SHEAR-WAVE REFLECTION MOVEOUT FOR AZIMUTHALLY ANISOTROPIC MEDIA

AbdulFattah Al-Dajani and M. Nafi Toksöz

Earth Resources Laboratory
Department of Earth, Atmospheric, and Planetary Sciences
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

The presence of azimuthal anisotropy causes shear wave propagation to split into fast and slow shear waves. The most common azimuthally anisotropic models used to describe fractured reservoirs are transverse isotropy with a horizontal axis of symmetry (HTI), and orthorhombic. In this paper, we study shear-wave reflection moveout in azimuthally anisotropic media with special attention paid to orthorhombic media with horizontal interfaces. In such cases the shear-wave reflection moveout is azimuthally variant and nonhyperbolic. We analyze the azimuthal dependence of normal moveout (NMO) velocity and we validate the accuracy of the conventional hyperbolic moveout equation. The azimuthal variation of NMO velocity is elliptical for both wave modes. In the presence of anisotropy-induced, nonhyperbolic moveout (NHMO), the hyperbolic moveout equation loses its accuracy with increasing offset (e.g., offset-to-depth ratio > 1). To study the azimuthal behavior of the NHMO for shear-wave reflections, we introduce an analytic representation for the quartic coefficient of the Taylor's series expansion of the two-way traveltime. In an orthorhombic medium the quartic coefficient for shear-wave reflections has a relatively simple form, especially in comparison to P-wave. The reflection moveout for each shear-wave mode in a homogeneous orthorhombic medium is purely hyperbolic in the direction normal to the polarization. The nonhyperbolic portion of the moveout, on the other hand, reaches its maximum along the polarization direction, and it reduces rapidly away from the direction of polarization. As a result, the anisotropy-induced, nonhyperbolic reflection moveout is significant in the vicinity of the polarization directions (e.g., $\pm 30^\circ$ and for large offset-to-depth ratios). The implementation of the NHMO equation and the utilization of the moveout coefficients allow for not only enhanced seismic imaging but also provide the link between seismic signatures and medium parameters.

INTRODUCTION

Reflection moveout is sensitive to the presence of azimuthal anisotropy. Transverse isotropy with a horizontal axis of symmetry (HTI) is the simplest azimuthally anisotropic model caused by vertical penny-shaped cracks embedded in an isotropic matrix. The orthorhombic model, on the other hand, is a better representative of a wide class of fractured reservoirs (e.g., orthogonal fracture systems in an isotropic matrix and a vertical fracture system in a transverse isotropic matrix such as shale). As a result, we give special attention to orthorhombic media. Both models are characterized by three orthogonal symmetry planes: (x_1, x_3), (x_2, x_3), and (x_1, x_2) given in a Cartesian coordinate system. We assume that the symmetry planes coincide with the coordinate system, and the (x_1, x_2) plane (i.e., the reflector) is horizontal. Figure 1 shows an example of an orthorhombic model. The orthorhombic symmetry, the focus of this paper, is defined through nine stiffnesses of the fourth-ranked stiffness tensor c_{ijkl} as: $c_{11}, c_{22}, c_{33}, c_{44}, c_{55}, c_{66}, c_{12}, c_{13}$, and c_{23}. Here, the stiffnesses c_{ijkl} are normalized by the medium density, ρ.

The presence of azimuthal anisotropy causes shear-wave propagation to split into fast and slow shear waves. Assuming a medium with a horizontal interface, the interest of our study, the particles displacements for the splitted shear waves are perpendicular to the propagation direction and are polarized parallel to the vertical symmetry planes. It is obvious that seismic surveys with a multicomponent source and receiver generate two perpendicularly polarized shear waves. In our notation, the two shear waves are called $S1$ and $S2$, where in a Cartesian coordinate system, $S1$ is polarized perpendicular to the (x_2, x_3) plane (i.e., parallel to the x_1 axis), and $S2$ is polarized perpendicular to the (x_1, x_3) plane (i.e., parallel to the x_2 axis).

Some studies on the kinematics of shear wave reflection moveout in azimuthally anisotropic media have been limited to zero-to-short offsets and weak anisotropy approximations (e.g., Sena, 1991; Li and Crampin, 1993). Other studies discuss the amplitude (i.e., energy) differences between the splitted shear waves (e.g., Thomsen, 1988). Key studies related to this paper include those by Thomsen (1986), Tsvankin and Thomsen (1994), Dix (1955), Tsvankin (1997a,b), Grechka and Tsvankin (1998), Al-Dajani and Tsvankin (1998), and Al-Dajani and Toksöz (1999).

Reflection moveout for P-wave propagation has been discussed in detail by Al-Dajani and Toksöz (1999). In this study, we focus our attention on shear-wave reflection moveout in azimuthally anisotropic media with orthorhombic symmetry and horizontal interfaces. Our goal is to develop analytical insights into the azimuthal behavior of shear-wave reflection moveout in common midpoint (CMP) gathers. We evaluate the NMO velocity equation of Grechka and Tsvankin (1998) for shear-wave propagation. As we should expect, the conventional hyperbolic normal moveout (NMO) equation parameterized by the stacking (NMO) velocity loses accuracy with increasing spreadlength due to the anisotropy-induced, nonhyperbolic moveout. As a result, we study the longspread nonhyperbolic reflection moveout (NHMO) for shear-wave propagation in azimuthally

anisotropic media. We introduce the exact representation for the quartic coefficient for shear-wave propagation and analyze the azimuthal behavior of the nonhyperbolic reflection moevout. Our study is valid for arbitrary strength of anisotropy. Ray-traced synthetic data are used to support this work.

ANALYTIC APPROXIMATIONS OF REFLECTION MOVEOUT

Figure 2 shows an example of an exact "ray-traced" reflection moveout for (a) an $S1$-wave and (b) an $S2$-wave propagation in an orthorhombic medium with a horizontal reflector at a depth of 1 km. The reflections are generated using a 3-D ray tracing code which is developed for 3-D laterally inhomogeneous anisotropic media (Gajewski and Pšenčik, 1987). The curves correspond to CMP azimuths 0°, 30°, 45°, 60°, and 90° (see Figure 3 for geometry and model parameters). The traveltime curves are displayed as a function of offset-to-reflector-depth (X/D) ratio. It is obvious that the reflection moveout for both shear waves is azimuthally variant.

The Normal Moveout (NMO) "Hyperbolic" Equation

Reflection moveout in common-midpoint (CMP) gathers is conventionally approximated by the hyperbolic equation:

$$t^2 = t_0^2 + A_2 x^2 \,, \tag{1}$$

where t is the reflection traveltime at source-receiver offset x, t_0 is the two-way zero-offset traveltime, and the quadratic moveout coefficient $A_2 = 1/V_{\rm nmo}^2$. $V_{\rm nmo}$ is the normal-moveout (NMO) velocity defined in the zero-spread limit.

The Normal Moveout (NMO) Velocity

To obtain the moveout coefficients for any (arbitrary) model, we express the two-way traveltime of any pure (non-converted) reflected mode as a double Taylor's series expansion in the vicinity of the zero-offset point. Keeping only the quartic and lower-order terms of the two-way traveltime squared, we obtain the quadratic (A_2) and quartic (A_4) coefficients for pure mode reflection in a homogeneous arbitrary anisotropic layer. The exact derivation is discussed in detail by Al-Dajani and Toksöz (1999).

The quadratic moveout coefficient A_2 (or the NMO velocity) in a single arbitrary anisotropic layer was introduced by Grechka and Tsvankin (1998) for pure mode propagation and arbitrary strength of anisotropy as an ellipse. After recasting, it is given as:

$$A_2(\alpha) \;=\; A_2^{(1)} \sin^2\alpha + A_2^{(2)} \cos^2 + A_2^{(x)} \sin\alpha\cos\alpha \,, \tag{2}$$

where the superscripts (1) and (2) indicate directions along the vertical planes (x_2, x_3) and (x_1, x_3), respectively. α is the azimuth of the CMP line from one of the vertical

planes (i.e., the (x_1, x_3) plane). $A_2^{(x)}$ is a cross-term which absorbs the mutual influence of all principal planes. For any horizontal, azimuthally anisotropic medium with a horizontal symmetry plane (e.g., HTI, orthorhombic, and monoclinic), $A_2^{(x)} = 0$ and equation (2) reduces to, after recasting

$$V^2_{\text{nmo}}(\alpha) = \frac{V^2_{\text{nmo},1} V^2_{\text{nmo},2}}{V^2_{\text{nmo},1} \cos^2\alpha + V^2_{\text{nmo},2} \sin^2\alpha}, \tag{3}$$

where the semi-axes of the NMO ellipse: $V_{\text{nmo},1}$ and $V_{\text{nmo},2}$ are the NMO velocities along the two vertical symmetry planes, (x_2, x_3) and (x_1, x_3), respectively. α is the azimuth of the CMP gather relative to one of the symmetry planes (e.g., the positive x_1-axis of the (x_1, x_3) plane). Here, we assume that the coordinate system coincides with the principal planes.

The NMO velocity for shear-wave propagation in an HTI medium is given in Tsvankin (1997a). Following the suggestion of Grechka and Tsvankin (1998) for P-wave propagation, the components of the NMO velocity for shear-wave propagation in an orthorhombic medium are obtained. They are given as:

- For $S1$-wave propagation:

$$V^2_{\text{nmo},1} = c66$$

$$V^2_{\text{nmo},2} = \frac{(c13^2 + 2c13c55 + c55^2 + c11(-c33 + c55))}{(-c33 + c55)}.$$

  In Tsvankin's (1997b) notation: $V^2_{\text{nmo},1} = V^2_{\text{S0}_2}(2\gamma^{(1)}+1)$, while $V^2_{\text{nmo},2} = V^2_{\text{S0}_2}(2\sigma^{(2)}+1)$.

- For $S2$-wave propagation:

$$V^2_{\text{nmo},1} = \frac{(c23^2 + 2c23c44 + c44^2 + c22(-c33 + c44))}{(-c33 + c44)}$$

$$V^2_{\text{nmo},2} = c66.$$

  Similarly, in Tsvankin's (1997b) notation: $V^2_{\text{nmo},1} = V^2_{\text{S0}_1}(2\sigma^{(1)}+1)$, while $V^2_{\text{nmo},2} = V^2_{\text{S0}_1}(2\gamma^{(2)}+1)$.

Note that V_{S0_2} is equivalent to Tsvankin's (1997b) V_{S0}, while $\sigma^{(i)} = (\epsilon^{(i)} - \delta^{(i)})V^2_{\text{P0}}/V^2_{\text{S0}_i}$, where $i = 1, 2$. V_{P0}, $\epsilon^{(i)}$, $\delta^{(i)}$, $\gamma^{(i)}$ are Tsvankin's (1997b) parameters for orthorhombic media, defined analogously to Thomsen's parameters for VTI. V_{S0_1} and V_{S0_2} are two vertical velocities for shear-wave propagation. It is clear that $V^2_{\text{S0}_1}(2\gamma^{(2)}+1) = V^2_{\text{S0}_2}(2\gamma^{(1)}+$

1). Note that the shear-wave splitting parameter, $\gamma^{(s)} \equiv (V_{S0_1}^2 - V_{S0_2}^2)/(2V_{S0_2}^2) = (\gamma^{(1)} - \gamma^{(2)})/(1 + 2\gamma^{(2)})$ (Tsvankin, 1997b).

Figure 4 shows a schematic diagram for the azimuthal variation of the quadratic coefficient A_2 and its inverse (the square of the NMO velocity). Note that Figures 4a,b correspond to the case of an arbitrary medium, while Figures 4c,d correspond to an azimuthally anisotropic medium with horizontal reflector and symmetry planes (e.g., HTI and orthorhombic). As seen in Figures 4b,d, the azimuthal variation of the NMO velocity in azimuthally anisotropic media is elliptical.

Next, let us verify the accuracy of the analytical representation of the NMO velocity (equation (3)) for $S1$- and $S2$-wave types for an orthorhombic medium. We also evaluate the hyperbolic moveout equation (1). The moveout velocity on finite spreads can be obtained by least-squares fitting of a hyperbolic moveout equation to the calculated traveltimes, i.e.,

$$V_{\rm mo}^2 = \frac{\sum_{j=1}^{N} x_j^2}{\sum_{j=1}^{N} t_j^2 - N\, t_0^2}, \tag{4}$$

where x_j is the offset of the j-th trace, t_j is the corresponding two-way reflection traveltime, t_0 is the two-way vertical traveltime, and N is the number of traces.

As seen in Figure 5, the moveout velocity obtained from the exact traveltimes using equation (4) in general coincides with the analytic NMO value (compare Figure 5 with Figure 6). Surprisingly, this is true even at large offset-to-depth (X/D) ratios and for most azimuth ranges. It is obvious that the conventional NMO equation (1), which is parameterized by the azimuthally-dependent NMO velocity (equations (2) and (3)), accurately represents the reflection moveout for shear-wave propagation (especially for conventional spreadlengths (X/D $\leq$ 1)). Interestingly, the anisotropy-induced deviations of the moveout curve from a hyperbola, a phenomenon which is well-known for P-wave propagation in azimuthally anisotropic media (Al-Dajani and Tsvankin, 1998; Al-Dajani and Toksöz, 1999), is rather different in the case of shear-wave propagation in orthorhombic media. To understand this difference we need to look at the nonhyperbolic portion of the reflection moveout for shear-wave propagation.

The Nonhyperbolic Moveout (NHMO) Equation

The reflection moveout for pure shear wave propagation in azimuthally anisotropic media is nonhyperbolic (except for the cases where we have elliptical anisotropy and/or fast shear-wave reflection moveout in HTI media). The nonhyperbolic moveout (NHMO) equation originally developed by Tsvankin and Thomsen (1994) for VTI media is used as a basis for our study:

$$t^2 = t_0^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A x^2}, \tag{5}$$

where A_4 is the quartic coefficient of the Taylor series expansion for $t^2(x^2)$, and $A = A_4/(1/V_{\rm hor}^2 - 1/V_{\rm nmo}^2)$; $V_{\rm hor}$ is the horizontal velocity. The denominator of the nonhyperbolic term ensures the convergence of this approximation at infinitely large horizontal offsets. Although equation (5) was originally designed for VTI media, it is generic and can be used in *arbitrary* anisotropic media if the appropriate coefficients (A_2, A_4, and A) were found, honoring the azimuthal anisotropic dependency. In fact, earlier studies by Al-Dajani and Tsvankin (1998) and Al-Dajani and Toksöz (1999) demonstrate the accuracy of equation (5) for P-wave propagation in azimuthally anisotropic media. Our objective is to study the validity of equation (5) for shear-wave propagation.

Figure 7 shows the time residuals (nonhyperbolic portion) of the reflection moveout as a function of offset-to-depth ratio (X/D) for an $S1$-wave and an $S2$-wave propagation in an orthorhombic layer, given in Figure 3. The time residuals are computed by taking the difference between the exact reflections (Figure 2) and the computed traveltimes using the hyperbolic moveout equation (1) which is parameterized by the analytic NMO velocity equation (3). The anisotropy-induced nonhyperbolic moveout causes deviations in the reflection moevout from a hyperbola (i.e., nonzero time difference). Interestingly, the nonhyperbolic reflection moveout (i.e., time residuals) is maximum along the direction parallel to the polarization and it rapidly decreases away from the polarization direction. Obviously, the reflection moveout for both wave types are purely hyperbolic along the directions that are perpendicular to their polarizations (Figure 7). This observation can be explained by studying the nonhyperbolic moevout coefficient (A_4).

The NHMO Coefficient

Application of the nonhyperbolic moveout equation (5) requires knowledge of the quartic moveout coefficient A_4. The quartic coefficient A_4 for pure mode reflection in a homogeneous arbitrary anisotropic layer is given by Al-Dajani and Toksöz (1999) as:

$$\begin{aligned} A_4(\alpha) &= A_4^{(1)} \sin^4\alpha + A_4^{(2)} \cos^4\alpha + A_4^{(x)} \sin^2\alpha \cos^2\alpha \\ &+ A_4^{(x1)} \sin\alpha \cos^3\alpha + A_4^{(x2)} \sin^3\alpha \cos\alpha\,, \end{aligned} \tag{6}$$

where $A_4^{(1)}$ and $A_4^{(2)}$ are the components of quartic coefficient along the two vertical principal planes (in Cartesian coordinates, (x_2, x_3) and (x_1, x_3), respectively). $A_4^{(x)}$, $A_4^{(x1)}$, and $A_4^{(x2)}$ are cross-terms which absorb the mutual influence from all principal planes. The components of the quartic coefficient are presented in terms of the medium parameters, while the azimuthal dependence is governed by trigonometric functions. Again, α is the azimuth of the CMP line from one of the vertical planes (i.e., (x_1, x_3) plane). No assumptions have been made about the type of symmetry to which the medium might pertain, nor about the wave type or the strength of anisotropy. The only assumptions made so far are the fact that we have a horizontal symmetry plane in order to have analytic representation, and that the medium symmetry (if any) coincides with our Cartesian coordinate system. In fact, equation (6) represents all sources of

nonhyperbolic moevouts (e.g., lateral heterogeneity, structure, anisotropy, etc.), and it has important applications in moveout analysis and seismic imaging. Our focus is on the anisotropy-induced nonhyperbolic reflection moveout. Because an anisotropic symmetry plane is transversely isotropic (TI), the moveout coefficients along the vertical symmetry planes are given analogously to the VTI, which is discussed in detail by Tsvankin (1997a,b) and Al-Dajani and Toksöz (1999). The problem, however, is to obtain the components of the moveout coefficients outside the symmetry planes.

As we expect, the more complicated the anisotropy model (lower symmetry), the more involved is the quartic coefficient given by equation (6). For example, in the case of VTI symmetry, equation (6) reduces to the known azimuthally independent quartic coefficient A_4 given by Tsvankin and Thomsen (1994). In the case of HTI, on the other hand, with a horizontal symmetry axis in the (x_1, x_3) plane, the components $A_4^{(x)}$, $A_4^{(x1)}$, $A_4^{(x2)}$, and $A_4^{(1)}$ vanish, and equation (6) reduces to the expression of Al-Dajani and Tsvankin (1998): $A_4 = A_4^{(2)} \cos^4 \alpha$. For such HTI symmetry and unlike $S1$-wave ($\equiv$ SV) propagation, the reflection moveout for $S2$-wave ($\equiv$ SH) propagation is purely hyperbolic (i.e., $A_4 = 0$). The quartic coefficient for shear-wave propagation in a HTI medium is provided in Al-Dajani and Tsvankin (1998).

In the case of a single homogeneous orthorhombic medium, both $A_4^{(x1)}$ and $A_4^{(x2)}$ vanish, and equation (6) reduces to:

$$A_4(\alpha) = A_4^{(1)} \sin^4 \alpha + A_4^{(2)} \cos^4 \alpha + A_4^{(x)} \sin^2 \alpha \cos^2 \alpha \,. \tag{7}$$

Equation (7) is valid for models with arbitrary strength of anisotropy and can be used for any pure-mode reflection moveout. Following the derivation of Al-Dajani and Toksöz (1999) for shear-wave propagation, we obtain that

- For $S1$-wave propagation, $A_4^{(1)} = 0$, while

$$\begin{aligned} A_4^{(2)} \;&= (c55(c13 + c55)^2(-c33 + c55) \\ &\quad (c13^2 + 2c13c55 + c33c55 + c11(-c33 + c55)))/ \\ &\quad ((c13^2 - c11c33 + c11c55 + 2c13c55 + c55^2)^4 t0^2) \end{aligned}$$

$$
\begin{aligned}
A_4^{(x)} &= -((c55(c12^2(c33-c55)^2 + c23^2c55^2 + \\
&\quad 2c23c44c55^2 + c44c55^3 - 2c23c33c55c66 - \\
&\quad 2c33c44c55c66 + 2c23c55^2c66 + 3c44c55^2c66 - \\
&\quad c55^3c66 + c33^2c66^2 - 2c33c55c66^2 + c55^2c66^2 + \\
&\quad c13^2(c23^2 + 2c23c44 + c44c55 + c44c66 - c55c66) - \\
&\quad 2c12(c33 - c55)(c23c55 + c44c55 - c33c66 + \\
&\quad c55c66) - 2c13(c12(c23 + c44)(c33 - c55) - \\
&\quad c23^2c55 - 2c23c44c55 - c44c55^2 + c23c33c66 + \\
&\quad c33c44c66 - c23c55c66 - 2c44c55c66 + \\
&\quad c55^2c66)))/((-c44 + c55)(c13^2 + 2c13c55 + \\
&\quad c55^2 + c11(-c33 + c55))^2c66^2t0^2)).
\end{aligned}
$$

- For $S2$-wave propagation, $A_4^{(2)} = 0$, while

$$
\begin{aligned}
A_4^{(1)} &= (c44(c23 + c44)^2(-c33 + c44) \\
&\quad (c23^2 + 2c23c44 + c33c44 + c22(-c33 + c44)))/ \\
&\quad ((c23^2 - c22c33 + c22c44 + 2c23c44 + c44^2)^4t0^2)
\end{aligned}
$$

$$
\begin{aligned}
A_4^{(x)} &= -((c44(c12^2(c33 - c44)^2 + c13^2(c23 + c44)^2 + \\
&\quad c23^2c44c55 + 2c23c44^2c55 + c44^3c55 - \\
&\quad c23^2c44c66 - 2c23c44^2c66 - c44^3c66 + \\
&\quad c23^2c55c66 - 2c23c33c55c66 + 4c23c44c55c66 - \\
&\quad 2c33c44c55c66 + 3c44^2c55c66 + c33^2c66^2 - \\
&\quad 2c33c44c66^2 + c44^2c66^2 - \\
&\quad 2c12(c33 - c44)(c23c55 + c44c55 - c33c66 + \\
&\quad c44c66) + 2c13(c23 + c44)(c12(-c33 + c44) + \\
&\quad c23c55 + c44c55 - c33c66 + c44c66))) \\
&\quad /((c23^2 + 2c23c44 + c44^2 + c22(-c33 + c44))^2 \\
&\quad (c44 - c55)c66^2t0^2)).
\end{aligned}
$$

Hence, the quartic coefficient for both modes of shear-wave propagation in an orthorhombic medium reduces to a simple form with only two components. Similar to the NMO velocity, we can conveniently represent the quartic coefficient for orthorhombic

media in terms of Tsvankin's (1997b) notation by applying the following substitutions:

$$\begin{aligned}
c33 &= V_{P0}^2 \\
c44 &= V_{S0_1}^2 \\
c55 &= V_{S0}^2 = V_{S0_2}^2 \\
c11 &= V_{P0}^2(2\epsilon^{(2)}+1) \\
c22 &= V_{P0}^2(2\epsilon^{(1)}+1) \\
c66 &= V_{S0_1}^2(2\gamma^{(2)}+1) = V_{S0_2}^2(2\gamma^{(1)}+1) \\
c13 &= \sqrt{(d1(2V_{P0}^2\delta^{(2)}+d1))} - V_{S0_2}^2 \\
c23 &= \sqrt{(d2(2V_{P0}^2\delta^{(1)}+d2))} - V_{S0_1}^2 \\
c12 &= \sqrt{(d3(2V_{P0}^2(2\epsilon^{(2)}+1)\delta^{(3)}+d3))} - \\
&\quad V_{S0_1}^2(2\gamma^{(2)}+1)\,,
\end{aligned}$$

where $d1 = V_{P0}^2 - V_{S0_2}^2$, $d2 = V_{P0}^2 - V_{S0_1}^2$, and $d3 = V_{P0}^2(2\epsilon^{(2)}+1) - V_{S0_1}^2(2\gamma^{(2)}+1)$. $\delta^{(3)}$ is Tsvankin's (1997b) notation, defined analogously to Thomsen's δ parameter for VTI. To perform the transformation, however, we need either the two vertical shear velocities and one γ or one vertical shear velocity and two γ(s).

Despite the complexity of the orthorhombic symmetry, it is interesting to note that the reflection moveout for any shear-wave mode is purely hyperbolic in the direction normal to the polarization, and that the nonhyperbolic portion of the moveout reduces rapidly away from the direction of the polarization. In fact, the quartic coefficient in the case of shear-wave propagation is simple compared to the case of P-wave propagation. Figure 8 shows a schematic diagram for the azimuthal variation of the quartic coefficient A_4. A sketch of the general representation (equation (6)) is given in Figure 8a. Figures 8b,c,d, on the other hand, show sketches for the azimuthal variation of the quartic coefficient for the three pure wave modes in the case of an orthorhombic medium with a horizontal interface (equation (7)). The shapes given in Figure 8 might vary depending on the relative magnitudes of the components of the quartic coefficient. For example, in some cases of orthorhombic media and according to equations (6) and/or (7), the quartic coefficient for P-wave reflections (Figure 8b) might appear as a flower-like shape (e.g., imagine Figures 8c and 8d combined). Here, we are interested in the quartic coefficient for shear-wave propagation. It is obvious from Figures 8c and 8d that the quartic coefficient, hence the nonhyperbolic reflection moveout, is significant in the vicinity of the polarization directions for $S1$- and $S2$-waves (e.g., $\pm 30°$ and for large offset-to-depth ratios). Away from the polarization directions, the quartic coefficient decreases rapidly and vanishes along perpendicular directions relative to the polarizations. This fact can be used, in addition to the ellipticity of the NMO velocity, to detect the orientation of the principal symmetry planes of the medium (hence, the fractures orientation) from shear-wave reflection moveout.

As mentioned earlier, the equivalence between orthorhombic and VTI media along the symmetry planes allows the application of VTI expressions along those directions.

This fact was useful to Tsvankin (1997b) when he introduced the anisotropic parameters for orthorhombic media, and following the notation of the well-known Thomsen's (1986) coefficients ϵ, δ and γ for vertical transverse isotropy. As seen above, the anisotropy-induced nonhyperbolic reflection moveout for shear-wave propagation is significant in the vicinity of the symmetry planes. An earlier work by Tsvankin and Thomsen (1994) on shear-wave reflection moveout for vertical transverse isotropy (VTI) demonstrated that the NHMO equation (5) is accurate for offset-to-depth ratios of 1.7–2. This conclusion remains valid for shear-wave propagation in orthorhombic media. In fact, the 3-D representation of both NMO velocity and the quartic coefficient, in azimuthally anisotropic media, makes the NHMO equation (5) more accurate for longer offset-to-depth ratios, especially outside the symmetry planes. In order to implement equation (5), we need to obtain the horizontal velocity V_{hor} as well. From the VTI equivalence along the horizontal symmetry plane, however, we can represent the phase velocity analytically, and hence obtain the horizontal group velocity which is evaluated at the azimuth of the source-to-receiver (CMP) gather. In multilayered anisotropic media, on the other hand, both the quadratic and quartic moveout coefficients are averaged using Dix-type equations and using the interval values which honor the azimuthal dependence of each layer (Al-Dajani and Tsvankin, 1998; Al-Dajani and Toksöz, 1999).

CONCLUSIONS

The use of multicomponent sources and receivers to acquire seismic data over azimuthally anisotropic media and/or the propagation of shear waves at normal incidence over fractured media cause the presence of two pure modes for shear waves: fast and slow waves. We have presented analytic descriptions for the quadratic (A_2) and quartic (A_4) coefficients of the Taylor's series expansion of the two-way traveltime for both modes of shear waves in azimuthally anisotropic media, with special attention paid to orthorhombic media with horizontal interfaces. In such media, the $S1$ and $S2$ shear waves are polarized parallel to the horizontal principal axes. The conventional NMO equation, which is parameterized by the azimuthally dependent NMO velocity for shear-wave propagation, accurately represents the reflection moveout (especially for conventional spreadlengths (X/D $\leq$ 1)). The azimuthal variation of the NMO velocity is elliptical for shear-wave propagation. On the other hand, the azimuthally-dependent quartic coefficient for pure shear-wave propagation, in a homogeneous arbitrary anisotropic layer with arbitrary strength of anisotropy, has a relatively simple trigonometric form. For an orthorhombic medium with a horizontal interface, the quartic coefficient for pure $S1$- and $S2$-wave reflection moveout has a simple azimuthal representation. The reflection moveout for any shear-wave mode in a homogeneous orthorhombic medium is purely hyperbolic in the direction normal to the polarization, and the nonhyperbolic portion of the moveout reduces rapidly away from the direction of polarization. As a result, the need for NHMO treatment for shear-wave propagation is significant in the vicinity of the polarization directions and for large offsets (e.g., $\pm 30^\circ$ and for offset-to-depth

ratio > 1). The NHMO equation, which is parameterized by the analytic NMO velocity and quartic coefficient for shear-wave propagation, provides more accurate moveout representation than the hyperbolic equation at large offsets (e.g., offset-to-depth ratios > 1). In multilayered anisotropic media, the moveout coefficients are averaged using Dix-type equations and using the interval values which honor the azimuthal dependence of each layer. The implementation of the NHMO equation and the utilization of the moveout coefficients allow for not only enhanced seismic imaging, but also provide the link between seismic signatures and the medium parameters.

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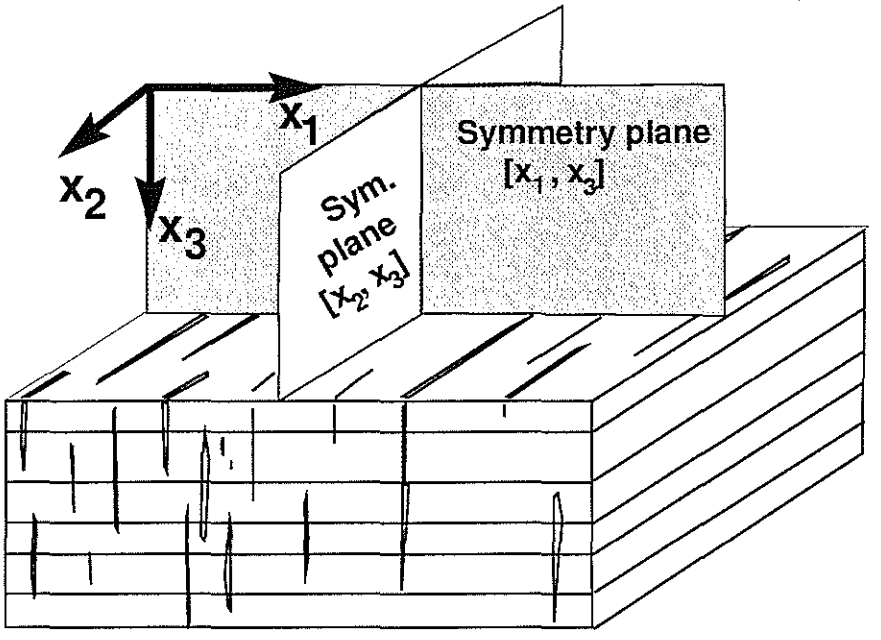

Figure 1: Orthorhombic media (shown above) have three mutually orthogonal planes of mirror symmetry. One of the reasons for orthorhombic anisotropy is a combination of parallel vertical cracks and vertical transverse isotropy (e.g., due to thin horizontal layering) in the background medium.

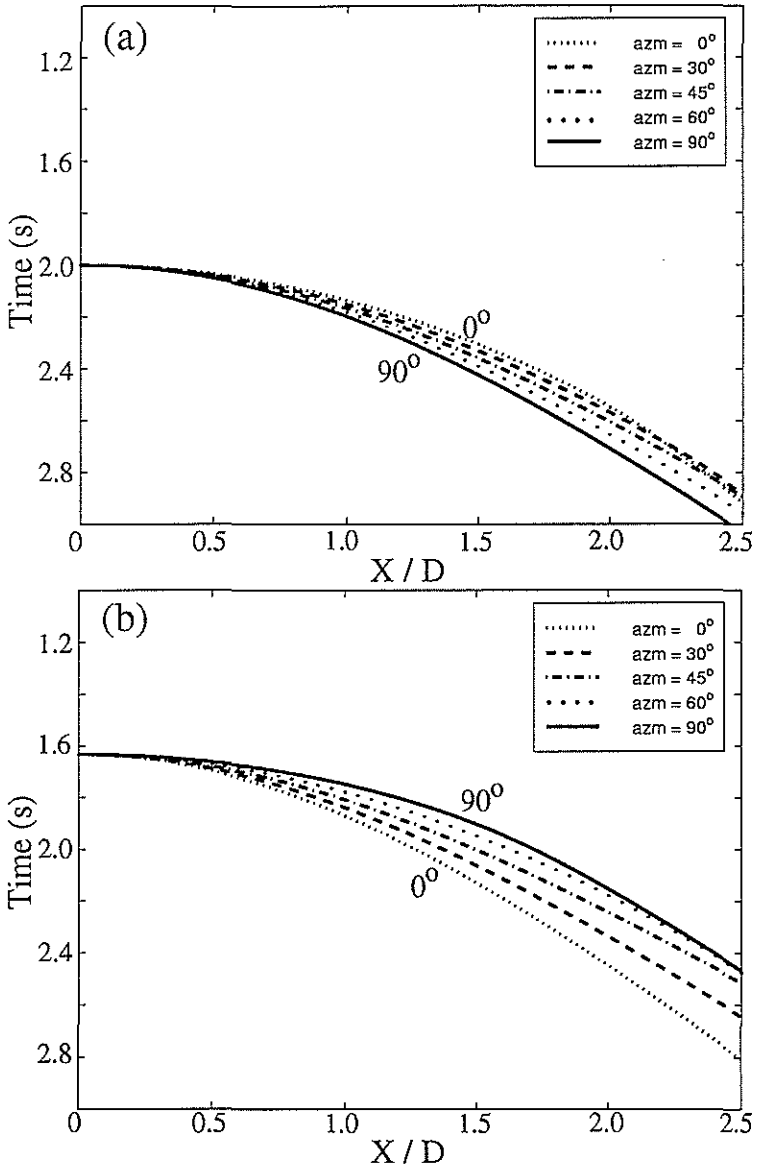

Figure 2: Exact ray-traced reflection moveout for (a) $S1$-wave and (b) $S2$-wave propagation. The curves correspond to CMP azimuths 0°, 30°, 45°, 60°, and 90°. The traveltime curves are displayed as a function of offset-to-reflector-depth (X/D) ratio. The geometry and model parameters are given in Figure 3.

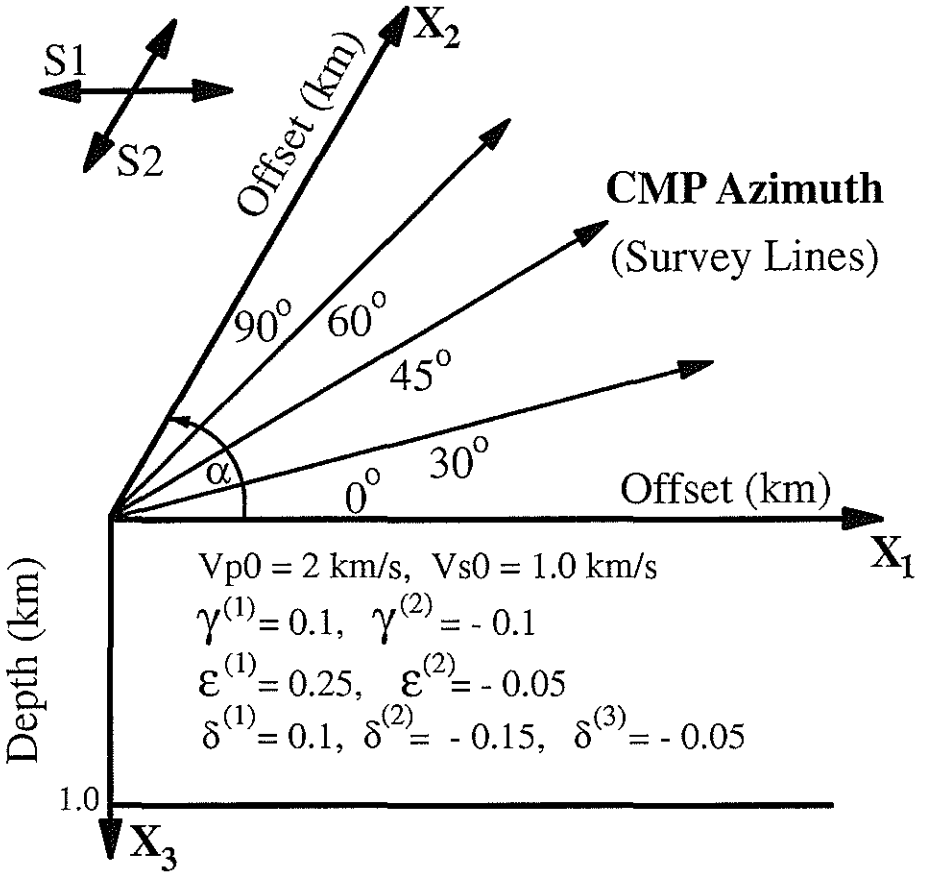

Figure 3: Orientation of the CMP azimuths (survey lines) over a horizontal orthorhombic layer. The model parameters are provided using Tsvankin's notation (see Tsvankin, 1997b; Al-Dajani and Toksöz, 1999).

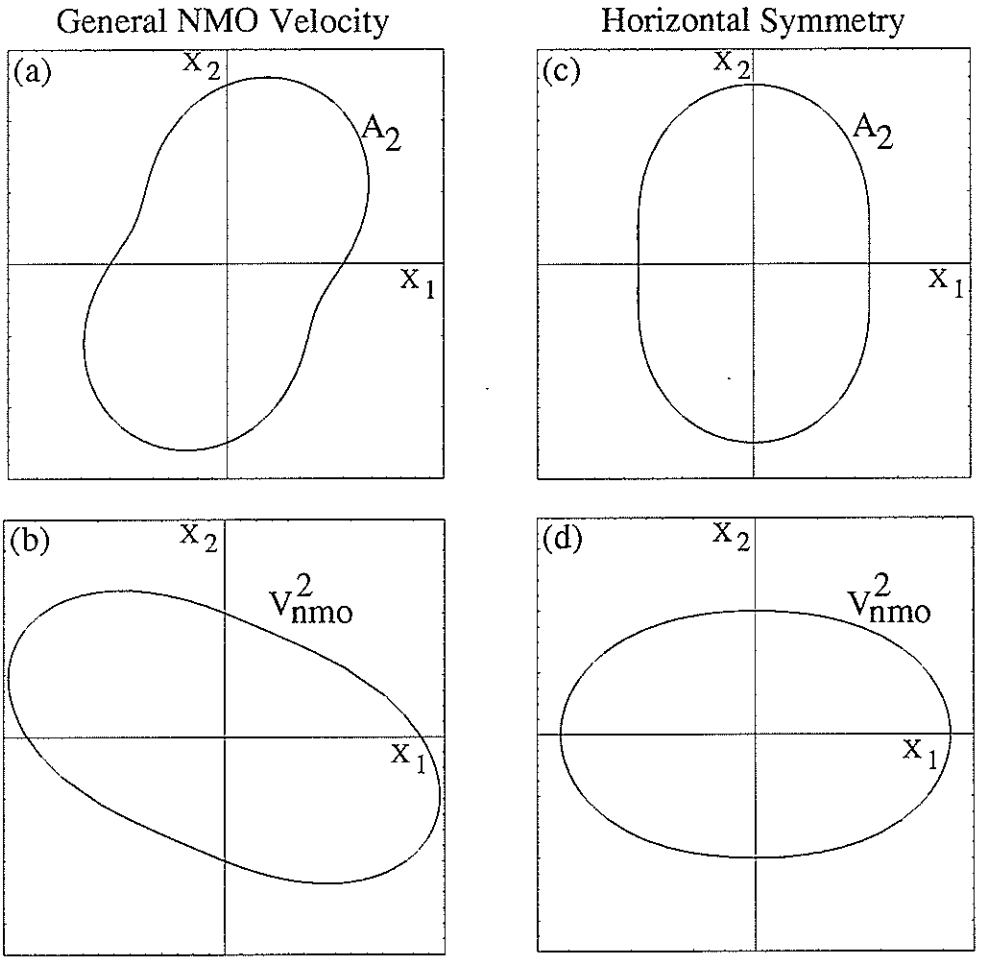

Figure 4: A plan view of the general behavior of the quadratic coefficient (A_2) and its inverse (V^2_{nmo}) for both modes of shear-wave propagation in an arbitrary medium (a and b), and in an orthorhombic medium with a horizontal reflector (c and d).

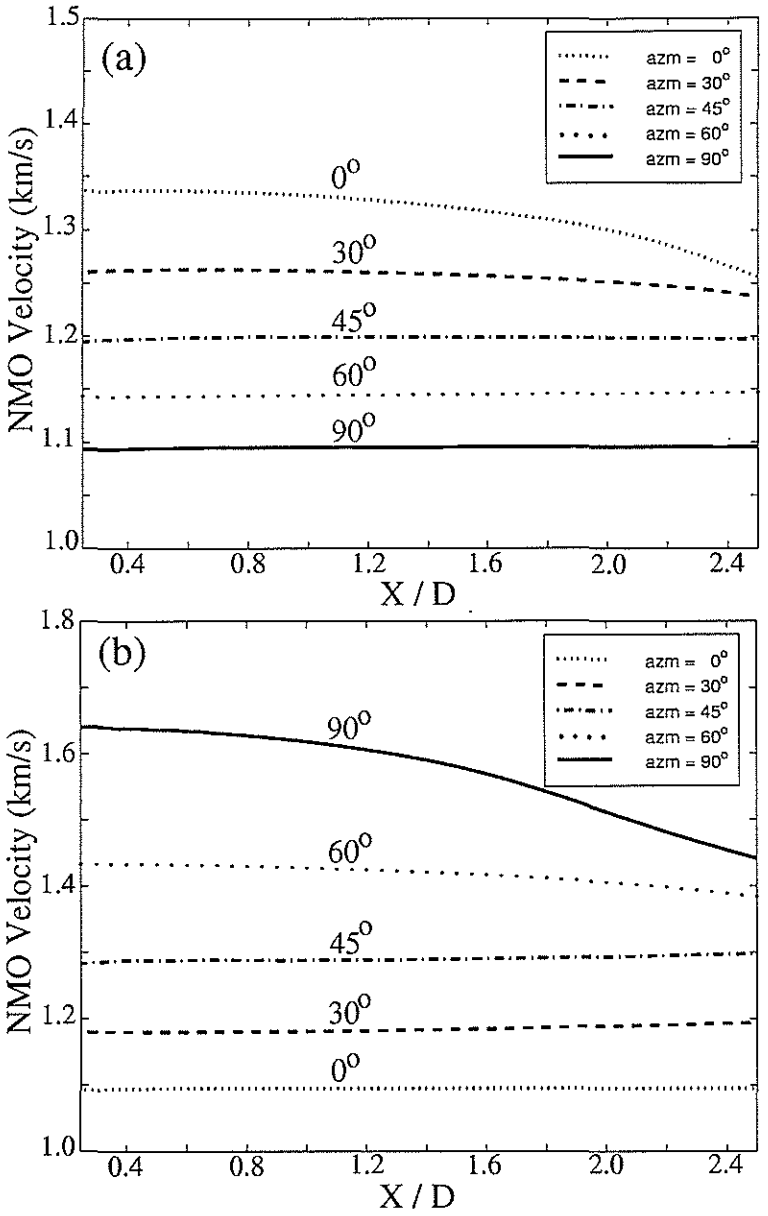

Figure 5: Estimated NMO velocity as a function of spreadlength-to-depth (X/D) ratio for (a) $S1$-wave (b) $S2$-wave propagation (equation (4)). The curves correspond to CMP azimuths 0°, 30°, 45°, 60°, and 90° (see Figures 2 and 3). The anisotropy-induced, nonhyperbolic moveout causes deviation (error) in the NMO velocity estimates. The error is proportional to the nonhyperbolic reflection moevout.

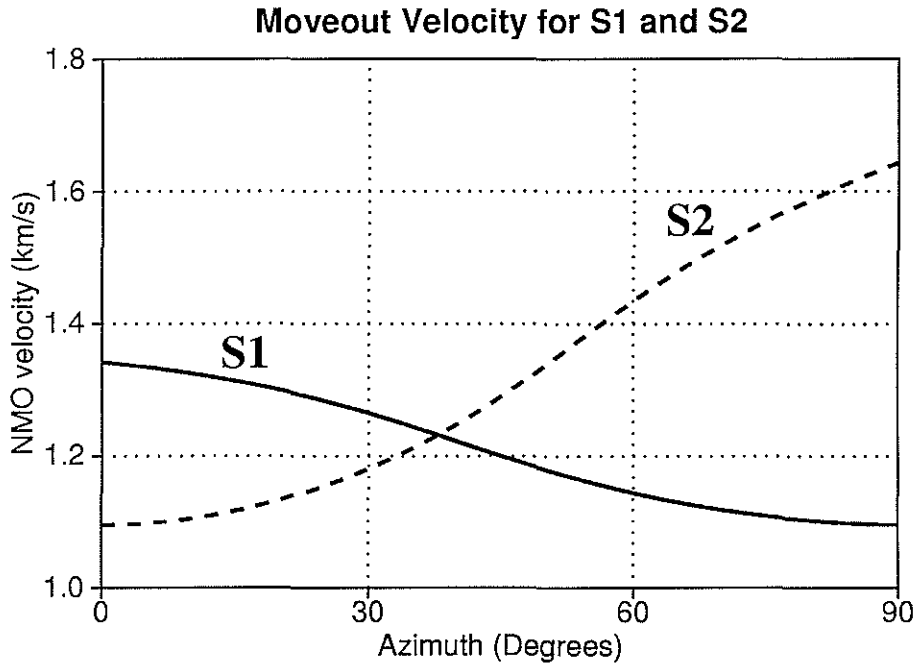

Figure 6: Exact azimuthal variation of NMO velocity for $S1$- and $S2$-wave modes, as computed using equation (3). The model parameters and geometry are given in Figure 3.

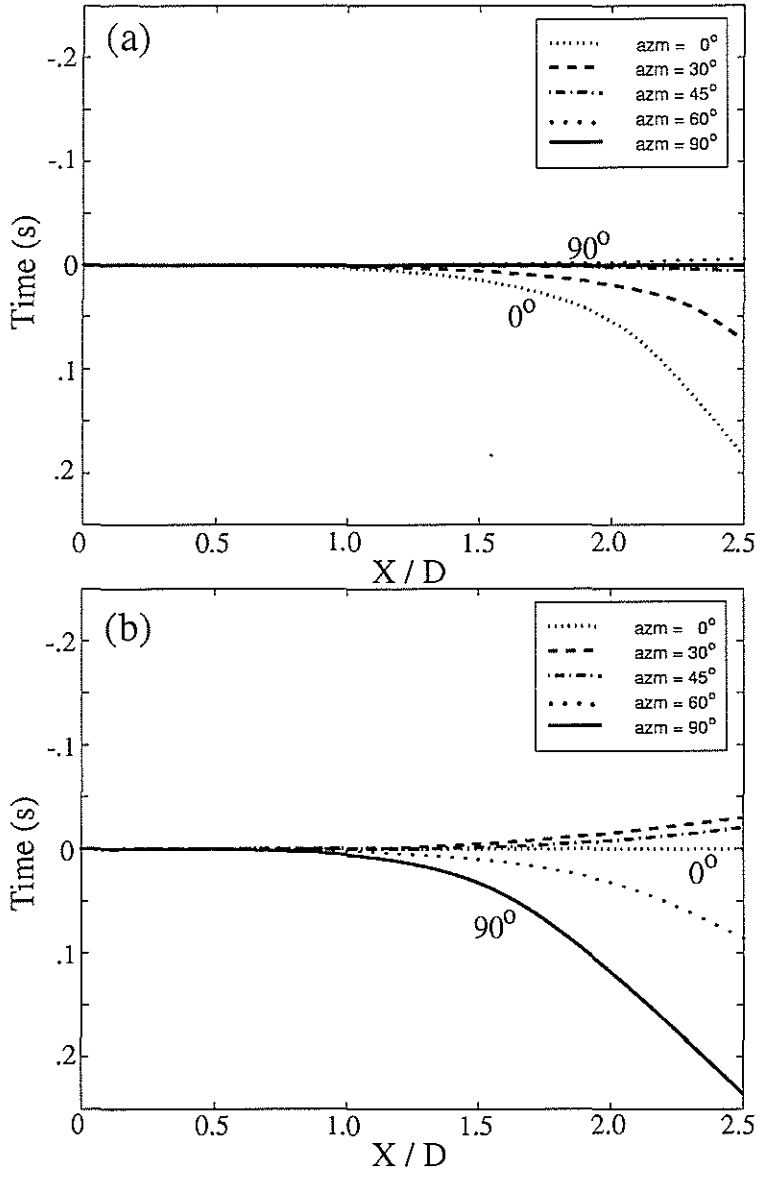

Figure 7: Time residuals (nonhyperbolic portion) of the reflection moveout as a function of offset-to-depth ratio (X/D) for (a) $S1$-wave and (b) $S2$-wave propagation. The anisotropy-induced nonhyperbolic moveout causes deviations in the reflection moevout from being hyperbolic (i.e., nonzero time residuals).

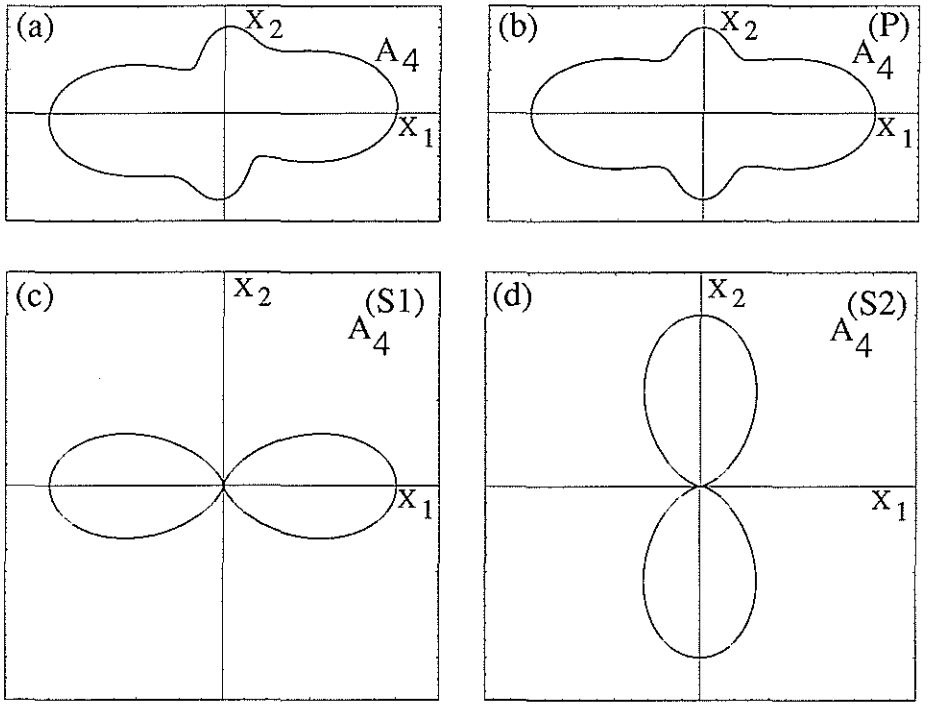

Figure 8: A plan view of the general behavior of the quartic coefficient A_4 in azimuthally anisotropic media. (a) shows a sketch of A_4 in an arbitrary medium, while (b), (c), and (d) show sketches of A_4 for P, $S1$, and $S2$, in an orthorhombic medium with a horizontal interface, respectively.