

SENSITIVITY ANALYSIS OF AMPLITUDE VARIATION WITH OFFSET (AVO) IN FRACTURED MEDIA

Mary L. Krasovec, William L. Rodi, and M. Nafi Toksöz

Earth Resources Laboratory
Department of Earth, Atmospheric, and Planetary Sciences
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

The variation in seismic P to P reflection amplitude with offset (AVO) caused by a system of fractures embedded in an isotropic background is investigated. Additionally, a sensitivity analysis of AVO parameters with respect to the fracture system parameters is made. The fracture system is assumed to be aligned vertically or horizontally and can be gas filled or fluid filled. Elastic constants are calculated by using formulations of Schoenberg (1988). From the elastic constants, the reflection amplitude as a function of angle is calculated using equations from Ruger (1997). Theoretical results for a single interface between fractured and unfractured media, both with and without lithology change, show opportunities for extraction of crack density information from seismic P-wave data collected in fractured geothermal or hydrocarbon reservoirs. For vertically oriented fractures, wide angle data ($> 30^\circ$) is crucial for the estimation of fracture parameters.

INTRODUCTION

The behavior of seismic P to P reflection amplitude versus offset (AVO) has been extensively studied in the petroleum industry for its usefulness as a hydrocarbon indicator. Laboratory and theoretical research has shown that the behavior of seismic P to P reflection amplitudes is anomalous when gas is present in the pore spaces of porous rocks. This result is obtained theoretically by combining the Gassman (1951) equation for bulk compressibility with the Zoeppritz (1919) equations, which predict reflection amplitudes caused by the velocity and density contrasts. The Gassman equations are not readily applicable to non-porous rocks, however. There has been less research specifically aimed

at understanding and applying AVO under conditions such as those found in geothermal fields or fractured reservoirs.

In this paper we describe a numerical forward model of AVO behavior in fractured reservoirs by combining the fracture representation in Schoenberg (1988) with the AVO approximation of Ruger (1997). We discuss the possibilities of AVO data inversion schemes by working through a sensitivity analysis, showing examples based on numerical results.

THE FORWARD MODEL

The behavior of seismic waves traveling through a fractured rock can be modeled by using crack compliances (Morland, 1974) to describe an effective medium. Schoenberg (1988) calculates compliances for cracks under the assumption that there is zero traction (both normal and tangential) on the internal crack surfaces, but that the cracks are thick enough to allow non-zero, normal displacement. Schoenberg's compliances are:

$$\begin{aligned} E_N &= \frac{4e}{3\Gamma_b} \left[1 - \Gamma_b + \frac{\kappa_f}{\pi\alpha\mu_b} \right]^{-1} \\ E_T &= \frac{16e}{3(3 - 2\Gamma_b)} \end{aligned} \quad (1)$$

where Γ_b is the squared ratio of S-wave to P-wave velocity in the background, μ_b is the background shear modulus, α is the fracture aspect ratio, e is fracture density per volume, and κ_f is the bulk modulus of the inclusion fluid.

When spatial variations in elastic properties due to fractures are smaller than the shortest wavelength of interest, the background Lamé constants and the two crack compliances discussed above can be used to describe an effective transversely isotropic (TI) medium which is equivalent to the fractured rock (Schoenberg, 1988). The fractures are assumed to lie in the x_1x_2 plane, with the axis of symmetry being the vertical axis x_3 , as in Figure 1A. The effective elastic constants (denoted by subscript e) have the following dependence on the two crack compliances and the background elastic constants:

$$\begin{aligned} C_{11e} &= C_{22e} = \lambda_b + 2\mu_b - \lambda_b^2 E_N / [(\lambda_b + 2\mu_b)(1 + E_N)] \\ C_{33e} &= (\lambda_b + 2\mu_b) / (1 + E_N) \\ C_{13e} &= C_{23e} = \lambda_b / (1 + E_N) \\ C_{44e} &= C_{55e} = \mu_b / (1 + E_T) \\ C_{66e} &= \mu_b. \end{aligned} \quad (2)$$

In the case of vertical fractures, the elastic constants in equation (2) are rotated so that x_1 is the axis of symmetry, as in Figure 1B.

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$$\begin{aligned}
 C'_{11e} &= (\lambda_b + 2\mu_b)/(1 + E_N) \\
 C'_{33e} &= C'_{22e} = \lambda_b + 2\mu_b - \lambda_b^2 E_N / [(\lambda_b + 2\mu_b)(1 + E_N)] \\
 C'_{12e} &= C'_{13e} = \lambda_b / (1 + E_N) \\
 C'_{44e} &= \mu_b / (1 + E_T) \\
 C'_{55e} &= C'_{66e} = \mu_b .
 \end{aligned} \tag{3}$$

The elastic constants in equations 2 and 3 are used to calculate Thomsen's (1986) parameters (ϵ, δ, γ) in the fractured medium, then the AVO response of the interface is modeled using expressions from Ruger (1997). The three term reflectivity is given by:

$$R(\theta) = A + B \sin^2 \theta + C \sin^2 \theta \tan^2 \theta.$$

The coefficients are, in the VTI case:

$$\begin{aligned}
 A_{VTI} &= \frac{\Delta Z}{2\bar{Z}} \\
 B_{VTI} &= \frac{1}{2} \left[\frac{\Delta V_{PV}}{\bar{V}_{PV}} - \left(\frac{2\bar{V}_{SV}}{\bar{V}_{PV}} \right)^2 \frac{\Delta G}{\bar{G}} + \Delta\delta \right] \\
 C_{VTI} &= \frac{1}{2} \left(\frac{\Delta V_{PV}}{\bar{V}_{PV}} + \Delta\epsilon \right)
 \end{aligned} \tag{4}$$

where Z and G are the P and S wave impedances, respectively, V_{PV} is the velocity of a P wave traveling along the vertical axis, and V_{SV} is the velocity of a horizontally traveling S wave velocity polarized in the vertical plane. A bar over a variable indicates the mean over the two layers, and a delta (Δ) indicates the difference in the variable between the layers. In the HTI case, we investigate only waves propagating parallel or perpendicular to the fracture set (see Figure 1). For waves propagating in the isotropy plane x_2x_3 , parallel to the fractures, the AVO coefficients are:

$$\begin{aligned}
 A_{ISO} &= \frac{\Delta Z}{2\bar{Z}} \\
 B_{ISO} &= \frac{1}{2} \left[\frac{\Delta V_{PV}}{\bar{V}_{PV}} - \left(\frac{2\bar{V}_{SV}}{\bar{V}_{PV}} \right)^2 \frac{\Delta G}{\bar{G}} \right] \\
 C_{ISO} &= \frac{\Delta V_{PV}}{2\bar{V}_{PV}} .
 \end{aligned} \tag{5}$$

For waves propagating in the symmetry-axis plane x_1x_3 , perpendicular to the fracture set:

$$A_{SYM} = \frac{\Delta Z}{2\bar{Z}}$$

rock type	P-wave velocity	S-wave velocity	Poisson's ratio	density
granite	6.5 m/s	3.4 m/s	.31	2.6 g/cc
sandstone (liquid-filled)	3.4	2.0	.24	2.3
sandstone (vapor-filled)	3.2	2.0	.18	2.2
shale	4.0	2.0	.33	2.4

Table 1: Background rock velocities used for the models in Figures 2 and 3.

$$\begin{aligned}
B_{SYM} &= \frac{1}{2} \left[\frac{\Delta V_{PV}}{\bar{V}_{PV}} - \left(\frac{2\bar{V}_{SV}}{\bar{V}_{PV}} \right)^2 \left(\frac{\Delta G}{\bar{G}} - 2\Delta\gamma \right) + \Delta\delta \right] \\
C_{SYM} &= \frac{1}{2} \left(\frac{\Delta V_{PV}}{\bar{V}_{PV}} + \Delta\epsilon \right).
\end{aligned} \tag{6}$$

RESULTS AND DISCUSSION

Comparison of theoretical AVO responses for various lithologies (see Table 1) has shown that two forward model parameters dominate the AVO behavior: Fracture density and fracture fluid modulus.

We begin with the simple case of an interface between fractured and unfractured granite. The lack of lithology change at the interface isolates the effect of the fractures. Figure 2 shows AVO A, B, and C parameters as functions of crack density for such an interface.

In all cases shown in Figure 2, AVO coefficients have a nearly linear dependence on fracture density, and in most cases gas filled fractures show a greater reflectivity than fluid filled fractures. In the VTI case, the AVO parameters A and B are sensitive to fracture density, but C is near zero for all fracture densities. The HTI isotropy plane shows little reflected energy for all fracture densities. In the HTI symmetry-axis plane, the AVO intercept A is very small, while B and C increase with fracture density. Thus, far offset data are crucial in the identification and characterization of reservoirs with vertical fractures.

Figure 3 shows AVO parameters for a shale overlying a fractured sand. The sandstone is modeled as porous (15% porosity) with different background velocities calculated by the Gassman equations, when gas saturated than when liquid saturated. The lithology change causes a reflection at zero crack density, shifting the AVO parameter plots vertically compared to Figure 2. It also causes separation between the liquid and gas cases in the HTI isotropy plane and in the normal reflection coefficient A in the HTI symmetry-axis plane. However, we note that the dependence of AVO parameters on fracture density is similar whether there is a lithology change at the interface or not.

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CONCLUSIONS

Numerical models of AVO parameters in fractured media suggest that the effects of an aligned fracture set on seismic AVO is similar in different types of rocks. We also see similar dependence on fracture density at interfaces with and without lithology changes. Fracture density and inclusion fluid type information can be determined in VTI media (horizontal fractures) from AVO intercept A and gradient B, and in HTI media (vertical fractures) from the usually neglected AVO coefficient C. Near offset data and HTI data from lines parallel to fractures contain little information about vertically oriented fracture sets.

Regarding inversion of AVO, we have found that (1) in many cases the forward model variables can be reduced to fluid modulus and crack density; in general, reflection coefficients are larger for gas than for liquid filled cracks, and increase nearly linearly with crack density; (2) VTI situations can be analyzed using the usual porous rock methods involving AVO slope and intercept; and (3) the AVO C coefficient is sensitive to fracture density in the case of gas filled, vertically aligned fractures.

ACKNOWLEDGMENTS

This work was supported by the Borehole Acoustics and Logging/Reservoir Delineation Consortia at the Massachusetts Institute of Technology.

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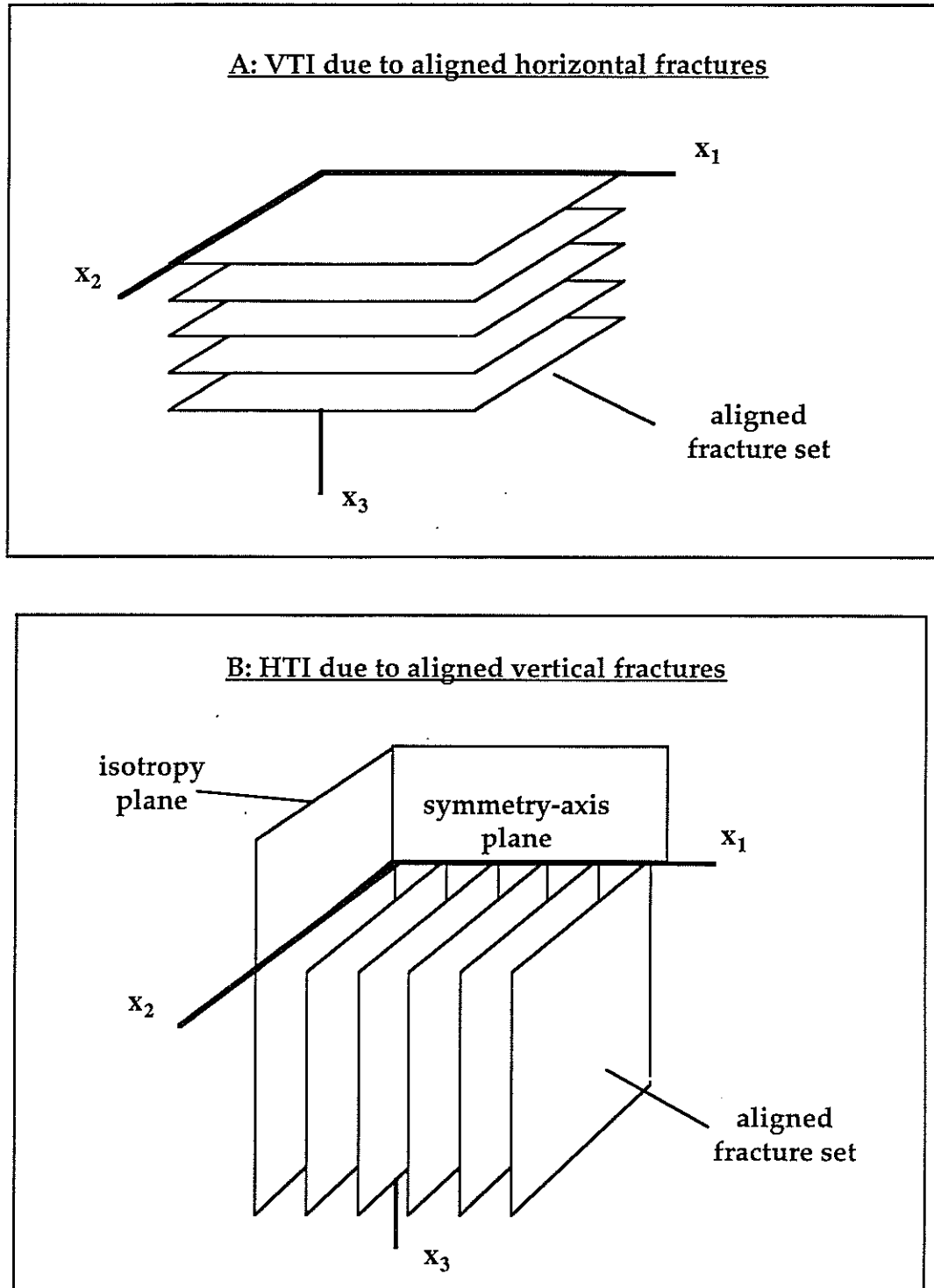


Figure 1: Fracture set orientations. (A) horizontal alignment leading to an effective VTI medium. (B) Vertical alignment leading to an effective HTI medium.

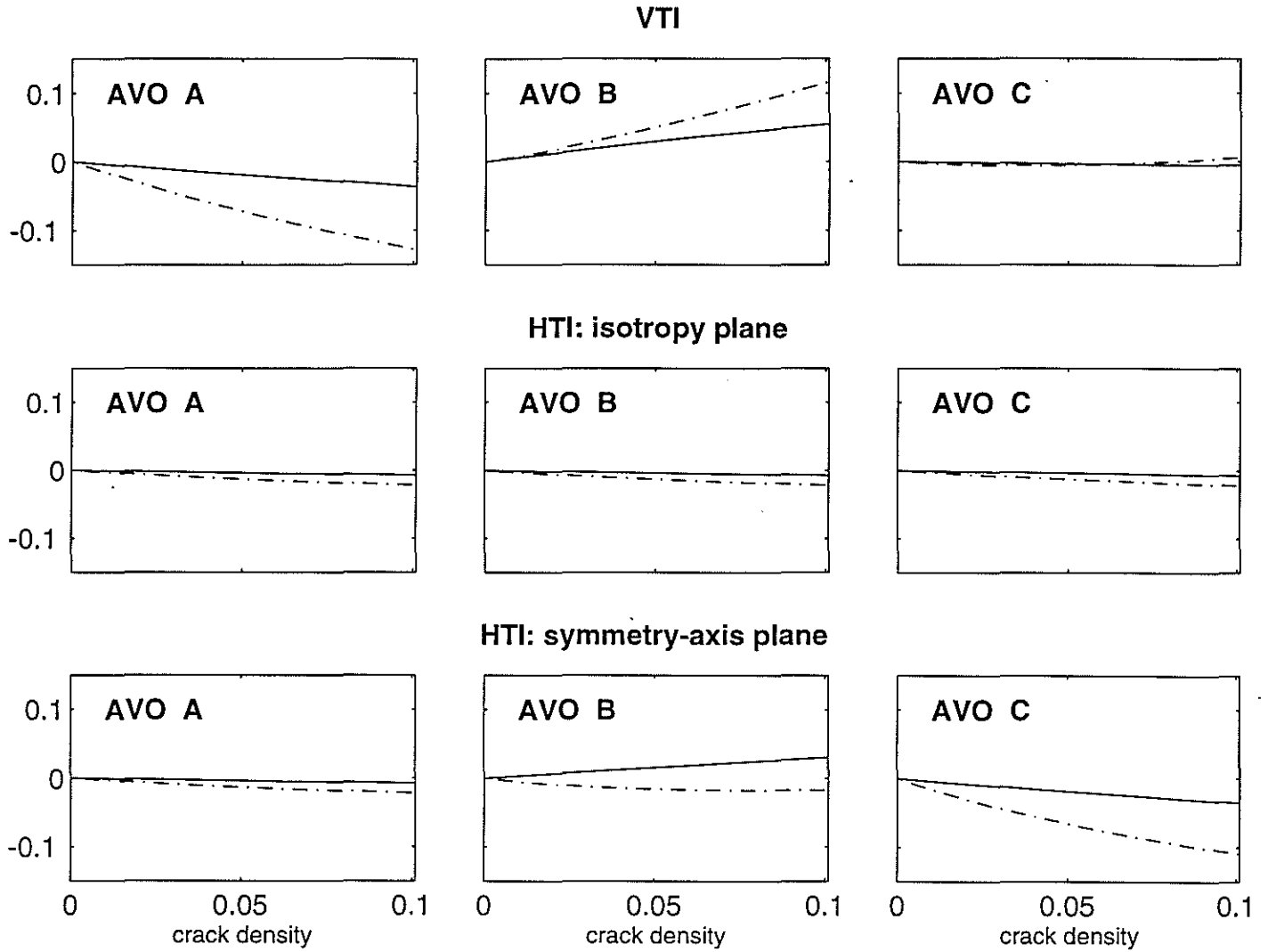


Figure 2: AVO A, B and C as functions of crack density and crack material (fluid shown as solid, gas is dashed) at an interface between solid granite and fractured granite. The fracture set can be horizontal (VTI) or vertical (HTI).

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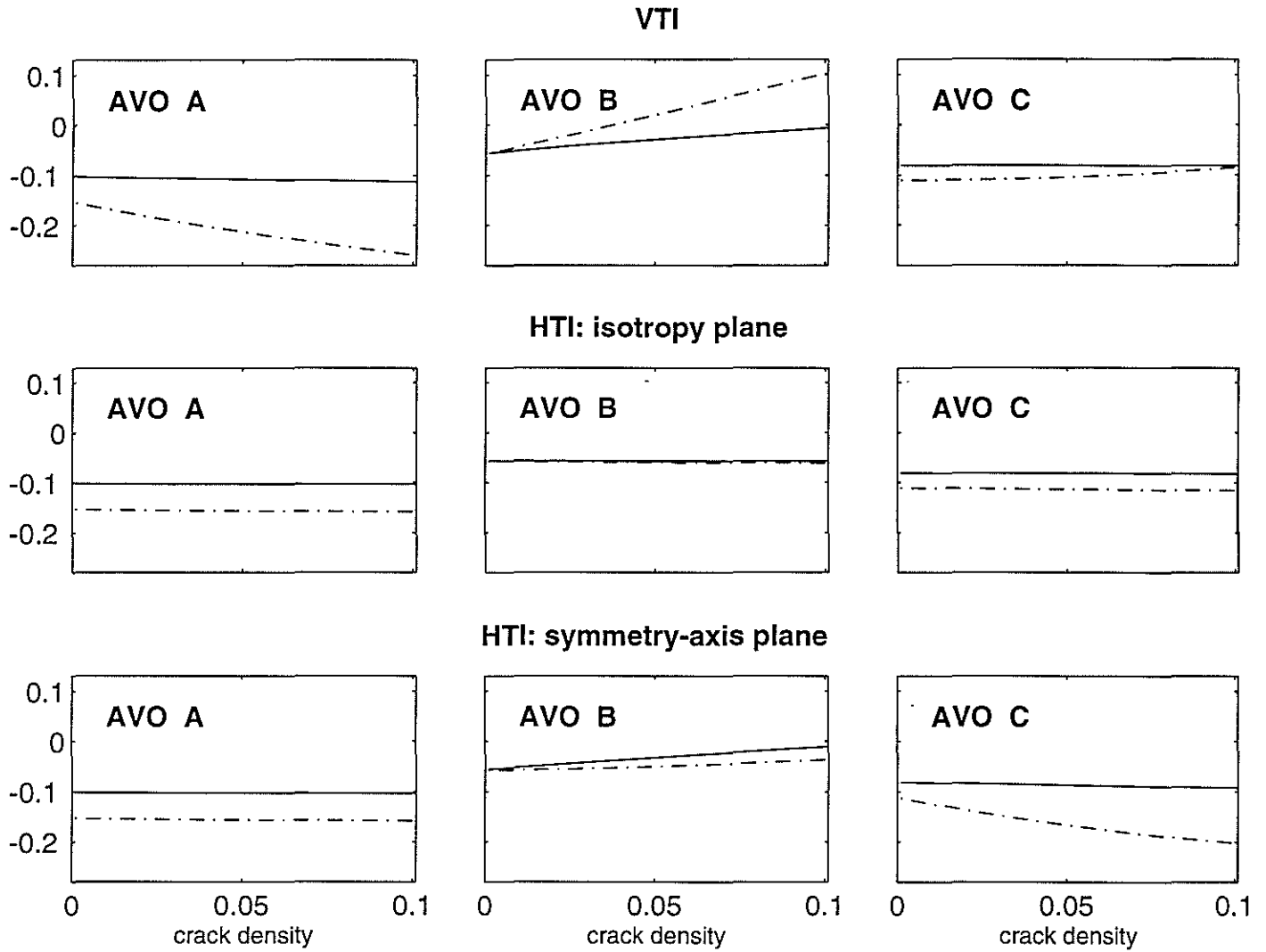


Figure 3: Similar results as in Figure 2 for an interface between a shale and a fractured sandstone. Note that the dependence of AVO parameters on crack density and crack fluid does not vary significantly because of lithology change.

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