BARGAINING WITH OPTIMISM

MUHAMET YILDIZ

1. INTRODUCTION

In real-world negotiations, the parties sometimes experience long, costly delays before reaching an agreement. In wage bargaining, the workers may strike or slow down the work before getting a new contract. The litigants may pay large sums in legal fees before reaching a settlement, and they may even end up in court. For example, Princeton University spent more than 40 million dollars in its legal defense against the Robertson family before reaching a settlement in 2008. More generally, in a large dataset on malpractice insurance cases, Watanabe (2006) finds that the settlement is delayed 1.7 years on average. Legislators may not be able to pass a necessary bill, such as a health-care reform bill, for decades. And wars may cause the death of thousands of people and scar generations while their leaders negotiate a peace agreement.

A prominent explanation for such costly delays is the parties' excessive optimism about their bargaining power in the future (Hicks (1932), Farber and Katz (1979), Shavell (1982)). The argument is simply that when the parties are excessively optimistic about the future, there may not be any agreement that can satisfy all parties' inflated expectations. In that case, there cannot be an agreement that all parties accept, making the delay inevitable. This explanation has been corroborated by a large body of evidence that suggests that optimism and self-serving biases are common (see, for example, Weinstein (1980)) and that even the seasoned negotiators exhibit these biases (Neale and Bazerman (1985) and Babcock *et al.* (1995), Babcock and Loewenstein (1997)).

While the explanation is compelling and the evidence for optimism is strong, recent studies have established that optimism plays a subtle role in bargaining and that excessive optimism alone may not explain the delays in real-world negotiations. For example, empirically, Farber and Bazerman (1989) argued that excessive optimism cannot explain the finding that the settlement rates in final-offer arbitration are higher than the settlement rates in conventional

arbitration. Theoretically, Yildiz (2003) introduces a bargaining model in which the players may be optimistic about the future bargaining power, which is modeled as the probability of making an offer in the future.[1] He shows that when the parties are to remain sufficiently optimistic for a sufficiently long future they must reach an immediate agreement in any subgame-perfect equilibrium. Hence, optimism alone cannot explain the delays observed in negotiations. Therefore, one needs a more careful analysis in order to understand the role of belief differences, such as optimism and pessimism, in bargaining.

Recently several authors have carefully examined the role of optimism in bargaining, analyzing dynamic models of bargaining in which players are optimistic about their bargaining power. In this study, I present the main findings of this literature. The rest of the introduction is devoted to a summary of these findings.

When there is a nearby deadline, the settlement is delayed to the last minute before the deadline. This *deadline effect* is commonly observed in real-world negotiations as well as in laboratory experiments (see Roth, Murnighan, and Schoumaker (1988) and the references therein). The first main finding is that the deadline effect naturally occurs in equilibrium of bargaining models with optimistic players (Simsek and Yildiz (2007)). The rationale is as follows. The cost of delay at the deadline is quite high, as the players cannot reach an agreement afterwards. Hence, in the last period, there is a wide range of individually rational agreements, and the players' bargaining power has a large impact on the terms of the settlements. Therefore, any optimism about the bargaining power in the last period is translated into a large amount of optimism about the shares at the deadline. In that case, there may not be any decision at the beginning that meets all players' inflated expectations from waiting until the deadline, in which case the players wait until the deadline to settle.

The second main finding is that when the parties can learn about their bargaining power during the negotiations, optimism may lead to long delays (Yildiz (2004)). The rationale for delay is as follows. If a Bayesian player i is optimistic about his bargaining power, then he is also optimistic that the information that they receive will vindicate his position. Hence, if players are expected to learn, an optimistic player i is also optimistic that the other player j will learn that i has a strong position in their bargaining and thereby be

---

[1] As it will be demonstrated in Section 3, in sequential bargaining without outside options, the players' equilibrium payoffs are equal to the discounted present value of all gains from trade at times when they make an offer in the future. Hence, in such a model, making an offer is the only sorce of bargaining power. In general, a player can get his bargaining power from many different sources, such as his outside options, his patience, and his ability to sway outside parties. While many of such factors can be modeled within sequential bargaining by considering a suitable stochastic process that determines the proposer at each instance, I will be agnostic about the source of bargaining power in this introduction.

persuaded to agree to i's terms. Hence, at the beginning of the negotiation when the players learn relatively quickly, each player waits in the hopes that the other player will learn and be persuaded to a reasonable agreement. As time passes, the learning slows down, and it becomes no longer worthwhile to wait for the other parties' learning. That is when they reach an agreement. This rationale for delay has been established in Yildiz (2004) in an abstract model of bargaining. The idea has been successfully applied in more applied models, such as pretrial negotiations (Watanabe (2006)), negotiation with optimism about the market conditions (Thanassoulis (2010)), and cross-license agreements (Galasso (2006)).

Note that the delay generated in this literature is significantly different from the delay due to incomplete information. First, the delay here is certain. Under the deadline effect, it is common knowledge at the beginning that the players settle only just before the deadline. Under learning, it is again common knowledge that the players will wait until t^* to settle. In contrast, in incomplete-information models, the delay is only a possibility; typically, there is a type that reaches an agreement immediately. Second, the delay here can be quite costly. Under the deadline effect, the players may lose approximately half of the pie in waiting. Under learning, they may lose approximately 17% of the pie in waiting.

The third finding is that when the optimism is persistent, the delay is short in the following sense. Under persistent optimism (without learning and deadlines), many results conclude that there must be an immediate agreement. Even in the studies that establish inefficient delays without learning and deadlines (such as Ali (2006) and Ortner (2010)), the amount of delay goes to zero in the continuous-time limit.[2] The rationale for this is as follows. When players are optimistic about the future, the range of individually rational agreements is quite narrow. Hence, the players' bargaining power does not have a large impact on the outcome. (It does not affect the outcome in cases of disagreement, and it has a small impact when there is an agreement.) Therefore, the players' optimism about their bargaining power is not fully translated into optimism about their shares in the future. In equilibrium, their optimism about the future shares becomes so small that the players reach an agreement relatively quickly–if not immediately.

The outline of the paper is as follows. In the next section, I present a static model of optimism. In this model, I present the traditional excessive-optimism explanation for disagreement and some important static applications. In Section 3, I present a dynamic model of bargaining with optimism. In Section 4, I present the deadline effect. In Section 5,

_____

[2]This is partly due to the fact that in such a limit the bargaining power becomes extremely transient (see Simsek and Yildiz (2007)).

I analyze the dynamic model under the assumption that the players do not learn about their bargaining power. In that section, I present the immediate-agreement results. In Section 6, I present the analysis in multilateral bargaining. In particular, I present Ali's (2006) result that optimism may make the backward induction unstable and cause a delay in multilateral bargaining. In Section 7, I explore the role of learning under optimism and present the main result of Yildiz (2004). In Section 8, I present some applications and empirical studies with learning and optimism, such as Watanabe (2006) and Thanassoulis (2010). In Section 9, I discuss the modeling assumptions and possible directions for future studies. Section 10 concludes.

2. Static Model

In this section, I present a simple static model of bargaining as in Nash (1950) in which the players may have heterogeneous priors on the disagreement outcome. Variations of this model have been analyzed by Landes (1971), Posner (1972), Farber and Katz (1979), and Shavell (1982). Within this model, I present the traditional excessive-optimism explanation for disagreement. The same explanation has been proposed informally for the bargaining delays in real life. I also review some of the significant applications of this model.

Let $N = \{1, 2\}$ be the set of players. The players want to make a joint decision. Let U be the set of all feasible expected utility pairs resulting from the joint decisions, and assume that U is compact and convex. If the players disagree, then the players' payoff vector is $x^d = (x_1^d,\ x_2^d)$, which is unknown. Each player has a subjective belief about x^d, and these beliefs may differ from each other. Write E_i for the expectation operator according to player i. Write also

$$d = (E_1\left[x_1^d\right], E_2\left[x_2^d\right])$$

for the expected disagreement payoff vector. Note that for each player i we take his own expectation of his continuation payoff as the expected disagreement payoff for player i.

Example 1. *In a pre-trial negotiation, one can take players 1 and 2 as the plaintiff and the defendant, respectively. The players negotiate a settlement s, which is paid to the plaintiff by the defendant. If they cannot settle, a judge (or an arbitrator) orders the defendant pay J to the plaintiff, and players incur litigation costs. Here, J is usually referred to as the judgement. In this model,*

$$U = \ \{(u_1(s),\ u_2\left(-s\right)) \mid s \in S \ \},$$

where u_i is the von-Neumann-Morgenstern utility function of player i and S is the set of possible settlement amounts. When the players are risk-neutral, $u_i(x) = x$ for each x. The

disagreement payoffs are

$$x_1^d = u_1(J - c_1) \ and \ x_2^d = u_2(-J - c_2),$$

where c_i is the litigation cost for player i.

The *contract zone* is defined as the set

$$U^d = \{u \in U \mid u \geq d\}$$

of all decisions that is at least as good as disagreement. This set is called the contract zone because the payoff vector from an agreement has to be in this set, as agreement requires the consent of both players. Following Nash (1950), the traditional models assume that the contract zone is non-empty. In that case, assuming no bargaining friction exists, one can conclude that players reach an agreement, which results in a payoff vector in the contract zone.

The contract zone may be empty in the model with heterogeneous beliefs. There may not be a decision that meets both players' expectations from disagreement even if the disagreement outcome x^d is dominated by some decision for every possible realization. This may happen when the parties' optimism about their disagreement payoffs offsets the costs associated with disagreement.

Example 2. *In the previous example, assume that the players are risk neutral. Suppose that the judgement is $\hat{J}$ if the judge finds the defendant guilty and 0 otherwise. Because of the litigation costs, the outcome in each case is dominated by a settlement. The disagreement outcome $(\hat{J} - c_1, -\hat{J} - c_2)$ in case of guilt is dominated by settlement $\hat{J}$, and the disagreement outcome $(-c_1, -c_2)$ without guilt is dominated by settlement 0. Nevertheless, the contract zone may be empty. To see this, let p_i be the probability the player i assigns to the event that the judge finds the defendant guilty. The disagreement payoff vector is*

$$d = (p_1\hat{J} - c_1, -p_2\hat{J} - c_2).$$

In this example, the players' optimism is measured by $p_1 - p_2$, the amount by which the plaintiff overestimates the likelihood of guilt according to the defendant. The contract zone is empty if and only if

(2.1) $$p_1 - p_2 > (c_1 + c_2)/\hat{J},$$

i.e.., the optimism exceeds the normalized cost of delay.

As the last example shows, when players are excessively optimistic (e.g., when $p_1 - p_2$ exceeds $(c_1 + c_2)/\hat{J}$), the contract zone may be empty, and there cannot be any decision

that can meet both players' inflated expectations. In that case the players necessarily disagree. This is the essence of the usual excessive-optimism explanation for disagreement in bargaining. This idea has been explored by Farber and Katz (1979), Shavell (1982) and Priest and Kline (1984). Note that disagreement simply follows from individual rationality and does not depend on the details of negotiation rules.[3]

Note that in this model the disagreement occurs only when it is Pareto-efficient. For otherwise the contract zone would not be empty. Therefore, although the parties agree that the disagreement is costly, given the parties' differing expectations, the outcome is the best plan they could come up with. It is tempting to generalize this finding to all bargaining models with optimism. It turns out that in dynamic models with optimism the delay can be highly Pareto-inefficient (see Remark 2 in Section 4).

Whether there is a disagreement crucially depends on the expected disagreement payoff d, which in turn is affected by the players' attitudes towards risk and the dispute-resolution mechanism used in case of disagreement. Farber and Katz (1979) analyze the role of risk-aversion. If the main uncertainty is about the way the judge or the arbitrator will rule in court, risk aversion decreases d without affecting the set U. That enlarges the contract zone and increases the settlement rate.

Much of the literature explores how agreement is affected by the role of different aspects of the legal system. For example, Shavell (1982) focuses on the allocation of legal costs, while Farber and Bazerman (1989) investigates the arbitration mechanism, comparing the final-offer arbitration, in which the arbitrator has to choose one of the offers submitted by the parties, to the conventional arbitration, in which the arbitrator can choose any decision. Empirically, the settlement rate is higher under final-offer arbitration, and Farber and Bazerman (1989) argue that the optimism alone cannot explain this fact.

More recently, Andreoni and Madoff (2007) show theoretically and experimentally that winner-take-all rules magnify the effects of optimism and diminish the likelihood of settling relative to judicial discretion.[4] This is illustrated in the following example.

---

[3]For a recent static model of negotiation with heterogenous priors and incomplete information, see Farmer and Pecorino (2002).

[4]Although the final-offer arbitration is a winner-take-all system and conventional arbitration allows judicial discretion, arbitration is distinct because the offers in the arbitration are endogenous.

Example 3 (Andreoni and Madoff (2007)). *The system in Example 2 is a winner-take-all. In that case, as we have seen, the contract zone is empty if and only if*

$$p_1 - p_2 > (c_1 + c_2)/J,$$

Suppose now that the judge assigns probability π on guilt and decides guilty if and only if $\pi > 1/2$. Let α_i be the probability density of π according to player i, so that $p_i = \int_{1/2}^1 \alpha_i(\pi) \, d\pi$. In accordance with optimism, assume that $\alpha_1(\pi) < \alpha_2(\pi)$ for $\pi < 1/2$ and $\alpha_1(\pi) > \alpha_2(\pi)$ for $\pi > 1/2$. Now consider a discretionary system in which the judge decides the judgement amount J rather than simply the guilt. In particular, he sets $J = \pi \hat{J}$. This system allows for judicial discretion. Then, the contract zone is empty if and only if

$$E_1[\pi] - E_2[\pi] > (c_1 + c_2)/\hat{J}.$$

But one can easily check that[5]

$$E_1[\pi] - E_2[\pi] \le p_1 - p_2.$$

Therefore, disagreement arises under the winner-take-all system whenever there is a disagreement under judicial discretion.

The above examples consider settlements that end the disputes, by transferring money from the defendant to the plaintiff. In practice, the negotiators sometimes choose a settlement that modifies the jury award, rather than settling the case. The settlement stipulates a high payment if the defendant is found guilty and a low payment if the defendant is found not guilty. Such high-low contracts seem counterintuitive as the parties go through costly litigation despite reaching an agreement. Prescott, Spier, and Yoon (2010) show that such high-low contracts can be optimal for risk-averse but optimistic players. Going to court allows them to bet on the outcome of the trial, utilizing the differing beliefs about the court decision, while bounding the payment by a contract insures the risk-averse parties against the extreme jury awards.

The static model here provides useful insights into the behavior of optimistic negotiators without compromising on tractability. It is also appropriate in pretrial negotiations in which the optimism is about a final decision in the court. Nevertheless, its reduced form does not allow one to investigate dynamic issues, such as the time of settlement and learning. More

---

[5]Indeed,

$$\begin{aligned} E_1[\pi] - E_2[\pi] &= \int (\alpha_1(\pi) - \alpha_2(\pi))\pi d\pi \le \int_{1/2}^1 (\alpha_1(\pi) - \alpha_2(\pi))\pi d\pi \le \int_{1/2}^1 (\alpha_1(\pi) - \alpha_2(\pi)) d\pi \\ &= p_1 - p_2, \end{aligned}$$

where the first inequality is by the assumption that $\alpha_1(\pi) < \alpha_2(\pi)$ for $\pi < 1/2$.

importantly, its insights may be misleading when the optimism is not about a decision by a judge but it is about the players' future bargaining power more broadly. In the remainder of the paper, I will review the studies that carefully explore the role of optimism in dynamic models.

3. Basic Dynamic Model

Rubinstein (1982) and Stahl (1971) have introduced a sequential bargaining model, which has been used as the canonical model of bargaining throughout economics.[6] Yildiz (2003) extends the Rubinstein-Stahl framework by allowing the players to be optimistic about their bargaining power in the future, where the bargaining power is measured by the probability of making an offer. Variations of the extended model have been used in the studies that I will review in the sequel. In this section, I will present the extended model.[7]

Two risk-neutral players want to divide a dollar. The players can strike a deal at dates in the set $T = \{t \in \mathbb{N} \mid t < \bar{t}\}$ for some $\bar{t} \leq \infty$.[8] Write $N = \{1, 2\}$ for the set of players and $U = \{u \in [0, 2]^2 \mid u^1 + u^2 \leq 1\}$ for the set of all feasible expected utility pairs.

Consider the following perfect-information game. At each $t \in T$, Nature recognizes a player $i \in N$; i offers an alternative $u = (u^1, u^2) \in U$; if the other player accepts the offer, then the game ends yielding a payoff vector $\delta^t u = (\delta^t u^1, \delta^t u^2)$ for some $\delta \in (0, 1)$; otherwise, the game proceeds to date $t + 1$, except for $t = \bar{t} - 1$, when the game ends. If no offer is accepted, then each gets 0. Write $\rho = (\rho_t)_{t \in T}$ for the recognition process, where ρ_t is the player who is recognized at date t. Write also $\rho^t \in N^t$ for a generic history of the recognized players before date t (i.e., on $\{0, 1, \ldots, t-1\}$). The players have heterogeneous beliefs about the recognition process. Write $p_t^i(\rho^s)$ for the probability player i assigns to the event that i will be recognized at date t given the history $\rho^s \in N^s$ with $s \leq t$. (Everything described in this paragraph is common knowledge.)

---

[6]See Binmore (1987) for a model with stochastic recognition process and Merlo and Wilson (1995) for the most general version of the Rubinstein-Stahl framework without incomplete information and belief heterogeneity.

[7]See Yildiz (2000) for a more general model and Yildiz (2003) for alternative models in which the players may be optimistic about their outside option or patience.

[8]I write $\mathbb{R}^k$ for a k-dimensional Euclidean space and $\mathbb{N}$ for the set of natural numbers.

Note that the recognition process is a stochastic process, in that each ρ_t is a random variable, defined over a state space. The underlying state space is fixed, and there are two possibly distinct probability distributions on the state space, one for each player.

The only departure from the framework of Rubinstein (1982) and Stahl (1971) is that there are two sets of beliefs, one for each player, and these beliefs may differ. In this model, as in Rubinstein-Stahl framework, the continuation value of a player can be written as the present value of the rents that he expects to extract when he is recognized in the future. Unlike in the Rubinstein-Stahl framework, the players here may be optimistic about their recognition in the future. Write

$$y_t(\rho^s) = p_t^1(\rho^s) + p_t^2(\rho^s) - 1$$

for *the level of optimism for t at ρ^s*. Note that $y_t(\rho^s)$ measures precisely how much a player j overestimates the probability of the event that j is recognized at date t according to the other player i. Indeed, according to i, the probability of that event is only $1 - p_t^i(\rho^s)$ while j assigns probability $p_t^j(\rho^s)$ to that event. The difference is $y_t(\rho^s)$. The players are said to be *optimistic for t at ρ^s* if $y_t(\rho^s) \geq 0$; they are said to be *pessimistic for t at ρ^s* if $y_t(\rho^s) \leq 0$.

Continuation values in Equilibrium. For finite $\bar{t}$, the bargaining game here can be solved by backward induction. Yildiz (2003) shows that even with infinite $\bar{t}$ the game is solvable by iterated elimination of conditionally-dominated strategies. (The elimination procedure is equivalent to backward induction in finite-horizon games.) This results in the following characterization of the continuation values under subgame-perfect equilibrium. (Here, continuation value at t is measured in terms of its equivalent dollar amount at t, so that the expected payoff is δ^t times the continuation value.)

Theorem 1. *For any (t, ρ^t, i), there exists a unique $V_t^i(\rho^t) \in [0, 1]$ such that, at any subgame-perfect equilibrium, the continuation value of i at the beginning of t given ρ^t is $V_t^i(\rho^t)$. Moreover,*

(3.1) $$V_t^i(\rho^t) = p_t^i(\rho^t)R_t(\rho^t, i) + \delta E^i[V_{t+1}^i \mid \rho^t],$$

where

(3.2) $$R_t(\rho^t, i) = \max\{1 - \delta S_{t+1}(\rho^t, i), 0\}$$

is the "rent" that is available to proposer at history (ρ^t, i) and

(3.3) $$S_t = V_t^1 + V_t^2$$

is the "perceived size of the pie" at the beginning of t – as a function of ρ^t.

The proof of this result can be found in Yildiz (2003). Here, $R_t(\rho^t, i)$ is the cost of delaying agreement one more period at history (ρ^t, i) under the possibly inflated expectations V_{t+1}^1 and V_{t+1}^2 from the future. When the cost is positive, they reach an agreement, and the proposer extracts the entire cost, as he is making a take-it-or-leave-it offer. When $1 - \delta S_{t+1}(\rho^t, i) < 0$, it is not possible to meet both parties' inflated expectation, and they disagree. Note that the responder is indifferent between agreement and delay. Hence, the continuation value of player i at the beginning of t is as in the difference equation (3.1): he expects that, in addition to δV_{t+1}^1, which may depend on the proposer, he will get $R_t(\rho^t, i)$ if he becomes the proposer, an event he assigns probability $p_t^i(\rho^t)$. I will describe the behavior in more detail momentarily.

Beforehand, I write the difference equation (3.1) in the integral form:

$$(3.4) \qquad\qquad V_t^i(\rho^t) = \sum_{s \geq t} \delta^{s-t} E^i[1_{\rho_s = i} R_s \mid \rho^t].$$

That is, the continuation value of i at the beginning of a period is the present value of all the rents he expects to extract as a proposer in the future. Hence, the recognition process is the only source of bargaining power in this model (and in other sequential bargaining models without outside option).[9]

There are at least three distinct reactions to this result. First, some take this result literally suggesting that "power to propose" is indeed the main source of bargaining power (see for example Baron and Ferejohn (1989)). This interpretation is reasonable in certain formal environments, such as congressional bargaining, where the congresspeople with power to shape the agenda have significant impact on the outcomes. Second, some take this result as a shortcoming of the model. In this view, the outcome ought to be determined by the actual power each party has in terms of what they can bring to the table and what they can do the other parties, rather than a procedural detail that is no more than a modeling device in many real-world negotiations with no procedure. A natural extension of this view is to endogenize the recognition process by allowing players strategically decide when to make an offer (see Perry and Reny (1993) and Sakovics (1993)). Since the proposers extract a rent, the parties rush to make an offer in these models. (See also Smith and Stacchetti (2001) for other interesting behavior in continuous time.) A third approach, which I subscribe to in this study, takes (3.1) and (3.4) to suggest that the recognition process is a way to model

---

[9]With outside options, the outside options also affect the players' equilibrium payoffs—only when the value of outside option exceeds the equilibrium share without the outside option. Discount factors also affect the equilibrium payoff in an intuitive way, but the effect of discount factor is mathematically equivalent to a change in the recognition process.

parties' actual bargaining power using the tools of game theory. As in Nash (1950), at each instance, each player has some relative bargaining power, which is defined as the share he would get from the gain from trade if they were to strike a deal at that moment. In light of (3.1), one models such a relative bargaining power by probability of making an offer at that moment. The bargaining power itself is determined by the forces on the ground. As the situation changes, the bargaining power changes, leading to a stochastic process (see Simsek and Yildiz (2007)).

Agreement and Disagreement Regimes in Equilibrium. I now describe the equilibrium behavior in a greater detail. There are two cases. The first one, namely the *disagreement regime*, is characterized by the inequality

$$\delta S_{t+1}(\rho^t, i) > 1.$$

In that case, the players do not reach an agreement at history (ρ^t, i). Indeed, if they do not agree at (ρ^t, i), then the continuation value of each player k at $t+1$ will be $V_{t+1}^k(\rho^t, i)$. In order for player k to agree on a division (u^1, u^2) of the dollar at t, he must be given at least $\delta V_{t+1}^k(\rho^t, i)$. That is, we must have $u^k \geq \delta V_{t+1}^k(\rho^t, i)$. Since an agreement requires the approval of both parties, this requires

$$u^1 + u^2 \geq \delta V_{t+1}^1(\rho^t, i) + \delta V_{t+1}^2(\rho^t, i) = \delta S_{t+1}(\rho^t, i).$$

Since there is only one dollar and $\delta S_{t+1}(\rho^t, i) > 1$, such u is not feasible. In other words, when $\delta S_{t+1}(\rho^t, i) > 1$, it is not possible to meet both parties' inflated expectations from the future, and the players cannot reach an agreement at history (ρ^t, i). In that case, when player i is recognized at the beginning of t, players anticipate that there will be no agreement at t, and each player's continuation value is the present value of waiting until date $t+1$. The available rent for the proposer is 0.

The second case is called the *agreement regime*, and characterized by the inequality

$$\delta S_{t+1}(\rho^t, i) \leq 1.$$

In that case, if they have not yet reached an agreement, the players agree at history (ρ^t, i) on a division that gives $1 - \delta V_{t+1}^j(\rho^t, i)$ to the proposer i, leaving the other player j his continuation value $\delta V_{t+1}^j(\rho^t, i)$. Note that the proposer gets more than his continuation value $\delta V_{t+1}^i(\rho^t, i)$ from delaying the agreement one more period. The difference,

$$R_t(\rho^t, i) = 1 - \delta V_{t+1}^j(\rho^t, i) - \delta V_{t+1}^i(\rho^t, i) = 1 - \delta S_{t+1}(\rho^t, i),$$

is the rent for the proposer.

4. Bargaining Delays under a Deadline—Deadline Effect

In this section, I take the deadline $\bar{t}$ finite and show that, when he players are sufficiently optimistic about their bargaining power at $\bar{t}-1$, they wait until $\bar{t}-1$ to reach an agreement. I then discuss a couple of basic properties of bargaining delays caused by optimism.

In real-world negotiations with a deadline, the agreement is often delayed until the very last minute before the deadline. This behavior is called the *deadline effect*. It is so common in real-world negotiations that there are multiple names for such agreements, such as *eleventh-hour agreement* and *settlement on the courthouse steps*. It is also commonly observed in laboratory experiments (see Roth, Murnighan, and Schoumaker (1988) and the references therein). The next result shows that the deadline effect naturally arises under optimism.

Theorem 2 (Simsek and Yildiz (2007)). *Assume that $\bar{t}$ is finite. If*

$$y_{\bar{t}-1}\left(\rho_0, \ldots, \rho_t\right) > (1 - \delta^{\bar{t}-1-t})/\delta^{\bar{t}-1-t} \qquad \left(\forall \left(\rho_0, \ldots, \rho_t\right) \in N^{t+1}, \forall t < \bar{t}-1\right),$$

then, in equilibrium, the players disagree at each $t < \bar{t}-1$ and reach an agreement at $\bar{t}-1$.

Proof. After the deadline, the players automatically receive zero: $V_{\bar{t}} = 0$. Hence, at $\bar{t}-1$, they reach an agreement in which the proposer gets everything. At any date $t < \bar{t}-1$, this leads to a disagreement. Indeed, at any history $(\rho_0, \ldots, \rho_t)$, since a player i can wait until $\bar{t}-1$, his continuation value from disagreement at t must be at least $\delta^{\bar{t}-1-t} p_{\bar{t}-1}^i\left(\rho_0, \ldots, \rho_t\right)$. In order to meet both parties' expectations, an agreement then requires

$$\delta^{\bar{t}-1-t} p_{\bar{t}-1}^1\left(\rho_0, \ldots, \rho_t\right) + \delta^{\bar{t}-1-t} p_{\bar{t}-1}^1\left(\rho_0, \ldots, \rho_t\right) = \delta^{\bar{t}-1-t}\left(1 + y_{\bar{t}-1}\left(\rho_0, \ldots, \rho_t\right)\right).$$

This exceeds 1 by the hypothesis. Therefore, there is a disagreement regime at t. $\qquad\square$

Recall that in Theorem 2, $y_{\bar{t}-1}\left(\rho_0, \ldots, \rho_t\right)$ is the level of optimism about $\bar{t}-1$ at history $(\rho_0, \ldots, \rho_t)$, while the threshold $(1 - \delta^{\bar{t}-1-t})/\delta^{\bar{t}-1-t}$ is the normalized cost of delaying the agreement from t to $\bar{t}-1$. Theorem 2 establishes that when the parties' optimism $y_{\bar{t}-1}$ about their bargaining power in the last period exceeds the cost, they wait until the very last period before the deadline to settle.

This result provides a simple rationale for the deadline effect. Since the players cannot reach an agreement after the deadline, the cost of delay is very large just before the deadline. Indeed, any division is individually rational. Hence, at $\bar{t}-1$, the players' bargaining power has a large impact on the terms of agreement, and the players' optimism about their bargaining power is directly translated into optimism about their shares. If the players are sufficiently optimistic about their bargaining power at $\bar{t}-1$, they become so optimistic about their

shares at $\bar{t}-1$ that no agreement can meet their expectations from waiting until $\bar{t}-1$. This results in the behavior described by the deadline effect.

In a model with durable bargaining power and stochastic deadlines, Simsek and Yildiz (2007) show that the strength of the deadline effect is increasing in optimism and the "durability" of bargaining power and decreasing in the amount of the uncertainty regarding of the arrival of deadline.

Theorem 2 exhibits some remarkable properties of delay under optimism. I will next discuss these properties and compare the delay here to the delay in usual models of bargaining.

Remark 1 (Delay is common knowledge). *In Theorem 2, it is common knowledge at the beginning that the players will not be able to reach an agreement before the last period. Despite this, they cannot reach an agreement because each player hopes that if they wait until the last period he will be vindicated and get a very good deal that will compensate him for the costs he incurs. In contrast, delay in bargaining models with incomplete information is only a possibility. In those models, there is often a type that reaches an agreement immediately.*

The delay in Theorem 2 is typically inefficient, as illustrated in the following simple case.

Example 4 (Delay is inefficient). *For all i, ρ^s and $t \geq s$, take $p_t^i(\rho^s) = p_t$ for some $p_t \in [0,1]$. Assume that $2p_{\bar{t}-1} > 1/\delta^{\bar{t}-1}$. Theorem 2 then concludes that the players wait until $\bar{t}-1$, when the proposer gets the entire dollar. Now consider the following contract: the players wait until $t = 1$ and the proposer at $t = 1$ gets the entire dollar. In expectation, this contract gives each player δp_1, while each player gets only $\delta^{\bar{t}-1} p_{\bar{t}-1}$ in equilibrium. The contract Pareto-dominates the equilibrium outcome whenever $p_1 > \delta^{\bar{t}-2} p_{\bar{t}-1}$, a condition that is easily satisfied when $\bar{t}$ is large.*

Remark 2 (Delay is inefficient). *Example 4 illustrates the general fact that the delay in dynamic bargaining models with optimism is typically Pareto-inefficient. In contrast, in the static model of Section 2, the disagreement occurs only when it is efficient. Likewise, in sequential bargaining models with complete information, delay arises in a Markov-perfect equilibrium only when it is Pareto efficient to wait; the only form of inefficiency in such a model is the lack of sufficient delay (Merlo and Wilson (1995)).*

5. Immediate Agreement with No Learning

When there is a nearby deadline, the optimistic players may delay the agreement to the very last period. Yildiz (2003) shows that if players are to remain optimistic for a sufficiently long future, then in equilibrium they reach an agreement *immediately*. Yildiz (2003) and

Ortner (2010) have obtained similar immediate-agreement results. These results show that optimism plays a subtle role in bargaining, and optimism alone cannot explain the bargaining delays. In this section, I present these results.

I maintain the following assumption, which states that players do not learn about the future recognitions as they observe which player gets to make an offer and when.

Assumption IND. *The players perceive the recognition process ρ to be independently distributed: $p_t^i(\rho^s) = p_t^i(\hat{\rho}^{s'})$ for all $(\rho^s, \hat{\rho}^{s'}, t, i)$ with $t \geq \max\{s, s'\}$.*

Under this assumption, p, y, V, S, and R are all deterministic. Hence, whether there is an agreement regime at a given date does not depend on the history. This simplifies the analysis dramatically.

In any disagreement regime, by (3.1), $V_t = \delta V_{t+1}$, and hence $S_t = \delta S_{t+1}$. Since $S_{t+1} \leq 2$, this implies that an interval of disagreement regimes can be at most as long as $\bar{L}(\delta)$ defined by $1 < 2\delta^{\bar{L}(\delta)} \leq 1/\delta$, yielding a uniform bound on possible delays. Note that the delay can be quite large: nearly half of the pie can be lost during the delay.

In any agreement regime, (3.1) becomes $V_t^i = p_t^i R_t + \delta V_{t+1}^i$, where $R_t = 1 - \delta S_{t+1}$. Adding this equation up for players yields

(5.1) $$S_t = 1 + y_t R_t.$$

This equation gives the main relation between the relative bargaining powers and the bargaining shares. It states that the discrepancy $S_t - 1$ between the perceived size of the pie and the actual size is proportional to the level y_t of optimism and to the rent $R_t = 1 - \delta S_{t+1}$ at t. When S_{t+1} is small, the rent R_t is large. In that case, the range of individually rational trades is large. The players' relative bargaining powers then affect the shares significantly, as the shares can vary as much as $1 - \delta S_{t+1}$. Then, the optimism y_t about the bargaining power is translated to the significant amounts of optimism about the shares, namely $y_t R_t$. Consequently, $y_t R_t$ may be so large that δS_t becomes larger than 1, causing a delay at $t-1$. On the other hand, when S_{t+1} is large, the rent R_t is small, allowing a narrow range of possible individually-rational trades. In that case, the bargaining power does not have a large impact on the shares at t. In that case, the level of optimism y_t about the bargaining power is translated to optimism about shares with multiplication by R_t, scaling down the amount of optimism about the shares significantly. Based on the above equation, the following lemma provides the main step.

Lemma 1. *Assume IND. Given any t with $y_t \geq 0$, if $S_{t+1} \in [1, 1/\delta]$, then $S_t \in [1, 1/\delta]$.*

Proof. Assume that $S_{t+1} \in [1, 1/\delta]$. Then, there is an agreement regime at t. Since $R_t = 1 - \delta S_{t+1} \in [0, 1 - \delta]$ and $y_t \in [0, 1]$, (5.1) yields $S_t = 1 + y_t R_t \in [1, 2 - \delta]$. Note that $2 - \delta \leq 1/\delta$. □

Lemma 1 can be spelled out as follows. Consider a date t at which the players are expected to reach an agreement (i.e., $S_{t+1} \leq 1/\delta$), but the expectations from the future are relatively high: $S_{t+1} \geq 1$. Lemma 1 first establishes that, since the expectations are high, the rent for the proposer is so low that the players prefer agreeing at $t-1$ to getting this rent at t. That is, $S_t \leq 1/\delta$. Secondly, the lemma establishes that, since the players are optimistic for t, their expectations about their shares at t are high: $S_t \geq 1$. This, of course, in turn leaves a small rent at $t-1$, so small that the prospect of getting the rent does not entice the players to delay the agreement at $t-2$. Iterative application of Lemma 1 then yields the following immediate-agreement theorem, which is the main result in Yildiz (2003).

Theorem 3 (Yildiz (2003)). *Assume IND. For any $\hat{t} \in T$, if $y_t \geq 0$ for each $t \leq \hat{t}$, then there is an agreement regime at each $t \in T$ with $t < \hat{t} - \bar{L}(\delta) - 1$.*

Proof. First note that, since $y_{\hat{t}} \geq 0$, $S_{\hat{t}} \geq 1$. There are two cases then. First consider the case $S_{\hat{t}} \leq 1/\delta$. In that case, $S_{\hat{t}} \in [1, 1/\delta]$. Hence, using Lemma 1 inductively, one can conclude for each $t \leq \hat{t} - 1$ that $S_{t+1} \in [1, 1/\delta]$ and hence there is an agreement regime. Now consider the case that $S_{\hat{t}} > 1/\delta$. In that case, there is an interval of disagreement regimes of length $L(S_{\hat{t}}, \delta) \leq \bar{L}(\delta)$ that ends at $\hat{t} - 1$. Now, assuming that $\hat{t}$ is sufficiently large, consider the last date with an agreement regime before $\hat{t} - 1$, namely $\tilde{t} = \hat{t} - 1 - L(S_{\hat{t}}, \delta)$. By definition, $S_{\tilde{t}+1} \leq 1/\delta$ and $S_{\tilde{t}+2} > 1/\delta$. By the latter inequality, $S_{\tilde{t}+1} = \delta S_{\tilde{t}+2} > 1$, i.e., $S_{\tilde{t}+1} \in [1, 1/\delta]$. Once again, using Lemma 1 inductively, one can conclude that $S_{t+1} \in [1, 1/\delta]$ at each $t \leq \tilde{t}$, showing that there is an agreement regime at each $t \leq \tilde{t}$. □

The main idea of Theorem 3 is illustrated in Figure 1. There may be an interval of disagreement regimes near the end of the game. Nevertheless, in such periods of disagreement, the players anticipate that they will not be able to reach an agreement, and hence their bargaining power does not have any value. If the anticipated delay is too long, then they would rather reach an agreement than commence a long delay, even if each player expects a high share at the end. This results in a uniform bound $L(\delta)$ on the length of such an interval of disagreement regimes. Now consider the day t just before the delay starts. Starting from the next day, the players are so optimistic that they would rather go through a long delay than reach an agreement, i.e., $S_{t+2} > 1/\delta$. Then, they must still have high expectations from future at t, even if their expectations are not so high that they wait. Indeed, $S_{t+1} = \delta S_{t+2} > 1$. In that case, the range of individually-rational agreements is small, and
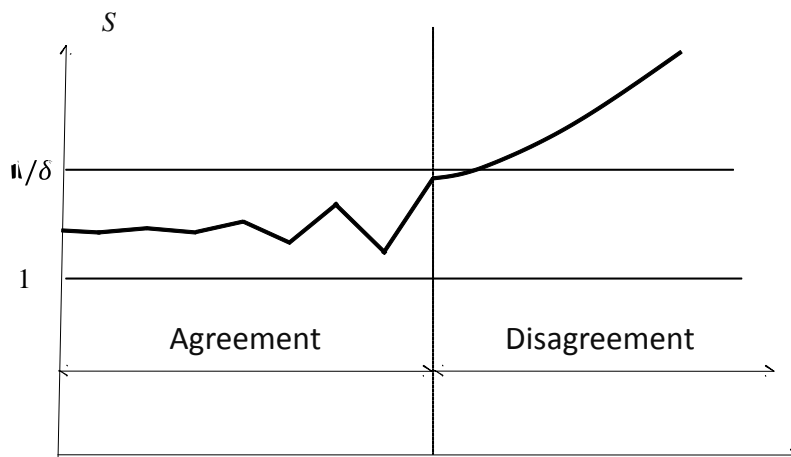
FIGURE 1. An illustration of Theorem 3, the main result of Yildiz (2003)

the the players' optimism about their bargaining power translates into a small amount of optimism about their shares. As illustrated by Lemma 1, this results in an agreement regime at each date prior to t.

It is crucial for this result that the optimism is persistent. As in Theorem 2, if the level of optimism drops suddenly at some t^*, then the players may wait until t^* to settle. (For example, if $y_t = -1$ for all $t > t^*$, each player thinks that he will not make an offer after t^* and behaves as if there is a deadline at t^*.) Yildiz (2003) shows that if there are no sharp declines in the level of optimism, there is an immediate agreement.

Ortner (2009) extends the optimism model in this section by allowing the size of the pie to be stochastic as in Merlo and Wilson (1995). He shows that the unique, subgame-perfect equilibrium may involve inefficient delays. He further shows that as the time delay between two consequent offers goes to zero, the length of delay goes to zero.

The immediate-agreement results here are not meant to refute the role of optimism in bargaining delays. They are meant to refute the naive idea that the agreement is delayed simply when the optimism is excessive. They illustrate that optimism alone cannot cause a delay. Whether it causes a delay depends on the details of how optimism varies with time and how it interacts with other factors, such as a deadline (or learning as we will see later). Therefore, if one wishes to understand the role of optimism in bargaining, he must carry out a careful analysis. His analysis must be more careful than the analysis of usual models because there is little received experience about the models with heterogeneous priors.

6. Multilateral Bargaining–Waiting to Settle

This section presents the main result of Ali (2006): in multilateral bargaining there may be some delay even under constant level of optimism because the backward induction process becomes unstable. I will also briefly present the main idea of a result by Galasso (2010) that establishes that in multilateral bargaining, optimism may increase or decrease the amount of delay depending on which aspect of bargaining power the player is optimistic about.

Take $n \geq 3$ players, and assume that the level of optimism is constant:

$$y_t \equiv p_t^1 + \cdots + p_t^n - 1 = \bar{y} \qquad (\forall\, t)$$

for some $\bar{y} > 0$. In order to avoid the "folk-theorem style" multiple equilibria in multilateral bargaining, Ali (2006) focuses on the finite-horizon case.

Consider a date t with agreement regime:

$$\delta S_{t+1} \leq 1.$$

The recognized player offers the other players their continuation values and keeps the rest for himself. Therefore, as in the previous section,

$$S_t = 1 + \bar{y}(1 - \delta S_{t+1}).$$

The backwards-difference equation is stable if and only if $|\bar{y}| < 1/\delta$. By definition,

$$\bar{y} \leq n - 1.$$

For $n = 2$, as in the previous section, this implies that $\bar{y} < 1/\delta$. In that case, S is a contraction mapping (backwards) and has an absorbing region with agreement. On the other hand, when $n > 2$ and δ is large, one can have

$$\bar{y} > 1/\delta.$$

In that case, S is exploding. Define $\bar{S} = (1 + \bar{y})/(1 + \delta\bar{y}) \in (1, 1/\delta)$ as the fixed point of the above equation, so that

$$S_t - \bar{S} = -\delta\bar{y}(S_{t+1} - \bar{S}).$$

When stable ($\delta\bar{y} \in (-1, 1)$), S_t converges to $\bar{S}$. But when $\delta\bar{y} > 1$, S_t goes away from $\bar{S}$ in the backward induction. Hence, unless $S_t = \bar{S}$, which happens only in knife-edge cases, S_t eventually goes outside of the agreement region, becoming $\delta S_t > 1$. In that case, there is a disagreement at $t - 1$.
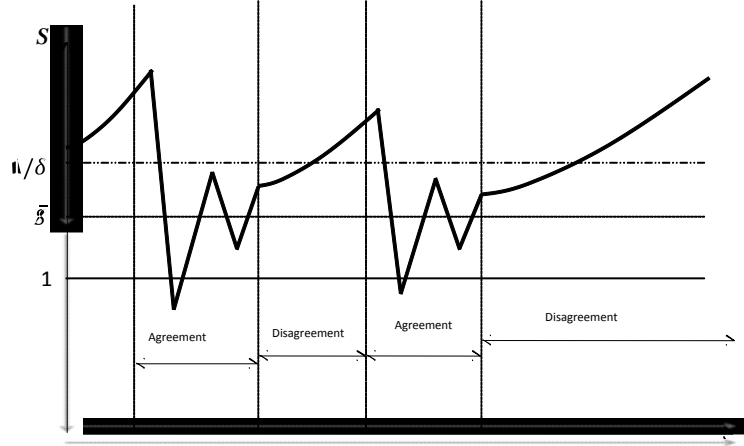
FIGURE 2. An illustration of Theorem 4, the main result of Ali (2006)

The dynamics in the disagreement regions remains as before:

$$S_t = \delta S_{t+1}.$$

Backward induction eventually takes players back to the agreement regime, where the unstable process starts all over again.

Figure 2 illustrates the behavior under $\bar{y} > 1/\delta$. There are periods that are "ripe for a settlement." These periods are separated by periods in which the players necessarily disagree. In the latter period, the players wait to settle in the next time where agreement is possible.

The following result must be clear from the previous discussion:

Theorem 4 (Ali (2006)). *Assume that $\delta\bar{y} > 1$. Then, for each $\tilde{t}$, there exists $\bar{t} > \tilde{t}$ such that there is a disagreement regime at $t = 0$.*

The inefficiency caused by the delay described in the previous result goes to zero as $\delta \to 1$, showing that the delay is much shorter than the one caused by a transient optimism, where half of the pie may disappear due to the delay. To see this, note that $S_{t+1} \geq 1$. Hence,

$$S_t \leq 1 + \bar{y}(1 - \delta).$$

Therefore, the length of any delay is uniformly bounded by

$$\tilde{L}(\delta, \bar{y}) = \frac{\log(1 + \bar{y}(1 - \delta))}{\log(1/\delta)}.$$

As $\delta \to 1$, $\delta^{\tilde{L}(\delta,\bar{y})} \to 1$.

The above finding appears to be quite general. Several other results, such as the results of Yildiz (2003) and Ortner (2009), establish that under perpetual optimism without learning,

the amount of delay due to optimism disappears as the players make the offers more and more frequently. There are two pieces of intuition for these results. First, these results consider the optimism about the instantaneous bargaining power in each period, which determines the allocation of the gain from not delaying the agreement one more period. That gain, however, goes to zero in that limit, as it is less than $1 - \delta$ under optimism. Of course, there are more periods to be optimistic about, and the amount of total gain remains constant. The second and deeper intuition is that, as in Theorem 3, the effect of optimism about the future is muted by strategic considerations, making the effect of the optimism about the bargaining power in future periods relatively negligible despite the large number of such periods.

Optimism with Trade Externalities. Ali (2006) has considered multilateral bargaining with a collective decision as in congressional bargaining. Galasso (2010) considers a situation in which a seller bilaterally negotiates with multiple buyers in order to sell a good to one of them. He shows that when there are trade externalities, the nature of optimism can be important in its effect on delay. He shows that optimism about the future trade opportunities may indeed decrease the amount of delay. The following example illustrates his result.

Example 5 (Galasso (2010)). *A firm is considering opening a factory in one of the two neighboring cities $i \in \{1, 2\}$. The value of the factory is 1 for the city in which it is located and θ in the neighboring city. The firm is negotiating the amount of municipal concession p it gets from the city where the factory will be located. The payoffs of the firm and the city are p and $1 - p$, respectively. There are only two periods. In the first period, the firm makes an offer to City 1. The firm finds it equally likely that it will negotiate with any of the cities in the second period and assigns probability $1/2$ to making an offer at that period. Each city assigns probability b for being approached in the second period and probability q for making an offer if approached. Note that b measures the optimism about the future trade opportunities, while q measures the optimism about the future bargaining power. In the second period, the proposer gets the entire gain from trade. Hence, at the end of the first period, the continuation value of the firm is $\delta/2$, and the continuation value of City 1 is $\delta(bq + (1-b)\theta)$. Hence, the agreement is delayed if*

$$bq + (1 - b)\theta > 1/\delta - 1/2.$$

Optimism q about future bargaining power always contributes towards a delay. When $\theta = 0$, optimism b about the future trade opportunities also contributes towards a delay. When $\theta > q$, however, optimism b about the future trade opportunities actually helps avoiding delay.

In a general dynamic model of negotiation, Galasso (2010) shows that the result in this example holds more generally, and the optimism about future trading opportunities may shorten the delay while optimism about the future bargaining power in terms of making an offer weakly increases the delay.

7. Learning under Optimism–Waiting to Persuade

Many of the results above have established that optimism alone cannot explain the bargaining delays. In a tractable learning model, Yildiz (2004) shows that there is a predetermined settlement date t^* such that the players wait until date t^* to settle. In this section I will present this result and explain the rationale it provides for bargaining delays, a rationale that is based on optimism *and* learning.

Consider the following simple form for the beliefs. Fix positive integers $\bar{m}_1$, $\bar{m}_2$, and n with $1 \le \bar{m}_2 < \bar{m}_1 \le n - 2$. Write (m, t) for the history ρ^t (at the beginning of date t) in which player 1 has made m offers and player 2 has made $t - m$ offers. Assume that, for any date s with $s \ge t$, at history (m, t) player i assigns probability

$$(7.1) \qquad\qquad P_i(\rho^s = 1 \mid \rho^t) = (\bar{m}_i + m)/(t + n)$$

to the event that Player 1 will make an offer at date s. This belief structure arises when each player believes that recognition at different dates are identically and independently distributed with some unknown probability μ of Player 1 making an offer at any date t, and μ is distributed with a beta distribution with parameters $\bar{m}_i$ and n.

The beliefs $p_s^i(m, t)$ take the following simple form:

$$
\begin{aligned}
p_s^1(m, t) &= \frac{\bar{m}_1 + m}{t + n} \equiv p^{t,1}(m) \\
p_s^2(m, t) &= 1 - \frac{\bar{m}_2 + m}{t + n} = p^{t,2}(m).
\end{aligned}
$$

Note that the period t beliefs about the recognition at future period s depend only on t—not s. Hence, optimism is measured at the time the beliefs are held without distinguishing which future recognition these beliefs are about. Write

$$y^t(m) \equiv y_s(m, t) = p_s^1(m, t) + p_s^2(m, t) - 1$$

for *the level of optimism at* (m, t). Note that

$$(7.2) \qquad\qquad y^t(m) = \frac{\bar{m}_1 - \bar{m}_2}{t + n} \equiv \frac{\Delta}{t + n} > 0$$

where $\Delta = \bar{m}_1 - \bar{m}_2$. Since $y^t(m) > 0$, the players are optimistic at each (m, t).

The level y^t of optimism is *deterministic*, i.e., y^t does not depend on m. (I will suppress m whenever a process is deterministic.) Yildiz (2004) shows that this results in deterministic perceived size S_t of pie and rent $R_t = \max\{1 - \delta S_{t+1}, 0\}$.

Consequently, (3.4) simplifies to

$$(7.3) \qquad\qquad V_t^i(m) = p^{t,i}(m)\Lambda_t$$

where

$$(7.4) \qquad\qquad \Lambda_t = \sum_{s=t}^{\infty} \delta^{s-t} R_s$$

is the present value of all future rents. The perceived size of the pie is

$$(7.5) \qquad\qquad S_t = (1 + y^t)\Lambda_t.$$

Notice that although S_t and Λ_t are deterministic, V_t^i is not deterministic. Indeed, the continuation value of a player i is proportional to the probability $p^{t,i}(m)$ that he assigns to making offers in future dates. This probability is an affine function of the number of times i has made an offer in the past.

The main objective of the analysis is to explore when there is an agreement regime (i.e. $\delta S_t \leq 1$) and when there is a disagreement regime (i.e. $\delta S_t > 1$). Since S is deterministic, whether there is an agreement regime is a function of time and does not depend on the history. By (7.5), there is an agreement regime at any $t - 1 \in T$ if and only if

$$(7.6) \qquad\qquad \Lambda_t \leq \frac{1}{\delta(1 + y^t)} \equiv D_t.$$

Since both Λ_t and D_t are deterministic, (7.6) implies that the settlement date t^* must be deterministic.

The next result, which is the main result of Yildiz (2004), states this fact and provides upper and lower bounds for the settlement date.[10] Note that the bounds are determined by the speed of learning, which is measured by the decline $y^t - y^{t+1}$ in optimism.

Theorem 5 (Yildiz (2004)). *There exists a predetermined date t^* such that, in equilibrium, players do not agree at any date $t < t^*$, and they reach an agreement at t^*. The settlement date t^* is common knowledge at the beginning of the game in equilibrium. Moreover,*

$$t_l \leq t^* \leq \max\{t_u, 0\}$$

---

[10]The result is stated differently because it corrects an algebraic mistake in the proof of Yildiz (2004). I thank Alex Wolitzky for realizing the mistake.

where

$$y^{t_u} - y^{t_u+1} = (1-\delta)/\delta$$
$$y^{t_l} - y^{t_l+1} = 2(1-\delta)/\delta.$$

Proof. Define $t^* = \min\{t \mid \Lambda_{t+1} \leq D_{t+1}\}$. Since both Λ and D are deterministic (i.e. independent of m), so is t^*. By (7.6), the players disagree on dates $t < t^*$ and settle at t^*.

To derive the lower bound t_l, I first establish a lower bound for Λ:

(7.7) $$\Lambda_t \geq 1/(1 + y^{t+1}).$$

To see the inequality, first consider a disagreement regime, so that $\delta S_{t+1} > 1$. In that case,

$$\Lambda_t = \delta \Lambda_{t+1} = \delta S_{t+1}/(1 + y^{t+1}) > 1/(1 + y^{t+1}),$$

where the first equality is by (7.4) and by the fact that $R_t = 0$ in a disagreement regime, the second equality is by (7.5), and the inequality is by $\delta S_{t+1} > 1$. Now, consider an agreement regime, i.e., $\delta S_{t+1} \leq 1$. In that case,

$$\Lambda_t = 1 - \delta S_{t+1} + \delta \Lambda_{t+1} = 1 - \delta S_{t+1} + \frac{\delta S_{t+1}}{1 + y^{t+1}} = \frac{1 + y^{t+1}(1 - \delta S_{t+1})}{1 + y^{t+1}}$$
$$\geq 1/(1 + y^{t+1}),$$

where the first equality is by (7.4) and by the fact that $R_t = 1 - \delta S_{t+1}$ in an agreement regime, the second equality is by (7.5), and the inequality is by $y^{t+1}(1 - \delta S_{t+1}) \geq 0$.

The bound (7.7) yields the lower bound t_l as follows. Since the speed $y^t - y^{t+1}$ of learning is decreasing in time, for any $t \leq t_l$,

$$y^t - y^{t+1} \geq 2(1-\delta)/\delta > (1 + y^t)(1 - \delta).$$

This inequality is equivalent to $\delta(1 + y^t) > (1 + y^{t+1})$. Hence, by (7.7),

$$\Lambda_t \geq 1/(1 + y^{t+1}) > 1/[\delta(1 + y^t)] = D_t,$$

showing that there is a disagreement regime at $t - 1$. Therefore, $t^* \geq t_l$.

The upper bound t_u has been derived in Yildiz (2004). Since the level of optimism goes to zero as $t \to \infty$, Yildiz (2004) observes that there must be agreement regimes after some date, i.e., the set

$$PA \equiv \{t \in T \mid \Lambda_s \leq D_s \ \forall s > t\}$$

is non-empty. On PA, by (7.4), $\Lambda_t = 1 - \delta y_{t+1}\Lambda_{t+1}$. Solving this stable difference equation forwards, he obtains an upper bound for Λ on PA:

$$\Lambda_t \leq 1/(1 + \delta y^{t+1}).$$

Comparing the upper bound to D_t, he finds the upper bound $\max\{t_u, 0\}$ to $\min PA$, which cannot be lower than t^*, by definition. □

When $t^* > 0$, the players know that they will have to wait until t^* for an agreement, but they cannot do anything to reach an agreement in an earlier date. This is because although the date t^* is known from the beginning, the players do not know what kind of an agreement they will reach at t^*. In fact, as we have seen earlier from (7.3), each player's share is roughly proportional to the number of times he will have been recognized by times t^*. Since each player i is optimistic about his own recognitions, he is then optimistic about his share at t^*. In summary, player i is hopeful that he will have the bargaining power frequently by the date t^* and thereby he will persuade the other player j that i will continue to have the bargaining power, persuading j to agree to i's terms.

Theorem 5 provides upper and lower bounds for the settlement date t^*. Indeed, there cannot be an agreement regime before the lower bound t_l and there cannot be a disagreement regime after the upper bound $\max\{t_u, 0\}$. Both bounds are given by a comparison of the speed of learning, $y^t - y^{t+1}$, to the normalized per-period cost of delay, $(1 - \delta)/\delta$:

$$
\begin{aligned}
y^{t_u} - y^{t_u+1} &= (1 - \delta)/\delta \\
y^{t_l} - y^{t_l+1} &= 2(1 - \delta)/\delta.
\end{aligned}
$$

As typical in a Bayesian learning model, at the beginning, the learning is fast and optimism drops fast. When the players are patient, i.e., when $(1 - \delta)/\delta \le (y^t - y^{t+1})/2$, this entices players to wait in the hopes that their opponents learn and agree to their terms. As time passes, the learning slows down and eventually it becomes too costly to wait for the opponent's learning. In particular, when the speed of learning goes below $(1 - \delta)/\delta$, the marginal cost $1 - \delta$ of waiting exceeds any gain a player expects from the other party's learning, and players reach an agreement. When the marginal gain from learning is equal to the marginal cost of delay, they reach an agreement. The above equalities give upper and lower bounds. Note that the delay here can be highly costly. From the lower bound t_l, one can compute that δ^{t^*} can be as low $\exp(-3/16) \cong 0.83$, i.e., 17% of the pie can be lost due to delay.

Theorem 5 establishes that when the players are optimistic and learning, they may try to persuade the other parties to their own terms by letting them receive more information, hoping that the information will vindicate them. This may lead to a long costly delay.

8. Applications with Learning and Optimism

In the previous section, I considered an abstract model of bargaining power in order to explore the role of optimism and learning in bargaining delays. While this form of bargaining power has a theoretical appeal (as established in Section 3) and may provide direct insights in some applications,[11] in practical applications the bargaining power is determined by the specific aspects of the problem. Analyzing the explicit model directly may provide further insights that may not be available in the reduced-form model above. In this section, I will explore two of such applications with learning. First, I will present a theoretical application Thanassoulis (2010) on the optimism regarding market conditions. Second, I will present a theoretical and empirical application by Watanabe (2006) to learning in pretrial negotiations.

Optimism about the Market Conditions. In many markets with highly differentiated products, the terms of trade are greatly affected by the existence of a second buyer, as many home buyers would readily attest. Thanassoulis (2010) investigates a bargaining model in which the parties are optimistic about the arrival of another buyer. I now present the solution in a simplified version of his model.

Using alternating offers, a seller and a buyer negotiate the price of a good. The value of the good is 0 to the seller and 1 to the buyer. A second buyer may arrive, in which case the competition between the two buyers drives the price to 1, seller receiving the entire gain from trade. If a second buyer exists, then it arrives with a Poisson distribution with arrival rate of $\lambda > 0$. The seller and the buyer assign probabilities p_S and p_B, respectively, to the existence of a second buyer. The level of optimism is measured by $p_S - p_B$. For example, in Figure 3, the players are optimistic in the area above the diagonal.

As the players negotiate without observing the arrival of a second buyer, each player lowers his probability on the existence of a second buyer and eventually becomes convinced that there is not a second buyer. (See Figure 3, for the trajectory of beliefs.) Hence, eventually, optimism becomes negligible, and learning slows down. Therefore, the players eventually agree. Thanassoulis (2010) shows that agreement may be delayed if players are initially very optimistic. For example, if the initial beliefs are in the disagreement region in Figure 3, the agreement is delayed until the beliefs go into the agreement region.

---

[11]For example, Galasso (2006) introduces a bargaining model similar to Yildiz (2004) in order to analyze the cross-license agreements, which allow parties to use each other's patented information, in semiconductor industry. He shows that a higher amount of capital intensity for firms leads to a lower incentive to litigate and delay a cross-license agreement. Using a data set on the US semiconductor industry, he shows that the data is consistent with the model's predictions.
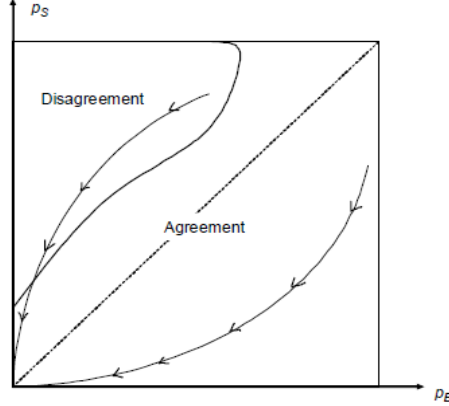
FIGURE 3. Belief trajectories (with arrows) and the agreement and disagreement regions in Thanassoulis (2010)

In order to determine the shares and the boundary between the agreement and the disagreement regions, let r be the continuous-time discount rate and V_S and $V_B = 1 - V_S$ be the shares of the seller and the buyer, respectively, in the agreement region. In the agreement region, a delay of dt costs to the buyer $rV_B dt + p_B \lambda V_B dt - \dot{V}_B dt$. Here, $rV_B dt$ is the cost due to discounting, $p_B \lambda V_B dt$ is the cost due to the fact that a second buyer may arrive in the meantime and the buyer may lose the entire V_B, and $-\dot{V}_B dt$ is the cost due to the change in the share. ($\dot{V}_B$ is the time derivative of V_B.) Similarly, the cost to the seller is $rV_S dt - p_S \lambda V_B dt - \dot{V}_S dt$. Since the players are splitting the cost equally in the continuous-time limit of alternating offer bargaining, by setting the costs for the buyer and the seller equal to each other, one obtains

(8.1) $$\dot{V}_B = -r/2 + [r + \lambda(p_S + p_B)/2]V_B.$$

This is the differential equation that governs the shares in the agreement region. In order to determine the region between the agreement and disagreement regions, note that, intuitively, there would be an agreement regime at $t - dt$ if and only if the cost is nonnegative: $rV_B dt + p_B \lambda V_B dt - \dot{V}_B dt \geq 0$. Substituting (8.1) into this inequality, one concludes that there is agreement at $t - dt$ if and only if

$$r \geq \lambda(p_S - p_B)V_B.$$

That is, the cost due to discounting exceeds the perceived additional value due to optimism. The boundary between the agreement and the disagreement regions is obtained when the cost is equal to the perceived additional value:

$$r = \lambda(p_S - p_B)V_B.$$

The boundary is plotted in Figure 3 in bold. Note that, by the last equality, disagreement requires a positive amount of optimism, and hence the disagreement region is above the diagonal. Note also that one may go out of disagreement region if one fixes the buyer's belief and make the seller more optimistic by increasing p_S towards 1. This is because in this model the speed of learning is proportional to $p(1 - p)$, and such an optimistic seller may learn so slowly that the buyer may just give in. Finally, if the initial beliefs are in the disagreement region, the players wait until their beliefs hit the boundary of the agreement region to agree, and the cost of such a delay may be quite high, as illustrated by Thanassoulis (2010).

Dynamics of Pretrial Negotiation. Watanabe (2006) develops a realistic dynamic model of pretrial negotiation in which the parties are optimistic about their winning in the court. As in Yildiz (2004), in his model, the parties may also receive information about the likelihood of each party's winning, and thus entice players to wait in order to persuade their opponents, causing delay. Using a rich data set on malpractice cases in Florida, he structurally estimates his model and shows that the model fits the data well.

In his model, a Plaintiff and a Defendant negotiate a settlement for a case with a statute of limitation at $\bar{T}$ after which the Plaintiff cannot file a case, and each gets 0. Before $\bar{T}$, the Plaintiff can file a case at any t_L, initiation a litigation stage of T periods. The players negotiate a settlement according to a standard bargaining protocol. At $t_L + T + 1$, a jury decides whether the Defendant is guilty, in which case the Defendant pays J to the Plaintiff. The likelihood π of Plaintiff's winning is not known. The players have optimistic views about this event and receive information about the event as in Yildiz (2004). In particular, binary information about π arrives with time-varying Poisson rates, where the information points either to the Plaintiff or to the Defendant. At time t, the Plaintiff and the Defendant assign probabilities

$$\frac{\bar{m}^P + m_t}{n_0 + n_t} \text{ and } \frac{\bar{m}^D + m_t}{n_0 + n_t},$$

respectively, to the event that Plaintiff wins, where n_t is the number of arrivals, m_t is the number of times the information points to the Plaintiff, n_0 is the firmness of the initial beliefs, and $\bar{m}^P/n_0$ and $\bar{m}^D/n_0$ are the initial beliefs of the Plaintiff and the Defendant, respectively. Note that this differs from the model in Yildiz (2004) only in two ways. First, the beliefs are about the probability of winning directly, rather than making an offer, which is an indirect proxy for the bargaining power. Second, information arrives everyday in Yildiz (2004), while it arrives only stochastically here. Watanabe (2006) shows that, when players do not learn (e.g. with 0 arrival rate), the players either agree in the first day or they go to the court (as in Theorems 3 and 2). When they learn they may settle after a delay (as in Theorem 5).
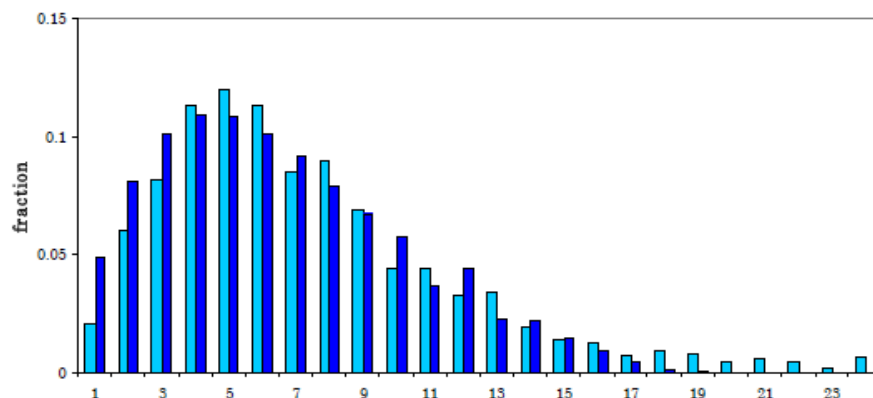
FIGURE 4. The histogram of time delay between the settlement and filing the case in Watanabe (2006). The delay is measured by quarters. The fitted and actual mean delays are 6.6 and 7.4 quarters, respectively.

Watanabe (2006) uses this model to analyze a data set that contains all of the malpractice cases in Florida between 1985 and 1999. It appears that the model fits the data well. For example, the histograms of the actual and the fitted time delay between filing and the settlement are plotted in Figure 4. As in this figure, the model's fitted data mirror the actual data well for the important parameters, such as settlement amount and delay. He also estimates that the players are initially optimistic and learn during the negotiation. Indeed, he estimates that, initially, the mean of Plaintiff's belief on probability of his winning is 0.9566, while the mean of defendant's belief on probability of plaintiff's winning is 0.2982. The frequency of that event is estimated to be 0.4979. Thereby he estimates that, initially, plaintiffs overestimate their winning probability by 92% on average, and the defendants overestimate their own winning probability by 40% on average.

9. COMMENTS ON THE MODELING ASSUMPTIONS

The literature above allows players to have heterogeneous beliefs and assumes that the belief difference is common knowledge. This sharply contrasts with the traditional view that attributes all belief differences to informational differences, an assumption that is known as the Common-Prior Assumption. In this section, I will explain the logic of the methodology the heterogeneous-prior literature employs and explain how the results are expected to change when the common-knowledge assumption is dropped.

Since Harsanyi (1967) and Auman (1976), economists have confined themselves to the realm of the common-prior assumption. Use of this assumption also coincided with the rise of game theory and information economics, perhaps because the common-prior assumption

allowed economists to zero in informational issues. In bargaining, starting with the semi-
nal work of Rubinstein (1982), economists developed a general theory of bargaining under
complete information and applied the theory to a wide range of economic areas from inter-
national economics (Bulow and Rogoff (1991)) to competitive markets (Gale (1986)). At the
same time, they explored the role of incomplete information in bargaining, exploring the im-
plications of screening, signaling, and the war of attrition.[12] In particular, they have shown
that signaling and the war of attrition can cause a long delay in reaching agreement, while
the delay in screening models becomes negligible in the continuous-time limit as conjectured
by Coase (1972).

Despite its spectacular success, the common-prior assumption has shown to be quite re-
strictive, both empirically and theoretically. First, empirical and experimental data as well as
casual observations suggest that the common-prior assumption is commonly violated, often
in a systematic way. For example, Aumann (1976) has proven that under the common-prior
assumption, if the beliefs are common-knowledge, then they must be equal. As Aumann
(1976) has recognized, this result "might be considered an evidence against [the common-
prior assumption], as there are in fact people who respect each other's opinions and never-
theless disagree heartily about subjective probabilities."[13] In the same vein, the No-Trade
Theorems (Milgrom and Stokey (1982)) establish that under the common-prior assumption,
rational players cannot trade on information or bet against each other. In particular, these
theorems show that having optimized their payoffs before receiving their information, the
rational players will not trade further after receiving their information. In contrast, it ap-
pears that the traders do continually trade after receiving information and bet against each
other. Experimental research and surveys on individuals' beliefs about the future life events
provide further direct evidence for prevalence of self-serving biases and optimistic outlook
(Weinstein (1980))[14].

Similar self-serving biases have been observed in the context of bargaining and the data
suggest that disagreements and bargaining delays are more common in environments with
larger room for such biases.[15] Furthermore, the survey results suggest that the seasoned

---

[12]See Kennan and Wilson (1993) for a detailed survey.

[13]Ironically, many economists took this result as an evidence *for* the the common-prior assumption. Note
also that Aumann pointed out that such disagreements may be caused by systematic errors in judgement,
which can be modeled using heterogenous priors.

[14]See Manski (2004) for a detailed discussion of the empirical research on expectations and further em-
pirical evidence for heterogenous expectations.

[15]See the partial survey by Babcock and Lowenstein (1997) for the findings referred to in this paragraph.

negotiators are also prone to such biases. For example, in a survey of union and school-board presidents regarding the salary negotiations for public-school teachers in Pennsylvania, Babcock, Wan, and Loewenstein (1996) find a statistically and economically significant level of self-serving bias. More interestingly, it appears that the subjects do recognize that the other people may exhibit such biases and strategically respond to this possibility, although they do not recognize that they themselves would also exhibit such biases. This is precisely the way the players react in a game theoretical model with heterogeneous priors.

Theoretically, the common-prior assumption has several shortcomings. First, since the beliefs represent the preferences of the players regarding acts with uncertain outcomes in game theoretical applications, there is a tension between this assumption and the basic tenet of the neoclassical economics that the preferences of the economic agents are given. One can, of course, question certain tastes and beliefs and analyze only a special class of them. For example, in the same way one can question the wisdom of an unhealthy breakfast and focus on convex preferences, one can question a belief that seem to contradict an overwhelming body of evidence or focus on the case of a common prior. Nevertheless, when the empirical data and common sense suggest a particular set of tastes and beliefs, such as increasing returns to scale in certain economies or an optimistic outlook in bargaining, it is imperative that we analyze the implications of such tastes and beliefs rather than dismissing them on religious grounds.

Second, as in most game theoretical applications, future bargaining power is related to a singular event rather than the frequency of certain events in a repeated experiments. In particular, it is often related to the behavior of a specific group of people, such as the way a mediator behaves in the negotiation, the way a particular judge rules in a particular case, the way individuals change their demand influencing the future prices, or the way the public sentiment shifts. For such events, there seems to be ample room for differing opinions that are consistent with the existing data. In such cases it is natural to think that the players entertain differing opinions even when they share the same information. For example, Ehud Barak and Yasser Arafat could have different opinions on how a terrorist attack, such as the one on September 11, 2011, would have affected the public sentiments in the United States regarding the Middle East policy of the United States. In that case, it is desirable to explore the implications of such belief differences.

Third, as suggested in the examples above, the beliefs about some outside events, such as the future bargaining power, are sometimes the beliefs about the behavior of some other players that are not explicitly modeled as players in the model. Hence, the common-prior assumption reduces to the assumption that the modeled players hold the same belief about

the unmodeled players, as in a Nash equilibrium. This is, of course, quite consistent with the traditional approach in game theory that focuses on equilibria. Nevertheless, today, the game theory is applied to a wide range of situations in which there is no reason to assume an equilibrium, and the theoretical research reveal that the foundations of equilibrium (and the common-prior assumption) are weaker than one might have assumed. Consequently, non-equilibrium analysis, such as rationalizability (Pearce (1985) and Bernheim (1985)), plays a central role in modern game theory. Allowing heterogeneous beliefs about these outside events corresponds to considering the non-equilibrium solution concepts in the broader game.

This raises a serious concern about the existing literature that allows heterogeneous priors, however. While the literature allows heterogeneous beliefs regarding outside events, including the behavior of the unmodeled players, it uses equilibrium as a solution concept. It is difficult to justify such a dichotomy as a result of learning (Dekel, Fudenberg, and Levine (2003)). Moreover, such a dichotomy may be internally inconsistent as one would have expected that the same factors that lead to systematic biases about the unmodeled behavior lead to the same systematic biases towards the behavior modeled by the strategies (Yildiz (2007)). Fortunately for the existing literature on the dynamic models of bargaining with optimism, the games they consider are solvable by iterated elimination of conditionally dominated strategies, and the results are robust to introducing heterogeneous priors regarding strategies.

Fourth, as mentioned above, recent results suggest that the theoretical foundations of the common-prior assumption is weaker than one might have assumed. In particular, a prominent justification of the common-prior assumption comes from the classical results on merging of opinions through learning (Blackwell and Dubins (1962)). These theorems suggest that two individuals who come from a similar background would have similar beliefs, approximating the common-prior model. In coming to such a strong conclusion, these theorems envision a situation in which the players observe the result of infinite number of repeated experiments in which the signal values take a finite set of possible values and the relationship between the underlying truth and the signals is common knowledge. Of course, none of these assumptions is satisfied in actual game theoretical applications, as each situation is unique in its own way. The players may only try to get an idea from the results in similar situations, where similarity is clearly a subjective concept. For example, in a tort case, the parties may be able to obtain a very good estimate of the frequency of the times a particular judge sides with the defendant, but this data may not be as useful if the plaintiff happens to be special in the plaintiff's own view (e.g. attractive, or disabled, or a minority, where one can add enough attributes to make the data insufficient). When one weakens the assumptions of the merging results to incorporate the realistic situations, however, the merging disappears. First, when the signal

space is infinite, the players' beliefs eventually merges only on a meager[16] set of parameters (Freedman (1965)). Since the players' general life experiences are about a much broader world in comparison to the specific negotiation at hand, this suggests that the players may start the negotiations with heterogeneous priors and learn their bargaining power eventually as the negotiations proceed as in Section 7. Second, when the players learn only from similar situations, the players' similarity notions may affect the resulting beliefs and the resulting behavior may be similar to the equilibrium behavior with heterogeneous priors (see for example Steiner and Stewart (2008)). Third, although the learning results are robust to the assumptions about relation between the underlying truth and the signals, the agreement results turn out to be quite fragile to these assumptions: for any situation in which the classical merging theorems apply, there is a nearby set of initial beliefs in which the players' beliefs diverge almost surely after learning (Acemoglu, Chernozhukov, Yildiz (2007)). In the nearby case, the players will behave according to a model with heterogeneous priors, rather than the one with common-prior assumption.

The literature I have discussed not only allows heterogeneous priors but also assumes that the players' beliefs are common knowledge. It is hard to verify such common knowledge assumptions, and one would expect to have both heterogeneous priors and incomplete information in actual situations. The rationale for the common knowledge assumption is methodological. Since we have a significant body of knowledge on the impact of incomplete information in bargaining, one assumes away any incomplete information in order to identify the role of heterogeneous priors alone. In particular, since the analysis of bargining models with incomplete information is tedious and the results are not straightforward due to the large multiplicity of equilibria, assuming away incomplete information is necessary if one wants to have a clear insight into the workings of the belief differences in bargaining.

Incorporating incomplete information to the analysis of bargaining under heterogeneous priors seems to be an important direction for further research. In particular, since optimistic and firm beliefs are beneficial for the player in equilibrium, one expects that when one drops the assumption that the beliefs are common knowledge, the players try to form a reputation for having optimistic and firm beliefs, leading to signaling and screening in equilibrium. One must, however, note that the resulting lessons will remain to be specific to the example one considers (regardless of the presence of heterogeneous priors). This is because the equilibrium behavior in sequential equilibrium is highly sensitive to the common knowledge assumptions and higher-order beliefs (i.e. the beliefs about beliefs about . . . beliefs about the underlying

---

[16]A set is said to be meager or Category 1 if it is countable union of nowhere dense sets. This is a topological notion of degeneracy.

world): for any date t and division x, one can find a world in which it is almost common knowledge that the game is as in Rubinstein (1982) but the unique rationalizable outcome is that the players wait until date t to settle on x (Weinstein and Yildiz (2009)).

10. CONCLUSION

The common-prior assumption is a central assumption in modern economic theory and has led to spectacular advances in economics. It is also a central assumption in bargaining theory with similar success. Nevertheless, the common-prior assumption turns out to be quite restrictive both theoretically and empirically. In particular, empirical research suggests that it is violated systematically. For example, optimistic and self-serving biases have been reported frequently. Such biased beliefs are also commonly observed in the context of bargaining—even sometimes among the seasoned negotiator. Therefore, it is imperative that we examine the role of systematic biases and in particular optimism in bargaining. Moreover, given the large body of research in bargaining under common-prior, one would expect that the marginal value of new insights in the unexplored area of bargaining with systematic biases would be higher.

The role of optimism in bargaining has been recognized by practitioners for a long while, and bargaining delays and disagreements are often casually attributed to such excessive optimism. More careful game theoretical analysis reveals that in a general dynamic framework, optimism and systematic biases play a quite subtle role, showing that exploring the implications systematic biases requires more careful analysis. For example, Theorem 3 shows that excessive optimism alone cannot explain the bargaining delays alone, as there will be immediate agreement under persistent optimism. In addition to this insight, other research reviewed above reveals two further insights. First, when there is a firm deadline in the near future, optimistic players will wait until the last minute before the deadline to settle, replicating the commonly observed behavior in real-life negotiations, a behavior that is called the deadline effect. Second, when players do learn about their future bargaining power during the negotiations, the optimistic players have a strong incentive to wait in the hopes that the other players would learn and be persuaded to more reasonable terms. In that case, the agreement is delayed until the learning slows down. Several authors explored the implications of optimism in more applied and empirical models, generating valuable insights into those problems. These results raises the hope that we can find many other valuable insights into the bargaining behavior by exploring the role of systematic biases in bargaining carefully.

11. REFERENCES

REFERENCES

[1] Acemoglu, Daron, Victor Chernozhukov and Muhamet Yildiz (2007): "Fragility of Asymptotic Agreement under Bayesian Learning," mimeo.

[2] Ali, Nageeb (2006): "Waiting to Settle: Multilateral Bargaining with Subjective Biases," *Journal of Economic Theory*, 130, 109-137.

[3] Aumann, Robert (1976): "Agreeing to Disagree," *Annals of Statistics*, 4, 1236-1239.

[4] Babcock, Linda and George Loewenstein (1997): "Explaining Bargaining Impasse: The Role of Self-Serving Biases," *Journal of Economic Perspectives*, 11-1, 109-126.

[5] Babcock, Linda, George Loewenstein, S. Issacharoff, and Colin Camerer (1995): "Biased Judgements of Fairness in Bargaining," *American Economic Review*, 85-5, 1337-1343.

[6] Babcock, Linda, Xianghong Wang, and George Loewenstein (1996): "Choosing the Wrong Pond: Social Comparisons that Reflect a Self-Serving Bias," *Quarterly Journal of Economics*, 111, 1-19.

[7] Baron, David and John Ferejohn (1989): "The Power to Propose," in *Models of Strategic Choice in Politics*, P.C. Ordeshook (ed.), University of Michigan Press, 343-366.

[8] Binmore, Kenneth (1987): "Perfect Equilibria in Bargaining Models," in *The Economics of Bargaining*, ed. by K. Binmore and P. Dasgupta. Oxford, U.K.: Basil Blackwell, 77-105.

[9] Bulow, Jeremy and Kenneth Rogoff (1991): "Sovereign Debt Repurchases: No Cure for Overhang," *Quarterly Journal of Economics*, 106(4), 1219-35.

[10] Coase, Ronald (1972): "Durability and Monopoly," *Journal of Law and Economics*, 15(1), 143–49.

[11] Farber, Henry and Max Bazerman (1989): "Divergent Expectations as a Cause of Disagreement in Bargaining: Evidence from a Comparison of Arbitration Schemes," *Quarterly Journal of Economics*, 104, 99-120.

[12] Farber, Henry S. and Harry C. Katz (1979): "Interest Arbitration, Outcomes, and the Incentive to Bargain," *Industrial and Labor Relations Review*, 33(1) (Oct., 1979), 55-63.

[13] Farmer, Amy and Paul Pecorino (2002): "Pretrial Bargaining with Self-serving Bias and Asymmetric Information," *Journal of Economic Behavior and Organization*, 48, 163-176.

[14] Freedman, David (1965): "On the Asymptotic Behavior of Bayes Estimates in the Discrete Case II," *The Annals of Mathematical Statistics*, 36, 454-456.

[15] Galasso, Alberto (2006): "Cross-License Agreements in the Semiconductor Industry: Waiting to Persuade?" University of Toronto Working Paper.

[16] Galasso, Alberto (2010): "Over-Confidence May Reduce Negotiation Delay," *Journal of Economic Behavior and Organization*, forthcoming.

[17] Gale, Douglas M (1986) "Bargaining and Competition Part I: Characterization," *Econometrica*, 54(4), 785-806.

[18] Hicks, John (1932): *The Theory of Wages*. New York: Macmillan Publishing Co.

[19] Kennan, John and Robert Wilson (1993): "Bargaining with Private Information," *Journal of Economic Literature*, 31, 45-104.

[20] Landes, William M. (1971): "An Economic Analysis of the Courts," *Journal of Law and Economics*, 14, 61-107.

[21] Manski, Charles (2004): "Measuring Expectations," *Econometrica*, 72, 1329-1376.

[22] Merlo, Antonio and Charles Wilson (1995): "A Stochastic Model of Sequential Bargaining with Complete Information," *Econometrica*, 63-2, 371-399.

[23] Milgrom, P. and N. Stokey (1982): "Information, Trade and Common Knowledge," *Journal of Economic Theory*, 26, 177-227.

[24] Nash, John (1950): "The Bargaining Problem," *Econometrica*, 18, 155-162.

[25] Neale, Margaret and Max Bazerman (1985): "The Effects of Framing and Negotiator Overconfidence on Bargaining Behaviors and Outcomes," *Academy of Management Journal*, 28-1, 34-49.

[26] Ortner, Juan M. (2010): "Optimism, Delay and (In)efficiency in a Stochastic Model of Bargaining," Princeton University Working Paper.

[27] Perry, Motty and Phillip Reny (1993): "A non-cooperative bargaining model with strategically timed offers," *Journal of Economic Theory*, 59, 50-77.

[28] Posner, Richard (1972): "The Behavior of Administrative Agencies," *Journal of Legal Studies*, 1, 305.

[29] Prescott, J.J., Kathryn E. Spier, and Albert H. Yoon (2010): "Trial and Settlement: A Study of High-Low Agreements," University of Michigan Working Paper.

[30] Priest, George L. and Benjamin Klein (1984): "The Selection of Disputes for Litigation The Selection of Disputes for Litigation," *Journal of Legal Studies*, 13-1 (Jan., 1984), 1-55.

[31] Rubinstein, Ariel (1982): "Perfect Equilibrium in a Bargaining Model," *Econometrica*, 50-1, 97-110.

[32] Sakovics, Jozsef (1993): "Delay in bargaining games with complete information," *Journal of Economic Theory*, 59, 78-95.

[33] Simsek, Alp and Muhamet Yildiz (2007): "Durability of Bargaining Power, Learning, and Deadline Effect," MIT Working Paper.

[34] Shavell, Steven (1982): "Suit, Settlement, and Trial: A Theoretical Analysis under Alternative Methods for the Allocation of Legal Costs" *Journal of Legal Studies*, 11-1 (Jan., 1982), 55-81.

[35] Smith, Lones and Ennio Stacchetti (2001): "Aspirational Bargaining," mimeo.

[36] Stahl, I. (1972): "Bargaining Theory," Stockholm School of Economics.

[37] Steiner, Yakub and Colin Stewart (2008): "Contagion through learning," *Theoretical Economics*, 3, 431-458.

[38] Thanassoulis, John (2010): "Optimal Stalling When Bargaining," *Journal of Economic Dynamics and Control*, 34, 101-120.

[39] Watanabe, Yasutora (2006): "Learning and Bargaining in Dispute Resolution: Theory and Evidence from Medical Malpractice Litigation," Northwestern University Working Paper.

[40] Watanabe, Yasutora (2010): "Estimating the Degree of the Experts' Agency Problem: The Case of Medical Malpractice Lawyers," Northwestern University Working Paper.

[41] Weinstein, Jonathan and Muhamet Yildiz (2009): "A Structure Theorem for Rationalizability in Infinite-horizon Games," mimeo.

[42] Weinstein, Neil (1980): "Unrealistic Optimism about Future Life Events," *Journal of Personality and Social Psychology*, 860-20.

[43] Yildiz, Muhamet (2000): *Essays on Sequential Bargaining*, Stanford University Ph.D. Thesis.

[44] Yildiz, Muhamet (2003): "Bargaining Without a Common Prior – An Immediate Agreement Theorem," *Econometrica*, 71-3, 793-811.

[45] Yildiz, Muhamet (2004): "Waiting to Persuade," *Quarterly Journal of Economics*, 119 (1), 223-249.

[46] Yildiz, Muhamet (2007): "Wishful thinking in strategic environments," Review of Economic Studies, 74 (1), 319-344, 2007.

MIT Economics Department

E-mail address: myildiz@mit.edu

URL: http://econ-www.mit.edu/faculty/myildiz/index.htm