Thermal Transpiration Flow

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Abstract Thermal transpiration is the macroscopic movement of rarefied gas induced by a temperature gradient. The gas moves from the lower to the higher temperature zone. An original method is proposed here to measure the mean macroscopic movement of gas in the case of a long circular cross-section glass micro-tube on to which a gradient of temperature is applied. The mass flow rate and the thermo-molecular pressure difference have been measured by monitoring the absolute pressure evolution in time at both ends of the capillary using high-speed response pressure gauges. Two gases Nitrogen and Helium are studied and three different temperature differences of 50, 60 and 70 Celsius degrees are applied to the tube. The analysed gas rarefaction conditions vary from transitional to slip regime.

Keywords: Micro Flow, Rarefied gas, Thermal Transpiration, Pumps

1 Introduction

It is well known that by applying a disequilibrium of temperature to a tube filled with a rarefied gas without any initial difference of pressure or any difference in chemical constitution the gas will macroscopically move from the lower to the higher temperature zone. This phenomenon was named *Thermal Transpiration* by Reynolds [1] in 1878. Reynolds, in an investigation with a plaster-of-Paris plug separating two regions 1 and 2 maintained at different temperatures, showed that at very low densities the equilibrium pressures on the two sides are related by the law

$$(1) \qquad \frac{p_2}{p_1} = \left(\frac{T_2}{T_1} \right)^{1/2}$$

Maxwell [2] closely followed this investigation up by mathematically analyzing the phenomenon using the, at that time, still controversial kinetic theory. Later on, in 1909 Knudsen [3] proved Reynolds law for the case of a tube by using both theoretical and experimental research. Knudsen's research led to the supposition that the above cited law had validity only at a zero-flow final equilibrium state which followed a transitional stage of gas displacement. Later Liang [4] in 1951 and Takaishi et al [5] in 1963 proposed semi-empirical formulas that predict how the thermo-molecular pressure ratio p_2/p_1 at the final equilibrium zero-flow state varies in function of the gas rarefaction, the gas physical properties and the applied temperature difference to the tube. Consequentially several experimental studies were realized: Edmonds et al [5] in 1965 compared their results with the semi-empirical equation of Liang for thin walls apertures finding a good agreement for helium; Watkins et al. [6] in 1966 compared their results with the semi-empirical equation of Weber et al. and found data with a satisfying qualitative agreement; Annis [7] in 1972 successfully compared his thermo-molecular pressure difference $p_2 - p_1$ (t.p.d.) experimental results with the numerical results of Loyalka et al. [8] who studied thermal transpiration on a cylindrical tube by using the Bhatnagar-Gross-Krook model. Finally, in 1978 Porodnov et al. [9] emphasized that despite the great number

of experimental works still no correct theory existed for the t.p.d. effect at arbitrary gas rarefaction conditions. This work [10] in combination with the work of Storvick et al. [11] gave a final good characterization of the thermo-molecular pressure difference for different gases at different rarefaction conditions by applying different temperature gradients to the tube. In recent times the advent of micro-electro mechanical systems (MEMS) made way for new perspectives on thermal transpiration. The possibility of using the pumping effect of thermal transpiration to create a micro-compressor without moving parts led to the experimental works of Vargo et al.[12] in 1999 and Han et al. [13] in 2007 in which the attention is mainly focused on the pressure increase due to the application of a temperature gradient along a channel.This final work of Han et al. [13] together with the work of York et al. [14] in 1999 gave a first characterization of the transitional stage of gas displacement by measuring the pressure variation in time before reaching the final zero-flow state. At the moment no real efforts have been made to measure the mass flow rate induced by thermal transpiration.

In this paper an original method for thermal transpiration mass flow rate (m.f.r.) measurements is proposed using Knudsen's intuition of a transitional gas displacement stage. Here we present the first results of the investigation conducted via measuring *in situ* the pressure evolution in time at both ends of the tube using two high-speed response pressure gauges. The rarefaction parameter delta

$$(2) \qquad \delta = \frac{1}{2} \frac{pD}{\mu \sqrt{2RT}}$$

indicates the rarefaction state of the gas and depends on thermodynamic parameters as pressure p and temperature T, the diameter D which is the characteristic length of the system and the gas physical properties as the viscosity μ and the specific gas constant R. Due to the tube's characteristic length and the applied pressure working conditions, which vary between 13.3 Pa and 465.5 Pa for Nitrogen

and between 13.3 Pa and 1330 Pa for Helium, the gas rarefaction conditions vary from transitional to slip regime: Helium $\delta=0.15$ to 15.3; Nitrogen $\delta=0.45$ to 15.6.

2 Theory

The main objective of this study is to measure the mass flow rate generated by thermal transpiration along the tube. This is done by exploiting the phenomenon's time dependance [15]. In the case of a tube, where a distribution of temperature is applied along its axis and which is connected to two infinite volume reservoirs at its inlet and outlet, the rarefied gas in it continuously flows, from the colder to the hotter side. By using this method, it is then possible to obtain a stationary, not-perturbed and fully developed flow. In order to understand the physical basic mechanism causing the movement of the gas, which is initially at rest when a temperature difference is still not imposed, it is necessary to regard the boundary conditions. It is possible to show, that the balance of momentum exchange between an elementary wall surface and the molecules of a fluid particle results in a force which is applied to the gas in the temperature gradient direction [16]. In other respects, the imposed gradient of temperature creates a gradient of density in direction of the tube's axis. The gas motion tends to reduce the density gradient: the gas flows from the higher to the lower density region id est from the cold-side to the hot-side reservoir. In the case where two finite reservoirs are positioned at the micro-tube's ends, the phenomenon changes and it becomes time-dependent. The present work uses the latter case configuration as in the initial experience of Reynolds [1]. By coupling to the tube two reservoirs with a finite volume, a final pressure difference is found when the phenomenon reaches its final zero-flow state [3]. The final zero-flow state is reached when the pressure difference fully equilibrates the thermal transpiration flow, creating a counter flow, which tends to balance the initial temperature gradient induced flow. This pressure difference variation in time is

extremely fast and only in recent works it has been possible to estimate its correct tendency and measure it [13,14]. The here used experimental apparatus has been built *in situ* in order to promptly capture this pressure evolution in time.

3   Experiments

3.1 Setup

The long circular cross-section glass micro-tube (D=490 ± 1μm; L=3.053 ± 0.01cm) is connected to two reservoirs (Figure 1). One reservoir is heated while the other is maintained at room temperature. The imposed temperature difference between both reservoirs creates a steep temperature gradient along the micro-tube. The temperature gradient was proved to have a parabolic shape by monitoring the temperature distribution on the tube's surface using an infrared camera. Each reservoir is coupled to a high-speed response time (30ms) capacitance diaphragm gauge (CDG). The hot-side CDG is especially performative for high gas-temperature working conditions. The temperature is monitored at both ends of the micro-tube and at the hot-side reservoir using three different thermocouples. The temperature is maintained stable during the whole duration of the experiment.

The internal ring of the system where micro-tube and reservoirs are positioned has two main functions: to connect the reservoirs twice, through the micro-tube and through a big diameter tube junction equipped with a below-sealed valve (valve A), and to dampen any pressure oscillations produced by the vacuum system having consequentially the function of a stabilizing chamber. The external open ring from one side supplies pure gas to the internal ring by opening the regulation butterfly-valves of two external high pressure tanks containing Helium and Nitrogen; from the other side it joints the internal ring a vacuum pump which can vacuum the system until 0.1 Pa. The pressure inside the internal ring is regulated by means of a below-sealed
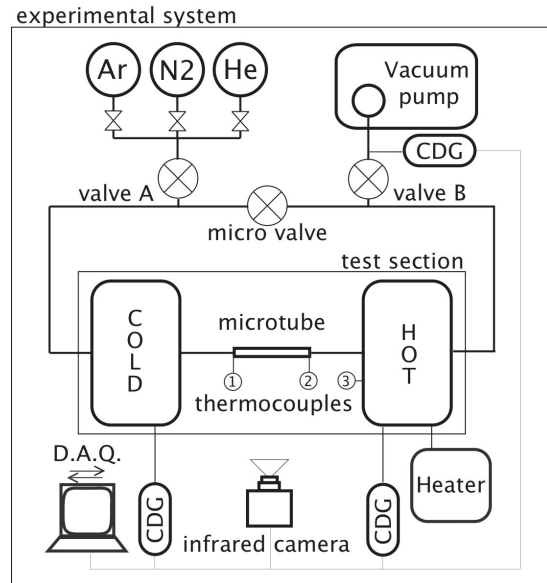
regulation valve (valve B).

Figure 1: *The experimental setup.*

3.2 Methodology

The methodology of the experiment is defined by four main stages (Figure 2). After imposing inside the stabilizing chamber the experimental gas pressure conditions valves B and C are closed (stage 1). These two valves will remain closed for the whole duration of the experience. At this moment the internal ring starts damping all the residual oscillations of pressure until a pressure equilibrium stage is reached. In order to have no pressure difference in the reservoirs valve A is kept open: this allows the pressure to distribute itself in a continuous and equal manner in the system. In this first stage the thermal transpiration phenomenon is already present. A stationary macroscopic gas displacement already exists along the tube due to the imposed temperature gradient.

At time t=0 valve A is closed. At time t=0+ the transitional stage (stage 2 and 3) of the experiment starts: the pressure variates in time increasing in the outlet reservoir while

decreasing in the inlet reservoir. This is the starting point of the experience and the pressure gauges monitor the pressure variation in time p(t). In the first phase of the transitional stage (stage 2) the pressure variation in time is linear thus the phenomenon is considered to be stationary. Subsequently the pressure-variation speed (p.v.s.) decreases (stage 3): this second phase of the transitional stage is characterized by a non-linear pressure variation in time thus the phenomenon is considered to be non-stationary. The p.v.s. decrease directly depends on the continuous increase of the pressure difference between the reservoirs created by the gas displacement. The pressure difference creates a flow which is directed contrarily to the thermal transpiration flow. At time t=0+ (stage 2) this counter-flow is zero and is negligible during the first phase of the transitional stage. Consequentially in the non-stationary phase of the experiment (stage 3) the flow driven by the created pressure difference is not negligible and modifies the mean mass flow rate intensity. The counter-flow will completely balance the thermal transpiration flow at the final stage of the experiment (stage 4). When the pressure difference arrives to its maximum the mean mass flow rate is zero. This is the final equilibrium stage known as the zero-flow state where the final f on initial i pressure ratio for the hot-side reservoir h is

$$(3) \qquad \frac{p_{fh}}{p_i} > 1$$

while for the cold-side reservoir c is

$$(4) \qquad \frac{p_{fc}}{p_i} < 1$$

The here adopted notations fh and fc substitute the initially cited notations of Reynolds law 2 and 1 respectively.
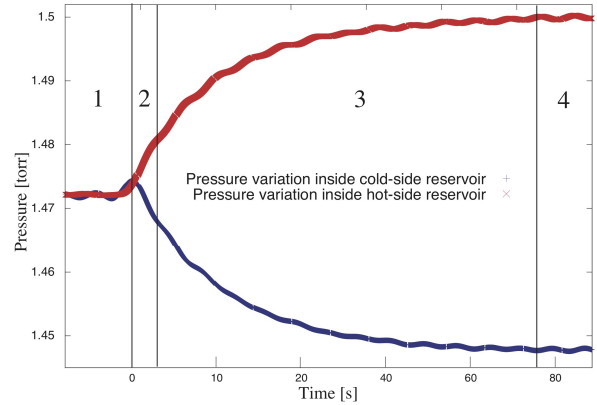

Figure 2: *The stages of the experiment.*

3.3 The exponential behaviour of the pressure evolution in time

The pressure variation p(t) and the pressure-variation speed dp(t)/dt behaviours depend on the gas nature, the applied temperature difference and the gas rarefaction conditions. This pressure variation in time is exponential in both reservoirs and a study is conducted using the fitting functions which characterize each experiment. Generally, the pressure variation in time is well defined by the functions,

$$(5) \qquad \begin{aligned} f_h(t) &= (p_{fh} - p_i)\left[1 - e^{-(t/\tau_h)}\right] + p_i \\ f_c(t) &= (p_i - p_{fc})\left[e^{(-t/\tau_c)} - 1\right] + p_i \end{aligned}$$

for the hot-side and cold-side respectively where τ is the only adjustable parameter. The average standard error of the pressure variation fitting is within ±1%. The two functions $f_h(t)$ and $f_c(t)$ are not perfectly mirror-symmetric in respect to the zero pressure axis since the volumes of the two reservoirs differ $V_h/V_c = 0.825$. Consequentially, the pressure variation in time in the reservoirs is different. The pressure-variation speed is then given by

$$\frac{df_h(t)}{dt} = \frac{1}{\tau_h}(p_{fh} - p_i)e^{-(t/\tau_h)}$$

(6)

$$\frac{df_c(t)}{dt} = -\frac{1}{\tau_c}(p_i - p_{fc})e^{-(t/\tau_c)}$$

It is possible to verify from the pvs equations the stationarity of the thermal transpiration flow right after the valve closure, at time t=0+. During the first second of the experiment the pvs can be considered constant and this assures the linearity of the pressure variation in time. This indicates that the thermal transpiration flow has not been yet affected by the counter poiseuille flow and it is considered to still be at its maximum intensity. For the first second the thermal transpiration flow is then negligibly perturbed by a difference of pressure generated counter flow.

4 Results

4.1 The thermal-molecular pressure difference

The classical approach to thermal transpiration advices us to look at the zero-flow equilibrium pressure difference at the end of the experience. This creates a comprehensive overview on the properties of the phenomenon in function of the gas physical properties, the applied temperature differences and the gas rarefaction conditions. In Figure 3 the tpd for Nitrogen and Helium are shown.

At the beginning of the transitional regime ($\delta \approx 0.44$) the pressure difference starts at a minimum value and then rapidly increases for higher values of delta. It is possible to observe how the thermal transpiration pumping effect touches a maximum value in the transitional regime. As expected, the intensity of this maximum is greater for higher applied temperature differences. Helium's maximum

thermo-molecular pressure difference is nearly 3 times greater than Nitrogen's maximal values. The maximum values are to be found for both gases at the same rarefaction conditions $\delta \approx 4 \div 5$. The tpd strongly decreases while exiting the transitional regime and entering the slip regime region.
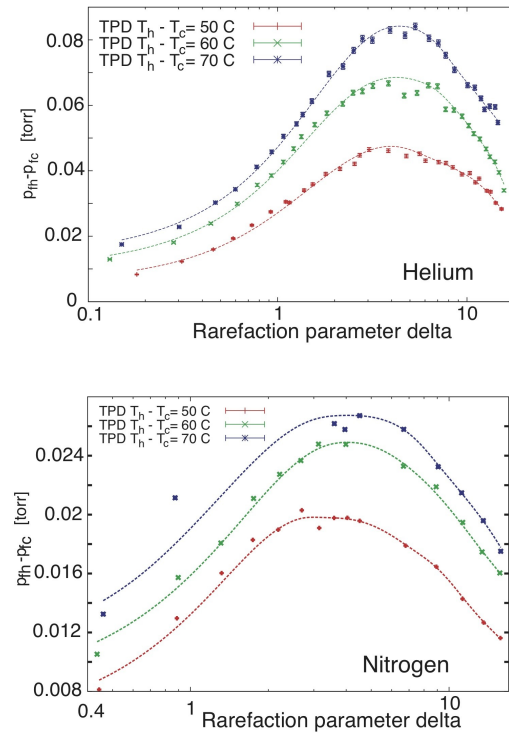

Figure 3: *Thermal molecular pressure difference.*

4.2 The mass flow rate

In this work it is shown that the adopted original measure mechanism leads to a time-dependent phenomenon since after time t=0+ the pressure-variation speed decreases with time. If the micro-tube's ends were connected to infinite volumes the pressure-variation speed will stay constant for the whole duration of the process leading to a constant flux of gas through the tube. Taking into account the stationary and not perturbed state of the flow before time t=0, the mass flow rate can also be considered stationary at time t=0+ after the

valve's closure. Here the mean mass flow rate is identified to be at its maximum intensity and has not yet been reduced by the created pressure difference. In the reservoirs the stationary mass flow rate is easily related to a constant pressure-variation speed [17] which here is given by the pressure variation speed at time t=0+. Thus from [17] we obtain:

$$(7) \qquad Q = \frac{V_c}{RT_c}\left(\frac{df_c}{dt}\right)_{t\to 0} = \frac{V_h}{RT_h}\left(\frac{df_h}{dt}\right)_{t\to 0}$$

The eq. gives the mean mass flow rate of the gas at the micro-tube's inlet or outlet leaving the cold-side reservoir or entering the hot-side reservoir. From the law of conservation of mass the same mean mass flow rate intensity is registered at the micro-tube's outlet in the hot-side reservoir: this reservoir is considered to be a gas storage volume. In Figure 4 the mass flow rate results for Helium and Nitrogen are shown for a wide spectrum of gas rarefaction conditions and for three different applied temperature differences.

As expected an increase of the mass flow rate intensity is achieved by increasing the rarefaction parameter values shifting them from transitional to slip regime conditions. Higher temperature differences lead to increased values of the mass flow rate intensity. The gas physical properties give as well different values of the m.f.r. which are higher for Nitrogen in transitional regime. In slip regime conditions the tendency seems to change at least for the higher applied temperature difference: Helium's mass flow rates are slightly higher than Nitrogen's. It is to be noticed that the pressure working conditions of the gases are different: it is possible to obtain higher m.f.r. intensities for Nitrogen if the same pressures are applied to the system (Figure 4 *box c*). Helium instead can be used at higher pressures in order to obtain same order m.f.r. intensities than in respect to Nitrogen. The measurements have a relative error of ±2.3%.
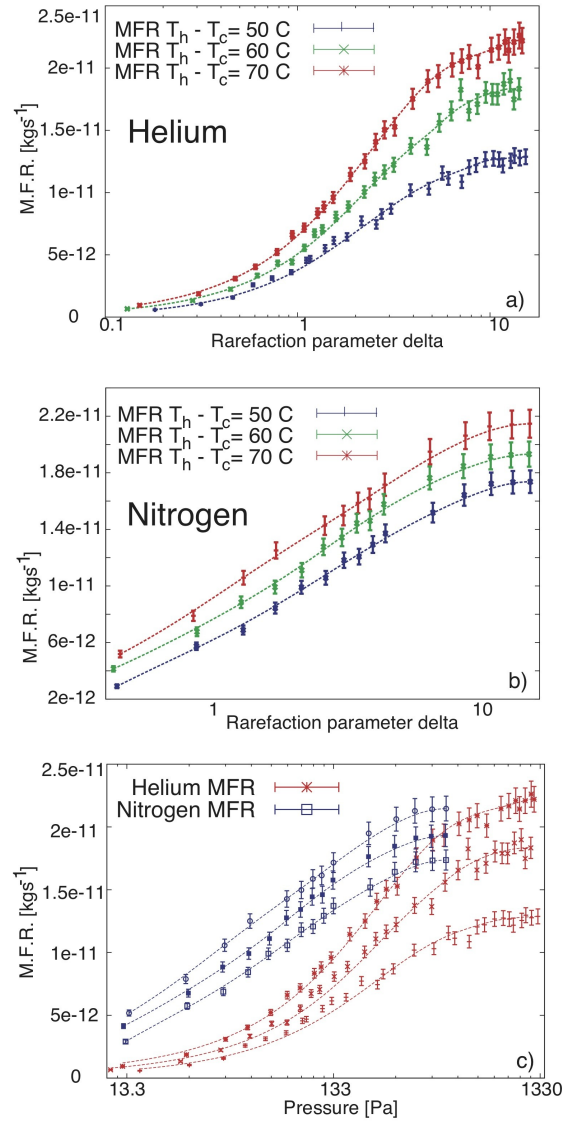

Figure 4: *Mass flow rate.*

5 Comments

The analyzed mass flow rate intensities are very small due to the small diameter of the here used micro-tube and the relatively low temperature differences applied to the tube. The here measured values of the mass flow rate vary from $5.8\cdot 10^{-13}[kg/s]$ for Helium at a temperature difference of 50° C in transitional regime conditions ($\delta=0.18$) to $2.2\cdot 10^{-11}[kg/s]$ for Helium at a temperature difference of 70° C in slip regime conditions (

$\delta = 14.7$). This means that in function of the applied temperature and the rarefaction working conditions the mass flow rate intensity can vary by almost 38 times. When lower pressures are applied to the system larger diameters dimensions can be used to obtain the same gas rarefaction conditions and thus increase the mass flow rate intensity. Thermal transpiration is as well prevalent in ambient pressure working devices with nano-scale characteristic lengths dimensions. This leads to the possibility of using thermal transpiration in a vast area of devices and working conditions which do not have to be necessarily of extremely small scale or work at extremely low pressure.

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