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Treatments of flows through micro-channels based on the Extended Navier-Stokes-Equations

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Abstract The paper briefly refers to the present treatments of micro-channel flows that are based on the existing Navier-Stokes-Equations and the employment of wall-slip boundary conditions. The Maxwell slip velocity is employed for this purpose. This theoretical treatment is questioned. It is shown by the authors that the existing Navier-Stokes-Equations are incomplete. They do not contain terms for the self diffusion of mass. Introducing these terms yields the extended Navier-Stokes-Equations that allow micro-channel flows to be treated without the assumption of Maxwellian slip velocities at the wall. A pressure driven slip velocity occurs at the wall and it results as part of the solution for flows in micro-channels by the "Extended Navier-Stokes Equations". Using these equations, analytical treatments of micro-channel flows are presented. Good agreement with existing experimental results is obtained.

Keywords: Micro-Channel Flow, Extended Navier-Stokes Equations

1. Introduction and Aim of Work

Micro-channel flows have excited fluid mechanics researchers all over the world and in the last few decades, a large number of publications treating this kind of flow have been published. Until very recently, the published theoretical treatments were based on the assumed wall slip velocity of Maxwell (1879), who reported the slip velocity to be

$$U_s = \left(\frac{2-\sigma}{\sigma}\right)\gamma\left(\frac{dU}{dx_2}\right)$$

This expression for the slip velocity is based on the assumption that a fraction of the molecules, interacting with the wall, is reflected in a deterministic way and the rest is reflected specularly. It is this approach that is seen in nearly all theoretical treatments of micro-channel flows.

It can be observed from the literature that in gaseous flows through micro-channels, under some conditions (Pfahler et al. (1991), Pong et al. (1994), Harley et al. (1995), Liu et al. (1995), Shih et al. (1995), Shih et al. (1996)), the measured mass flow rates can be higher than those computed from the Classical Navier-Stokes Equations (CNSE), for certain given inlet and outlet pressures. Considerations show that the differences between experimental and theoretical results exist when the Knudsen number of the flow is high, at least in parts of micro-channels. Moreover, in a gaseous flow through a micro-channel, one observes that the pressure no longer decreases linearly along the length of the channel. The presented treatment of micro-channel flow also results in a slip velocity at the channel walls, but this has not been introduced in an empirical manner. It is derived as part of the analytical treatment of the flow.

In some of the recently published articles, Durst and co-workers (2006) have questioned the Maxwell slip velocity approach. They have shown that the CNSE are valid for constant fluid property flows only. If there are densities, pressure or temperature gradients in the fluid domain, self-diffusion will set in and this is not taken into account by the CNSE. In order to overcome this difficulty, Durst and co-workers derived the Extended Navier-Stokes-Equations (ENSE), e.g. see Durst et al. (2006), and employed them for the numerical treatment of micro-channel flows using the commercial software FLUENT, e.g. see Sambasivam & Durst (2010). Furthermore, very recently analytical treatments of fluid flows through micro-channels became available, e.g. see Filimonov et al. (2010), using the ENSE. The velocity profiles at every location of the micro-channel could be derived and the expressions for these profiles were integrated to yield convective, diffusive and total mass flow rates. The latter were compared with corresponding experimental results to verify the applied solution approach. Very good agreement with available experimental data was obtained for the derived total mass flow rate.

However, the velocity profile alone does not bring out the micro-channel flow physics and its distinctive characteristics. It is important to understand the special features of the pressure distribution along the flow direction and the role of the characteristic pressure in the flow process. The semi-analytically derived pressure results are compared with the corresponding experimental data and numerical simulations available in the literature and, again, very good agreement is achieved. This funding encouraged the present authors to derive analytical results for other properties of micro-channel flows. The characteristic pressure is introduced for channel flows and its physical meaning is considered and explained. With the presentation, the authors' theoretical research work on isothermal micro-channel flows is summarized. A new approach to handle micro-channel and micro-capillary flows was taken. The suggested approach will also allow heat and mass transfer to be taken into consideration. Micro-channel flows with and without changes in their cross-sectional areas will become, in this way, theoretically treatable in the same manner as other fluid flows through channels with larger cross-sections and at lower pressure gradients than are usually employed in micro-channel flows. The resulting equations can be employed to yield new results for micro-channel flows.

2. Analytical Treatments of Micro-Channel Flows

2.1 Order of Magnitude

The gas flow is assumed to be two-dimensional for micro-channels in rectangular coordinate system. Further, it is also assumed to be steady and isothermal in nature. Since the flow is isothermal, the viscosity is also considered to be constant. The extended Navier-Stokes equations in the total velocity form can be written for steady, isothermal gas flows, see Sambasivam and Durst (2010), as given below:

Continuity equation $$\frac{\partial\left(\rho U_i^T\right)}{\partial x_i}=0 \qquad (2.1)$$

Momentum equations

$$\frac{\partial}{\partial x_i}\left[\rho U_i^T U_j^T\right]= -\frac{\partial P}{\partial x_j}-\frac{\partial}{\partial x_i}\left[\tau_{ij}^C-\dot{m}_i^D U_j^D-\frac{2}{3}\delta_{ij}\dot{m}_k^D U_k^C\right] \qquad (2.2)$$

with the density ρ calculated from the equation of state $\rho = P/RT$ and molecular momentum transport given by:

$$\tau_{ij}^C=-\mu\left[\frac{\partial U_j^C}{\partial x_i}+\frac{\partial U_i^C}{\partial x_j}\right]+\frac{2}{3}\mu\delta_{ij}\frac{\partial U_k^C}{\partial x_k} \qquad (2.3)$$

Further, equations (2.1) and (2.2) can be expanded for a two-dimensional flow situation, employing equation (2.3), as given below:

Continuity equation: $$\frac{\partial\left(\rho U_1^T\right)}{\partial x_1}+\frac{\partial\left(\rho U_2^T\right)}{\partial x_2}=0 \qquad (2.4)$$

$x_{1/2}$ - momentum equation:

$$\frac{\partial\left(\rho U_1^T U_{1/2}^T\right)}{\partial x_1}+\frac{\partial\left(\rho U_{1/2}^T U_2^T\right)}{\partial x_2}=-\frac{\partial P}{\partial x_{1/2}}-$$
$$-\frac{\partial}{\partial x_{1/2}}\left[\begin{array}{c}-\frac{4}{3}\mu\frac{\partial U_{1/2}^C}{\partial x_{1/2}}+\frac{2}{3}\mu\frac{\partial U_{2/1}^C}{\partial x_{2/1}}-\dot{m}_{1/2}^D U_{1/2}^D \\ -\frac{2}{3}\left(\dot{m}_1^D U_1^C+\dot{m}_2^D U_2^C\right)\end{array}\right]$$
$$-\frac{\partial}{\partial x_{2/1}}\left(-\mu\frac{\partial U_1^C}{\partial x_2}-\mu\frac{\partial U_2^C}{\partial x_1}-\dot{m}_{2/1}^D U_2^D\right) \quad (2.5)$$

For the case of gas flows through straight micro-channels, there is no diffusion transport of mass in the cross-stream direction since the pressure is constant in this direction. Similarly, the convection velocity in the cross-stream direction is also zero since the flow is fully developed. Further, it can be shown that the convective acceleration terms are negligible in the momentum equations for the case of gas flows through micro-conduits. Therefore, equations (2.4) to (2.5) can be simplified as follows:

$$\frac{\partial\left(\rho U_1^T\right)}{\partial x_1}=0 \quad (2.6)$$

$$0=-\frac{\partial P}{\partial x_1}-\frac{\partial}{\partial x_1}\left[\begin{array}{l}-\frac{4}{3}\mu\frac{\partial U_1^C}{\partial x_1}-\dot{m}_1^D U_1^D \\ -\frac{2}{3}\dot{m}_1^D U_1^C\end{array}\right]- \quad (2.7)$$
$$-\frac{\partial}{\partial x_2}\left(-\mu\frac{\partial U_1^C}{\partial x_2}\right)$$

$$0=-\frac{\partial P}{\partial x_2}-\frac{\partial}{\partial x_1}\left(-\mu\frac{\partial U_1^C}{\partial x_2}\right)-$$
$$-\frac{\partial}{\partial x_2}\left[\frac{2}{3}\mu\frac{\partial U_1^C}{\partial x_1}-\frac{2}{3}\left(\dot{m}_1^D U_1^C\right)\right] \quad (2.8)$$

Further, one can employ the order of magnitude analysis to further simplify equations (2.6) – (2.8). The characteristic velocity and length scales for this analysis are as follows. The convective velocity can be scaled with the average velocity at the exit of the channel $\overline{U}$ and the diffusion velocity is scaled as $\sim \mu/\rho L$ based on the expression for the diffusion transport of mass given by equation:

$$\dot{m}_i^D=-\underbrace{(D\rho)}_{\mu}\left[\frac{1}{\rho}\frac{\partial\rho}{\partial x_i}+\frac{1}{2T}\frac{\partial T}{\partial x_i}\right]=$$
$$=-\mu\left[\frac{1}{P}\frac{\partial P}{\partial x_i}-\frac{1}{2T}\frac{\partial T}{\partial x_i}\right]=U_i^D\rho \quad (2.9)$$

Further, the characteristic length for the streamwise and cross-stream directions are the length L and height H of the channel, respectively where $H \ll L$. Furthermore, one can scale the pressure with $\rho\overline{U}^2$. Subsequently, one can perform the order of magnitude estimates for the various terms in equation (2.8) as given below:

$$\frac{\partial P}{\partial x_1}\sim\frac{\rho\overline{U}^2}{L} \quad (2.10a)$$

$$\frac{\partial}{\partial x_1}\left(-\frac{4}{3}\mu\frac{\partial U_1^C}{\partial x_1}\right)\sim\mu\frac{\overline{U}}{L^2}$$
$$\frac{\partial}{\partial x_1}\left(-\dot{m}_1^D U_1^D\right)\sim\frac{\mu^2}{\rho L^3}$$
$$\frac{\partial}{\partial x_1}\left(-\frac{2}{3}\dot{m}_1^D U_1^C\right)\sim\mu\frac{\overline{U}}{L^2} \quad (2.10b)$$

$$-\frac{\partial}{\partial x_2}\left(-\mu\frac{\partial U_1^C}{\partial x_2}\right)\sim\mu\frac{\overline{U}}{H^2} \quad (2.10c)$$

The terms in equation (2.10b) can all be neglected and equation (2.7) can be rewritten as:

$$0=-\frac{\partial P}{\partial x_1}+\frac{\partial}{\partial x_2}\left(\mu\frac{\partial U_1^C}{\partial x_2}\right) \quad (2.11)$$

Similarly, the order of estimate analysis of the various terms in equation (2.8) is given below:

$$-\frac{\partial P}{\partial x_2}\sim\frac{\rho\overline{U}^2}{H} \quad (2.12a)$$

$$\frac{\partial}{\partial x_1}\left(-\mu\frac{\partial U_1^C}{\partial x_2}\right)\sim\mu\frac{\overline{U}}{HL}$$
$$\frac{\partial}{\partial x_2}\left(\frac{2}{3}\mu\frac{\partial U_1^C}{\partial x_1}\right)\sim\mu\frac{\overline{U}}{HL}$$
$$\frac{\partial}{\partial x_2}\left[-\frac{2}{3}\left(\dot{m}_1^D U_1^C\right)\right]\sim\mu\frac{\overline{U}}{HL} \quad (2.12b)$$

Neglecting the terms from equation (2.12b), equation (2.8) can be reduced to the following form.

$$\frac{\partial P}{\partial x_2} = 0 \quad (2.13)$$

It is evident from equation (2.13) that the pressure is only a function of the streamwise coordinate, x_1 in micro-conduits. It is well known that the diffusion velocity, defined by equation (2.10) is only a function of the streamwise coordinate x_1 and hence one can write the following expression:

$$\frac{\partial U_1^D}{\partial x_2} = 0 \quad (2.14)$$

Employing equation (2.14), equation (2.7) can be written as:

$$-\frac{\partial P}{\partial x_1} + \frac{\partial}{\partial x_2}\left(\mu \frac{\partial U_1^C}{\partial x_2}\right) + \frac{\partial}{\partial x_2}\left(\mu \frac{\partial U_1^D}{\partial x_2}\right) =$$

$$-\frac{\partial P}{\partial x_1} + \mu \frac{\partial}{\partial x_2}\left(\frac{\partial U_1^T}{\partial x_2}\right) = 0 \quad (2.15)$$

since the dynamic viscosity μ is a constant for isothermal gas flows through micro-channels. The final set of simplified governing equations for gas flows through micro-channels is given below:

Continuity Equation $$\frac{\partial(\rho U_1^T)}{\partial x_1} = 0$$

Momentum Equation $$\mu \frac{\partial}{\partial x_2}\left(\frac{\partial U_1^T}{\partial x_2}\right) - \frac{\partial P}{\partial x_1} = 0$$

$$\frac{\partial P}{\partial x_2} = 0$$

3. ANALYTICAL TREATMENTS OF MICRO-CHANNEL-FLOWS

3.1 The Derivations of the Velocity Distribution

The final equations of section 2 describe the flow in a two-dimensional micro-channel, where ρ is the fluid density, U^T is the total velocity and μ is the viscosity of the fluid, P is the pressure and x_1 and x_2 are the coordinates in the flow and cross-flow direction, respectively.

U^T consists of the convective velocity U^C and the diffusive velocity U^D:

$$U^T = U^C + U^D \quad (3.1)$$

According to the diffusion term, shown in equation (2.9), derived by Durst et al. (2006), one can write

$$\rho U^D = -\frac{\mu}{P}\frac{\partial P}{\partial x} \quad (3.2)$$

Noting that P is a function of the flow direction only (because of the boundary layer approximation of the derived equations), equation (2.2) can be integrated in the x_2 −direction (normal to the flow), to give

$$\rho U^T = -\frac{\rho}{2\mu}(\mathrm{h}^2 - x_2^2)\frac{dP}{dx_1} - \frac{\mu}{P}\frac{dP}{dx_1} =$$

$$= \rho\left\{-\underbrace{\left[\frac{1}{2\mu}(\mathrm{h}^2 - \mathrm{y}^2)\frac{\mathrm{dP}}{\mathrm{dx}}\right]}_{(\mathrm{I})} - \underbrace{\left[\frac{\mu \mathrm{RT}}{\mathrm{P}^2}\frac{\mathrm{dP}}{\mathrm{dx}}\right]}_{(\mathrm{II})}\right\} \quad (3.3)$$

where P and (dP/dx_1) are the local pressure and pressure gradient inside the micro-channel respectively. Hence, there is the actual expression for the velocity distribution in isothermal micro-channel flow on the right hand side. The ideal gas law $P=\rho RT$ has been used to eliminate the density ρ in equation (3.3) which readily shows that the velocity profile consists of two parts:

Term (I): Parabolic part of the velocity profile due to convection

Term (II): Constant part of the velocity profile due to diffusion

At every x_1-location in the micro-channel, where P and dP/dx_1 are known, the total velocity U^T can be computed with the help of equation (3.3). From the second part of this equation, we can see that the diffusive part of the velocity profile will only make strong contributions to the follow for low pressures P or for high temperatures T. Hence, to achieve “micro-channel effects” at room temperatures, the pressure at the exit of the micro-channel has to be chosen very low.

3.2 Derivation of the Pressure Distribution

The total mass flow rate, flowing through any cross-section of the micro-channel, is obtained by integrating the mass flux over the cross-sectional area of the channel, e.g. one can write:

$$\dot{M}^T = w\int_{-h}^{h}(\rho U^T)\,dx_1 =$$

$$= -2hw\left(\frac{h^2P}{3\mu RT}+\frac{\mu}{P}\right)\frac{dP}{dx_1} \qquad (3.4)$$

Due to the principle of conservation of mass, $\dot{M}^T$ is a constant in the x-direction.

Therefore, $d\dot{M}^T/dx_1 = 0$. Hence from these considerations, a differential equation for the pressure results:

$$\frac{d}{dx_1}\left(-2hw\left(\frac{h^2P}{3\mu RT}+\frac{\mu}{P}\right)\frac{dP}{dx_1}\right)=0$$

Carrying out the differentiation with respect to x_1 yields:

$$2hw\left[\frac{dP}{dx_1}\left(\frac{h^2}{3\mu RT}\frac{dP}{dx_1}-\frac{\mu}{P^2}\frac{dP}{dx_1}\right) + \frac{d^2P}{dx_1^2}\left(\frac{h^2P}{3\mu RT}+\frac{\mu}{P}\right)\right]=0$$

A further simplification and some rearrangements of the terms yield the following equation:

$$\frac{d^2P}{dx_1^2}\left(\frac{2wh^3P}{3\mu RT}+\frac{2hw\mu}{P}\right)+ + \left(\frac{dP}{dx_1}\right)^2\left(\frac{2wh^3}{3\mu RT}-\frac{2hw\mu}{P^2}\right)=0 \qquad (3.5)$$

Equation (3.5) is the governing equation for the pressure profile in the fluid along the flow direction of the channel with boundary conditions P_{in} and P_{out} specified according to the experimental conditions. To yield the required pressure information, equation (3.5) was solved in this work using a Runge-Kutta method and the results were validated against data available in the literature.

3.2 Characteristic Pressure of Micro-Channel Flow

Equation (3.4) describes the total mass flow rate through any cross-section of a micro-channel. As mentioned earlier, it consists of two terms. The first term on the right-hand side represents the convective mass flux and the second term represents the diffusive mass flux. Hence equation (3.4) can be rewritten by taking the term for the diffusive mass flux out of the brackets, yielding an expression for $\dot{M}^T$ in following form:

$$\dot{M}^T = \underbrace{-2hw\frac{\mu}{P}\frac{dP}{dx}}_{\dot{M}^D}\left(\frac{h^2P^2}{3\mu^2RT}+1\right) \qquad (3.6)$$

Further, by rearranging the terms, we obtain:

$$\frac{\dot{M}^D}{\dot{M}^T} = \frac{1}{\frac{h^2P^2}{3\mu^2RT}+1} = \frac{1}{\left(\frac{P}{P_c}\right)^2+1} \qquad (3.7)$$

where the characteristics pressure P_c, as suggested by Adachi et al. (2010) was introduced

$$P_c = \frac{\mu\sqrt{3RT}}{h} \qquad (3.8)$$

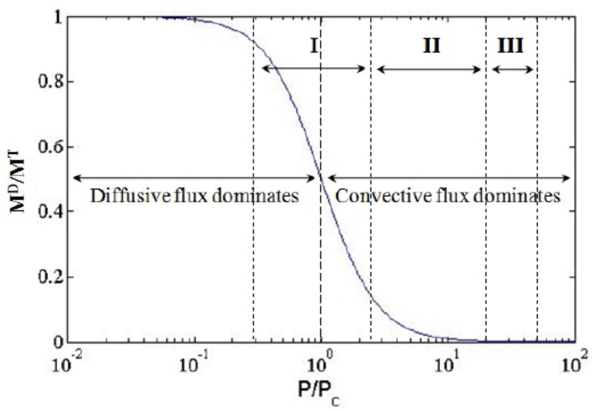


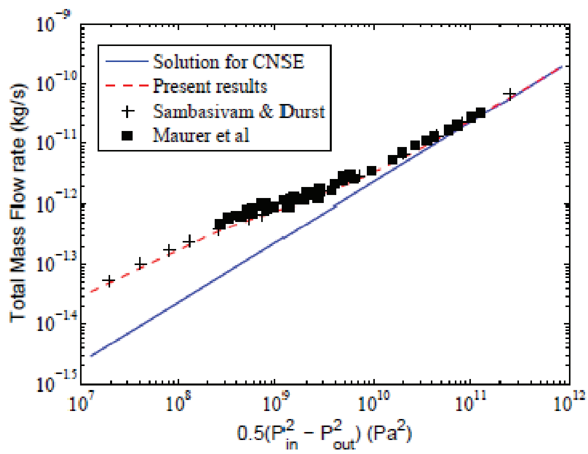
Fig. 1: *Fraction of the diffusive mass flux on the total mass flow with experimental ranges of I – Maurer et al. (2003), II – Arkilic et al. (1994) and III – Colin et al. (2004).*

From equation (3.7) we can see that for $P = P_c$ the diffusive mass flux is equal to 50% of the total mass flux. This means that in this case the convective and the diffusive parts are of the same magnitude. With decreasing pressure P, along the flow direction of a micro-channel, first the convective mass flux dominates the total mass flux (for $P > P_c$) and after reaching the characteristic pressure (for $P < P_c$) the diffusive part starts to dominate the total mass flux.

This is sketched in Figure 1, which shows the ratio M^D/M^T as a function of P/P_c. The figure also contains the pressure ranges of the experimental data of Maurer et al. (2003), Arkilic et al. (1994) and Colin et al. (2004). As mentioned earlier, Maurer et al. (2003) obtained significant deviations from the classical theory, which can now be explained by the strong diffusion effects in the pressure range of their experiments. In the pressure range of Arkilic et al. (1994) the diffusion effects are present but they are quite small whereas in the case of Colin et al. (2004) there are no diffusion effects at all and therefore good agreement between experimental results and the classical channel flow data must exist.

4. RESULTS AND COMPARISONS WITH EXPERIMENTS

As already mentioned, equation (3.5) was solved in the current work semi-analytically using a fourth order Runge-Kutta method. Pressure and pressure gradient profiles obtained as the solution were subsequently used to calculate the total mass flow rate, given by equation (3.4), for different inlet and outlet pressure values. In Figure 2, experimental results from Maurer et al. (2003) for total mass flow rates are shown together with the semi-analytical solutions obtained in this work using ENSE and analytical solutions using CNSE. In this case, the abscissa is a product of the mean pressure $P_{avg} = 0.5\,(P_{in} + P_{out})$ and the pressure difference $\Delta P = (P_{in} - P_{out})$. It can be seen that there is good agreement between the experimental data and the solution based on ENSE over the whole range of pressure ratios. Good agreement can also be seen between the present total mass flow rate and the corresponding data of Sambasivam & Durst (2010). In comparison with the ENSE solution, the results based on the classical NS equations shows closer agreement only for higher differences between the inlet and outlet pressures. It is important to realize that the comparison in Figure 2 is a good basis for the conclusion that the present treatment of micro-channel flow is physically correct for the computed mass flow rate, the corresponding velocity profile and the pressure.



<u>Fig. 2:</u> T*otal mass flow rate for semi-analytical (present results) and numerical (Sambasivam & Durst (2010)) solution of ENSE and the experimental data from Maurer et al. (2003), plotted against* $P_{avg}\Delta P$.

In Figure 3 the local mass flow rate is presented showing its composition of the convective and diffusive parts. The pressure along the micro-channel is also presented normalized by the characteristic pressure. In the graph below, i.e. in Figure 3 the development of the axial velocity profile is presented and again the normalized pressure along the flow direction is given to show how the velocity develops in the flow direction.

For better visualization and comparison with corresponding pressure and pressure gradient profiles, distributions of the diffusive and the convective velocity along a micro-channel are illustrated in Figures 5(a) and 5(b), respectively. Comparing Figures 5(b) and 5(d), one can easily see that the maximum of the convective velocity occurs at the same axial position as the minimum of the pressure gradient, as already mentioned.

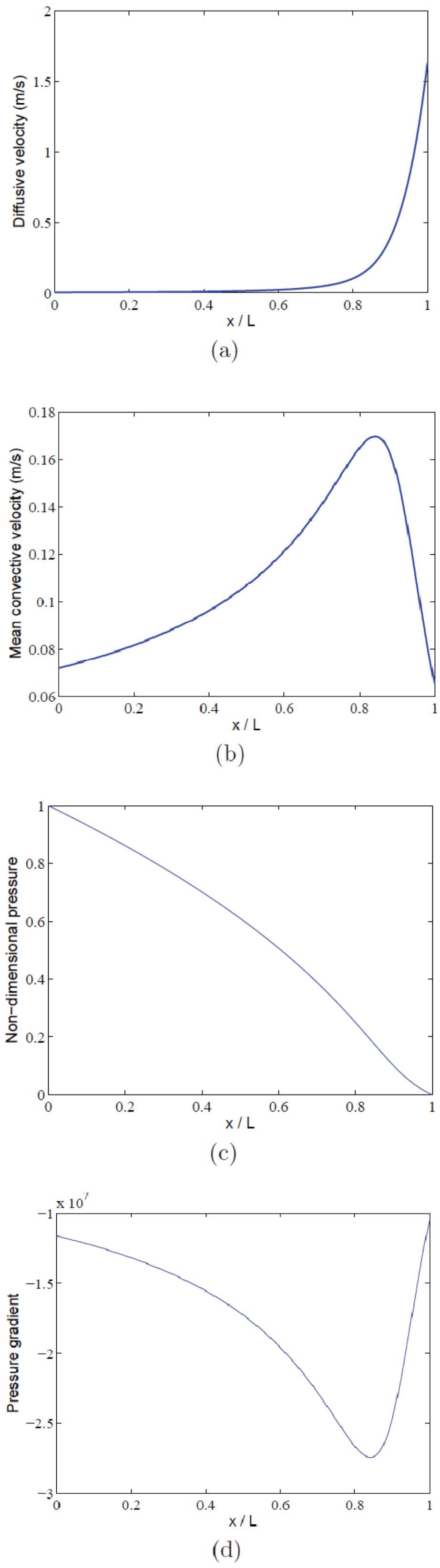


Fig. 5: *Diffusive velocity (a), mean convective velocity (b), pressure (c) and pressure gradient (d): axial profiles for a micro-channel with* $P_{in} = 4.5\ P_C$, $P_{out} = 0.2\ P_C$ *and* $P_c = 41484.5\ Pa$.

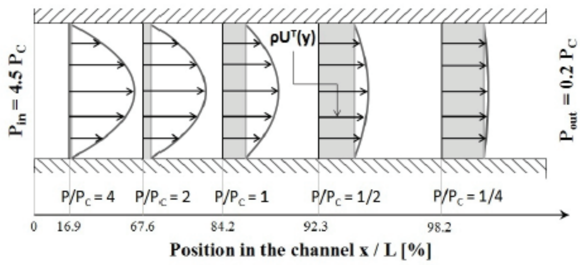


Fig. 3: *Mass flow rate in the micro-channel represented by* $\rho U^T(x_2) = \rho U^D(x_2) + \rho U^C(x_2)$*:* $\rho U^T(y)$*- black arrows,* ρU^D*- grey colored*

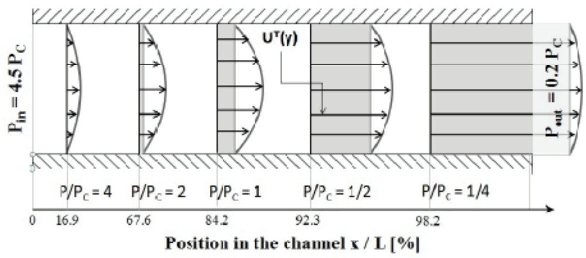


Fig. 4: *Corresponding velocity profiles along the micro-channel (compare figure 3).*

For the diffusive velocity, shown in Figure 5(a), one can note its considerable increase close to the outlet of the micro-channel for the given inlet and outlet pressures. In the same region, there is a change of the curvature of the pressure profile (Figure 5(c)) due to the minimum in the pressure gradient (Figure 5(d)). As mentioned before, the minimum in the pressure gradient is reached at the characteristic pressure P_c. At this point the diffusive mass flux is equal to the convective mass flux and the diffusive effects start to dominate the flow through the micro-channel further downstream. This behavior was already discussed and explained in Figure 1.

Figure 5 makes clear that it is very difficult to foresee the fluid and flow property variation in micro-channel flows, without the theoretical treatment of the flow, either analytically or numerically. Hence, the present treatment provides a sound basis for understanding of flows through micro-channels.

5. CONCLUSIONS AND FINAL REMARKS

For micro-channel flows the local velocity profile depends on the local pressure and also on the local pressure gradient in the flow direction. Usually, this information is not known, since only the inlet and the outlet pressures are given. For this reason, in the present paper, an ordinary differential equation for the pressure is derived and it is solved with the help of a fourth-order accurate Runge-Kutta integration method. Through this, the pressure in the flow direction in the micro-channel and the corresponding pressure gradient are known at every location in the flow for given inlet and outlet pressures. Hence the gas flows in micro-channels can be treated semi-analytically for the pressure distribution and the corresponding velocity profile at every axial location in the channel.

The results presented in this paper were obtained by solving the extended Navier-Stokes-Equations for micro-channel flows without any assumption of slip velocities at the channel walls. The slip velocity comes out as part of the solution for the velocity profile. To confirm the validity of the chosen solution approach, comparisons of various data, obtained by solving the derived semi-analytical solution, are made with corresponding experimental and numerical results available in the literature. Very good agreement is obtained. This confirms that the slip velocity obtained in the micro-channel flows is not due to Maxwellian interactions of the molecules with the wall but is due to a pressure-driven diffusive mass flux building up in the channel, in addition to the convective mass flux of the flow.

Utilizing the derived equations, the authors were able to show that the micro-channel flow possesses a characteristic pressure. At such a characteristic pressure, the flow properties such as pressure, pressure gradient, mass flow rate, etc. start to behave differently. Using the analytical solution for the velocity and the semi-analytical solution for the pressure, the authors were able to fix the value of the characteristic pressure.

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