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Monitoring a Time Series Approach**

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# Arbitrage, Market Definition and Monitoring a Time Series Approach.

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## Abstract

This article considers the application to regional price data of time series methods to test stationarity, multivariate cointegration and exogeneity. The discovery of stationary price differentials in a bivariate setting implies that the series are rendered stationary by capturing a common trend and we observe through this mechanism long-run arbitrage. This is indicative of a broader market definition and efficiency. The problem is considered in relation to more than 700 weekly data points on gasoline prices for three regions of the US and similarly calibrated simulated series. The discovery of a single common trend is consistent with competitive pricing and a broad market definition, but the finding of a single weakly exogenous variable affects this conclusion.

*Key Words:* Arbitrage, Cointegration, Competition, Equilibrium Price Adjustment, Stationarity, Stochastic Trend, Weak Exogeneity.

*JEL Classification:* C32, D18, D40.

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## Introduction

The article considers the problem of analyzing product price inter-relatedness to determine whether markets are competitive. This notion goes back to earlier analysis undertaken in the 70s (Maunder, 1972), but these studies pre-date the application of tests of stationarity and a well developed understanding of this issue in the economics literature. An interesting survey of the application of time series methods to explain pricing discrepancies and test for the existence of a causal relation that underlies pricing behaviour was prepared by LecG in 1999 for the United Kingdom Office of Fair Trade (OFT). Forni (2004) reported on a study that he prepared on milk pricing across Italy. It was suggested that tests under the alternative and the null of stationarity can be applied as an effective and efficient way of analyzing anti-trust cases associated with market concentration and non-competitive pricing. Tests of stationarity in the context of a price differential are essentially tests of cointegration between price series and cointegration is viewed in this sense as a signal of a broad market definition that is consistent with arbitrage (Forni, 2004). The cointegration case has been used to determine competitiveness in a number of studies, see De Vany and Walls (1999), Hendry and Juselius (2001) and La Cour and Møllgaard (2002).

In this article we discuss further the developments in the literature previously summarized in Hendry and Juselius (2001). We find that beyond the narrow regional and geographical arguments that apply to the study of Forni (2004), it would seem difficult to concede that stationarity tests applied to determine that price proportions are stationary can dominate the richer analysis that can be drawn from the cointegration approach. We feel that the approach that is developed from testing stationarity has some potential pitfalls that may not be recognized when all one is interested in is the behaviour of a ratio. As is explained by Hendry and Juselius, there are occasions when tests of stationarity may be more effective and more efficient, but they are conditioned on the prior observation that the model underlying this approach is valid.

Firstly we provide a brief summary of the literature on stationarity and error correction, but draw the reader's attention to the quite considerable literature on the use of statistical methods to assess anti-trust cases.<sup>1</sup> Then we consider a trivariate extension of the model analyzed by Hendry and Juselius (2001). Finally we develop an empirical example to motivate our analysis and then make our conclusions.

### Stationarity tests and measures of market definition

The notion that an analysis of price behaviour might be used to detect irregularities in pricing can be found in studies such as Maunder (1972). As is correctly explained by Forni (2004), this discussion predates an appropriate understanding of the pitfalls that arise when inference is undertaken on time series that are non-stationary. Forni (2004) has addressed this issue by suggesting that some form of test of stationarity can be applied to logarithmic data on prices. Specifically, Forni applies his analysis to data on a homogenous product to determine whether there are local markets for milk in Italy. The analysis is undertaken on a relatively small data set, the bivariate comparisons are applied inter-regionally and they relate to homogenous products where local legislation places a limit on the capacity to sell the product easily across Italy. Forni, considers two forms of test and sets up a framework by which the combination of tests under the

null of stationarity and the null of non-stationarity might be used to determine a market definition. This operates from narrow through to less narrow to broad depending on the level at which the tests are accepted. Here, we will concentrate on the model that underlies the well known Dickey Fuller test (Dickey and Fuller, 1979) as they correspond best with the methods used to determine cointegration (Engle and Granger, 1987) and Johansen (1995).

Firstly, we will consider the reasons behind the approach of Forni (2004); subsequently we will compare that with the approach of De Vany and Walls (1999), then place this in the context of the discussion in Hendry and Juselius (2001) and consider further cointegration and exogeneity.

Forni (2004) suggests the use of stationarity and non-stationarity tests of the log price ratio to determine a broad (stationary) as compared with a narrow (non-stationary) market. If the prices respond to each other, then the market is competitive, otherwise it is not and the market is narrowly defined.<sup>ii</sup> More recent studies apply univariate and panel tests (Giulietti et al, 2010, and Hunter and Tabaghdehi 2012).

There is an extensive literature on Purchasing Power Parity (PPP) that has tested this proposition in the long-run by determining whether the real exchange rate is stationary. Let:

$$\eta_t = e_{12t} - p_{1t} + p_{2t}$$

measure the real exchange rate, then we can apply a number of tests to determine the proposition that  $(\eta_t)$  is either stationary or non-stationary. It is generally accepted that either the series  $e_{12t}$ ,  $p_{1t}$  and  $p_{2t}$  need all to be I(1) or that  $e_{12t}$  and  $(p_{1t} - p_{2t})$  are I(1).<sup>iii</sup> When this approach is applied to the case of regulation, then:

$$\eta_t = p_{1t} - p_{2t}$$

The Dickey-Fuller test associated with both the cases above is related to the following short-run dynamic model:

$$\Delta \eta_t = \pi_0 + \pi_1 \eta_{t-1} + \varepsilon_t \quad (1)$$

$\eta_t$  is stationary when  $\pi_1 < 0$  or the price ratio defines a stationary combination so the two prices respond to each other. Otherwise,  $\eta_t$  is non-stationary when  $\pi_1 = 0$  and

$$\Delta \eta_t = \pi_0 + \varepsilon_t$$

Hence  $\eta_t$  follows a random walk with drift when  $\pi_0 \neq 0$  and:

$$\eta_t = \pi_0 + \eta_{t-1} + \varepsilon_t$$

The often made conclusion when the null hypothesis is accepted is that real exchange rates are non-stationary or in the regulation case that the prices do not define an appropriate market. However, the t-test applied to (1) is not a simple test of the proposition that a series is stationary

(under the alternative) it is a test of the proposition that two or more series are stationary in combination or rather deducting part of one series from another renders the first series stationary. Firstly, we consider the problems that arise by not considering (1) above to be a model of either price, then we move on to discuss the approach adopted in De Vany and Walls (1999). The latter study realizes that the model estimated by the application of the Dickey Fuller test is one of a sequence of restrictions associated with the proposition that the series are cointegrated or rendered stationary by some linear combination of variables. The Dickey-Fuller test applied to the log proportion of two series relates to a restricted dynamic econometric model:

$$\Delta \eta_t = \Delta(p_{1t} - p_{2t}) = \pi_0 + \pi_1(p_1 - p_2)_{t-1} + \varepsilon_t.$$

It appears clear that Forni (2004) views this model (underlying the Dickey Fuller test) as a model of  $p_{1t}$ , because he also undertakes the reverse test that should relate to  $-\eta_t$ .

Kremers et al (1992) show that the model underlying the Dickey Fuller test applied by Forni (2004) is a special case of an error correction model (see Davidson et al (1978)):

$$\Delta p_{1t} = \pi_0 + \delta_0 \Delta p_{2t} + \pi_1(p_{1t-1} - p_{2t-1}) + \varepsilon_t. \quad (2)$$

When compared with (1), (2) assumes that  $\delta_0 \neq 1$ . The test of stationarity for the price proportions is still based on a null of non-stationarity when  $\pi_1 = 0$  as compared with an alternative of stationarity when  $\pi_1 < 0$ , but in this instance when the sample is large enough Kremers et al (1992) show that the t-test can be compared with points on the normal distribution (-1.64). For smaller samples the critical values correspond more closely with those of Dickey and Fuller (1979) as applied to the cointegration case (Engle and Granger, 1987). However, efficient inference on all the coefficients of the model depends on the proposition that  $\Delta p_{2t}$  is weakly exogenous for the parameters of interest ( $\pi_1$ ), this requires a sequential cut in the parameter space, between the parameters of the model driving the price series (see Engle, Hendry and Richard, 1983 or Ericsson and Irons, 1994). If the null is accepted, then there may still be a short-run relation between prices when  $\pi_1 = 0$ , but this is between price inflation series for different firms or regions. Therefore:

$$\Delta p_{1t} = \pi_0 + \delta_0 \Delta p_{2t} + \varepsilon_t.$$

Secondly, there may be a non proportional relationship between prices in the long-run (Hosken and Taylor, 2004) so the error correction model to be estimated is:

$$\Delta p_{1t} = \pi_0 + \delta_0 \Delta p_{2t} + \delta_1(p_{1t-1} - \beta p_{2t-1}) + \varepsilon_t.$$

Hence, the test used by Forni (2004) might not be able to reject the null hypothesis that  $(p_{1t-1} - p_{2t-1})$  is non-stationary, when either  $\delta_0 \neq 1$  and the Dickey-Fuller test is not appropriate or  $\beta_0 \neq 1$ , and neither the Dickey-Fuller nor the Kremers et al (1992) error correction tests are correctly formulated. In the latter case an estimate of elasticity which is non-unity may still imply slightly inelastic price response or an infinitely elastic price response.<sup>iv</sup>

Further, the regulation literature does not come to the conclusion, that stationarity tests alone can

be used to detect discriminatory or unfair pricing. In the PPP literature, Banerjee et al (2005) suggest that the single equation or panel approach are not always correct, because they may miss out on the important cross sectional dependence between price series, the exchange rate and other fundamental factors that can also influence the real exchange rate.<sup>v</sup> There is also some evidence from the empirical literature based on multivariate tests of cointegration that the proportionality restriction associated with PPP can be accepted when series are modelled jointly (Johansen and Juselius, 1992, and Juselius, 1995). The latter point seems very important for the validity of univariate analysis of relative price behaviour as unlike PPP there may be additional reasons for price movements not to match each other.

In the case of PPP it is important that the basket of goods be comparable. Lothian (2012) who analyses 400 years of price and exchange rate data for Great Britain and Holland provides evidence for the importance of the nature of the price series and compares a range of different historical price series across this period. The key issue in the context of pricing of products relates to the homogeneity of the product. A similar point is made by Hosken and Taylor (2004) in their reply to Forni and they suggest for non-homogenous products that it may be likely to find non-proportionality in the price relations. The price discrepancies may be amenable to transformation and this relates to the discussion in Hunter et al (2001) of the use of hedonic pricing to correct for quality differences in data. This would provide a basis for differences between the short and the long-run coefficients. While for the case of mature products such distinctions may relate to constant and dummy corrections though there may be slow moving effects related to vintages that may not be captured in this type of way.

## Cointegration and Exogeneity

De Vany and Walls (1999) apply a different approach to the problem, firstly they test individual price series to determine the order of integration, then subject to finding that the series are at most I(1) they test the proposition that combinations of prices are stationary and then they determine whether the response is unitary elastic. This has the advantage over the analysis of Forni (2004) that it does not presume an order of integration as it makes no assumption about the normalization of prices in the long-run or about the exogeneity of the variables. Consider the conventional Vector Auto-Regressive model in error correction form:

$$\Delta x_t = \Pi x_{t-1} - \sum_{i=1}^{q-1} \Gamma_i \Delta x_{t-i} + \varepsilon_t. \quad (3)$$

Now  $x_t$  may be viewed as containing variables for product  $i$  and  $j$ , which for the case considered by Forni (2004) and De Vany and Walls (1999) means that  $x_t = [p_{1t} \ p_{2t}]'$ . Now there are  $N$  potentially cointegrating relationships, though only one of them will be valid when the series are of the same order of integration and there are  $N=2$  price series to be analyzed as is the case in the previous section. When the underlying distribution is normal, then the VAR can be transformed into a first order model and that can be viewed as a multivariate generalization of the model analyzed by Dickey and Fuller (1979). So least squares can be used to estimate of  $\Pi$  from the multivariate residual regression:

$$R_{1,t} = \Pi R_{0,t} + error.$$

From the point of view of maximum likelihood estimation, this is equivalent to concentrating the likelihood function. As long as a Gaussian likelihood is used, the maximum likelihood estimator of  $\Pi$  is also unaffected, even under the restriction that the matrix is of reduced rank,  $r < N$ . The estimates of the long-run derive from the decomposition:

$$\Pi = \alpha\beta'$$

where  $\alpha$  and  $\beta$  are  $N \times r$  matrices of loadings and cointegrating relationships respectively. The requirement for Gaussianity means that the disturbances,  $\varepsilon_t$  must be jointly normally distributed, an important assumption. Therefore the likelihood ratio test determines the null ( $H_0$ ) and alternative ( $H_A$ ) hypotheses, for  $j=1, 2, \dots, N$  where:

$$H_0 : r \leq j-1$$

$$H_A : r \leq j.$$

The test statistic is:

$$LR(j-1, N) = -T \left[ \sum_{i=j}^N \log(1 - \lambda_i) \right] = \lambda_{trace}(j-1).$$

Johansen (1995) presents critical values for the trace test for cointegration, which follows a non-standard distribution under the null of non-stationarity. In general there is an  $N$  vector of variables  $x_t$  assumed to define one or more long-run relationships ( $\beta'x_t$ ) that are assumed to be stationary.

To consider exogeneity in the long-run we analyze the matrix of long-run parameters ( $\Pi$ ) in (3), the Vector Auto-Regression (VAR) in error correction form. Partitioning  $\Pi$  into blocks of cointegrating vectors associated with  $y_t$  (endogenous variables) and those related to  $z_t$  (exogenous variables):

$$\Pi = \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,1} & \Pi_{2,2} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}' = \begin{bmatrix} \alpha_1\beta_1' & \alpha_1\beta_2' \\ \alpha_2\beta_1' & \alpha_2\beta_2' \end{bmatrix}$$

where  $\alpha_1$  is  $N_1 \times r$ ,  $\alpha_2$ ,  $N_2 \times r$ ,  $\beta_1$  is an  $N_1 \times r$ , and  $\beta_2$  an  $N_2 \times r$  matrix. A necessary condition for weak exogeneity of  $z_t$  for a sub-block  $\beta_{2,1} = [\beta_{2,1} \ \beta_{2,1}]$  is a block triangular  $\alpha$  matrix:

$$\alpha = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ 0 & \alpha_{2,2} \end{bmatrix}.$$

However, the triangular form is not sufficient for weak exogeneity, which is combined with the condition:

$$\alpha_{1,2} = \Sigma_{1,2}(\Sigma_{2,2})^{-1}\alpha_{2,2} = 0 \quad \text{or} \quad \alpha_{1,2} - \Sigma_{1,2}(\Sigma_{2,2})^{-1}\alpha_{2,2} = 0.$$

Hence, we require further restrictions for appropriate long-run inference in a conditional model.

These two conditions are necessary and sufficient for weak exogeneity of  $z_t$  for the sub-block  $\beta_j$ . It follows that weak exogeneity for a sub-block occurs when the condition  $\alpha_{2,j}=0$  is extended to a quasi diagonal form:

$$\alpha = \begin{bmatrix} \alpha_{1,1} & 0 \\ 0 & \alpha_{2,2} \end{bmatrix}.$$

Where  $\alpha_{1,2} = \Sigma_{1,2}(\Sigma_{2,2})^{-1}\alpha_{2,2} = 0$  or either  $\alpha_{2,2}$  or  $\omega = \Sigma_{1,2}(\Sigma_{2,2})^{-1} = 0$ . Otherwise  $\alpha_{1,2} = \omega\alpha_{2,2}$  and:

$$\alpha = \begin{bmatrix} \alpha_{1,1} & \omega\alpha_{2,2} \\ 0 & \alpha_{2,2} \end{bmatrix}.$$

The loadings in the first and second blocks are to  $\omega$  proportional to each other. In the case where  $\alpha_{1,2} \neq 0$ , then weak exogeneity of  $z_t$  for the matrix  $\beta$  occurs when  $\alpha_{2,2} = 0$  and:

$$\begin{aligned} \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,1} & \Pi_{2,2} \end{bmatrix} &= \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \beta'_{1,1} & \beta'_{2,1} \\ \beta'_{2,1} & \beta'_{2,2} \end{bmatrix} \\ &= \begin{bmatrix} \alpha_{1,1}\beta'_{1,1} + \alpha_{1,2}\beta'_{2,1} & \alpha_{1,1}\beta'_{2,1} + \alpha_{1,2}\beta'_{2,2} \\ 0 & 0 \end{bmatrix}. \end{aligned}$$

As a result the cointegrating vectors are estimated from the block of equations related to  $y$ . Notice that there are at most  $N-r$  weakly exogenous variables in this sense or the fundamental criterion for cointegration is broken.

Cointegrating exogeneity augments the triangular  $\alpha$  with non-causality between  $y$  and  $z$  at the level of the system. Hence, the long-run relationships for  $z$  do not depend on the levels of  $y$ . It follows that  $z_t$  is cointegrating exogenous for the sub-vector  $\beta_1$ , if and only if:

$$\Pi_{2,1} = 0.$$

Following Hunter (1990) this form of separating cointegration is quite arbitrary in that any orthogonal combination of  $\beta_1$  and  $\alpha_2$  sub-blocks satisfy (ii). However, when  $\beta_1$  defines the parameters of interest, then the partition  $\alpha_2 = [0 : \alpha_{1,1}]$  and  $\beta_1 = [\beta_{1,1} : 0]$  are the only ones that are relevant. This gives rise to the following matrix of long-run parameters (Hunter, 1992):

$$\begin{aligned} \Pi &= \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} \\ \Pi_{2,1} & \Pi_{2,2} \end{bmatrix} = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ 0 & \alpha_{2,2} \end{bmatrix} \begin{bmatrix} \beta'_{1,1} & \beta'_{2,1} \\ 0 & \beta'_{2,2} \end{bmatrix} \\ &= \begin{bmatrix} \alpha_{1,1}\beta'_{1,1} & \alpha_{1,1}\beta'_{2,1} + \alpha_{1,2}\beta'_{2,2} \\ 0 & \alpha_{2,2}\beta'_{2,2} \end{bmatrix}. \end{aligned}$$

With CE,  $\alpha_{i,j}$  and  $\beta_{i,j}$  are dimensioned  $(r_j \times N_j)$  and the following vectors:  $\eta_{1t} = \beta'_{1,1}x_t$  and



$\eta_{2t} = \beta'_{2,2} z_t$  define  $r_1$  and  $r_2$  blocks of stationary variables.

If conditions (i) and (ii) above hold, then cointegrating exogeneity in this form is an exact analogue of strong exogeneity (see Engle et al, 1983).

Next we consider cointegration in the confines of the system due to De Vany and Walls (1999) and provide a more general case related to equilibrium price targeting (Burke and Hunter, 2011).

## Simple Cointegration tests of Market Definition

Let us assume that a single relationship of the variety tested in the papers by Forni (2004) and De Vany and Walls (1999) exists, then:

$$\beta'_i x_t = [1 \ -1] \begin{bmatrix} p_{1t} \\ p_{2t} \end{bmatrix} \sim I(0).$$

Hence, the hypothesis accepted is  $p_{1t} - p_{2t} \sim I(0)$ . In the case considered by both Forni (2004) and De Vany and Walls (1999) the system contains two equations for each price pair analyzed. This implies for two  $I(1)$  variables there is a maximum of  $r=1$  meaningful long-run relations. The Johansen test statistic, tests for the number of long-run relations, but without restriction they are not automatically identified. In the case where  $x'_t = [p_{1t-1} \ p_{2t-1}]$ , then the cointegrating vector is identified by a normalization therefore an acceptable matrix is  $\beta' = [1 \ \beta_{12}]$ . Once the normalization is imposed, then a t-test of the proposition that  $\beta_{12} = -1$  can be undertaken using the critical values for the parameters in  $\alpha$  and  $\beta$  derived by Doornik (1998).<sup>vi</sup> For the case where the order of integration is the same, the model analyzed by Forni (2004) is a special case of that analyzed by De Vany and Walls (1999). For ease of exposition we will consider the bivariate VAR(2), with  $x$  and  $\beta$  defined above,  $\Pi = \alpha\beta'$  and one cointegrating vector:

$$\Delta x_t = \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \end{bmatrix} [1 \ \beta_{12}] x_{t-1} - \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \Delta x_{t-i} + \epsilon_t.$$

This is observationally equivalent to a non-singular transformation matrix  $A$  that following pre-multiplication gives rise to the following system.

$$\begin{aligned}
A\Delta x_t &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_{11} \\ \alpha_{12} \end{bmatrix} [1 \quad \beta_{12}] x_{t-1} \\
&\quad - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \Delta x_{t-i} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \varepsilon_t \\
\Delta \begin{bmatrix} p_{1t} - p_{2t} \\ p_{2t} \end{bmatrix} &= \begin{bmatrix} \alpha_{11} - \alpha_{12} \\ \alpha_{12} \end{bmatrix} [1 \quad \beta_{12}] x_{t-1} \\
&\quad - \begin{bmatrix} \gamma_{11} - \gamma_{21} & \gamma_{12} - \gamma_{22} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \Delta x_{t-i} + \begin{bmatrix} \varepsilon_{1t} - \varepsilon_{2t} \\ \varepsilon_{2t} \end{bmatrix}.
\end{aligned}$$

Notice, that the transformation makes no difference to the long-run equations. For inference in the single equation to be equivalent to inference that arises in the VAR we require the following restrictions to hold,  $\gamma_{12} - \gamma_{22} = 0$ ,  $\alpha_{12} = 0$  and  $\beta_{12} = -1$ :

$$\begin{aligned}
\Delta \begin{bmatrix} p_{1t} - p_{2t} \\ p_{2t} \end{bmatrix} &= \begin{bmatrix} \alpha_{11} \\ 0 \end{bmatrix} [1 \quad \beta_{12}] x_{t-1} - \\
&\quad \begin{bmatrix} \gamma_{11} - \gamma_{21} & 0 \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} \varepsilon_{1t} - \varepsilon_{2t} \\ \varepsilon_{2t} \end{bmatrix}.
\end{aligned}$$

In the case of Forni (2004), the Dickey-Fuller test imposes the restriction  $\beta_{12} = -1$  on the long-run and also imposes the same restrictions on the short-run dynamics of an error correction model that contains no cross price terms or  $\gamma_{12} = \gamma_{22}$ . When the VAR is compared with the single equation error correction framework then inference is optimal in the long-run as this is efficiently estimated on a single equation when  $p_{2t}$  is weakly exogenous for the cointegrating vector  $\beta$  or the long-run is only derived from the first equation or  $\alpha_{12} = 0$ . If it makes no difference whether or not:

$$A^* = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

is applied to the VAR, then the same results should arise from the Dickey Fuller test irrespective of the nature of the price comparison, except that the condition for weak exogeneity is required and  $\gamma_{11} - \gamma_{21} = 0$ .

The model discussed by Hendry and Juselius (2001) is a simple generalization of De Vany and Walls (1999) where the price of a substitute is included, the price of oil. The first issue that arises in the study by Hendry and Juselius (2001), which would appear to apply to an analysis of competitive markets is that the price series have a common trend. To enliven our discussion we will assume that the two prices considered relate to gas. If there is a single common trend across related prices, then all prices should be driven by the common trend for the market to be competitive. Following Forni (2004), De Vany and Walls (1999), and Hendry and Juselius (2001)

it is suggested that the single cointegrating vector takes the form  $[1 \ -1]$  that is suggested as being consistent with a broad market definition. Both Forni (2004), and De Vany and Walls (1999) develop studies that though bivariate in nature relate to a number of inter-related prices. And these authors suggest that the same behaviour is required for competitiveness for all the products without considering the likely interactions that might arise when cross prices are analyzed. Both these studies end up rejecting the broad market definition on the basis of their bivariate analysis without apparently considering that the specification might be limiting. Hendry and Juselius (2001) introduce a third related product, which is a comparable price based on the broad industry classification of energy and extraction industries and on the basis of the mode of extraction that might be viewed as having a similar underlying cost structure to gas. Oil the third price has features in common with gas and the nature of such features can be explored. Specifically, the market for gas and oil might be driven by a single common trend, not a hypothesis posited by Hendry and Juselius, but the gas market might be conditioned on oil and this has implications for efficient estimation. Finally for the energy market to be efficient, the gas market ought to react to shocks in the oil market. Hendry and Juselius also consider the role of the trend in the cointegrating relationship and the dynamic model, and consider the nature of leader follower behaviour. In the next section we will consider the role of a trivariate model similar in nature to that of Hendry and Juselius (2001) and extend this to multivariate models which might yield slightly different conclusions to those offered by Forni (2004), and De Vany and Walls (1999).<sup>vii</sup>

## **Multi-price correspondence and Market Definition**

In this section the question of price comparison is addressed firstly in the confines of a trivariate price system similar to that analyzed by Hendry and Juselius (2001). The principle espoused in the confines of the bivariate case by Hendry and Juselius (2001) that competition implies a common trend driving corresponding prices across homogenous product markets appears to generalize to a multi-price framework. We pay particular attention to the role of the common trend in the context of these models and the role of exogeneity in explaining the competitive structure and price inter-relatedness in a market.

Following Hendry and Juselius (2001) we consider the conventional VAR (2) model with three inter-related market prices. For cointegration when all the series are  $I(1)$  there are at most  $r = N - 1$  cointegrating relationships and corresponding to them  $N - r = 1$  common trends. The Johansen trace test outlined in the previous section determines the number of cointegrating vectors  $r$  and the number of common trends when the series are not of a higher order of integration. For the trivariate case we will start with  $r=2$ . Therefore:

$$\Delta \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \end{bmatrix} \begin{bmatrix} 1 & \beta_{21} & \beta_{31} \\ \beta_{12} & 1 & \beta_{32} \end{bmatrix} \begin{bmatrix} p_{1t-1} \\ p_{2t-1} \\ p_{3t-1} \end{bmatrix} - \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix}.$$

As is stated in Hendry and Juselius (2001), the above model has unrestricted matrices  $\alpha$  with  $rN$  parameters and  $\beta$  that after normalization has  $r(N-1)$  parameters. As is discussed in Burke and Hunter (2005) there is a simple order condition that requires  $j = r^2 - r$  restrictions to the parameters in the matrix pair  $[\alpha \ \beta]$ . Prior to such restrictions  $\alpha$  and  $\beta$  are not identified. This is before any consideration of the nature of the cointegrating vectors that are consistent with some form of market definition. Following, Boswijk (1992), Hunter and Simpson (1996) and Bauwens and Hunter (2000) it is possible to identify either via restrictions on  $\alpha$  alone,  $\beta$  alone, and  $\alpha$  and  $\beta$ . However, some combinations of cross restrictions may not be valid (Toda and Phillips, 1995).

Here, we will explain the primary condition required for a competitive market, then the relationship that the primary condition has with long-run weak and cointegrating exogeneity of variables in the system and the link with identification. A further discussion of identification is considered by Burke and Hunter (2011).

**Proposition** *In the case where the market is competitive and prices are  $I(1)$ , then all prices in a broadly defined market respond positively and in proportion to a single common trend  $p_t^* \approx \alpha'_\perp x_t$  or  $N-r=1$  and  $p_{it} - p_t^* \sim I(0)$  defines a set of  $N$  dependent vectors.<sup>viii</sup>*

**Proof** *Assuming that all prices are  $I(1)$  then the series has the following common trends representation (Johansen (1995)):*

$$x_t = Cx_0 + C \left( \sum_{i=1}^t \epsilon_i + \mu \right) + \alpha (\beta' \alpha)^{-1} \sum_{i=0}^{\infty} (I + \beta' \alpha)^i \beta' (\epsilon_i + \mu) \quad (4)$$

where  $C = \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$ ,<sup>ix</sup>  $\mu$  defines a drift parameter and  $x_t$  will be an  $N$  vector of prices. Then the model with  $N$  price series can be decomposed into two blocks that is  $N-r$  trends and  $r$  cointegrating vectors. We isolate the trend component by multiplying (4) by  $\alpha'_\perp$  where  $\alpha'_\perp \alpha = 0$ .

$$\begin{aligned} \alpha'_\perp x_t &= \alpha'_\perp Cx_0 + \alpha'_\perp C \left( \sum_{i=1}^t \epsilon_i + \mu \right) + \alpha'_\perp \alpha (\beta' \alpha)^{-1} \sum_{i=0}^{\infty} (I + \beta' \alpha)^i \beta' (\epsilon_i + \mu) \\ &= \alpha'_\perp x_0 + \alpha'_\perp \left( \sum_{i=1}^t \epsilon_i + \mu \right). \end{aligned}$$

Setting the initial condition to zero and for the purposes of exposition  $\mu=0$ , then the common trend can be characterized by:

$$\alpha'_{\perp} x_t = \alpha'_{\perp} \sum_{i=1}^t \epsilon_i.$$

Therefore with  $N-r=1$  a single common trend drives prices. It follows from the Granger representation theorem that this single common trend is annihilated by the cointegrating vectors that define  $r$  stationary variables when the condition for cointegration  $\beta' C = 0$  is satisfied. To see this multiply (4) by  $\beta'$  :

$$\begin{aligned} \beta' x_t &= \beta' C x_0 + \beta' C \left( \sum_{i=1}^t \epsilon_i \right) + \beta' \alpha (\beta' \alpha)^{-1} \sum_{i=0}^{\infty} (I + \beta' \alpha)^i \beta' (\epsilon_i) \\ &= \sum_{i=0}^{\infty} (I + \beta' \alpha)^i \beta' (\epsilon_i). \end{aligned}$$

This defines  $r$  stationary series without restriction on the cointegrating relationships. The stationary variables,  $\beta' x_t$ , are linear combinations of the prices that return the system to equilibrium. For a broad market definition we require the prices to move in proportion to a single common trend and  $r=N-1$  cointegrating relationships. It follows from the common trends decomposition of the VAR that  $C = \beta_{\perp} (\alpha'_{\perp} \Gamma \beta_{\perp})^{-1} \alpha'_{\perp}$  is an  $N \times N$  matrix and  $\alpha'_{\perp}$  is  $N \times N-r$  matrix used to define the single common trend. If an adjustment is made for the initial conditions, then the common trend can be estimated by  $\tau_t = \alpha'_{\perp} x_t$ . When  $\tau_t = \alpha'_{\perp} x_t$  defines a common trend and the market is competitive, then: i)  $\alpha'_{\perp} 1 = 1$ , where  $1$  is an  $N \times 1$  vector of 1s, ii) for  $i=1, 2, \dots, N$ ,  $p_{it} - p_t^* \sim I(0)$ . Consider the price differential, when  $p_t^* \approx \alpha'_{\perp} x_t$ , and  $i_i$  is a vector of zero's except for element  $i$  set equal to 1, then:

$$\begin{aligned} p_{it} - p_t^* &= p_{it} - \alpha'_{\perp} x_t \\ &= (\alpha'_{\perp} i_i - \alpha'_{\perp}) x_t \\ &= \alpha'_{\perp} (i_i - I_n) x_t. \end{aligned}$$

If  $(i_i - I_n) = R_i$ , then there are  $N$  vectors of the form  $\beta_i = \alpha'_{\perp} (i_i - I_n)$  that are dependent. To see this consider a trivariate system with  $\alpha'_{\perp} = [\alpha_{1\perp} \quad \alpha_{2\perp} \quad \alpha_{3\perp}]$ :

$$\beta'_n = \begin{bmatrix} \beta_{.1} \\ \beta_{.2} \\ \beta_{.3} \end{bmatrix} = \begin{bmatrix} \alpha'_{\perp} R_1 \\ \alpha'_{\perp} R_2 \\ \alpha'_{\perp} R_{i3} \end{bmatrix}$$

$$= \begin{bmatrix} \alpha_{2\perp} + \alpha_{3\perp} & -\alpha_{2\perp} & -\alpha_{3\perp} \\ -\alpha_{1\perp} & \alpha_{1\perp} + \alpha_{3\perp} & -\alpha_{3\perp} \\ -\alpha_{1\perp} & -\alpha_{2\perp} & \alpha_{1\perp} + \alpha_{2\perp} \end{bmatrix}$$

where  $R_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ ,  $R_2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}$  and

$$R_3 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

After normalization:

$$\beta'_N = \begin{bmatrix} 1 & \frac{-\alpha_{2\perp}}{\alpha_{2\perp} + \alpha_{3\perp}} & \frac{-\alpha_{3\perp}}{\alpha_{2\perp} + \alpha_{3\perp}} \\ \frac{-\alpha_{1\perp}}{\alpha_{1\perp} + \alpha_{3\perp}} & 1 & \frac{-\alpha_{3\perp}}{\alpha_{1\perp} + \alpha_{3\perp}} \\ \frac{-\alpha_{1\perp}}{\alpha_{1\perp} + \alpha_{2\perp}} & \frac{-\alpha_{2\perp}}{\alpha_{1\perp} + \alpha_{2\perp}} & 1 \end{bmatrix}.$$

Selecting  $N-1$  cointegrating vectors from  $\beta'_N$ :

$$\beta' = \begin{bmatrix} 1 & \beta_{21} & \beta_{31} \\ \beta_{12} & 1 & \beta_{32} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{-\alpha_{2\perp}}{\alpha_{2\perp} + \alpha_{3\perp}} & \frac{-\alpha_{3\perp}}{\alpha_{2\perp} + \alpha_{3\perp}} \\ \frac{-\alpha_{1\perp}}{\alpha_{1\perp} + \alpha_{3\perp}} & 1 & \frac{-\alpha_{3\perp}}{\alpha_{1\perp} + \alpha_{3\perp}} \end{bmatrix}.$$

This leaves  $r=N-1$  restrictions that exactly identify the cointegrating vectors:

$$\beta_{21} + \beta_{31} = -1 \text{ and } \beta_{12} + \beta_{32} = -1$$

or for all  $i = 1, 2, \dots, N$ ,

$$\sum_{j \neq i} \beta_{ji} = -1.$$

These restrictions are consistent with  $p_{it} - p_t^* \sim I(0)$  for  $i=1, 2, \dots, N$ , and  $\alpha'_{\perp} t = 1$ . ■

This is similar to the model in Hendry and Juselius (2001) who state that the cointegrating vectors without restriction have four free parameters after normalization. Prior to any further restriction it

is of interest to note that economic theory implies that all firms ought to react to the underlying competitive behaviour, that is per-force associated with a common trend when the series are all I(1). The existence of the common trend imposes no restriction on the parameters of the cointegrating vectors unless the common trend can be isolated. This might arise when one of the equations is viewed as a generalized random walk that occurs when one equation, say the third contains no long-run relationships or  $\alpha_{31}=\alpha_{32}=0$  and  $p_{3t}$  is weakly exogenous for  $\beta$ ; for similar discussion see Kurita (2008). If a weakly exogenous variable were to be extracted from the system, then a less competitive market is observed and the following is a valid normalization:

$$\begin{bmatrix} 1 & 0 & \beta_{31} \\ 0 & 1 & \beta_{32} \end{bmatrix}.$$

The latter implies that the normalization is not imposed on a variable that can be long-run excluded from the system or is not well defined in the respective long-run equation. One might anticipate that the cointegrating vectors defined by the system might be driven by one variable. Were this to be the case, then one might consider the following structural form:

$$\begin{aligned} & \begin{bmatrix} 1 & \omega_{12} & \omega_{13} \\ \beta_{12} & 1 & \beta_{32} \\ 0 & 0 & 1 \end{bmatrix} \Delta \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} \\ = & \begin{bmatrix} \alpha_{11} & \alpha_{12} - \omega_{12} \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \beta_{21} & \beta_{31} \\ \beta_{12} & 1 & \beta_{32} \end{bmatrix} \begin{bmatrix} p_{1t-1} \\ p_{2t-1} \\ p_{3t-1} \end{bmatrix} - \\ & \begin{bmatrix} \gamma_{11} + \gamma_{21}\omega_{12} & \gamma_{12} + \gamma_{22}\omega_{12} & \gamma_{13} + \gamma_{23}\omega_{12} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & 0 & 0 \end{bmatrix} \Delta x_{t-1} \\ & + \begin{bmatrix} \epsilon_t + \omega_{12}\epsilon_{2t} + \omega_{13}\epsilon_{3t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix} \end{aligned}$$

to elaborate further, the system potentially has a single equation, followed by the primary error correction term and then the common trend. Notice that this can be transformed to a VAR by inverting the matrix of parameters on the dependent variables. Therefore:

$$\Delta \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} & a_{13} \\ \beta_{12} & 1 & \beta_{32} \\ 0 & 0 & 1 \end{bmatrix}^{-1} \times \left\{ \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \beta_{21} & \beta_{31} \\ \beta_{12} & 1 & \beta_{32} \end{bmatrix} \begin{bmatrix} p_{1t-1} \\ p_{2t-1} \\ p_{3t-1} \end{bmatrix} - \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ 0 & 0 & 0 \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} \epsilon_{1t} \\ \epsilon_{2t} \\ \epsilon_{3t} \end{bmatrix} \right\}$$

$$\Delta \begin{bmatrix} p_{1t} \\ p_{2t} \\ p_{3t} \end{bmatrix} = \begin{bmatrix} -\frac{\alpha_{11}}{a_{12}\beta_{12}-1} & -\frac{a_{12}}{a_{12}\beta_{12}-1} - \frac{\alpha_{12}}{a_{12}\beta_{12}-1} \\ \alpha_{11} \frac{\beta_{12}}{a_{12}\beta_{12}-1} & \frac{1}{a_{12}\beta_{12}-1} + \alpha_{12} \frac{\beta_{12}}{a_{12}\beta_{12}-1} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \beta_{21} & \beta_{31} \\ \beta_{12} & 1 & \beta_{32} \end{bmatrix} \begin{bmatrix} p_{1t-1} \\ p_{2t-1} \\ p_{3t-1} \end{bmatrix} - \begin{bmatrix} -\frac{\gamma_{11}}{a_{12}\beta_{12}-1} + a_{12} \frac{\gamma_{21}}{a_{12}\beta_{12}-1} & -\frac{\gamma_{12}}{a_{12}\beta_{12}-1} + a_{12} \frac{\gamma_{22}}{a_{12}\beta_{12}-1} & -\frac{\gamma_{13}}{a_{12}\beta_{12}-1} + a_{12} \frac{\gamma_{23}}{a_{12}\beta_{12}-1} \\ -\frac{\gamma_{21}}{a_{12}\beta_{12}-1} + \beta_{12} \frac{\gamma_{11}}{a_{12}\beta_{12}-1} & -\frac{\gamma_{22}}{a_{12}\beta_{12}-1} + \beta_{12} \frac{\gamma_{12}}{a_{12}\beta_{12}-1} & -\frac{\gamma_{23}}{a_{12}\beta_{12}-1} + \beta_{12} \frac{\gamma_{13}}{a_{12}\beta_{12}-1} \\ 0 & 0 & 0 \end{bmatrix} \Delta x_{t-1} + \begin{bmatrix} -\frac{\epsilon_t}{a_{12}\beta_{12}-1} + a_{12} \frac{\epsilon_{2t}}{a_{12}\beta_{12}-1} + \frac{\epsilon_{3t}}{a_{12}\beta_{12}-1} (a_{13} - a_{12}\beta_{32}) \\ \beta_{12} \frac{\epsilon_t}{a_{12}\beta_{12}-1} - \frac{\epsilon_{2t}}{a_{12}\beta_{12}-1} + \frac{\epsilon_{3t}}{a_{12}\beta_{12}-1} (\beta_{32} - a_{13}\beta_{12}) \\ \epsilon_{3t} \end{bmatrix}.$$

It is also suggested that the cointegrating relations still only apply to the first two equations. Broadly speaking the analysis in Hendry and Juselius (2001) is similar to that described above. The model is essentially the differential between the two equations estimated by the Johansen procedure, that restricts any price terms in the first equation to be zero and imposes the condition that the second variable is weakly exogenous. Then a bivariate conditional VAR might be estimated without affecting the nature of the results on cointegration. However, this had to be



tested prior to this re-arrangement.

The hypothesis  $p_{1t} - \beta_{12} p_{2t} \sim I(0)$  may not give rise to a unit coefficient, because the prices have a different response or the prices relate to variables that are not homogenous or are differentiated in some way or other (Hosken and Taylor, 2004).

There might be other prices or other variables, which might legitimately influence the long-run relationship.<sup>x</sup> In the context of PPP, interest rates are often required to make the real exchange rate stationary (Johansen and Juselius, 1992 and Juselius, 1995).

## Empirical Analysis

Here we consider similar data to Hosken and Taylor (2004), Hunter and Tabaghdehi (2012) and Kurita (2008) to analyze the stationarity and cointegration properties of regional gasoline prices in the US. The data is similar to that analyzed by Kurita (2008) who took the same regions for the period 1990 (week 38) - 2004 (week 52) and this is a subset of the data analyzed by Hunter and Tabaghdehi (2012) who consider a broader range of regions. Here the price series are tested for stationarity and these results are compared with those related to the Johansen procedure. The data are for three regions of the US and the product is a homogenous product (gasoline) except for the location. Here three regions are selected, New York (NY), the Gulf Coast (GC) and Lower Atlantic (LA). The regions are similar, they are all coastal regions, they each had facilities by which crude oil can be imported and refined.

In Figure 1 the logarithms of gasoline prices are given for the data used by Kurita (2008) who considers three prices from the regions, New York (NY), Gulf Coast (GC) and Lower Atlantic (LA):

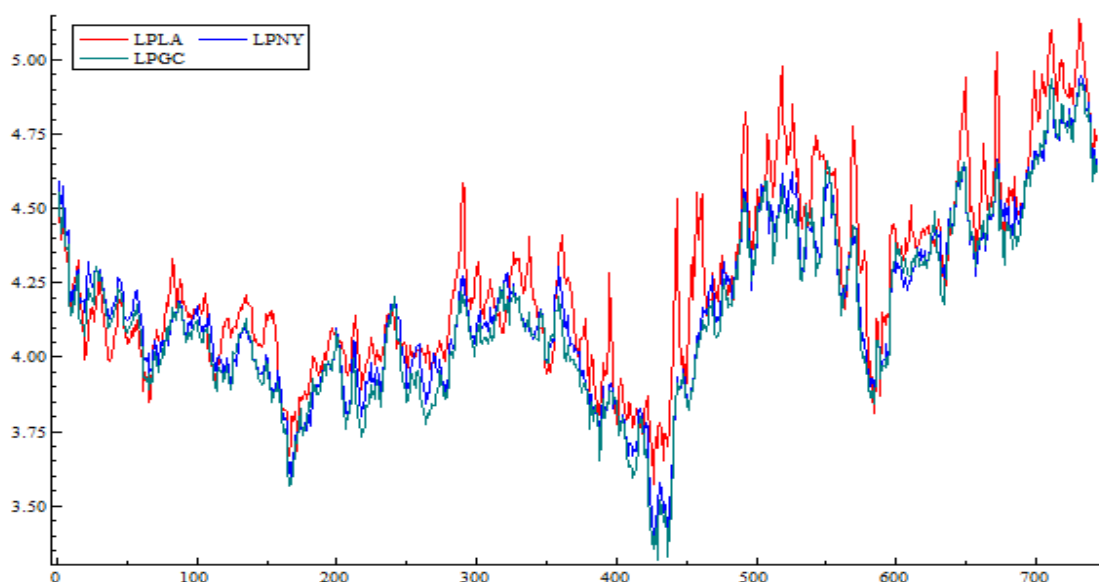


Figure 1: The logarithm of gasoline prices for NY, GC and LA

In Figure 2, the series have also been transformed into log price differentials, so as is explained in section 2, the notion of cointegration is tested using Dickey Fuller regressions (Forni, 2004) and Error Correction Models (Kremer et al, 1992).

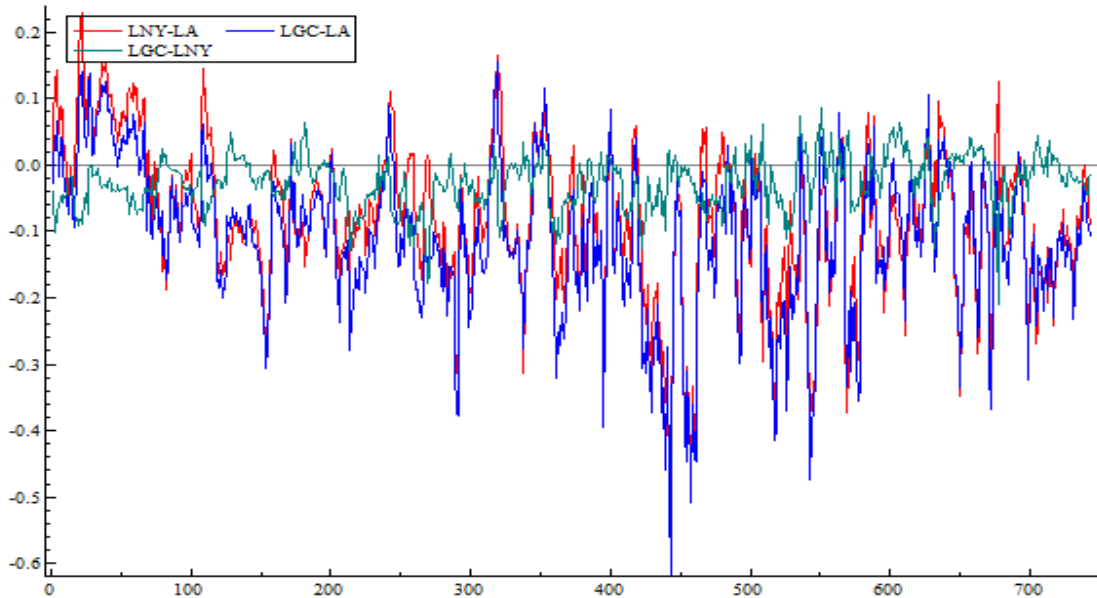


Figure 2: Logarithmic price differentials: NY and LA, GC and LA, GC and NY.

In Table 1, tests of stationarity are derived from single equations and this is done for comparison with the results that derive from the VAR. The effective sample includes  $T=721$  observations and the results relate to Augmented Dickey Fuller (ADF) tests and Error Correction Models (ECM) with  $q$  lags in the estimations.

**Table 1 Unit root and ECM tests of price-proportions**

Correction terms	$t$ -ADF( $q$ )	$t$ -ecm( $q$ )  $p_{LA}$	$t$ -vecm( $q$ )  $p_{LA}$
$P_{NY-LA}$	-4.701**(11)	-3.72**(11)  $p_{LA}$	-3.30**(11)  $p_{LA}$
$P_{GC-LA}$	-4.601**(11)	-3.39**(11)  $p_{LA}$	-1.47(11)  $p_{LA}$
$P_{GC-NY}$	-5.983**(17)	-5.31**(17)  $p_{NY}$	-5.42**(17)  $p_{NY}$
$P_{LA-NY}$	-4.701**(11)	-4.45**(11)  $p_{NY}$	-3.30**(11)  $p_{NY}$

\* Significant at the 5% level and \*\* significant at the 1% level.

The results in the first column of Table 1 imply that the price proportions are stationary in each case as the critical values are significant when compared at the 5 / 1% level with a one sided critical value of  $-2.87 / -3.44$  as computed in OxMetrics Professional (Doornik and Hendry, 2009). Significance implies that the series in this case price proportions are stationary. Stationarity relates to price series that follow the same stochastic trend. If that is the case, then the series move in proportion to each other in the long-run. Here arbitrage would seem to hold in the long-run, but these tests are unable to determine whether the series follow each other. These tests simply imply that the log price series render each other stationary and give no direction to the correction. To understand the latter it is necessary to analyze all equations in the system. It is of interest to

observe that the results in the first and the final element of this column are the same as the latter is the reverse regression. This suggests that the method developed in Forni (2004) should be limited to either the upper or lower triangle of results. Hunter and Tabaghdehi (2012) show that there is an exact correspondence to this result for the Dickey Fuller (DF) test and the estimations in the first column suggest that this generalizes when  $q$  lags appear in the ADF test estimations.

There are instances where it may be appropriate to test using the ADF test. This arises either when the sample is small or when the price proportions exhibit less complex time series behaviour. If there are common features in the data that cancel then the stationarity tests may reveal an effective methodology to determine the nature of the market. As can be observed from Table 1 the dynamic in some cases is restricted to 11 and 17 lags, this compares to the VAR that is estimated with 20 difference terms. However, the approach is limited to finding whether prices move in proportion and is not able to determine anything about causality or conditioning. Further, the autoregressive (AR) model that underlies the ADF test imposes a common factor restriction on the ECM (Burke and Hunter, 2005).

The test is efficient when the model can be restricted so that the prices move in proportion in both the long and the short-run. This restriction can be removed by augmenting the DF test with either of the price inflation terms. The second column in Table 1 is based on the model underlying the ADF test extended by the contemporaneous inflation terms from the second price in each of the correction terms ( $\Delta p_{it}$ ) and their  $q$  lags. For conventional inference to be efficient, the augmenting price inflation variable is required to be weakly exogenous. However, this is not a problem for the test of stationarity on the error correction term as the estimated coefficient is super consistent (Davidson and MacKinnon, 2004). Here, these tests are not enhanced by augmentation with the price inflation terms, though it cannot be concluded from this that the market is also efficient in a short-run sense. Though, the inclusion of the contemporaneous inflation term may be important, econometric inefficiency may arise in these estimations, because many of the  $q-1$  lagged price inflation terms are likely not to be significant.

For further comparison, the last column in Table 1 relates to the estimation of an associated error correction model. In this case the dependent variable is the difference of the first price in the correction term, but the model is autoregressive as the contemporaneous price inflation term ( $\Delta p_{it}$ ) is excluded and to permit the possibility that the cointegrating rank is  $N-1$  a second correction term is included. It should be noted that the first and final elements in this column relate to the price correction terms  $p_{NY-LA}$  and  $p_{LA-NY}$  these test statistics are the same, and again this reflects that these are reverse regressions. The second element in this column is not significant and this emphasizes the difference between the estimation that arises from the correction term ( $p_{GC-LA}$ ) and a single equation here related to ( $\Delta p_{GCt}$ ) that arises from a vector error correction model. Here the error correction term is testing for the importance of cointegration related to a single price equation, but insignificance can either arise as the cointegrating relation does not exist or the cointegration cannot be detected from that equation. So estimation of a single equation cannot discriminate between these two possibilities. The penultimate element in this column yields  $t$ -statistics that appear not to be materially different across the different models.

The single equation estimation is efficient when they are conditioned on one or more exogenous

variable, but this cannot be determined by directly estimating one equation and for the equation to be well defined, the cointegrating rank should also be determined in advance. As a result further discussion of the error correction model is considered in relation to the simulated data as the nature of the model is known in that case.

The system estimated is a trivariate unrestricted VAR(21) in error correction form. It can be observed from the first row of results in Table 2 that the null of the Johansen trace test can be rejected at both the 1% and 5% level. The subsequent element in the trace test relates to cointegrating rank of  $r=3$ , but as the test is not significant (p.value=[0.281]) so it follows that the  $rank(\Pi) = N - 1 = 2$ . To preclude the possibility of I(2) trends it makes sense to undertake a test that will confirm the rank result (Johansen, 1995, and Burke and Hunter, 2005). This is a rank test for the null of non-stationarity with respect of the second differences and this is a sufficient condition for cointegration. If the I(2) test related to Paruolo (1996) is applied, then with  $r=2$  cointegrating vectors and  $N=3$  variables, here the key test for I(2) relates to the final statistic on the diagonal of the I(2) test table the test term is  ${}_1Q_{2,1} = 49.586[0.0000]$ . The clear significance of this test implies that there is a single stochastic trend and with  $r=2$  there is no room for I(2) trends.

**Table 2 Tests of cointegration, WE, LE and Parallel Pricing.**

Hypothesis <sup>†</sup>	Null	Statistic [p.value]
Cointegration  $r=1$	$r \leq 2$	$-T\Sigma \ln(1 - \lambda_i) = 20.344[0.007]$
(WE)  $r=1$ $p_{GC}$	$\alpha_{1i} = 0$ , for $i = 1, 2$	$\chi^2(2) = 5.9656[0.0507]$
$p_{NY}$	$\alpha_{2i} = 0$ , for $i = 1, 2$	$\chi^2(2) = 7.1725[0.0277]$
$p_{LA}$	$\alpha_{3i} = 0$ , for $i = 1, 2$	$\chi^2(2) = 4.7231[0.0943]$
(LE)  $r=1$ $p_{LA}$	$\beta_{j1} = 0$ , for $j = 1, 2$	$\chi^2(2) = 17.699[0.0001]$
$p_{NY}$	$\beta_{j1} = 0$ , for $j = 1, 2$	$\chi^2(2) = 31.400[0.0000]$
$p_{GC}$	$\beta_{j1} = 0$ , for $j = 1, 2$	$\chi^2(2) = 29.701[0.0000]$
Normalization (N) +(WE) $p_{LA} r = 2$	$\beta_{11} = \beta_{22} = 1$ , $\beta_{21} = \beta_{12} = 0$ $\alpha_{3i} = 0$ , for $i = 1, 2$	$\chi^2(2) = 4.7231[0.0943]$
(CE) $p_{GC} (N)$ +(WE) $p_{LA} r = 2$	$\alpha_{21} = 0$	$\chi^2(3) = 0.4182[0.5178]$
Parallel Pricing   (CE) $p_{GC}$ +(N) + (WE) $p_{LA} r = 2$	$\beta_{13} = \beta_{23} = -1$	$\chi^2(2) = 3.7136[0.1562]$

<sup>†</sup> Cointegrating Exogeneity (CE), Weak Exogeneity (WE) and Long-run Exclusion (LE). \*Significant at the 5% level and \*\* significant at the 1% level.

The next block of results in Table 2 relate to WE and from the p.value it can be determined that the log price for GC and LA might be WE for  $\beta$ . While the test for the price for the NY is significant

and so WE is rejected at the 1% level of significance. In the next block long-run exclusion (LE) is tested (Juselius, 1995). The clear significance of these tests emphasizes the rank condition and the likely robustness of inference associated with any normalization of either cointegrating vectors. The equations are ordered in relation to the WE test as is suggested in Hunter and Simpson (1996). WE is tested for LA on a model normalized on the price for NY in the first equation and the GC in the second equation.

Identification requires there to be  $r-1$  restrictions per long-run relation (Hunter and Simpson, 1996). This can be undertaken in a number of ways, but one that is straightforward relates to what is termed the normalization rule by Boswijk (1996). Therefore  $\beta' = [I_2 \quad b]$  where  $I_2$  is the  $2 \times 2$  identity matrix and  $b' = [b_1 \quad b_2]$ . This restriction is not binding as it exactly identifies each cointegrating vector so as might be observed for the next line in the table the WE test is not affected by applying the normalization rule to  $\beta'$ . The next row tests for CE by imposing the restriction  $\alpha_{2,1} = 0$ . This gives rise to a triangular block in  $\Pi$  so the long-run equations for GC and LA are not long-run caused by the NY price. However, this is seen in Hunter and Simpson (1996) as trivial in terms of the first long-run relation as the GC price does not appear in this vector.<sup>xi</sup> In the final block the notion of parallel pricing is tested by imposing the restrictions  $b' = [-1 \quad -1]$ . As a result these tests give rise to the same conclusions as the stationarity tests.

As Forni (2004) correctly points out the test based on the error correction term simultaneously tests parallel pricing along with cointegration. However, it does not consider WE and CE or the orientation of the long-run. These issues are considered in more detail when the simulations are analyzed. There is further discussion of exogeneity and identification for this type of model in Burke and Hunter (2011).

The final set of results in Table 2 are accepted at the 5% level and they yield the following  $\alpha$  and  $\beta$  matrices, and the associated restricted  $\Pi$  matrix conditioned on the GC price that is cointegrating exogenous and the LA price that is weakly exogenous for  $\beta$ :

$$\alpha = \begin{bmatrix} -0.18686 & -0.11187 \\ (0.035941) & (0.040008) \\ 0 & -0.079938 \\ & (0.021095) \\ 0 & 0 \end{bmatrix} \text{ and } \beta' = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} -0.18686 & -0.11187 & 0.29873 \\ 0.0 & -0.079938 & 0.079938 \\ 0.0 & 0.0 & 0.0 \end{bmatrix}$$

The final long-run relation exhibits parallel pricing, but in the long-run NY and GC prices depend on the prices in LA and there is a long-run structure that is driven by GC prices. That prices in LA are WE for  $\beta$  implies that they do not respond in the long-run to prices in NY and GC. The error correction may be more appropriately seen as a measure of the degree of mispricing and the

long-run relations seen as arbitrage correction terms. The results in Table 2 tell us that these responses are observed to flow in the direction LA to the other regions and this would suggest that the market is not informationally efficient (Hunter and Burke, 2007, and Kurita, 2008). This analysis would be strengthened were we to have information on prices at the firm level, but further information might be extracted from regional details on the supply of gasoline. Further, these results imply that the regulatory authorities would need to be careful to permit any further concentration in the industry especially in terms of gas station ownership or refinery capacity in LA. Similarly, a further causal feature relates to the GC price being cointegrating exogenous for  $\beta_{,1} = [1 \ 0 \ -1]$  the first cointegrating vector.

Next the simulated data are considered and the following  $\alpha$  matrix imposes WE and  $\beta'$  is fixed so that there is parallel pricing. The same cointegrating vectors apply in the simulated case as occurs with the actual data:

$$\alpha_s = \begin{bmatrix} -.09 & .04 \\ 0.11 & -.17 \\ 0 & 0 \end{bmatrix} \text{ and } \beta'_s = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}.$$

The following series are calibrated in a similar way to the NY, GC and LA gasoline price series. The processes are different from the actual data as they are generated by independent errors that are normal, but they behave in a remarkably similar way to the actual data.

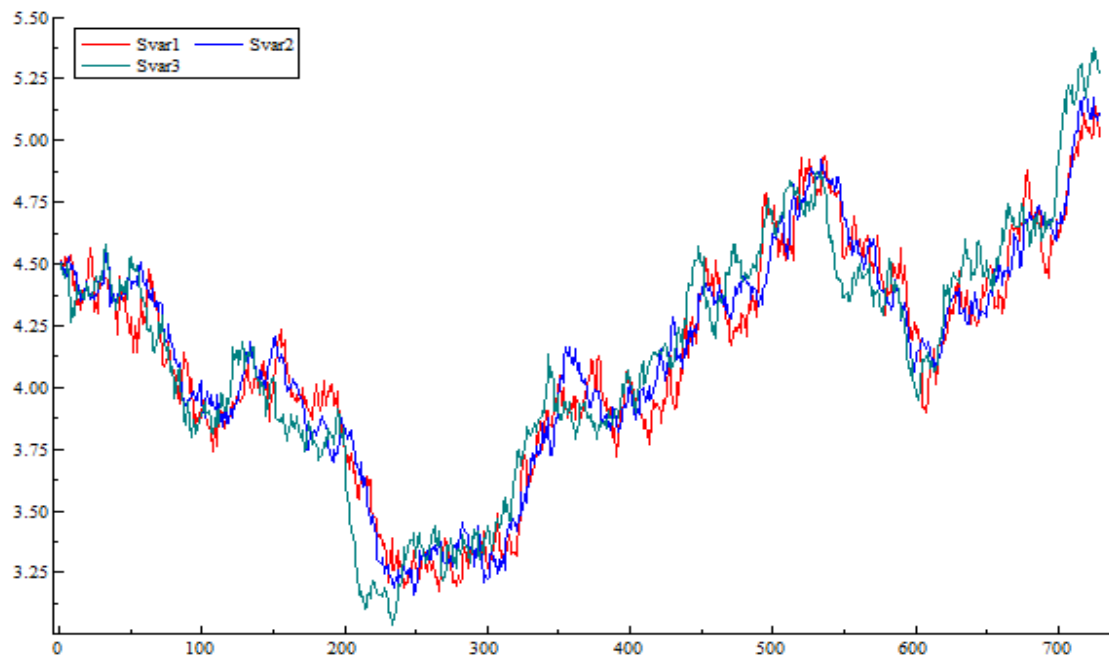


Figure 3: Simulated gasoline price series.

Next the same tests are applied to the simulated data. It exhibits similar features to the actual data

though it is based on a simpler time series structure.<sup>xii</sup> The series are computed as a system, but the table below yields single equation results that follow from the simulated data. It is of interest that based on simulated data Haug (1996) found that there was no correlation between the Johansen test and the residual based cointegration tests. However, the simulations in Haug were generated under the null while the results presented in Table 3 relate to a cointegration case.

**Table 3, Unit root and ECM tests of price-proportions for simulated data.**

Correction terms	$\hat{\pi}_1 + 1$	$t - DF$	$t - ecm$
$p_{1-3}$	0.89927	-5.974 **	-8.09 **
$p_{2-3}$	0.92059	-5.247 **	-9.15 **
$p_{3-1}$	0.89927	-5.974 **	0.0426
$p_{1-2}$	0.78766	-9.114 **	-7.19 **

\* Significant at the 5% level and \*\* significant at the 1% level.

The data are calibrated using the model previously estimated so it is to be anticipated that the computed DF tests give rise to results that satisfy the alternative of stationarity. Hunter and Tabaghdehi (2012) suggest that cross restrictions apply to the sequence of tests. As can be seen from the DF test on the price proportions in the third column, the test statistics based on the simulated data are exactly the same for the price proportions  $p_{1-3}$  and those for the reverse regression using  $p_{3-1}$ . Hunter and Tabaghdehi (2012) show the exactitude of this result in the case where the series follow random walks. This implies that the analysis is at least limited to  $N(N-1)/2 = 3$  combinations. The lower triangle of results is presented in the table above, but these relate to three primary equations for each of the prices.

Under cointegration there are only  $r = N - 1 = 2$  long-run relations and this suggests that a similar result applies to these regressions for stationarity as arises in the case of cross exchange rates derived in Smith and Hunter (1985). Firstly, these equations are coherent when the coefficients on the error correction terms are the same. Otherwise any discrepancy implies that the results may differ by the extent of the discrepancy in the coefficient on the lagged level. It follows that when coherent models are estimated in all cases, only  $N-1$  are considered unique. In this case the coefficients on the Dickey Fuller test are relatively similar and so any discrepancy is likely to be small. As can be observed from the test that applies to  $p_{1-2}$  this is similar in scale to the value that comes from the sum of the tests on  $p_{1-3}$  and  $p_{3-2}$  so the t-value (-9.114) is not materially different from -11.221. This calls into question the ordering of tests presented in Forni (2004). Firstly, the process has to be limited to the upper or lower triangle of results. Secondly, it seems likely that only  $N-1$  of these tests do not depend on the behaviour of the underlying price relations.

The same problem does not apply to the approach that arises when an error correction system is estimated. Firstly, this approach corresponds to the estimation of three time series relations. Secondly, the test based on a test of an error correction term can consider both CE and WE. Finally, the model can cope with more than one error correction term. As can be observed from Table 3 above, similar conclusions can be drawn in relation to cointegration in terms of the significance of the error correction term as can be observed from the p values related to the first

two elements in the last column. The next p.value in the column is associated with a small test statistic and this implies insignificance; this arises as the third variable is weakly exogenous for  $\beta$ . By sequentially estimating the error correction models it is possible to determine whether a series is weakly exogenous or not, but this may not be the best approach and it relates to a specific choice of  $r$  in each case.

It is now possible to turn to the VAR and the Johansen test applied to the simulated data to see whether this similarly calibrated data set gives rise to similar results.

**Table 4 Tests of cointegration, WE, LE and Parallel pricing for simulated data.**

Hypothesis	Null	Statistic [p.value]
Cointegration $r = 1$	$r \leq 2$	$-T\sum \ln(1 - \lambda_i) =$ 77.365[0.000]**
(WE)  $r = 2$	$p_1$	$\alpha_{1i} = 0, \text{ for } i = 1, 2$
	$p_2$	$\alpha_{2i} = 0, \text{ for } i = 1, 2$
	$p_3$	$\alpha_{3i} = 0, \text{ for } i = 1, 2$
(LE)  $r = 2$	$p_1$	$\beta_{j1} = 0, \text{ for } j = 1, 2$
	$p_2$	$\beta_{j2} = 0, \text{ for } j = 1, 2$
	$p_3$	$\beta_{j3} = 0, \text{ for } j = 1, 2$
Normalization (N) +(WE) $p_3 r = 2$	$\beta_{11} = \beta_{22} = 1,$ $\beta_{21} = \beta_{12} = 0$ $\alpha_{3i} = 0, \text{ for } i = 1, 2$	$\chi^2(2) = 1.6819[0.4313]$
Parallel Pricing (N) +(WE) $p_3 r = 2$	$\beta_{13} = \beta_{23} = -1$	$\chi^2(4) = 2.2849[0.6835]$

\* Significant at the 5% level and \*\* significant at the 1% level.

In Table 4 we find that there are two cointegrating vectors as occurs in the case of the actual data. It can be observed from the p values that the test is significantly different from zero in the case of the first two prices. The last price can be observed to be weakly exogenous for  $\beta$  as the test is small and the p.value exceeds .05, so it is not significant. The next block of tests is all consistent with each column in  $\beta'$  being significantly different from zero. This implies that none of the prices may be long-run excluded. This also makes it possible to normalize the first vector on the first price and the second vector on the second price a requirement for identification according to Boswijk (1996). The knowledge that the third price is weakly exogenous implies that it is appropriate to condition the first and second vectors on the third price. Bauwens and Hunter (2000) term this an orientation, so the system is orientated towards the first two prices and the finding that the third simulated price is weakly exogenous for  $\beta$  implies that this orientation is unique (Burke and Hunter, 2005).



## Conclusion

There is a simplicity associated with the method that applies tests of stationarity to the problem (Forni, 2004). However, as is shown in Hunter and Tabaghdehi (2012) behind the simplicity are a number of problems. In particular, testing the reverse results and repeated testing when it is known that there are  $N-1$  cointegrating vectors at most. Then the common factor restriction inherent in the DF/ADF test binds the same behaviour to the short-run and the long-run. However, when the time series is not extensive then it may be that there is no alternative that make sense and it is also possible to extend the approach to the panel context (Beirne et al, 2007, Hadri, 2000, and Giuliatti et al, 2010). Error correction models may have a similar felicity of application and possibility of extension to the panel case and they also have some further benefits in regard to testing (Kremers et al, 1992).

However, stationarity might have been rejected in the context of DF equations, because: the tests perform poorly with relatively small samples (Podivinsky and King, 2000); the test has a less appropriate null (Hosken and Taylor, 2004) or the proportionality restriction required to test stationarity as compared with cointegration is too restrictive (Hunter, 2003). Key variables may also be omitted from the long-run so the dynamic equations do not meet the conditions required for separate estimation and inference (Hunter and Burke, 2007 and Kurita, 2008). The method may be reasonable when price proportions well characterize the data, the information sets are short and the geographic context as is discussed by Hosken and Taylor (2004) is the primary concern.

If more data are available, then a much richer piece of analysis is possible. Cointegration across the system gives rise to a set of long-run relations that are tested jointly. This approach can be further analyzed in terms of long-run exogeneity to distinguish between parallel pricing and aggressive price leadership considered by Markham (1951) and as relates to the notion of weak exogeneity (Hunter and Burke, 2007 and Kurita, 2008). Furthermore, long-run causality may also be considered in the context of CE (Hunter, 1990) that is an essential pre-requisite to determining the short-run concept of Granger Causality (Giannini and Mosconi, 1992). This can be analyzed in terms of separability in the market and with further study of the market structure this form of analysis may be able to pick out anti-trust behaviour, with a geographic dimension. As Hosken and Taylor (2004) suggest this may even occur when it may not be possible more generally to distinguish between arbitrage pricing and price leadership.

Here all the analysis seems to accept that there is arbitrage in the long-run. However, the tests for WE imply that these relations are driven by the LA prices. This implies that the prices in LA do not reflect behaviour in the other regions and the price behaviour in LA can be seen as defining the stochastic trend. It seems unusual that the long-run is forced by LA, but this finding may be useful as an early warning to trigger further investigation. Furthermore, regulatory bodies would be well advised not to consider further concentration in the industry and to provide incentives to stimulate procedures by which the price behaviour in the other regions is better reflected in those that apply to LA.

As with the concept of causality the notion of weak exogeneity is embedded in the system. So extending the data set to more regions may give rise to different conclusions. A further concern is

that the GC can in isolation be seen as weakly exogenous for  $\beta$ , but subject to LA being weakly exogenous, it is then only possible for GC prices to be cointegrating exogenous for  $\beta_1$ . This implies that the long-run relations associated with GC prices are not reflecting what happens in NY and this is a further signal of imperfection. It should be noted that finding a system with  $N-1$  long-run relations all satisfying the pure error correction terms related to parallel prices is also consistent with what is termed Long-run Equilibrium Price Targeting by Burke and Hunter (2011).

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<sup>i</sup> See the discussion by Forni (2004) of the residual demand approach and any analysis that draws on the calculation of elasticities. Hosken, O'Brien, Scheffman and Vita (2002) consider demand studies based on systems to analyze the appropriateness of horizontal mergers. And Froeb and Werden (1998) discuss the use of the difference in the Herfindahl Hirschman index to assess welfare gains through mergers in homogenous product markets. There is also discussion of consumer surplus and following that consumer detriment (Hunter et al, 2001).

<sup>ii</sup> The Dutch regulator (Nederlandse Mededingingsautoriteit) has applied tests of non-stationarity to challenge mobile service providers (KPN, Orange and Vodafone) over the competitiveness of their pricing. However, the report by London Economics that analyses the Dutch market has been strongly criticized by Hunter (2003). The methods applied were incoherent and the data not sufficient to prove the case.

<sup>iii</sup> Forni (2004) makes the valid point that following Engle and Granger (1987) the series ought to be of the same order of integration for the test to be appropriately applied, but this suggestion gives the approach a broader appeal. So it should be anticipated that these series ought to be similar when they relate to an appropriately defined market. Furthermore, in the context of I(2) series as is explained in Johansen (1995) and Burke and Hunter (2005) the stationary combination may exist but not in the form considered by Engle and Granger (1987). Hence, the broad market definition may be wrongly rejected, because the wrong estimator

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is applied. Similarly, the series may be fractionally integrated and again cointegration may be wrongly rejected when the I(1) approach is incorrectly thought to provide an acceptable approximation.

<sup>iv</sup> It should be noted that Forni (2004), cited by London Economics, applies both the ADF and the KPSS test, which uses the null of stationarity. When stationarity is an issue both of these tests should be considered. However, the stationarity test due to Leybourne and McCabe (1994) and the KPSS test used by Forni (2004) came under significant scrutiny by Caner and Killian (2001) who suggest that the tests under the null of stationarity perform relatively poorly. Sekioua and Karanasos (2006) provide evidence for the superior performance of the Generalised Least Squares Dickey Fuller test.

<sup>v</sup> Beirne et al (2007) suggest for the case of the real exchange rate that the relevant concept is that a sequence of exchange rates or relative prices are stationary on average. This leads to a tension between applying univariate tests where one may concentrate on specification of an underlying time series model and define corrected specifications, as compared with the panel approach where key aspects of the specification are assumed to be homogeneous. For the panel estimations presented in Beirne et al (2007) they argue when the test used applies the null of stationarity that the test due to Hadri (2000) works well with excellent power and size properties when the time dimension exceeds 50. It should be noted that Giuliatti et al (2010) have had less success in this context though the electricity price data they use is more likely to be sensitive to non-linearity.

<sup>vi</sup> More generally, the restrictions can be imposed and the likelihood re-evaluated to test propositions on the cointegrating vectors using a likelihood ratio test (Johansen and Juselius, 1992, and Giannini and Mosconi, 1992), this will be considered in more detail in the next section.

<sup>vii</sup> Hendry and Juselius (2001) might be viewed as an analysis of a price system conditioned on other variables, more generally these variables and the underlying identification of long-run demand phenomena would seem to direct one to a more structural analysis (see La Cour and Møllgaard, 2002). Hunter and Ioannidis (2001) considered the role of demand systems developed from non-stationary data.

<sup>viii</sup> Underlying this proposition is the idea that competitive pricing is driven in the long-run by cost with relatively homogenous products without significant quality differences. Though using the hedonic pricing framework should be able to capture primary differences in quality (see discussion in Hunter et al, 2001).

<sup>ix</sup> In the case of the VAR( $q$ ) model  $\Gamma = -(I - \Gamma_1 \dots \Gamma_{q-1})$ .

<sup>x</sup> The notion that prices within a market move in proportion is somewhat simplistic, but when stationarity is used to test for inappropriate pricing by firms the range of factors that might be introduced to generate a stationary process is likely to be restricted. For example, in the case of PPP, the Balassa-Samuelson effect would suggest the inclusion of a trend in PPP models when the data compares developed with less developed economies. The trend captures a gap in the rate of technological progress between the two economies. But a trend that explains differences in sectoral price differentials might be inappropriate as it is likely to be viewed illegitimate not to pass gains due to technological change on to the consumer.

<sup>xi</sup> Here the focus is on parallel pricing as the cointegration and stationarity approaches correspond exactly, but CE is not trivial when the problem is looked at in terms of Long-run Equilibrium Price Targeting (LEPT), which in relation to this algebra is an exact analogue of parallel pricing. For this reason Hunter and Burke (2007) impute with LEPT the concept of CE best relates to the case where there are two stochastic trends or  $rank(\Pi) = N - 2$ .

<sup>xii</sup> It may make more sense to view the simulated data as relating to the information corrected for short run dynamics by the Johansen methodology that transforms the underlying relations to multivariate random walks (Johansen, 1995).