GEOSTATISTICAL SEISMIC INVERSION USING WELL LOG CONSTRAINTS

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ABSTRACT

Information about reservoir properties usually comes from two sources: seismic data and well logs. The former provide an indirect, low resolution image of rock velocity and density. The latter provide direct, high resolution (but laterally sparse) sampling of these and other rock parameters. An important problem in reservoir characterization is how best to combine these data sets, allowing the well information to constrain the seismic inversion and, conversely, using the seismic data to spatially interpolate and extrapolate the well logs.

We have developed a seismic/well log inversion method that combines geostatistical methods for well log interpolation (i.e., kriging) with a Monte Carlo search technique for seismic inversion. Our method follows the approach used by Haas and Dubrule (1994) in their sequential inversion algorithm. Kriging is applied to the well data to obtain velocity estimates and their variances for use as *a priori* constraints in the seismic inversion. Further, inversion of a complete 2-D seismic section is performed one trace at a time. The velocity profiles derived from previous seismic traces are incorporated as "pseudo well logs" in subsequent applications of kriging. Our version of this algorithm employs a more efficient Monte Carlo search algorithm in the seismic inversion step, and moves progressively away from the wells so as to minimize the kriging variance at each step. Numerical experiments with synthetic data demonstrate the viability of our seismic/well data inversion scheme.

INTRODUCTION

Conventional seismic processing techniques produce a distorted image of the subsurface in two way travel time as opposed to depth. Inversion must be performed upon the seismic data to obtain the true velocity field. A potentially useful yet underutilized constraint on seismic inversion problems comes in the form of well logs, and limited local samples of the true velocity field.

Recently, geostatisticians have begun to make better quantitative use of well log data, primarily through a set of techniques known collectively as "kriging" (Doyen, 1988). These methods globally estimate the value of subsurface parameters when only sparse local samples are available. A method by Haas and Dubrule (1994) attempts to move one step further and combine kriging with post-stack seismic inversion. They use a convolutional model to simulate seismic wave propagation and a Monte Carlo method to do inversion. Other related methods attempt to invert post-stack seismic data via Monte Carlo methods to obtain a velocity model of the subsurface but do not address the use of well data (Mosegaard and Vestergaard, 1991; (Vestergaard and Mosegaard, 1991).

This paper attempts to improve upon the method put forth by Haas and Dubrule (1994) by making better use of well data and using a more efficient Monte Carlo inversion method to invert a 2-D seismic data set.

GEOSTATISTICS

Given a field of parameters to be estimated, say a 2-D velocity field, geostatistics seeks to model each parameter as a random variable (RV). This leads to the definition of a probability density function at each parameter location and a covariance between each two parameters. If the field is discrete, the covariance between each of these RV's can be put into a covariance matrix. If each RV is additionally constrained to be Gaussian, the covariance matrix, along with a mean, fully characterizes the random field.

A function can be defined which describes the covariance vs. spatial separation (lag). It is known as the (auto)covariogram or, if normalized, the (auto)correlogram. The distance it takes for the function to almost reach 0 is known as the range or correlation length. This function can take a number of forms. Two popular ones used in geophysical applications are Gaussian and exponential. Gaussian and exponential correlograms have the following forms, respectively:

$$e^{x^2/a^2} \quad , \quad e^{-|x|/a}$$

where a is the correlation length and x is the lag.

It is simple to create realizations possessing an exponential or Gaussian autocovariance function. One must generate Gaussian white noise with variance of the desired stationary process in the state domain, transform it into the frequency domain, multiply it by the square root of the desired power spectrum, and then transform back to the state domain. The realization will have the desired covariance.

Geostatistical Seismic Inversion

The choice of which correlogram to use depends on what the application is. In trying to model the heterogeneity of geology in a realistic manner, an exponential correlogram is preferred because it contains more high frequency components than the Gaussian, which is more representative of the intermittancy visible in geologic sections. In this paper, only exponential covariograms are used.

A geostatistical estimation technique known as kriging is seeing routine use in geophysics recently. Given sparse spatial samples of a geophysical variable, it allows for interpolation as well as error bars on the interpolated value. It involves construction of a square covariance matrix with dimensionality equal to the number of data within a prescribed distance of the estimated point. Such a matrix must be inverted for every point to be estimated. As such it can be a very expensive operation when a large field needs to be estimated and/or a large number of data exist.

There are many different versions of kriging in the current literature. They are categorized according to their assumptions: "Simple kriging" assumes a known covariance function and a known mean of the underlying process; "ordinary kriging" assumes only a known covariance function, its mean is a constant unknown function; "universal kriging" assumes a known covariance and an unknown mean which is a linear combination of known functions. There are many other forms of kriging (see Cressie, 1993, for a summary). In this work ordinary kriging is used as the estimation methodology.

Following Cressie (1993), we let

$$v(z) = \mu + n(z), \qquad z \in D$$

be the random field of interest where μ is the unknown constant mean, $n(z)$ is a stationary random process, and D is the spatial domain of interest. Assume that the estimate is a linear function of the observed RV's,

$$\hat{v}(z_0) = \sum_{i=1}^{n} \lambda_i v_i(z_i), \qquad \sum_{i=1}^{n} \lambda_i = 1.$$

Then,

$$\hat{v}(z_0) = \lambda^{\mathrm{T}} \mathbf{v}$$

and

$$\sigma^2_{OK} = C(0) - \lambda^{\mathrm{T}} \mathbf{d} + \mathbf{m},$$

where $C(0)$ is the variance of any random variable in the field (i.e., the value of the covariance function at 0 lag) and $\mathbf{d}$ is the vector of covariance values between the estimated point and the data points($C(\mathbf{z_i} - \mathbf{z_0})$). The weights, λ, and Lagrange multiplier, m, of the ordinary kriging estimate at location $\mathbf{z_0}$ will be,

$$\lambda^{\mathrm{T}} = (\mathbf{d} + \mathbf{1}\frac{(1 - \mathbf{1}^{\mathrm{T}}\mathbf{C}^{-1}\mathbf{d})}{\mathbf{1}^{\mathrm{T}}\mathbf{C}^{-1}\mathbf{1}})^{\mathrm{T}}\mathbf{C}^{-1}, \quad m = \frac{(1 - \mathbf{1}^{\mathrm{T}}\mathbf{C}^{-1}\mathbf{d})}{\mathbf{1}^{\mathrm{T}}\mathbf{C}^{-1}\mathbf{1}},$$

where $C = C(\mathbf{z_i} - \mathbf{z_j})$ is the matrix of covariance values between all sampled RV's in the random field.

An example of an application of these kriging equations is shown in Figures 1 and 2. Figure 1 is a realization of a 2-D exponential random field, $v(\mathbf{z})$. The field represents the P-wave velocity of the subsurface. It is anisotropic, having a range of 100 ft/s in the horizontal direction, and 10 ft/s in the vertical direction. Also, the variance of each RV, $C(\mathbf{0})$, is 250 ft/s. Two vertical samples (i.e., well logs) are extracted from this field at trace locations 10 and 40. Figure 2 shows the results of kriging these well locations when the autocovariance of the media is assumed known. Finally, Figure 3 shows the velocity field (Figure 1) converted into a seismogram via a convolutional model.

MONTE CARLO INVERSION

In recent years, adaptive search methods have gained popularity as a tool for finding optimal solutions to geophysical inverse problems. Their popularity has been motivated by vast increases in computing power and the fact that these methods are more effective than traditional gradient-based methods in finding globally, rather than just locally, optimal solutions to nonlinear problems. Recent applications of adaptive search methods to seismic inversion include genetic algorithms (Stoffa and Sen, 1991), simulated annealing (Rothman, 1985; Sen and Stoffa, 1991; Vestergaard and Mosegaard, 1991), and Monte Carlo sampling (Haas and Dubrule, 1994).

The search algorithm employed by Haas and Dubrule (1994) is a simple Monte Carlo method, whereby the velocity profile, $v(z)$, fit to a given seismic trace, $u(t)$, is obtained as a realization of a specified random process. Many realizations of the process are generated and the one yielding the smallest error of fit, E, is chosen as the solution. E is defined as $E = \int (u(t) - u_{\text{cal}}(t))^2 \, dt$ where $u_{\text{cal}}(t)$ denotes the synthetic seismogram calculated from $v(z)$, computed using the standard convolutional model. Haas and Dubrule generate velocity realizations by sampling a fixed random process whose mean and variance are derived from the posterior mean and variance resulting from kriging of previous well data. However, they ignore correlations between the velocity at different depths.

Our Monte Carlo search algorithm differs in two main respects. First, we retain correlation information in the kriging variance, i.e., our realizations are from a correlated process. To accomplish this efficiently, we approximate the process as stationary, which is a reasonable approximation given the geometry of our kriging problem. Second, we borrow a very useful concept from simulated annealing: we allow the process from which realizations are generated to change adaptively during the search. In simulated annealing, this is accomplished by specifying a temperature schedule, whose choice is one of the most difficult aspects of the algorithm. Our approach is to gradually shift the mean and reduce the variance of the sampling process such that, like simulated annealing, the search becomes more and more local as better solutions are found. The "annealing" in our algorithm is not controlled by a temperature schedule but, rather,

is determined only by the total number of trials, which must be specified as input.

Proposed Monte Carlo Method

A new Monte Carlo inversion method presented in this paper is easy to apply (no temperature schedule needed) and provides good results for the convolutional seismic problem. The crux of this method is based on some properties of the following process

$$n(z) = \alpha r(z) + \beta s(z)$$

where $r(z)$ and $s(z)$ are independent Gaussian processes. The covariance matrix, Λ_n, of $n(z)$, therefore, can be expressed as

$$\Lambda_n = \alpha^2 \Lambda_r + \beta^2 \Lambda_s.$$

If we further impose the constraint that $\Lambda_n = \Lambda_r = \Lambda_s = \Lambda$, the constants α and β must satisfy $\alpha^2 + \beta^2 = 1$. We therefore have three zero mean Gaussian random processes, n, r, s, with the same covariance matrix. The purpose of constructing $n(z)$ in such a way will be made clear below.

The algorithm proceeds as follows:

1. Perform kriging at a location adjacent to a well log using only points within three correlation lengths of each estimated point. This is done to reduce computational burden. This will give a kriged mean, $\mu(z)$ at every point in a pseudo well log along with a kriged variance matrix, Λ, which is approximately constant as a function of depth.

2. Specify the total number of iterations N_{tot} to be performed

3. For $i = 1 \ldots N_{total}$ let

   $$v_i(z) = \mu(z) + n_i(z)$$

   where

   $$n_i \sim N(\alpha_i r_i(z), \beta_i \Lambda)$$

   and

   $$\begin{aligned} r_i &= n_{i-1} \text{if} E_i < E_{i-1} \text{or} \\ r_i &= r_{i-1} \text{if} E_i \geq E_{i-1}. \end{aligned}$$

4. As the algorithm proceeds, let $\alpha^2 = \frac{N_{tot}-i+1}{N_{tot}}$ and $\beta^2 = \frac{i-1}{N_{tot}}$. Thus, more and more weight will be applied to $s(z)$ while still ensuring that all realizations of $n(z)$ have covariance Λ.

APPLICATION

We now apply the proposed method to the 2-D problem. Using the model presented in Figure 1, we extract the two well logs and assume that we know the correct 2-D covariance function for purposes of kriging. Kriging is applied at a location adjacent to a well location. This provides a probability distribution from which to draw realizations. The inversion algorithm is then applied for 1000 iterations at this location. The model that provides the best fit to the seismic data in Figure 3 is kept and folded back into the data set at that location as a "pseudo well log." Kriging is then performed again at the next adjacent location using only the previous pseudo well log and the next closest well as data. The algorithm proceeds this way until every seismic trace has been inverted. The entire inversion of 50 traces requires approximately 45 minutes with $Matlab^{TM}$ on a 400 MHz PC.

This sequential method bears resemblance to the method proposed by Haas and Dubrule (1994) and, in fact, was based upon it. However, their methodology applies kriging at locations away from (not adjacent to) the existing well logs thus giving them a much larger variance when implementing their Monte Carlo algorithm. Their algorithm performs sequential inversion by leap frogging through the field. They also limit their parameter space to 30 samples in the vertical direction corresponding to a total of 120 ms. This corresponds to a few wiggles of the seismic trace, thus making it easier to fit the data. In contrast, the models used in this work are sampled in depth at 1 ft intervals over 512 ft depth. Thus there are 512 parameters to invert for.

The results for the 2-D inversion are presented in Figures 4 through 6. Figure 4 shows the result of the inversion when we know the true covariance function of the underlying process. The next figures show the sensitivity of the inversion method to changes in the assumed covariance function. Figure 5 incorrectly assumes that the vertical correlation length is twice what it actually is. Figure 6 incorrectly assumes that the vertical correlation length is half what it actually is. As can be seen, all the models closely approximate the true velocity model of Figure 1.

CONCLUSIONS

We have presented a method for inverting post-stack seismic data given limited well log data. Numerous methods exist to estimate petrophysical properties away from wells. Many methods also exist to invert seismic data. However, few workers attempt to combine these methodologies in a coherent way.

The Monte Carlo method introduced proves itself to be easy to implement as well as robust in the sense that it still finds satisfactory solutions even when *a priori* assumptions are bad. It has an advantage over simulated annealing in that it doesn't require a cooling schedule to invert the data.

ACKNOWLEDGMENTS

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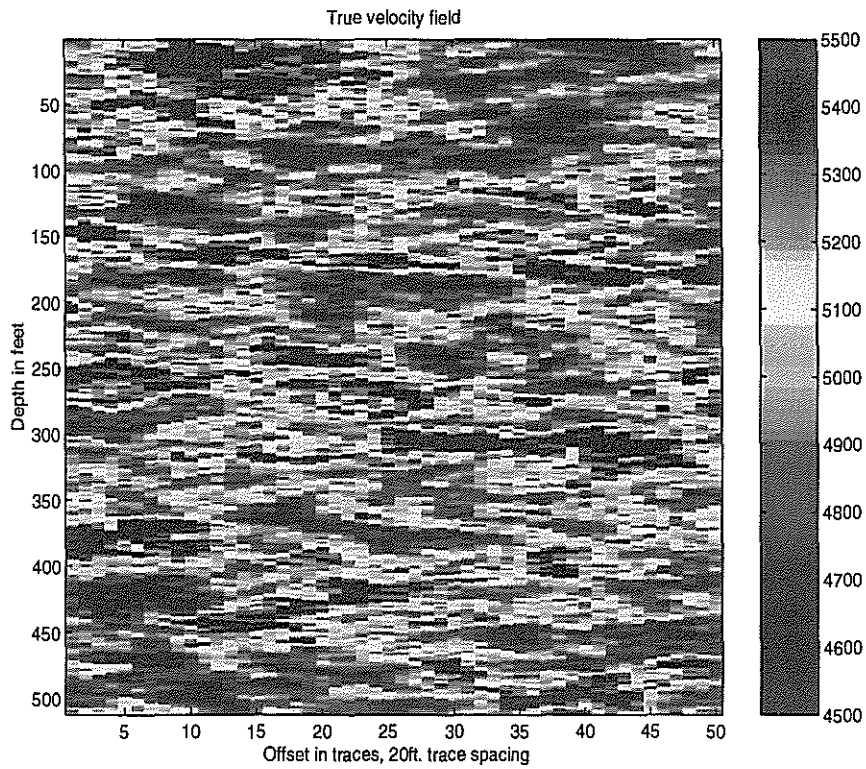
Geostatistical Seismic Inversion

True velocity field

Figure 1: 2-D exponential random field.
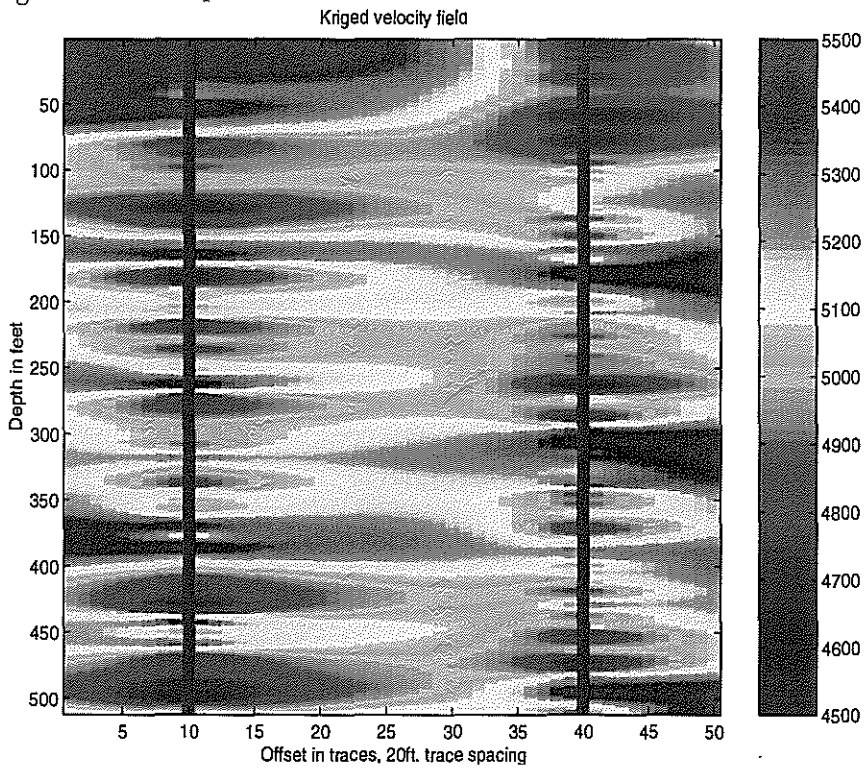
Kriged velocity field

Figure 2: Kriging applied to wells assuming known covariogram.
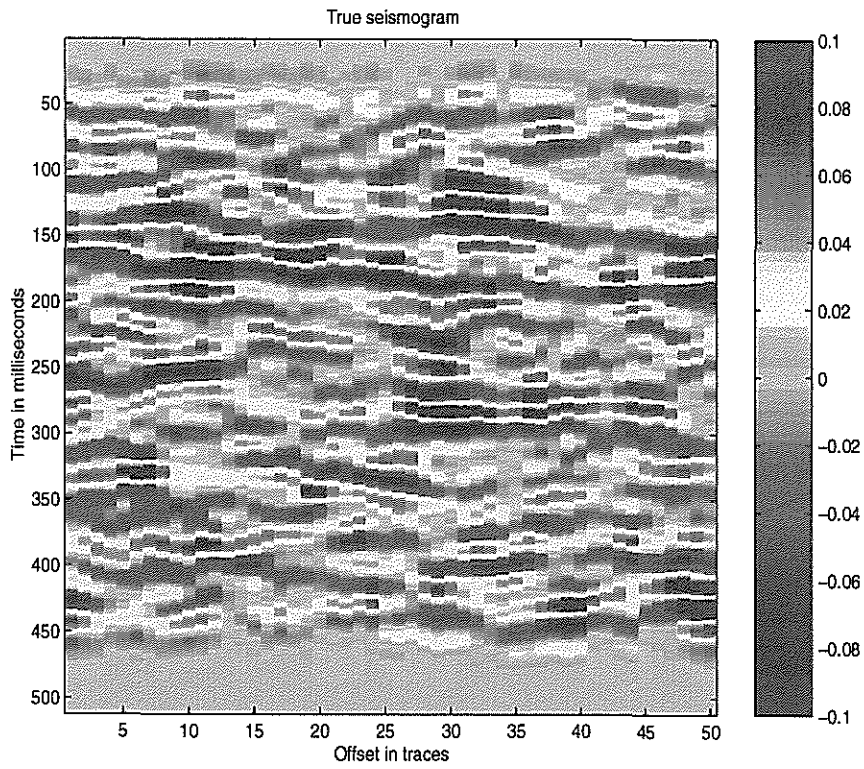
True seismogram

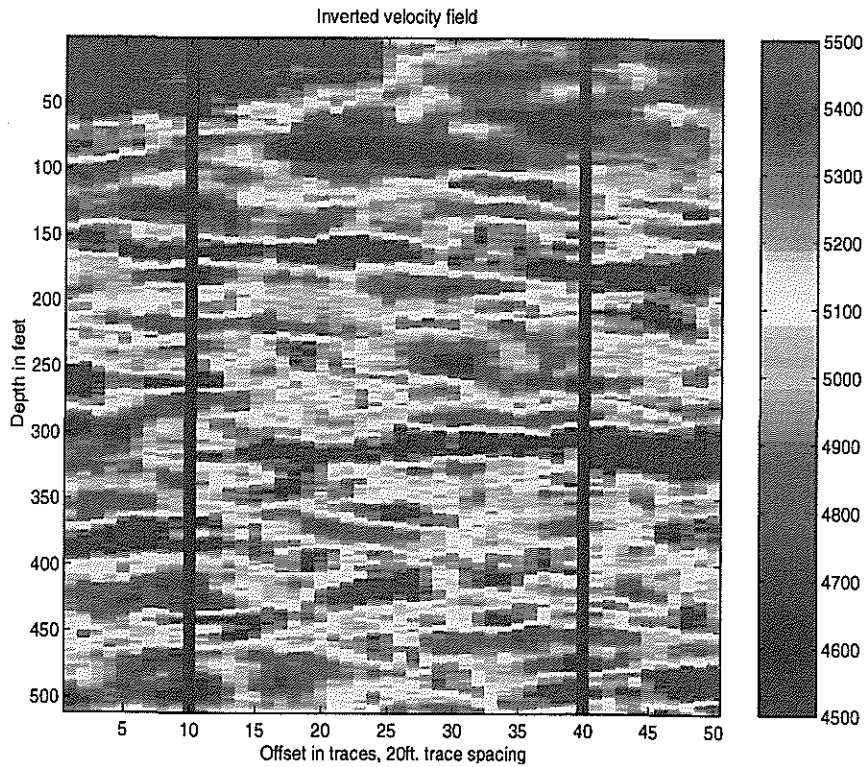Figure 3: Seismogram converted from 2-D field via convolutional model.

Inverted velocity field

Figure 4: Inversion results assuming correct parameters.

Inverted velocity field
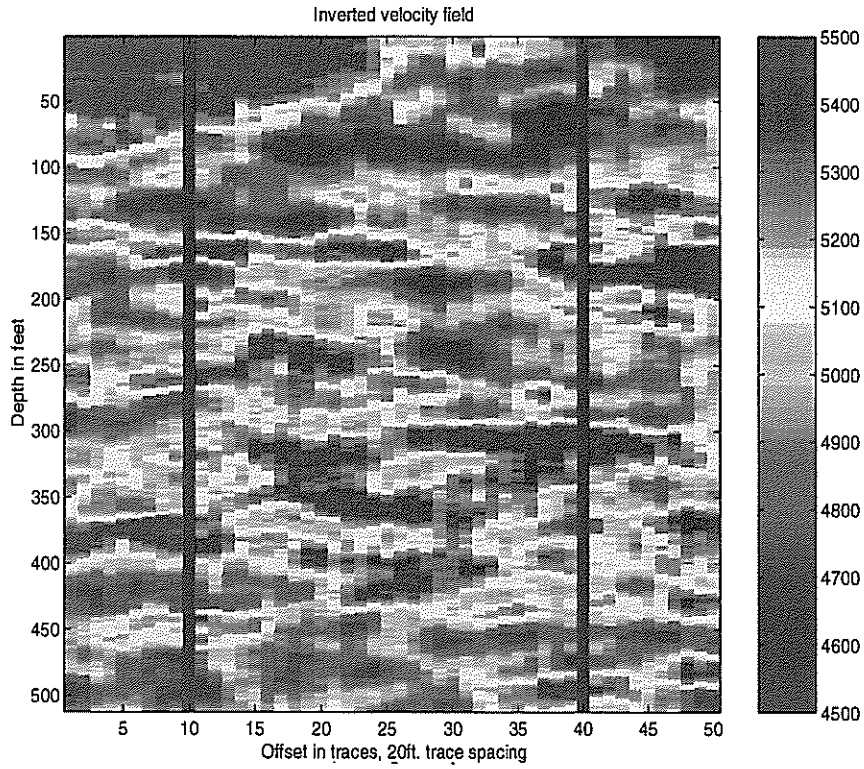
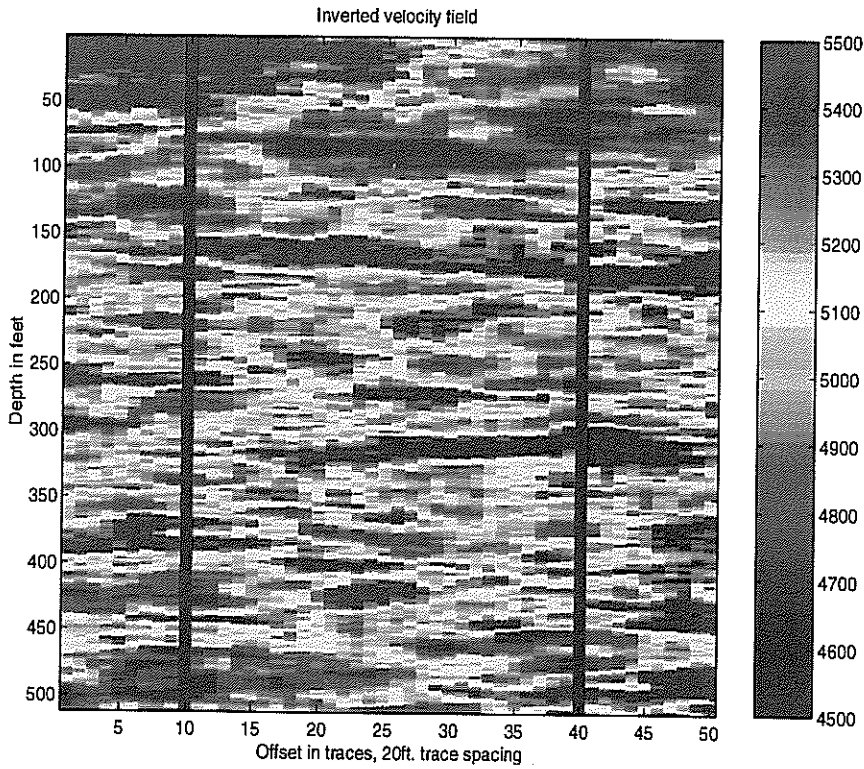Figure 5: Inversion results assuming twice true vertical correlation.

Inverted velocity field

Figure 6: Inversion results assuming half true vertical correlation.