artificial machines are designed with a single purpose, while animals are capable of unlimited tasks. Many biological functions which are often neglected by **CMS** research, such as feeding, breeding, language and vision, depend on motor control. Besides walking, swimming and many other styles of locomotion, we utilize many tools, such as cars, skates, bicycles and tennis rackets.

Following traditional control methods, it seems that unlimited resources need to be allocated for motor control, while biological research shows motor control requires very few mental resources.

- **Performance** Although the problem of biological motor control is more complex, the resulting performance surpasses artificial machines in many aspects. Natural motions are more

  1. **Robust:** A human can maintain walking stability on rough terrains which would be inaccessible for vehicles.

  2. **Manoeuvrability and speed:** Typical modern aeroplanes travel at a maximum of $32\ body\ length/sec$ and yaw at $720\ deg/sec$. While pigeons may travel at $75\ body\ length/sec$, yaw at about $5000\ deg/sec$(Byl, 2008).

  3. **Energy Efficiency:** The energy consumed by a walking human is only 5% of that for the world famous humanoid ASIMO(Collins *et al.*, 2005).

1.3 Motor Invariant Theory

1.3.1 Utilizing Natural Dynamics

Biological motor control has achieved a delicate balance of robustness, controllability and energy efficiency. The real-time performance may further suggest that the biological method is simple and requires little computational load. These are the dreaming properties for **CMS** research and the explanation that how biological systems achieve this forms the genesis of this thesis.

At first, the natural dynamics of interactions between the body and environment is very complex. In most **CMS** research studies, some complex non-linear properties of natural dynamics are treated perturbations for planning, and are cancelled by control effort. However from an evolutionary perspective, the mechanical structures are a product of natural selection, which has evolved alongside with the environment for millions of years. These structures are an advantage rather than a handicap. Without the need to consider stability, energy efficiency and real-time constraints, motion can be synthesized by natural dynamics without any control effort. Thus a new idea is that motor control is based on natural dynamics. The neural system plays a minor role in planning; it simply utilizes natural dynamic properties. From this perspective, the key question to be answered by Motor Invariant Theory (**MoIT**) is how to utilize the natural dynamics in a systematic manner.

1.3.2 Motor Invariant Theory

This thesis proposes a new idea for the underlying reason for superiority of biological motor control. It seems that in the process of motion adaptation, some valuable properties of natural dynamics are kept invariant. The conjecture is that: instead of the detection and cancellation all kinds of perturbations, biological systems rely the success of motor control on certain invariant properties of natural dynamics. This is *Motor Invariant Theory(**MoIT**)*.

MoIT incorporates the motion primitive conjecture. In dynamics, invariant properties are stable properties. From a dynamic perspective, not all the motions generated by natural dynamics are stable, only a few are stable, which can be utilized as templates and become motion primitives. The following question is how the motor control system utilizes these templates to synthesize new motion.

MoIT proposes that when facing a new situation, humans do not solve motor control problems from the ground up. Instead, our control system utilizes successful experience in similar situations. In dynamics, adapted motions are qualitatively the same with the motion primitives or templates, and there is a one to one mapping relationship between the adapted motion and the motion primitive. This similarity in dynamics is called topological conjugacy.

This idea can be illustrate in Figure 1.1.

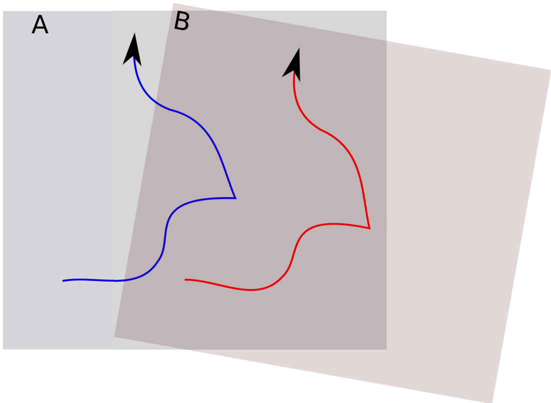


Figure 1.1: *The Transformation Idea of Motion Invariant Theory*

In dynamic **CMS** research, a motion is represented by a curve $x(t)$ parameterized by t. $x(t)$ is the solution to the equation (Equation 1.1) that describes the dynamics between the body and environment.

$$\dot{x} = F(x) \tag{1.1}$$

The state x must be defined in some coordinate system. Suppose it x is defined on coordinate system A, and the curve of Equation 1.1 is the blue(left) one.

To illustrate adaptation, we define a transformation T that translate the state value x.

$$\tilde{x} = T(x)$$

In this way, each equation can be described in two coordinate systems. Suppose x is the state value on coordinate A and $\tilde{x}$ is the state value on coordinate B. As an example, the red(right) curbe can be described by Equation 1.3 and Equation 1.2.

$$\dot{\tilde{x}} = F(\tilde{x}) \tag{1.2}$$

$$\dot{x} = \tilde{F}(x) \tag{1.3}$$

Since such two equations describe the same motion, the solution of one equation can be achieved by transforming the solution of the other. Supposing $x'(t)$ is the solution to Equation 1.3 and $\tilde{x}(t)$ is the solution to the Equation 1.2, then we have

$$x'(t) = T^{-1}(\tilde{x}(t))$$

Equation 1.2 and Equation 1.1 have the same F, thus:

$$\tilde{x}(t) = x(t)$$

Then we have

$$x'(t) = T^{-1}(x(t))$$

By transformation, we obtain a new motion $x'(t)$ from $x(t)$.

The transformation method has many advantages: it is much less computationally expensive and leaves many important properties untouched. For example, if the original system F is stable, then the transformed system $\tilde{F}$ should also be stable. In mathematical language, if there exists a continuous one-one mapping between the two dynamic systems, then the two are *topological conjugate*. This relationship is presented by $F \simeq \tilde{F}$. F and $\tilde{F}$ are called *analogous systems*, which share the same topological structure. The existence of one-one mapping is a necessary and sufficient condition for sharing topological structure. Based on this, two approaches for motion adaptation are developed. Transformation can be specified explicitly or implicitly by maintaining the topology.

If the perturbation does not violate the topology, the corresponding one-one mapping will modify the motion without changing it qualitatively. In dynamics, the topology preserving ability is an intrinsic property of many dynamic systems: *structural stability*.

One strategy of motor control is to enhance the structural stability. By this approach, when the qualitative property is preserved by the control system, the one-one mapping that transforms motions is automatically specified. However, in many cases, working out the details of one-one mapping maybe be difficult or computationally expensive.

Therefore this approach is qualitative.

In **MoIT**, this approach models involuntary motion adaptations which are low level functions of the neural control system. The topological structure is one important property that should be kept invariant, and it becomes a motor invariant in **MoIT**: the *Global Motor Invariant*.

Also if the transformation is known, then the two systems must be topologically equivalent. Therefore, another approach is to directly specify the transformation. This method modifies motion with precision and **MoIT** applies it to high level voluntary motor control. In many situations, to achieve a desired transformation T, control effort needs to be applied. When applying this method, how to select a proper transformation T is the most challenging question.

In **MoIT**, the selection of T is based on two principles.

- Parameters of transformation T should be easy to detect and formulate.
- The transformation T should be energy efficient. For a differential dynamic system, some transformation explores the natural dynamics and requires little or no energy input.

When specifying transformation directly, some quantitative properties will be unchanged during transformation, they are *Local Motor Invariant*. This idea is similar to motion parametrization, but there is a clear difference. Traditional motion parameterization paramterize motion curves in the configuration space, while in **MoIT**, transformations are applied on the dynamic system. The dynamic system are parameterized with a concern of energy efficiency and stability.

Although the new mathematical language seems obscure at first glance, the properties that it describes are universal in physical world, with or without life. The underlying idea is intuitive and can be explained well through commonly observed phenomena.

1.3.3 The Floating Ship: An Example of Stability

The floating ship example shows the idea of structural stability and topological conjugacy. In real life, typical ships have bigger height than width, as shown in Figure 1.2.

An interesting question is when floating on waves, how the ship maintains its configuration or "posture".

Through analysing the topology and structural stability, we see that it requires little effort to maintain this posture. This conclusion applies to different ships since their dynamics are qualitatively the same, or topologically conjugate.

Dynamics

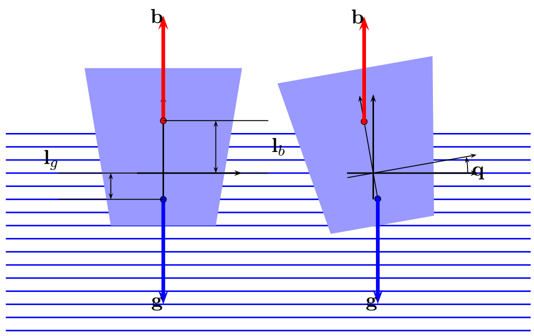


Figure 1.2: *The Floating Ship Example*

The sway motion of the ship shown in Figure 1.2 can be described by Equation 1.4

$$J\ddot{q} + d\dot{q} = \tau(q)_g + \tau(q)_b + \tau_u \tag{1.4}$$

where q is the swaying angle, J is the inertia, d is the damping coefficient, and τ_g,τ_b,τ_u are the corresponding torques of gravity, buoyancy and external control.

When a ship is at sea, its motion is mainly governed by the two forces, buoyancy b and gravity g. If $\tau_u = 0$, the ship motion is totally governed by the natural dynamic forces. Such a system is *autonomous*.

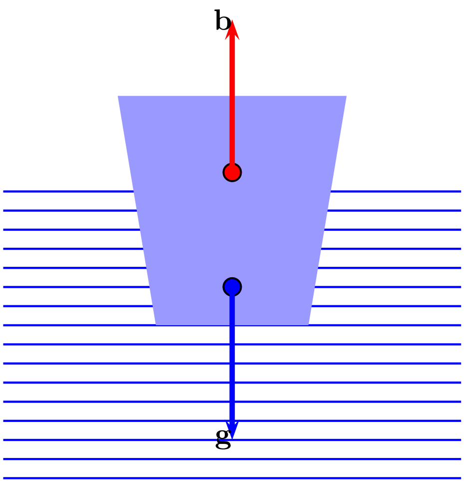
To make it consistent with the discussions in the following chapters, Equation 1.4 is reformulated. By defining the *state* variable $\mathbf{x} = [q, \dot{q}]$, Equation 1.4 becomes

$$\dot{\mathbf{x}} = F_{J,d}(\mathbf{x}) + Du$$

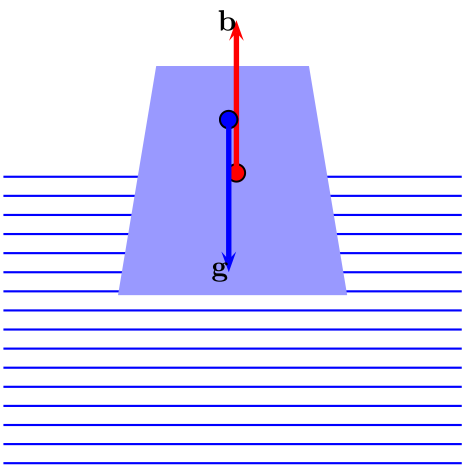
where F is a function of $\mathbf{x}$, the subscripts J and d are *system parameters*, D is a matrix, which describes how the control effort is applied, and u is *control input*. For this example u is τ_u, which is 0.

Equilibrium Postures

A ship will only rest at the postures where $\tau_g + \tau_b + \tau_u = 0$, which are called *Equilibrium* Postures. The only two possible ones are shown in Figure 1.3(a) and Figure 1.3(b).



(a) The Stable Equilibrium Posture



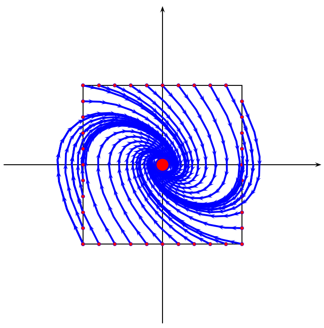
(b) The Unstable Equilibrium Posture

Figure 1.3: *The Equilibrium Postures*

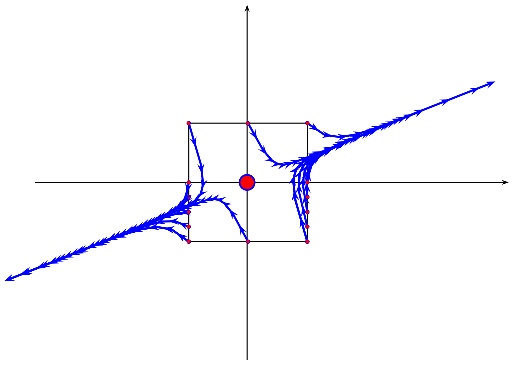
However, the two postures are different, which is illustrated with the *phase plot*. On the phase plot, the horizontal axis represents q; and the vertical axis represents velocity $\dot{q}$. On the phase plot, the motion of the ship is shown as a curve, which is called *flow*.

The posture in Figure 1.3(a) is *attractive* or *stable*. If a small perturbation moves the ship away from the left posture, it will return to the equilibrium posture automatically as shown in Figure 1.4(a).

Whereas the posture in Figure 1.3(b) is *repelling* or *unstable*, if the ship is moved away from the equilibrium posture, by natural dynamics, it will move away even further, as shown in Figure 1.4(b).



(a) The Stable Equilibrium Posture



(b) The Unstable Equilibrium Posture

Figure 1.4: *Phase Plots of The Equilibrium Postures*

A Simple Task

All the flows form the *phase portrait* of the dynamic system, which illustrates all the possible motions. The discovery is that all the flows start from the repelling posture and end at the attractive posture. Several curves are shown in Figure 1.5. This means that no matter what the current posture, the ship will return to the normal stable posture automatically.

This is an intrinsic property of natural dynamics, and thanks to this, balancing is a simple task which requires no control effort. This property is determined by the qualitative structure design criterion which demands the centre of buoyancy is above the centre of gravity.

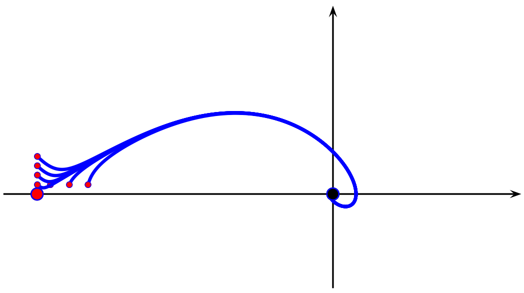


Figure 1.5: *Global Properties of the Flows: All the curves start from the repelling posture (Red) and end at the attractive one(Blue)*

Generalization of the Ship Example

This conclusion is independent of the shape, size, weight or material of the ship. In general cases, the same wave perturbation will result in different sway motions for different ships. However, as long as the qualitative structure design criterion is maintained, balancing remains "easy". The phase portraits of all ships share following properties.

- one repelling point
- one attractive point
- all flows start from repelling point and end at the attractive point.

In mathematical terms, all the phase portraits share the same topological structure of Figure 1.6.

This phenomenon illustrates the principal idea of motion adaptation in **MoIT**. When the variations among individuals or situations result in motion variations, the qualitative dynamics or topological structure of the dynamic system remains invariant.

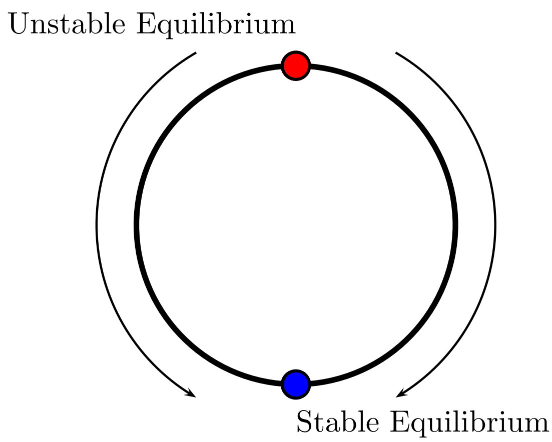


Figure 1.6: *the topology of the phase portraits of ship dynamic*

1.3.4 The Mass Spring System: Symmetry Transformation

Despite the complexity of the body structure, biological motor control is fast and accurate. Such quantitative properties pose another puzzle in motor control research, as solving the complex dynamics directly would require prohibitively long computational time and excessive mental resources.

MoIT proposes a new method to achieve speed and accuracy in motor control. This efficient strategy is based on the ideas of transformation and symmetry. New motions are achieved through transforming template motions,without solving the dynamics. To keep the motion natural looking, the control system chooses the transformation directions that are energy efficient, or using an alternative, allowed by the natural dynamics.

Such ideas can be illustrated by the following mass spring example, shown in Figure 1.7. The mass spring system is selected because it captures some important properties of biological dynamics. The compliant actuators of muscles work like springs, and rigid bones are modelled as mass.

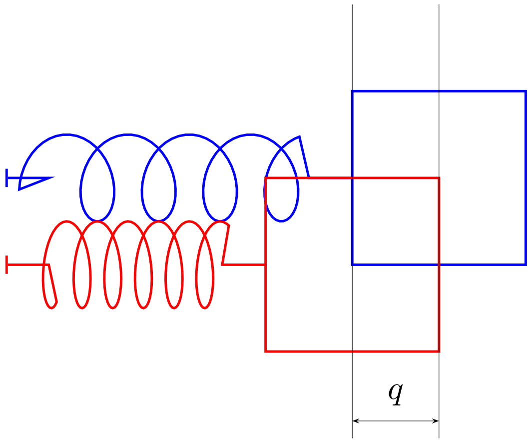


Figure 1.7: *the mass spring system*

Dynamics

The canonical equation of a mass spring system is Equation 1.5

$$\ddot{q} + q = 0. \tag{1.5}$$

where q is the offset distance.

By defining the *state variable*, $\mathbf{x} = [q, \dot{q}]$, Equation 1.5 can also be reformulated in the form as

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

Figure 1.8 shows two flows passing through different states x and x' on the phase plot.

Symmetry and Transformation

The mass spring system has some "symmetrical properties". To an intuitive eye, different flows share the same circle "Shape". Without solving the Equation 1.5, new flows (the solid one) can be obtained by scaling the original (the dotted) flow.

From a mechanical viewpoint, this is because the flows of a mass spring system pre-

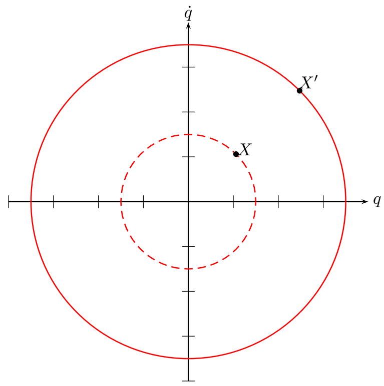


Figure 1.8: *Mass Spring Phase Plot: two flows pass through different states ($\mathbf{x}$ and $\mathbf{x}'$)*

serve energy. To see this, we can define the energy function

$$E = \frac{1}{2}(m\dot{q}^2 + kq^2)$$

where k is the stiffness, m is the mass. Since E is a constant, we make $E = c$, When $m = 1, k = 1$, we obtain

$$q^2 + \dot{q}^2 = 2c$$

The equation above is the implicit function of a circle.

Therefore, given a template flow that passes through $\mathbf{x}$, the flow passes through $\mathbf{x}'$ can be obtained by enlarging the original template flow. In this manner, we determine the future motion after $\mathbf{x}'$, without solving the dynamics.

Dynamic Perception and Local Motor Invariant

The idea of "transformation and symmetry" may shed light on the dynamic perception problem. It is highly unlikely that animals solve Equation 1.5 to understand the the mass spring system. As an alternative, the dynamics can be encoded in a different manner: a motion template and the symmetry property. If so, observed motions can be validated by checking them against our memorized motion templates.

To make it better, it is even unnecessary to work out the transformation. In fact, it is enough just to check some properties invariant under transformation. For the example of mass spring system, we can check the “shape” of the flow. For a mechanical perspective, this means to check the energy preserving property.

The invariant properties like preserving energy or shape can be quantitatively measured. Since they are invariant only when flows move in a specific direction, they are called *Local Motor Invariant*.

1.3.5 The Rimless Wheel

The third example is a mechanical system with a more complex structure, the rimless wheel. The complexity of the mechanical structure provides an opportunity to test various control ideas and compare them.

Dynamics

The Simple 2D model is shown in Figure 1.9. Where α is the angle between the spokes, γ is the angle of the slope, L is the length of the spoke,g is gravity.

The dynamics of the system includes two phases: the rolling phase and the striking phase.

During the rolling phase, the rimless wheel works like an inverted pendulum, the dynamics is as follows:

$$\ddot{\theta} = \frac{\mathrm{g}}{L}\sin(\theta - \gamma)$$

When another spoke hits the ground, a strike happens. The impulse equation is

$$\dot{\theta}^{+} = \cos(\alpha)\dot{\theta}^{-}$$

$+, -$ means after and before collision.

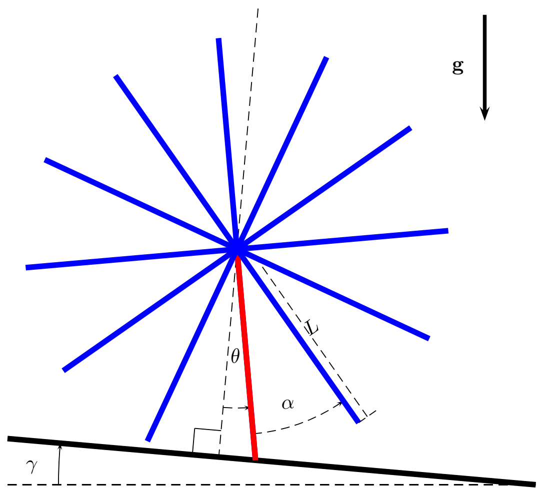


Figure 1.9: *The Rimless Wheel*

Comparing with the mass spring system, the motion of a rimless wheel is more complex. Depending on the initial condition, rimless wheel can roll uphill, roll downhill, stand with one spoke or stand with two spokes. As the rimless wheel continues its motion, the final results of motion may be any of the following:

- rolling down the hill at a constant speed.
- rolling down the hill at ever increasing speed.
- stopping with one spoke as support.
- stopping with two spokes as support.

The first one is of much interest. In dynamics, constant rolling speed means the flow forms a limit cycle. Figure 1.10 shows the limit cycle on a phase plot.

The Qualitative Approach

The motion of a rimless wheel can be controlled by many methods. The first method explores the topological invariant property. For the rimless wheel system, the angle α

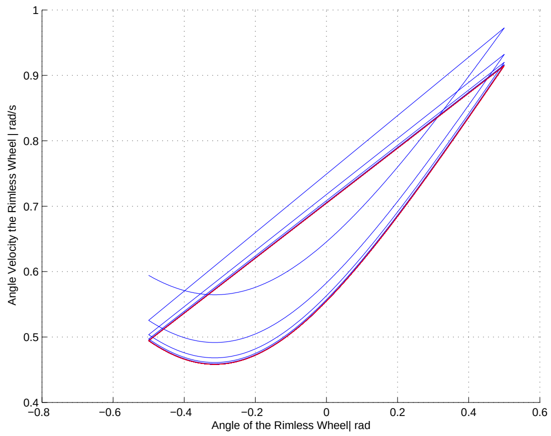


Figure 1.10: *The Limit Cycle of The Rimless Wheel*

between spokes and the slope angle γ can be changed. By doing this, we can change the stable rolling speed of the rimless wheel. This will result in a series of dynamic systems analogous to the original one. By gradually changing the parameter, on the phase plot, the limit cycle changes its shape accordingly. The limit cycles of different mechanical parameters are shown in Figure 1.11.

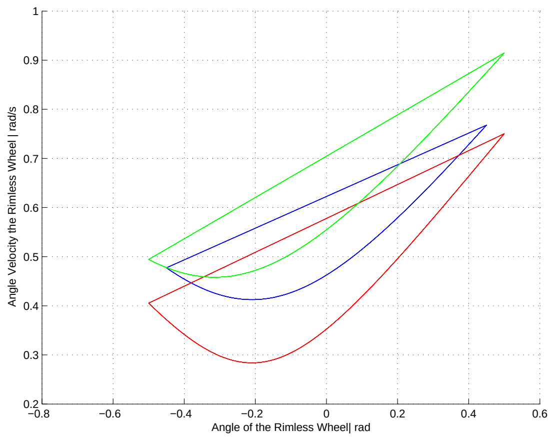


Figure 1.11: *Different Mechanical Parameters result in Different Rimless Wheel*

This is the qualitative approach; motion can be adapted by changing the parameter of the mechanical system. This method requires no control energy input to maintain the new motion; it is energy efficient and easy to implement. However, the relationship between system parameters and the deformation of the limit cycle is hard to find, which prevents applying this method for tasks that require precision.

For example, given a state on the state space, it is difficult to make the limit cycle pass through the state by changing the parameters.

The Quantitative Approach

Another approach to control the rolling speed is by applying control force. For example, we apply control u to the dynamic system, this can be achieved by adding a rotating motor to the center of the rimless wheel, then the equation becomes

$$\ddot{\theta} = \frac{g}{L}\sin(\theta + \gamma) + u$$

if we set $u = \varepsilon\frac{g}{L}\sin(\theta + \gamma)$, where ε is a parameter, then the rolling speed of the rimless wheel will be a parameter of u. Figure 1.12 shows limit cycles of different ε parameters on a phase plot.

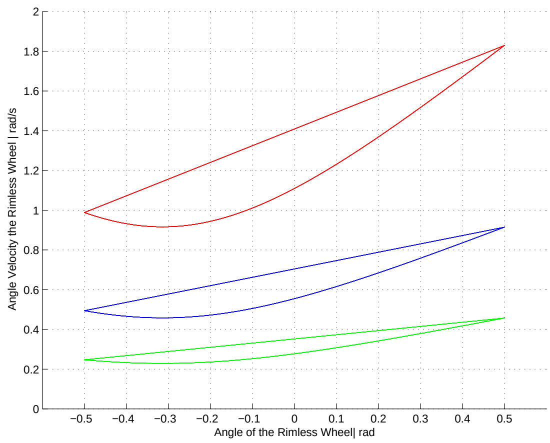


Figure 1.12: *Different Limit Cycles with Different Control*

As shown in Figure 1.12, the limit cycle is stretched vertically. The relationship between ε and the rolling speed is simple, making this method computationally efficient and suitable for precise tasks. To make the limit cycle pass through a state $(\theta, \dot{\theta})$, if the state of same θ on the limit cycle is $(\theta, \dot{\theta}')$, then we have

$$\varepsilon = (\frac{\dot{\theta}}{\dot{\theta}'})^2 - 1$$

The disadvantage of this method is it require energy input. Therefore for a large deformations,this method is not energy efficient.

The Difference and Comparison

These two methods are different but related. Neither methods will change the dynamics qualitatively. The systems after parameter modification, or the controlled systems are still able to run uphill, down hill, stop with one or two spokes and roll at a constant speed.

This demonstrates the underlying topology is not changed. Both methods try to transform the phase portrait. The different transformation require different computational or energy cost.

There is another reason for choosing the rimless wheel as a example, its dynamics resemble that of animals' locomotion behaviour. As further development, we propose this idea for motion control of dynamic characters.

1.4 Contribution

Based on the biological idea, this research proposes an more efficient framework for animation production. Natural motion features are maintained by adopting biological inspired control techniques.

In application, the new framework is capable of synthesize motions automatically without any manual key frame work or motion capture.

MoIT introduces topological conjugacy as the foundational theory that unifies different biomechanic research ideas in a new framework. In **MoIT**, Motion Primitives are identified by the *structural stability* property. Entrainment and Lie Group Transformation are introduced as control techniques efficient in terms of energy and computation.

This combination implies a new control hierarchy framework and has a good biological meaning: **CPG** comes from the research of spinal cord, the low level control system; and the transformation idea comes from research of the cortex, which models the high level control system. The low level system maintain the stability utilize some robust and qualitative measures like entrainment; while high level system control the precision, which adapts the stable motion for specific purpose.

Compared with current **CMS** methods, the new approach has several advantages:

1. **More Types of Adaptation.** Most dynamic methods only focus on generating responsive motions to dynamic perturbations. Adaptations across different characters are treated as an independent research topic (motion re-targeting) and are tackled with very different methods. **MoIT** solves the two problems with one approach. The mathematical idea of topological conjugacy incorporates both motion re-targeting and perturbation responses in a unified framework. Thus **MoIT** is capable of generating more types of adaptation.

2. **Better Usability.** For many **CMS** methods, each **DOF** is controlled independently. When modifying motions, the animator has to modify each **DOF**, which is tedious work.

   In **MoIT**, adaptation is achieved by applying transformation. Each type of transformations can be parameterized by one parameter, and there are only a few types of transformations available for a specific motion task. By specifying very a few parameters for the transformation, control inputs of all **DOF**s are modified automatically, making this method easier to use.

3. **Noval Motion Generation.** **MoIT** relies on the dynamics of body and environment. Motion Capture Data are not needed as reference input. In some situations, this method can generate new motion that cannot be captured.

4. **Computationally Efficient.** This motion synthesis approach requires little computation time and memory, therefore it suits real-time applications.

5. **Dynamic Motion Transition.** Transitional motions can also be simulated dynamically, and in this research such methods have been developed upon solid theoretical foundation.

Because of its biological foundation, algorithms and simulation results of **MoIT** might shed light on biology research questions. Some conclusions and control techniques developed in this thesis provide alternative ideas for biological motor control, and have potential theoretical value.

1. The Motion Primitive Hypothesis is an old idea in biological research, but there is no agreement on the definition and underlying reasons. Biological research has tried to identify motion primitives by exploring neural anatomy, EMG signals or muscle activation patterns.

   MoIT examines motion primitives from the dynamic viewpoint. The discovery and conclusion are more logical and complete. Besides pointing out a motion primitive, **MoIT** also explains why certain motions become primitive, how many primitives exist, and how they are formed.

2. Many biological research ideas like **CPG** and invariant based perception are proposed empirically. For a complete theory, much necessary detailed information is still missing. As a contrast, **MoIT** is based on rigid mathematical theory, for many biological ideas, **MoIT** provides workable mathematical machinery.

1.5 Organization of the Thesis

This thesis is organised as follows.

In Chapter 2, previous research on motion synthesis and biological motor control which are the motivation and justification for **MoIT** are discussed, .

In Chapter 3, *The Qualitative Dynamics Theory* is introduced to explain motion primitives. Biological based methods for maintaining the global motor invariant are developed.

Chapter 4 focuses on the idea of Local Motor Invariant and Symmetry. Lie Group theory is introduced to analyse the symmetrical properties in motion dynamics. Symmetry Controllers are developed to provide necessary energy input for adapting motions.

Chapter 5 discusses the combination problems. For a single motion primitive, strategies are developed to preserve both the global and local motor invariant simultaneously. Motion primitive transition is discussed. Methods for combining motion elements into more complex motions are developed. Finally, in oder to develop an animation system, the software architecture and work flow are discussed.

Chapter 3, 4 and 5 lay down the theoretical foundation of **MoIT**. The following chapter provides experimental verification.

Chapter 6 focuses on the synthesizing adaptive motions for one primitive. Bipedal walking, which is commonly observed but poses great challenges for current **CMS** research,is chosen as the example,. Methods based on **MoIT** successfully boost the stability and generate adaptive gaits, and further validation shows the synthesized gaits comply with natural observation.

In Chapter 7, motion transition is discussed. A new balancing motion primitive is developed. Adaptive transitional motions from stance to walk and walk to stance are generated dynamically.

In Chapter 8, motor invariant theory is extended to more complex characters. Three strategies are developed to simplify the problem for different situations.

This thesis ends with Chapter 9. After discussion of new finding arising from this research, some new questions and ideas for graphics and neural science are proposed for further research.

Chapter 2

BACKGROUND

Current **CMS** methods have different ideas of motor control. Many current **CMS** research studies adopted the control hierarchy of artificial systems. No matter whether the control method is based on tracking or optimization, in such systems, there is a clear separation of planning and execution. The body is treated as the mechanical apparatus, which execute the motor commands from the neural system.

Motor Invariant Theory(**MoIT**) is based on the integrative theory of motor control(Dickinson *et al.*, 2000): It does not separate motion execution from motion planing. For biological systems,it is believed that the planning and execution can not be separated distinctively. In the integrative theory framework, neural system plays a limited role in the planning. Body and environment are taken into consideration and motor control can only be understood from a broader perspective.

In this chapter, limitations of current **CMS** methods are discussed first, which motivate this research. New theory is developed because these limitations can not be overcome without breaking the current theoretical framework. Supporting biological research studies are discussed later, which serve as justifications for **MoIT**.

2.1 A survey of CMS

Many methods are developed in **CMS**, making it impossible to include all the work in this chapter. For a short discussion, **CMS** methods are categorized by the principal control model: memory based or computation based. Memory based control ideas are the foundation of the many data-driven techniques; while procedural methods are computation based. Pros and cons of methods are discussed category by category.

2.1.1 Data Driven

Data-driven methods are based on ready motion data, generated by Key-frame or Motion Capture(Mocap). In practice, motion data are segmented into short time clips. An animation is synthesized by selecting motion clips and connecting them together(Kovar and Gleicher, 2003; Parent, 2002).

Like other example based methods, data driven methods can generate good results if similar motion clips are available, but difficult to generate adaptation or novel motion, either for a different character or scenario. The "re-targeting" problem is a big challenge in **CMS** research.

In practise, motion versatility requires a large data base. As a consequence data management becomes another problem. Due to this reason, the Annotation Database (Arikan *et al.*, 2003) and the Motion Graph (Kovar *et al.*, 2008) were proposed. Currently, the problem of catalogue and search of motion data are not trivial and remain open(Keogh *et al.*, 2004; Müller *et al.*, 2005).

2.1.2 Procedural Method

For physics based **CMS**, different procedural approaches have been proposed.

- **Tracking Controllers.**

  Some early research applied classical **PD** controller for dynamic motion synthesis (Raibert and Hodgins, 1991). Later research (Hodgins *et al.*, 1995) applied the same method for different tasks like running, bicycling, vaulting and balancing. For high dimensional characters, **PD** controllers need to track predefined motion curves(Yin *et al.*, 2007) in configuration space.

A **PD** controller is shown in Equation 2.1.

$$u = K(q - q_d) + d\dot{q} \tag{2.1}$$

where u is the control effort, K is the stiffness, q_d is the desired or reference position, and d is the damping efficient. **PD** based methods can run in real-time and generate adaptive responses to small perturbation. But large perturbation responses or deviations from the reference trajectory are difficult to achieve.

Most **PD** based controllers use motion capture data as references. As an alternative, Laszlo *et al.* (1996) introduced Limit Cycle (**LC**) as tracking reference for periodic locomotion animation. Current research studies(Coros *et al.*, 2010, 2009; Laszlo *et al.*, 1996) track fixed limit cycles. Limit cycles are defined on the phase space, thus such method can be seen as curve tracking in the phase space. Phase space curve tracking methods share many characteristics with **PD**tracking controller of configuration space, which promise real-time speed but lack adaptation, and the results are stereotype looking.

- **Optimization.** The redundant **DOF**s make motion planning non-deterministic. Optimization has been introduced to **CMS** for this problem. The idea is to choose the "best" one among all the possible motions.

  Many merits have been proposed for **CMS**. For dynamic methods, a reasonable merit is the energy cost E.

$$\mathbf{E} = \int_{t_0}^{t_1} f_a(t)^2 dt \tag{2.2}$$

  where f_a is the active force generated by actuators like motors or muscles. This is introduced to **CMS** research as the influential Spacetime Constraints(Witkin and Kass, 1988). It is based on the hypothesis that the natural looking trajectory costs minimum energy, which closely relates to the idea of Darwin's Theory of

Evolution.

Optimization based methods produced believable motions for variable tasks. Jain *et al.* (2009) provided an example of locomotion. Macchietto *et al.* (2009) found a method for balance maintaining movement. Liu (2009) proposed a method for object manipulation.

Drawbacks of Optimization

Optimization is a popular method for physics based animation. It generated the best motion results in current research. But this method has several drawbacks.

- **Numerical Stability** Optimization methods promise the energy efficiency of the motion results. But in practise, it is difficult to design a stable numeric scheme to find optimal motion solution.

  The motion results are sensitive to the accuracy of the model and the proximity of the initial guess. Liu (2005) points out that the original spacetime constraint methods only suit high energy motions, like jumping and running. For low energy tasks (such as walking) the results are not natural looking.

- **Computational Complexity:** Optimization methods like spacetime constraints is a variational optimization problem in nature. For a complex character, it might take prohibitively long time,thus the applications is limited to problems which are computationally feasible. In addition, little is known about how to reuse a computation result for motion adaptation.

2.1.3 Hybrid Methods

There are many research attempts to make tracking controllers more adaptive or optimization faster. One popular idea is to mix the two methods: optimization is done offline for planning the reference trajectory, while tracking controllers are adopted as online real-time controllers. Many methods start to train the controller with motion capture data (Coros *et al.*, 2010; de Lasa *et al.*, 2010; Lee *et al.*, 2010a,b; Levine *et al.*, 2011; Liu *et al.*, 2010; Wang *et al.*, 2010; Wei *et al.*, 2011; Wu and Popović, 2010; Ye and Liu, 2010).

Also new research propose use simplified dynamic models for optimization planning(Mordatch *et al.*, 2010).

These attempts may remove some limitations of tracking or optimization, and make them feasible for certain applications. But **CMS** problems can not be solved completely in this manner. Learning based methods are complex and sensitive to training examples, the stability of such controllers can not be strictly proved. In addition, offline optimization does not reduce the computational burden in nature.

2.1.4 Biological Constraints

The problems of **CMS** has also been spotted earlier in biological motor control research. Biological researchers have dropped traditional artificial control ideas long ago, because they violate the biological constraints. Although the mechanism behind information processing remains obscure, some characteristics of biological information processing are well recognized, making **CMS** methods above questionable(Glynn, 2003).

- **Sensing and Control Limitations:** Motor control is not only a mechanical problem, but also a complex process involving chemical, electrical and mechanical changes. Many crucial mechanical parameters and variables such as mass, inertia, force, are inaccessible to the neural system and can only be approximated. Some important control variables (such as torque) are controlled indirectly by the neural system through a complex process. Also body and environmental measurements are noisy and time varying, making methods that are sensitive to errors unsuitable for control biological system.
- **Neural Computation:** The neural system is powerful, but inferior in speed and accuracy when compared with digital computers. Neural signals are of only hundreds of Hz and their transmission speed is slow. In addition there is a long delay between firing a neural signal and generating force in the muscles. It may cost about half a second from seeing an object to force generation in arm (Latash,

2008). This makes it impossible for the neural system to carry out the complex computation necessary for realtime optimization.

  Following the idea of optimization control, the dynamics of fluid environment and deformable body are more difficult to optimize. But most primitive life forms live in the sea and have limited intelligence.

- **Memory Capacity:** Some argue that motion control is not based on computation, but based on memory. This idea avoids the question of computation speed, but it faces another problem of the memory capacity. Since motion varies greatly, if we store every variation of motion in our brain, brain will run out of memory space.

Because of such constraints, researchers have started to look for different strategies.

2.2 Motion Primitives

At first, researchers are reminded that logical think or mental conscious plays little role in motor planning. Animals including human exhibit complex motion behaviours after birth or at early ages, abilities like breathing, heat beating and child bearing are inborn without learning.

Some suggests that motor ability are inborn and organized in blocks(Bizzi *et al.*, 2002, 1995). Strong evidences come from the experiment where stimulating of a single spinal motor afferent triggers a complete sweeping motion(Bizzi *et al.*, 1995). A new theory, Motion Primitive Conjecture, was proposed. In this theory, motion is built from a limited number of building blocks, which are called *motion primitives*. Complex motions are combinations of motion primitives, just like we connect alphabets into sentences. Motion Primitive Conjecture also provides insight into the motion perception. Gallese *et al.* (1996) have found action and perception trigger similar reactions in a group of neurons.

2.2.1 Dynamic Motion Primitives

The Conjecture of motion primitive is supported by both the behaviour study and anatomy of natural animals. For dynamic **CMS**, the puzzle is how motion primitives idea can simplify dynamic motor control.

A proposed answer is that every motion primitive has some valuable dynamic properties, like stability and efficiency, which is determined by the natural dynamics. Some researchers point out that motion style is closely related to the body structure and environment. They have not been changed much by the evolution of neural system, for example, the whales swim more like fish than other mammals. Animals do not move the way they want, but rather the way they can. A further explanation is that the body and the environment play the most important role in motor control: they form the basic pattern of motion (Nishikawa *et al.*, 2007). For neural control, the responsibility is not to plan the trajectory from ground up, but modifying or tweaking basic patterns to meet specific purpose. Several theories are proposed for the neural control mechanism.

Experiments have shown that even under the same conditions, the motions still vary. Some **DOF**s are not controlled and freely influenced by the environment. For this phenomenon, *Uncontrolled Manifold Hypothesis(**UMH**)*(Latash, 2008) proposes that only the final results is the concern of motor control, trajectory is not.

*Equilibrium Point Hypothesis(**EPH**)*(Feldman, 1986) explained below can be seen as a specification of **UMH**. This idea comes from properties of differential equations. For a dynamic system

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

the equilibrium points $\mathbf{x}_e$ satisfies the condition $F(\mathbf{x}_e) = 0$. **EPH** suggests the neural system does not control motion trajectory, but the position of the equilibrium point.

Impedance Control (Hogan, 1985) refines the idea of **EPH** by providing a model for effects of the extra **DOF**s as explained below. At an equilibrium point $\mathbf{x}_e$,

$$F(\mathbf{x}_e) = 0$$

Impedance Control proposed that the extra **DOF**s provide a way to control the stability and admittance of the equilibrium point $\mathbf{x}_e$. The mathematical description is

$$F(\mathbf{x}_e + E_r) = KE_r \tag{2.3}$$

where E_r is the offset error vector, K is stiffness matrix or impedance,which determines the stability. The extra **DOF**s provide the neural system a way to tune the direction of K according to the purpose. This mechanism will provide the actors a way for avoiding obstacles or risks. Experiments (Franklin *et al.*, 2007) have proved this idea by showing that the measured matrix K has anisotropic properties.

2.2.2 Neural Control Mechanism

Motor control involves little mental work, and current idea of neural science is that motor control is a low level intelligent activity and can be controlled without brain input. Research studies have proposed several neural activities related to its role in "tweak" motion primitives.

- In vertebrate animals, Central Pattern Generator (**CPG**) serves important functions in locomotion, respiration, swallowing and other rhythm behaviours. Cohen (1988) argues that locomotion is the result of the interaction between neural and mechanical oscillators via a process called **entrainment**. Neural systems modify the motion by adjusting frequency and amplitude of the rhythmic neural signal.

- Some research studies find out that motion will change in a uniform manner(Viviani and Stucchi, 1992).Flash and Handzel (2007) proposed modelling motion adaptation through *affine transformation*. This idea is inspiring for the fact that affine transformation group is closely related to vision perception system. This theory implies a close relationship between motor control and vision.

2.2.3 Bionic Robotic Research

Ideas from biological research also inspired many robotic engineering experiments,which show the feasibility of new control principles. Such robots utilize the natural dynamic rather than the tracking or optimization strategy. Here are some important research studies reported.

- **Limit Cycle in Walking.** A very important discovery is the bipedal walking can happen without any control(McGeer, 1990). Under specific conditions, a mechanic structure can walk down a slope passively, with natural looking gaits. Further research have shown that such a mechanical system can walk on a plane with a very simple control strategy(Collins *et al.*, 2005).

- **CPG and entrainment** The **CPG** based entrainment is applied for robotic research(Williamson, 1999), the results show the **CPG** will boost the system stability and can maintain motion in unpredictable situations. Fukuoka *et al.* (2003) has applied **CPG** for quadrupedal walking.

  Taga (1995) had applied the idea for bipedal walking control, little is known about how to tuning the parameters to generate desired motion adaptations.

- **Passive based Control.** The control and mechanics community also starts thinking about passive based control methods that utilize the natural dynamics. Many techniques such as (Asano and Yamakita, 2001; Pratt *et al.*, 1997) have been developed to control redundant systems, However early methods are usually limited to its application or may be not efficent in computational time or energy.

  These techniques are generalized as a systematic method(Spong, 1998, 1996), which provides a solid mathematical theory and can be applied to mechanic systems with more complex properties(Spong and Bullo, 2005)

Chapter 3

GLOBAL MOTOR INVARIANT

Motions are similar but vary greatly. For example, different people will walk with different gaits. An interesting question is how the word “walk” refers to different gaits. Motor Invariant Theory(**MoIT**) proposes an answer: despite differences in gaits, we agree on the word “walk” because in essence, we all walk in the same manner. Intuitively, the gaits are periodic, energy efficient and stable. The variations come from the differences in body, environment or purpose. From dynamic perspective, all the gaits dynamics share the same structure, or the qualitative properties of walking are invariant. In **MoIT**, the qualitative invariant properties are *Global Motor Invariant*.

For the biological perspective, we believe the walking ability is inborn and encoded in the body structure. What “Walk” means is one motion primitive. In **MoIT**, the motion primitives are identified by the global motor invariant. This claim will be justified in Section 3.2.

In theory, it is difficult to define the gait similarity mathematically. Topology is introduced for a clear definition of global invariant. Topologically equivalent means that the dynamic systems are qualitatively the same. Basic ideas of topology and qualitative dynamics are introduced in Section 3.1.

Entrainment is the biologically based method to maintain the global motor invariant. We will discuss the theory and experiments in Sections 3.3 and Section 3.4.

3.1 Introduction to Qualitative Dynamics

Motion Primitives are "trivial" motion tasks. The evolution process equips animals with a body structure that suits many motion tasks. As a result, such motion tasks can be accomplished by exploring the natural dynamics without too much control effort. For the dynamic perspective, the delicate design of body structure permits several patterns when animals interact with the living environment. Such patterns exists across detailed variations in body structure such as the tall and short characters and environment like rough or plane ground. They are robust or *structurally stable* in the dynamic term. In **MoIT**, the identification of motion primitives and adaptation are based on the structural stability. This section serves as a short introduction to concepts and prepared mathematics.

Qualitative dynamic properties are analysed with the tool of differential topology. This idea can be traced back to Poincare(Poincaré, 1885; Poincaré and Magini, 1899) and was laterly developed by the Smale School(Smale, 1970). There is no enough space to include the whole subject, please refer to book (Abraham and Marsden, 1978) for more details. Throughout this thesis, the geometrical perspective is adopted as it is more intuitive. Some primary knowledge of topology and manifold is required which can be found in (Abraham and Marsden, 1978). For the sake of completeness, this thesis will provide a rough and intuitive explanation below. Intuitively speaking, *topology* studies the geometry properties that are preserved through continuous deformations, such as twistings and stretchings of objects. Discontinuous deformations like tearing will break the topology. Due to this reason, in the topological space, a circle is topologically equivalent to an ellipse because stretching a circle can deform it into an ellipse and a sphere is equivalent to an ellipsoid.

A *manifold* is a topological space that locally looks like the Euclidean space of a specific dimension. A line and a circle are one-dimensional manifolds, a plane and sphere are two-dimensional manifolds, and so on into high-dimensional space.

A dynamic system is usually described as a differential equation, from the geometrical perspective, the differential equation also describes a differentiable manifold. Qualitative properties can be obtained by analysing the topological property of geometry.

Global Motor Invariants are identified by the topological structure.

3.1.1 Dynamic Systems and Differentiable Manifold

Motions of a mechanical system are determined by its configuration q in configuration space Q and generalized speed $\dot{q}$ in the tangent space T_qQ. Define the state value $\mathbf{x} = [q, \dot{q}] \in M$, where M is the state space, or state manifold. A motion is a trajectory $t \mapsto q(t)$ in the configuration space parameterized by time t. For a dynamic system, $q(t)$ usually is derived from the state trajectory $\mathbf{x}(t)$, which is described by a differential equation.

For every point $\mathbf{x} \in M$, F and u determine a derivative vector $\dot{\mathbf{x}}$ in the *Tangent Space* $T_{\mathbf{x}}M$. Vectors over the full space of $\mathbf{x}$ form the *vector field* $\mathbf{V}$, described by Equation 3.1.

$$\dot{\mathbf{x}} = F_\alpha(\mathbf{x}) + u \tag{3.1}$$

where u is the control effort, α is the system parameters, and F is determined by the system's natural property. If $u = 0$, no control effort is applied. Such systems are *autonomous systems*.

A solution to Equation 3.1 is an *integral curve*. *Flow* $\Phi(\mathbf{x})$ of $\mathbf{V}$ is the *integral curve* through $\mathbf{x}$. Flows are usually visualized by a *phase plot*. All the flows make up the *phase portrait*, which illustrates all the possible motions of the dynamic system.

Example

For a mass-spring system, state variable $\mathbf{x} = [q, \dot{q}]$ is defined, and Equation 1.5 can be transformed into Equation 3.2.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x} \tag{3.2}$$

3.1.2 Basin of Attraction

Intersections of flows are *equilibria*. At each **equilibria**, the local space can be divided into subspaces: *centre manifold*, *stable manifold*, and *unstable manifold*.

centre manifold For a flow ϕ_c passing through a point $\mathbf{x}_c$ on centre sub manifold W_c, ϕ_c will remain on the Centre Manifold.

$$\phi_c(t) \in W_c, t \in R$$

stable manifold For a flow ϕ_s passing through a point $\mathbf{x}_s$ on stable sub manifold W_s, ϕ_s will finally converge to a flow ϕ_c on centre manifold.

$$\phi_s(+\infty) = \phi_c$$

unstable manifold For a flow ϕ_u passing through a point $\mathbf{x}_u$ on unstable manifold W_u, ϕ_u will be repelled from ϕ_c on centre manifold, the inverse of ϕ_u converges to ϕ_c.

$$\phi_u(-\infty) = \phi_c$$

Attractors are the equilibria where the whole local space is stable, or the dimension of unstable manifold is zero. **Repellors** are the equilibria where the whole local space is unstable, or the dimension of stable manifold is zero.

In theory, only the attractors of the dynamic systems can be observed and are of interest in motor control:

1. Fixed Point or equilibrium point, a phase plot is show in Figure 1.4(a).
2. Limit Cycle, a phase plot is shown in Figure 3.1. The attractor of a limit cycle has the shape of a cycle, which implies self sustained oscillations or periodic behaviours. An attractive limit cycle will attract the neighbouring flows spirals into it over the time. Such a system is stable, if any perturbation move the state off the limit cycle, the system will return to the limit cycle automatically.

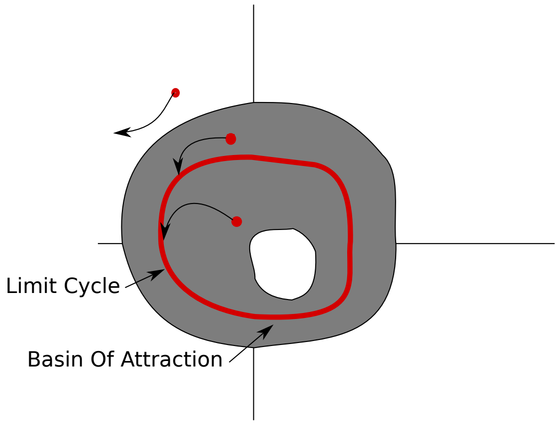


Figure 3.1: *Limit Cycle*

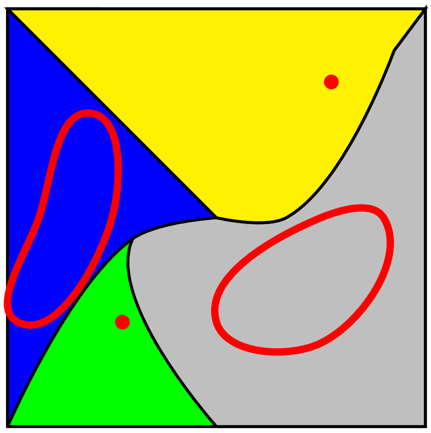


Figure 3.2: *Cellular Structure of Phase Space*

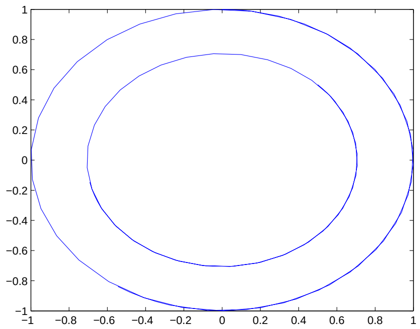
For non-linear dynamic systems, there may exist many attractors. The phase plane is divided into different regions, resulting in a cellular structure. Within each region, all the flows converge to one attractor $\mathcal{A}$, and the corresponding region is the *basin of attraction* $\mathcal{B}(\mathcal{A})$. Figure 3.2 shows the landscape of phase portrait of a dynamic system, in which the basins of attraction are coloured differently.

3.1.3 Topological Conjugacy

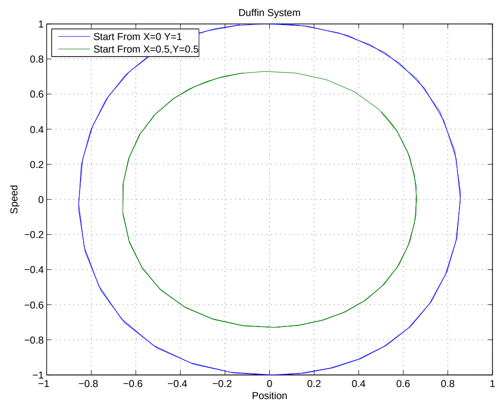
The topological structure of a dynamic system can be described by the type of equilibria and the connectivity of their basins of attraction. Many dynamic systems share same the topological structure. For example, the Duffin system described by Equation 3.3 is different from the mass-spring system.

$$\ddot{q} + q + q^3 = 0 \tag{3.3}$$

However, the two systems share the same topology. Phase plots of the two systems are shown in Figure 3.3(a) and Figure 3.3(b). Flows of the two systems are similar, and we can “deform ” one into another. This equivalent relationship is *topological conjugacy*.



(a) Mass Spring System



(b) Duffin System

Figure 3.3: *Topological Conjugacy*

Definition. *Let M and M' be topological spaces, and let $F\colon M \to M$ and $F'\colon M' \to M'$ be continuous functions. We say that F is* topologically conjugate *to F', if there exists a continuous one-one continuous and invertible mapping $h\colon M \to M'$ such that $h(F(M)) = F'(h(M))$. h is a* topological conjugation *between F and F'. if two systems are topological conjugate, they are* analogous systems.

3.2 Global Motor Invariant and Motion Adaptation

Qualitative dynamic properties are determined by the attractors and their basins of attraction. **MoIT** establishes the relationship between motion primitives and dynamic theory. In **MoIT**, each attractor and its basin of attraction define a motion primitive. “ triviality” of primitive tasks relies on the attraction. If the attractor is a fixed point, then the motion will be terminated. If the attractor is a limit cycle, then the motion will be periodic. Larger basin of attraction means motion is more stable, while narrow basin of attractions means the fragile stability. Qualitative property is the *Global Motor Invariant*

Definition. Global Motor Invariant *is the tuple of attractors and their basin of attraction*

Motions vary because of different perturbations. In **MoIT**, perturbations are classified in two categories and treated with different control strategies.

- **State perturbation**

  Perturbations that only affect the state $\mathbf{x}$ are *State Perturbations*. State Perturbations change the current state, but not the underlying dynamic system.

  If the perturbed state $\mathbf{x}'$ remains in the basin of attraction, the perturbed flow will converge to the same attractor. For the walking example, state perturbations can model the push and recovery motion. Such a kind of motion adaptation is *Responsive adaptation*.

  To make the character more responsive without motion failure, The motion controller should enlarge the basin of attraction.

- **Structure Perturbation** Structure Perturbations affect the dynamic system. For biological systems, such perturbations are very common, when a man puts a heavy box on his shoulder or has been injured, the walking dynamics will change due to the structural perturbations.

  For some dynamic systems, structural perturbations only deform the phase portrait and result in an analogous system. This will result in motion variations but will not change motion stability. This kind of motion adaptation is called *system adaptation*. For **CMS**, " motion retargeting" can be seen as an example of system adaptation.

  In some cases, topological structure may not be maintained. Some perturbations will result in *bifurcation*s that violates the topology of the underlying dynamic system. Such an example is that the damping perturbations on the mass spring system will change the dynamics qualitatively. As show in Figure 3.4, damping

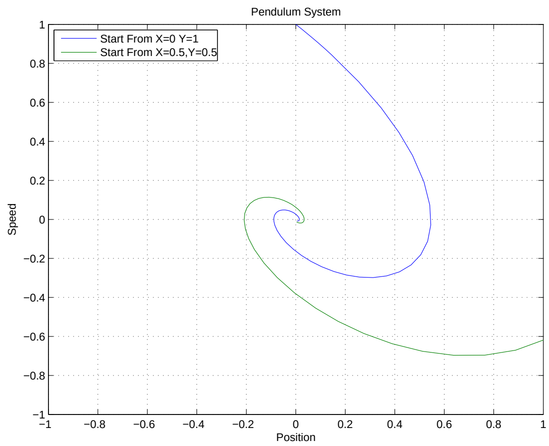


Figure 3.4: *damping perturbation on mass spring system*

changes the topology the periodic flows into a fixed point attractor.

The ability of a dynamic system maintaining its topology structure is *structural stability*. To make motions adaptive to environment and body changes, controller should boost the structural stability of the motion and prevent bifurcations.

These ideas can be seen as a different mathematical interpretation of biological research principles. For the Uncontrolled Manifold Hypothesis(**UMH**), the basin of attraction of an motion primitive can serve as the uncontrolled manifold of **UMH**. State Perturbations are not controlled and motion is freely influenced. For Equilibrium Point Hypothesis(**EPH**), attractor of motion primitive is a generalization of equilibrium points. Impedance control can be seen as adjusting the basin of attraction.

3.2.1 Biological Meaning of Structural Stability

For **CMS** research, Structural Stability is a new idea , but there are good reasons behind it. In natural environment, perturbations and uncertainty are everywhere. Because of the sensing and computation limitations, feedback idea can't cope with all types of perturbations. In **MoIT**, the alternative idea is such perturbations can be neglected. If the motion primitive is structurally stable, even without control effort, motion and the underlying dynamics will not change qualitatively. Such an idea can reduce much of the computational burden and provide a framework for motion adaptation.

For biological research questions, the structural stability idea and the qualitative perspective provide better explanations than optimization and feedback theory.

The first is the control difficulty and evolution of swimming and walking. From the quantitative perspective, fluid dynamics is more difficult to compute than rigid body dynamics. This seems to suggest that the swimming is more difficult and walking.

But in biological evolution, swimming seems easier, for it is developed earlier and many primitive life forms inhabitant in fluid environment.

The qualitative perspective comply with the biological facts, fluid is continuous and uniform, the dynamics have simple topological structure. Stability control for such dynamic systm may become trivial and fish can maintain its posture with little neural effort.

On the other side, although the rigid body dynamics for walking are quantitatively easier, the topological structure of walking dynamics is much more complex. On the phase plane, there exist many equilibria, and the basin of attraction of walking primitive has limited area, thus the stability of walking is fragile and needs more complex control measures.

MoIT also explains the body similarity for animals that move through similar environment in a similar manner despite their far distance in the evolution chain. The similarity in body structure promises the same dynamic topology. We are also reminded that motion primitive is closely related to the environment. It is meaningless to talk about walking when the character floating on water, even with the same control strategy, body and environment cannot form the desired dynamic topology.

Further **MoIT** suggests the direction of evolution. For one motion primitive, body may evolve to make the primitive more structurally stable.

3.3 Global Motor Invariant Control

In real-life, natural dynamics can be extremely complex. The corresponding manifolds have a complex topological structure, which provides many motion primitives.

For **CMS** applications, the question arises whether different motion primitives can be controlled with a simple and unified method. The idea is that even there are many motion primitives, attractors can be catalogued in very limited number of types. Also even the dimension of dynamic system is large, the dimension of the attractors is not.

- Fix point is of zero dimension.
- Limit cycle is of one dimension.

It is still under hot debate which type of attractor serves as the foundations for motor control(Degallier and Ijspeert, 2010). The current idea is that limit cycle is necessary. Based on a limit cycle, a fix point can be achieved by:

1. terminate a limit cycle.
2. approximated by a limit cycle with small amplitude.
3. bifurcate a limit cycle.

Currently only the limit cycle is considered in **MoIT**, mainly due to the following two reasons:

- **periodic behaviour is common** Besides the periodic motions such as swimming and running, other biological activities like heart beating, waking and sleeping are periodic. A periodic system has the potential to simulate more types of motion and integrate with other biological simulation.
- **similar results** For animations, periodic motions look similar to the terminated motion when the amplitude of limit cycle is small. If the oscillation amplitude can be controlled, both types of motion trajectories can be synthesized within one framework.

Control strategies are designed based on the type of attractor. For the fix point attractors, traditional **PD** controllers are simple and efficient. For the limit cycle attractors, entrainment controllers are proposed as an efficient method.

3.3.1 Neural Oscillator and its Stability

3.3.2 CPG and Entrainment

Biology research suggested that motions are mainly controlled by the organ called *Central Pattern Generator*. **CPG** is a small autonomous network that generates rhythmic signals. From the dynamic perspective, the idea of controlling motion by rhythmic signals can be modelled as entrainment (González-Miranda, 2004). When coupling two oscillation system together, entrainment happens when two systems oscillate in synchronize. This effect is also known as a resonant which will enhance the oscillating behaviour.

Only two neurons are needed with mutual inhibitive property, as shown in Figure 3.5.

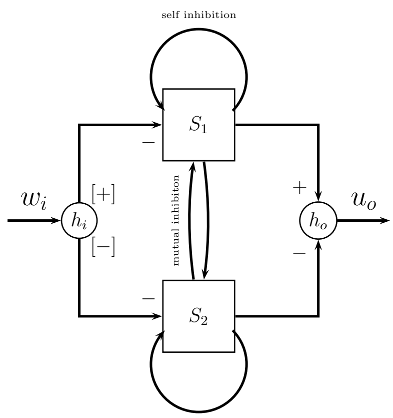


Figure 3.5: *Neural Oscillator Structure*

One oscillation model was developed by Matsuoka (1985) and was extensively studied later on. This model can be described as Equation 3.4.

$$
\begin{aligned}
\tau_1 \dot{s}_1 &= c_1 - s_1 - c_2 l_1 - c_3[s_2]^+ - \sum_j h_{ij}[w_j]^+ \\
\tau_2 \dot{l}_1 &= [s_1]^+ - l_1 \\
\tau_1 \dot{s}_2 &= c_1 - s_2 - c_2 l_2 - c_3[s_1]^- - \sum_j h_{ij}[w_j]^- \\
\tau_2 \dot{l}_2 &= [s_2]^+ - l_2
\end{aligned} \tag{3.4}
$$

where $[t]^+ = \max(0,t)$, $[t]^- = \min(0,t)$. $s_{1,2}$ and $l_{1,2}$ are state variables. c_1,c_2,c_3 are parameters of the oscillator which are kept constant$[c_1, c_2, c_3] = [1, 2, 2]$ in this research. Values of $\tau_{1,2}$ control the oscillation frequency, and their ratio controls the shape of waves. In this research $\frac{\tau_1}{\tau_2} = 0.5$. The output signal u_o is defined in Equation 3.5:

$$u_o = h_o([s_1]^+ - [s_2]^+) \tag{3.5}$$

where h_o is the output amplifying coefficient.

Matsuoka oscillator is an autonomous oscillator, which can start to oscillate without any control effort. Figure 3.6 shows the natural oscillator output.

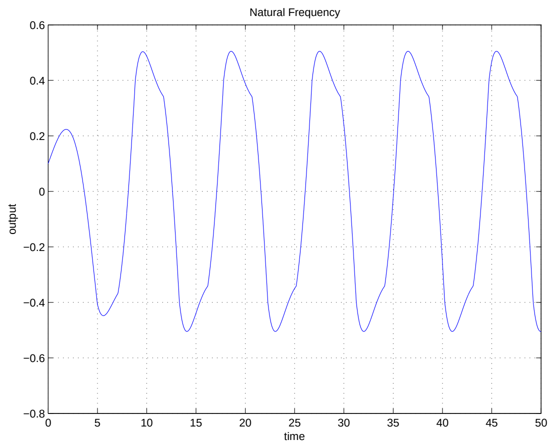


Figure 3.6: *Natural Oscillation*

Matsuoka oscillator is adaptive; entrainment can happen when it is coupled with different oscillators. Figure 3.7 shows the entrainment oscillation,where Matsuoka oscillator synchronises with the input signal.

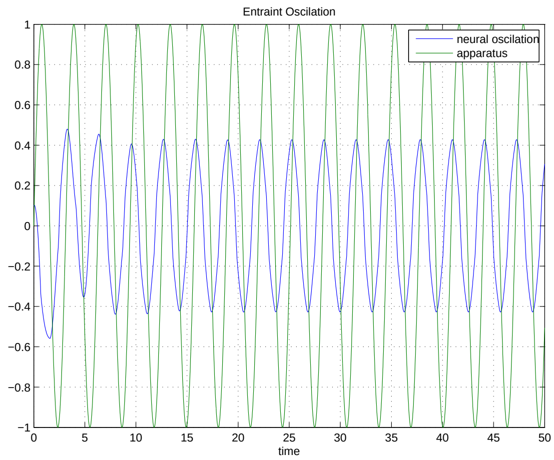


Figure 3.7: *Entrainment Oscillation*

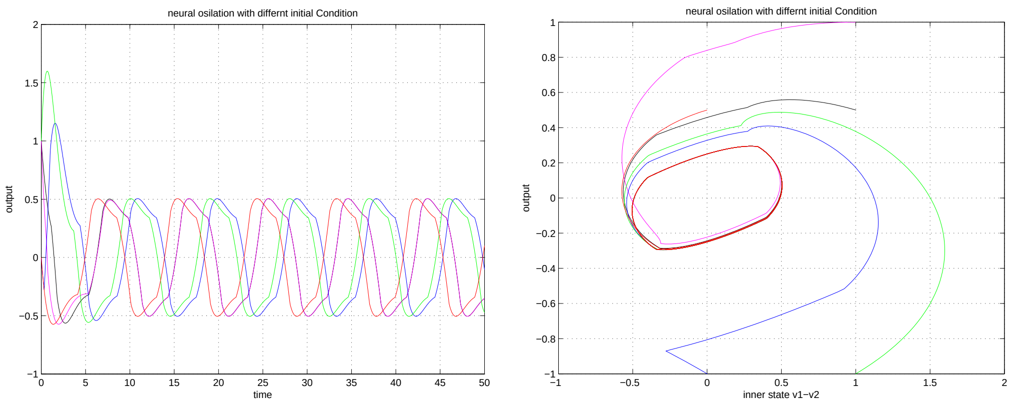
Because of the non-linear properties, its behaviour has not been completely understood. Matsuoka (1987) analysed the adaptive properties by investigating the location of the roots of the characteristic equation. Williamson (1998) analysed the properties in frequency domain. Futakata and Iwasaki provided a rigid analysis of energy efficiency and stability for some specific examples. This research study investigates the qualitative property with empirical methods.

After examining many simulation results, the Matsuoka Oscillator shows three important properties:

- Simple Topological Structure. The topology structure of a neural oscillator is simple: it includes one attractive limit circle and one fix repellor.
- Large Basin of Attraction. All the simulations which we carried out converge to the same limited circle.
- Fast Converging Speed. In most cases, the flow will converge to the limit circle within one period time.

The above features are shown in Figure 3.8.

The large area of basin of attraction means the final behaviour is totally determined by the system parameters. The initial conditions will have no effect on the stable oscilla-



(a) state plot (b) Phase Plane

Figure 3.8: *Neural output with different initial positions*

tion behaviour. Matsuota oscillator can be treated as a single input single output(SISO) system. The output signal is controlled by three system parameters and input signal. Equation 3.4 can be reformed as Equation 3.6.

$$u_{\mathrm{o}} = S_{[h_{\mathrm{i}}, h_{\mathrm{o}}, \tau]}(u_{\mathrm{i}}) \tag{3.6}$$

where $u_{\mathrm{i}} = \sum_j h_j[w_j]$, is the weighted sum of all the input signal.

The converging speed can be seen as a quick recovery ability, which is very valuable for motor control. When an impulse perturbation happens, it will recover in one period time.

3.4 Example:Maintain Ball Bouncing Height

The Bouncing Ball system is shown in Figure 3.9, where a ball is bouncing on a moving paddle. This system is of simple dynamics, but difficult to control with optimization or **PD** methods.

The bouncing ball system captures the complex discontinuous dynamics of body and environment interaction. It can be treated as a template model for many motion tasks like jumping, running and ball playing. This example demonstrates how limit cycle

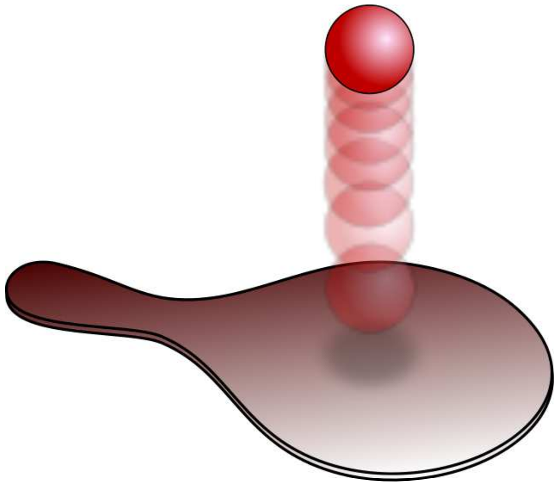


Figure 3.9: *The Bouncing Ball System*

arises through entrainment.

Dynamics

The bouncing ball system is of hybrid dynamic, which involves two phases.

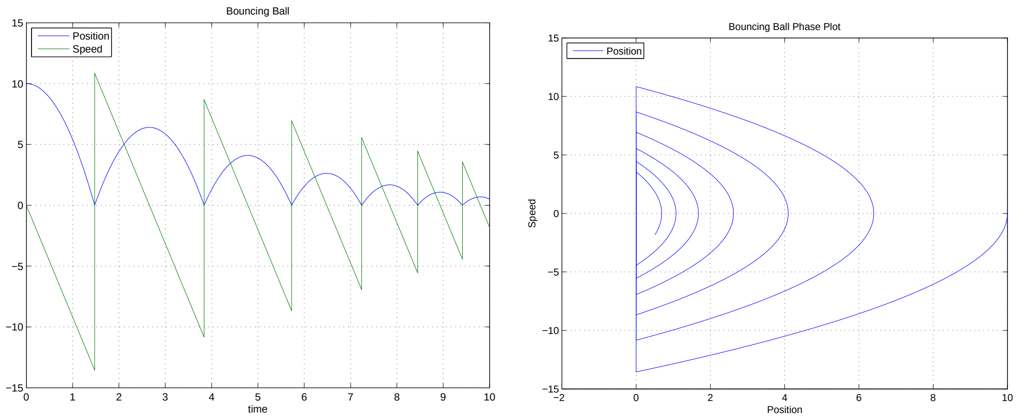
- **The Continuous Flying Phase:** When the ball is flying, it is only affected by the gravity.
- **The Discontinuous Strike Phase:** When the ball hits the paddle, the speed of the ball is changed instantly.

The natural dynamics of bouncing ball system are described by Equation 3.7.

$$\begin{aligned} \ddot{q}_{ball} &= -\mathrm{g} && \text{if } q_{ball} > q_{paddle} \text{ (free flying)} \\ \dot{q}^{+}_{\text{ball}} - \dot{q}^{+}_{\text{paddle}} &= \epsilon(\dot{q}^{-}_{\text{ball}} - \dot{q}^{-}_{\text{paddle}}) && \text{if } q_{ball} \leq q_{paddle} \text{ (paddle strike)} \end{aligned} \tag{3.7}$$

where $\ddot{q}_{ball}$ is the acceleration, g is the gravity, q_{ball}, q_{paddle} are the positions of the ball and paddle, $\dot{q}^{+}_{ball,paddle}$ are the speed after a paddle strike and $\dot{q}^{-}_{ball,paddle}$ are the speed before the strike, ϵ is collision coefficient $-1 < \epsilon < 0$.

Figure 3.10 shows plots of the system. After each strike, the ball will bounce with a smaller height.



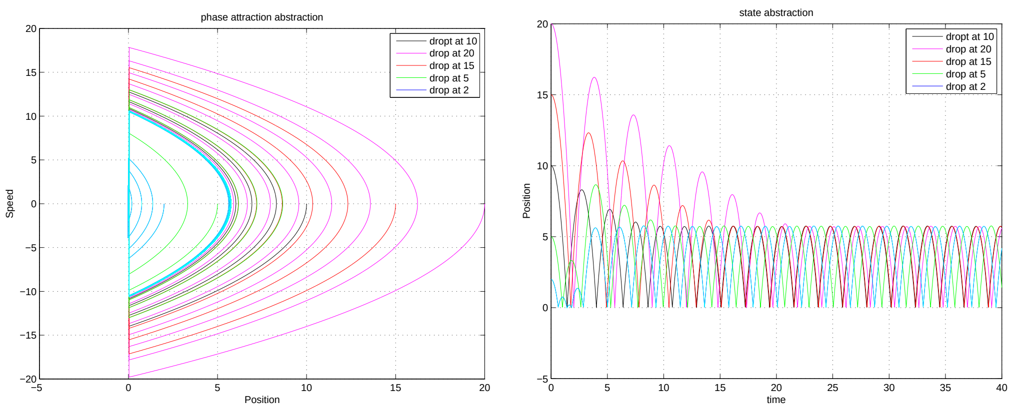
(a) state plot (b) Phase Plane

Figure 3.10: *Original Bouncing Ball System*

Emergence of Limit Cycle

The bouncing ball system has only one fixed point attractor and its basin of attraction covers the whole phase space. However, its behaviour is near periodic. Or alternatively, it can be seen as a bifurcation of a limit cycle. Neural Oscillator can be applied to recover the limit cycle through entrainment.

The input of the neural oscillator is the velocity $u_{\mathrm{i}} = \dot{q}_{ball}$, the output drives the paddle position $q_{paddle} = u_{\mathrm{o}}$. Neural controller will move the paddle up and down. The movement of the paddle is limited to a small range $[-0.1, 0.1]$, compared with the bouncing height of the ball (more than 5), the height variation of the paddle can be almost neglected. Dropped from different positions, the ball will maintain the bouncing height of 5 units after several strike, as shown in Figure 3.11.



(a) State Plot (b) Phase Plane

Figure 3.11: *The Attractive Limited Circle of the Coupled Bouncing Ball System*

Chapter 4

LOCAL MOTOR INVARIANT

It is not enough that animals are able to maintain the global motor invariant. For a fish, preserving *Global Motor Invariant* means the swimming is stable and can be sustained. However, a fish also needs to adjust the speed and direction during swimming, which is of crucial importance for survival. In real-life, an animal can adapt motion primitives according to its purpose precisely. In this chapter, we will develop the control strategies for tweaking motion patterns according to the motion purposes.

It is important to remember that such tweaking strategies are also constrained by the computation and memory capacity of the neural system, and should explore natural dynamics as the basic motion primitive theory. For **CMS**, it is of no meaning developing walking pattern by exploring natural dynamics but using optimization to adjust the walking speed. To meet such requirements, **MoIT** adopted different ideas.

At first, when tweaking motion patterns, stability should not be violated. As stated in the previous chapter, a topological conjugation (one-one continuous invertible mapping) maintains the topology thus maintains the qualitative stability. Thus the “tweaking” action should be a topology conjugation. In an alternative perspective, such operations form a group and permit a combination operation.

According to Group Theory, this means if two tweaking actions preserve the stability separately, the combination of the two actions also preserve the stability. The space of topology conjugation is very large. Currently, **MoIT** only investigates a subset called

The Lie Transformation Group that is supported(Flash and Handzel, 2007) by the biological research studies and can be calculated efficiently. The selected groups can be divided into orthodox subgroups, each of which is continuous and can be parameterized by one parameter. In **CMS**, such parameters are closely related to motion purposes such as walking speed or swimming direction.

From the dynamic perspective, "tweaking" should also explore natural dynamics (passive based) as primitives. Methods adopted in **MoIT** belong to a popular passive-based control principle, which carries many names: Controlled Symmetry, Controlled Lagrange, or Potential Shaping. Different names reflect the fact that this method can be developed through different ways. Roughly speaking, the original dynamic system is transformed according to motion purpose, the kinematics is untouched and control is applied by modifying the potential energy. Such methods suit biological actuators like muscles and are also computationally efficient: Closed form formula are developed for converting tweaking parameters to control effort.

This chapter is laid out in this way: Section 4.1 introduces the basic idea of group and symmetry from intuitive geometry examples to more abstract algebraic formulation. Section 4.2 investigates application of the Controlled Lagrange Method. At last an example is provided in Section 4.3 to illustrate the idea.

In theory the ideas of group and invariant are closely related, like the two sides of a coin. Group are the transformations which keep certain property invariant. When searching for the group transformation, the invariant property is also determined.

In Motor Invariant Theory, the quantitative properties that are preserved during group transformation are called *Local Motor Invariant*.

4.1 Group and Symmetry

For the more traditional geometrical perspective, "Symmetry" means a geometry is the same after certain transformation. For example, a square remains the same shape after 90 degree clockwise rotation, as shown in Figure 4.1.

Actions that preserve the square shape can be combined. For example, if the action of

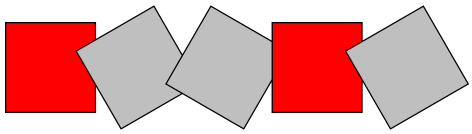


Figure 4.1: *Symmetry of The Square*

90 degree clockwise rotation preserves the shape, then the action of rotating twice, i.e., 180 degree clockwise rotation also preserves the shape.

All the actions that can preserve the symmetry form a group G. A group has the following properties.

1. For any g_a, g_b in G, $g_a * g_b$ belongs to G. (The operation "$*$" is closed).

2. For any $g_a, g_b, g_c \in G$, $(g_a * g_b) * g_c = g_a * (g_b * g_c)$. (Associativity of the operation).

3. There is an element $e \in G$ such that $g_a * e = e * g_a = g_a$ for any $g_a \in G$. (Existence of identity element).

4. For any $g_a \in G$ there exists an element g_h such that $g_a * g_h = g_h * g_a = e$. (Existence of inverses).

For the square example, all the actions preserve the square shape form the group G. g_1 is 90 degree clockwise rotation, identity element e is the action of no rotation, $g_2 = g_1 * g_1$ is the action of rotating 90 degree clockwise twice. Since g_2 preserves symmetry, g_2 is an element of the group G,

From the algebraic perspective, "Symmetry" means the value of function is invariant after transformation. For a function $I(x)$, the group transformation is define by $\tilde{x} = g_a(x)$. By symmetry, we mean $I(x) = I(\tilde{x})$. $I(x)$ is an invariant function of group G.

Note that shapes invariant by actions in G are not unique. Many shapes are invariant, and their combinations are also invariant, as shown in Figure 4.2. In the algebraic sense, invariant functions of group G form a space, the invariant space I^G.

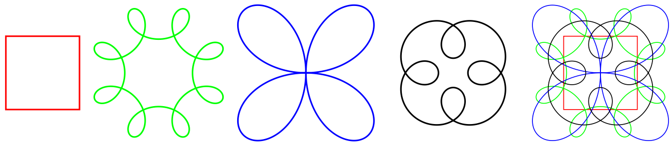


Figure 4.2: *Two invariant Shapes and the invariant combination*

4.1.1 Lie Group and Differential Equation

Physically-based motions are usually described by differential equations, and motion is the solution of the equation. Same as the square shape, there are also symmetry groups that keep the differential equations invariant. An important property of such a group is that its elements can transform the solution of differential equations from one into another(Olver *et al.*, 1986). For **CMS**, this property can potentially help reduce computational burden: new motions can be achieved through applying transformation to the dynamic equations of motion primitives.

In mathematical theory, *Lie Group* is continuous group, which is also a manifold. Since it is a manifold, coordinate system can be assigned to a Lie Group and each elements can be parameterized. For example, the symmetry rotation group of square is discrete, while symmetry group of circle is continuous. For the symmetry group of the circle, each element can be parameterized by the the rotation angle. In the following discussions, ε is the parameter of a element g in the group G.

Theory of Lie group comes from the study of differential equations. For the differential equation in Equation 4.1.

$$\dot{\mathbf{x}} = F(\mathbf{x}) \tag{4.1}$$

Invariant function I can be defined as:

$$I(t, \mathbf{x}, \dot{\mathbf{x}}) = F(\mathbf{x}) - \dot{\mathbf{x}}$$

Solutions of the differential equation are the kernel of the invariant function I :

$$I(t, \mathbf{x}, \dot{\mathbf{x}}) = 0$$

The group transformation will act on all the variables of the invariant function. Therefore t, $\mathbf{x}$ and $\dot{\mathbf{x}}$ are all transformed.

$$(t, \mathbf{x}, \dot{\mathbf{x}}) \mapsto (\tilde{t}, \tilde{\mathbf{x}}, \dot{\tilde{\mathbf{x}}})$$

If the group G is symmetrical, then value of the function I will be invariant. Therefore the kernel is transformed into kernel, and the transformed variables are still solutions to the original differential equations.

$$I(t, \mathbf{x}, \dot{\mathbf{x}}) = I(\tilde{t}, \tilde{\mathbf{x}}, \dot{\tilde{\mathbf{x}}}) = 0$$

Note that the $\dot{\mathbf{x}}$ is not independent which depends on the t and $\mathbf{x}$,

$$\dot{\tilde{\mathbf{x}}} = \frac{d\tilde{\mathbf{x}}}{d\tilde{t}}$$

From the geometrical perspective, it is not easy to present the transformation of t. Instead, we define two actions on the state space and tangent space. In the state space, we define the action g that transforms the state.

$$g(\mathbf{x}) = \tilde{\mathbf{x}}$$

In the tangent space, we define the *lift action* Tg

$$Tg(\dot{\mathbf{x}}) = \dot{\tilde{\mathbf{x}}}$$

Tg can be worked out by formatting the derivatives in the original coordinate system. For example, the translation g_ε

$$(x, y) \mapsto (x + \varepsilon, y + \varepsilon)$$

Tg_ε is

$$(\dot{x}, \dot{y}) \mapsto (\dot{x}, \dot{y})$$

Tg is the identity element e.

In the general cases, g transforms Equation 4.1 into Equation 4.2

$$Tg(\dot{\mathbf{x}}) = F(g(\mathbf{x})) \tag{4.2}$$

If g is symmetrical, Equation 4.1 and Equation 4.2 are equivalent

For example, The scaling action is applied to the state space of the mass spring system of Equation 3.2.

$$\tilde{\mathbf{x}} = g_\varepsilon(\mathbf{x}) = [\varepsilon q, \varepsilon \dot{q}]$$

then the lift action is

$$\tilde{\mathbf{x}} = Tg_\varepsilon(\mathbf{x}) = [\varepsilon \dot{q}, \varepsilon \ddot{q}]$$

by substitution $\mathbf{x} \mapsto \tilde{\mathbf{x}}$, the original system becomes

$$\dot{\tilde{\mathbf{x}}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \tilde{\mathbf{x}}$$

which is

$$\varepsilon \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \varepsilon \mathbf{x} \tag{4.3}$$

Equation 4.3 is equivalent to Equation 3.2. If $\mathbf{x}(t)$ is a solution, so is $\tilde{\mathbf{x}}(t)$.

To verify the group property. define $*$ as:

$$g_{\varepsilon_1} * g_{\varepsilon_2}(\mathbf{x}) = [\varepsilon_1 \varepsilon_2 q, \varepsilon_1 \varepsilon_2 \dot{q}]$$

The inverse is:

$$g_{\varepsilon}^{-1} = g_{\frac{1}{\varepsilon}} \ \varepsilon \in R^{+}$$

Definition. *For a group G, the invariant function of state $I(\mathbf{x})$ is called a* local motion invariant *of G.*

Invariant functions $I(\mathbf{x})$ has important meaning in dynamics. According to **Noether's Theorem**, each $I(\mathbf{x})$ corresponds to a conservative law.

4.2 Lie Group and Controlled Lagrange

It is not enough for animals only to explore symmetry groups of natural dynamics for motion adaptation. For a dynamic system, the symmetry group is quite restricted. Working out the symmetry group might be a non-trivial task. In real-life, animals usually exert control effort during motion adaptations.

MoIT theory proposes the idea that control effort can make a non symmetrical group become symmetrical, and introduce the *Controlled Lagrange* technique. Based on biological research(Flash and Handzel, 2007), some simple groups are selected the symmetry group for motor control. When such group is applied to the dynamic system, control efforts are applied to ensure the symmetry.

Usually a dynamic system is represented as by Euler-Lagrange Equation 4.4(Goldstein *et al.*, 2002).

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0 \tag{4.4}$$

where $L = K - V$, L is the Lagrange, K is the kinetic energy, V is the potential energy, q is the generalized coordinates, and $\dot{q}$ is the generalized velocity.

By applying the group transformation g, both the generalized coordinates and generalized velocity will be changed:

$$g(\mathbf{x}) = \tilde{\mathbf{x}} = [\tilde{q}, \dot{\tilde{q}}]$$

The Euler-Lagrange equation for the transformed dynamic system is described by

Equation 4.5. If control is applied, the Euler-Lagrange equation of the controlled dynamics is described by Equation 4.6. If symmetry is persevered, the two equation should be equivalent. Then symmetry control input u_1 can be calculated by comparing the two equations.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\tilde{q}}} - \frac{\partial L}{\partial \tilde{q}} = 0, \tag{4.5}$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = u_1. \tag{4.6}$$

When the two equations are equivalent, their Lagrange L, Kinetic Energy K and potential energy V should be the same or of the same scale factor. Thus in theory, two strategies exist and will result in two different u_1: we can calibrate the kinetic by scaling and apply control effort to compensate the difference in potential energy, or calibrate potential energy and compensate the kinetic energy. **MoIT** adopts the potential shaping strategy, for it is computational efficient and suitable for muscle like biological actuators. As a special case, potential energy shaping for homogeneous group or affine group promises a close form formulation. Several groups and their potential shaping control effort are as below:

Offset Action

Offset actions modify the generalized coordinate q by a constant, while speed and time remain unchanged. Given the offset parameter ε, the mapping will be in the following form:

$$(t, q, \dot{q}) \mapsto (t, q + \varepsilon, \dot{q})$$

The corresponding state transformation and lift action are

$$g_f(\mathbf{x}) = [q + \varepsilon, \dot{q}] \tag{4.7}$$

$$Tg_f(\dot{\mathbf{x}}) = \dot{\mathbf{x}} = [\dot{q}, \ddot{q}] \tag{4.8}$$

On the phase plot, the configuration q is usually represented by the horizontal axis, and the generalized speed $\dot{q}$ is represented by the vertical axis. From the geometrical perspective, offset actions will move the phase portrait horizontally as shown in Figure 4.3.

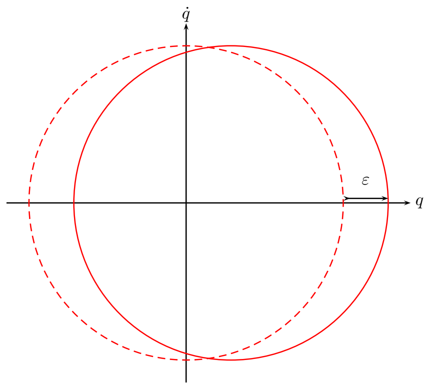


Figure 4.3: *Offset Action*

Substituting the transformed q and $\dot{q}$ into Equation 4.5 and Equation 4.6, the control input can be worked out in the following closed form formula:

$$u_1(q) = \frac{\partial}{\partial q}\left(V(q) - V(\tilde{q})\right). \tag{4.9}$$

Taking the mass spring system of Equation 1.5 as an example, the transformed equation and control equation are as follows.

$$\ddot{\tilde{q}} + \tilde{q} - \varepsilon = 0$$
$$\ddot{q} + q = u_1$$

By comparing the two equations, we work out that:

$$u_1(q) = \varepsilon$$

Time Scaling

Time scaling actions divide the time variable by a factor ε. The generalized coordinates are kept unchanged, and the generalized speed will be multiplied by ε. For the action of parameter ε, the action mapping is:

$$(t, q, \dot{q}) \mapsto (\frac{t}{\varepsilon}, q, \varepsilon\dot{q})$$

The corresponding state transformation and lift action are

$$g_{\mathrm{t}}(\mathbf{x}) = [q, \varepsilon\dot{q}]$$
$$Tg_{\mathrm{t}}(\dot{\mathbf{x}}) = [\varepsilon\dot{q}, \varepsilon^2\ddot{q}]$$

From a geometrical perspective, time scaling will stretch the phase portrait vertically, as shown in Figure 4.4.

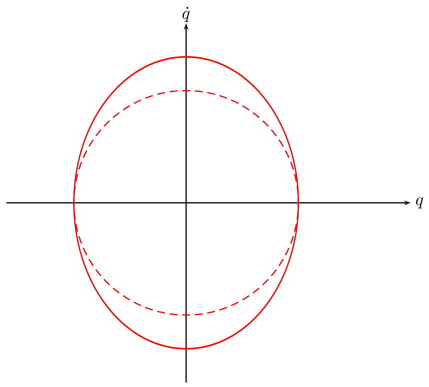


Figure 4.4: *Time Scaling Action*

The control input can be worked out in the same manner as offset actions. There is also a closed form formula for control input.

$$u_l(q) = (1 - \varepsilon^2)\frac{\partial V(q)}{\partial q}. \tag{4.10}$$

Again, taking the mass spring system of Equation 1.5 as an example, the transformed and controlled equations are

$$\frac{\ddot{\tilde{q}}}{\varepsilon^2} + \tilde{q} = 0$$
$$\ddot{q} + q = u_l$$

The local control input is:

$$u_l = (1 - \varepsilon^2)q$$

Energy Scaling

For the dynamic system of the conservative field, the energy is preserved in motion and different motions are characterized by their energy. For such a system, motion can be adapted by modifying the energy of the dynamic system.

Energy Scaling action is introduced to adapt motions. The scaling transformation has the following property:

$$E(\tilde{\mathbf{x}}) = \varepsilon^2 E(\mathbf{x})$$

where E is the energy, defined as $E(\mathbf{x}) = K + V$, K is the kinetic energy, and V is the potential energy.

Further suppose that both the potential and kinetic energy are transformed uniformly.

$$K(\tilde{\mathbf{x}}) = \varepsilon^2 K(\mathbf{x})$$
$$V(\tilde{\mathbf{x}}) = \varepsilon^2 V(\mathbf{x})$$

When mass inertia matrix is constant, the energy scaling transformation is linear as follows:

$$(t, q, \dot{q}) \mapsto (\frac{f(\varepsilon)}{\varepsilon}t, f(\varepsilon)q, \varepsilon\dot{q})$$

$f(\varepsilon)$ is a function of ε, which is determined by the conservative field. Geometrically, an energy scaling action enlarges the phase portrait,as shown in Figure 4.5.

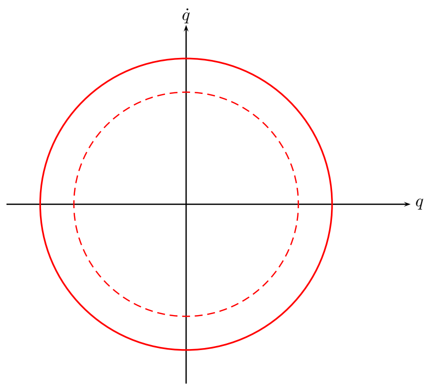


Figure 4.5: *Energy Scaling Action*

The corresponding state transformation and lift action are:

$$\begin{aligned} g_e(\mathbf{x}) &= (f(\varepsilon)q, \varepsilon\dot{q}) \\ Tg_e(\dot{\mathbf{x}}) &= (\varepsilon\dot{q}, \frac{\varepsilon^2}{f(\varepsilon)}\ddot{q}) \end{aligned} \tag{4.11}$$

u_l can by worked out in the same manner as the above actions. Rather than write down the closed form formula, the thesis prefers an alternative process. Energy Scaling can be seen as a combined action of two actions: scaling the generalized coordinates and scaling the time variable. Separate formula can be developed for two actions independently. This principle generates modular code structure.

The mass spring system of Equation1.5 is selected again as an example. For the mass spring system, Energy is defined as $E = \frac{1}{2}(q^2 + \dot{q}^2)$. If the energy is scaled up by ε^2, the potential energy is scaled up by ε^2. Because $V = \frac{1}{2}q^2$, and $\varepsilon^2 V = \frac{1}{2}(f(\varepsilon)q)^2$, thus

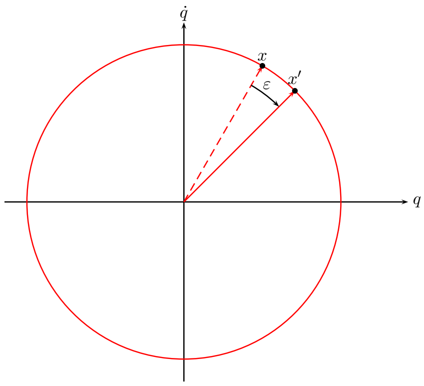


Figure 4.6: *Offset Action*

$f(\varepsilon) = \varepsilon$.

The control input can be worked out in the same manner as the above actions. However, when object moved in the conservative field, energy scaling is a symmetry group of the original dynamic system, thus no control effort is needed.

$$u_1 = 0$$

Time Offset

Time offset actions modify the time variable t by the parameter ε. The map is as follows

$$(t, q, \dot{q}) \mapsto (t + \varepsilon, q, \dot{q})$$

For a system oscillating with limit cycle, time offset action will modify the phase, as shown in Figure 4.6.

For a dynamic system, time offset is symmetrical for all dynamic system. At the first look, no control effort is needed. In practise, time offset is achieved by applying time scaling twice, after applying time scaling ε for sometime, and then apply the inverse action(time scaling of $\frac{1}{\varepsilon}$).

4.2.1 Action Selection

There are many actions available for motion adaptation. In certain situations, there are many different ways to satisfy the motion constraints, causing the problem which action should be applied. Different groups will result in different motion styles. This idea is supported by lots of examples in Chapter 6. In practise, this is left for the animator to decide. Usually, the symmetry of natural dynamic is preferred, for such actions are energy efficient.

4.3 Example: Symmetry of the Bouncing Ball System

Symmetry is a common property among many dynamic systems, even for the hybrid systems like the bouncing ball system of Equation 3.7. It is shown in this section that by utilizing the symmetry group, complex motions can be predicted in an computationally efficient way.

The bouncing ball system of 3.7 has a energy scaling symmetry.

The energy function of the bouncing ball system is

$$E = \mathrm{g}q + \frac{1}{2}m\dot{q}^2$$

If the energy is scaled up by ε^2, potential energy is scaled up by ε^2. Because $V = \frac{1}{2}\mathrm{g}q$, and $\varepsilon^2 V = \frac{1}{2}f(\varepsilon)q$, thus:

$$f(\varepsilon) = \varepsilon^2$$

the energy scaling transformation is

$$g_{\mathrm{e}}(\mathbf{x}) = [\varepsilon^2 q, \varepsilon\dot{q}]$$

For the bouncing ball system, the energy of a system can be characterized by the initial dropping height.

Given the motion of a ball dropped at 5 as shown in Figure 4.7, we set $\varepsilon = \sqrt{2}$ and obtained the motion dropped from 10 through the transformation as shown in Figure 4.8. Figure motion dropped from 10 is shown in Figure 4.9.

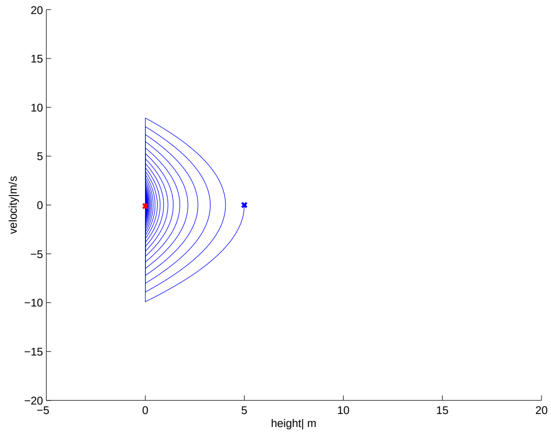


Figure 4.7: *Drop at 5*

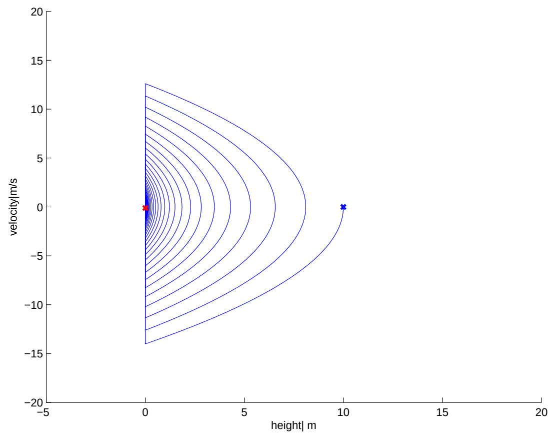


Figure 4.8: *Drop at 10 by transformation*

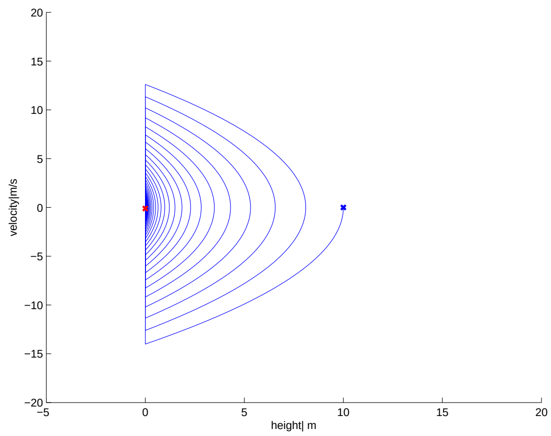


Figure 4.9: *The simulation result of dropped from 10*

Chapter 5

MOTION SYNTHESIS FRAMEWORK

The principal ideas of **MoIT** are discussed in previous two chapters. The stability of motion is controlled by maintaining the topology. For the periodic motions, neural oscillator can be used to enhance the structural stability. And group Transformation provides a mechanism to modify motion with precision.

Questions arise when these ideas are being applied to **CMS**. The first question comes from combining the controller of neural oscillator and symmetry controller. We must ensure that the combination will violate neither the symmetry nor the topology. This question is discussed in details in Section 5.1.

Section 5.3 provides more detailed information of the pipeline, or the procedure of applying this idea in **CMS** applications.

5.1 Combined Invariant Control

5.1.1 Combine Invariant Control

Neural control u_{o} maintains the topology, and local control u_{l} maintains the symmetry. Combining two controllers must violate neither the global or local invariant.

In order to adjust the combined controller for practical applications, **CPG** is applied first to maintain the topology against the structural perturbation. Then symmetry controllers are applied afterwards to meet application specific constraints.

From the perspective in Chapter 3, the inclusion of symmetry control must not violate the topology. It is easy to prove that controlled symmetry maintains the topology. For the controlled symmetry's effect on topology, we have the following theorem:
Theorem. *Transformation of Control Symmetry is Topological Conjugation*

From the perspective in Chapter 4, we must ensure the inclusion of neural oscillator control u_{l} will not break the controlled symmetry.

For this, the parameters of **CPG** need to be modified accordingly to maintain the symmetry property. This is called *Adjoint Transformation*.

5.1.2 Adjoint Transformation of CPG

Adjoint Transformation modifies the parameters of neural oscillator to maintain the symmetry.

For a dynamic system

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

when controlled by neural oscillator, it becomes

$$\dot{\mathbf{x}} = F(\mathbf{x}) + Du_{\mathrm{o}} \tag{5.1}$$

where D is the connection matrix, which describes how the neural oscillator is connected to mechanical system.

When group action g is applied, Equations 5.1 is transformed into

$$Tg(\dot{\mathbf{x}}) = F(g(\mathbf{x})) + DTg(u_o) \tag{5.2}$$

If symmetry is preserved, the Equation 5.3 and Equation 5.2 should be equivalent.

$$\dot{\mathbf{x}} = F(\mathbf{x}) + u_l + D\tilde{u_o} \tag{5.3}$$

where $\tilde{u_o}$ is the output of neural system after adjoint transformation.

As shown in Equation 3.6, since u_o is a complex function of u_i, it is difficult and not computational efficient to develop a closed form formula. As an alternative, the idea is to utilize the symmetry property of Matsuoka Oscillator. In this way, CPG can be transformed by modifying the parameters. The transformation scheme is based on the following proposition.

Proposition. *By modifying parameter* $\tau_{1,2}$

$$\tau_{1,2} \mapsto \varepsilon\tau_{1,2}$$

is equivalent to time scaling of the neural oscillator by parameter ε*.*

This proposition can be easily proved by substituting $\tilde{\tau}_{1,2} = \varepsilon\tau_{1,2}$, and $\tilde{t} = \frac{t}{\varepsilon}$ into the Matsuoka Oscillator(Equation 3.4), the equation will remain the same. Based on above the proposition, a scheme of the adjoint transformation is proposed that modifies the parameters $\tau_{1,2}$,h_i,h_o and maintains the symmetry of the coupled system. The input and output of neural are chosen to maintain the shape.

1. Modify τ by the time scaling parameter $\tau \mapsto \varepsilon\tau$.
2. the input variable w and input efficient h_i are modified to make sure the input function satisfies the time scaling symmetry $u_i(t) \mapsto u_i(\frac{t}{\varepsilon})$
3. Parameters of h_o are modified according to the connection matrix D, or how the mechanical system is driven. If u_o drives the position variable q then, h_o should be multiplied by the position scale factor. If u_o drives the velocity,h_o should be multiplied by the speed scale factor. If the h_o is force and acting on

the acceleration $\ddot{q}$, then h_{o} should be multiplied by the acceleration scale factor.

According to this adjoint transformation strategy, we can get the following theorem **Theorem.** *For a transformation group G, if the parameters of the neural oscillator are modified according to the adjoint transformation, combined system preserves symmetry I^G.*

To prove it, readers can check the symmetry by substituting transformed variables into the original system. With such a treatment, both the Local Motor Invariant and Global Motor Invariant are maintained. For the specific symmetry types proposed in Chapter 4, several examples of adjoint transformations are provided

Offset Symmetry.

For offset symmetry:

$$(t, q, \dot{q}) \mapsto (t, q + \varepsilon, \dot{q})$$

there is no time scaling effect. To maintain the symmetry, the simplest way is to select u_{i} and u_{o} from the functions in the invariant space I^G. For example, when all q is transformed by a constant, the difference and the velocity will not be transformed. Thus, the input of the neural oscillator is chosen to be the angle difference between the joints or velocity.

Time Scaling

For time scaling:

$$(t, q, \dot{q}) \mapsto (\frac{t}{\varepsilon}, q, \varepsilon\dot{q})$$

Adjoint Transformation $\tau \mapsto \varepsilon\tau$. The input coefficient h_{i} and output coefficient h_{o} are scaled accordingly. if the output u_{o} is applied as a force, then it should be scaled by the acceleration factor

$$h_{\mathrm{o}} \mapsto \varepsilon^2 h_{\mathrm{o}}$$

Energy Scaling

Energy Scaling is a combined action of time scaling and posture scaling:

$$(t, q, \dot{q}) \mapsto (\frac{f(\varepsilon)}{\varepsilon}t, f(\varepsilon)q, \varepsilon\dot{q})$$

the time scaling factor is $\frac{\varepsilon}{f(\varepsilon)}$,

The parameters $\tau_{1,2}$ are transformed

$$\tau_{1,2} \mapsto \frac{\varepsilon}{f(\varepsilon)}\tau_{1,2}$$

The input coefficient is scaled to make the amplitude of the input signal maintained.

$$h_{\mathrm{i}} \mapsto \frac{h_{\mathrm{i}}}{\varepsilon}$$

The output coefficient is scaled according to the connection of the control, if the output drive the velocity, then the output is h_{o}

$$h_{\mathrm{o}} \mapsto \varepsilon h_{\mathrm{o}}$$

5.1.3 Example: Height Control of Bouncing Ball

The bouncing ball system has the energy scaling symmetry, and a limit cycle emerged when coupled with a neural oscillator. When energy transformation is applied to the limit cycle, the bouncing height can be adjusted according to the purpose. By combining both motor invariant controllers, stability is maintained and motion can be adjusted precisely.

Adjoint Transformation

Supposing the coupled system is bouncing at height of 5 For the energy scaling:

$$(t, q, \dot{q}) \mapsto (\varepsilon t, \varepsilon^2 q, \varepsilon \dot{q})$$

the time scaling factor is ε, and we have:

$$\tau_{1,2} \mapsto \varepsilon \tau_{1,2}$$

The input to the neural oscillator is $\dot{q}$,

$$h_{\mathrm{i}} \mapsto \frac{h_{\mathrm{i}}}{\varepsilon}$$

Neural Oscillator drives the position of the paddle, the output u_{o} needs to be scaled by the position scale value. For $q \mapsto \varepsilon^2 q$, we have

$$h_{\mathrm{o}} \mapsto \varepsilon^2 h_{\mathrm{o}}$$

When $\varepsilon^2 = 3$, the ball will bounce at height of 15, and it maintains its topological structure, which is a limit cycle, as shown in Figure 5.2. With this method, arbitrary bouncing height can be controlled.

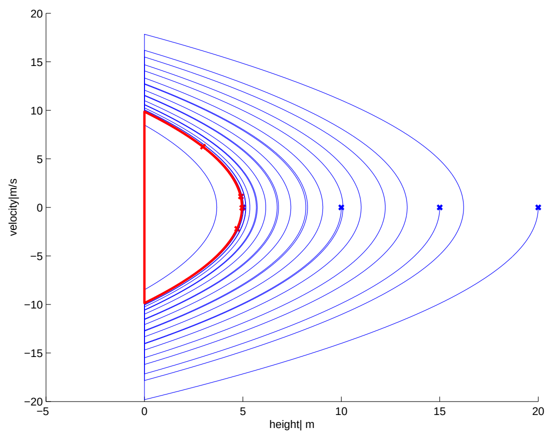


Figure 5.1: *Energy Scalling*

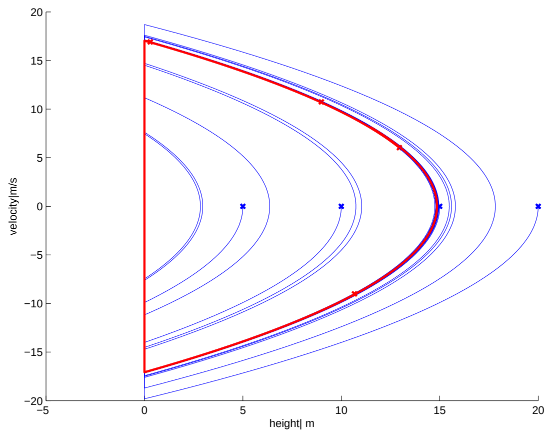


Figure 5.2: *Energy Scaling*

5.2 Combine Motion Primitives

5.2.1 Dynamic Motion Graph

Virtual characters are capable of many types of motions and switch between them fluently. *Motion Graph*(Kovar *et al.*, 2008) is proposed for data-driven **CMS**: basic motion tasks are recorded, and a graph describes how a character can change from one motion into another motion. For the transitional motions, the most popular synthesizing method is blending.

MoIT implies an idea similar to the motion graph but from a different direction. Usually, traditional *motion graph*s are manually designed, while **MoIT** proposes an idea which generates the motion graph from the dynamics automatically. In theory, the topological structure of a dynamic system can be represented by a graph. Each motion primitive is represented as a node, and two nodes are connected only if their basins of attraction(**BoA**s) are in neighbour.

In dyanmic research, many methods have been proposed to identify the topological structure of a dynamic system automatically (HSU, 1980). They can be used in **MoIT** to identify motion primitives and their connectivity.

For example, Figure 5.3 shows the phase portrait of a hypothetical dynamic system. Its phase space is divided into four regions of different colors. The four **BoA**s, within

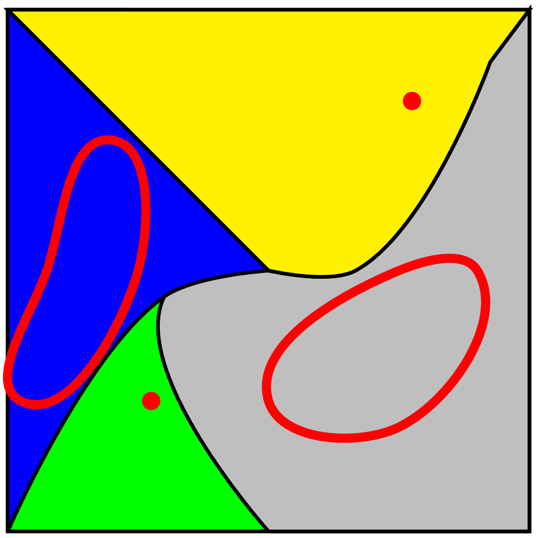


Figure 5.3: *Phase Plot of Motion Primitives*

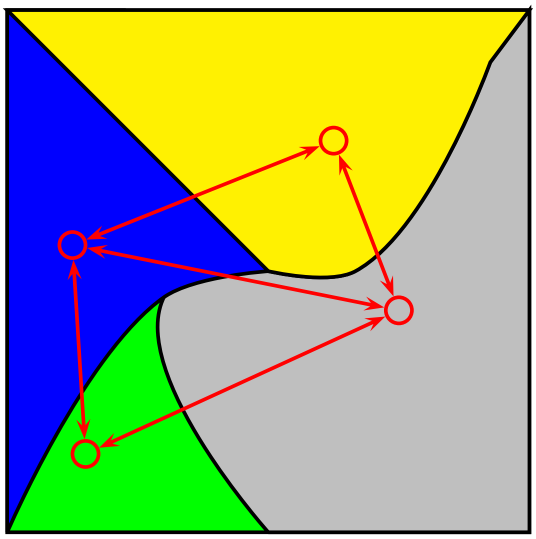


Figure 5.4: *The Graph Structure of A Dynamic System*

each region, there is an attractor(red). The graph in Figure 5.4 shows the corresponding graph structure, in which each node represents the **BoA**, the connecting edge means the basin of **BoA**s of connected motion primitives are in neighbour, which can also be verified by Figure 5.3.

5.2.2 Dynamic Motion Transition

In real life, the transition of motion is an adaptive and interesting phenomenon. However, Blending techniques tend to generate motions with little variations.

While based on the control method for maintaining motion primitives, **MoIT** proposes a physics based method for generation of transitional motion.

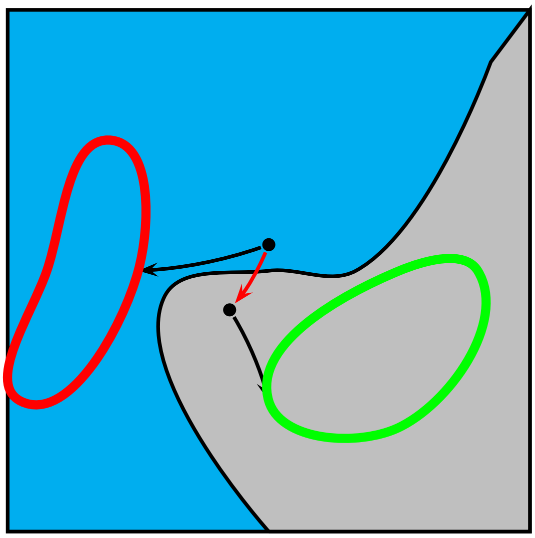


Figure 5.5: *Motion Primitive Transition*

From the geometrical perspective, motion transition means putting the current $\mathbf{x}$ out of one **BoA** into another. This process is illustrated in Figure 5.5 where the current state represented by the black dot lies in the left region of **BoA** and will converge to the red limit cycle over time.

The neighbouring region is the **BoA** of another primitive, in which if the current state lies, will converge to the green limit cycle. Because two basins of attractions do not overlap, the transition will not happen automatically without effort. From a geometrical viewport, to make motion transition happens, a small action is needed to push the state across the boundary, represented by the red line. This can be achieved by many efficient methods.

Entrainment Overlap

Empirically,when a **CPG** is applied for one motor primitive $\mathcal{A}$, the basin of attraction $\mathcal{B}(A)$ is enlarged. Supposing the enlarged basin of attraction is represented by $\mathcal{B}(A')$, if **CPG**s are applied for two motion primitives $\mathcal{A}_1$,$\mathcal{A}_2$ in neighbour, the enlarged basins of attraction ($\mathcal{B}(A_1')$ and $\mathcal{B}(A_2')$) will overlap.

$$O = \mathcal{B}(A_1') \bigcap \mathcal{B}(A_2') \neq \varnothing$$

where O is the overlapping region.

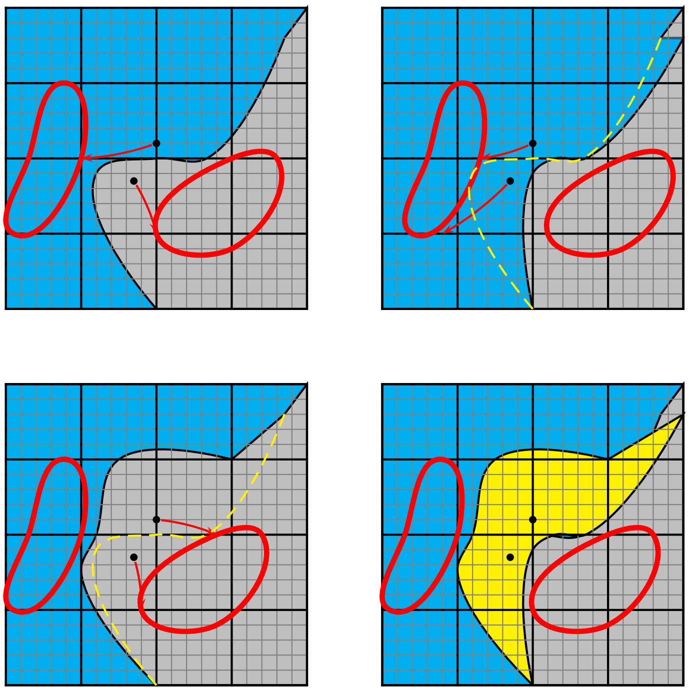


Figure 5.6: *Motion Transition based on Motion Primitives Overlap*

If state $\mathbf{x}$ lies in the O, the dynamic system will converge to a different attractor by switching the **CPG** controller. Figure 5.6 shows the idea through an example. The phase plot shows two motion primitives which are connected. Basins of attraction of natural dynamics are separated by the dotted line, which do not overlap. When **CPG** is applied, two basins of attraction are enlarged, and the shared region is coloured in yellow color. When the current state lies in O, the state will converge to the left limit cycle if the **CPG** of the left region is activated and converge to the right limit cycle if the right **CPG** is activated. Motion Primitive can be switched in this manner.

Transform Method

Controlled Symmetry can also be applied for motion primitive transition. We can change the **BoA** where the current state lies by transforming the phase portrait.

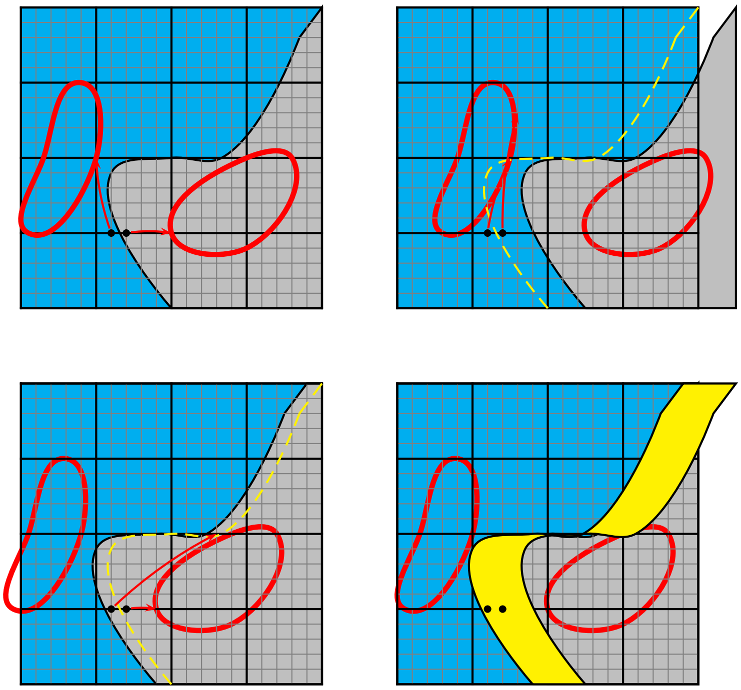


Figure 5.7: *Offset Transition*

As shown in Figure 5.7, the phase portrait of natural dynamic system is the same as that of Figure 5.6. The current state converges to the left (red) limit cycle. By applying offset action to the dynamic system, the phase portrait moves leftward, which makes current state lie in the right **BoA**. Over time, current state will converge to the right limit cycle, the motion primitive is changed accordingly.

5.2.3 Combined Method

Both methods utilize the natural dynamics and result in a physically realistic transition. However, both methods require x lies in the overlapping region. In the motor invariant theory, the current state x is not directly controlled. The measure is to make the overlap region O cover part of both attractors.

As shown in Figure 5.8, the overlap region covers both attractors $\mathcal{A}$, $\mathcal{A}'$, bidirectional transitions are possible when motion converge to to the limit cycle.

More importantly, when transformation is applied, the action is applied to the dynamic system. Thus both motion primitives are transformed, called the the *connection transformation* . As shown in Figure 5.8, when a speed action transformation is applied, both motion primitives are modified.

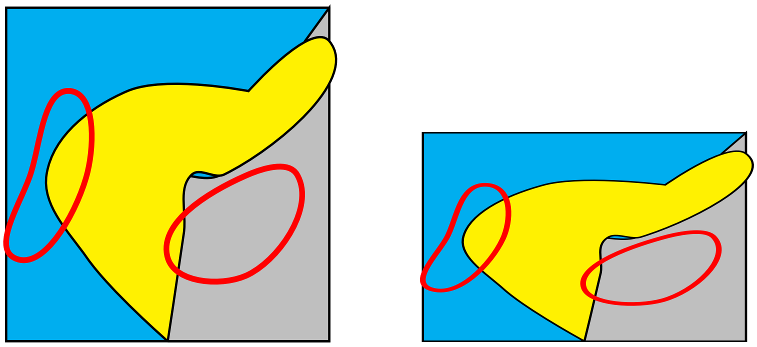


Figure 5.8: *Combined Method*

5.3 Motion Synthesis Framework

While this procedure may appear mathematically complex, applying this method for motion synthesis is straightforward.

CMSonly requires:

1. a mechanical oscillator $F(\mathbf{x})$ which describes the body and environment dynamics.
2. a neural oscillator (for example, the Matsuoka oscillator in Equation 3.4) and associated parameters that generate entrainment.
3. an action $g \in G$ which adapts the problem to the current environment (three possible operators are proposed in Section 4.1). The adjoint system transformation is applied to the neural oscillator.
4. an integrator to solve the system (we use the fourth order Runge–Kutta method provided in the MATLAB function *ode45*).

In the following chapters, this method is applied to generating adaptive motions.

Chapter 6

MOTION PRIMITIVE TWEAKING:BIPEDAL WALKING

The examples of bouncing ball and mass spring systems explain the idea well. However, they are are too simple for **CMS** applications. This chapter focus on controlling more complex mechanical systems which have great application value. Details are given about how to adapt a motion primitive for environmental and application specific constraints. Combination and transitions of motion primitives are discussed in the next chapter.

The motion primitive under study in this chapter is *bipedal walking*, which is a topic of great application value for both the graphic and robotic engineering. Although many methods have been applied to the bipedal walking in the past decades, human bipedal walking ability still has not been achieved. The early belief is that bipedal walking is unstable in nature, and many control methods are developed based on trajectory tracking principle. The turning point is the discovery of the passive dynamic walking machine, which shows that under specific conditions, walking can happen naturally without the need of any control effort. This makes us believe that the walking ability is inborn, and most control problems have already been solved by the mechanical structure.

From the perspective of **MoIT**, bipedal walking is a motion primitive. In this chapter, the passive walking gait is treated as the motion template. Neural Oscillator and Symmetry Control efforts are applied to tweaking the template while maintaining the global and local motor invariants. This method is capable of generating adaptive and stable gaits in real-time. This process may provide a clear example of application of the **MoIT** idea.

6.1 the Bipedal Walking Primitives

The word "Bipedal" comes from Latin which means "two feet" , Here, "bi" for "two" and "ped" for "foot". With two legs, animals can walk, run and jump. Relatively few modern animals use two legs for normal locomotion. Biological research believes that human bipedalism is developed well before the large human brain or the development of stone tools, so human are capable of bipedal walking long before the age of intelligence, and bipedal walking ability is not closely related to the human mental power.

The walking of human is characterized by the switch of the stiff supporting leg, which moves like an "inverted pendulum". Walking is identified there is a two leg supporting phase during each step.

As for secondary motion in walking, the hip rotates around the axis of the spine to increase stride length, and also rotates around the horizontal axis to improve balance during stance.

In **MoIT**, walking is treated as an independent motion pattern. To illustrate the idea without unnecessary complexity, the walking dynamics is simplified.

As shown in Figure 6.1, motion is projected into three spaces:the sagittal plane, coronal plane and transverse plane. For bipedal walking, yaw and roll motion are relatively small and usually treated as secondary motion or totally neglected, the main motion happens in the *sagittal plane*.

This chapter focuses on the lower body motion in sagittal plane only. The motion of upperbody in figures are added simply for visualization purpose, of which the simulation and control will not dicussed in this chapter. Along with other **DOF**s, such as

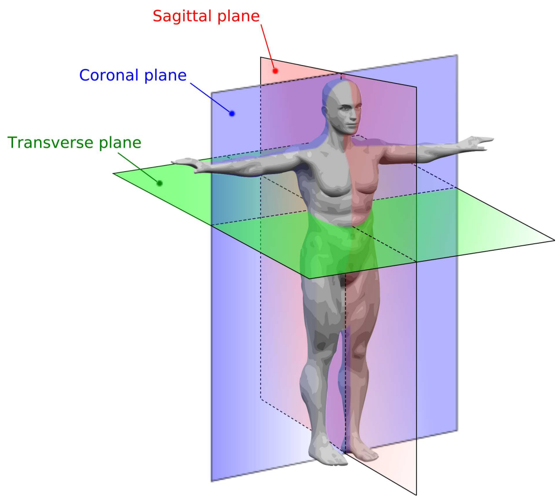


Figure 6.1: *Sagittal Plane, courtesy of Yassine Mrabet*

turning motion in coronal plane and sway motion in transverse plane, torso and arm simulations are discussed in Chapter 8.

This is because it is more convenient to explain ideas in with a simple model and perfect symmetrical properties. The motions of some **DOF**s are treated as perturbations, for they make the "symmetry" not so perfect An ealy discussion may cause confusion.

Dynamics

The simplified walking model is shown in Figure 6.2.

The walking model of Figure 6.2 is based on rigid body dynamics. The supporting leg is kept straight. In the figure, L is the length of the leg, q_1 is the angle of the supporting leg, m_t and m_s are the mass of the shank and thigh, q_2 and q_3 are the corresponding angles of the swinging shank and thigh, b_1, a_1 and b_2, a_2 describe the relative position of gravity center, m_h represents sum mass of the body and hip .

Like the bouncing ball system, this dynamic system is hybrid(Ames and Sastry, 2006) and includes both continuous and discrete dynamics. Passive walking with knees includes four phases(Chen, 2007).

- **Free Swing Phase** The support leg (the blue one) is kept straight. During this

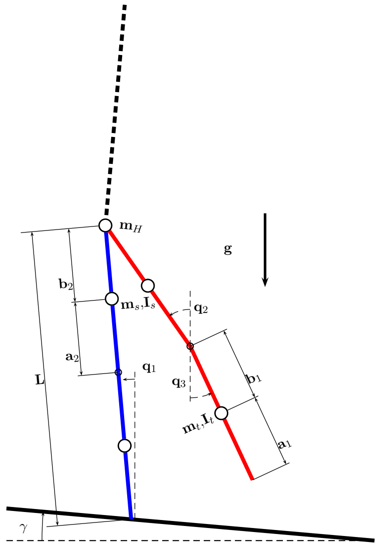


Figure 6.2: *A Passive Walking Model with Knee*

phase, the knee of the swing leg is bended, and the thigh and shank swing freely.

- **Knee Strike Phase** The knee joint of the swing leg has a limit. When the knee angle reaches the limit, a collision happens. After the collision, the swing leg is kept straight.

- **Knee Lock Swing Phase** During this swing phase, both the swing and support leg are kept straight.

- **Heel Strike Phase** When the heel of the swing leg hits the ground, a collision happens. After that the swing and support legs are switched.

Figure 6.3 shows the gaits of four phases.

- **Flying Phases** Both the free and locked knee swing phases are described by the continuous dynamics. Both equations are in the form of Equation 6.1.

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N(\mathbf{q}) = 0 \tag{6.1}$$

where $q = [q_1, q_2, q_3]$, $\dot{q} = [\dot{q_1}, \dot{q_2}, \dot{q_3}]$, M is the initial mass matrix, and C and N

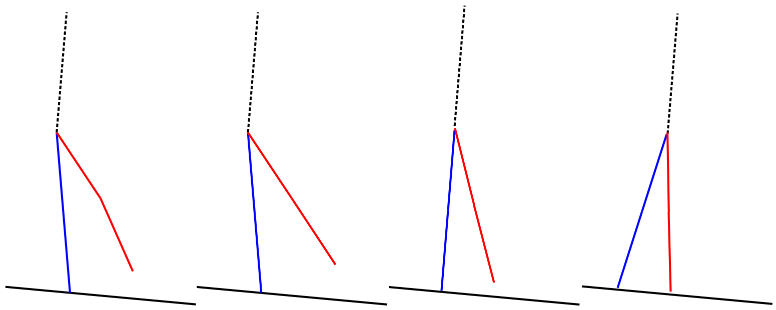


Figure 6.3: *The four phases in Walking*

are the centrifugal force matrix and gravity respectively. For Knee Free Phase, M and C are 3 by 3 matrix, and N is 3 by 1 vector. for Knee Lock Phase, M and C are 2 by 2 matrix, and N is 2 by 1 vector. Putting them into the standard form, and define $\mathbf{x} = [q, \dot{q}]$, Equation 6.1 is transformed into Equation 6.2 Then the function is in the form.

$$\dot{\mathbf{x}} = -\begin{bmatrix} \mathbf{1} & 0 \\ 0 & M \end{bmatrix}^{-1} \begin{bmatrix} 0 & \mathbf{1} \\ 0 & C \end{bmatrix} \mathbf{x} - \begin{bmatrix} \mathbf{0} \\ N \end{bmatrix} \tag{6.2}$$

- **The Strike Phases** The knee strike and heel strike phases are modelled based on discrete dynamics. Collision equations are developed based on momentum preserving principle. Both collision equations are in the form of Equation 6.3.

$$J^{+}\dot{\mathbf{q}}^{+} = J^{-}\dot{\mathbf{q}}^{-} \tag{6.3}$$

where J is the matrix of angular momentum inertia, and the superscripts $+, -$ represent those after and before collision respectively. For Knees Strike,J^{-} is a 3 by 2 matrix, J^{+} is 2 by 2 matrix; For Heel Strike, both $J^{+,-}$ are 2 by 2 matrix.

Dynamic equations are developed based on Lagrange Mechanics (Goldstein *et al.*, 2002). For details of calculating the dynamic equation, please refer to (Chen, 2007) For the components of each matrix, please refer to the appendix.

With special initial conditions(Chen *et al.*, 2007), the passive walker can walk down the slope with a stable gait. On the phase plot, a limit cycle emerges. Figure 6.4 shows

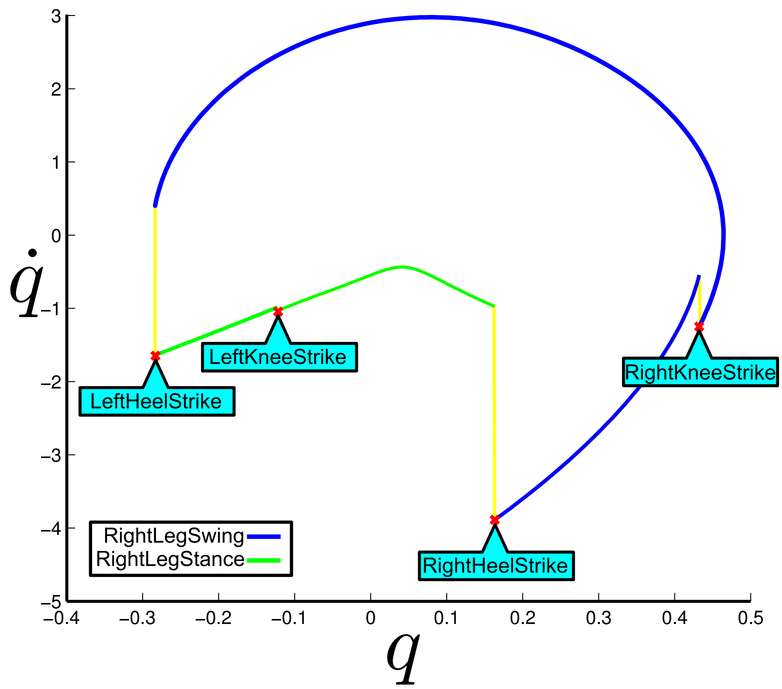


Figure 6.4: *Four Phases Marked on a Walking Cycle*

the phase plot of one thigh for a stable walking cycle. where the events that separate the four phases are marked.

In theory, the generalized coordinates for walking have 4 degrees of freedom, with angle for shank and thigh for each leg. Since the state space is 8 dimension, it is not possible to draw the phase portrait on a picture. Only 2 variables can be plotted.

Considering that motions of the two legs are almost the same, it is enough to show one leg motion, thus the state space is reduced to 4 dimensions. Chapter 8 shows that the knee motion is not very important since the motion of the thigh captures the most valuable information. The phase plot of the thigh of one leg is selected to illustrate the walking. Other selection is possible since all the **DOF**s are simulated and controlled.

Figure 6.4 only shows the motion of the right leg. The green plot shows the stance phase. During this phase, the right leg is supporting the body. The blue parts show the swing phase. During this phase, the right leg is swinging and the left leg is supporting the body. The yellow lines mark the 4 collision events during walking. Note that during the collision, the walking dynamics is discontinuous, and the speed of walking is changed suddenly without changing the position. This means the yellow segments are not on the limit cycle. If the walker starts from the state in the middle of the yellow segment, it will fall.

6.2 Global Motor Control and Adaptive Gaits

The Passive Dynamic Walker exhibits a natural looking gait. However, the walking motion is not stable. In **MoIT**, the repetitive walking motion suggests that the natural walking dynamic forms a limit cycle. It is believed that humans utilize the limit cycle for walking for energy efficiency(Collins and Ruina, 2005).

To overcome the fragile stability, **CPG** is applied with the hope to make the walking more stable through entrainment. Experiments have shown that stability is enhanced and different perturbations result in varied and natural looking responsive motions.

6.2.1 Entrainment

For walking, only one neural oscillator is applied to maintain the stability of limit cycle. The output of neural oscillator works as torque applied to hip angle (angle between the two thighs). The dynamics are shown in Equation 6.4

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N(\mathbf{q}) = Du_{\mathrm{o}} \tag{6.4}$$

For the knee lock phase, $D = [1, -1]^T$. For the knee free phase, $D = [1, -1, 0]^T$. This means the neural oscillator controls the thigh, and the knee is left to swing freely.

CPG prefers periodic, continuous signals, the hip angle is a convenient choice.

$$u_{\mathrm{i}} = h_{\mathrm{i}}(q_1 - q_2)$$

τ_1, τ_2 are set to make the oscillating frequency close to the walking frequency. The output coefficient h_{o} is set to a small value to make the walking energy efficient.

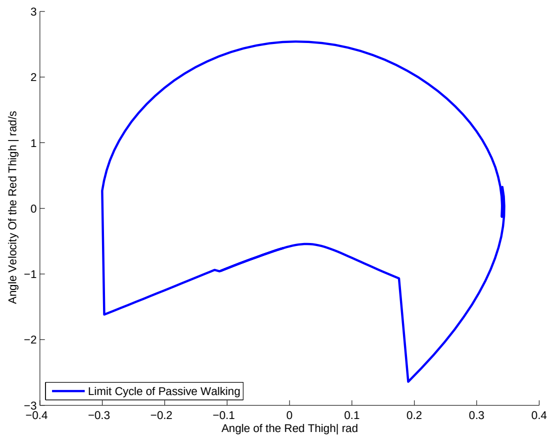


Figure 6.5: *Limit Circle And Different Phase in Passive Walking*

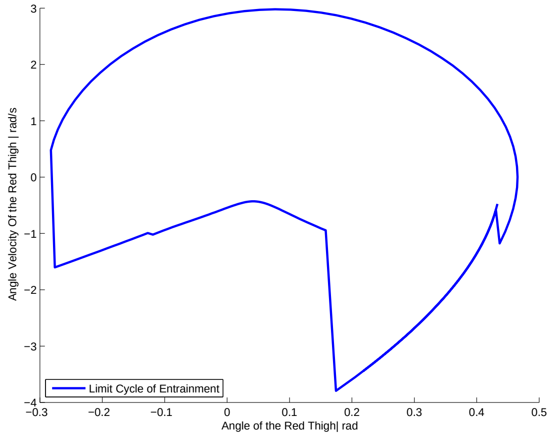


Figure 6.6: *The gait with neural controller*

When the drive force is small, the limit cycle of entrainment system is similar to the original passive one. Both limit cycles are shown in Figure 6.5 and Figure 6.6. Walking gaits are shown in Figure 6.8 and Figure 6.7. Both figures are sampled by the same time interval. The controlled gait looks a little sparser. It means that with the neural control input, the character walks a bit quicker.

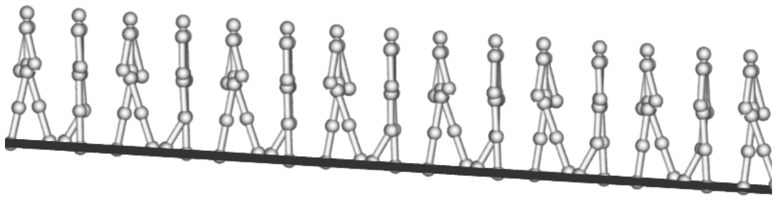


Figure 6.7: *The Passive Walking Gait*

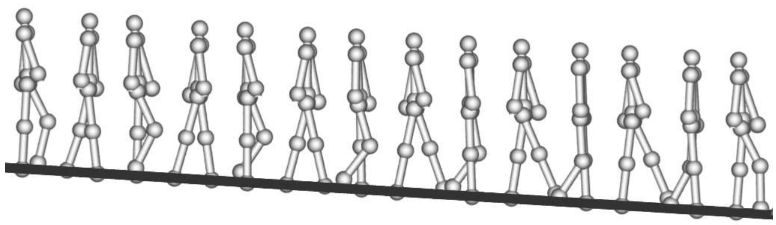


Figure 6.8: *Passive Walking with Neural Control*

By comparing the limit cycles and the walking gaits, we find out that the controlled gait and passive gait are quite similar. The controlled gaits are a bit faster and the step size is slightly bigger. Visually, the two gaits are almost the same. Although both are natural looking and very hard to detect control effort, the dynamics has been changed greatly, especially the stability.

Structural Stability

Entrainment boosts the structural stability of walking. The passive walking can not be maintained on plane, because such a structure perturbation of slope angle has violated the topology. The consequence is that limit cycle does not exist any more.

When passive walker walks on a plane, the step-size decreases after each step. After several steps, the walker will stop or fall over, as shown in Figure 6.9.

After coupling with a neural oscillator, the walker maintains walking with a small step size, as shown in Figure 6.10. To maintain the energy efficient property of natural motion, u_o is limited to small, leading to a small step size accordingly.

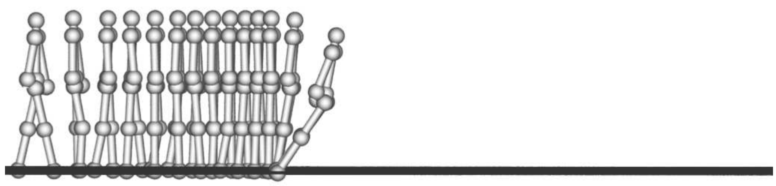


Figure 6.9: *The Passive Gait On Plain*

In Figure 6.11, the walking cycle is kept shrinking over time, resulting in a gait of walking to stop intention. But after several steps, the walking gaits reach a limit cycle (shown in red). The new walking limit cycle is of a smaller size, which means a smaller step.

Area of Basin of Attraction

Another measurement for stability is to size of the basin of attraction. Passive walking is fragile, which means the basin of attraction is very narrow. If the walker is pushed, it will fall.

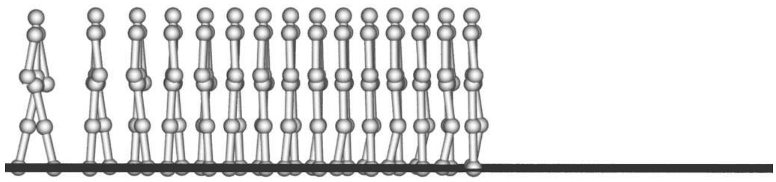


Figure 6.10: *Entrainment Gait On Plane*

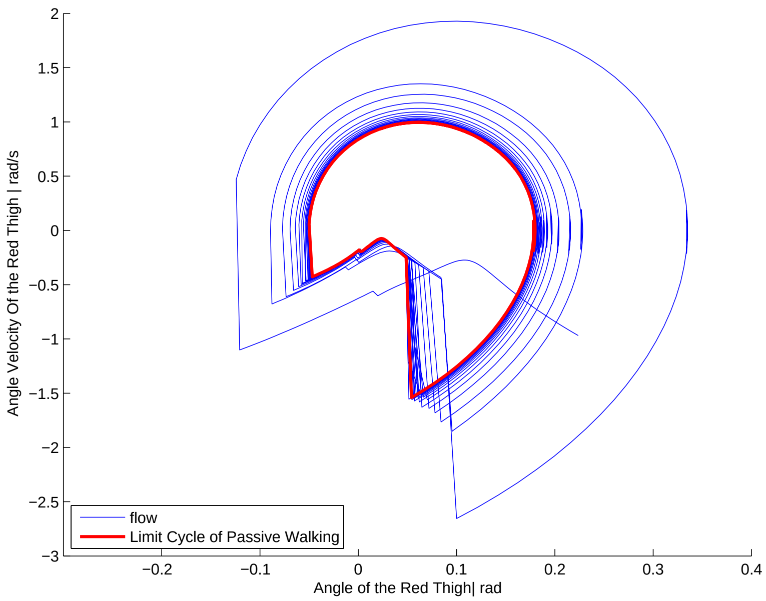


Figure 6.11: *Limit Cycle of entrainment gait on plane*

Entrainment greatly enlarges the basin of attraction of the walking limit cycle. In Figure 6.11, the initial position is far from the limit cycle. It indicates that the basin of attraction has been enlarged.

A better test is to push or pull the walking character. When push and pull are applied to the character, the state is moved away from the limit cycle. The harder the push or the pull is, the further it moves away. The gaits of being pushed or pulled are shown in Figure 6.12 and Figure 6.13. The push and pull are applied at the end of the first step, the moment when the leftmost character figures are rendered on the pictures. For both cases, the characters start walking with normal stable gaits.

When the character is pushed, the supporting leg move forward while the motion of the swing leg remain almost the same. As a result, the push effect increases the hip angle, which is the input signal of the neural oscillator. Due to the increase of input, the neural oscillator will generate a bigger torque output, which increase the hip angle and drive the character to take a big step. As time goes on, the state will converge to limit cycle and the character will return to the normal gait. When the character is pulled backward, the character will take a smaller step or even step backwards for one or two steps. After that it will gradually return to the normal walking gait.

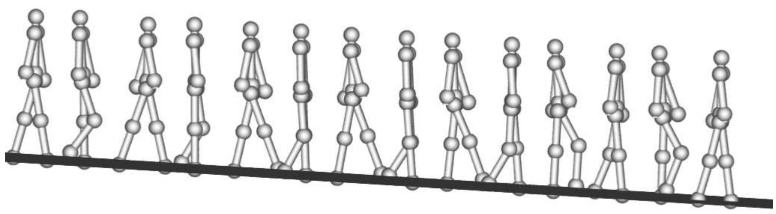


Figure 6.12: *The Push Perturbated Gait*

Figure 6.14 and Figure 6.15 show the flow converging towards the limit cycle. When the character is pushed, it takes a big walking cycle. However because of the entrainment, hlconverges to the limit cycle within next a few period. The pull effects make the

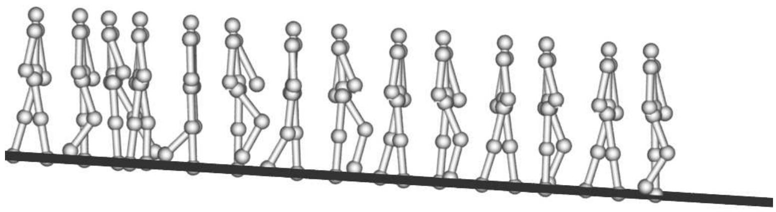


Figure 6.13: *The Pull Perturbated Gait*

character take a smaller step size in the next several steps. The walker takes a bigger or smaller step to adjust walking and finally returns to the normal walking gait.

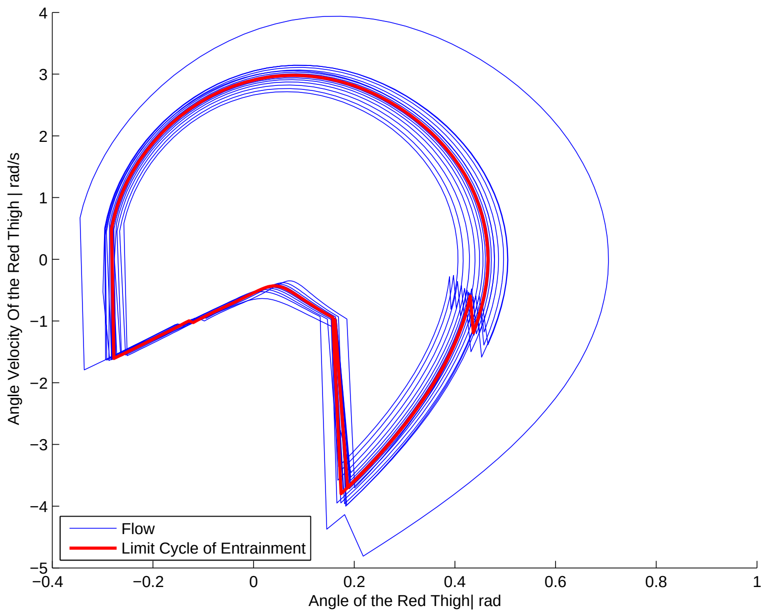


Figure 6.14: *The Pushed Gait Phase Plot*

The initial step size can also be changed, and the walker will adjust it automatically. Figure 6.16 and Figure 6.17 show the gaits. Figure 6.18 and Figure 6.19 show the phase plots.

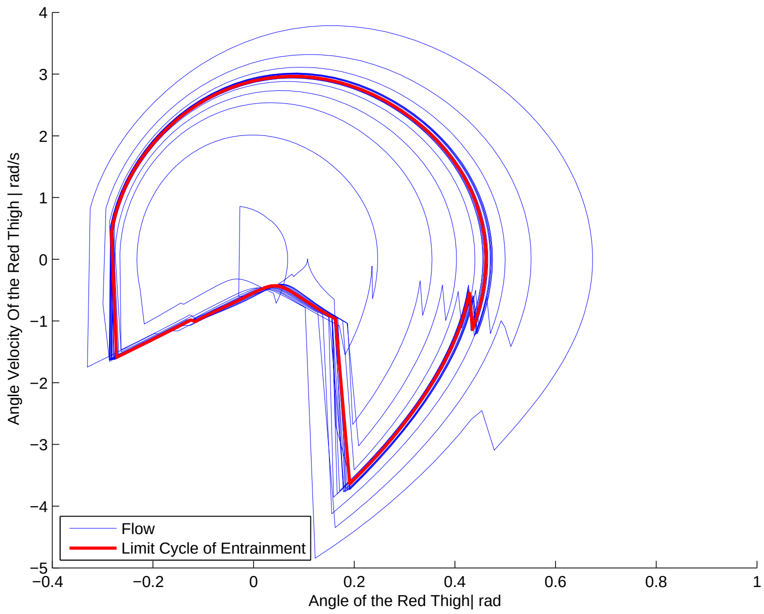


Figure 6.15: *The Pulled Gait Phase Plot*

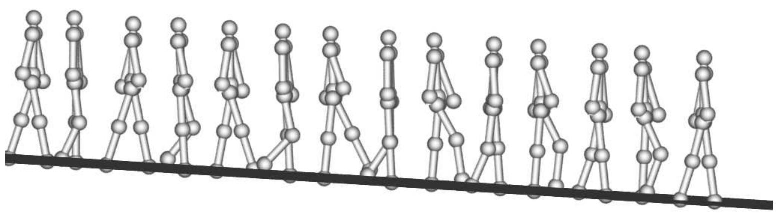


Figure 6.16: *Big Initial Step Size*

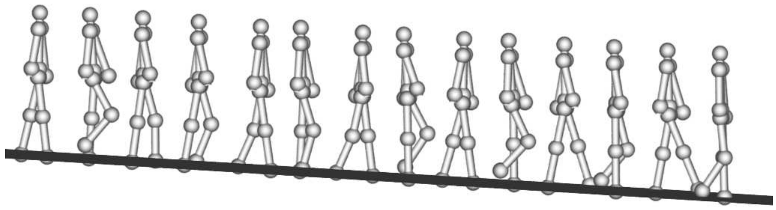


Figure 6.17: *Small Initial Step Size*

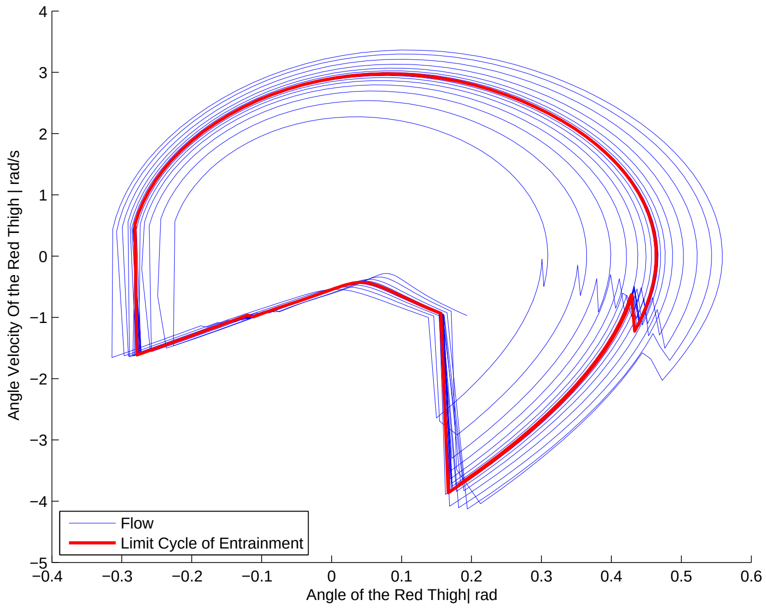


Figure 6.18: *Big Initial Step Initial Phase Plot*

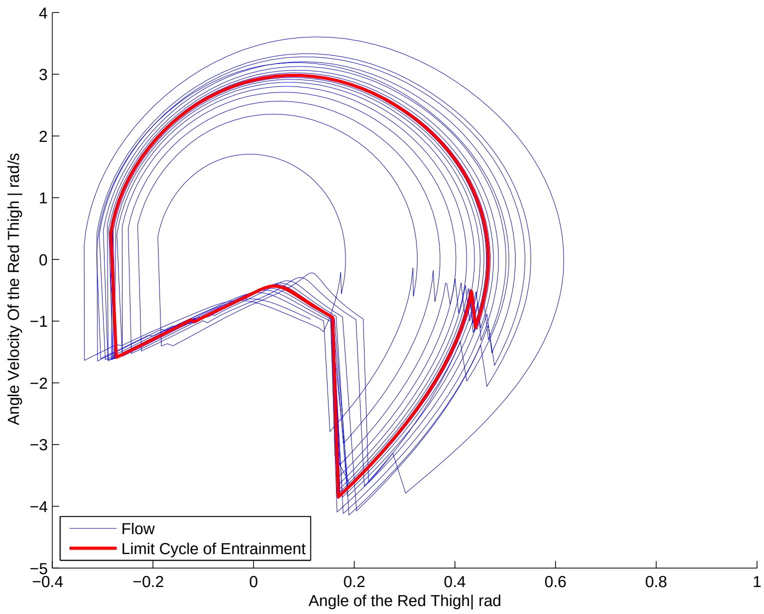


Figure 6.19: *The Small Initial Step Gait Phase Plot*

The entrainment of **CPG** greatly enlarges the basin of attraction. If the walker starts with very different postures, the character will return to normal walk.

6.2.2 Walking Re-targeting

Transferring the gait of one character to another is a challenging job. **MoIT** theory provides a method for physics based motion re-targeting. **CPG** will maintain the topology of the dynamics. When the dynamic parameters are changed, the topological conjugacy will result in a varied motion.

The passive walker has many parameters, like mass and leg length. Different parameters will result in a different dynamics systems. But all these dynamic systems share the same topology. There is a limit cycle and the characters are capable of periodic gaits. Some interesting gaits are shown and discussed below in this section.

If all the parameters are scaled uniformly, the gait will remain the same, only the velocity will be changed. To demonstrate different gaits, the parameters are modified relatively. The motion variation is generated by adjusting the mass ratio and mass distribution ratio, the total mass and total leg length of all examples are kept the same.

Mass Distribution Ratio

When the total mass is maintained, Mass Distribution Ratio is defined as the hip mass over leg mass.

$$\alpha_m = \frac{m_h}{m_s}$$

where m_h is the mass of the hip and m_t is mass of the thigh. The mass ratios of shank and thigh is kept unchanged.

Different α_m will result in different gaits. Bigger α_m result in gaits to that look burdened. The different limit cycles are shown in Figure 6.20.

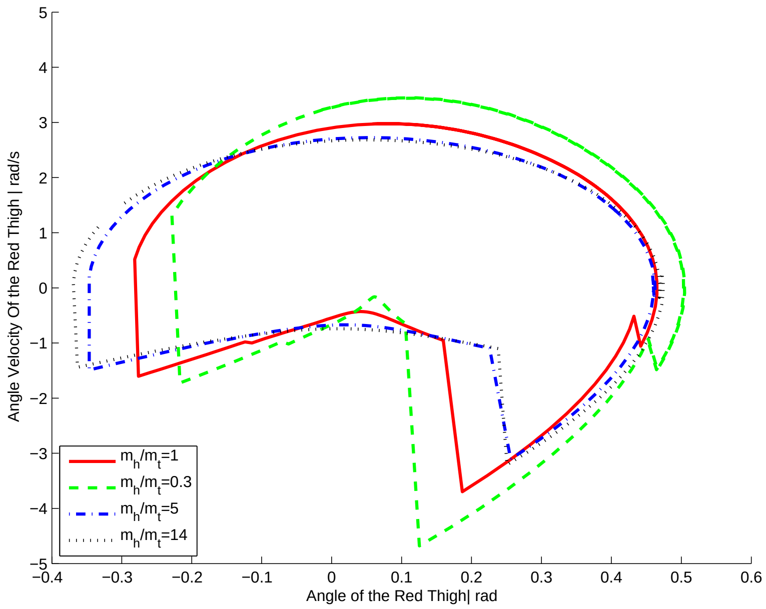


Figure 6.20: *Different Gait Resulting from the Different Mass Ratio*

For bigger α_m, the walker will walk with a bigger step but a slow speed($\dot{q}$ is lower). For smaller α_m, character will walk more quickly($\dot{q}$ is bigger), the swing leg will swing with a bigger amplitude.

Different gaits are shown in Figure 6.21, Figure 6.22 and Figure 6.23.

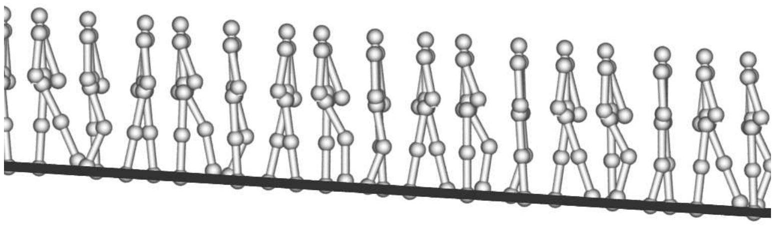


Figure 6.21: *Gait with* $\alpha_m = 0.3$

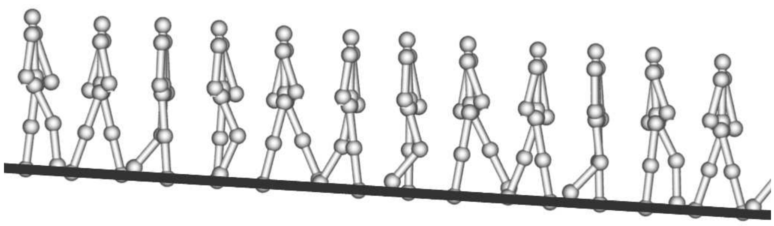


Figure 6.22: *Gait with* $\alpha_m = 5$

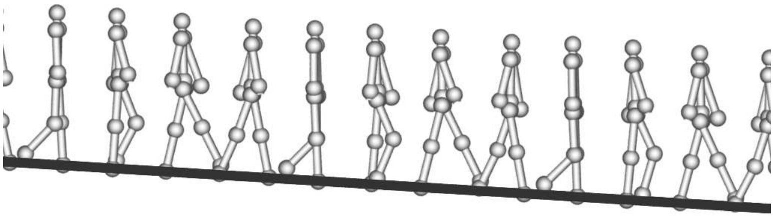


Figure 6.23: *Gait with* $\alpha_m = 14$

Leg Length Distribution Ratio

Except for the change of the ratio parameter $\alpha_l = \frac{l_t}{l_s}$, the leg length is kept unchanged. By changing α_l motion for different characters are generated. This demonstrates the motion re-targeting results.

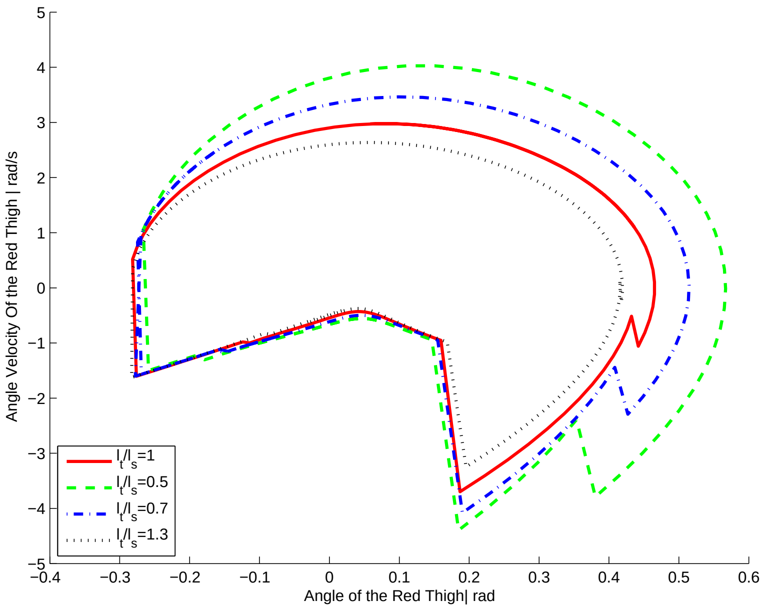


Figure 6.24: *Different Gait Resulting from the Different Mass Ratio*

The limit cycle in Figure 6.24 implies something important about leg length in walking.

Basically, the motions of the supporting leg and the step size are almost kept the same, while different leg length rations will result in different swing motions. The longer the shank, thigh has to swing quickly and with a bigger amplitude. There are also bigger impulses during the strike phase. For both the knee and heel strike, larger impulse is generated. This result may indicate the effects of high heel shoes for walking.

Figure 6.25, Figure 6.26 and Figure 6.27 show the different gaits.

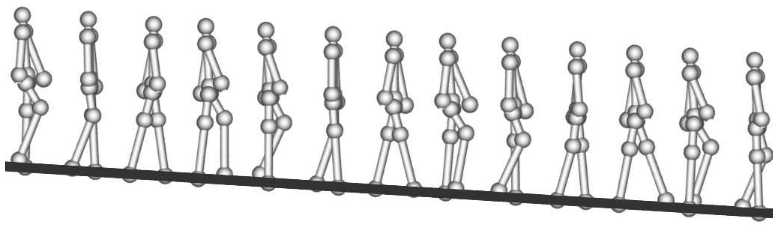


Figure 6.25: *gait of* $\alpha_l = 0.5$

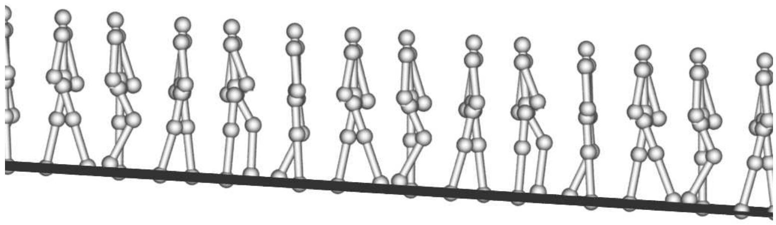


Figure 6.26: *gait of* $\alpha_l = 0.7$

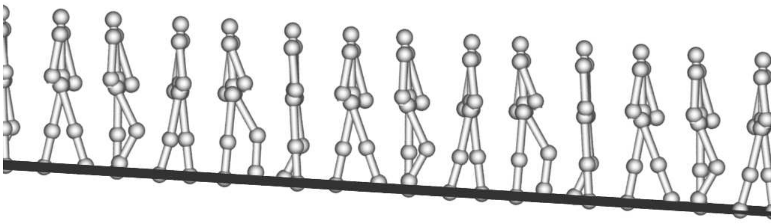


Figure 6.27: *gait of* $\alpha_l = 1.3$

Unbalanced Mass Ratio

Also define the *Unbalanced Mass Ratio*

$$\alpha_b = \frac{\text{Left Leg Mass}}{\text{Right Leg Mass}}$$

.

As shown in Figure 6.28, when α_b is increased, two legs swing differently and the limit circle is splitted into two. Bigger α_b will result in a cripple like gait, as shown in Figure 6.29

Different Slopes

Usually, changing the angle of the slope may not seen as motion re-targeting. But in **MoIT**, changing slope means changing the parameter of the dynamic equation, which can be analysed in the same manner as as changing body parameters.

Figure 6.30 shows the limit cycle of walking on different slopes. For different slopes, entrainment maintains the limit cycle, but the limit cycle changes its shape. Different stable limit cycles are show in Figure 6.30. Basically, the bigger the slope, the bigger the step size, and the higher the speed. Slope changing has similar effects to energy

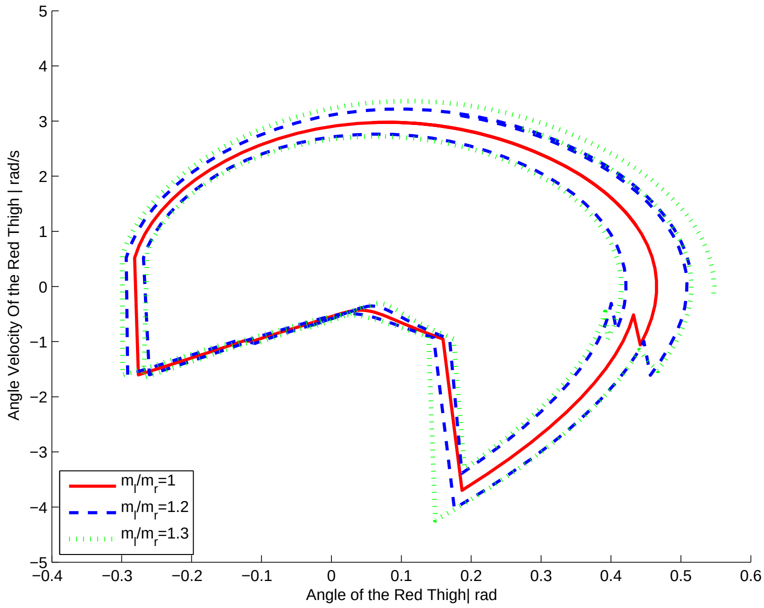


Figure 6.28: *Different Leg Mass Stable Gaits*

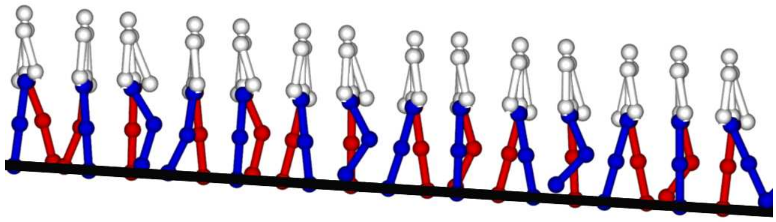


Figure 6.29: *Gait of* $\alpha_b = 1.3$

scaling.

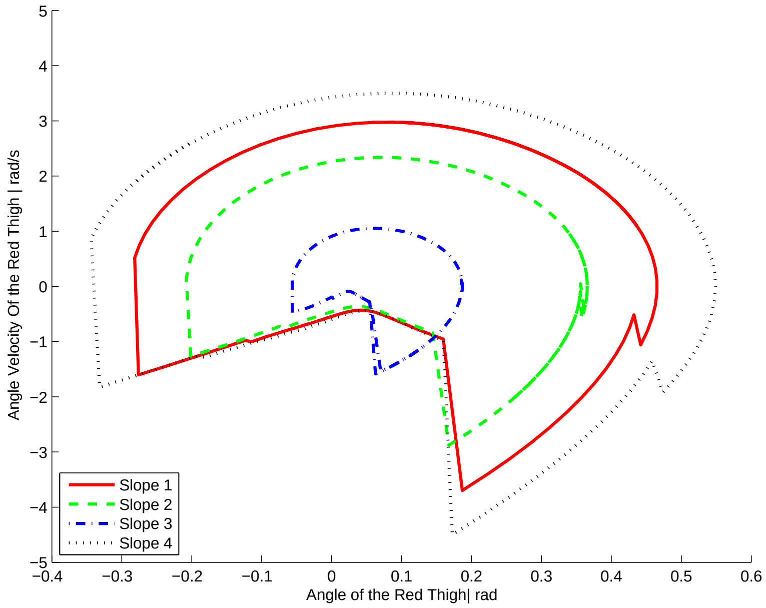


Figure 6.30: *Walking on Different Slopes*

Figure 6.31,Figure 6.32 and Figure 6.33 show different gaits.

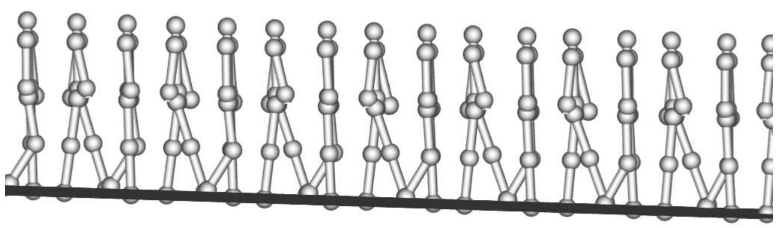


Figure 6.31: *Gait On Slope 1*

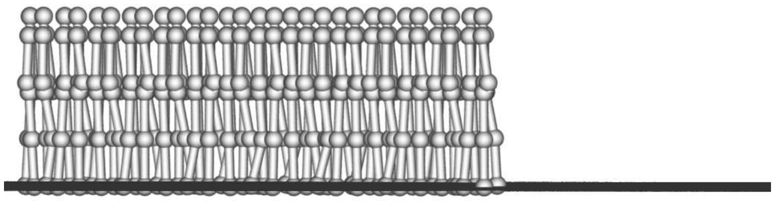


Figure 6.32: *Gait On Slope 2*

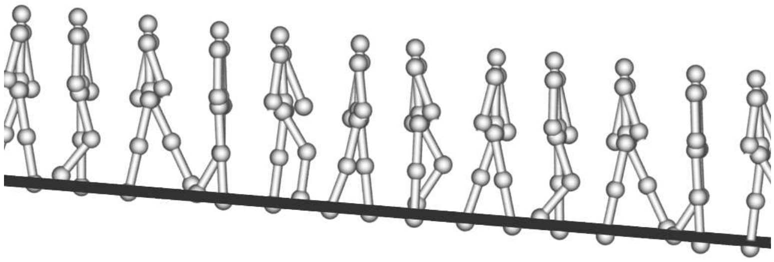


Figure 6.33: *Gait On Slope 3*

6.3 Local Motor Invariant Control

Neural Oscillator boosts the stability. Sometimes stability becomes a limitation in motion. For the walking example, if the basin of attraction covers the whole space, then the passive walker can't walk upslope. If the walker is trying to walk upslope, he or she will begin to walk backward down slope after a few steps as shown in Figure 6.34. In addition, it is not convenient to adjust the speed of walking,since the limit cycle is fixed.

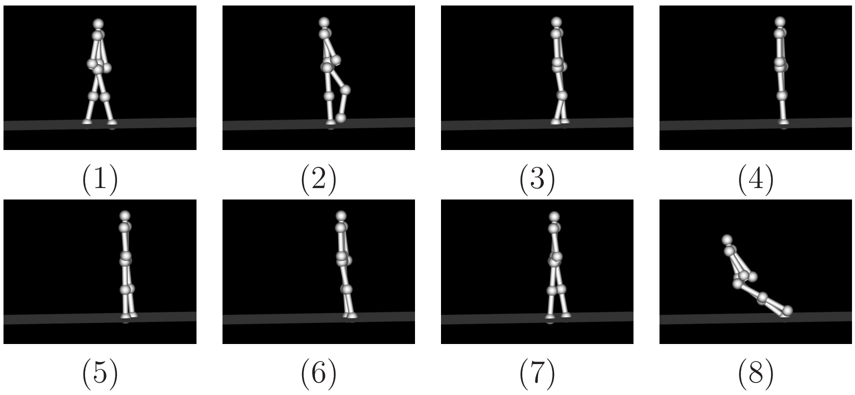


Figure 6.34: *Failure of walking upslope*

Local Motor Invariant provides a mechanism to adapt motion according to the environment and application-specific purpose. For the bipedal walking, group actions provides a mechanism to adjust the walking slope and walking speed in precision.

The original system does not have energy scaling symmetry. Energy Scaling is approximated by a combined method as discussed in section 6.4.1.

When active group actions are applied to the passive walker, it may require all the **DOF**s to be actuated. This involves actuating the q_1, q_2 and q_3. With our dynamic model, q_2 and q_3 are controllable by actuating the knee and hip joints. However, q_1 is not controllable. To actuate q_1, the walker needs feet and motors to drive the ankle joint. The feet are neglected mainly to simplify the collision and contact dynamics. This control scheme is achievable with real human like walker, thus transform action will not result in visually artifects for normal walking condition. However, such simplification will result dyanmic artefacts in extreme cases, because the limited friction force, the toque applied at the ankle should be limited within a range. For a large slope, real human can not generate enough ankle torque mainly because of limited friction.

6.3.1 Group Actions

Equation 6.5 describes walking with local control.

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N(\mathbf{q}) = u_l \tag{6.5}$$

Lie group actions are developed for two types of symmetry.

- **Offset Action**. Offset Action moves the phase plot horizontally. This will make the passive walking on terrains of different slopes. For the bipedal walking, according the Equation 4.9, the offset action is:

  $$u_l = N(q) - N(q + \varepsilon)$$

  where ε is the slope angle change.

- **Speed Action** Speed Action maintains the gait, but modifies the walking speed. According to Equation 4.10, the local control is:

  $$u_l = (1 - \varepsilon^2)N$$

  where ε is the time scaling factor.

For the original system, energy scaling is not a simple, linear transformation. Energy Scaling is approximated by a combined method discussed later.

Figure 6.35 demonstrates different limit cycles after applying Lie group actions. The red one is the original limit cycle. Green ones are applied offset actions and blue ones are applied speed actions.

By applying the offset action, the passive walker can walk upslope, as shown in Figure 6.36

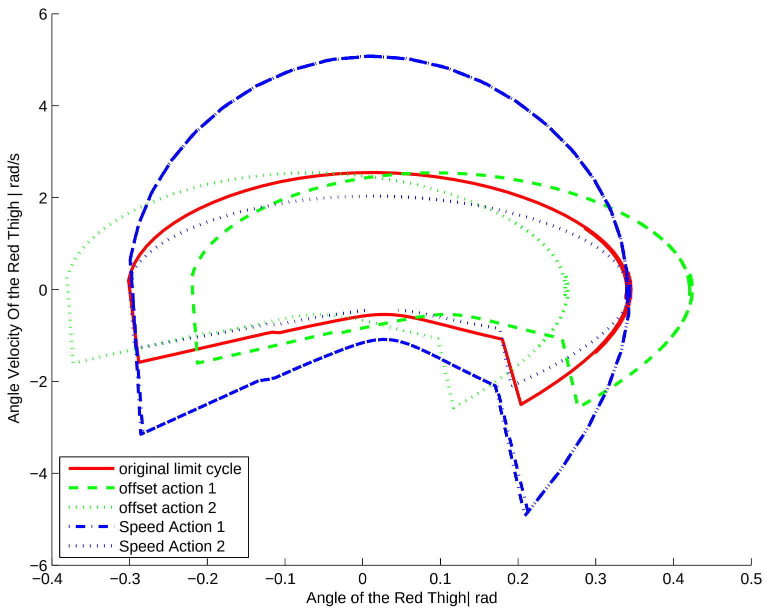


Figure 6.35: *Lie Group Actions on the Phase Plot*

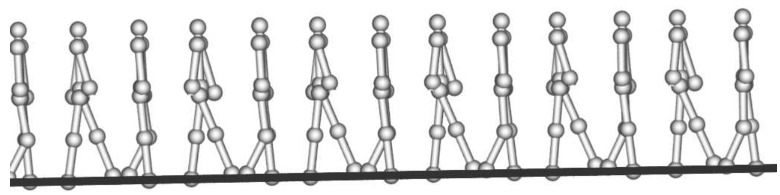


Figure 6.36: *Up slope Gait Generate by Lie Group offset Action*

6.4 Application of Combined Method

Global Motor Invariant Control boosts the walking stability. However, the resulting motion does not meet application's needs sometimes. Local Motor Invariant Control can adapt the walking to application purpose, but it can't boost the stability. Combining the two controllers make it possible to take the strengths of the two methods.

The combined method is described by Equation 6.6

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N(\mathbf{q}) = Du_{\mathrm{o}} + u_{\mathrm{l}} \tag{6.6}$$

In applications, animator can generate different gaits through adjusting parameters of the neural oscillator and the body first, and then transform the different gaits by Lie group actions. For animators, this method is efficient, natural looking and easy to use.

Such combinations will achieve unlimited variations of gaits. We will demonstrate below how gait variations can be achieved in this manner.

6.4.1 Step Size Adjust

The first example shows how a character can adjust his step size realistically. When the character walks down different slopes, a steeper slope will result in a bigger step size as shown in Figure 6.30. If offset Lie group actions are applied, we can transform the gaits of different slopes on the plane. In this way we can achieve different step gaits on the plane.

Figure 6.37 shows limit cycles of different step size on the plane.

And the different gaits are shown in Figure 6.38,Figure 6.39 and Figure 6.40.

6.4.2 Varying Slopes

Neural Oscillator can maintain walking on varying slopes, but can't make a character walk up slope. An offset Lie Group action will allow the character to walk up a slope

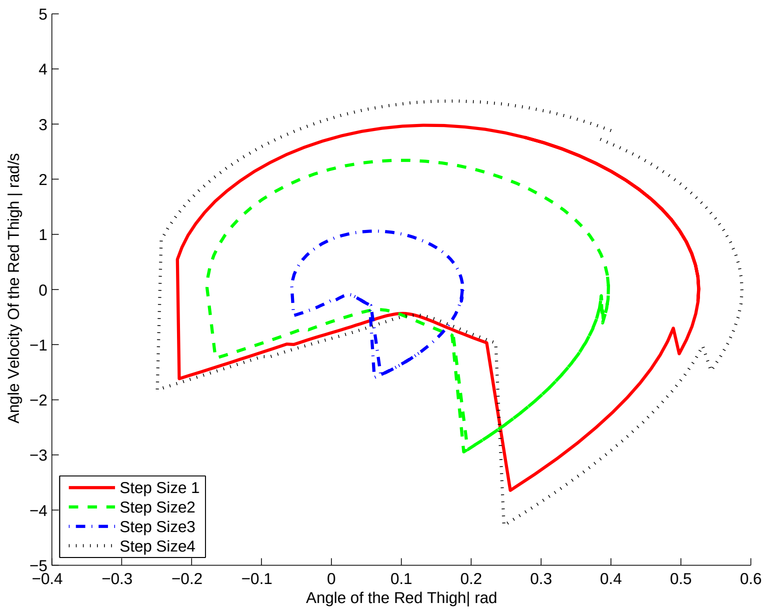


Figure 6.37: *Limit Cycles of Different Step Size Gaits*

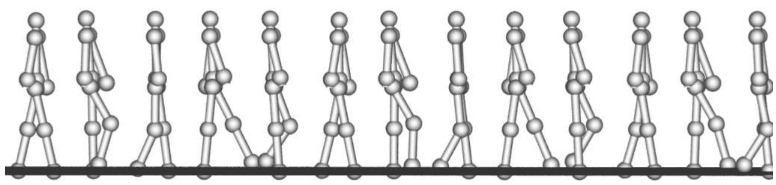


Figure 6.38: *gait with step size 1*

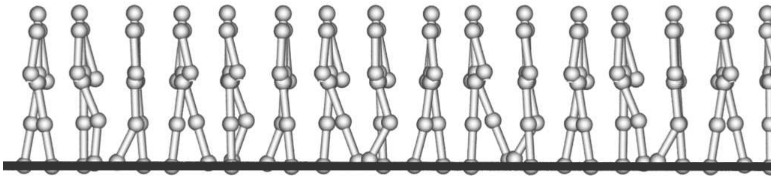


Figure 6.39: *gait with step size 2*

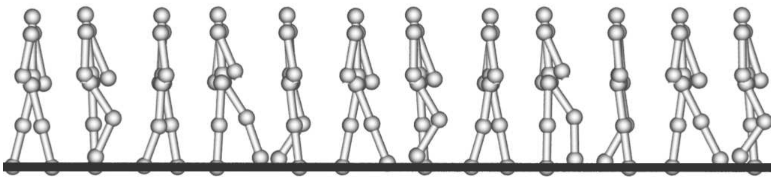


Figure 6.40: *gait with step size 4*

with a constant angle. However varying the slope will result in walking failure. By combining the two methods, the passive walker can walk on terrains of varying slopes.

The control strategy is straight forward, basin of attraction of the walking limit cycle is transformed to capture the current state. When walking on varying slopes, the offset action remains constant when the slope is constant. During slope transitions, the controller looks ahead and sets offset parameter according to the slope of next step.

It is at the moment of transition, the state will move far away from the stable limit cycle. The character needs to take a few steps to return to normal gait.

After the first step in transition, the state will be farthest away form the limit cycle. This is the time when character may fail. More complex control method can be designed transformation the basin of attraction to capture the state. However, in our experiment, the basins of attraction provided by entrainment is already big enough. In our experiments, the state has never escaped from it.

Figure 6.41 and Figure 6.42 show the gaits on smooth slopes. The phase plot of gaits in Figure 6.41 is shown in Figure 6.43.

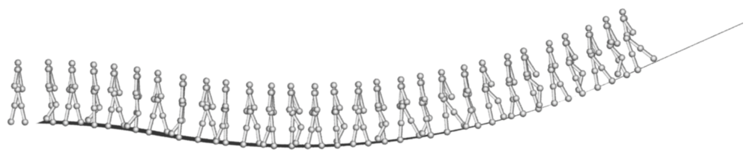


Figure 6.41: *Continuous Varying Slope*

Figure 6.44 show gaits on non-smooth terrain. The slope angles in radians are 0.08,0.17,0.28,0.4. Figure 6.45 shows the phase plot of gaits in Figure 6.44, where the phase plots on different slopes are marked with different colors .

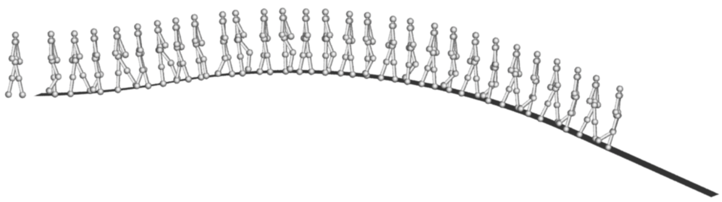


Figure 6.42: *Continous Varying Slope*

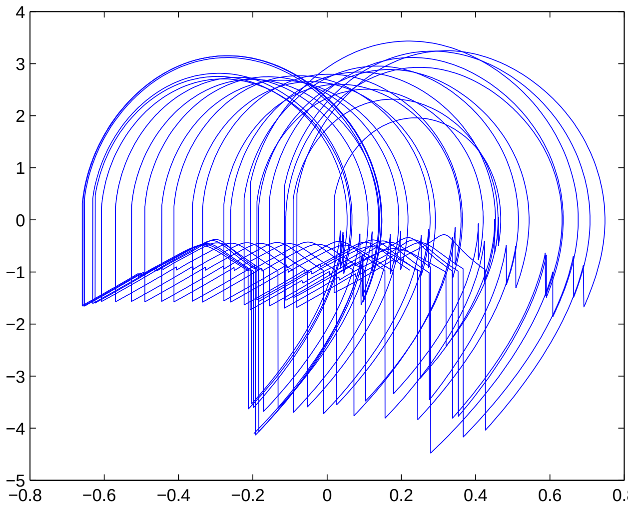


Figure 6.43: *Continous Varying Slope*

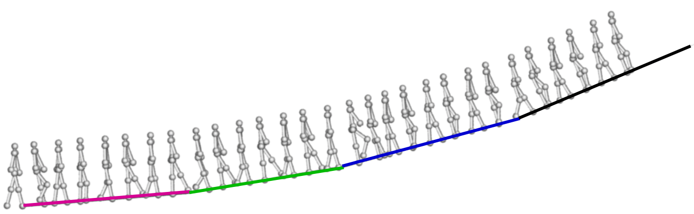


Figure 6.44: *Non-smooth Terrain coloured*

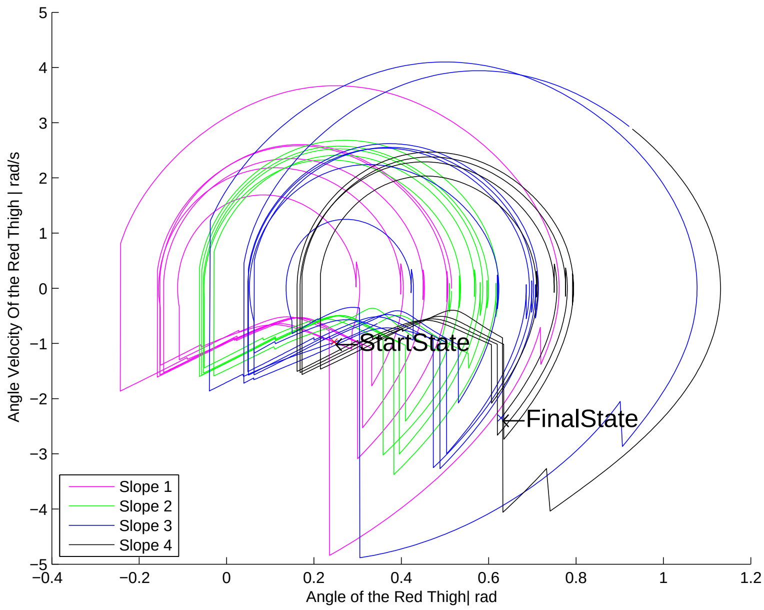


Figure 6.45: *The Phase Plot of non-smooth terrain*

6.5 Verification

In this section, we discuss stability, energy efficiency and the biological justification for the proposed approach. The stability is demonstrated by numerically approximating the basin of attraction of the passive walking model under environmental perturbations and under different initial conditions. The energy cost of each controller is evaluated with various gradient and offset action conditions. In order to link our results to the biological observations, we will analyse the captured motion data of a human walker adapting to environmental perturbations which are similar to those demonstrated in the above sections.

6.5.1 Stability analysis

The stability is analysed numerically by considering the basin of attraction of the passive dynamic walking model. The improved stability of our proposed approach is demonstrated in Figure 6.46 . The simulation runs from the foot strike phase (the bottom left corner of the plot) until it either converges towards the limit cycle or diverges.

The initial conditions, which are the starting angular velocity of the leg for this case, are incrementally increased and decreased and the result is re-plotted until the motion is unstable. Only stable cycles are displayed. The passive walker is stable when walks down a slope of 0.06 radians (Figure 6.46 (a)) but considerably less stable when walks down a slope of 0.03 radians (Figure 6.46 (b)).

In Figure 6.46 (c) the stability of the system (as demonstrated by the size and shape of the basin of attraction) is greatly improved by coupling the **CPG**.

By applying the offset group action with $\varepsilon = 0.03$ to the system in Figure 6.46 (d), the step size is adjusted to compensate for the change in slope angle, which improves the stability further.

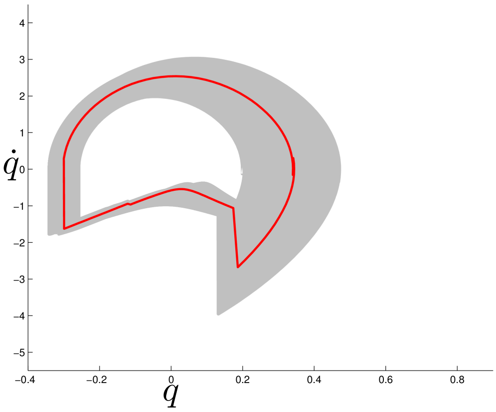
6.5.2 Energy efficiency

Since the passive walker uses no energy, the energy consumed in the system depends on the control variables u_{o} and u_{l} only. We compute the individual cost of transport (Collins and Ruina, 2005) of each controller as $\int |\omega u_{\mathrm{o}}(\mathbf{x}_c)|$ for the neural controller and $\int |\omega u_{\mathrm{l}}(\mathbf{x})|$ for the local controller, where $u_{\mathrm{o}}(\mathbf{x}_c)$ and $u_{\mathrm{l}}(\mathbf{x})$ are local and global invariant control effort and ω is the angular velocity.

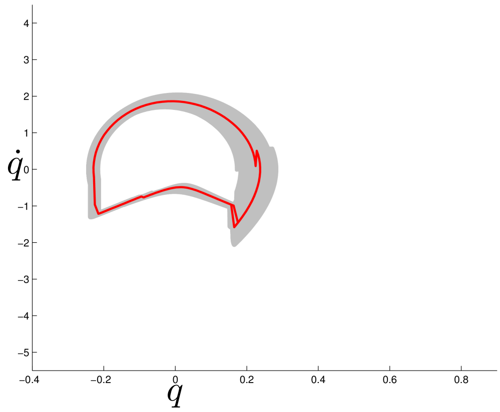
Since these may affect each other, the resultant cost may be less than the total energy applied by the controllers. If these two controllers have independent actuators, then we should consider the sum of the absolute controller torque output from the controllers. We assume that there is only a single actuator, implying that only the resultant torque is appropriate. Therefore the resultant (net) cost of transport c_{et} applied by the controllers in our method is described by the following formula:

$$c_{et} = \int |\omega \left(u_{\mathrm{o}}(\mathbf{x}_c) + u_{\mathrm{l}}(\mathbf{x})\right)| dt. \tag{6.7}$$

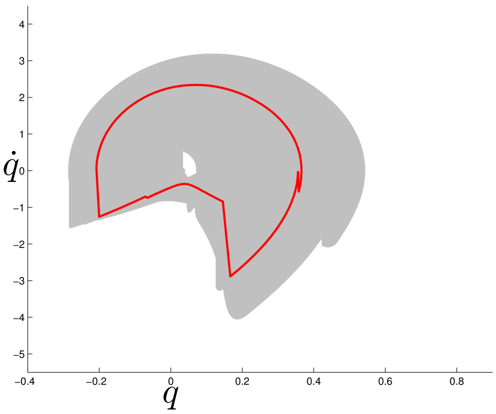
We evaluate this energy over a stable limit cycle by varying the gradient and the value of the offset controller in Table 6.1. Applying the offset action corresponds to altering the step size of the walking model. We observe that the energy cost associated with applying the Lie group action increases linearly with the offset value. The energy cost



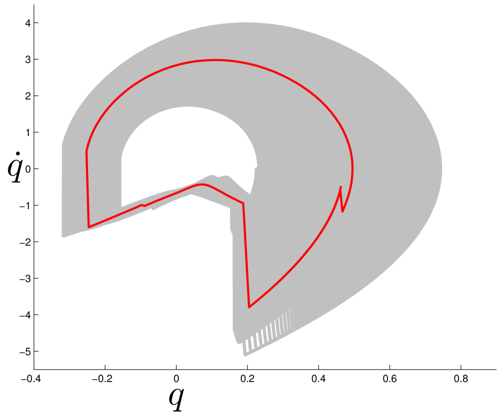
(a) Passive walker (0.06 radians)



(b) Passive walker (0.03 radians)



(c) +Neural control



(d) +Group action

Figure 6.46: *Sensitivity analysis demonstrating the stability of the walking model under perturbations of initial angular velocity.*

		Cost of transport c_{et}		
Gradient (rads)	**Offset** r	**Action cost**	**Neural cost**	**Net cost**
-0.060	0.000	0.000	0.021	**0.021**
-0.030	0.000	0.000	0.020	**0.020**
-0.030	0.030	0.030	0.021	**0.028**
0.000	0.000	0.000	0.029	**0.028**
0.000	0.030	0.030	0.020	**0.026**
0.000	0.060	0.061	0.021	**0.047**
0.000	0.080	0.081	0.021	**0.068**
+0.020	0.080	0.081	0.021	**0.065**

Table 6.1: *Cost of transport for the global and local controllers and of the system as a whole.*

of applying the neural controller seems to be relatively constant. Note that the optimal solution for planar walking is to use an offset action with $\varepsilon = 0.03$, which results in a smaller step size. Compared with a state of the art real robot walking on a plane (Collins and Ruina, 2005) with no local controller, our method uses approximately half the energy, probably due to the lower dimensionality and lack of damping in our system. Such results is not enough to prove our method is most energy efficient one, but it shows the new method belongs to the energy efficient class.

6.5.3 Biological justification

In order to provide a biological justification, we performed a simple experiment by capturing the walking motion of a single person using a commercial grade motion capture system The participant walked on a calibrated mechanical treadmill under two separate environmental conditions in three increments. We varied the speed using the treadmill settings and the elevation by lifting one side of the treadmill. The motion of the walker was captured for a minute under each condition. The resulting data was cleaned from noise and smoothed before analysis. In Figure 6.47, we show the results of plotting the angle against angle gradient in the sagital plane between a vertical direction and the line from the hip to the ankle of the participant, which approximately corresponds to the variables q_1, q_2, q_3 in our dynamic system. Minimal data processing was necessary to tease out this result a standard 1-D filter to remove small local peaks, and the entire path was divided into motion segments and aligned by finding peaks in the cycle corresponding to the foot striking the ground.

In Figure 6.1(b,d), The motion flows vary and cover an area on the phase plot. which can be seen as states moves around the limit cycles because of environmental noise. For a different setup, the area shift its postion and shape slight, but maintain its basic shape. This phenomenon agrees with idea of global invariant in **MoIT**.

For biological system, the precise limit cycle is unattainable. The mean cycle of the walking motion flows are treated as an approximation limit cycle. Figure 6.1(c,e) are the mean cycles of Figure 6.1(b,d).

Changes in treadmill speed clearly caused the participant to increase the energy in the

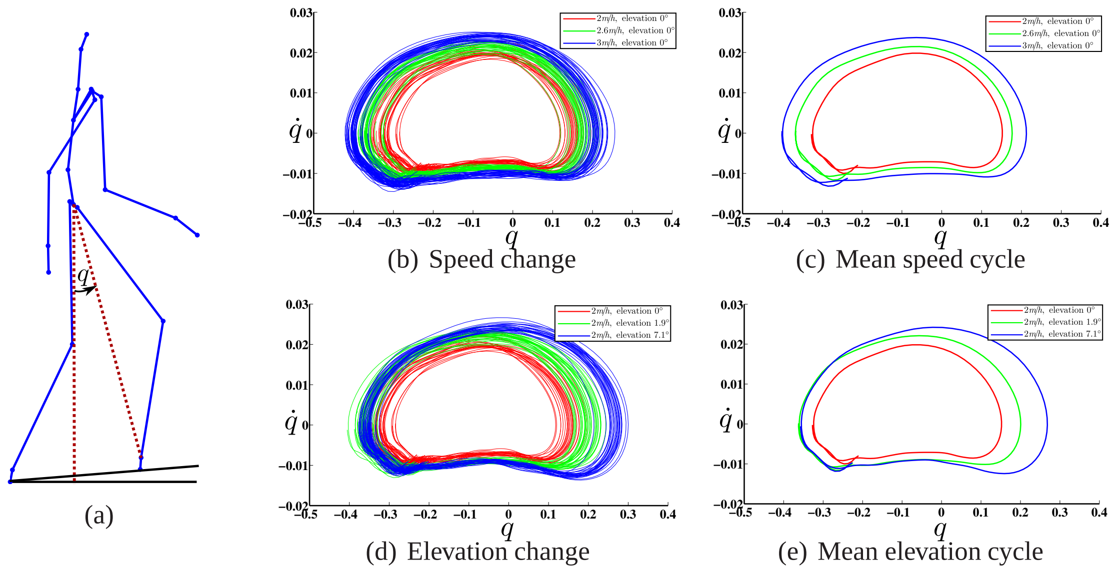


Figure 6.47: *On the phase plot, we can demonstrate how a real human adjusts to changes in the environment. The red, green and blue lines represent data captured under different elevation or speed conditions. q is the angle in radians between an orthogonal to the horizon and the line from the hip to the ankle of one leg.*

dynamic system, analogous to the energy scaling action. When the elevation of the treadmill was altered, the participant adapted by both increasing the step size transformation (presumably in order to maintain the same speed) and adapted to the change in gradient by applying an offset operator.

There are distinct differences between a fully actuated biological human system and the passive walking model. A human will adopt an ankle strategy to minimize the strike momentum and therefore reduce energy loss, which explains why there is no significant spike in the real limit cycle when the foot strikes the ground. In spite of this, the experiment result support the idea of invariant and transformation of **MoIT**.

6.6 Animation Practise

Based on the realistic walking patterns generated by dynamic simulation, animators can further tweak various parameters for the animation purpose. This process can be done in a systematic manner.

The first step is to adjust the walking periold. This involves specifying the height and mass of the character. The τ parameters of the **CPG** can be adjusted automatically by the computer, because τ is propotional to $\frac{m}{L}$.

For the second step, animator needs to specify h_{i} and h_{o} of **CPG** to determine the coupling intensity of the **CPG** and the walker. Smaller values mean weak coupling, result in unstable but efficient looking gait; while bigger values means strong coupling, the motion will be more stable but energy consumming.

For the third step, animators can specify the speed, step size and direction by applying a single or comibnations of group actions.

For the last step, animator may add style variation for the character by modifying the mass ration a_m,mass distribution ratio a_l and etc.

Animation will be an iterative process. However, because the low computation cost of the method, computer can provide motion feedback in realtime.

Chapter 7

MOTION PRIMITIVE TRANSITION:WALK AND STANCE

This chapter focuses on synthesizing transitional motions. Another motion primitive:the stance is developed in Section 7.1. The transitional motions from walking to stance and from stance to walking are discussed in Section 7.3.

7.1 The Stance Primitives

For passive walkers, if the walking velocity is not big enough after a heel strike, the passive walker will stop walking and rest at the double support posture. This stable posture is shown in Figure 7.1.

On phase plot, such motions have the topology of a fixed point attractor, which is another motion primitive: the stance.

7.1.1 Simplified Dynamics

When people stand, the two legs are almost straight. Instead of the four linked rigid body model, the stance for this case can be simplified as a point mass supported by

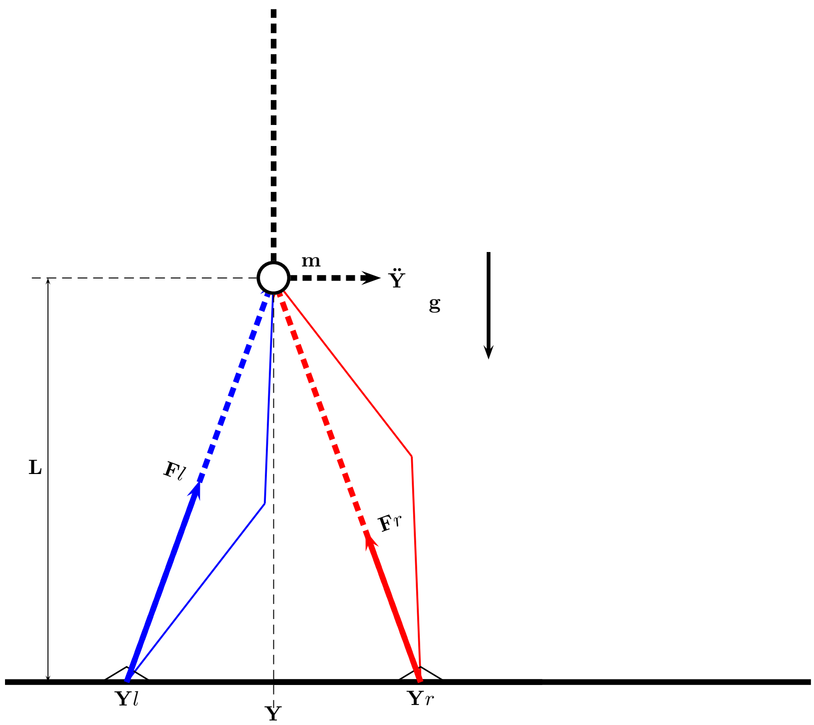


Figure 7.1: *The Stance Motion Primitives*

two straight legs. Stephens and Atkeson (2009) proposed the height of waist is almost constant and can be neglected. Therefore, the simplified dynamic model has only one degree of freedom, i,e. the horizontal displacement. Given the horizontal displacement, configurations of shank and thigh can be worked out through *inverse kinematic* methods.

The stance dynamic is not continuous and the phase space can be divided into three regions. The postures of different regions are shown in Figure 7.2.

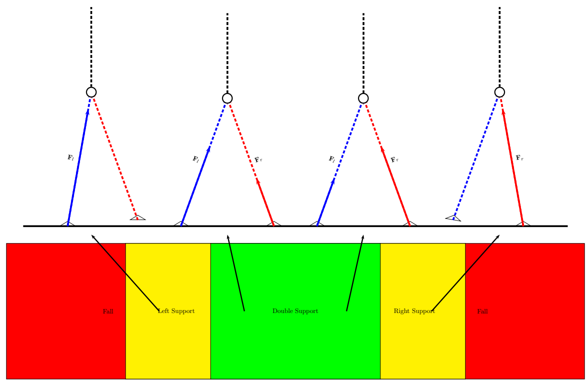


Figure 7.2: *discontinuous dynamics of stance*

- **Double Support** When the off center displacement is small, the body is supported by two legs. the motion is governed by the gravity.

  $$\ddot{q} = \frac{\mathrm{g}}{L}(q - y_r) + \frac{\mathrm{g}}{L}(q - y_l)$$

  where q is the off center displacement, L is the height of the mass point, and g is gravity.

  Torques are generated by the two legs to maintain stability. Intuitively, the left torque is increased when the centre moves left, and the same is true with the right torque. We suppose the relationship between torques and centre position is linear. Dynamic Equation 7.1 incorporates the control strategy.

  $$\ddot{q} = \frac{\mathrm{g}}{L}w_r(q - y_r) + \frac{\mathrm{g}}{L}w_l(q - y_l) + \frac{\tau_L + \tau_R}{mL} \tag{7.1}$$

where w_l and w_r are the weight of the two torques. We have $w_l + w_r = 1$.

- **Single Leg Support** For a big horizontal displacement, people stand on a single leg. The passive dynamic is

$$\ddot{q} = \frac{g}{L} q$$

Equation 7.2 incorporates the torque generated by legs.

$$\ddot{q} = \frac{g}{L} q + \frac{y_{L,R}}{L} \tau_{L,R} \tag{7.2}$$

- **Fall and Walk** For even bigger displacement, the stance posture can not be maintained. The phase space region where human can maintain the stand posture is called "support region". The width of the "support region" depends on the height and the step size. When moving out of the "support region", the stance posture can't be maintained, and a human will either walk or fall.

Without damping effects, the original system is similar to a mass spring system. It will vibrate endlessly, and the flow is a cycle, as shown in Figure 7.3. If the speed is high, then the state will move out of the basin of attraction. Maintaining stance is to maintain the horizontal displacement within the support region.

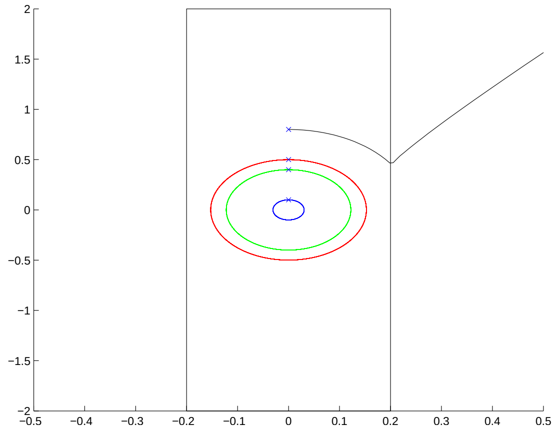
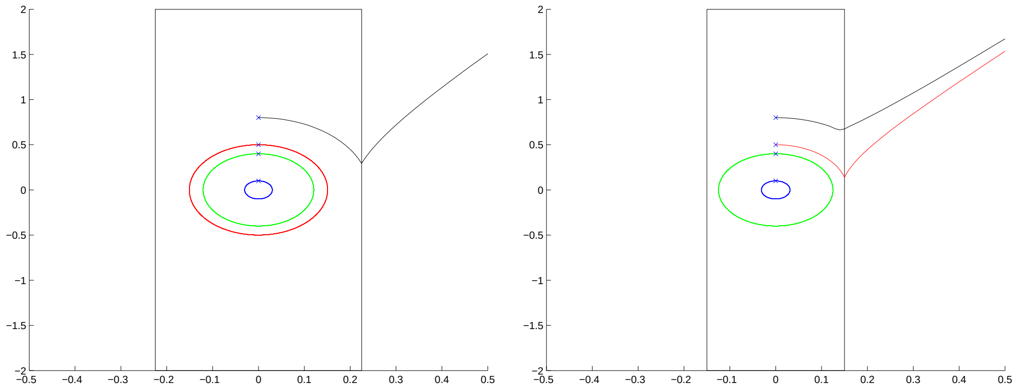


Figure 7.3: *uncontrolled motion*

The support region is propotional to the distance between the supporting legs. Figure 7.4 shows the phase plot and supporting regions with different step size.



(a) Wider Supporting Region

(b) Narrow Suporting Region

Figure 7.4: *Topological Conjugacy*

7.2 Motor Invariant Control

7.2.1 Entrainment

By coupling the dynamic system with the neural oscillator, the position of the centre is fed into the neural oscillator and the output of the neural oscillator drives the torque generated by the legs.

$$u_{\mathrm{i}} = h_{\mathrm{i}}(q); u_{\mathrm{o}} = \tau_{L,R}$$

Entrainment happens and a limit cycle is formed. However, since entrainment will no modify the boundary of the support region, entrainment does not boost the stability. Because it is impossible for mechanical system to converge to the limit circle within 1/4 period, and the neural oscillator will not modify the boundary.

7.2.2 Local Invariant Control

All the three group actions can be applied. However, only two group actions among the three are useful and affect the stability.

Time Scaling

Time scaling action will stretch the phase plot in the velocity direction, as shown in Figure 7.5. It will enlarge the basin of attraction to include high speed state.

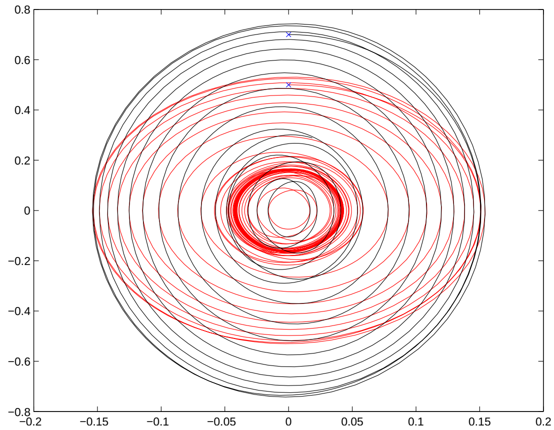


Figure 7.5: *Time Scaling*

Energy Control

Energy scaling action will modify the size of the limit cycle, which modifies the wobbling amplitude. Figure 7.6 shows the energy action effect on the limit cycle. When energy action is applied, the limit cycle shrinks.

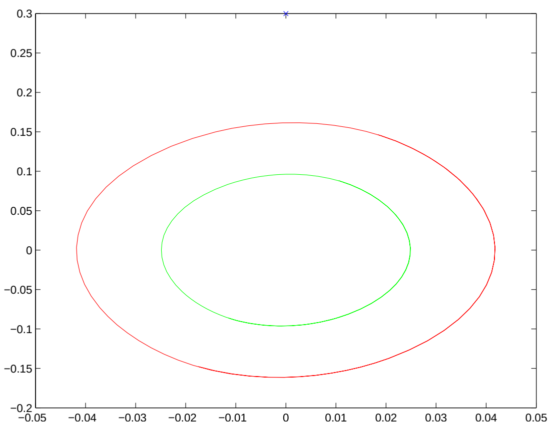


Figure 7.6: *Energy Scaling*

Fast Convergence Control

By applying speed and energy scaling actions sequentially, wobbling can converge to the limit cycle and stop quickly. In Figure 7.7, the speed action is first applied to include the high speed state for 1/4 period. When the state reach the pos that the speed is zero, the energy scaling is applied for next $1/4$ period to shrink the limit cycle size. For the next $1/4$ period, the speed action is applied, and so on.

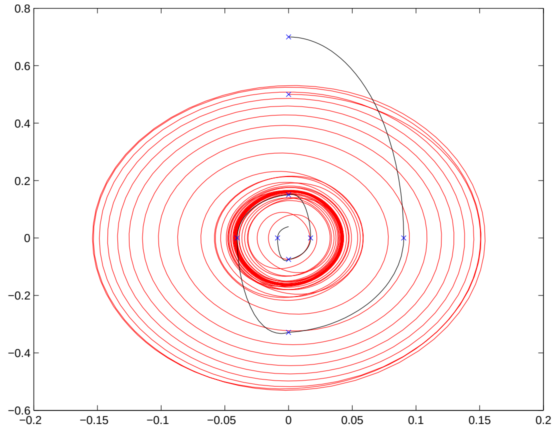


Figure 7.7: *Fast Converge*

7.2.3 Stability

Motions of stance are put together for comparison. Without any control, the character fails as shown in Figure 7.8.

In Figure 7.9, the speed action is applied, and the character maintains its stance motion, but wobbles endlessly.

In Figure 7.10, both speed action and energy action are applied, and the character maintains the stance and vibrates with a shrinking amplitude.

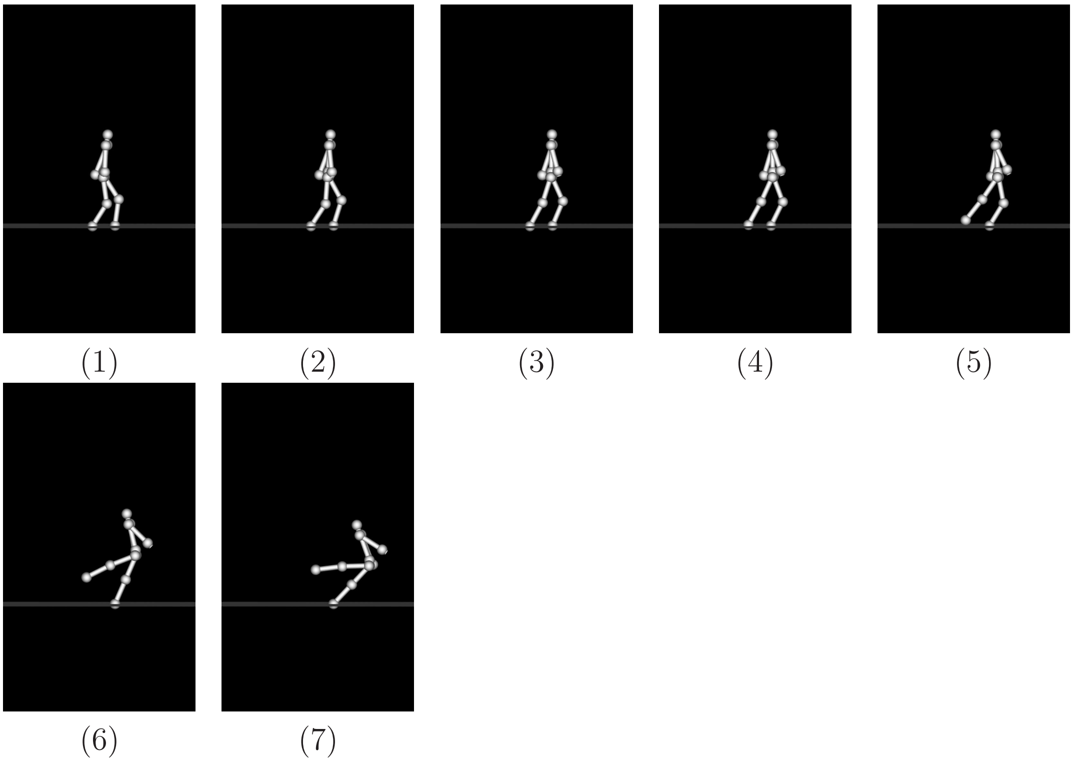


Figure 7.8: *Balance Motion without Neural Control*

7.3 Walking and Stance Transition

Both limit cycles of walking and stance are shown in Figure 7.11. The phase plot here shows the supporting leg, and the swing leg is indicated in shadow red. Motion transition means make the state transform from one limit cycle into another.

7.3.1 Walk to Stance

Walk to stance transition happens at the heel strike phase. Without control effort, the bipedal machine will continue to walk. As shown in Figure 7.11, if we switch on the stance motion primitive controller, the current state will fall into the basin of attraction of stance with a proper group transform action. Two legs will start to vibrate with smaller amplitude, this is the walk to stance transition.

The walking step length is closely related the supporting region for stancing. A bigger

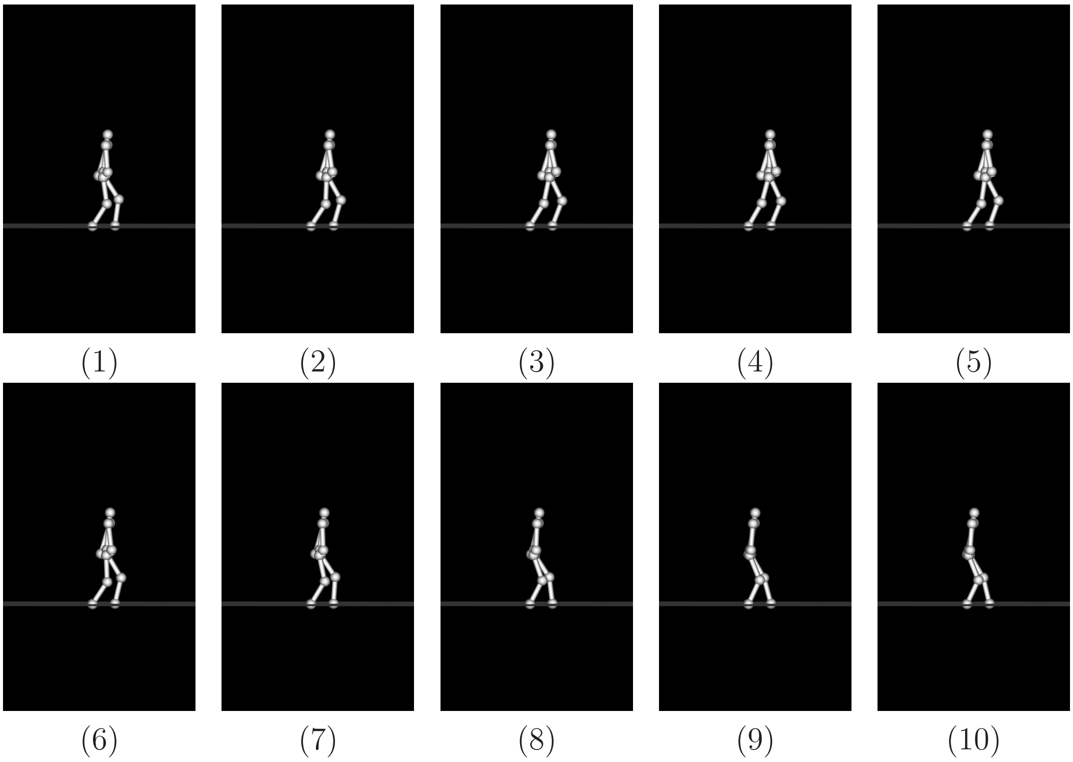


Figure 7.9: *Wobbling Stance*

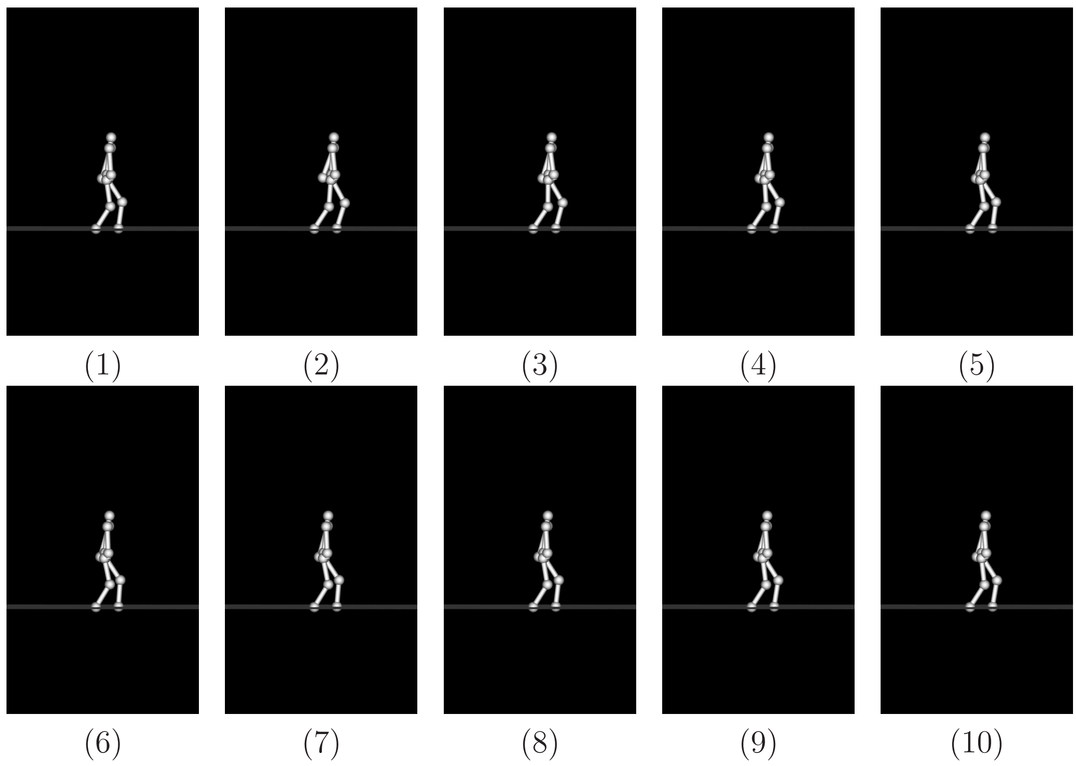


Figure 7.10: *Stable Stance*

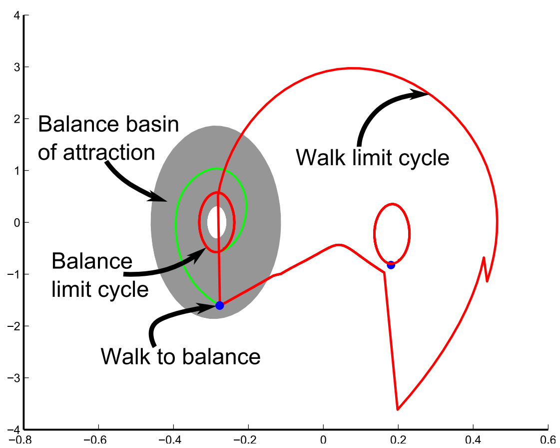


Figure 7.11: *Walk to Stance Transition*

stepsize will result in a bigger supporting region for stancing. While a smaller stepsize will result a narrow supporting region for stancing. With local invariant controller, stance can be maintained no matter how big the stepsize is. However, a smaller stepzie will require a bigger time scalling or more control effort.

Knee Bending Scheme

During walk to stance transition, the two legs are straight when the heel strikes. At this time, the support region is very small. Any push of the figure, it will move out of the two support region. To enlarge the basin of attraction, the walkers have to bend legs and lower the height. There are many ways for bending the legs.

- **One Leg Bending** walker can bend one leg while keeping the other leg straight.
- **Double Leg Bending** walker can make the two leg bend.

Since the knees is not necessary straight when a human walk, it is very difficult to tell which one is more realistic. These two schemes are extreme cases. Motion of Double Leg Bending is shown in Figure 7.12.

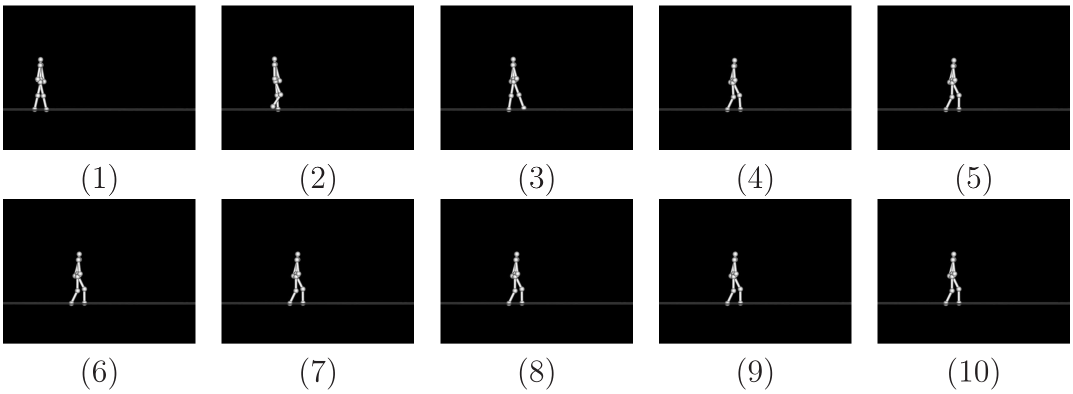


Figure 7.12: *Stop Walking with Two Legs Bend*

7.3.2 Stance to Walk

When the stance to walk transition happens, the current state should be close to the walking limit cycle. Due to this reason, stance to walk happens when the legs are moving forward at maxim speed and the position of the hip is in the middle. At this time, we switch on the walker controller, and the character starts walking. Figure 7.13 shows the process on phase plot.

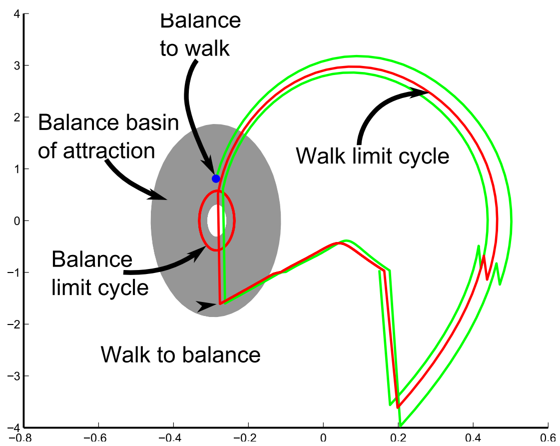


Figure 7.13: *The Phase Plot for Stance to Walk*

From stance to walk, the height has to be increased. Only one scheme exists for straightening the knees. The scheme which we use is to keep the front leg straight and make the hind leg from bend to straight.

Another non-trivial problem is is that when switching stance to walk, it is impossible to put both legs on the limit cycle. The supporting leg has been given the priority, for the supporting leg is more important for maintaining stability.

7.3.3 Smooth Transition by Speed Action

When transiting from walk to stance, the basin of attraction must include the heel strike state. However, the original basin of attraction of stance does not. A speed action is needed to enlarge the basin of attraction. As an alternative, we can lower the walker speed. In this way, walking to stance may become easier.

For the transition from stance to walk, if little effort is exerted, the initial position will be far from the walking limit cycle. To maintain the walking stability, speed actions are applied to decrease the walking speed. To make both limit cycles connected each other, the speed action of stance and walking mus have a constant ratio, This phenomenon is common for our daily experience. **MoIT** gives it a mathematical meaning.

Chapter 8

TOWARDS HIGH DIMENSION

8.1 Introduction

In the previous chapters, motions are dynamically synthesized for characters with simplified dynamics. A question arises whether the motor invariant theory(**MoIT**) is applicable to characters with higher degrees of freedom. For walking and stance examples, high degree systems will incorporate the motions of the torso and arms. Also for snakes and fishes, synthesising motion for a flexible spine may also be challenging. **MoIT** provides a different perspective, and some of the challenges can be solved in a very different manner.

Redundant **DOF** is the key challenge in motion synthesis. From the theoretical perspective of **MoIT**, redundant **DOF**s do not increase the computational burden exponentially. **MoIT** explores the natural dynamics of the body and the redundant **DOF** can move passively. The computation cost of one neural oscillator remains constant when coupling with different mechanical models. As long as the symbolic equations of a dynamics is given, symbolic expression for each group action can be derived. Thus the computation of controlled symmetry action is trivial and increase linearly with the number of **DOF**s.

However, the symmetrical controller requires symbolic expression of the dynamic systems. With high dimensional systems, obtaining the symbolic expression is not trivial and finding the basin of attractions is even more challenging. This chapter focuses on techniques that avoid developing high **DOF** symbolic equations. Different strategies are developed to utilized low dimensional dynamic equations to simulate high dynamic systems.

- **Negligible DOFs** For characters with high **DOF**s, some **DOF**s can be simply neglected. **MoIT** is based on two concepts, the qualitative property and symmetry actions. **DOF**s can be neglected for two reasons: first for some **DOF**s in some motion primitives, their motion is minor and has little effect on the system's dynamic property. For such **DOF**s, controller systems can be designed according to the simple model. The high dimensional model can be used for simulation, but will not affect the burden of control calculation.

  Second for some other **DOF**s, their effects are equivalent to some group transformation. If a group action controller is developed, the effects of such **DOF**s can also be neglected.

- **Mechanical Coupling** In certain circumstances, the divide and conquer strategy works. Instead of simulating and developing controllers for a complex mechanical system, the complex system is divided into many components with low **DOF**s, and controllers are developed for each of them.

- **Time Offset** In some cases, the motions of some **DOF**s are similar or mimic each other, the dynamics can be simplified as controlling just one **DOF**, and synthesizing other **DOF**s by mimicking it.

8.2 Negligible DOFs and Reduction

8.2.1 Negligible DOF

Although biological mechanical structures have high degrees of freedom, many **DOF**s will not affect the topology or qualitative properties. For the walking example, Raibert *et al.* (1986) pointed out that walking is the same as a ball rolling down a slope while running is the same as a ball bouncing down a slope. In our research, a control strategy is developed based on the compass gait model, as shown in Figure 8.1. The degree of knees in Figure 6.2 and foot in Figure 8.2 will have little effect on the qualitative properties.

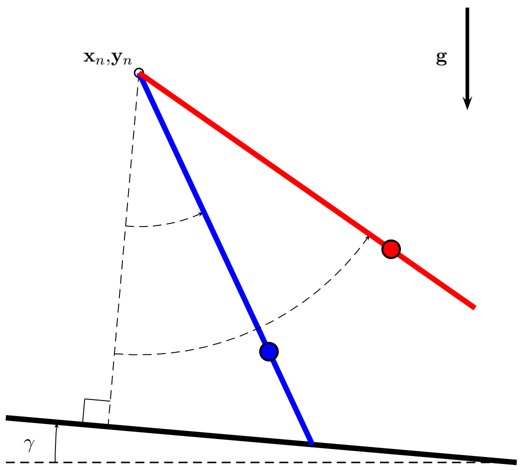


Figure 8.1: *Compass Gait*

Although the compass gait and arc foot model are different from our walker with knees, the three models are all capable of passive walking and show limit cycles of similar shapes, as shown in Figure 8.3 and Figure 8.4.

From geometry perspective, the low dimensional phase portrait can be seen as the skeleton of the a high dimension phase portrait, the introduction of new **DOF**s will provide space for possible new attractors or motion primitives. However, if the motion range of the extra **DOF** is very limited, then the extra space will be very small and cover only a small area. Furthermore by applying control effort, basins of attraction of the original attractors are enlarged and may use up any new space.

Motions of some **DOF**s are relatively small, or have little effect on the topology. From an alternative perspective, such motions are treated as perturbations, which can be processed by the perturbation or averaging techniques(Khalil and Grizzle, 2002). As an example, the equation of the walker with knee is very different from the compass

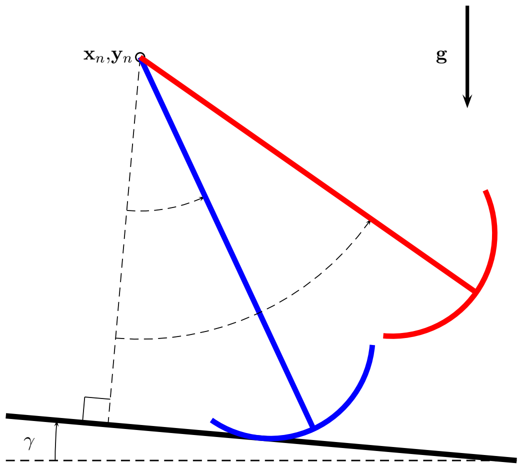


Figure 8.2: *Arc Foot Walker*

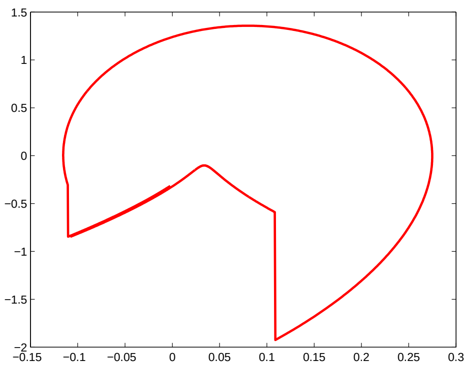


Figure 8.3: *the limit cycle of compass gait*

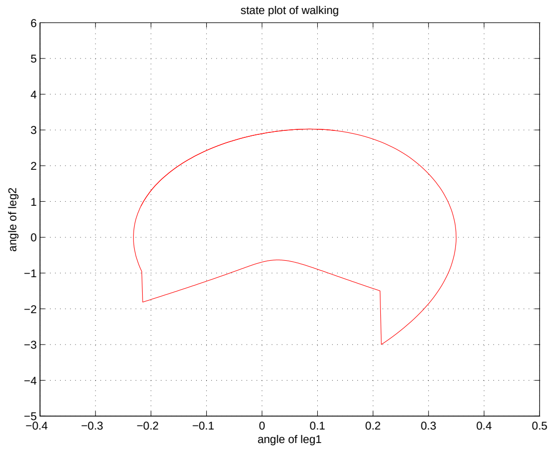


Figure 8.4: *the Limit Cycle of arc foot*

gait model. However, from a different perspective, the relatively small motion of the knee can be seen as perturbations to the leg length and the mass position parameters.

Feet are added to the original walker following this principle. For normal walking, the motion range of the ankle is very small. According to experience, the feet will boost the stability. At current, we did not taken the complex feet shape and collision dynamics into account. However, the bigger contact region will prolong the double supporting time, which allows the walker adjust the stability for the next step.

For simulation, for each step, after heel collsion, we get the new state $[q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_3]$. Feet actuations will push the current state towards the limit cycle.

The effect of ankle actuation is modelled by the simplified liner model, as shown in Equation 8.1.

$$\dot{q}^f = (1-r)\dot{q} + r\dot{q}^{desir} \tag{8.1}$$

where the $\dot{q}$ is the state after the heel strike, $\dot{q}^f$ is the state after foot actuation. r is the linear ratio. $\dot{q}^{desir}$ is the desired state, or the state on the limit cycle. If foot action pushes the walker towards the limit cycle perfectly, then $r = 1$,

It is easy to prove that with foot actation

$$\dot{q}^f - \dot{q}^{desir} = (1-r)(\dot{q} - \dot{q}^{desire})$$

This can been as the distance from the current state to the limit cycle is scaled down by $1 - r$, or can be also intepreted as the basin of attraction is enlarged by a scale factor of $\frac{1}{1-r}$ at the heel strike time. Both explanation shows the walking more stable.

Adding feet will change the shape of the limit cycle slightly, the gait is shown in Figure 8.5.

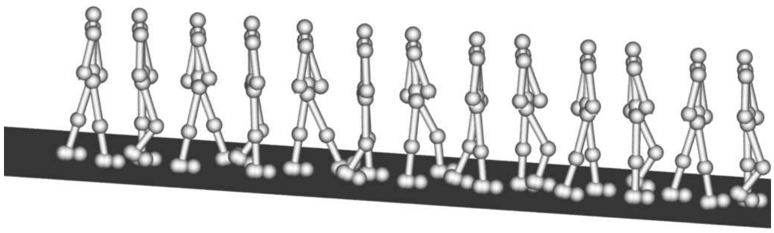


Figure 8.5: *Walking with Feet*

8.2.2 Symmetry Reduction

For a dynamic system of high dimension, in some cases, the **DOF**s can be divided in a specific manner: a lower dimensional dynamic system which captures the key properties of motion, and some extra **DOF**s that place the lower dimensional dynamics in higher dimensional space(Marsden *et al.*, 1990). The extra **DOF**s have the same effect as group actions, and the dynamics can be controlled with a lower dimensional model.

This idea helps to extend the 2D walker into 3 dimensions. Rather than developing the full 3D dynamics, a 3D walker is developed based on the 2D walker. Motions in the coronal plane and transverse plane transform sagittal plane dynamics. The motion in the coronal plane and transverse plane can be simplified as rigid body simulation, which places the 2D walker at a correct position in 3D space.

Lateral Motions

An illustrative example shows the sway motions in the coronal plane.

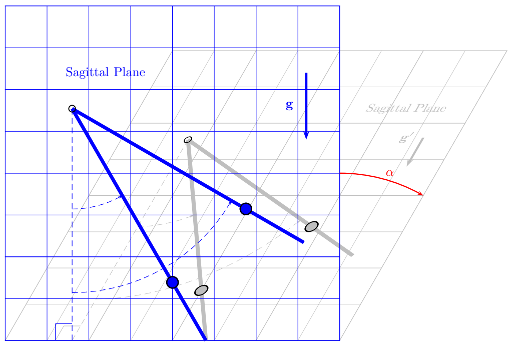


Figure 8.6: *Sideway*

As shown in Figure 8.6, when the passive 2D walker walks on a terrain with a sway angle α. The gravity force on the sagittal plane is decreased.

$$\mathrm{g}' = \cos(\alpha)\mathrm{g}$$

where g' is the projected gravity force on the sagittal plane. By substituting the projected gravity in the dynamic equations, we have

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + N'(\mathbf{q}) = 0 \tag{8.2}$$

The external force N becomes $N' = cos(\alpha)N$

This has the same effect as applying speed action of the parameter ε, where $\varepsilon^2 - 1 = \cos(\alpha)$. The effects of sway motion on the 2D dynamics can be simulated by adjusting the speed action parameters ε. For a walker with a speed action controller, this effect can be totally compensated.

The sway motion on the coronal plane is not based on passive dynamics and unstable in nature(Kuo, 1999). Great effort is executed at the ankle and waist for maintaining posture. Such motions are closely related to the character's motion purpose and not mainly governed by natural dynamics, thus are left to the animators. For procedural method, we can use a **PD** based method to make the walker sway about the centre position.

The passive walker is put to walking on the plane. When walking on the plane, sway motion will result in an early heel collision, which may treated as a noise to the 2D passive walker.

Figure 8.7 show the lateral way motion and walking motion. The lateral sway angle synchronizes with walker motion. Figure 8.8 the lateral motion effects on the walking limit cycle. The walking limit cycle split in two and seems sugest that the period of walking is doubled

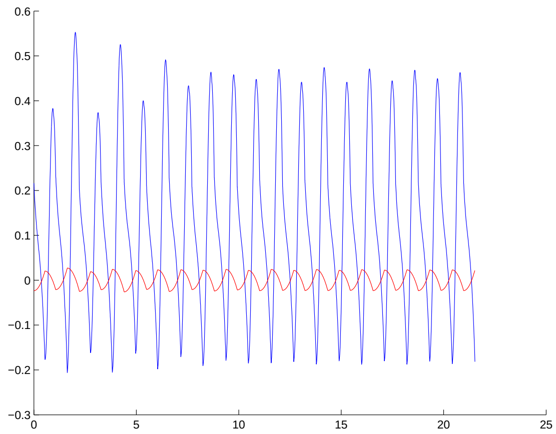


Figure 8.7: *Sway Motion and Leg Motion*

Turning Motion

The rotation on the transverse plane has no effect on the 2D dynamic walking model. If the ground is rotating around the transverse plane at constant speed, the dynamics on the sagittal plane will remain the same. In three dimensions, the difference is that centrifugal force is generated perpendicular to the sagittal plane, which is compensated by the friction of the foot.

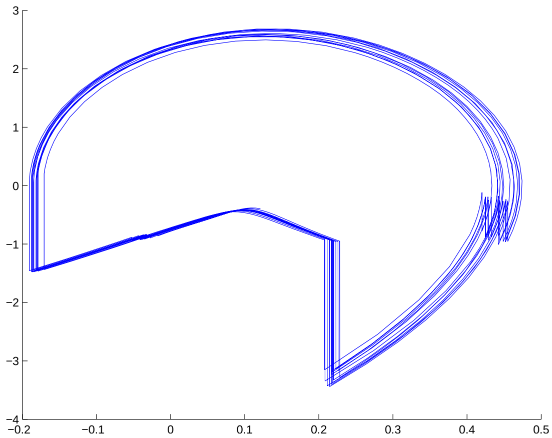


Figure 8.8: *Lateral Sway Motion Effects on Walking Limit Cycle*

For the walker, a turning means rotating the sagittal plane, this can be achieved by actuating the hip joint of the supporting leg, as shown in Figure 8.9.

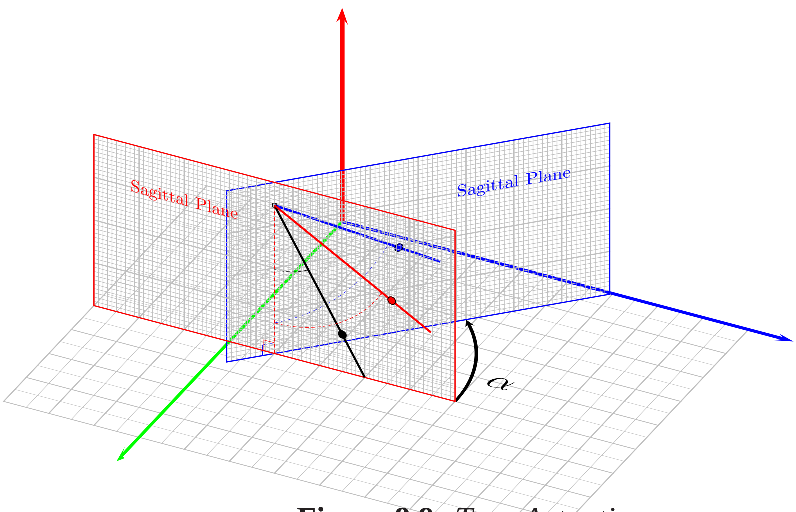


Figure 8.9: *Turn Actuation*

The same as for the lateral motion, turning is not achieved by exploring natural dynamics, but determined by the animator's purpose. The animator determines the turning angle and speed. As a simplification, during the turning, the dynamic equations of 2D walker remain the same. Turning gaits are shown in 8.10.

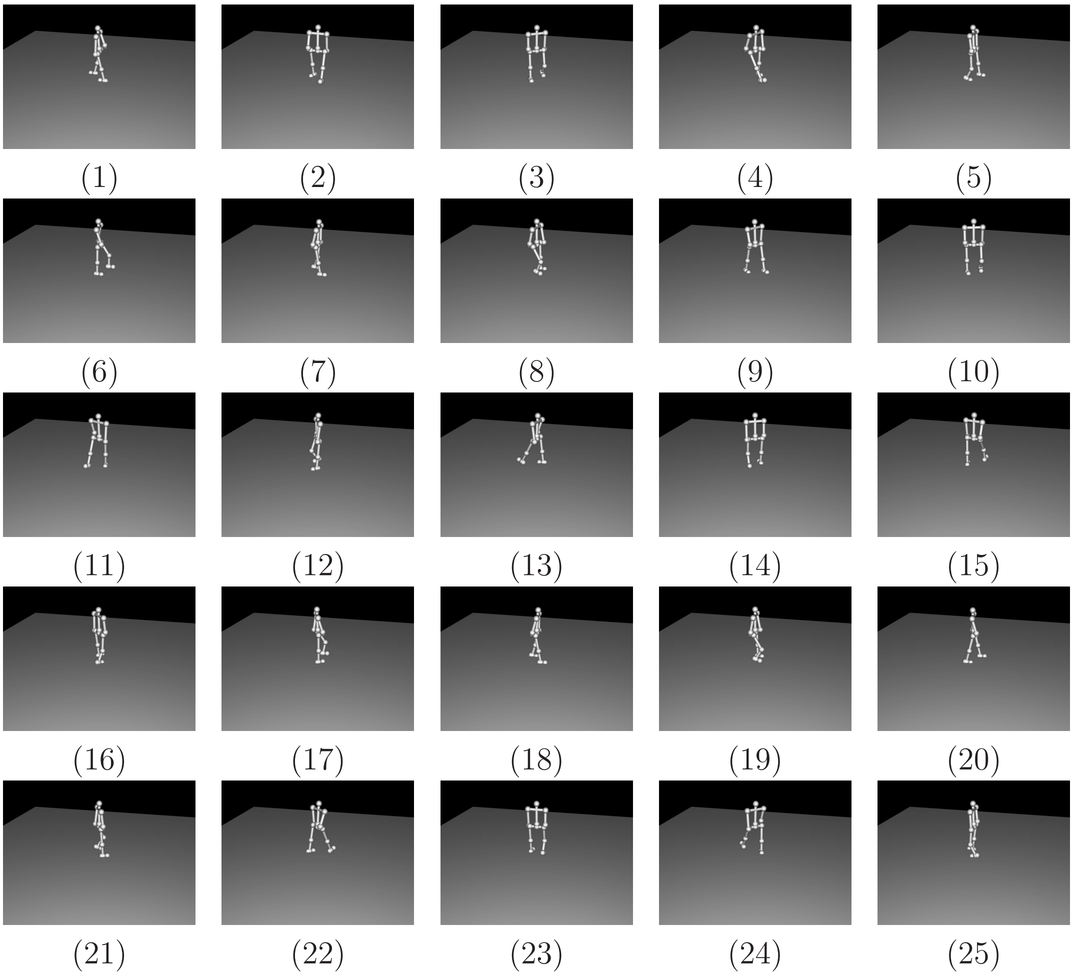


Figure 8.10: *Walk And Turn*

8.3 Mechanical Coupling

Many high **DOF**s systems have a tree topology, which is composed of many branches. For such systems, the divide-and-conquer strategy is utilized to avoid the difficulty of developing a complex dynamic equation of high dimensions.

The mechanical system can be seen as many different simple components connected together. Different components can be simulated independently, and the interactions between different parts form mechanical coupling.

If a mechanical system is in the following form

$$\dot{\mathbf{x}} = F(\mathbf{x})$$

the state is $\mathbf{x} = [q_1, q_2, \dot{q}_1, \dot{q}_2]$ we can reform the dynamic equation in a different manner $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2]$ where $\mathbf{x}_1 = [q_1, \dot{q}_1]$ $\mathbf{x}_2 = [q_2, \dot{q}_2]$

and the original system can be seen as two systems coupled together

$$\begin{aligned}\dot{\mathbf{x}}_1 &= F_1(\mathbf{x}_1) + C_1(\mathbf{x}_1, \mathbf{x}_2)\\ \dot{\mathbf{x}}_2 &= F_2(\mathbf{x}_2) + C_2(\mathbf{x}_1, \mathbf{x}_2),\end{aligned}$$

if $C_{1,2} \ll F_{1,2}$, then the dynamic will be dominated by $F_{1,2}$ and $C_{1,2}$ can be treated as perturbations. Controllers are designed according to $F_{1,2}$.

Mechanical Structure with Branches

In fact any mechanical system can be reformulated as an entrainment network, a proper division should separate the system at the places where the coupling is weak. The weak coupling joints can be identified through the mechanical structure. Usually, the joints where the system branches are a good choice.

If the mechanical system has the structure shown in Figure 8.11.

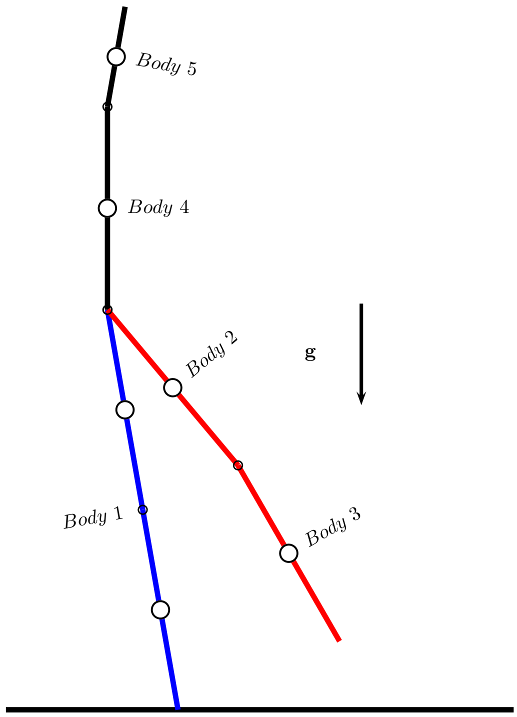


Figure 8.11: *Mechanical Structure with Branches*

The 5 **DOF**s dynamic system is in the following form

$$M \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_4 \\ \ddot{q}_5 \end{bmatrix} + C \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \end{bmatrix} + \begin{bmatrix} N_1(q_1) \\ N_2(q_2) \\ N_3(q_3) \\ N_4(q_4) \\ N_5(q_5) \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}$$

where $q_{1,2,3,4,5}$ are the configuration coordinates of 5 links, and the mass matrix is

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{12} & m_{22} & m_{23} & m_{24} & m_{25} \\ m_{13} & m_{23} & m_{33} & m_{34} & m_{35} \\ m_{14} & m_{24} & m_{34} & m_{44} & m_{45} \\ m_{15} & m_{25} & m_{35} & m_{45} & m_{55} \end{bmatrix}$$

and

$$C = \begin{bmatrix} 0 & c_{12}\dot{q}_2 & c_{13}\dot{q}_3 & c_{14}\dot{q}_4 & c_{15}\dot{q}_5 \\ -c_{12}\dot{q}_1 & 0 & c_{23}\dot{q}_3 & c_{24}\dot{q}_4 & c_{25}\dot{q}_5 \\ -c_{13}\dot{q}_1 & -c_{23}\dot{q}_2 & 0 & c_{34}\dot{q}_4 & c_{35}\dot{q}_5 \\ -c_{14}\dot{q}_1 & -c_{24}\dot{q}_2 & -c_{34}\dot{q}_3 & 0 & c_{45}\dot{q}_5 \\ -c_{15}\dot{q}_1 & -c_{25}\dot{q}_2 & -c_{35}\dot{q}_3 & -c_{45}\dot{q}_4 & 0 \end{bmatrix}$$

For the branch structure in Figure 8.11, the coefficient of unconnected links will be zero, thus

$$M = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} \\ m_{12} & m_{22} & m_{23} & 0 & 0 \\ m_{13} & m_{23} & m_{33} & 0 & 0 \\ m_{14} & 0 & 0 & m_{44} & m_{45} \\ m_{15} & 0 & 0 & m_{45} & m_{55} \end{bmatrix}$$

and

$$C=\begin{bmatrix}0 & c_{12}\dot{q}_2 & c_{13}\dot{q}_3 & c_{14}\dot{q}_4 & c_{15}\dot{q}_5\\ -c_{12}\dot{q}_1 & 0 & c_{23}\dot{q}_3 & 0 & 0\\ -c_{13}\dot{q}_1 & -c_{23}\dot{q}_2 & 0 & 0 & 0\\ -c_{14}\dot{q}_1 & 0 & 0 & 0 & c_{45}\dot{q}_5\\ -c_{15}\dot{q}_1 & 0 & 0 & -c_{45}\dot{q}_4 & 0\end{bmatrix}$$

This matrix of dynamic equation can be grouped in the following manner: where

$$M=\left[\begin{array}{ccc|cc}m_{11} & m_{12} & m_{13} & m_{14} & m_{15}\\ m_{12} & m_{22} & m_{23} & 0 & 0\\ m_{13} & m_{23} & m_{33} & 0 & 0\\ \hline m_{14} & 0 & 0 & m_{44} & m_{45}\\ m_{15} & 0 & 0 & m_{45} & m_{55}\end{array}\right]=\begin{bmatrix}M_{33} & M_{c32}\\ M_{c32} & M_{22}\end{bmatrix}$$

and

$$C=\left[\begin{array}{ccc|cc}0 & c_{12}\dot{q}_2 & c_{13}\dot{q}_3 & c_{14}\dot{q}_4 & c_{15}\dot{q}_5\\ -c_{12}\dot{q}_1 & 0 & c_{23}\dot{q}_3 & 0 & 0\\ -c_{13}\dot{q}_1 & -c_{23}\dot{q}_2 & 0 & 0 & 0\\ \hline -c_{14}\dot{q}_1 & 0 & 0 & 0 & c_{45}\dot{q}_5\\ -c_{15}\dot{q}_1 & 0 & 0 & -c_{45}\dot{q}_4 & 0\end{array}\right]=\begin{bmatrix}C_{33} & C_{c32}\\ C_{c32} & C_{22}\end{bmatrix}$$

The coupling network of two dynamic equations is

$$M_{33}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\\ \ddot{q}_3\end{bmatrix}+C_{33}\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\end{bmatrix}+\begin{bmatrix}N_1(q_1)\\ N_2(q_2)\\ N_3(q_3)\end{bmatrix}=\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}-\begin{bmatrix}m_{14}\ddot{q}_4+m_{15}\ddot{q}_5\\ 0\\ 0\end{bmatrix}-\begin{bmatrix}c_{14}\dot{q}_4^2+c_{15}\dot{q}_5^2\\ 0\\ 0\end{bmatrix}$$

$$M_{22}\begin{bmatrix}\ddot{q}_4\\ \ddot{q}_5\end{bmatrix}+C_{22}\begin{bmatrix}\dot{q}_4\\ \dot{q}_5\end{bmatrix}+\begin{bmatrix}N_4(q_4)\\ N_5(q_5)\end{bmatrix}=\begin{bmatrix}u_1\\ u_2\end{bmatrix}-\begin{bmatrix}m_{14}\\ m_{15}\end{bmatrix}\ddot{q}_1-\begin{bmatrix}-c_{14}\\ -c_{15}\end{bmatrix}\dot{q}_1^2$$

From a mechanical perspective, this is equivalent to simulating two branches of the mechanical structure independently and coupling is treated as perturbation effects. Figure 8.12 shows how the mechanical structure is decoupled.

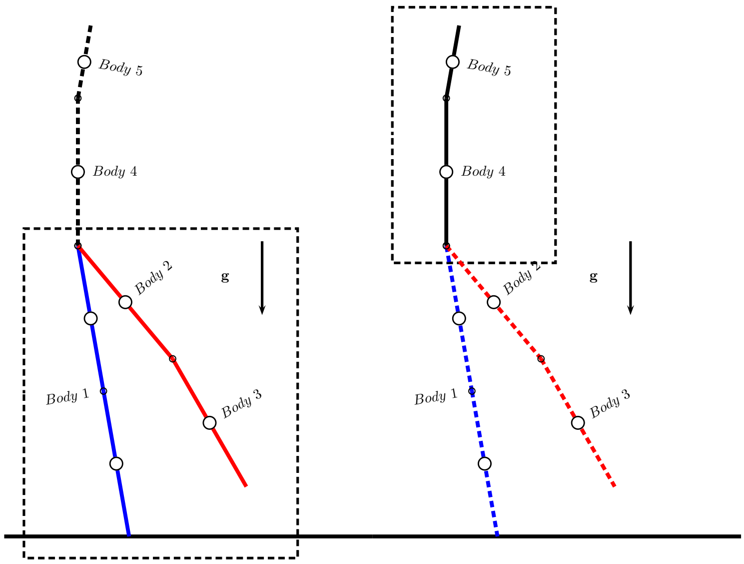


Figure 8.12: *mechanical coupling*

8.3.1 Torso And Arm

Using this mechanical coupling idea, the arm and torso motions are incorporated in our simulation. Three variables are added for the torso, the angle q_{tor}, the mass m_{tor} and the distance from the hip is l_{tor}. With the upper body, the equation for walking becomes

$$M\ddot{q}+C\dot{q}+N = u-\begin{bmatrix} m_{tor}l_{tor}Lcos(q_1-q_{tor})\ddot{q}_{tor} \\ 0 \\ 0 \end{bmatrix}-\begin{bmatrix} m_{tor}l_{tor}L\sin(q_1-q_{tor})\dot{q}_{tor}^2 \\ 0 \\ 0 \end{bmatrix} \tag{8.3}$$

From the Equation 8.3, if the torso is kept still, lower body walking will not be effected. In real life walking, the upper body is usually kept straight upward, so the coupling input from the upper body is very small.

The dynamics of the torso can be modelled as an inverted pendulum perturbed by the lower body dynamics, as follows.

$$m_{tor}l_{tor}^2\ddot{q}_{tor} = m_{tor}l_{tor}(gsin(q_{tor}) - Lcos(q_1 - q_{tor})\ddot{q}_1 - Lsin(q_1 - q_{tor})\dot{q}_1^2)) \quad (8.4)$$

by analysing the Equation 8.4, torso motion is unstable in nature, so control effort must be exerted to maintain its posture. Such a control task is trivial, **PD** controllers will work for maintaining stability, but the resulting swaying may be not natural looking.

The control method adopted by this research is based on the controlled Lagrange method. Although an inverted pendulum is not stable, a pendulum is stable. Through shaping the potential energy by control effort, we turn the inverted pendulum into a pendulum. The control input for the torso is

$$u = -km_{tor}l_{tor}(gsin(q_{tor}))$$

where k is a constant. When $k > 1$, it will turn the upper body dynamic from inverted pendulum to a pendulum. A bigger k will make the sway motion smaller, and keep the lower body motion untouched. A smaller k will make the upper body motion swing more and generate more perturbations to the lower body. For stable walking, the upper body motion is restricted to a small value.

When the stable pendulum is coupled with the walking motion, stable entrainment happens, so the torso and walking motion coordinate naturally. Figure 8.13 shows the entrainment of the torso motion and walking, where the body sway and walking are synchronized. To keep the stability, we set K to make the torso vibrate with a small amplitute. Figure 8.14 shows the effects of torso movement on walking. In our test, walking motion never converge to the limit cycle, but wobble around it.

Note that in real-life, the torso is closely related to the motion purpose and not governed by natural dynamic properties. For animation application, it is unnecessary to control the upper body dynamically. We can use procedure or other “IK” methods to generate primary motion of the upper body; walking dynamics perturbations are added for secondary motions. The motion of arms can be incorporated by following the same principle, it is just another level of complexity.

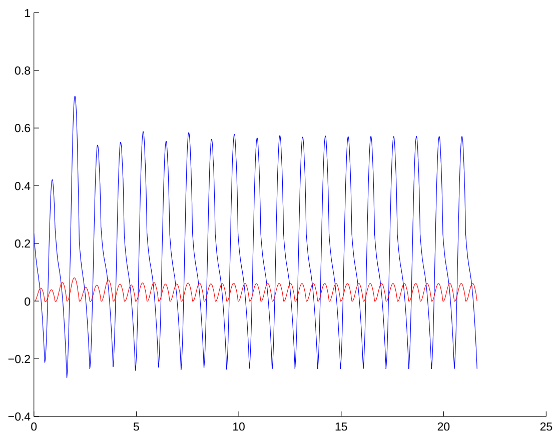


Figure 8.13: *The Mechanical Entrainment of Leg And Torso*

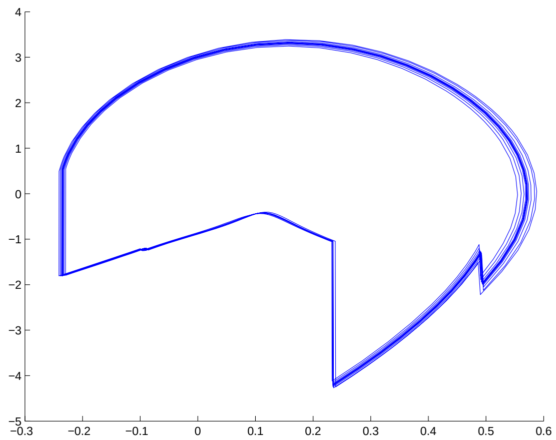


Figure 8.14: *The Torso Motion Effects on Walking*

8.4 Time Shift

For reptiles and fish, the main challenge rests in synthesizing the flexible spine which is composed of many **DOF**s. Such **DOF**s are similar and equally important, it is not appropriate to reduce any **DOF** through symmetry or mechanical coupling.

For such mechanical structures, an ad-hoc method is proposed. Each controller controls just one joint. The hypothesis states that since the joints are similar, their dynamics and motion should also be similar. Thus the same control strategy is applied for every joint. Motions of each joints are differentiated by the Time Shift group action.

Fish Swimming

These ideas are applied to synthesising the fish swimming motion. In this application, the group action is the Time Shift. The fish is made up of 8 links, and each **DOF** is controlled by a neural oscillator. The 8 **CPG** have the same parameters, but have different initial positions. Thus they have the same limit cycle, but different phases, as shown in Figure 8.15

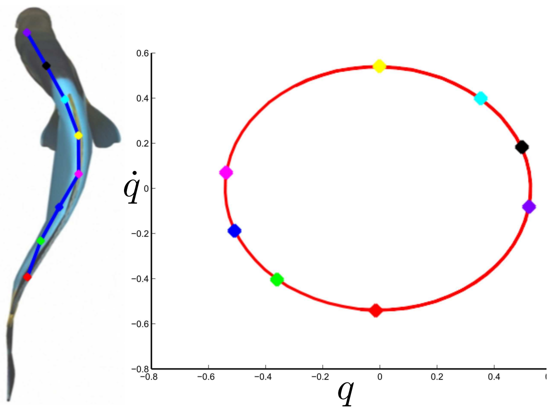


Figure 8.15: *CPG for Fish*

A simplified dynamic model is used. Each joint is modelled as a spring system, as in Equation 8.5

$$\ddot{q} = Kq \tag{8.5}$$

where q is the joint angle.

Figure 8.17 shows the gait of a fish swimming in line.

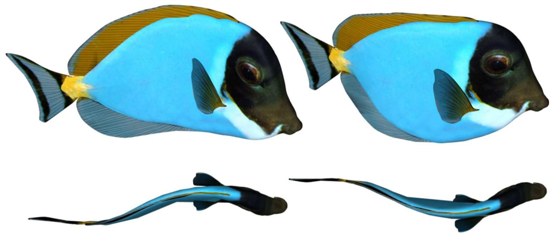


Figure 8.16: *Swimming Motion by our method*

8.4.1 Swimming Motion Tweaking

The swimming motion is divided into two space, the world space in which the position and orientation are specified, the local space in which the shape of the fish is specified. A simple fluid dynamic model is adopted for the relationship between local space and world space.

In the world space, the swimming trajectory is described the curvature K and length L. The trajectory curvature K is proportional to the sum of the joint angle. $K = c\sum_{i=1}^{n} q_i$ The swimming velocity is proportional to the velocity of the joint oscillation $v = c(\sum_{i=1}^{n} \dot{q_i}^2)$.

When a group action is chosen, the action is applied to all the **DOF**s. There are many group actions available for tweaking the fish swimming motion. Offset Action will result in the turning, Speed Action will make the fish swim faster. Energy Action will modify the swimming intensity. Figure 8.17 show the swimming in line gait. Figure 8.18 shows the phase plot of 4 segments, as time goes, the phase plot of the 4 **DOF**almost overlap. Figure 8.19 shows the state evolution over time, where all the state oscilate with the same amplitute but differentiate by a time offset.

Figure 8.20 show the turning gait, where an offset action is applied. This will make the body bend and turn the swimming direction. Figure **??** shows the phase plot when

Figure 8.17: *Fish Swim*

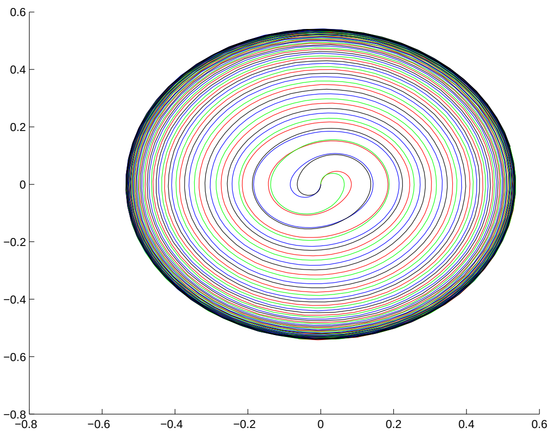


Figure 8.18: *Phase Plot of* 4 *segments*

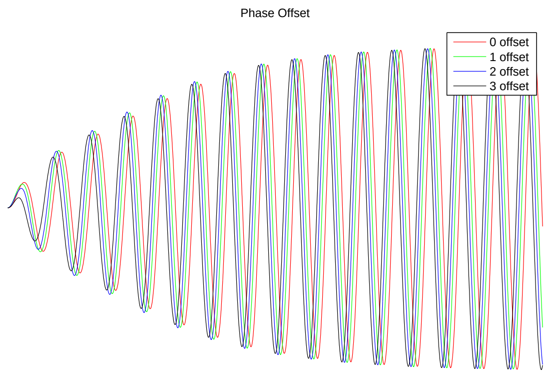


Figure 8.19: *The State of* 4 *segment*

all the **DOF**are applied offset action. Figure 8.22 show the state of the **DOF**after the offset action.

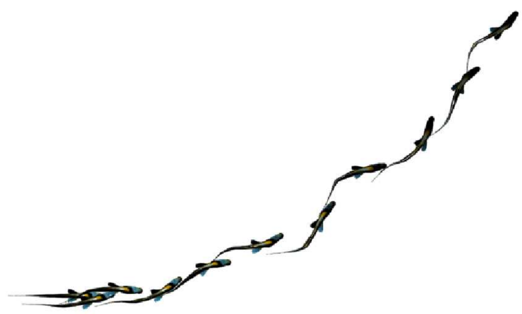


Figure 8.20: *Fish Swim Turn*

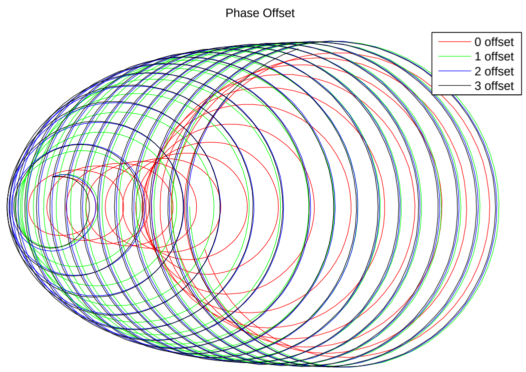


Figure 8.21: *Offset Action on All the **DOF***

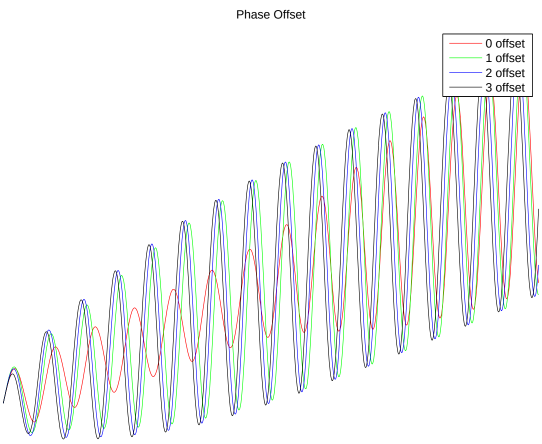


Figure 8.22: *Fish Swim*

Chapter 9

CONCLUSION AND FURTHER WORK

9.1 Conclusion

Physics based methods for synthesizing character animation have attracted much research interest in recent years. However, efficient methods for natural looking motion are still out of reach. This is mainly because of the complex structure of body dynamics. For physics based methods, the planning and inverse dynamic problems are very challenging. Optimization or Data Driven based methods are proposed, but such methods often require prohibitive computational time or extensive motion data that easily runs out of memory.

Taking a different perspective, the underlying question of motor synthesis research is how animals move in a complex and variable environment. This topic is more valuable and interesting, and, in fact, attracts even more research beyond the computer graphics community. Biological and robotic researcher investigated motor control from a very different perspective, and discovered some more properties which may be more crucial for understanding animal motions than the visual properties that are the main concern of graphic researchers. They have identified the limited neural activity, stability and energy efficiency of motor control.

The current idea from biological science and robotic engineering experience rejects the popular ideas of graphic researchers, because the sensing, computation and actuation systems of real animals are not suitable for optimization or database management. Animals in nature must adopt a very different strategy for moving. The inspiration from biology and robotic research is an explanation of the complexity of body dynamic. The complexity of body dynamics is not to challenge the neural control system, on the contrary, the complexity reflects the sophistication of nature. A sophisticate mechanical system may ease the control difficulties of many daily motion tasks. The new idea is that in fact most of the motion problems have already been solved by nature. Evolution has equipped animals with very handy mechanical apparatus, so that many motion tasks can be accomplished without any effort. To meet a specific purpose, animals only need to modify basic motion behaviours in a clever way.

These ideas inspired this research to develop animation methods considering of the biological facts. The belief is that if our animation methods follow the biological principle, potentially our characters in the virtual world will move and react in a more natural manner. Such a goal has been partly achieved in this research. In addition, more valuable results arise from this process. To develop simulation programs, intuitive biological ideas are tested for their computational efficiency and logical soundness. As a consequence, a new mathematical interpretation and many algorithms are proposed in this research. These new ideas are summarized as the Motor Invariant Theory. The new theory is more detailed and accurate compared with current biological ideas, and is applicable to controlling real robots. If it can be proved by further biological research and experiment, this theory may have significant meaning.

Motor Invariant Theory is composed of several interconnecting ideas. The theory unifies these ideas in a very different perspective of dynamics. The traditional force - motion perspective is not insightful for understanding natural dynamics, because it provides little information about the stability and energy efficiency of motion.

Motor Invariant Theory adopts the geometrical perspective. The concept of phase space is introduced and the dynamic system is transformed into a geometrical structure: the phase portrait. After this transformation, motion dynamics can be studied with many geometrical tools. On a phase plot, the dynamic system is divided into different regions. There is an attractor in each region which attracts all the states in the surrounding states toward it. Motor Invariant Theory proposes that animal motion utilizes these attractors for motor control. Because attractors promise stability and energy efficiency, they will greatly reduce control difficulties.

This idea has support from biological research. The idea of organized motions in blocks is proposed as the motion primitive hypothesis. And the idea of utilizing attractors has been proposed by the equilibrium point hypothesis. Such ideas may be new for graphic researchers, but the principles are long established in biological research.

The novelty of Motor Invariant Theory is the idea of the adaptation mechanism. Given that the attractors are the starting point for motion planning, the following question is how the neural control system tweaks the dynamics to achieve specific motions. During this process, the challenge is that stability must be maintained, energy cost must be minimized and the computation should not last long. Optimization based methods are not suitable. Also the tracking controllers are not appropriate for motor control, because motions vary greatly. The idea of local stability control that constrains the motion within a small error range from the reference will make motion lack variation. Motor Invariant Theory proposes that the stability property should be controlled qualitatively. Large deviations from the reference should be allowed while stability is controlled. In the geometrical perspective, this means the shape and position of the attractor does not matter, the controller only needs to maintain the attractor and the current state within the basin of attraction. This idea is modelled by the mathematical language of topology. Maintaining the attractor without considering the shape and position means the topology remained the same. In motor invariant theory, changing the shape and position of attractors is not only allowed but utilized as a powerful tool. The idea of changing the shape and position of the attractors not only generates adaptive motions, but also promises stability and energy efficiency and computation efficiency.

Two methods have been developed following this principle. The first idea is entrainment. This idea applies to almost all periodic systems. For entrainment systems, the periodic behaviour will be enhanced and perturbations are rejected. From the geometrical perspective, the entrainment will maintain the topology of limit cycle and enlarge the basin of attraction. In addition, the idea of entrainment is well supported by biological research. Also the method is computationally efficient. Another method is based on symmetry and the preserving law of mechanical systems. Natural dynamic systems tend to preserve many properties during motion, like energy or momentum.

Transforming motions in a way that preserves such invariant properties will promise energy efficiency. Such transformation actions form another important mathematical structure, the Lie group.

It is easy to prove that a Lie group transformation will not alter the topology, thus the stability of transformed motions is guaranteed. This provides animators with a direct method for modifying the motion without concerns about stability. Also this method is easy to use. Because Lie group transformation can be parameterized with a few parameters. Animators can modify motions by specifying very few parameters of Lie group, instead of each **DOF** of the character. As examples, three Lie groups are developed, the offset group which changes the locator positions, which changes the direction of motion; the time scaling group which modifies the speed of motion, and also the energy scaling group which modifies the energy of motion. With such tools, given a motion primitive, animators are allowed to modify the position, speed and amplitude of motion, without worrying about the stability. As for the computation cost, this research found that for rigid body systems, control input of each group element has a close form formula, and the computational cost is trivial to compute. The idea of Lie Group is also supported by biological research, which found that the motion trajectory has many transformation invariant properties.

Because the **CPG** entrainment and Lie Group transformation are based on the topological invariant principles, these two controllers can be combined. Such operations will change the shape and location of the locator, resulting in many types of variations in motion. If the basin of attraction is modified to capture the current state, the current motion primitive can be maintained. However, there are also important applications for changing the shape and position of the locator to avoid current state. As a result, the motion will diverge, and finally converge to a different attractor. The important application is in motion transition. We can tweak the neighbour attractor to capture the current state, which will generate stable transitional motion. This shows how motor invariant theory can be easily extended to explain more natural motion phenomena.

Such methods have been applied to control various mechanical systems and characters. The bouncing ball example shows how the entrainment forms an attractive limit cycle and how group action changes the shape. In this process the bouncing height is maintained and can be stabilized against many perturbations. Another example is bipedal walking. Although bipedal walking seems difficult to control, it can happen naturally because a limit cycle exists. With the entrainment method, the periodic behaviour is enhanced and the basin of attraction is enlarged. This makes passive walking more stable. This qualitative control approach can generate different gaits with different body structures and environment conditions. When Lie group actions are applied, the passive walker is capable of walking on different terrains (offset action), at different speeds (time scaling) or with different step sizes (energy scaling). For the balancing motion primitive, entrainment will turn the dynamic system attractive and group operators will adjust the size of basin of attraction and the time needed to stabilize. Also the transitional motion of walking and balance can be synthesized with an energy efficient method requiring little control effort.

Such simulation results are compared with real life data and they comply with the observed facts.

This research provides an answer to the way animals achieve computational efficiency, energy efficiency and stability against various perturbations. For animation researchers, motor invariant theory proposes a method that generates adaptive and natural looking motions in a computationally efficient and reliable way.

9.2 Unsolved Question

But as a new theory, there are still many unanswered questions.

Finding the attractors in a high dimension dynamic system is not an easy task. At the end of the research, several methods are proposed to simplify the dynamic space to make the task of finding locators easier. We propose neglecting degrees of freedom in minor motions; dynamic space can be reduced according to the symmetrical properties or exploring the similarity and time shift properties in many mechanical structures. Such methods help to add more detail to the synthesized motion, like the rotation, body and arm swing motions. Also the method can be extended for more applications like crowd and swimming simulation. But this question is not answered completely in this research.

Nature seems to outsmart us. Even though we have learned a lot from nature, we still have much to learn.

For computer animation, current methods of **MoIT**are capable of generating physically realistic motion adaptation in real-time, however, at current stage, this method have several drawbacks preventing its production application.

The method is fully automatics, but requries symbolic differential equations. For animators, adjust animation by tweaking the aparameters of a differential equation is not an intuitive process. Also at current, number of motion primitives is very limited. However, the idea of Lie group transformation and topological conjugacy is generative that can be applied to any differential system.

In theory, symbolic equations are not necessary. From the geomtrical perspective, as long as the phase portrait can be obtained, this method can be applied.

In the further work, more types of animation systems can be developed based on different models of the dynamic system. Key frame and motion capture date maybe incoporate to genearate dynamic systems by machine learning technology. Also intutive tools can be developed which allow the animator to sculpt the phase portrait directly.

9.3 Further Work

Motor Invariant Theory is not an improvement on existing **CMS** techniques, it is a different paradigm. This thesis does not explore the full implication and potential of this new theory. There is room for improvement, new techniques to be developed and even new questions to be answered. This section lists several potential topics that may interest computer graphic or biological research communities.

9.3.1 Stable Templates of Motion Primitives

This research started with a unstable system, where stability is enhanced by adding control effort. Motor control is a complex task. In many cases, it is impossible to model all the control efforts that turn an unstable system into a stable one.

An alternative method is to start from a stable system and modify its shape to match the observation. Such methods may lose the details of motion but provide better stability and controllability. For games or film production, this idea may be important, animators require controllability and stability over physical realism. For characters performing acrobatics, the characters must not fall even though the dynamic system is unstable in nature. Compared with traditional method like **PD**controller, this method will be more robust.

9.3.2 More Types Of Symmetry

More types of symmetry will generate more types of transformation that can be applied to adapt motion. All the group actions adopted in this research are linear transformation group, which are easy to compute. But the types of transformation are very limited. Exploring further types of symmetry may provide different adaptation schemes and may expand the theory to different motion primitives.

- **Discrete Symmetry Properties** Bipedal walking motions is synthesized in this research, an interesting idea is motions for four or more legs be synthesized based on the bipedal walking strategy.

  This can be done by exploring another type of symmetry: discrete symmetry. For dogs, the hind leg and font leg will move in synchronization or in antiphase.

- **Non-linear Symmetry from Structural Parameter Tuning** Non-linear symmetry preserving transformation will generate more types of adaptation. Since non-linear transformation is more difficult to find, it remains questionable how a biological system perceives it and applies it to motion adaptation. However non-linear transformation is suitable for modelling the transformation resulting from tweaking system parameters. From the idea of structural stability we know the results of tweaking system parameters are equivalent to having a one-one mapping transformation. Further research results from non-linear transformation may potentially completely solve the motion re-targeting problem

- **Symmetry of Partial Differential System**

  All the methods developed are for ordinary differential equations, which is good enough for rigid body dynamics. In fact the topological properties and symmetrical properties also apply to partial differential equations. A famous example is the Lorenz transformation group and Maxwell equation.

  Symmetries of partial differential equations are important for they may extend the control strategy to control the motion of elastic bodies or locomotions in fluids. Such motions are more expensive and are rarely addressed by current **CMS** research.

To explore more types of symmetry, reformulating the form of equations may ease the task. Current dynamic equations are based on a fixed coordinates frame. It is helpful to formulate the equations in a coordinate free manner or in the local frame.

9.3.3 Transform the Motion Capture Data

For computer animation, even though methods for simulating high dimensional characters are proposed. It may be impractical to synthesize all types of motions by procedural methods. An alternative method is to use dynamic simulation to modify motion capture data, which is well addressed in many research studies in the computer graphics community.

Based on the idea of topological equivalence, motion primitive of different persons or motions of different situations should have the property of topological equivalence. In state space, there should exist a one-one mapping transformation function. Motion Data can be converted into the state space and transformed by one-one mapping.

We can use the low dimensional model to find the one to one mapping relationship, which is applied to transform the high dimensional motion capture data. Potentially, this method may retain the motion details and involve little computational work.

9.3.4 Muscle Actuation

In the thesis, control effort is applied directly to each **DOF** of the mechanical system. In biological research, this process is not so direct. The neural system generates some chemicals which affect the material properties of muscles, and force is generated as an indirect side effect.

The question of muscle actuation is untouched in this research, but with further thought, **MoIT** could also provide an alternative idea of muscle action. If transformation is the reason for applying control effort, the actuation of muscles can be calculated directly from the transformation, without considering the force generated. From this perspective, muscle actuation can be easier than calculating the forces. The reason is transformation can be achieved by two methods, either control effort or by changing the system parameters.

For the simple mass spring system, offset can be implemented by changing the rest length parameter d. Speed action can be implemented by changing the stiffness K. and energy scaling can be achieved by adjusting the stiffness K and then restoring it.

For biological systems, the method of changing parameters may be better as it will help motor control system get rid of the necessary feedback and computation. In fact most control effort in the thesis is potential energy shaping, which only involves modifying the potential energy. If muscles are modelled as springs, then potential energy shaping can also be achieved through modifying spring parameters.

The complex muscle structure may provide a mechanism for fine tuning the deformation of the phase portrait and the attractor can be changed into any possible shape. This idea may provide a conjecture for further biological research. For graphic research, incorporating muscles in this manner will have no effect on motion synthesis or computational work. The potential benefit is that the parameters of muscles can affect the skin deformation.

9.3.5 Perception based Dynamics

Motion perception is a high level capacity; it is based upon our object recognition ability and our dynamic reasoning ability. Many physiological questions in computer graphics may ultimately rely on recognition and perception research in neural science. The introduction of a motion synthesis method also touches on the question of dynamic motion perception and encoding problems in intelligence. The topological equivalence and symmetry may also provide an understanding of the perception problem.

Based on the idea of topology equivalence, the neural system may not need to encode the details of dynamic system, the neural system can form an analogous dynamic systems in our brain which is analogous to the real dynamic systems. Such model will lack the detailed accuracy, but get the qualitative properties right.

Based on the idea of symmetry, neural system may store some experience and the symmetrical property of dynamics in the memory. Our brain may verify dynamics by transforming our experience to match our observation.

We are still not sure which method is better, but for our brain, both methods are more practical than forming a symbolic equation solving it numerically. Maybe a new dynamic simulator can be designed to test this hypothesis.

A dynamic simulator can be built upon the topology and symmetry property. Animators can animate by specifying the attractor and the transformation being applied. If the hypothesis is true, even though the method will generate physically inaccurate results, the audience will not notice it.

Appdx A:Dynamic Equation for Passive Walking

Knee Free Phase

During the knee free swing phase, the passive walker can be seen as a triple inverted pendulum. The dynamic system is a constrained rigid body dynamic system.

It will takes the following form.

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + N = \tau$$

where M is mass-inetia matrix. C is the centrifugal matrix. N is the gravity force.

τ is the external control input. q is the configuration vector $q = (q_1, q_2, q_3)$.

Each symbol is illustrate in Figure 6.2. q_1 is the supporting leg angle. q_2 is the angle of the swinging thigh. q_3 is the angle of the swing shaft. L is the leg length. a_1, b_1, a_2, b_2 specify the position of gravity centre.

m_H is the mass of the hip. m_s is the mass of the shaft. m_t is the mass of the thigh.

In certain situations, the mass of legs are not symmetrical. Thus for mass of the thigh and shaft, upperscript is used to specify whether the leg is the swing one(SW) or the supporting one(ST).

The mass matrix is definite and symmetrical:

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix}$$

The elements are as follows.

$$M_{11} = m_s^{st} a_1^2 + m_t^{st}(l_s + a_2)^2 + (m_h + m_t^{sw} + m_s^{sw})L^2$$

$$M_{12} = -(m_t^{sw} b_2 + m_s^{sw} l_t) L cos(q_2 - q_1)$$

$$M_{13} = -m_s^{sw} b_1 cos(q_3 - q_1)$$

$$M_{22} = m_t^{sw} b_2^2 + m_s^{sw} l_t^2$$

$$M_{23} = m_s^{sw} l_t b_1 cos(q_3 - q_2)$$

$$M_{33} = m_s^{sw} b_1^2$$

The centrifugal matrix is anti-symmetrical.

$$C(q, \dot{q}) = \begin{bmatrix} 0 & C_{12}\dot{q}_2 & C_{13}\dot{q}_3 \\ -C_{12}\dot{q}_1 & 0 & C_{23}\dot{q}_3 \\ -C_{13}\dot{q}_1 & -C_{23}\dot{q}_2 & 0 \end{bmatrix}$$

where the elements are as follows:

$$C_{12} = -(m_t^{sw} b_2 + m_s^{sw} l_t) L sin(q_1 - q_2)$$

$$C_{13} = -m_s^{sw} b_1 L sin(q_1 - q_3)$$

$$C_{23} = m_s^{sw} l_t b_1 sin(q_3 - q_2)$$

The N is the generalized force generated by gravity.

$$N = \begin{bmatrix} -(m_s^{st} a_1 + m_t^{st}(l_s + a_2) + (m_h + m_s^{sw} + m_t^{sw})L) g \sin(q_1) \\ (m_t^{sw} b_2 + m_s^{sw} l_t) g sin(q_2) \\ m_s^{sw} b_1 g sin(q_3) \end{bmatrix}$$

where g is the gravity coefficient.

Knee Strike

Knee Strike happens when the swing knee joint reaches the limit. The dynamic assumption is that after the knee strike, the knee joinst will be locked and the triple inverted pendulum system of knee free dynamics will become a double inverted pendulum system.

The following equations are established based on the rotation momentum preservation property of the dynamic system.

$$J^+ \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}^+ = J^- \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}^-$$

J are the rotation inertia matrix. J^+ is the rotation inertia after the collision. J^- is the rotation inertia before the collision.

After the collision, the knee joinst is locked, the swing thigh and shank will rotate together, thus we have

$$q_3^+ = q_2^+$$

Because during the collision, only rotation moment of two centre (the hip centre and supporting toe centre) are preserved, two rotation momentums are preserved. So J^- is an 3 by 2 matrix

$$J^- = \begin{bmatrix} J_{11}^- & J_{12}^- & J_{13}^- \\ J_{21}^- & J_{22}^- & J_{23}^- \end{bmatrix}$$

and J^+ is an 2 by 2 matrix.

The elements are as follows:

$$J^+ = \begin{bmatrix} J_{11}^+ & J_{12}^+ \\ J_{21}^+ & J_{22}^+ \end{bmatrix}$$

$$J_{11}^- = -(m_s^{sw} l_t + m_t^{sw} b_2) Lcos(q_1 - q_2) - m_s^{sw} b_1 cos(q_1 - q_3) + (m_t^{sw} + m_s^{sw} + m_h) L^2 + m_s^{st} a_1^2 + m_t^{st} (l_s + a_2)^2$$

$$J_{12}^- = -(m_s^{sw} l_s + m_t^{sw}) Lcos(q_1 - q_2) + m_s^{sw} b_1 l_t cos(q_2 - q_3) + m_t^{sw} b_2^2 + m_s^{sw} l_t^2$$

$$J_{13}^- = -m_s^{sw} b_1 Lcos(q_1 - q_3) + m_s^{sw} b_1 l_t cos(q_2 - q_3) + m_s^{sw} b_1 b_2$$

$$J_{21}^- = -(m_s^{sw} l_t + m_t^{sw} b_2) Lcos(q_1 - q_2) - m_s^{sw} b_1 Lcos(q_1 - q_3)$$

$$J_{22}^{-}=m_s^{sw}b_1l_tcos(q_2-q_3)+m_s^{sw}l_t^2+m_t^{sw}b_2^2$$

$$J_{23}^{-}=m_s^{sw}b_1l_tcos(q_2-q_3)+m_s^{sw}b_1^2$$

$$J_{11}^{+}=J_{21}^{+}+m_t^{st}(l_s+a_2)^2+(m_h+m_t^{sw}+m_s^{sw})L^2+m_s^{st}a_1^2$$

$$J_{12}^{+}=J_{21}^{+}+m_s^{sw}(l_t+b_1)^2+m_t^{sw}b_2^2$$

$$J_{21}^{+}=-(m_s^{sw}(b_1+l_t)+m_t^{sw}b_2)Lcos(q_1-q_2)$$

$$J_{22}^{+}=m_s^{sw}(b_1+l_t)^2+m_t^{sw}b_2^2$$

Knee Locking Phase

For the knee locking swing phase, the walker can be seen as a double inverted pendulum system. The equation of this rigid body dynamic system also has the following form:

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+N=\tau$$

The mass inertia matrix is a 2 by 2 symmetrical matrix.

$$M(q)=\begin{bmatrix} M_{11} & M_{12} \\ M_{12} & M_{22} \end{bmatrix}$$

The elements are as follows:

$$M_{11} = m_s^{st} a_1^2 + m_t^{st}(l_s + a_2)^2 + (m_h + m_t^{sw} + m_s^{sw})L^2$$

$$M_{12} = -(m_t^{sw} b_2 + m_s(l_t + b_1))Lcos(q_2 - q_1)$$

The centrifugal matrix is 2 by 2 and anti-symmetrical.

$$C(q, \dot{q}) = \begin{bmatrix} 0 & C_{12}\dot{q}_2 \\ -C_{12}\dot{q}_1 & 0 \end{bmatrix}$$

and the only non-zero element is:

$$C_{12} = (m_t^{sw} b_2 + m_s(l_t + b_1))Lsin(q_1 - q_2)$$

The general force vector of gravity has only two elements:

$$N = \begin{bmatrix} -(m_s^{st} a_1 + m_t^{st}(l_s + a_2) + (m_h + m_s^{sw} + m_t^{sw})L)g\sin(q_1) \\ (m_t^{sw} b_2 + m_s^{sw}(l_t + b_1))gsin(q_2) \end{bmatrix}$$

Heel Strike Phase

The heel strike happens when the swing heel touch the ground. The impact dynamics equation is also based on the rotation momentum conservation law. Thus we have the dynamic equation as the following form:

$$J^+ \dot{q}^+ = J^- \dot{q}^-$$

After the heel strike, the passive walker will start in the knee free phase, At the beginning, although the walker walks in knee free model, the shank and thigh have the same rotating speed.

$$\dot{q}_3^+ = \dot{q}_2^+$$

Thus J^+ is 3 by 2 matrix There is a switch between the supporting and swing leg, so the J^+ is:

$$J^+ = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

Based on rotation momentum preservation, the J^- is:

$$J^- = \begin{bmatrix} J_{11}^- & J_{22}^- \\ J_{21}^- & 0 \end{bmatrix}$$

The elements of J^- are:

$$J_{11}^- = J_{21}^- + (m_h L + m_t^{st}(a_2 + l_s) + m_s^{st} a_1 + m_t^{sw}(a_2 + l_s) + m_s a_1) L cos(q_1 - q_2)$$

$$J_{12}^- = -m_s^{sw} a_1 (l_t + b_1) - m_t^{sw} b_2 (l_s + a_2)$$

$$J_{21}^- = -m_s^{st} a_1 (l_t + b_1) - m_t^{st} b_2 (l_s + a_2)$$

$$J_{11}^+ = J_{21}^+ + (m_s^{st} + m_t^{st} + m_h) L^2 + m_s^{sw} a_1^2 + m_t^{sw}(a_2 + l_s)^2$$

$$J_{12}^+ = J_{21}^+ + (m_s^{st} + m_t^{st} + m_h) L^2 + m_s^{sw} a_1^2 + m_t^{sw}(a_2 + l_s)^2$$

$$J_{21}^{+} = -(m_s^{st}(b_1 + l_t) + m_t^{st} b_2) Lcos(q_1 - q_2)$$

$$J_{22}^{+} = m_s^{st}(l_t + b_1)^2 + m_t^{st} b_2^2$$

References

Abraham R. and Marsden J., 1978. *Foundations of mechanics*.

Ames A. and Sastry S., 2006. *A categorical theory of hybrid systems*, volume 68.

Anderson F. and Pandy M., 2001. Dynamic Optimization of Human Walking. *Journal of Biomechanical Engineering*, **123**, 381.

Arikan O., Forsyth D. and O'Brien J., 2003. Motion synthesis from annotations. *ACM Transactions on Graphics (TOG)*, **22**(3), 402–408.

Asano F. and Yamakita M., 2001. Virtual gravity and coupling control for robotic gait synthesis. *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on*, **31**(6), 737–745.

Bizzi E., dAvella A., Saltiel P. and Tresch M., 2002. Book review: Modular organization of spinal motor systems. *The Neuroscientist*, **8**(5), 437.

Bizzi E., Giszter S., Loeb E., Mussa-Ivaldi F. and Saltiel P., 1995. Modular organization of motor behavior in the frog's spinal cord. *Trends in Neurosciences*, **18**(10), 442–446.

Byl K. *Metastable legged-robot locomotion*. PhD thesis, Massachusetts Institute of Technology, 2008.

Chen H. and others , 2007. Passive dynamic walking with knees: A point foot model.

Chen V. F. H. Passive dynamic walking with knees: A point foot model. Master's thesis, Massachusetts Institute of Technology, 2007.

Cohen A. H., 1988. Evolution of the vertebrate central pattern generator for locomotion. *Neural control of rhythmic movements in vertebrates*, 129–166.

Collins S., Ruina A., Tedrake R. and Wisse M., 2005. Efficient bipedal robots based on passive-dynamic walkers. *Science*, **307**(5712), 1082.

Collins S. and Ruina A., April 2005. A bipedal walking robot with efficient and human-like gait. In *Robotics and Automation, 2005. ICRA 2005. Proceedings of the 2005 IEEE International Conference on*, 1983–1988.

Coros S., Beaudoin P. and van de Panne M., 2010. Generalized biped walking control. *ACM Transactions on Graphics (TOG)*, **29**(4), 130.

Coros S., Beaudoin P. and van de Panne M., 2009. Robust task-based control policies for physics-based characters. In *SIGGRAPH Asia '09: ACM SIGGRAPH Asia 2009 papers*, New York, NY, USA. ACM, 1–9.

de Lasa M., Mordatch I. and Hertzmann A., 2010. Feature-based locomotion controllers. *ACM Transactions on Graphics (TOG)*, **29**(4), 131.

Degallier S. and Ijspeert A., October 2010. Modeling discrete and rhythmic movements through motor primitives: a review. *Biol. Cybern.*, **103**, 319–338.

Dickinson M., Farley C. and others , 2000. How animals move: an integrative view. *Science*, **288**(5463), 100.

Feldman A., 1986. Once more on the equilibrium-point hypothesis(λ model) for motor control. *Journal of motor behavior*, **18**(1), 17–54.

Flash T. and Handzel A., 2007. Affine differential geometry analysis of human arm movements. *Biological cybernetics*, **96**(6), 577–601.

Franklin D., Liaw G., Milner T., Osu R., Burdet E. and Kawato M., 2007. Endpoint stiffness of the arm is directionally tuned to instability in the environment. *Journal of Neuroscience*, **27**(29), 7705.

Fukuoka Y., Kimura H. and Cohen A., 2003. Adaptive dynamic walking of a quadruped robot on irregular terrain based on biological concepts. *The International Journal of Robotics Research*, **22**(3-4), 187.

Futakata Y. and Iwasaki T. Entrainment to natural oscillations via uncoupled central pattern generators. *Automatic Control, IEEE Transactions on*, (99), 1–1.

Gallese V., Fadiga L., Fogassi L. and Rizzolatti G., 1996. Action recognition in the premotor cortex. *Brain*, **119**(2), 593.

Glynn I., 2003. *An anatomy of thought: The origin and machinery of the mind*. Oxford University Press, USA.

Goldstein H., Poole C., Safko J. and Addison S., 2002. *Classical mechanics*, volume 70.

González-Miranda J., 2004. *Synchronization and control of chaos: an introduction for scientists and engineers*. Imperial College Pr.

Hodgins J. K., Wooten W. L., Brogan D. C. and O'Brien J. F., 1995. Animating human athletics. In *SIGGRAPH '95: Proceedings of the 22nd annual conference on Computer graphics and interactive techniques*, New York, NY, USA. ACM, 71–78.

Hogan N., 1985. Impedance control-An approach to manipulation. I-Theory. II-Implementation. III-Applications. *ASME, Transactions, Journal of Dynamic Systems, Measurement, and Control (ISSN 0022-0434)*, **107**.

HSU C., 1980. A theory of cell-to-cell mapping dynamical systems. *ASME, Transactions, Journal of Applied Mechanics*, **47**, 931–939.

Jain S., Ye Y. and Liu C. K., 2009. Optimization-based interactive motion synthesis. *ACM Trans. Graph.*, **28**(1), 1–12.

Keogh E., Palpanas T., Zordan V., Gunopulos D. and Cardle M., 2004. Indexing large human-motion databases. In *Proceedings of the Thirtieth international conference on Very large data bases-Volume 30*. VLDB Endowment, 780–791.

Khalil H. and Grizzle J., 2002. *Nonlinear systems*, volume 122. Prentice hall Upper Saddle River, NJ.

Kovar L. and Gleicher M., 2003. Flexible automatic motion blending with registration curves. In *Proceedings of the 2003 ACM SIGGRAPH/Eurographics symposium on Computer animation*. Eurographics Association, 214–224.

Kovar L., Gleicher M. and Pighin F., 2008. Motion graphs.

Kuo A., 1999. Stabilization of lateral motion in passive dynamic walking. *The International Journal of Robotics Research*, **18**(9), 917.

Laszlo J., van de Panne M. and Fiume E., 1996. Limit cycle control and its application to the animation of balancing and walking. In *SIGGRAPH '96: Proceedings of the 23rd annual conference on Computer graphics and interactive techniques*, New York, NY, USA. ACM, 155–162.

Latash M., 2008. *Neurophysiological basis of movement*. Human Kinetics Publishers.

Lee Y., Kim S. and Lee J., 2010a. Data-driven biped control. *ACM Transactions on Graphics (TOG)*, **29**(4), 129.

Lee Y., Wampler K., Bernstein G., Popović J. and Popović Z., 2010b. Motion fields for interactive character locomotion. In *ACM Transactions on Graphics (TOG)*, volume 29. ACM, 138.

Levine S., Lee Y., Koltun V. and Popović Z., 2011. Space-time planning with parameterized locomotion controllers. *ACM Transactions on Graphics (TOG)*, **30**(3), 23.

Liu C. K., 2009. Dextrous manipulation from a grasping pose. In *SIGGRAPH '09: ACM SIGGRAPH 2009 papers*, New York, NY, USA. ACM, 1–6.

Liu C.-Y. K. *Towards a generative model of natural motion*. PhD thesis, Seattle, WA, USA, 2005.

Liu L., Yin K., van de Panne M., Shao T. and Xu W., 2010. Sampling-based contact-rich motion control. *ACM Transactions on Graphics (TOG)*, **29**(4), 128.

Macchietto A., Zordan V. and Shelton C. R., 2009. Momentum control for balance. In *SIGGRAPH '09: ACM SIGGRAPH 2009 papers*, New York, NY, USA. ACM, 1–8.

Marsden J., Montgomery R. and Ra?iu T., 1990. *Reduction, symmetry, and phases in mechanics*. Number 436. American Mathematical Society.

Matsuoka K., 1985. Sustained oscillations generated by mutually inhibiting neurons with adaptation. *Biological Cybernetics*, **52**(6), 367–376.

Matsuoka K., 1987. Mechanisms of frequency and pattern control in the neural rhythm generators. *Biological Cybernetics*, **56**(5), 345–353.

McGeer T., 1990. Passive dynamic walking. *The International Journal of Robotics Research*, **9**(2), 62.

Mordatch I., de Lasa M. and Hertzmann A., 2010. Robust physics-based locomotion using low-dimensional planning. *ACM Transactions on Graphics (TOG)*, **29**(4), 71.

Müller M., Röder T. and Clausen M., 2005. Efficient content-based retrieval of motion capture data. In *ACM Transactions on Graphics (TOG)*, volume 24. ACM, 677–685.

Nishikawa K., Biewener A., Aerts P., Ahn A., Chiel H., Daley M., Daniel T. and others , 2007. Neuromechanics: an integrative approach for understanding motor control. *Integrative and Comparative Biology*.

Olver P., Olver P., Olver P., Olver P. and Mathematician G., 1986. *Applications of Lie groups to differential equations*, volume 107. Springer-Verlag.

Parent R., 2002. *Computer animation: algorithms and techniques*. Morgan Kaufmann Pub.

Poincaré H., 1885. Sur les courbes définies par les équations différentielles. *J. Math Pures et Appl.(4. série)*, **1**, 167–244.

Poincaré H. and Magini R., 1899. Les méthodes nouvelles de la mécanique céleste. *Il Nuovo Cimento (1895-1900)*, **10**(1), 128–130.

Pratt J., Dilworth P. and Pratt G., 1997. Virtual model control of a bipedal walking robot. In *Robotics and Automation, 1997. Proceedings., 1997 IEEE International Conference on*, volume 1. IEEE, 193–198.

Raibert M. and others , 1986. *Legged robots that balance*. MIT press Cambridge, MA.

Raibert M. and Hodgins J., 1991. Animation of dynamic legged locomotion. *ACM SIGGRAPH Computer Graphics*, **25**(4), 349–358.

Smale S., 1970. Differentiable dynamical systems. *Uspekhi Matematicheskikh Nauk*, **25**(1), 113–185.

Spong M., 1998. Underactuated mechanical systems. *Control Problems in Robotics and Automation*, 135–150.

Spong M. W., 1996. Energy based control of a class of underactuated mechanical systems. In *IFAC World Congress*, 431–435.

Spong M. and Bullo F., 2005. Controlled symmetries and passive walking. *Automatic Control, IEEE Transactions on*, **50**(7), 1025–1031.

Stephens B. and Atkeson C., 2009. Modeling and control of periodic humanoid balance using the linear biped model. In *Humanoid Robots, 2009. Humanoids 2009. 9th IEEE-RAS International Conference on*. IEEE, 379–384.

Taga G., 1995. A model of the neuro-musculo-skeletal system for human locomotion. *Biological cybernetics*, **73**(2), 97–111.

Viviani P. and Stucchi N., 1992. Biological movements look uniform: evidence of motor-perceptual interactions. *J Exp Psychol Hum Percept Perform.*, **18**(3), 603–623.

Wang J., Fleet D. and Hertzmann A., 2010. Optimizing walking controllers for uncertain inputs and environments. In *ACM Transactions on Graphics (TOG)*, volume 29. ACM, 73.

Wei X., Min J. and Chai J., 2011. Physically valid statistical models for human motion generation. *ACM Transactions on Graphics (TOG)*, **30**(3), 19.

Williamson M., 1998. Exploiting natural dynamics in robot control.

Williamson M. *Robot arm control exploiting natural dynamics*. PhD thesis, Massachusetts Institute of Technology, 1999.

Witkin A. and Kass M., 1988. Spacetime constraints. In *SIGGRAPH ’88: Proceedings of the 15th annual conference on Computer graphics and interactive techniques*, New York, NY, USA. ACM, 159–168.

Wu J. and Popović Z., 2010. Terrain-adaptive bipedal locomotion control. *ACM Transactions on Graphics (TOG)*, **29**(4), 72.

Ye Y. and Liu C., 2010. Optimal feedback control for character animation using an abstract model. *ACM Transactions on Graphics (TOG)*, **29**(4), 74.

Yin K., Loken K. and van de Panne M., 2007. Simbicon: simple biped locomotion control. *ACM Trans. Graph.*, **26**(3), 105.