# The Effect of Flow Coefficient on the Design of Miniature Centrifugal Impeller

Guiqin LIU [1*], Weng-Kong CHAN [1]
* Corresponding author: Tel: 0065-83053002; Email: liug0007@ntu.edu.sg
1: School of Mechanical and Aerospace Engineering, Nanyang Technological University, SG, 639798

**Abstract** A systematic and simple design methodology of miniature centrifugal impeller is proposed. In the design of miniature centrifugal impeller, the flow coefficient $\phi_1$ plays a significant role. Theoretically, the geometric parameters, including inner radius, blade angles and blade height can be expressed as functions of the flow coefficient. Accordingly, the theoretical head and energy losses are also influenced by the flow coefficient. To investigate the effect of flow coefficient, a series of miniature impellers are designed for different flow coefficients, and CFD simulations are conducted. Both theoretical analysis and CFD simulations show similar trends. Initially, the pressure generated increases with increasing flow coefficient. Upon reaching a maximum, it will subsequently decrease with increasing flow coefficient. Hence, an optimal flow coefficient should be chosen to achieve the best performance. From the theoretical results, the maximum pressure generated occurs when the flow coefficient is approximately 2.8, while for CFD, it is approximately 1.3. The difference between the theoretical analysis and CFD simulation shows that the theoretical model should be further improved to enhance its accuracy.

**Keywords:** Miniature centrifugal impeller design, flow coefficient

## Introduction

Miniature pumps have many potential applications, such as in the pharmaceutical, electronic cooling and medical industries because of their size. Miniature centrifugal pump generates higher pressure head and can be more accurately controlled than other miniature pumps. However, the design approach of miniature centrifugal pump is still not well established and most published papers are mainly based on empirical data.

For miniature pumps, the Reynolds number is small, which will cause a significant difference in performance.

In the present centrifugal pump design method, the flow coefficient $\phi_1$ is a critical parameter, which is usually given empirically or selected from empirical charts. The impeller geometrical parameters can be derived based on a selected value of flow coefficient. In the same way, it can also play an important role in the design of miniature impellers, but the effect of flow coefficient may be different because of the size involved. The energy model should be examined to determine the effect of flow coefficient on pump performance.

## Impeller design

In the design of miniature pump impeller, the macro-size pump design method is used. But the value of the flow coefficient is not selected from empirical chart, since it is of interest to determine the effect of flow coefficient. To minimize fabrication complications, the shroud and shaft at the inlet side are eliminated and the eye radius is assumed to be the same as the inner radius. The blade is 2-D (meridional direction and axial direction) with constant blade height and angle. It is assumed that the inlet flow follows the blade profile perfectly. For the ease of fabrication, a semi-open impeller is selected.

The design requirements are given as follows:

Flow rate: $Q$=1 L/min

Outer diameter: $D_2$=10 mm

The required rotating speed $\omega$ can be obtained from the "Cordier" diagram(Balje, 1981). For centrifugal pumps, the specific speed $n_s$ can be selected as 1 and the specific diameter $d_s$ can be selected as 3. The speed $\omega$ and expected head $H$ can be calculated from the $n_s$ and $d_s$ equations as follows:

$$d_s = \left(D_2 \left(gH\right)^{1/4}\right) \Big/ \sqrt{Q} \tag{1}$$

$$n_s = \omega\sqrt{Q}/(gH)^{3/4} \quad (2)$$

Rotational speed: $\omega$=450 rad/s
Pump head: $H$=2254 Pa

The inlet radius can be determined from the definition of the eye and inlet flow coefficients, which are expressed as:

$$\phi_e = \frac{V_{e1}}{U_e} = \frac{Q}{\pi\omega R_e^3} \quad (3)$$

$$\phi_1 = \frac{V_{m1}}{U_1} = \frac{Q}{R_1 b_1 \omega(2\pi R_1 - Zt/\sin\beta_1)} \quad (4)$$

Generally, the flow coefficient is defined as the ratio of radial velocity and the tip speed, which indicates the relationship between the average flow rate and the geometrical shape. The eye flow coefficient is the ratio of inlet velocity and the eye radius rotational speed. A flow rate needs a suitable eye radius to handle it, so the eye flow coefficient should be selected according to some charts. The eye flow coefficient is equal to the inlet flow coefficient based on mass conservation. Hence:

$$R_1 = R_e = \sqrt[3]{\frac{Q}{\pi\omega\phi_1}} \quad (5)$$

$$b_1 = \frac{\tau_1}{2} R_1 = \frac{\tau_1}{2}\sqrt[3]{\frac{Q}{\pi\omega\phi_1}} \quad (6)$$

Where $\tau_1$ is the blade blockage factor and is defined as:

$$\tau_1 = \frac{2\pi R_1}{2\pi R_1 - \frac{Zt}{\sin\beta_1}} \quad (7)$$

From the physical meaning and definition of $\phi_1$, the inlet blade angle is defined as:

$$\beta_1 = \beta_{f1} = \tan^{-1}\phi_1 \quad (8)$$

The outlet parameters are given as:

$$b_2 = b_1 = b \quad (9)$$

$$\beta_2 = \beta_1 = \beta \quad (10)$$

The parameters listed above are expressed in terms of the inlet flow coefficient $\phi_1$, flow rate $Q$, and rotational speed $\omega$. For other parameters, empirical selection based on macro-size pump design approach is applied. The blade thickness $t$ is given by $t$=0.04$R_2$. The clearance gap is selected as 0.2mm, and the number of blades is 5.

For a given flow coefficient, the inner radius $r_1$, blade height $b$ and angle $\beta$ are unique, as illustrated in Fig. 1. If the impeller geometry is determined, the energy transferred from the rotating blade to fluid, and energy losses can be obtained. The effect of the flow coefficient on the impeller design using the energy model analysis can be derived accordingly.

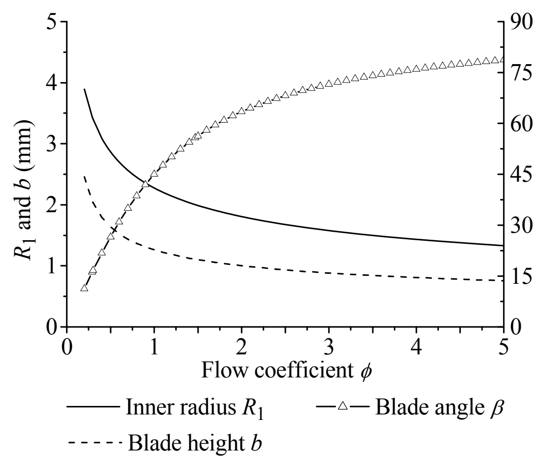


**Fig. 1.** Geometrical variables flow coefficient

As the flow coefficient increases, the inner radius $R_1$ and the blade height $b$ decrease, and the blade angle $\beta$ increases, but all these geometrical variables approach a constant value. A large flow coefficient means a relatively large radial velocity and a small tip speed. For a given flow rate, a large radial velocity implies a small inner radius, and a small blade height.

Table 1 Simulation models and pressure rise

| No. | $\phi_1$ | $r_1$(mm) | $b$(mm) | $\beta$(°) | $\Delta P$(Pa) |
|---|---|---|---|---|---|
| 1 | 3.49 | 1.50 | 0.84 | 74.0 | 624 |
| 2 | 2.99 | 1.58 | 0.88 | 71.5 | 638 |
| 3 | 2.49 | 1.68 | 0.93 | 68.1 | 740 |
| 4 | 1.99 | 1.81 | 1.00 | 63.3 | 797 |
| 5 | 1.47 | 2.00 | 1.11 | 55.8 | 829 |
| 6 | 1.41 | 2.03 | 1.12 | 54.6 | 803 |
| 7 | 1.29 | 2.09 | 1.15 | 52.3 | 782 |
| 8 | 0.90 | 2.36 | 1.31 | 41.9 | 638 |
| 9 | 0.75 | 2.50 | 1.34 | 37.0 | 586 |
| 10 | 0.44 | 3.00 | 1.72 | 23.6 | 174 |

For theoretical analysis, the flow coefficient is varied from 0.2 to 5. For CFD simulations, ten impeller models are designed for different values of flow coefficient, and their geometrical parameters and simulated pressure rise are listed in Table 1.

### Theoretical head analysis

The theoretical energy transfer from the rotating blades to the fluid can be obtained based on 1-D flow analysis in the meridional direction. The theoretical head is the Euler's head, and it is defined as the rate of energy transfer per unit mass of fluid flowing in meters. It is expressed as:

$$E_{\mathrm{th}}=\frac{T\omega}{gQ}=\frac{1}{g}\left(V_{\mathrm{u2}}U_2-V_{\mathrm{u1}}U_1\right) \tag{11}$$

For centrifugal pumps, flow enters the impeller channels only in the radial direction, so the inlet circumferential velocity $V_{\mathrm{u1}}$ is zero. Hence, we have:

$$E_{\mathrm{th}}=\frac{1}{4g}D_2^{\,2}\omega^2-\frac{\omega Q}{2\pi g b_2}\cot\beta$$
$$=\frac{\omega^2}{g}\left(R_2^{\,2}-\left(\frac{Q}{\pi\phi_1\omega}\right)^{2/3}+\frac{Zt}{2\pi\sin\tan^{-1}\phi}\left(\frac{Q}{\pi\phi_1\omega}\right)^{1/3}\right) \tag{12}$$

### Slip loss

Since the number of blades in a centrifugal pump is finite, the fluid at the exit of the impeller channel is incompletely guided. This causes a phenomenon called slip which is an inviscid phenomenon. The slip velocity at the outer radius is illustrated in Fig. 2. Many researches show that the slip factor is mainly influenced by the blade angle and the number of blades (Bothmann and Reffstrup, 1983; Paeng and Chung, 2001; Stodola, 1927). For a miniature impeller, the inner radius has a significant effect on the flow field, so the radius ratio should be considered. Von Backstrom's (von Backstroem, 2006) equation is selected. The slip factor is:

$$\sigma_{\mathrm{s}}=1-\frac{V_{\mathrm{slip}}}{U_2}=1-\frac{1}{1+5\dfrac{\left(1-R_1/R_2\right)Z}{2\pi\left(\cos\beta\right)^{0.5}}} \tag{13}$$

The slip loss $E_{\mathrm{slip}}$ is:

$$E_{\mathrm{slip}}=\frac{U_2^{\,2}}{g\left(1+5\dfrac{\left(1-R_1/R_2\right)Z}{2\pi\left(\cos\beta\right)^{0.5}}\right)} \tag{14}$$

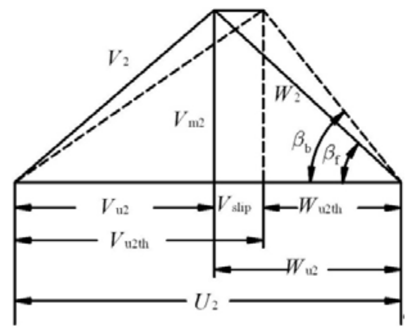


**Fig. 2.** Velocity triangle at impeller outlet

From equation (14) we can conclude that, the bigger the inner radius $R_1$ and the smaller the blade angle $\beta$, the bigger the slip loss. This phenomenon can be explained by the physical meaning of velocity slip. For impeller with smaller inner radius, the flow is not well guided along the blade, so slip is larger and the slip loss is bigger. As $R_1$ and $\beta$ are expressed as functions of $\phi_1$ (equations (5) and (8)), slip loss can also be expressed as a function of $\phi_1$. As $\phi_1$ increases, the slip loss will decrease.

### Leakage loss

The pressure difference between the impeller trailing edge and leading edge will cause leakage flow in the gap between the rotating impeller and stationary housing. Theoretical analysis of leakage flow were conducted by Chan (Chan et al., 2000) and Teo (Teo et al., 2010) based on simplified Navier-Stokes equations of leakage flow in cylindrical coordinates. For gaps of constant height, the tangential velocity and leakage flow rate are:

$$V_\theta=\frac{r\omega z}{\delta} \tag{15}$$

$$Q_{\mathrm{Lk}}=\frac{\pi\delta^3}{6\mu\ln\left(R_2/R_1\right)}\left(\Delta P-\frac{3\rho\omega^2\left(R_2^2-R_1^2\right)}{20}\right) \tag{16}$$

Where $\Delta P$ is the pressure difference between the trailing and leading edges.

The leakage flow rate is related to the gap size, the inner radius and outer radius. A larger leakage flow rate is due to a bigger gap size. The effect of the impeller inner radius and outer radius should be further researched, because $\Delta P$ is related to them.

## Disk friction loss

Fluid viscosity will cause shear stresses corresponding to the velocity gradient in the gap between the rotating disk and the stationary casing. The disk friction loss of a rotating disk is defined as(Gulich, 2010):

$$P_{\mathrm{Df}} = \frac{1}{2}\rho C_{\mathrm{m}}\omega^3 R_2^{\,5}\left(1-\left(\frac{R_1}{R_2}\right)^5\right) \quad (17)$$

Where $C_m$ is the friction coefficient. For the miniature pump, it can be expressed as(Daily and Nece, 1960):

$$C_{\mathrm{m}} = 1.85\left(\frac{\delta}{R_2}\right)^{1/10}\mathrm{Re}_{\mathrm{d}}^{\,-1/2} \quad (18)$$

The disk friction loss is:

$$E_{\mathrm{Df}} = \frac{P_{\mathrm{Df}}}{\rho g Q_{\mathrm{Lk}}} = \frac{1}{2}\frac{C_{\mathrm{m}}\omega^3 R_2^{\,5}}{g Q_{\mathrm{Lk}}}\left(1-\left(\frac{R_1}{R_2}\right)^5\right) \quad (19)$$

The wetted area between rotating disk and fluid and friction coefficient are the main factors affecting disk friction. The disk friction loss will be larger for a small inner radius $R_1$. As $\phi_1$ increases, the disk friction loss increases also.

## Tip clearance loss

For semi-open impellers, fluid flows from the pressure side to the suction side through the clearance between casing and the blades. Hence, the kinetic energy of the leakage flow is largely dissipated. Gap size, Reynolds number, velocity distribution will also influence the tip leakage loss. The pressure loss due to the tip clearance is mainly induced by the leakage flow through the tip (Senoo and Ishida, 1986).

The average relative velocity is:

$$\overline{W} = \frac{V}{\sin\beta} = \frac{Q}{2\pi r b \sin\beta} \quad (20)$$

The tip velocity from pressure side to suction side is:

$$V_{\mathrm{tip}} = \sqrt{2\left(P_{\mathrm{p}}-P_{\mathrm{s}}\right)/\rho} = \sqrt{2\overline{W}\left(W_{\mathrm{s}}-W_{\mathrm{p}}\right)} \quad (21)$$

$$W_{\mathrm{s}}-W_{\mathrm{p}} = \left(2\pi r\sin\beta/Z\right)\times \left(2\omega\frac{dr}{dm}-\frac{1}{\sigma}\left(\frac{\overline{W}_0}{r}\cos\beta\frac{dr}{dm}+\cos\beta\frac{d\overline{W}_0}{Dm}\right)\right) \quad (22)$$

So the tip velocity is:

$$V_{\mathrm{tip}} = \sqrt{\frac{4Q\omega\sin\beta}{bZ}} \quad (23)$$

Hence, the leakage flow rate is:

$$Q_{\mathrm{tip}} = Z\delta\left(R_2-R_1\right)\sqrt{\frac{4Q\omega}{bZ\sin\beta}} \quad (24)$$

So:

$$E_{\mathrm{tip}} = \frac{\omega Z\delta\left(R_2-R_1\right)\cot\beta}{\pi g b_2}\sqrt{\frac{Q\omega}{bZ\sin\beta}} \quad (25)$$

The tip clearance loss is mainly caused by the blade length and pressure difference between the pressure side and suction side. So a smaller inner radius will cause a higher tip loss. A bigger flow coefficient will have result in higher tip clearance loss.

## Impeller friction loss

Viscosity will cause friction loss near blade walls, and it can be approximated by using the standard pipe friction model(Tuzson, 2000). The impeller friction loss is:

$$E_{\mathrm{Imf}} = C_{\mathrm{f}}\frac{\left(R_2-R_1\right)\left(W_2+W_1\right)^2}{4gD_{\mathrm{h}}\sin\beta} \quad (26)$$

The default value of friction coefficient $C_f$ is 0.005. Hydraulic diameter and relative velocities are defined as:

$$D_{\mathrm{h}} = \frac{b\left(\pi D_2/Z\right)\sin\beta}{b+\left(\pi D_2/Z\right)\sin\beta},\quad W_1 = \frac{V_{\mathrm{m1}}}{\sin\beta},\quad W_2 = \frac{V_{\mathrm{m2}}}{\sin\beta}$$

So the friction loss can be expressed as:

$$E_{\mathrm{Imf}} = \frac{C_f\left(R_2-R_1\right)Q^2}{16g\pi^2 b^2\sin^3\beta}\left(\frac{1}{2\pi R_2\sin\beta/Z}+\frac{1}{b}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2 \quad (27)$$

Impeller friction loss is mainly influenced by the friction coefficient and wetted area between the fluid and impeller walls. So a smaller $R_1$ and longer impeller channel will cause higher friction loss. A larger flow coefficient will result in a higher impeller friction loss. The current model is inadequate as it does not consider flow separation losses in the impeller passage.

**Blockage loss**

Blades thickness will cause flow stagnation and flow separation near the blade leading edge, especially for miniature pumps. The flow near the blade leading edge can be treated as water flowing from a large pipe to a small pipe. The energy loss due to sudden contraction can be treated as blockage loss.

$$E_{\mathrm{Blk}} = \frac{K}{2g} V_1^2 \tag{28}$$

Where $V_1$ is velocity after the blockage, $K$ is loss coefficient.

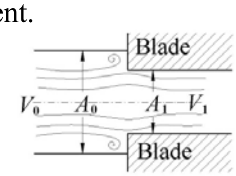


**Fig. 3.** The loss due to blade thickness blockage

For the miniature pump in this study, $D_1/D_2$ is between 1 to 1.22. A linear relationship can be selected between $K$ and $D_0/D_1$.

$$K = 0.45\left(\frac{D_0}{D_1} - 1\right) \tag{29}$$

So the blockage loss is:

$$E_{\mathrm{Blk}} = 0.45\left(\frac{D_0}{D_1} - 1\right)\frac{V_2^2}{2g}$$

$$= \frac{0.225}{g}\left(\frac{Q}{\left(2\pi r_1 - \frac{Zt}{\sin\beta}\right)b}\right)^2 \left(\sqrt{\frac{1}{1 - \frac{Zt}{2\pi r_1 \sin\beta}}} - 1\right) \tag{30}$$

With given blade thickness $t$, the smaller the inner radius $R_1$, the larger the diameter ratio, and the larger the blockage loss. A smaller flow coefficient will have a larger blockage loss.

**Diffusion loss**

When the fluid flows in the impeller channels, the expansion of the channels will cause a diffusion loss. The diffusion loss should be considered when the relative velocity ratio $W_1/W_2$ is larger than 1.4(Tuzson, 2000), and the loss expression is:

$$E_{\mathrm{Diff}} = \frac{0.25 W_1^2}{2g} \tag{31}$$

A small flow coefficient will have a big relative velocity, and hence a high diffusion loss.

**Numerical model**

To validate the theoretical results, ten selected models as in Table 1 are solved numerically using Fluent (version 6.3.26). The entire simulation model is divided into four zones; inlet, rotating impeller, gap and volute zones. Wedge-shaped mesh is selected for the rotating impeller zone, while hexahedron mesh is selected for other zones. The total meshes are approximately 1 million.

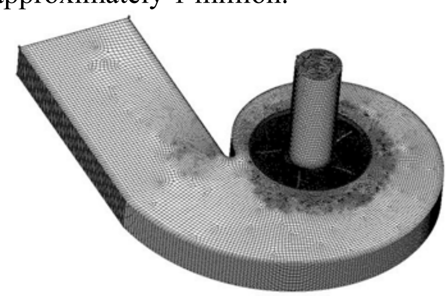

**Fig.4.** Volume meshes built in GAMBIT

Grid independency was investigated using models with meshes from 0.75 million to 1.25 million, and it was found that the result obtained by using 1 million is adequate.
The k-ω turbulent model is selected as the Reynolds number is approximately 45000, and second order spatial discretization is used in the iteration process.

**Results and discussions**

The theoretical pressure rise of the centrifugal impeller is:

$$E = E_{\mathrm{th}} - E_{\mathrm{slip}} - E_{\mathrm{Lk}} - E_{Df} - E_{\mathrm{tip}} - E_{\mathrm{Imf}} - E_{\mathrm{Blk}} - E_{\mathrm{Diff}}$$

The energy losses are displayed in Fig. 4 and 5, and the Euler head, energy losses, theoretical pressure and CFD simulation results are illustrated in Fig.5-7.

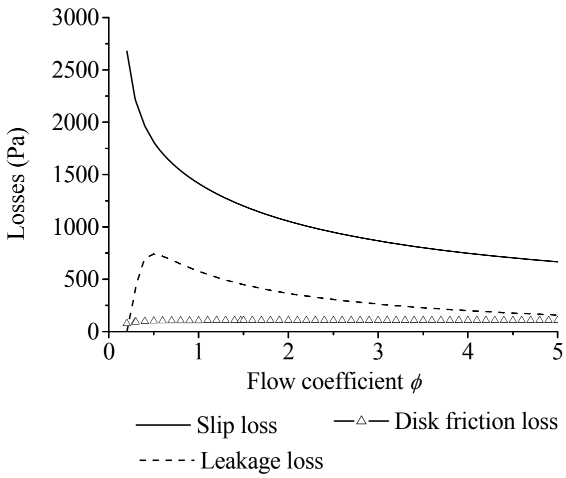


**Fig. 5.** The influence of the flow coefficient on slip loss, leakage loss and disk friction loss

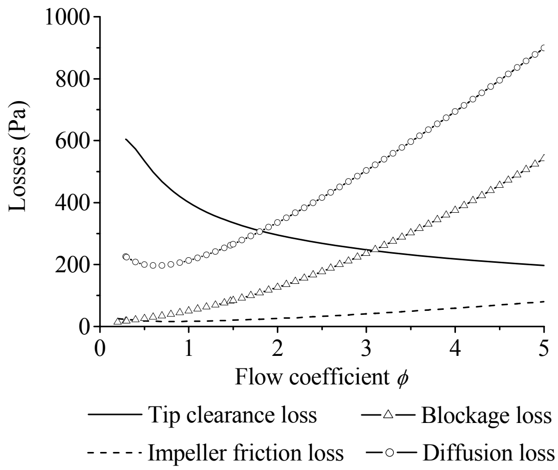


**Fig. 6.** The influence of the flow coefficient on tip clearance loss, impeller friction loss, blockage loss and disk friction loss

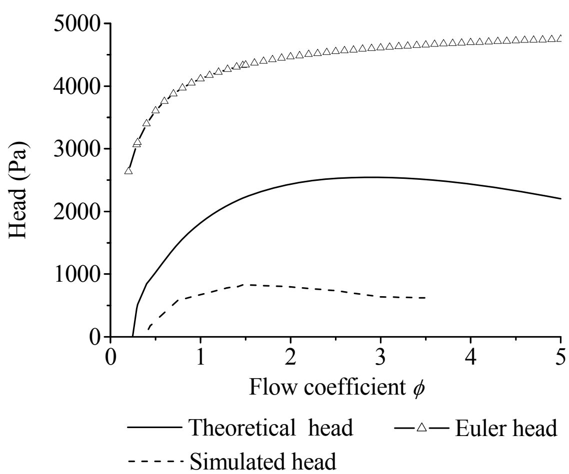


**Fig. 7.** The influence of the flow coefficient on theoretical and simulated pressure rise

The trend of the analytical and simulated head is similar. The pressure generated increases with increasing flow coefficient. Upon reaching a maximum, it will subsequently decrease with increasing flow coefficient. However, the value of flow coefficient corresponding maximum analytical and simulated head is different. The maximum analytical pressure occurs when the flow coefficient is approximately 2.8, while for CFD, it is approximately 1.3. This may be due to that both the inlet and volute losses are not included. Refinements to this model are necessary, but the current model indicates that an optimized value of flow coefficient exists to achieve best pump performance.

## Conclusions

This paper presents a systematic and effective analysis of the impeller design and energy models. The effect of the flow coefficient is investigated in detail. Key geometrical parameters and energy models can be expressed as functions of the flow coefficient. An optimized value of flow coefficient exists from CFD results and the analysis of Euler head and energy loss models.

## Nomenclature

| | | |
|---|---|---|
| $b$ | blade height | [mm] |
| $d_s$ | specific diameter | |
| $g$ | gravity acceleration | [m/s$^2$] |
| $l$ | blade length | [m] |
| $n_s$ | specific speed | |
| $t$ | blade thickness | [m] |
| $C_f$ | friction coefficient of impeller | |
| $C_m$ | friction coefficient of disk | |
| $D$ | diameter | [m] |
| $D_r$ | hydraulic diameter of impeller | [m] |
| $E$ | head loss | [Pa] |
| $E_{th}$ | Euler head | [Pa] |
| $H$ | expected head | [Pa] |
| $K$ | loss factor of blockage loss | |
| $P$ | power | [W] |
| $Q$ | flow rate at design point | [m$^3$/s] |
| $R$ | radius | [m] |

| | | |
|---|---|---|
| Re | Reynolds number $\mathrm{Re}=\rho\omega D_2^2/\mu$ | |
| $\mathrm{Re}_d$ | disk Reynolds number $\mathrm{Re}_d=\rho\omega R_2^2/\mu$ | |
| $U$ | circumferential velocity | [m/s] |
| $V$ | absolute velocity | [m/s] |
| $V_0$ | impeller inlet velocity | [m/s] |
| $W$ | relative velocity | [m/s] |
| $Z$ | number of blades | |
| $\beta$ | blade angle | [°] |
| $\beta_f$ | flow angle | [°] |
| $\delta$ | gap size | [m] |
| $\mu$ | water viscosity | [Pa] |
| $\rho$ | water density | [$m^3$/s] |
| $\sigma_s$ | slip factor | |
| $\tau$ | blockage factor | |
| $\phi$ | flow coefficient | |
| $\omega$ | rotational speed | [rad/s] |

## Subscripts, superscripts and abbreviations

| | |
|---|---|
| e | impeller eye |
| i | impeller inlet |
| p | pressure side |
| s | suction side |
| tip | gap between semi-open blade and housing |
| u | circumferential component |
| Blk | blockage loss |
| Df | disk friction |
| Diff | diffusion loss |
| Imf | impeller friction |
| Lk | gap leakage between disk and housing |
| θ | circumferential component of gap flow |
| 1 | impeller blade leading edge |
| 2 | impeller blade trailing edge |

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