

# Electrical Conductivity for Copper Oxide (CuO) nanofluids in the superconducting phase. A generalization of Type II superconductivity hydrodynamics behavior

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**Keywords:** Superconductors, two-phase conductivities, nanofluids

**Abstract** Copper oxide superconducting nanofluids exhibit a lot of very interesting technological properties and their behaviour is typical of a two-phase nanofluid. Near the superconductivity transition temperature, their electrical conductivity is the sum of a normal conductivity component and a flux flow superconducting contribution from the unpinned motion of vortices within the sample. Armed with recent experimental results for regular type II superconductivity nanosamples, we review the corresponding expected behaviour for CuO High Temperature Superconducting (HTSC) systems. The equivalent Navier-Stokes equations that go under the name Ginzburg–Landau equations for the superconducting density are briefly reviewed and their solutions are presented in a clear way for the particular problem. Contribution of fluctuations of the structural vortex lattice, which is a stable solution of the Time Dependent Ginzburg-Landau (TDGL) equations, to the flux flow two-phase conductivity is briefly presented. The corresponding discussion for the two-phase thermal conductivity of a superconducting nanosample is going to be presented in a separate future publication.

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## 1 Introduction

Superconducting Electronics is a rapidly growing field which spans over many very useful applications in e.g. small magnetic field measurements ( brain waves ), green energy projects (wind farms generators, power transmission wires) etc. In order to establish the correct use of High Temperature Superconducting materials (HTSC) in superconducting electronics circuits and construct useful units out of them we need to understand and analyze the effect of noise on these circuits. One of the main reasons for the existence or not of noise in HTSC circuitry is the fluctuations of the order parameter ( wavefunction amplitude ) near the transition temperature of the HTSC material  $T_c$ , which is near the liquid nitrogen temperature area (77 K) and higher. In this paper we will try to summarize exactly the behavior of the order parameter and thus the conductivity near  $T_c$ . The paper is organized as follows: In Section 2 we present the basic equations that describe the phenomenology of the model that we are using for the electronic transport and go under the name of Ginzburg-Landau theory. In Section 3 we develop the contribution of the vortex conductivity fluctuations to the total conductivity tensor. In Section 4 we apply the above theory to the two-fluid electron gas HTSC material near  $T_c$  and discuss the signature of the fluctuation corrections. In Section 5 we present our conclusions and in Section 6 we give a list of our references. Finally in Section 7 we present a list of all our symbols and parameters used.

## 2 Time Dependent GL Theory

An important development of the theory of superconductivity has been the investigation of the dynamic behaviour of magnetic flux structures and the discovery of the connection between flux motion and the transport properties of superconductors. Encouraged by the success of the GL theory for treating the case of thermodynamic equilibrium, time-dependent generalizations of the GL theory have been studied for describing variations of the order parameter with time ([1],[2]). However the existence of an energy gap and the interconversion between normal excitations and superfluid due to the probe frequency, constitute major difficulties in writing a simple form for the corresponding GL equations or for repeating the same approximations as in the static case. Following the work of references [3]-[8], the resulting equation is of diffusion type

$$\gamma^{-1} \frac{d\psi(\vec{r},t)}{dt} + \frac{df}{d\psi(\vec{r},t)} = 0 \quad (2.1)$$

where  $\gamma$  is the scattering rate of the order parameter  $\psi(\vec{r},t)$  and  $f(\vec{r},T,H)$  is the free energy density of the nanosample (  $\vec{r}$  is the position of the electron in space,  $T$  is the local temperature and  $H$  is the local magnetic intensity)

$$f(\vec{r},T,H) = \frac{1}{2} \left( \vec{\nabla} - \frac{e}{\hbar c} \vec{A} \right) \psi \cdot \left( \vec{\nabla} - \frac{e}{\hbar c} \vec{A} \right) \psi + \alpha(T) |\psi|^2 + \frac{\beta(T)}{2} |\psi|^4 + \dots \quad (2.2)$$

where  $m$  is the mass of the electron,  $\vec{A}$  is the magnetic vector potential,  $\alpha(T)$  and  $\beta(T)$  are the phenomenological GL parameters. We look for a time-dependent order parameter in the form

$$\psi(t) = \psi_0 + \delta\psi(t) \quad (2.3)$$

with  $\psi_0$  being the time-independent equilibrium solution and  $\delta\psi(t)$  the time-varying deviation from equilibrium, assuming

$$\left| \frac{\delta\psi(t)}{\psi_0} \right| \ll 1 \quad (2.4)$$

Using (1.3) we find for zero magnetic field

$$\frac{d\delta\psi(t)}{dt} \approx 4\gamma\alpha(T) \delta\psi(t) \quad (2.5)$$

i.e.  $\psi(\vec{r},t)$  relaxes exponentially with a relaxation time

$$\tau = -\frac{1}{4\gamma\alpha(T)} = -\frac{1}{\dots} \quad (2.6)$$

Now we allow for spatial variation of the equilibrium solution and we turn on a magnetic field along the z-direction. Restoring gauge invariance we obtain from (2.1), (2.5) and Ohm's law for the normal current [9]

$$D^{-1} \left( \frac{\partial}{\partial t} + i \frac{e}{\hbar c} \vec{A} \cdot \nabla \right) \psi + \xi^{-2} (|\psi|^2 - 1) \psi + \dots = 0 \quad (2.7)$$

$$\vec{j} = \sigma \cdot \left( -\vec{\nabla} \varphi + \frac{1}{c} \vec{E} \right) + \text{Re} \left\{ \psi \cdot \left( \vec{\nabla} - \frac{2e}{\hbar c} \vec{A} \right) \cdot \psi \right\} \frac{1}{c} \quad (2.8)$$

ignoring the difference of the chemical potential for  $\vec{E}=0$  and  $\vec{E} \neq 0$ , where  $D^{-1} = \frac{\hbar^2}{2m} \gamma^{-1}$  is the diffusion coefficient,  $\varphi$  is the electric (scalar) potential,  $\vec{H} = \vec{\nabla} \times \vec{A}$ ,  $\xi$  is the T-dependent coherence length and  $\vec{E}$  is the electric intensity vector.

These equations were derived also microscopically, for a gapless superconductor in [3] and generalized in [8]. They don't include the quasiparticle contributions (normal excitations) which for some transport coefficients (e.g. thermal conductivity) can be substantial (see [10]). They include only the effect of the dynamical perturbation on the order parameter (Aslamazof-Larkin process [11], [12]). This omission can by no means be explained till the present, and the only way of remedying it is a direct but difficult full microscopic calculation.

### 3 The Effect of a Transport Current in the Vortex State

This is a highly non trivial problem and of great practical importance. If we try to pass a dc current through the vortex array, there is a Lorentz force per unit volume  $\mathbf{j} \times \mathbf{H} / c$  acting on the vortices. This leads to motion of the flux lattice vortices (FLs) except if they are pinned by sample inhomogeneities or scattered by other fluctuating FLs that wander inside the sample. A measure of how freely the FLs are moving, is given by the experimental voltage drop which arises as follows: We consider a frame of reference moving with the velocity of the vortices  $\mathbf{u}_L$ . In this frame, vortices are stationary and the Abrikorov solution describes their arrangement [13], while the electric field is everywhere zero since there is no FL motion. Transform back to the laboratory frame, for typical FL velocities  $\sim 10^3 \text{ cm/sec}$  the relativistic Lorentz factor be small, we get:

$$E_x = E_x - u_{Ly} H_z / c = 0 \quad (3.1)$$

assuming that the transport current flows in the x-direction, where the primed coordinate frame is the one that moves with the FL. Using Ohm's law

$$E_x = \rho_f J_x \quad (3.2)$$

we get

$$\rho_f = u_{Ly} H_z / c J_x \quad (3.3)$$

Substituting typical values  $u_L = 1000 \text{ cm/sec}$ , critical current density  $10^6 \text{ A/cm}^2$  at  $1000 \text{ Oe}$  we get  $\rho_f \sim 10^{-5} \text{ Ohm-cm}$  which is close to an expected value for the normal state resistivity at low T for superconducting alloys.

Let us now give a more concrete study of flux motion with the experimental situation deferred to the next chapter. Consider our usual geometry with a bulk superconductor in a magnetic field along the z direction. If an applied external dc current (transport current) flows along the x-direction, then the vortex structure moves with velocity  $\mathbf{u}_L$ . The angle between the y axis and  $\mathbf{u}_L$  is called the Hall angle for obvious reasons. The FLs motion is governed by the following balance equation for the forces per unit length on the FL:

$$\mathbf{f}_L + \mathbf{f}_M + \mathbf{f}_r - \mathbf{f}_p = 0 \quad (3.4)$$

(i)  $\mathbf{f}_L = \mathbf{j} \times \mathbf{H} / c$  is the Lorentz force per unit length which arises from the magnetic hydrostatic pressure plus an additional tension along the FL.  $\mathbf{j}_x$  is the transport current density,  $\phi_0$  is the flux quantum and c is the light velocity.

(ii)  $\mathbf{f}_M = -n_s (\mathbf{u}_L \times \phi_0 \mathbf{e}_z) / c$  is the Magnus force as derived in [14], which is a consequence of restoration of Galilean invariance in a neutral superfluid.  $n_s$  is the superfluid density and  $\mathbf{u}_L$  is the supercurrent velocity. Notice that the Magnus force gives a velocity component parallel to the transport current and thus contributes to the Hall effect [15].

(iii)  $\mathbf{f}_r = -\eta \mathbf{u}_L$  is the sum of all the damping forces (e.g. dissipating currents within the core and possibly quasiparticle damping) where  $\eta$  is a phenomenological coefficient.

(iv)  $\mathbf{f}_p$  is the pinning force which opposes vortex motion so it is negative compared to  $\mathbf{f}_L$ . Pinning arises from local minima in the free energy functional due to defects in the crystal structure of the sample and it turns out to be very important in the HTSC.

Equation (3.4) describes also the flux motion for an applied time dependent magnetic field for zero transport current. Notice that when extensive pinning is present there is no dissipation due to flux motion since the flux line lattice (FLL) is pinned and thus superconductivity persists to higher external currents, giving rise to useful technological applications. Depending on which term wins in (2.4) we see different kinds of flux motion.

In this communication we will examine the flux flow regime: When  $f_L + f_p > 0$ , then the vortex motion is retarded only by viscous damping. The flow state, results when the current exceeds a critical value (pinning) with vortices moving perpendicular to the direction of the current. A voltage appears following a linear rise with increasing current, which however is much smaller than if the material were normal. The slope  $dV/dI$  defines the flux flow resistance  $R_f$ . Notice that while  $I_c$  depends on the pinning force,  $R_f$  (slope) does not. Thus  $R_f$  is characterized only by the parameters that affect the vortex lattice formation. The nonlinear region of the  $V(I)$  curve can be explained by non uniform FLL motion due to spatial variation in  $f_p$ . According to GL theory the critical current decreases with increasing magnetic field and temperature.

To obtain a quantitative expression for the flux flow resistivity we use eq. (3.1), following reference [16], for the resistive voltage due to vortex motion which we write as

$$-\nabla V = -\mathbf{u}_L \times \mathbf{H} / c \quad (3.5)$$

For our geometry  $u_{Ly}$  causes the longitudinal (resistive) voltage whereas  $u_{Lx}$  causes the transverse (Hall) voltage. Neglecting for the moment pinning in (3.4), we see that if there is no normal core damping ( $\eta \rightarrow 0$ ) then  $\mathbf{u}_L = \mathbf{u}_s$  and there is only a Hall response since the FL follows the supercurrent. If there is no Magnus force then  $\mathbf{u}_L \perp \mathbf{u}_s$  and there is only longitudinal response present, which gives a resistivity

$$\rho_f = \phi_0 H / \eta c^2 \quad (3.6)$$

which is the slope of increase of the longitudinal electric field with the current density  $\mathbf{j}_x$ . This leaves us with calculating  $\eta$ , bearing in mind that the final results should be consistent with the Bardeen-Stephen formula [17]:

$$\rho_f = \frac{H}{\dots} \quad (3.7)$$

for low fields and low  $\kappa$  superconductors, where  $H_{c2}(0)$  is the upper critical magnetic field at zero temperature. Bardeen

and Stephen derived the above relation under the following assumptions

- (i) Local electrostatics
- (ii) Fully normal vortex core of extent  $\xi$  (superconducting coherence length)
- (iii) Dissipation occurs only by normal resistive processes in this core (quasiparticle scattering by the lattice)
- (iv)  $T \ll T_c$ , i.e. neglect the electrons outside the normal core
- (v) The transport current density and the normal current density generated from the flux motion are assumed small with respect to the supercurrent densities circulating around the vortex cores i.e.  $u_l \tau \ll \xi$
- (vi) No pinning  $\Rightarrow$  normal current density in the core = applied transport current density.

They found

$$\eta = \varphi_0 H / c^2 \rho_n \quad (3.8)$$

where  $\rho_n$  is the normal state resistivity, and inserting this into (3.6) they derived (3.7) which is quite successful for small T (i.e.  $T \ll T_c$ ). To complete the discussion we give the microscopic result from the work of Caroli and Maki [5] for the slope of the flow curve near  $H_{c2}$

$$\frac{dM}{dT} = \frac{\rho_n}{\dots} \quad (3.9)$$

Usui et al [18] have calculated by fitting to their data for V, values of  $\kappa(T)$ , which then plugged into the GL expression for the magnetization

$$\left(\frac{dM}{dT}\right) = \frac{1}{\dots} \quad (3.10)$$

( $\beta = 1.16$  for a triangular Abrikosov FLL) and then fitted magnetization slope results to their experimental data with excellent agreement. In the dirty limit the expressions of Caroli and Maki were corrected by Thomson [7] and gave fairly good agreement with experiments.

Finally there is an interesting consistency between the BS result (3.7) and its assumption (iii) which can be written in terms of the coherence length and the inter-vortex distance  $d$  as

$$\frac{\rho_f}{\rho_n} = \frac{\xi^2}{d^2} \times (\text{numerical factor} \sim 1)$$

Thus  $\rho_f/\rho_n$  is approximately equal to the volume fraction occupied by the vortex core in direct agreement with the assumption of quasiparticle scattering dissipation in the normal cores only. In other words the current must pass through the normal cores and as they are normal the work is dissipated in driving the current across them. When the vortices are stationary and not too close to  $T_c$  or  $H_{c2}$  there is plenty of space for the current to move through superconducting region, thus dissipating no energy. However

when the vortices move, there is a "frustration" for the current in choosing between superconducting and normal region, which interchange rapidly due to the viscous vortex motion. Then there is current passing through the normal cores and thus dissipation, resulting to finite longitudinal flux flow resistivity. The Nozières-Vinen model [15] (Magnus force) gives essentially the same result for the flux flow resistivity.

## 4 Two fluid Model

We consider now a superconductivity nanofluid that is isotropic, unpinned, low temperature and follows the Bardeen-Cooper-Schrieffer (BCS) standard superconductivity mechanism (see [19]). Although the mechanism of superconductivity for the CuO HTSC doesn't exactly follow the BCS standard mechanism, their electromagnetic properties are very closely following the BCS electromagnetic response.

Following TDGL theory (Vekris & Pelcovits [20]) we write for the two fluid conductivity:

$$\sigma_f = \sigma_n + \sigma_n \frac{H_{c2}(T) - B}{vB} \quad (4.1)$$

where  $\sigma_f$  is the flux flow conductivity,  $\sigma_n$  is the normal fluid conductivity,  $H_{c2}(T)$  is the upper critical magnetic field (temperature dependent) and  $B$  is the local magnetic field within the nanofluid body. As  $B \rightarrow H_{c2}(T)$  the first term in (3.1) becomes negligible.  $v$  is a fitting factor in the spirit of references Liang and Kunchur ([22],[23]). They apply the mean field result (i.e. no fluctuations) equation (4.1) and plot the resistivities vs T for an adequate temperature range right below  $T_c$ . They use the expression

$$H_{c2}(T) = H_{c2}'(T - T_{c0}) \quad (4.2)$$

where  $H_{c2}' = -9.125 \text{ T/K}$  and  $T_{c0} = 5.55 \text{ °K}$  for their  $\text{Mo}_{0.79}\text{Ge}_{0.21}$  LTSC samples A and B. After plotting their data for R vs B (Figure 1) and deciding on the best  $v$  (Figure 2), a strong agreement between experimental values and theory is observed for the LTSC MoGe conductivity  $\sigma_f$  vs T away from  $T_c$  (Figure 3). There is clearly a strong discrepancy between their experimental data and the fitted formula (4.1) (for  $v=0.4$ ) near  $T_c$ . Notice that this formula can be casted as

$$\frac{R_f}{R_n} = \frac{1}{1 + \frac{1-B}{vB}} \quad (4.3)$$

where

$$b = \frac{B}{H_{c2}(T)} \quad (4.4)$$

and  $R_f$  is the flux flow state resistance,  $R_n$  is the normal state resistance.

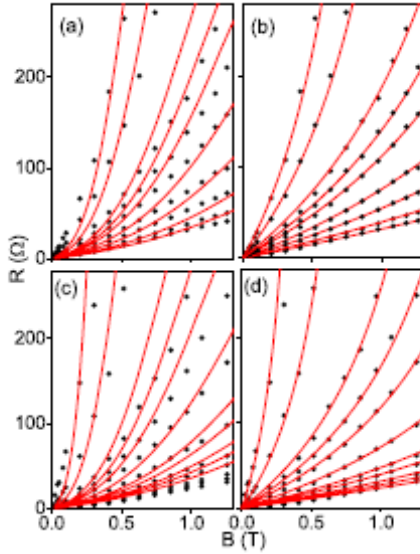


Figure 1: (a) and (b) Resistance versus magnetic field data (symbols) for sample A at temperatures (bottom to top):  $T = 3.25, 3.74, 4.24, 4.56, 4.79, 4.90, 5.00, 5.21,$  and  $5.31$  K. (c) and (d) Resistance versus magnetic field data (symbols) for sample B at temperatures (bottom to top):  $T = 3.17, 3.48, 3.71, 4.01, 4.21, 4.53, 4.77, 4.88, 4.98, 5.19,$  and  $5.30$  K. (a) and (c) show fits to LO theory (Ref. [24]). (b) and (d) show fits to the TDGL theory [Eq. (4.1)] using  $\nu$  as a fitting parameter for each curve (i.e., for each  $T$ ). The resulting values of  $\nu$  are shown in Fig. 2. All fits use the same  $H_{c2}(T)$  given by  $H_{c2}(T) = H'_{c2}[T - T_c]$  and the measured value  $H'_{c2} = -3.125$  T/K.

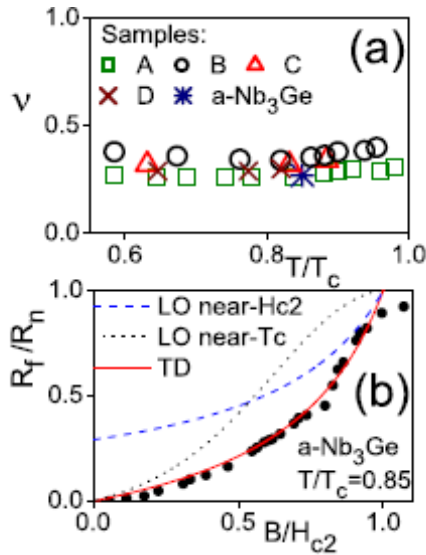


Figure 2: (Color online) (a) Experimentally deduced parameter  $\nu$  [of Eq. (2)] and its variation with temperature for MoGe samples A-D. The asterisk shows a  $\nu$  value for the  $Nb_3Ge$  data of Ref. 15 plotted in panel (b). (b) Normalized resistance versus normalized field for amorphous  $Nb_3Ge$  films from Berghuis *et al.* (Ref. [25])  $t = T/T_c = 0.85$  with  $T_c = 2.93$  K. Solid red line represents a TDGL curve [Eq. (4.2)] with  $\nu = 0.27$ . Blue dashed and black dotted lines correspond to the LO theory for the condition "close to  $H_{c2}$ " (Ref. [24]).

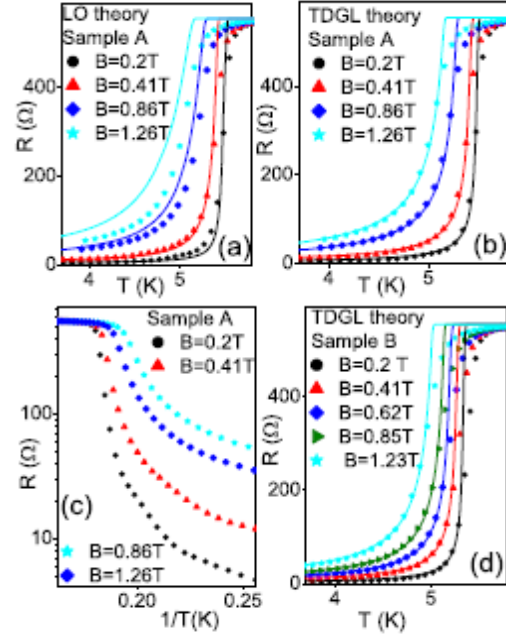


Figure 3: (Color online) Resistive transitions in magnetic fields. Symbols show experimental data. Solid lines are theoretical curves. (a) Sample A with Larkin-Ovchinnikov theory curves (Ref. [24]). (b) Sample A with TDGL theory curves [Eq. (4.1)] with  $\nu = 0.34$  for all  $B$  and  $T$ . (c) Arrhenius plots of the same resistive-transition data for Sample A. (d) Sample B with TDGL theory curves [Eq. (4.1)] with  $\nu = 0.26$  for all  $B$  and  $T$ .

The discrepancy is clearly an over exaggeration of  $R_f$  near  $T_c$ . Is there a mechanism that could explain this? We propose the thermal fluctuations of the vortex lattice that are present and very strong near  $T_c$ . Following [20] and [21] we estimate the corrected formula to be:

$$\frac{R_f}{R_n} = \frac{1}{1 + \frac{(t-b)}{pb} [1 - 2\pi\epsilon_G (1-t)^{-\frac{1}{2}} b (1-b)^{-\frac{1}{2}}]} \quad (4.5)$$

where  $t = T/T_{c0}$ , and  $\epsilon_G$  is the Ginzburg number for the nanofluid (analogous to the Reynolds numbers for viscosity problems).

We plot the results in Figure 4 which we expect to be a much better fit for HTSC nanosamples. Usually  $\epsilon_G \approx 1$  for HTSC  $CuO$  nanofluids, which tells us that for  $T \sim T_{c0}$  these fluctuations are extremely important, since the correction factor becomes very large. However, in the case of Liang and Kunchur, their sample is a low  $T_c$  superconducting nanofluid where  $\epsilon_G \sim 10^{-6}$ , thus only right on  $T_c$  things would get interesting.

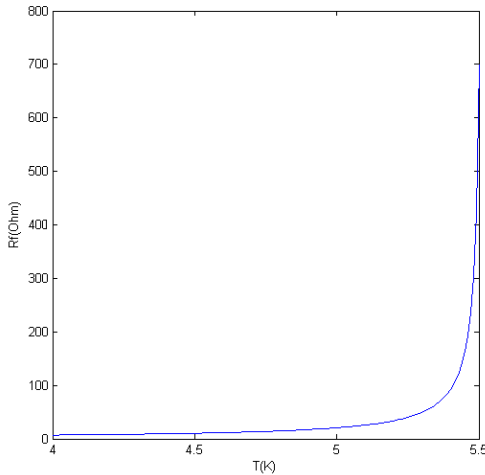


Figure 4: Plot of the flux flow two-phase conductivity  $R_f$  vs  $T$  from (3.5) (i.e. including fluctuation corrections).

## 5 Conclusion

We present a review of the flux flow conductivity status knowledge for a CuO HTSC two-phase nanofluid, viewed from the perspective of the very recent experimental results of Liang and Kunchur extrapolated for a HTSC. We give a plausible explanation for the correction of standard TDGL flux flow contribution due to thermal fluctuations that distort, but do not melt, the vortex lattice FLL in the mixed phase of the nanofluid. More experimental evidence is needed to validate our assertion. The technological advantages of a definite answer to the above question would be critical for the construction of dissipationless lab-on-a-chip systems.

## 6 References

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## 7 Appendix

$\vec{A}$	total magnetic vector potential (Oe·cm)
$\vec{B}$	magnetic induction (Tesla)
$b$	dimensionless ratio of magnetic fields
$D$	diffusion coefficient (gr/(erg <sup>2</sup> ·sec))
$d$	intervortex spacing in Abrikosov lattice (cm)
$\vec{E}$	electric intensity (statvolt/cm)
$e^*$	charge of the superconducting electronic excitation (esu)
$f(\vec{r}, T, H)$	free energy density of nanosample
$\vec{F}_L$	Lorentz force (gr·(cm/sec <sup>2</sup> ))
$\vec{F}_M$	Magnus force (gr·(cm/sec <sup>2</sup> ))
$\vec{F}_P$	pinning force (gr·(cm/sec <sup>2</sup> ))
$\vec{F}_V$	damping force (gr·(cm/sec <sup>2</sup> ))
$\vec{H}$	magnetic field (Oe)
$H_{c2}(T)$	temperature dependent upper critical magnetic field (Oe)
$H'_{c2}(T)$	slope of the upper critical magnetic field (Oe/°K)
$\vec{j}$	current density (A/cm <sup>2</sup> )
$m$	mass of the electron (gr)
$n_s$	superfluid density (cm <sup>-3</sup> )
$R_n$	normal resistance (Ohm)
$\vec{r}$	position of excitation in space (cm)
$T$	temperature (°K)
$T_c$	transition temperature (°K)
$t$	time (sec)
$\vec{v}_f$	vortex velocity (cm/sec)
$\vec{v}_s$	supercurrent velocity (cm/sec)
$V$	voltage (statvolt)
$\alpha(T)$	first Landau-Ginzburg coefficient
$\beta(T)$	second Landau-Ginzburg coefficient
$\beta$	Abrikosov flux lattice parameter (dimensionless)
$\gamma$	scattering rate of the order parameter (sec <sup>-1</sup> )
$\delta\psi(\vec{r}, t)$	order parameter variation (dimensionless)
$\epsilon_G$	Ginzburg number (dimensionless)
$\eta$	damping coefficient (gr/sec)
$\kappa$	Ginzburg-Landau parameter (dimensionless)

$\lambda$	wavelength of excitation (cm)
$\xi(T)$	temperature dependent coherence length (cm)
$\rho_n$	normal state resistivity (Ohm cm)
$\rho_f$	flux flow resistivity (Ohm)
$\sigma_f$	flux flow conductivity (inverse resistivity units)
$\tau$	relaxation time (sec)
$\phi$	electronic scalar potential (statvolt)
$\phi_0$	superconducting flux quantum (Oe/cm <sup>2</sup> )
$\psi(\vec{r}, t)$	order parameter (dimensionless)
$\psi_0(\vec{r}, t)$	equilibrium order parameter (dimensionless)