Transient micro heat transfer in a gas confined between parallel plates due to a sudden increase of the wall temperature

Olga BUCHINA [1,*], Manuel VARGAS [2], Dimitris VALOUGEORGIS [1], Stefan STEFANOV [2]

* Corresponding author: Email: buchina@mie.uth.gr
1: Department of Mechanical Engineering, University of Thessaly, Greece
2: Institute of Mechanics, Bulgarian Academy of Sciences, Bulgaria

Abstract The transient heat transfer problem through a rarefied gas confined between parallel plates maintained at different temperatures is investigated. The theoretical formulation is based on the linear BGK kinetic model subject to Maxwell diffuse reflection. The governing time dependent equation is solved by using the discrete velocity method. Results are presented for the number density, macroscopic velocity, heat flux and temperature distributions in the whole range of the Knudsen number. The results are compared with direct Monte Carlo simulations and good agreement is found for small temperature variations.

Keywords: Kinetic Theory, Rarefied Gas Dynamics, Micro Heat Transfer, Knudsen Number

1. Introduction

Steady-state heat transfer through a rarefied gas confined between parallel plates is one of the main benchmark problems in the field of rarefied gas dynamics and it has been extensively used to test and validate several computational approaches. However, the corresponding unsteady heat transfer problem has received much less attention. Time dependent heat transfer configurations are common in gaseous micro devices and may be produced by time dependent boundary cooling or heating. Recently in [1,2] the transient heat transfer in a gas confined in a small-scale slab due to the instantaneous change of a wall temperature as well as due to periodic boundary heating has been investigated. In these works semi-analytical approaches have been applied in the free molecular and hydrodynamic limits, while the DSMC method has been used in the transition regime.

In the present work, analysis of the time response of a rarefied gas confined between two infinite parallel plates due to a sudden jump in the temperature of one of the boundaries is presented based on linear kinetic theory. The time dependent heat transfer is modelled by the linearized unsteady BGK equation subject to Maxwell purely diffuse boundary conditions and accordingly chosen initial conditions.

Results are presented for the time evolution of all macroscopic quantities (number density, bulk velocity, temperature, heat flux) from the initial state all the way to the steady-state conditions. Since a kinetic approach is implemented the results are valid in the whole range of the Knudsen number. It is interesting to note that the sudden change in the thermal properties of a boundary causes a gas flow with non-zero bulk velocity perpendicular to the plates as well as a space dependent heat flux. After some time which depends on the degree of gas rarefaction the velocity tends to zero and the heat flux between the plates becomes constant recovering gradually the well known steady-state behaviour. The accuracy of the results is validated by solving the same micro heat transfer configuration by the DSMC method obtaining a very good agreement for small temperature variations.

Overall, the kinetic solution provides a detailed description of the evolution of the heat flow field with regard to time from the starting point, where the gas is at rest up to a certain time where almost steady-state conditions are recovered. Also, the present work provides an estimate of how fast a

rarefied heat flow, depending upon its rarefaction, will respond to a sudden change of the wall temperature. It is noted that since the present analysis is based on linear kinetic theory it is limited to small temperature differences.

2. Statement of the problem

Consider two infinite parallel plates, fixed at $y = \pm H/2$, confining a monatomic gas at pressure P_0. Both plates and the gas are initially at temperature T_0. Then, at some time the temperature of the plate at $y = -H/2$ is suddenly raised to $T_1 = T_0 + \Delta T$ and maintained at this temperature, while the temperature of the plate at $y = +H/2$ is kept at T_0. As a consequence of this temperature increase of one of the plates, a net time-dependent heat flux will be exchanged among the plates and the rarefied gas, which gradually will grow and, as time tends to infinity, will approach the steady-state heat flux.

The main dimensionless quantity determining the heat flux is the reference rarefaction parameter referring to the distance H between plates and defined as

$$\delta = \frac{P_0 H}{\mu_0 \upsilon_0}, \qquad (1)$$

where P_0 is the reference pressure, μ_0 is the gas viscosity at reference temperature T_0 and $\upsilon_0 = \sqrt{2RT_0}$ is the most probable molecular velocity, with R denoting the gas constant. The rarefaction parameter is proportional to the inverse Knudsen number. The dimensionless space and time independent variables

$$y = \frac{y'}{H}, \quad t = \frac{\upsilon_0 t'}{H}, \qquad (2)$$

respectively, where $-1/2 \le y \le 1/2$ and $t > 0$ are introduced. The macroscopic quantities of number density, temperature, heat flux and velocity may be written in dimensionless form as

$$\rho = \frac{n - n_0}{n_0} \frac{T_0}{\Delta T}, \tau = \frac{T - T_0}{\Delta T}, q = \frac{q'}{P_0 \upsilon_0} \frac{T_0}{\Delta T}, u = \frac{u'}{\upsilon_0} \frac{T_0}{\Delta T}$$

$$\qquad (3)$$

respectively. It is noted that q and u are the y-components of the corresponding vectors which are nonzero, while $n_0 = \int_{-1/2}^{1/2} n(y) dy$ is an average density.

By considering a limited temperature difference between the plates according to $|\Delta T| \ll T_0$, the heat flow is modelled by the unsteady linearized BGK model equation, which may be given by the two coupled integrodifferential equations:

$$\frac{\partial \phi}{\partial t} + \upsilon_y \frac{\partial \phi}{\partial y} + \delta \phi = \delta \left[\rho + 2\upsilon_y u + \tau \left(\upsilon_y^2 - \frac{1}{2} \right) \right],$$

$$\frac{\partial \psi}{\partial t} + \upsilon_y \frac{\partial \psi}{\partial y} + \delta \psi = \delta \tau \qquad (4)$$

Here, $\phi = \phi(t, y, \upsilon_y)$ and $\psi = \psi(t, y, \upsilon_y)$ are the reduced distributions functions, with $\upsilon_y \in (-\infty, \infty)$ denoting the y-component of the molecular velocity, while the macroscopic moments at the right hand side of Eqs. (4) are given by

$$\rho = \frac{1}{\sqrt{\pi}} \int \phi e^{-\upsilon_y^2} d\upsilon_y,$$

$$u = \frac{1}{\sqrt{\pi}} \int \upsilon_y \phi e^{-\upsilon_y^2} d\upsilon_y,$$

$$\tau = \frac{1}{3\sqrt{\pi}} \int \left[2\psi + \phi (2\upsilon_y^2 - 1) \right] e^{-\upsilon_y^2} d\upsilon_y,$$

$$q = \frac{1}{2\sqrt{\pi}} \int \left[2\psi + \phi (2\upsilon_y^2 - 3) \right] \upsilon_y e^{-\upsilon_y^2} d\upsilon_y. \qquad (5)$$

The interaction between the particles and the walls is modelled according to Maxwell purely diffuse boundary conditions, which are written for the outgoing distributions at the walls as

$$\phi^+ \left(t, -\frac{1}{2}, \upsilon_y \right) = (\upsilon_y^2 - 2) + 2\int_0^\infty \phi \left(t, -\frac{1}{2}, -\upsilon_y \right) \upsilon_y e^{-\upsilon_y^2} d\upsilon_y$$

$$\psi^+ \left(t, -1/2, \upsilon_y \right) = 1 \qquad (6)$$

and

$$\phi^+ \left(t, 1/2, \upsilon_y \right) = 2\int_0^\infty \phi \left(t, 1/2, \upsilon_y \right) \upsilon_y e^{-\upsilon_y^2} d\upsilon_y$$

$$\psi^+ \left(t, 1/2, \upsilon_y \right) = 0 \qquad (7)$$

The initial conditions are

$$\phi(0, y, \upsilon_y) = \psi(0, y, \upsilon_y) = 0. \qquad (8)$$

3. Method of solution

The governing Eqs. (4) and (5) subject to the boundary and initial conditions (6-8) are solved numerically in the whole range of the rarefaction parameter δ. The computational scheme is fully deterministic and all spaces are accordingly discretized. In particular, the discretization in the molecular velocity space is performed by using the discrete velocity method. The continuum spectrum is substituted by a discrete set of velocities υ_m, $m = 1, 2, ... M$, which are taken to be the roots of the Legendre polynomial, with M denoting the degree of the polynomial, accordingly mapped into the interval of interest. The physical space is divided into I segments and it is consisting of $i = 1, 2, ... I + 1$ nodes, while the discretization is performed by a second order central difference scheme. Finally, the discretization in time $t > 0$ is implicit and at its time step $t_n = 1, 2, ... N$ a first order backward difference scheme is applied. A similar approach has been recently applied to solve unsteady fully developed flow through rectangular and circular microchannels [3,4].

The discretized set of Eqs. (4) is solved at each time step and for each molecular velocity marching through the physical space, while at each time step the macroscopic quantities defined by Eqs. (5) are estimated by a Gauss-Legendre quadrature.

Since the scheme is implicit any size of time step will provide stable results, which however will not be necessarily accurate enough. To capture the proper evolution of the macroscopic quantities it is required to have the dimensional time step less than the collision time, which is defined as $\tau_c = \mu_0 / P_0$. By using (2) and introducing the dimensionless time step one can obtain the following condition, which must be satisfied:

$$\Delta t \times \delta < 1. \qquad (9)$$

Based on the above the results presented in the next section, have been obtained with $M = 300$ and $I = 200$. Also, in order to have the same accuracy for all δ the time step is taken as $\Delta t = 10^{-4}$ in all cases, which always

satisfies condition (9). The evolution with respect to time is concluded at some total time, where the computed unsteady macroscopic quantities reach up to 99% of their corresponding steady-state distributions. This discretization ensures grid independent results up to at least two significant figures.

4. Results

Results are presented for the macroscopic quantities in terms of time in the whole range of the rarefaction parameter δ.

Figures 1-4 show the time evolution of the number density $\rho(y)$, macroscopic velocity $u(y)$, temperature $\tau(y)$ and heat flux $q(y)$ distributions in terms of the space coordinate y for specific values of time $t > 0$ and for $\delta = 0$, 1, 10 and 100, which roughly correspond to the free molecular, transition, slip and continuum regimes respectively. The corresponding steady-state solution at each δ is also included (dashed line).

Initially, at $t = 0$, the gas is at rest all macroscopic quantities are uniform, with the bulk velocity and heat flux equal to zero. Then, due to the sudden increase of the temperature of the plate located at $y = -1/2$ non-uniform distributions start to develop. As time is increased all macroscopic quantities keep developing up to some certain time and then they are reaching asymptotically the corresponding steady-state profiles. It should be noted that every macroscopic quantity takes its own time in order to reach the steady state and in all cases this time depends on the gas rarefaction.

In Fig. 2, in the transient phase, there is a nonzero bulk velocity which has an oscillatory damping behavior with time. This effect can be observed in the whole range of the gas rarefaction. This behavior of the gas may be explained by the density wave travelling with varying bulk velocity from the hot to the cold wall and backwards. Its amplitude is decreased as time is increased and finally at sufficiently large times it is damped.
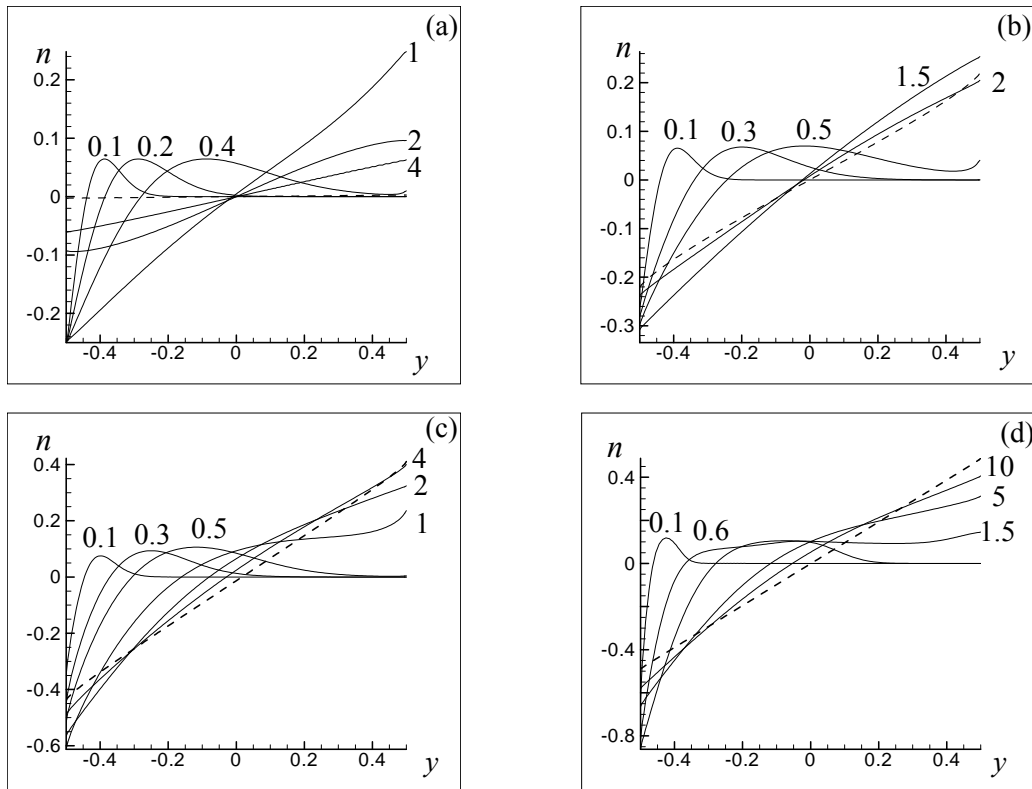
Fig. 1: Time evolution of number density at (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 10$ and (d) $\delta = 100$.
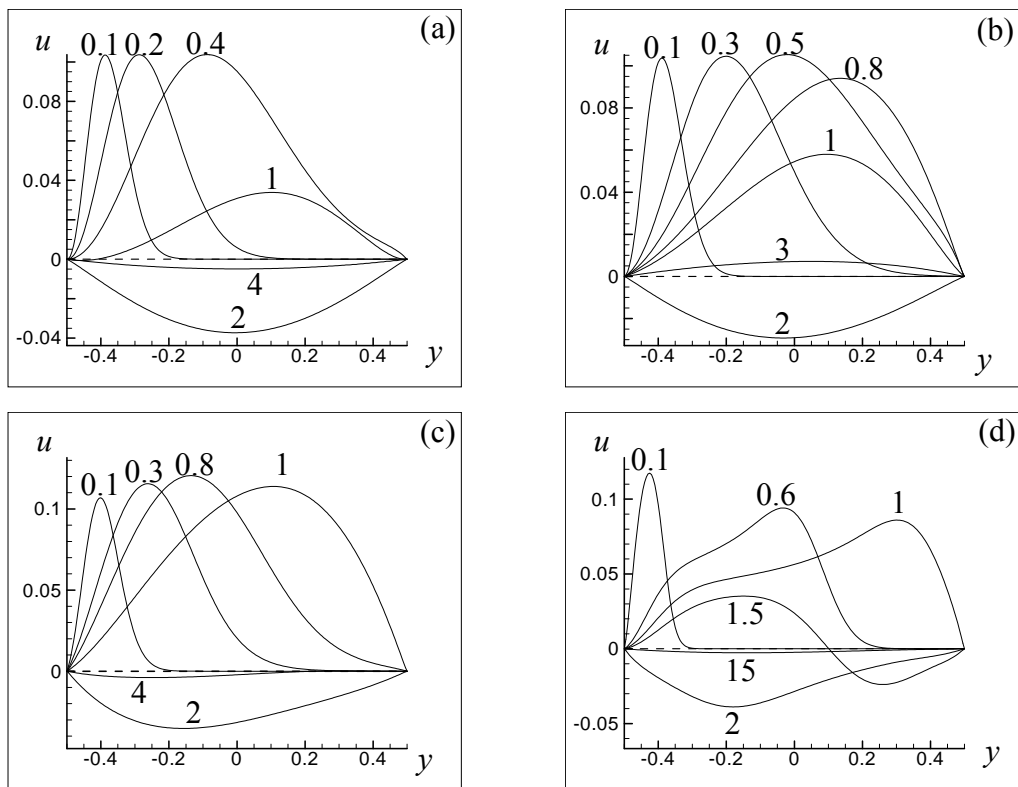

Fig. 2: Time evolution of bulk velocity at (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 10$ and (d) $\delta = 100$.
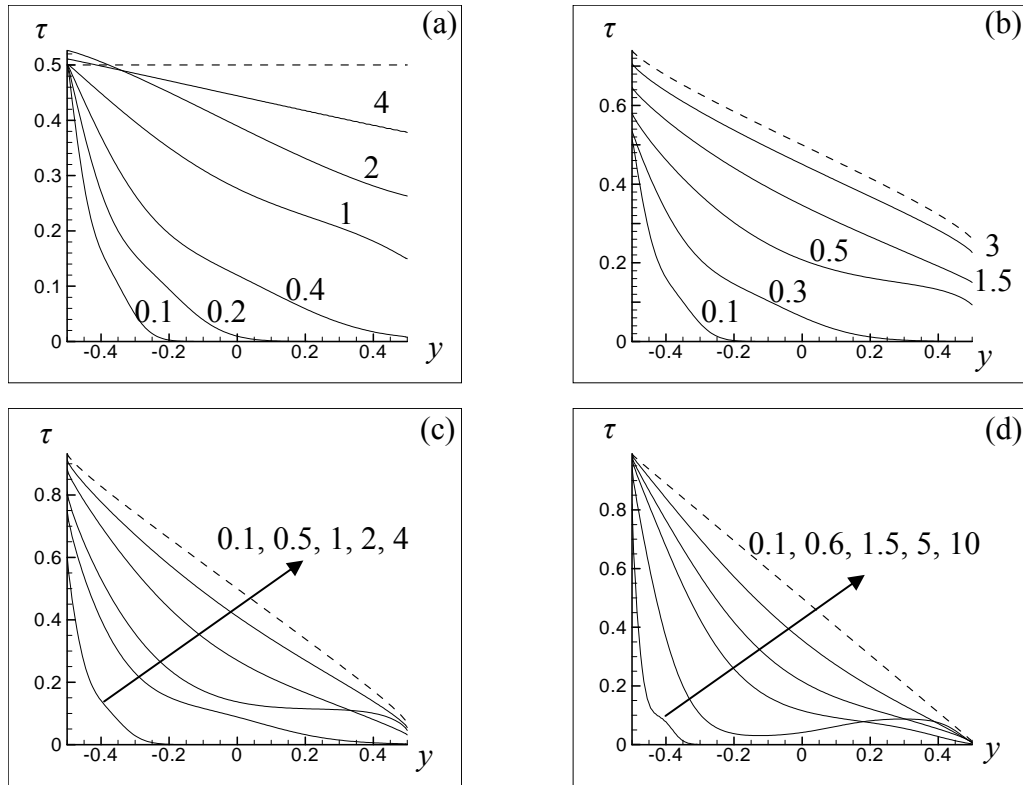
Fig. 3: Time evolution of temperature at (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 10$ and (d) $\delta = 100$.
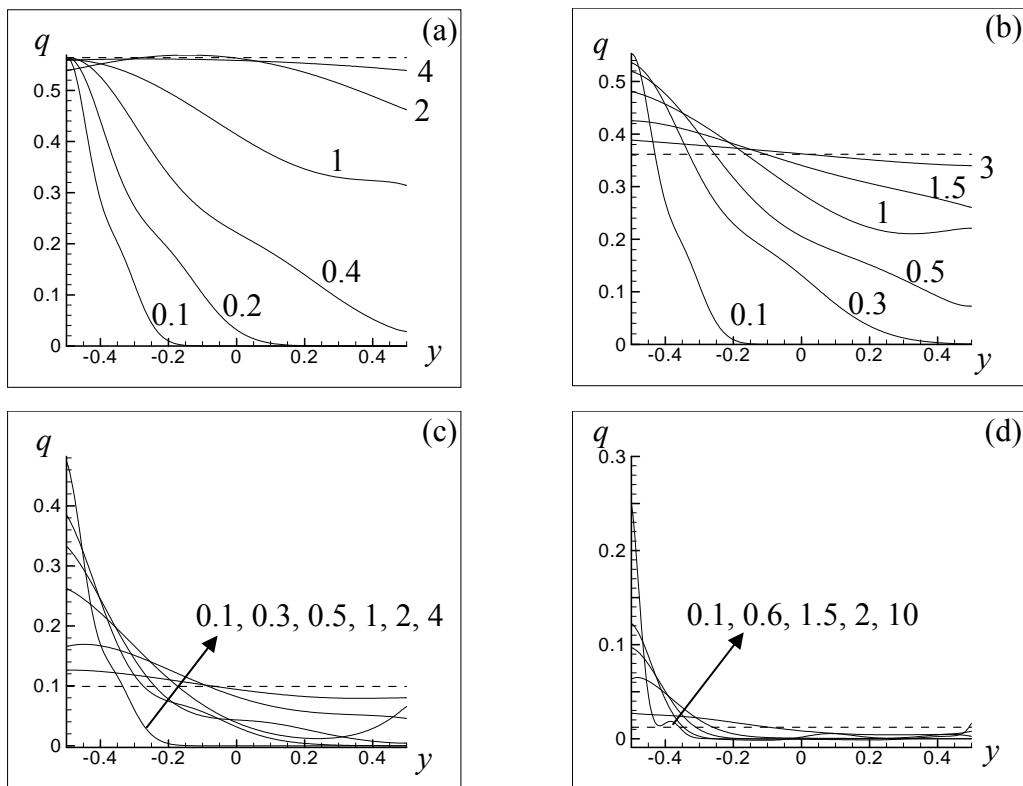

Fig. 4: Time evolution of heat flux at (a) $\delta = 0$, (b) $\delta = 1$, (c) $\delta = 10$ and (d) $\delta = 100$.
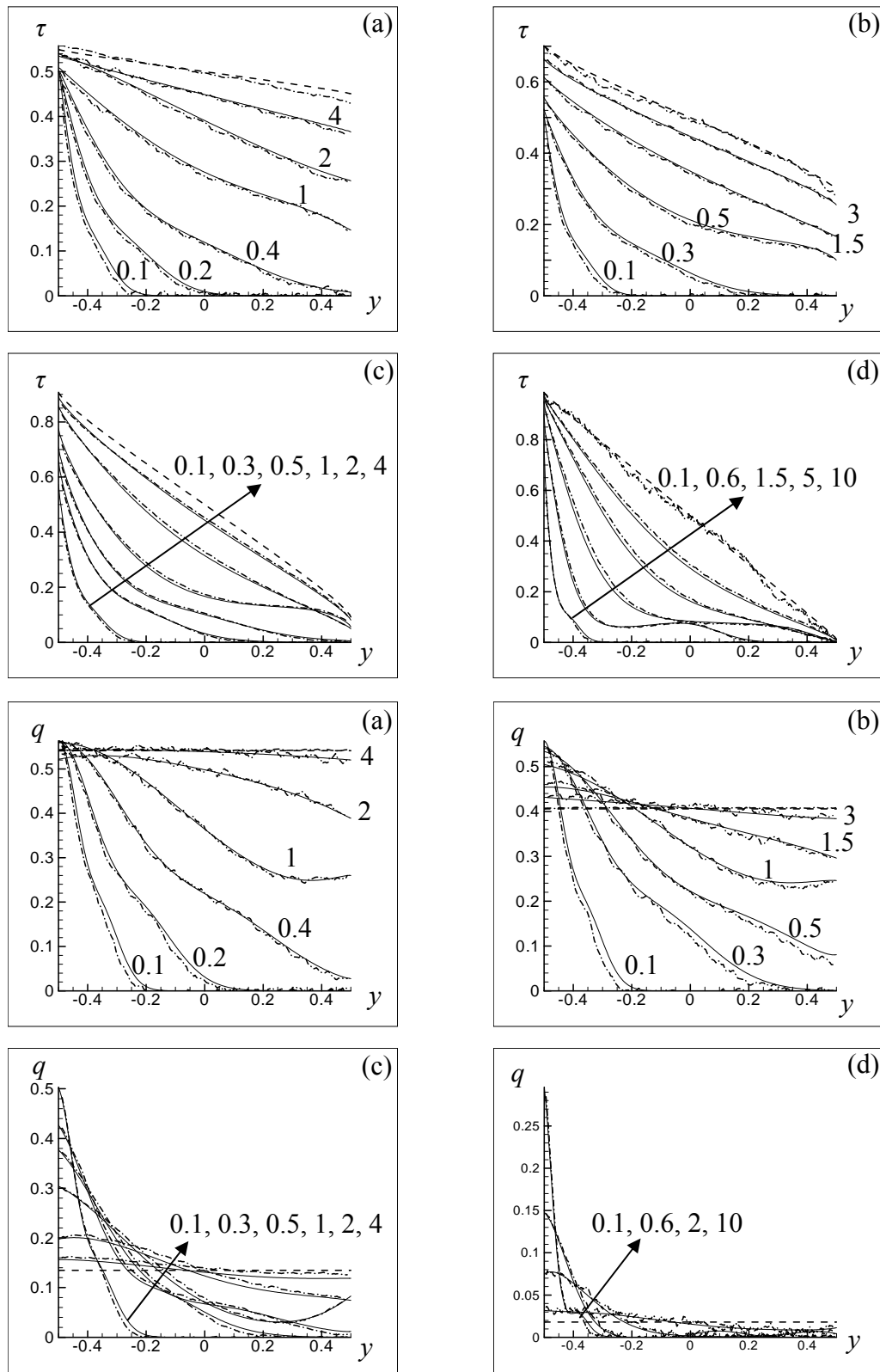
Fig. 5: Comparison of the temperature (upper four graphs) and of the heat flux (lower four graphs) distributions between kinetic and DSMC results at (a) $\delta = 0.067$, (b) $\delta = 0.67$, (c) $\delta = 6,67$ and (d) $\delta = 66.67$; dashed-dotted lines refer to DSMC results.

In parallel the heat flux varies between the plates, i.e., it is space dependent and at large times when the velocity magnitude becomes zero, the heat flux tends to become constant recovering its steady state value. Also it is interesting to note in Figs. 1 and 3 that the gas near the hot wall has lower number densities and higher temperatures than that at the cold wall. Especially this effect is more evident if we consider the gas state far from the free molecular one (e.g. Figs 1,3 c,d), because in this case the molecules going towards the cold wall lose their heat energy due to increasing number of collisions with outgoing molecules.

In order to validate the accuracy of the kinetic solution the comparison with the corresponding results obtained by the DSMC solutions of the Boltzmann equation for a hard-sphere gas is performed. The y-direction is discretized using $n_c = 400$ computational cells for $\delta = 100$ and $n_c = 200$ for $\delta < 100$, while the time step is chosen to be a small fraction of the cell traversal time $H / (\upsilon_0 n_c)$. Also 2000 particles per cell are used. The results are ensemble averaged until the desired statistical uncertainty is obtained. In accordance with our problem description, the gas is initially at equilibrium with the wall temperature T_0, while diffuse boundary conditions are used. Also the temperature ratio is $T_1 / T_0 = 1.05$.

Figure 5 shows the space and time variations of the temperature and the heat flux respectively at $\delta = 0.067$, 0.67, 6.67 and 66,67. The dashed-dotted lines correspond to DSMC results and the thin solid lines mark the kinetic results. In all cases the dashed line denotes the steady-state limit. It follows that a good agreement is obtained and therefore the kinetic solution provides an accurate description of the transient heat transfer at small temperature difference. For this specific problem the required computational effort for the kinetic solution is considerably smaller than for the DSMC procedure.

5. Conclusions

The transient heat transfer through a rarefied gas confined between parallel plates is investigated on the basis of the linear BGK kinetic model subject to Maxwell diffuse boundary conditions. The problem is solved in a fully deterministic manner. Results are presented for the number density, velocity, temperature and heat flux in the whole range of the Knudsen number. The accuracy of the kinetic results is validated by comparing them successfully with corresponding DSMC results. The obtained solution will be used, in the near future, in the design of micro-Pirani sensors.

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