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Do Labyrinthine Legal Limits on Leverage Lessen the Likelihood of Losses?

An Analytical Framework[†]

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A common theme in the regulation of financial institutions and transactions is leverage constraints. Although such constraints are implemented in various ways—from minimum net capital rules to margin requirements to credit limits—the basic motivation is the same: to limit the potential losses of certain counterparties. However, the emergence of dynamic trading strategies, derivative securities, and other financial innovations poses new challenges to these constraints. We propose a simple analytical framework for specifying leverage constraints that addresses this challenge by explicitly linking the likelihood of financial loss to the behavior of the financial entity under supervision and prevailing market conditions. An immediate implication of this framework is that not all leverage is created equal, and any fixed numerical limit can lead to dramatically different loss probabilities over time and across assets and investment styles. This framework can also be used to investigate the macroprudential policy implications of microprudential regulations through the general-equilibrium impact of leverage constraints on market parameters such as volatility and tail probabilities.

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I. Introduction

A significant portion of financial regulation is devoted to ensuring capital adequacy and limiting leverage. Examples include Regulation T,¹ the Basel Accords,² the National Association of Insurance Commissioners (NAIC) rules,³ the U.S. Securities and Exchange Commission (SEC) Net Capital

1. Regulation T is the common name for 12 C.F.R. § 220 (2011), which imposes limits on leverage by regulating the credit that may be extended by brokers and dealers. This regulation is discussed in further detail in Part II, *infra*.

2. The Basel Accords have been promulgated by the Basel Committee on Banking Supervision of the Bank for International Settlements. BASEL COMM. ON BANKING SUPERVISION, BANK FOR INT'L SETTLEMENTS, *BASEL III: A GLOBAL REGULATORY FRAMEWORK FOR MORE RESILIENT BANKS AND BANKING SYSTEMS* (rev. 2011) [hereinafter *BASEL III CAPITAL RULES*], available at <http://www.bis.org/publ/bcbs189.pdf>; BASEL COMM. ON BANKING SUPERVISION, BANK FOR INT'L SETTLEMENTS, *INTERNATIONAL CONVERGENCE OF CAPITAL MEASUREMENT AND CAPITAL STANDARDS: A REVISED FRAMEWORK* (comprehensive ver. 2006) [hereinafter *BASEL II*], available at <http://www.bis.org/publ/bcbs128.pdf>; BASEL COMM. ON BANKING SUPERVISION, BANK FOR INT'L SETTLEMENTS, *INTERNATIONAL CONVERGENCE OF CAPITAL MEASUREMENT AND CAPITAL STANDARDS* (1988) [hereinafter *BASEL I*], available at <http://www.bis.org/publ/bcbs04a.pdf>. These accords establish capital adequacy requirements for banks, and many of the principles of the accords have been adopted in the United States. For a recent discussion of the status of the adoption of the accords in the United States, see STAFF OF THE JOINT COMM. ON TAXATION, 112TH CONG., NO. JCX-56-11, *PRESENT LAW AND ISSUES RELATED TO THE TAXATION OF FINANCIAL INSTRUMENTS AND PRODUCTS 12* (2011), available at http://www.jct.gov/publications.html?func=download&id=4372&chk=4372&no_html=1 (explaining that the capital requirements are “generally based on accords promulgated by the Basel Committee on Banking Supervision”); FED. RESERVE SYS., *COMPREHENSIVE CAPITAL ANALYSIS AND REVIEW: SUMMARY INSTRUCTIONS AND GUIDANCE 4*, 18 (2011), available at <http://www.federalreserve.gov/newsevents/press/bcreg/bcreg20111122d1.pdf> (discussing the Federal Reserve’s adoption of Basel III standards and the expectations those standards place upon banks). The implementation of the accords in the United States is discussed in further detail in Part II, *infra*.

3. The NAIC promulgates model laws, and these include risk-based capital (RBC) regulations. See *RISK-BASED CAPITAL (RBC) FOR INSURERS MODEL ACT § 312-1 to -10* (Nat’l Ass’n of Ins. Comm’rs 2012) (assuring adequate capitalization of insurance companies by setting corrective actions to be taken if a company reports inadequate capital). These regulations have been adopted as law in many states. See *RISK-BASED CAPITAL (RBC) FOR INSURERS MODEL ACT § ST-312-1 to -7* (listing model act adoption by states).

Rule,⁴ and various exchange-mandated margin requirements.⁵ These regulations impose a plethora of leverage constraints across banks, broker-dealers, insurance companies, and individuals, all with the same intent: to limit losses.⁶ Financial leverage is akin to a magnifying lens, increasing the return from profitable investments but also symmetrically increasing the losses to unprofitable ones.⁷ Therefore, limiting leverage places an upper bound on the potential losses of regulated entities. However, because it also places an upper bound on the potential profits of such entities, leverage constraints are viewed by financial institutions as boundaries to be tested and obstacles to be circumvented.⁸

While this tension is an inevitable consequence of most regulatory supervision, the outcome is generally productive when the leverage constraints operate as they were intended. By preventing institutions and

4. This rule appears in the Code of Federal Regulations as 17 C.F.R. § 240.15c3-1 (2011), and it mandates capital requirements for brokers and dealers. For further discussion of this rule, as well as a description of some recent misinterpretations of the rule by prominent individuals, see Andrew W. Lo, *Reading About the Financial Crisis: A Twenty-One-Book Review*, 50 J. ECON. LITERATURE 151, 175–76 (2012) (discussing various popular misunderstandings in the press and literature about 15c3-1’s role in the financial crisis).

5. For example, the Standard Portfolio Analysis of Risk (SPAN) methodology of the CME Group is “the official performance bond (margin) mechanism of 50 registered exchanges, clearing organizations, service bureaus and regulatory agencies throughout the world.” SPAN, CME GROUP, <http://www.cmegroup.com/clearing/risk-management/span-overview.html>.

6. See WALTER W. EUBANKS, CONG. RESEARCH SERV., R4146, THE STATUS OF THE BASEL III CAPITAL ADEQUACY ACCORD 1, 7 (2010) (describing the Basel III capital accord’s role as the first broad international agreement to introduce a leverage ratio requirement in addition to provisions governing the amount of capital banks must hold as a cushion against loss and insolvency); U.S. GEN. ACCOUNTING OFFICE, GAO/GGD-98-153, RISK-BASED CAPITAL: REGULATORY AND INDUSTRY APPROACHES TO CAPITAL AND RISK 130–32 (1998) (“The primary purpose of [the SEC net capital rule] is to ensure that registered broker-dealers maintain at all times sufficient liquid assets to (1) promptly satisfy their liabilities—the claims of customers, creditors, and other broker-dealers; and (2) to provide a cushion of liquid assets in excess of liabilities to cover potential market, credit, and other risks if they should be required to liquidate.”) (footnotes omitted); Mark Mitchell et al., *Limited Arbitrage in Equity Markets*, 57 J. FIN. 551, 559 (2002) (noting that “Regulation T sets boundaries for the initial maximum amount of leverage that investors, both individual and institutional, can employ,” and that “[i]f security prices move such that the investor’s position has less than the required maintenance margin, . . . [the investor] will be required to, at a minimum, post additional collateral or reduce his position so as to satisfy the maintenance margin requirements”); *Risk-Based Capital General Overview*, NAT’L ASS’N INS. COMM’RS (2009), http://www.naic.org/documents/committees_e_capad_RBCoverview.pdf (“The NAIC risk-based capital (RBC) system was created to provide a capital adequacy standard that is related to risk . . .”).

7. Stefan Thurner et al., *Leverage Causes Fat Tails and Clustered Volatility* 1 (Cowles Found. Discussion Paper No. 1745R, rev. 2011), available at http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1963337.

8. See Thomas F. Cosimano & Dalia S. Hakura, *Bank Behavior in Response to Basel III: A Cross-Country Analysis* 6 (Int’l Monetary Fund Working Paper WP/11/119, 2011) (discussing how Basel III requirements could be circumvented and how Basel II was circumvented by banks); Viral Acharya et al., *Securitization Without Risk Transfer* 3 (Nat’l Bureau of Econ. Research Working Paper No. 15730, Feb. 2010) (“These results suggest, and indeed we document, that the effective leverage of commercial banks was significantly larger than that implied by their on-balance sheet leverage or their capitalization from a regulatory standpoint.”).

individuals from overextending themselves during normal market conditions, these constraints reduce the likelihood of unexpected and unsustainable losses during market dislocations. In doing so, such constraints promote financial stability by instilling confidence in the financial system. The same logic suggests that ineffective constraints and inadequate capital can yield financial instability and a loss of confidence in the financial system. The recent financial crisis has provided a series of illustrations of this possibility.⁹

In this paper, we propose that such instability can be attributed to a growing mismatch between static regulatory constraints on leverage and the dynamic nature of market risk in the financial system. Because regulations are slow to change and financial risk can shift almost instantaneously, the likelihood of unintended consequences is virtually certain. Moreover, the breakneck speed of financial innovation and corporate transmutation enables institutions to maneuver nimbly around fixed regulations via regulatory arbitrage. This extreme form of “jurisdiction shopping” allowed AIG Financial Products—one of the most sophisticated and well-funded proprietary trading desks in the world—to be regulated by the Office of Thrift Supervision (OTS),¹⁰ an organization designed to supervise considerably less complex institutions. If risks can change abruptly, and financial institutions can skirt leverage restrictions via creative financial engineering and corporate restructuring, fixed capital requirements and leverage limits will seem too onerous during certain periods and inadequate during others. This oscillation is due to the cyclical nature of risk in a dynamic (as opposed to a static) economy and is related to—but not the same as—business and credit cycles.

We develop a simple statistical framework for studying capital requirements and leverage constraints that yields several interesting implications for regulating leverage, managing risk, and monitoring financial stability. This framework begins with the observation that all capital adequacy requirements and leverage limits are designed to control the probability of loss of a certain magnitude.¹¹ This observation implies that five quantities lie at the heart of such policies: the leverage constraint or capital requirement, the maximum allowable probability of loss, the magnitude of the loss under

9. See JOSEPH STIGLITZ, *FREEFALL: AMERICA, FREE MARKETS, AND THE SINKING OF THE WORLD ECONOMY* 27 (2010) (describing the freefall of America’s economy in the recent financial crisis as stemming from deregulation that allowed “the reckless lending of the financial sector, which had fed the housing bubble, which eventually burst”).

10. According to AIG’s 2007 10-K, “AIG [was] subject to OTS regulation, examination, supervision and reporting requirements.” Am. Ins. Grp., Annual Report (Form 10-K) 13 (Feb. 8, 2008), available at <http://www.sec.gov/Archives/edgar/data/5272/000095012308002280/y44393e10vk.htm>.

11. See *infra* Part II; Thurner et al., *supra* note 7, at 3 (“Regulating leverage is . . . good for everyone, preventing [risky] behavior that all are driven to yet none desire.”).

consideration, and the mean and variance of the probability distribution used to compute the loss probability.¹²

This result leads to the main thrust of our paper: there is a specific mathematical relation among these five quantities so that if one of them—say, the leverage constraint—is constant over time, then the remaining four must either be fixed as well, or they must move in lockstep as a result of the fixed leverage constraint. This almost-trivial observation has several surprisingly far-reaching implications. As risk varies over time, fixed leverage constraints imply time-varying loss probabilities, in some conditions greatly exceeding the level contemplated by the constraint. Even if risk is relatively stable, as expected returns vary over the business cycle, the probability of loss will also vary and provide incentives for all stakeholders to consider changing the constraint in response. Finally, if asset-return correlations across regulated entities can change over time, as they have in the past, loss probabilities will change as well, spiking during times of financial distress.

These implications underscore the inadequacy of current regulatory restrictions on leverage, which are almost always fixed parameters that require significant political will to change. Although drawing clear bright lines may be good practice from a legal and rule-making perspective, such an approach is not ideal from an economic perspective. Rigid capital requirements and leverage limits may be easier to implement than more flexible and adaptive rules, but they will achieve their intended objectives only in relatively stable environments. Our loss-probability approach provides a systematic framework for integrating microprudential regulation of individual institutions with macroprudential policies for promoting financial stability.

We begin in Part II, An Overview of Leverage Constraints and Related Literature, with a brief overview of existing leverage limits across the banking, brokerage, insurance, and asset-management industries, and provide a short literature review. We introduce our basic framework for determining capital requirements in Part III, An Analytical Framework, by deriving an explicit expression for the leverage constraint as a function of the entity's asset-return distribution, and provide several numerical examples to develop intuition for the determinants of leverage constraints. In Part IV, The Dynamics of Leverage Constraints, we study the behavior of leverage constraints over time and show that changes in market conditions can change significantly the efficacy of such constraints if they are static. Using the S&P 500 index as an illustrative case, we find that a fixed leverage limit can imply a loss probability of less than 1% in one period and well over 25% in an adjacent period. We

12. Throughout this paper we consider only probability distributions that are completely determined by their mean and variance. This is the case, for example, for both normal and t distributions, with a specified number of degrees of freedom, and these will be the two main examples we consider. For a description of normal distributions, see 1 WILLIAM FELLER, AN INTRODUCTION TO PROBABILITY THEORY AND ITS APPLICATIONS, 174 (3d ed. 1968). For a description of t distributions, see 2 WILLIAM FELLER, AN INTRODUCTION TO PROBABILITY THEORY AND ITS APPLICATIONS 48 (1966).

conclude with some suggestions for improving current leverage-related regulation.

II. An Overview of Leverage Constraints and Related Literature

Investments are often partially financed with borrowed funds. For example, an investor may borrow from his broker to purchase stock, a bank may use amounts deposited with it to invest in various assets, and an individual may secure a mortgage to purchase his home. These and many other borrowing transactions are subject to complex sets of rules that attempt to limit the risk that the borrowed funds will not be repaid. Some rules are imposed directly by governmental regulation, and others are imposed privately by individual parties to transactions or by broad groups engaged in similar types of transactions. Accordingly, the laws and literature on capital requirements and leverage constraints are vast, spanning many contexts, types of financial institutions, and industries. The traditional focus has been on banks, securities firms, and insurance companies. The collection of articles in *Capital Adequacy Beyond Basel: Banking, Securities, and Insurance* provides an excellent survey of the practices and challenges surrounding capital adequacy in these industries.¹³

A theme common to nearly all borrowing rules and regulations is the idea that the borrower should be required to finance a certain amount of the total investment himself without recourse to borrowing. This concept may be expressed as an upper limit on leverage, where leverage is defined as the ratio of the total investment value to the portion of the value financed directly by the borrower. Alternatively, the same idea may be expressed in terms of a minimum margin requirement, where margin is defined as the ratio of the value financed directly by the borrower to the total investment value. The notions of leverage and margin are two sides of the same coin, and, in fact, their numerical values are mathematical reciprocals. Hence, an upper bound on leverage is equivalent to the reciprocal lower bound on margin.

In this section, we consider certain aspects of two prominent sets of regulations—those applicable to lending by broker-dealers and those applicable to banking institutions—to provide context and background for our subsequent modeling and analysis. We start with the well-known example of a governmental margin requirement contained in Regulation T, which concerns extensions of credit by brokers and dealers to investors.¹⁴ The regulation sets a minimum margin of 50% for the initial purchase of an equity security by an investor through an account with a broker or dealer.¹⁵ The minimum margin required under Regulation T does not fluctuate with changes

13. CAPITAL ADEQUACY BEYOND BASEL: BANKING, SECURITIES, AND INSURANCE 204 (Hal S. Scott ed., 2005).

14. See *supra* note 1 for the definition of Regulation T.

15. This requirement applies to “margin equity securit[ies],” subject to certain exceptions. 12 C.F.R. § 220.12(a) (2011).

in the market price of a purchased security that continues to be held by the investor.¹⁶ Ongoing maintenance margins that adjust with fluctuations in value are put in place by private groups and entities, however, and these rules produce a complex tapestry of applicable margin requirements for equity securities. For example, a 25% minimum maintenance margin is required by the Financial Industry Regulation Authority (FINRA)—a private self-regulatory organization of securities firms¹⁷—and individual firms subject to FINRA rules may, and often do, establish higher maintenance margins.¹⁸

The 50% margin requirement of Regulation T does not apply to all investments. For example, the required margin for “exempted” securities and nonequity securities may be established by a creditor in “good faith.”¹⁹ There are also additional margin rules for investments beyond those covered in Regulation T, such as those for securities futures.²⁰ Moreover, applicable margin requirements have not stayed constant over time but instead have fluctuated significantly. In fact, the initial margin requirement under Regulation T has been as low as 40% and as high as 100% since a limit was first established in 1934.²¹ Figure 1 illustrates the margin requirement level as well as the volatility of equity markets from June 2, 1926 through December 31, 2010. The margin value has been set at 50% since 1974.

16. *Id.* § 220.3(c)(1) (2011).

17. FIN. IND. REGULATORY AUTH. R. 4210(c) (2010), *available at* http://finra.complinet.com/en/display/display.html?rbid=2403&record_id=12855&element_id=9383&highlight=4210%28c%29#r12855.

18. For example, the TD Ameritrade Margin Handbook states, “[l]ike most brokerage firms, our clearing firm sets the minimum maintenance requirement higher than the 25% currently required by FINRA.” TD AMERITRADE, MARGIN HANDBOOK 5 (2011), *available at* <https://www.tdameritrade.com/forms/AMTD086.pdf>.

19. 12 C.F.R. § 220.12(b) (2011).

20. *See* 117 C.F.R. §§ 242.400–406 (2011) (containing regulations of customer margin in the futures context established pursuant to the Securities and Exchange Act of 1934); 17 C.F.R. §§ 41.41–49 (2011) (containing regulations established pursuant to the Commodities Exchange Act, including customer margin requirements for futures in § 41.42).

21. The data for the historic margin levels under Regulation T were obtained from the New York Stock Exchange (NYSE) Factbook. NEW YORK STOCK EXCHANGE FACTBOOK, <http://www.nyxdat.a.com/factbook> (select the “Margin Debt and Stock Loan” chapter and then select “FRB initial margin requirements”). We use 50% for the 1962 margin level, however, because previous authors have found that the NYSE Factbook has listed this level incorrectly at 90% since 1981. Dean Furbush & Annette Poulsen, *Harmonizing Margins: The Regulation of Margin Levels in Stock Index Futures Markets*, 74 CORNELL L. REV. 873, 878 n.25 (1989). The historic volatility is based on 125-day rolling windows of daily market-weighted returns using Center for Research in Securities Prices (CRSP) data. *Center for Research in Securities Prices*, WHARTON RESEARCH DATA SERVICES, <http://wrds.wharton.upenn.edu>. For another analysis of historic changes in margin requirements, see generally Peter Fortune, *Margin Requirements, Margin Loans, and Margin Rates: Practice and Principles*, NEW ENG. ECON. REV., Sept.–Oct. 2000, at 19.

Date of Margin Change	New Margin	Volatility of 125-Day Period Before Change	Volatility of 125-Day Period After Change	% Change in Volatility Around Margin Increases	% Change in Volatility Around Margin Decreases
15-Oct-34	45%	23.2%	14.9%		
1-Feb-36	55%	13.8%	16.3%	18.2%	
1-Nov-37	40%	30.4%	34.8%		14.2%
5-Feb-45	50%	7.9%	9.1%	15.6%	
5-Jul-45	75%	9.5%	11.0%	15.6%	
21-Jan-46	100%	11.6%	14.6%	25.6%	
1-Feb-47	75%	23.8%	13.4%		-43.8%
30-Mar-49	50%	13.4%	9.0%		-33.2%
17-Jan-51	75%	12.6%	10.1%	-19.9%	
20-Feb-53	50%	6.8%	8.6%		26.6%
14-Jan-55	60%	9.5%	11.3%	18.2%	
23-Apr-55	70%	11.3%	10.2%	-9.9%	
16-Jan-58	50%	16.6%	10.9%		-34.4%
5-Aug-58	70%	7.9%	9.0%	13.9%	
16-Oct-58	90%	7.5%	9.4%	25.4%	
28-Jul-60	70%	9.2%	10.5%		14.9%
10-Jul-62	50%	19.8%	21.5%		8.9%
6-Nov-63	70%	6.1%	9.1%	48.6%	
8-Jun-68	80%	10.4%	8.9%	-14.6%	
6-May-70	65%	12.4%	19.6%		58.0%
6-Dec-71	55%	12.3%	10.1%		-18.3%
24-Nov-72	65%	8.0%	10.1%	26.0%	
3-Jan-74	50%	17.1%	18.4%		7.8%
Number of Changes in Margin:				12	10
Fraction of Volatility Increases:				75%	60%
Average Percentage Change in Volatility:				13.6%	0.1%

Table 1. Changes in Regulation T margin requirements and equity market volatility before and after each change from 1934 through 1974.²²

22. For explanation of data sources, see *supra* note 21.

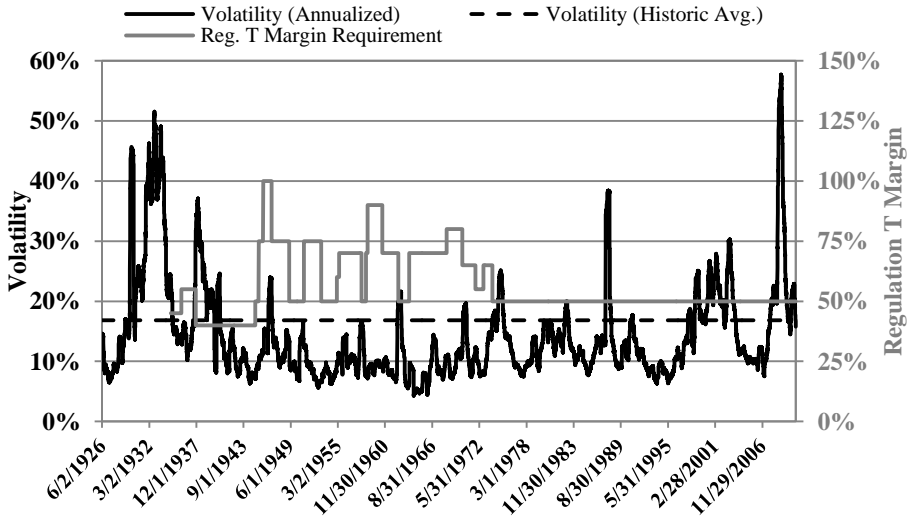


Figure 1. Margin requirements and 125-day rolling-window volatility from June 2, 1926 to December 31, 2010. The horizontal dashed line represents the historic average level of volatility.²³

A principal purpose of minimum margin requirements is to protect lenders from the risk of nonrepayment.²⁴ Accordingly, it may be expected that minimums would be higher when volatility is higher, and consistent with this expectation, Figure 1 shows clearly that the periods of highest volatility occurred before and after the era of high margin requirements that lasted from the mid-1940s to the early 1970s. We can also analyze the relationship between margins and volatility during the era of high margins.

Table 1 summarizes the change in market volatility from the period 125 days before historic changes in the Regulation T margin requirement to the period 125 days after such changes. There have been twelve increases in the margin requirement since its establishment in 1934, and 125-day volatility has increased in nine of these instances. The average percentage change in volatility around these twelve events was a 13.6% increase. By contrast, there have been ten historic decreases in the requirement, and volatility has decreased in four of these instances, with an average percentage change in volatility of 0.1%. Unfortunately, because of the small data set, the differences between times around margin increases and margin decreases are

23. For explanation of data sources, see *supra* note 21.

24. See Peter Fortune, *Is Margin Lending Marginal?*, REGIONAL REV., 3d quarter 2001, at 3, 4 (recounting how one of the main motives for establishing margin requirements in the wake of the 1929 stock market crash was the belief that margin credit led to risky investments and losses for lenders).

not statistically significant.²⁵ However, the results are suggestive and compatible with the idea that increases in the requirement occur more frequently at times when volatility is increasing, while decreases generally occur at more random times with respect to changes in volatility.

A different type of regulation sets restrictions on the leverage ratio of assets to capital for banking institutions. The applicable definitions of capital and assets are complex and have largely been derived from the principles developed in the accords issued by the Basel Committee on International Banking Supervision.²⁶ The first accord, known as Basel I, occurred in 1988, and its principles were made applicable in the United States through regulations finalized in 1989.²⁷ The second accord, known as Basel II, was published in 2004, and many of its principles have now been made applicable in the United States through further regulation.²⁸ To develop an appreciation of the complexity of leverage constraints in these various industries, consider Paragraph 624 of Basel II which contains the “Supervisory Formula” used to determine the level of capital for a given bank for purposes of satisfying capital requirements (see Figure 2).²⁹ Although such formulas may be readily understood and implemented by financial-engineering experts, they provide little transparency to regulators and policymakers charged with managing systemic risk in the banking industry.

The third accord, known as Basel III,³⁰ was agreed upon in 2010 but has not yet been implemented in the United States.³¹ The complexity of its capital requirements—which incorporate more contingent capital requirements (see Table 2)—will likely be even greater, and will be phased in gradually by various financial institutions over the next few years (see Table 3).

25. For example, the two-sample *t*-statistic comparing the percentage changes in volatility around increases in margin to those around decreases in margin is $t = 1.16$, corresponding to a *p*-value of 0.26.

26. See generally BASEL COMM. ON BANKING SUPERVISION, CORE PRINCIPLES FOR EFFECTIVE BANKING SUPERVISION (2011) (setting forth principles to guide countries in the development of regulations and supervisory practices related to banks and banking systems).

27. BASEL I, *supra* note 2. The implementation of Basel I into United States regulations can be found in 12 C.F.R. § 3, app. A (2011).

28. BASEL II, *supra* note 2. The implementation of Basel II into United States regulations can be found in 12 C.F.R. § 3, app. C (2011). According to the Financial Stability Institute, Basel II has been or will be adopted by 112 countries by 2015. FIN. STABILITY INST., OCCASIONAL PAPER NO. 9, 2010 FSI SURVEY ON THE IMPLEMENTATION OF THE NEW CAPITAL ADEQUACY FRAMEWORK 6 (2010).

29. BASEL II, *supra* note 2, at 139.

30. BASEL III CAPITAL RULES, *supra* note 2.

31. BASEL COMM. ON BANK SUPERVISION, PROGRESS REPORT ON BASEL III IMPLEMENTATION 7–8 (2011); see also Edward Wyatt, *Fed Proposes New Capital Rules for Banks*, N.Y. TIMES (Dec. 20, 2011),

http://www.nytimes.com/2011/12/21/business/fed-proposes-new-capital-rules-for-banks.html?_r=2 (reporting that Basel III standards are not expected to be phased in until at least 2016).

$$S[L] = \begin{cases} L & \text{when } L \leq K_{IRB} \\ K_{IRB} + K[L] - K[K_{IRB}] + (d \cdot K_{IRB} / \omega) \left(1 - e^{\omega(K_{IRB} - L) / K_{IRB}} \right) & \text{when } K_{IRB} < L \end{cases}$$

where

$$h = (1 - K_{IRB} / LGD)^N$$

$$c = K_{IRB} / (1 - h)$$

$$v = \frac{(LGD - K_{IRB})K_{IRB} + 0.25(1 - LGD)K_{IRB}}{N}$$

$$f = \left(\frac{v + K_{IRB}^2}{1 - h} - c^2 \right) + \frac{(1 - K_{IRB})K_{IRB} - v}{(1 - h)\tau}$$

$$g = \frac{(1 - c)c}{f} - 1$$

$$a = g \cdot c$$

$$b = g \cdot (1 - c)$$

$$d = 1 - (1 - h) \cdot (1 - \text{Beta}[K_{IRB}; a, b])$$

$$K[L] = (1 - h) \cdot (1 - \text{Beta}[L; a, b])L + \text{Beta}[L; a + 1, b]c$$

Figure 2: Paragraph 624 of the Basel II Accord (2006) specifies the “Supervisory Formula,” which is used to determine the level of capital for a financial institution for purposes of capital-adequacy supervision.³²

In addition to regulatory rules, the Dodd-Frank legislation provides a statutory mandate for general leverage capital requirements and risk-based capital requirements.³³

Although the various Basel Accord rules are complex, the typical capital adequacy requirement for banks has been that capital of specified types must have a value equal to at least 8% of an adjusted asset number.³⁴ The adjusted asset number factors are “risk weighting” in the sense that particular assets are assigned weights, with safe assets getting a zero or relatively low number, and riskier assets getting a relatively high number.³⁵ The weighted average of the assets is then computed to determine the adjusted asset number.³⁶ The result is

32. BASEL II, *supra* note 2, at 139.

33. Dodd-Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank Act) § 171, 12 U.S.C. § 5371 (2006 & Supp. IV 2011).

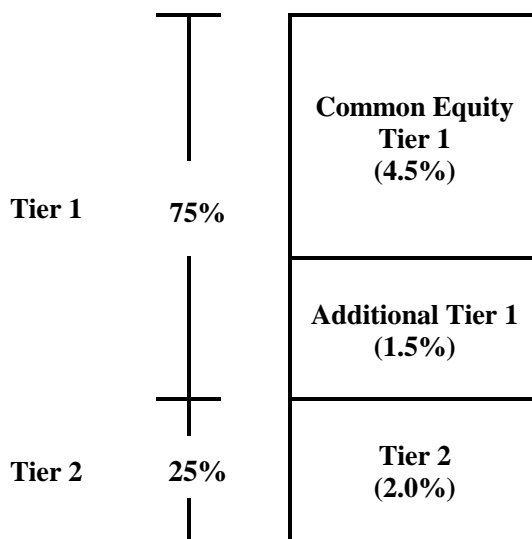
34. See *infra* Table 2.

35. See BASEL III CAPITAL RULES, *supra* note 2; Peter King & Heath Tarbert, *Basel III: An Overview*, BANKING & FIN. SERVS. POL’Y REP., May 2011, at 1, 4–5.

36. King & Tarbert, *supra* note 35, at 2.

a margin/leverage requirement, with a blended requirement level across various assets, depending upon their risk levels.

Minimum Common 8% Total Capital Ratio



Capital Conservation Buffer

Common Equity Tier 1 Ratio (percent)	Existing Buffer (percent)	Minimum Capital Conservation Ratio (percentage of earnings banks are required to hold to rebuild buffer)	Percentage of earnings available for discretionary distributions
4.5 – 5.125	0 – 0.625	100%	0%
> 5.125 – 5.75	0.625 – 1.25	80%	20%
> 5.75 – 6.375	1.25 – 1.875	60%	40%
> 6.375 – 7.0	1.875 – 2.5	40%	60%
> 7.0	2.5	0%	100%

Table 2. Capital requirements of the Basel III Accord.³⁷

The capital adequacy regulations contemplate a significant amount of complexity and sophistication in risk-weighted asset calculations. In particular, banks are permitted to use internal models to calculate quantities

37. See BASEL III CAPITAL RULES, *supra* note 2; King & Tarbert, *supra* note 35, at 1, 4–5.

such as “value-at-risk” (VaR)³⁸ and to use these quantities in determining the value of risk-weighted assets.³⁹

While insurance companies face a different set of capital requirements, the motivation for such requirements is similar to that for banks: to reduce the likelihood of defaulting on promised payments to customers. However, Scott E. Harrington has argued that extending the Basel framework to insurance and reinsurance companies is ill-advised because these institutions are less systemically important and face greater market forces that impel them to maintain adequate capital reserves.⁴⁰ During the 1991 to 1994 period, U.S. insurance companies migrated from minimum absolute capital requirements to minimum risk-based capital (RBC) requirements developed by the National Association of Insurance Commissioners.⁴¹ Martin Eling and Ines Holzmüller provide a useful overview of RBC standards in the United States, Europe, New Zealand, and Switzerland for property–casualty insurers and conclude that there is great heterogeneity in how risk-based capital is defined and the types of regulations used to ensure capital adequacy (see Table 4).⁴²

The recent financial crisis has taught us that insufficient capital is also problematic for money market funds, pension funds, hedge funds, governments, and any nonfinancial institution that is a counterparty to over-the-counter (OTC) derivatives transactions.⁴³ In these cases, the notion

38. See BASEL III CAPITAL RULES, *supra* note 2; value-at-risk is a measure of risk that is equal to the level that performance will equal or exceed in all but $x\%$ of cases, where $x\%$ is a tolerance parameter that must be specified. VaR is defined as the maximum portfolio loss that can occur during a specified period within a specified level of confidence. NEIL D. PEARSON, RISK BUDGETING: PORTFOLIO PROBLEM SOLVING WITH VALUE-AT-RISK 3–4 (2002).

39. King & Tarbert, *supra* note 35, at 7.

40. Scott E. Harrington, *Capital Adequacy in Insurance and Reinsurance*, in CAPITAL ADEQUACY BEYOND BASEL, *supra* note 13, at 87, 88.

41. *Id.* at 100–01.

42. Martin Eling & Ines Holzmüller, *An Overview and Comparison of Risk-Based Capital Standards*, J. INS. REG., Summer 2008, at 31.

43. See generally *Reform of the Over-the-Counter Derivative Market: Limiting Risk and Ensuring Fairness: Hearing Before the H. Comm. on Fin. Servs.*, 111th Cong. 10–11 (2009) (statement of Henry Hu, Director, Div. of Risk, Strategy, and Fin. Innovation, U.S. Sec. & Exch. Comm’n) (explaining that “the recent financial crisis revealed serious weaknesses in the U.S. financial regulation,” including “the lack of regulation of OTC derivatives,” creating risks that “can lead to insufficient capital, inadequate risk management standards, and associated failures cascading through the global financial system”); *id.* at 10–14, 24–32, 45–46 (discussing approaches to the regulation of nonfinancial entities that use swaps and customized OTC derivatives and the implications of regulation on the behavior of market participants, including financial and nonfinancial institutions); FIN. CRISIS INQUIRY COMM’N, THE FINANCIAL CRISIS INQUIRY REPORT 354 (2011) (describing the market failure immediately following the collapse of Lehman Brothers and noting that “[a]mong the first to be directly affected were the money market funds and other institutions that held Lehman’s \$4 billion in unsecured commercial paper,” and that “[o]ther parties with direct connections to Lehman included the hedge funds, investment banks, and investors who were on the other side of Lehman’s more than 900,000 over-the-counter derivatives contracts The Lehman bankruptcy caused immediate problems for these OTC derivatives counterparties”); *id.* at 428 (identifying “insufficient capital” as one of the “common risk management failures” that led to the financial crisis); Michael S. Barr, *The Financial Crisis and the Path of Reform*, 29 YALE J. ON REG.

of liquidity—particularly “funding liquidity”—becomes more relevant, as Markus Brunnermeier and Lasse Pedersen⁴⁴ as well as John Dai and Suresh Sundaresan⁴⁵ argue. These authors show that during times of financial distress, insolvency is not the only challenge to financial institutions; the ability to maintain liquidity in the face of forced unwindings can be equally challenging, as we learned in 1998 from the case of Long-Term Capital Management (LTCM).⁴⁶

In fact, hedge funds, money market funds, and insurance companies are now often referred to collectively as being part of the “shadow banking system” since they perform many of the same functions as traditional banks but are not subject to the same regulatory supervision and oversight as the banking industry. Moreover, while we have certain indirect measures of the amount of leverage employed by hedge funds and insurance companies and the illiquidity risk they are exposed to,⁴⁷ these institutions currently have no reporting requirements to federal regulators regarding their degree of leverage

91, 94, 104 (2012) (asserting that in the lead up to the financial crisis derivatives were traded “with insufficient capital to back the trades,” and that “[t]he lack of information on derivative exposures led firms to withdraw from counterparties” and to demand “more margin protection from their remaining counterparties, which put further downward pressure on underlying asset prices”); Willa E. Gibson, *Clearing and Trade Execution Requirements for OTC Derivatives Swaps Under the Frank-Dodd Wall Street Reform and Consumer Protection Act*, 38 RUTGERS L. REC. 227, 230 (2011) (noting that “[i]n 2009 . . . , 90% of Fortune 500 companies used customized [OTC derivatives],” and describing the Dodd-Frank regulatory approach, whereby “a non-financial entity utilizing OTC derivatives swap contracts for mitigating or hedging risks . . . is exempted from clearing” its customized contracts, and those counterparties who are not exempted are subjected to a bifurcated regulatory approach that “was designed with the expectation that the capital and margin requirements would suffice to manage the systemic as well as counterparty risk that has historically threatened the integrity and stability of the OTC derivatives markets”).

44. Markus K. Brunnermeier & Lasse Heje Pedersen, *Market Liquidity and Funding Liquidity*, 22 REV. FIN. STUD. 2201–02 (2009).

45. John Dai & Suresh Sundaresan, *Risk Management Framework for Hedge Funds: Role of Funding and Redemption Options on Leverage 1–2* (Mar. 21, 2010) (unpublished manuscript) (on file with author).

46. See generally PRESIDENT’S WORKING GRP. ON FIN. MKTS., HEDGE FUNDS, LEVERAGE, AND THE LESSONS OF LONG-TERM CAPITAL MANAGEMENT (1999) (recounting the lessons learned from the near-collapse of Long-Term Capital Management, including the effects of liquidity shocks on distressed firms).

47. See generally, e.g., Mila Getmansky et al., *An Econometric Model of Serial Correlation and Illiquidity in Hedge Fund Returns*, 74 J. FIN. ECON. 529 (2004) (describing measures of illiquidity risk); Monica Billio et al., *Econometric Measures of Connectedness and Systemic Risk in the Finance and Insurance Sectors*, 105 J. FIN. ECON. (forthcoming 2012) (describing network measures of systemic risk).

Phase-in Arrangements

(shading indicates transition periods – all dates are as of 1 January)

	2011	2012	2013	2014	2015	2016	2017	2018	As of 1 January 2019
Leverage Ratio		Supervisory Monitoring							
Minimum Common Equity Capital Ratio			1 Jan 2013 – 1 Jan 2017	Parallel run 1 Jan 2017, Disclosure starts 1 Jan 2015				Migration to Pillar 1	
Capital Conservation Buffer			3.5%	4.0%	4.5%	4.5%	4.5%	4.5%	4.5%
Minimum common equity plus capital conservation buffer						0.625%	1.25%	1.875%	2.50%
Phase-in of deductions from CET1 (including amounts exceeding the limit for DTAs, MSRs and financials)			3.5%	4.0%	4.5%	5.125%	5.75%	6.375%	7.0%
Minimum Tier 1 Capital				20%	40%	60%	80%	100%	100%
Minimum Total Capital			4.5%	5.5%	6.0%	6.0%	6.0%	6.0%	6.0%
Minimum Total Capital plus conservation buffer			8.0%	8.0%	8.0%	8.0%	8.0%	8.0%	8.0%
Capital instruments that no longer qualify as non-core Tier 1 capital or Tier 2 capital			8.0%	8.0%	8.0%	8.625%	9.25%	9.875%	10.5%
Phased out over 10 year horizon beginning 2013									
Liquidity coverage ratio	Observation period begins				Introduce minimum standard				
Net stable funding ratio	Observation period begins							Introduce minimum standard	

Table 3. Proposed phase-in timetable for various components of the Basel III Accord.

System	RBC standards	Solvency II	Self-Regulatory Framework	Swiss Solvency Test
1. General information				
Country of application	USA	European Union	New Zealand	Switzerland
Years of introduction	1994	2012 (expected)	1994	2006
Main pillars	1. Rules-based RBC formula 2. RBC model law on intervention authority	1. Quantitative regulations for capital requirements 2. Qualitative supervisory review 3. Public disclosure	1. Fair Insurance Code 2. Insurance and Savings Ombudsman Scheme 3. Insurance Council's Solvency Test	1. Principles-based target capital model 2. Risk management report 3. Quality assessment
Regulated companies	Insurers (domestic & foreign); no reinsurers	Insurers and reinsurers (domestic & foreign)	P&C insurers (domestic & foreign); no life insurers or reinsurers	Insurers and reinsurers (domestic & foreign)
Consideration of management risk	No	Rudimentarily addressed by pillar II	Yes	No
Public disclosure requirements	Yes	Yes	Yes	No
2. Definition of capital required				
Model typology	Static factor model	Static factor + dynamic cash-flow model	Static factor model	Static factor + dynamic cash-flow model
Rules- versus Principles-based	Rules-based	Principles-based	Principles-based	Principles-based
Total balance sheet approach	No	Yes	Yes	Yes
Time horizon	1 year	1 year	1 year	1 year
Risk measure/calibration	No risk measure	Value at risk/99.5% confidence level	A.M. Best: Expected policyholder deficit S&P: Value at risk	Expected shortfall/99% confidence level
Consideration of operational risk	Not explicitly (implicit via off-balance-sheet items—R0)	Quantitatively	A.M. Best: No explicit consideration S&P: Quantitatively	Qualitatively
Consideration of catastrophe	No	Yes (as part of underwriting risk)	Yes	Yes
Use of internal models	No	Appreciated	No	Appreciated for insurers; required for reinsurers
3. Definition of available capital				
Definition based on market or book values	Book values	Market values	Market values	Market values
Classification of available capital	No	Yes (three tiers)	No	No
Consideration of off-balance-sheet items	No	Yes	Yes	No
4. Intervention				
Levels of intervention	4	2	No intervention by regulator, but market discipline	2
Clarity of sanctions	Strict, clear rules	Not clear yet	No direct sanctions	Not clear yet

Table 4. Comparison of risk-based capital standards in property–casualty insurance companies in the United States, Europe, New Zealand, and Switzerland.⁴⁸

48. *Id.* at 47.

and illiquidity.⁴⁹ Because risk is not static—it is, in fact, determined endogenously by the ongoing interactions of all market participants, as shown by Jon Danielsson, Hyun Shin, and Jean-Pierre Zigrand⁵⁰—the lack of transparency of the shadow banking system makes it virtually impossible for regulators to either anticipate or respond to changes in systemic risk with any degree of precision or speed.

One other strand of literature that is relevant to leverage constraints attempts to quantify the economic cost of insuring the solvency or liquidity of an entity. Its relevance is clear: by measuring the cost of such insurance, we can prioritize the need for capital requirements among the most costly entities. The standard approach is due to Robert C. Merton, who observed that financial guarantees are equivalent to put options on the assets of the insured entity,⁵¹ hence the cost of the guarantee is the market price of the put option or the production cost of synthetically manufacturing such a derivative contract by a dynamic portfolio strategy.⁵² This powerful paradigm provides explicit valuation formulas for many types of guarantees and has been applied to the valuation of deposit insurance,⁵³ the management of risk capital in a financial institution,⁵⁴ the valuation of implicit government guarantees to Fannie Mae and Freddie Mac,⁵⁵ and the measurement of sovereign risk.⁵⁶ The approach

49. The first leverage and liquidity reporting requirements for hedge fund advisors are scheduled to come into effect for fiscal years ending on or after June 15, 2012. Reporting by Investment Advisers to Private Funds and Certain Commodity Pool Operators and Commodity Trading Advisors on Form PF, Release No. IA-3308, 76 Fed. Reg. 71,128, 71,157 (Nov. 16, 2011); Press Release, Sec. & Exch. Comm'n, SEC Approves Confidential Private Fund Risk Reporting (Oct. 26, 2011), <http://www.sec.gov/news/press/2011/2011-226.htm> (summarizing the new reporting requirements, including disclosure of leverage and liquidity information for certain hedge funds). In the case of insurance companies, regulation is generally at the state level, but certain companies may be subject to federal regulation for reasons not specifically related to their insurance business. For example, AIG was subject to supervision by the Office of Thrift Supervision until March 2010, and if currently proposed rules are adopted, AIG will be subject to Federal Reserve Board regulation as a systemically important financial institution under the Dodd-Frank Act. Am. Ins. Grp., Annual Report (Form 10-K) 18–19 (2011), *available at* http://www.aigcorporate.com/investors/2011_February/December_31_2010_10-K_Final.pdf.

50. See Jon Danielsson et al., *Balance Sheet Capacity and Endogenous Risk 2* (Aug. 2011) (unpublished working paper) (discussing a model for equilibrium and market risk as functions of aggregate bank capital).

51. Robert C. Merton, *An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees*, 1 J. BANKING & FIN. 3, 4 (1977).

52. Robert C. Merton, *On the Application of the Continuous-Time Theory of Finance to Financial Intermediation and Insurance*, 14 GENEVA PAPERS ON RISK & INS. 225, 228 (1989).

53. Merton, *supra* note 51, at 8–9.

54. See Robert C. Merton & André Pérol, *Theory of Risk Capital in Financial Firms*, J. APPLIED CORP. FIN., Fall 1993, at 17 (defining “risk capital” as the amount invested to insure the firm’s net asset value and demonstrating that the amount of risk capital necessary depends on the riskiness of the net assets and not on the form of financing).

55. Deborah Lucas & Robert McDonald, *Valuing Government Guarantees: Fannie and Freddie Revisited*, in MEASURING AND MANAGING FEDERAL FINANCIAL RISK 131, 135 (Deborah Lucas ed., 2010).

taken in these studies is highly complementary to ours. We make use of the same statistical inputs: the probability distribution of asset returns. However, our focus is not on pricing guarantees—which requires more economic structure, such as an equilibrium or arbitrage-pricing model—but on the less ambitious task of relating capital requirements and leverage constraints to other aspects of an entity’s financial condition.

III. An Analytical Framework

If the primary purpose of leverage constraints is to limit the potential losses of an entity, the most natural way to frame such a concept is in terms of loss probabilities. For a given portfolio, define the following quantities:

$$\begin{aligned}\pi_t &\equiv \text{date-}t \text{ profit and loss of the portfolio} \\ K_t &\equiv \text{date-}t \text{ assets under management} \\ I_t &\equiv L_t \times K_t = \text{leveraged assets, } L_t \equiv \text{leverage ratio} \geq 1 \\ R_t &\equiv \frac{\pi_t}{I_t} = \text{portfolio return}\end{aligned}$$

The probability that this portfolio will lose more than a fraction δ of its capital K_t on any date t is given by:

$$\text{Prob}(\pi_t < -\delta K_t) \tag{1}$$

A leverage constraint can be viewed as an attempt to impose an upper bound on this probability, i.e.,

$$\text{Prob}(\pi_t < -\delta K_t) \leq \gamma \tag{2}$$

for some predefined level of γ , e.g., 5%.⁵⁷ This expression may also be used to define the amount of collateral or margin required by a counterparty (such as a broker or supervisory agency) so as to cover a potential loss of up to δK_t with probability $1-\gamma$. For larger values of δ , larger potential losses can be covered; for smaller values of γ , the likelihood of coverage is greater.

56. Dale Gray, *Modeling Financial Crises and Sovereign Risks*, 1 ANN. REV. FIN. ECON. 117, 129 (2009).

57. The condition in (2) is closely related to the concept of Value-at-Risk (VaR), a measure of risk that is used in many contexts, including banking capital adequacy rules. See *supra* note 38 and accompanying text. The formula in (2) is thus equivalent to the statement that VaR for the investment portfolio, with specified confidence level γ , is a loss that is no larger than δK_t . See, e.g., Risk-Based Capital Guidelines, 12 C.F.R. § 3, app. B (“[E]nsur[ing] that banks with significant exposure to market risk maintain adequate capital to support that exposure.”); Capital Adequacy Guidelines for Banks, 12 C.F.R. § 3, app. C (establishing “[m]inimum qualifying criteria for . . . bank-specific internal risk measurement . . . processes for calculating risk-based capital requirements”).

A. Leverage Limits

An analytic expression for a constraint on leverage L_t corresponding to the upper bound γ may be readily obtained by inverting (2) under the assumption of normality for the portfolio's profit-and-loss π_t :

$$\begin{aligned}
 \text{Prob}(\pi_t < -\delta K_t) &= \text{Prob}(R_t L_t K_t < -\delta K_t) & (3) \\
 &= \text{Prob}(R_t < -\delta/L_t) \\
 &= \text{Prob}\left(\frac{R_t - \mu}{\sigma} < -\frac{\delta/L_t + \mu}{\sigma}\right) \\
 &= 1 - \Phi\left(\frac{\delta/L_t + \mu}{\sigma}\right) \leq \gamma \\
 1 - \gamma &\leq \Phi\left(\frac{\delta/L_t + \mu}{\sigma}\right) \\
 \Phi^{-1}(1 - \gamma) &\leq \frac{\delta/L_t + \mu}{\sigma} \\
 L_t \leq \bar{L} &\equiv \frac{\delta}{\sigma \Phi^{-1}(1 - \gamma) - \mu} & (4)
 \end{aligned}$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function and $\Phi^{-1}(\cdot)$ is its inverse.⁵⁸

Inequality (4) relates the leverage constraint \bar{L} to four parameters: the loss threshold δ , the expected return μ , and volatility σ , of the portfolio, and the loss probability γ . For a fixed loss probability γ , the leverage constraint is tighter (lower) for higher volatility σ , lower expected return μ , and a smaller loss threshold δ . With the other parameters fixed, a smaller γ implies a more reliable leverage constraint, i.e., one that is more likely to be satisfied. These are the trade-offs that policymakers and regulators must consider when they impose statutory limits on leverage.

A conservative approach to using (4) would be to assume a zero or negative value for the expected return μ (since it is mainly under adverse market conditions that a leverage constraint is most useful). From the financial institution's perspective, assuming $\mu = 0$ may penalize high-return strategies, but if a portfolio's expected return is difficult to ascertain *ex ante* with any degree of precision, an assumption of zero may not be an

58. The standard normal distribution has mean zero and standard deviation one, and its cumulative probability density is given by

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-x^2/2} dx.$$

1 WILLIAM FELLER, AN INTRODUCTION TO PROBABILITY AND ITS APPLICATIONS 174 (3d ed. 1968).

unreasonable starting point. In such cases, (4) reduces to a particularly simple expression that involves only δ , σ , and γ :

$$\bar{L} = \frac{\delta}{\sigma\Phi^{-1}(1-\gamma)} \quad (5)$$

For more risk-averse financial institutions such as savings and loans, money market funds, and other depository entities, a smaller δ —one commensurate with the lower-volatility assets held by these institutions—is appropriate. More speculative entities such as hedge funds may allow for larger δ , corresponding to more risky portfolio holdings. These risk-dependent considerations can be incorporated directly into the leverage constraint by simply specifying δ in units of return standard deviation: $\delta = \kappa\sigma$. This specification makes the loss threshold a multiple κ of the standard deviation of the portfolio's return, hence riskier assets will have higher loss thresholds and less risky assets will have tighter thresholds. This risk-sensitive leverage constraint seems to be a more accurate reflection of the motivation underlying such laws.

In this case, the expression for the leverage limit \bar{L} takes on a particularly transparent form:

$$\bar{L} = \frac{\kappa}{\Phi^{-1}(1-\gamma) - SR}, \quad SR \equiv \mu/\sigma \quad (6)$$

where SR denotes the Sharpe ratio (relative to a 0% risk-free interest rate).⁵⁹ This version of the leverage constraint states that if leverage does not exceed \bar{L} , then the probability of a κ -standard-deviation loss of capital is at most γ . Higher Sharpe-ratio strategies will have greater freedom to take on leverage, implying larger values for \bar{L} . Lower Sharpe-ratio strategies will be more severely leverage-constrained. This relation between Sharpe ratios and leverage is intuitive, but it should be noted that certain illiquid investment strategies may yield inflated Sharpe-ratio estimates due to serial correlation in their monthly returns.⁶⁰ These estimates must be adjusted for such effects before being used to establish leverage constraints.⁶¹

59. See William F. Sharpe, *The Sharpe Ratio*, J. PORTFOLIO MGMT., Fall 1994, at 49, 50 (“In [the *ex ante* version of the Sharpe ratio], the ratio indicates the expected differential return per unit of risk associated with the differential return.”).

60. See generally Andrew W. Lo, *The Statistics of Sharpe Ratios*, 58 FIN. ANALYSTS J. 36, 36–52, (2002).

61. *Id.*

B. Numerical Examples

To develop intuition for the leverage constraints (4) and (6), Table 5 reports values of \bar{L} for a variety of expected-return/volatility/loss-threshold/loss-probability combinations under normally and t -distributed portfolio returns. The magnitudes of these leverage limits seem broadly consistent with common intuition for various financial institutions. For a loss ratio of, say, $\delta = 20\%$ and annual volatility of $\sigma = 15\%$ —which would be appropriate for portfolios of hedge funds—assuming an expected return of $\mu = 0\%$ and a loss probability of $\gamma = 0\%$ under normality implies a leverage limit of 9.1. This limit is within the typical range of leverage that prime brokers offer their hedge fund clients.⁶²

For a more conservative institution, such as a savings bank or money market fund, a loss ratio of 1% and an annual volatility of 5% may be more appropriate, and assuming an expected return of 0% and a loss probability of 1%, Table 5 reports a leverage limit under normality of 1.4. For such institutions, this may not be conservative enough, especially in the presence of “black swan” events.⁶³ To capture this aspect of portfolio exposure, we can use an alternative to the standard normal distribution in (4), such as Student’s t -distribution, which approximates the normal distribution for large degrees of freedom, but which exhibits leptokurtosis or “fat tails” for smaller degrees of freedom.⁶⁴ For our alternative calculations, we use a t distribution with two degrees of freedom; this distribution has such fat tails that moments beyond the first are no longer finite.⁶⁵ Under this more conservative assumption, the leverage limit declines from 1.4 to 0.5, virtually no leverage allowed.

For the intermediate case of a retail equity investor, a loss ratio of 10% and an annual volatility of 25% may be appropriate. An expected return of 0% and a loss probability of 1% under the normal distribution with two degrees of freedom implies a leverage limit of 0.9 in this case, which is close to the limit of 2 imposed by Regulation T.⁶⁶

62. Andrew Ang et al., HEDGE FUND LEVERAGE 38 (2011).

63. See, e.g., Eugene F. Fama, *The Behavior of Stock Market Prices*, in STATISTICAL MODELS OF ASSET RETURNS 34, 34 (Andrew W. Lo ed., 2007) (discussing the random-walk model as a model of stock behavior that relatively devalues prior information as a predictor of stock behavior); see generally NASSIM NICHOLAS TALEB, *THE BLACK SWAN: THE IMPACT OF THE HIGHLY IMPROBABLE* (2007).

64. Thomas Lux, *The Stable Paretian Hypothesis and the Frequency of Large Returns: An Examination of Major German Stocks*, 6 APPLIED FIN. ECON. 463, 465 (1996).

65. The probability density function for this distribution is $(1/\sigma) f_{t,2}((t - \mu)/\sigma)$ where $f_{t,2}(t) = (1 + t^2/2)^{-3/2}/(2\sqrt{2})$. The expected value of this distribution is equal to μ , but the variance is infinite. Nevertheless, we still refer to the parameter σ^2 in the same way as we would usually refer to variance, since this value provides a measure of the scale of the distribution that is entirely analogous to the variance value for distributions with finite variance.

66. See *supra* Part II (stating that the 50% initial margin imposed by Regulation T corresponds to a reciprocal leverage value of 2).

Maximum Leverage $L(\mu, \sigma, \delta, \gamma)$ Under Normality												
$\gamma = 5\%$				$\gamma = 1\%$								
σ	1%	5%	10%	20%	μ	σ	1%	5%	10%	20%	δ	
1%	9.7	48.3	96.5	193.0	1%	6.8	34.1	68.2	136.5	1%	6.8	34.1
5%	1.9	9.7	19.3	38.6	5%	1.4	6.8	13.6	27.3	5%	1.4	6.8
10%	1.0	4.8	9.7	19.3	10%	0.7	3.4	6.8	13.6	10%	0.7	3.4
15%	0.6	3.2	6.4	12.9	15%	0.5	2.3	4.5	9.1	15%	0.5	2.3
20%	0.5	2.4	4.8	9.7	20%	0.3	1.7	3.4	6.8	20%	0.3	1.7
25%	0.4	1.9	3.9	7.8	25%	0.3	1.4	2.7	5.5	25%	0.3	1.4
50%	0.2	1.0	1.9	3.9	50%	0.1	0.7	1.4	2.7	50%	0.1	0.7
$\mu = 0\%$												
1%	11.9	59.7	119.4	238.7	1%	7.9	39.5	78.9	157.8	1%	7.9	39.5
5%	2.0	10.0	20.1	40.1	5%	1.4	7.0	14.0	28.1	5%	1.4	7.0
10%	1.0	4.9	9.8	19.7	10%	0.7	3.5	6.9	13.8	10%	0.7	3.5
15%	0.7	3.3	6.5	13.0	15%	0.5	2.3	4.6	9.2	15%	0.5	2.3
20%	0.5	2.4	4.9	9.7	20%	0.3	1.7	3.4	6.9	20%	0.3	1.7
25%	0.4	1.9	3.9	7.8	25%	0.3	1.4	2.7	5.5	25%	0.3	1.4
50%	0.2	1.0	1.9	3.9	50%	0.1	0.7	1.4	2.7	50%	0.1	0.7
$\mu = 5\%$												
1%	15.6	78.2	156.4	312.8	1%	9.4	46.8	93.6	187.2	1%	9.4	46.8
5%	2.1	10.5	20.9	41.8	5%	1.4	7.2	14.4	28.9	5%	1.4	7.2
10%	1.0	5.0	10.0	20.1	10%	0.7	3.5	7.0	14.0	10%	0.7	3.5
15%	0.7	3.3	6.6	13.2	15%	0.5	2.3	4.6	9.3	15%	0.5	2.3
20%	0.5	2.5	4.9	9.8	20%	0.3	1.7	3.5	6.9	20%	0.3	1.7
25%	0.4	2.0	3.9	7.8	25%	0.3	1.4	2.8	5.5	25%	0.3	1.4
50%	0.2	1.0	1.9	3.9	50%	0.1	0.7	1.4	2.7	50%	0.1	0.7
$\mu = 10\%$												
1%	226.8	1133.9	2267.7	4535.5	1%	21.1	105.6	211.2	422.5	1%	21.1	105.6
5%	2.4	11.9	23.9	47.7	5%	1.6	7.9	15.8	31.6	5%	1.6	7.9
10%	1.1	5.3	10.7	21.3	10%	0.7	3.7	7.3	14.6	10%	0.7	3.7
15%	0.7	3.4	6.9	13.7	15%	0.5	2.4	4.8	9.5	15%	0.5	2.4
20%	0.5	2.5	5.1	10.1	20%	0.4	1.8	3.5	7.1	20%	0.4	1.8
25%	0.4	2.0	4.0	8.0	25%	0.3	1.4	2.8	5.6	25%	0.3	1.4
50%	0.2	1.0	2.0	3.9	50%	0.1	0.7	1.4	2.8	50%	0.1	0.7

Maximum Leverage $L(\mu, \sigma, \delta, \gamma)$ Under t_5 Distribution												
$\gamma = 5\%$				$\gamma = 1\%$								
σ	1%	5%	10%	20%	μ	σ	1%	5%	10%	20%	δ	
1%	5.4	27.2	54.4	108.7	1%	2.3	11.4	22.8	45.6	1%	2.3	11.4
5%	1.087	5.4	10.9	21.7	5%	0.5	2.3	4.6	9.1	5%	0.5	2.3
10%	0.5	2.7	5.4	10.9	10%	0.2	1.1	2.279	4.6	10%	0.2	1.1
15%	0.4	1.8	3.6	7.2	15%	0.2	0.8	1.5	3.0	15%	0.2	0.8
20%	0.3	1.4	2.7	5.4	20%	0.1	0.6	1.1	2.3	20%	0.1	0.6
25%	0.2	1.1	2.2	4.3	25%	0.1	0.5	0.9	1.8	25%	0.1	0.5
50%	0.1	0.5	1.1	2.2	50%	0.0	0.2	0.5	0.9	50%	0.0	0.2
$\mu = 5\%$												
1%	6.1	30.5	60.9	121.9	1%	2.4	11.9	23.9	47.7	1%	2.4	11.9
5%	1.1	5.6	11.1	22.2	5%	0.5	2.3	4.6	9.2	5%	0.5	2.3
10%	0.5	2.7	5.5	11.0	10%	0.2	1.1	2.3	4.6	10%	0.2	1.1
15%	0.4	1.8	3.7	7.3	15%	0.2	0.8	1.5	3.0	15%	0.2	0.8
20%	0.3	1.4	2.7	5.5	20%	0.1	0.6	1.1	2.3	20%	0.1	0.6
25%	0.2	1.1	2.2	4.4	25%	0.1	0.5	0.9	1.8	25%	0.1	0.5
50%	0.1	0.5	1.1	2.2	50%	0.0	0.2	0.5	0.9	50%	0.0	0.2
$\mu = 10\%$												
1%	6.9	34.7	69.3	138.6	1%	2.5	12.5	25.1	50.1	1%	2.5	12.5
5%	1.1	5.7	11.4	22.7	5%	0.5	2.3	4.6	9.3	5%	0.5	2.3
10%	0.6	2.8	5.6	11.1	10%	0.2	1.2	2.3	4.6	10%	0.2	1.2
15%	0.4	1.8	3.7	7.4	15%	0.2	0.8	1.5	3.1	15%	0.2	0.8
20%	0.3	1.4	2.7	5.5	20%	0.1	0.6	1.1	2.3	20%	0.1	0.6
25%	0.2	1.1	2.2	4.4	25%	0.1	0.5	0.9	1.8	25%	0.1	0.5
50%	0.1	0.5	1.1	2.2	50%	0.0	0.2	0.5	0.9	50%	0.0	0.2

$\mu = 25\%$												
σ	1%	5%	10%	20%	μ	σ	1%	5%	10%	20%	δ	
1%	11.8	59.0	118.0	236.0	1%	2.9	14.7	29.5	58.9	1%	2.9	14.7
5%	1.2	6.1	12.2	24.4	5%	0.5	2.4	4.8	9.5	5%	0.5	2.4
10%	0.6	2.9	5.7	11.5	10%	0.2	1.2	2.3	4.7	10%	0.2	1.2
15%	0.4	1.9	3.8	7.5	15%	0.2	0.8	1.5	3.1	15%	0.2	0.8
20%	0.3	1.4	2.8	5.6	20%	0.1	0.6	1.2	2.3	20%	0.1	0.6
25%	0.2	1.1	2.2	4.4	25%	0.1	0.5	0.9	1.8	25%	0.1	0.5
50%	0.1	0.5	1.1	2.2	50%	0.0	0.2	0.5	0.9	50%	0.0	0.2

Table 5. Leverage constraints under loss probabilities γ of 5% and 1% for various levels of expected return μ , return volatility σ , and loss ratios δ , assuming that profits and losses are normally or t -distributed, with two degrees of freedom.

Maximum Leverage $L(\mu, \sigma, \delta, \gamma)$ Under Normality														
$\gamma = 5\%$														
SR	κ				SR	κ				SR	κ			
	2	5	10	20		2	5	10	20		2	5	10	20
0.00	1.2	3.0	6.1	12.2	0.00	0.9	2.1	4.3	8.6	0.00	0.7	1.7	3.4	6.8
0.10	1.2	3.1	6.1	12.2	0.10	0.9	2.2	4.3	8.6	0.10	0.7	1.7	3.4	6.9
0.20	1.2	3.1	6.1	12.3	0.20	0.9	2.2	4.3	8.6	0.20	0.7	1.7	3.4	6.9
0.50	1.2	3.1	6.2	12.4	0.50	0.9	2.2	4.4	8.7	0.50	0.7	1.7	3.5	6.9
0.75	1.3	3.1	6.3	12.5	0.75	0.9	2.2	4.4	8.8	0.75	0.7	1.7	3.5	7.0
1.00	1.3	3.2	6.3	12.6	1.00	0.9	2.2	4.4	8.8	1.00	0.7	1.8	3.5	7.0
1.50	1.3	3.2	6.5	12.9	1.50	0.9	2.2	4.5	9.0	1.50	0.7	1.8	3.5	7.1
2.00	1.3	3.3	6.6	13.2	2.00	0.9	2.3	4.5	9.1	2.00	0.7	1.8	3.6	7.2
5.00	1.5	3.8	7.5	15.0	5.00	1.0	2.5	5.0	9.9	5.00	0.8	1.9	3.8	7.7
10.00	2.0	4.9	9.9	19.7	10.00	1.2	2.9	5.9	11.8	10.00	0.9	2.2	4.4	8.7
20.00	5.2	13.0	26.0	52.0	20.00	1.9	4.7	9.4	18.8	20.00	1.2	3.0	6.0	12.0

Maximum Leverage $L(\mu, \sigma, \delta, \gamma)$ Under t_2 Distribution														
$\gamma = 5\%$														
SR	κ				SR	κ				SR	κ			
	2	5	10	20		2	5	10	20		2	5	10	20
0.00	0.7	1.7	3.4	6.8	0.00	0.3	0.7	1.4	2.9	0.00	0.3	0.7	1.4	2.9
0.10	0.7	1.7	3.4	6.9	0.10	0.3	0.7	1.4	2.9	0.10	0.3	0.7	1.4	2.9
0.20	0.7	1.7	3.4	6.9	0.20	0.3	0.7	1.4	2.9	0.20	0.3	0.7	1.4	2.9
0.50	0.7	1.7	3.5	6.9	0.50	0.3	0.7	1.4	2.9	0.50	0.3	0.7	1.4	2.9
0.75	0.7	1.7	3.5	7.0	0.75	0.3	0.7	1.4	2.9	0.75	0.3	0.7	1.4	2.9
1.00	0.7	1.8	3.5	7.0	1.00	0.3	0.7	1.4	2.9	1.00	0.3	0.7	1.4	2.9
1.50	0.7	1.8	3.5	7.1	1.50	0.3	0.7	1.5	2.9	1.50	0.3	0.7	1.5	2.9
2.00	0.7	1.8	3.6	7.2	2.00	0.3	0.7	1.5	2.9	2.00	0.3	0.7	1.5	2.9
5.00	0.8	1.9	3.8	7.7	5.00	0.3	0.8	1.5	3.0	5.00	0.3	0.8	1.5	3.0
10.00	0.9	2.2	4.4	8.7	10.00	0.3	0.8	1.6	3.2	10.00	0.3	0.8	1.6	3.2
20.00	1.2	3.0	6.0	12.0	20.00	0.4	0.9	1.8	3.5	20.00	0.4	0.9	1.8	3.5

Table 6. Leverage constraints under loss probabilities γ of 5% and 1% for various Sharpe Ratios $SR = \mu/\sigma$, and loss thresholds of κ standard deviations, assuming that profits and losses are normally or t -distributed, with two degrees of freedom.

These examples have been constructed under the assumption that the expected return μ of the portfolio is 0%, which we adopted to be conservative. However, for many financial institutions, this is clearly not an accurate reflection of their investment returns. For example, traditional broker–dealers earn relatively steady profits because they are paid for market-making services, with the risk of occasional large losses in the presence of strong market trends.⁶⁷ The impact of these considerations on leverage constraints is most readily observed in the Sharpe ratio version of \bar{L} calculations, which is found in (6) and is numerically evaluated in Table 6.

The entries in Table 6 show that as the Sharpe ratio increases, more leverage is allowable because the likelihood of significant losses decreases. With a loss threshold κ of 2 standard deviations and a loss probability γ of 1%, a Sharpe ratio of 0 yields a leverage constraint of 0.9 under the normality assumption, implying not only that no leverage is possible, but that 10% of this portfolio must be held in cash to ensure a loss probability of no more than 1%. However, a loss threshold of 20 standard deviations and a Sharpe ratio of 20 implies a leverage constraint of 18.8.

This may seem like an irrelevant set of parameters and an absurdly high amount of leverage, but recall that SR is the ratio of μ to σ with no risk-free rate subtracted from the numerator. Therefore, low-risk fixed-income investments such as a money market fund can have an extremely large SR because the standard deviation is so low. In fact, because money market funds are allowed to quote a fixed \$1 per share net asset value (NAV),⁶⁸ the volatility associated with their capital appreciation is 0, implying an infinite SR . Of course, the volatility of holding-period returns is not 0 due to dividend payments, but it is very small, hence an SR of 20 is not unrealistic for such institutions.

The main insight from Table 6 is that heterogeneity in the risk-reward ratios of financial institutions implies heterogeneity in the leverage constraints imposed on them. The analytical framework described in (4) and (6) provides intuition for the range of leverage limits and margin requirements imposed across financial institutions, securities, and markets. It will also allow us to identify potential weaknesses in existing regulations on leverage, and point the direction for making improvements.

67. See Memorandum from NYSE Euronext to Bd. of Governors of the Fed. Reserve Sys. et al. 2, 4–5 (Feb. 9, 2012), available at http://www.federalreserve.gov/SECRS/2012/February/20120224/R-1432/R-1432_021012_100681_346800234732_1.pdf (describing the market-making business and noting that market-makers assume risks that can lead to losses).

68. Under SEC Rule 2a-7, 17(a)(20), C.F.R. § 270.2a-7 (2011), money market funds need not treat net asset value as different from \$1 provided that the fair estimate of NAV does not deviate by more than one half of 1% from \$1, i.e., one half of one penny. For more detailed discussion of the ability of money market funds to treat their NAV as fixed, see generally Jill Fisch & Eric D. Roiter, *A Floating NAV for Money Market Funds: Fix or Fantasy?*, (NELLCO Legal Scholarship Repository No. 390, Aug. 25, 2011), available at http://lsr.nellco.org/cgi/viewcontent.cgi?article=1395&context=upenn_wps.

IV. The Dynamics of Leverage Constraints

The analytical framework described in Part III for determining limits to leverage was derived under the assumption that the parameters of the constrained entity's asset returns were constant over time. One of the most important observations from constraints (4) and (6) is that if the leverage constraint \bar{L} is to be constant over time (as opposed to being a time-varying function of market conditions), then either its inputs $(\mu, \sigma, \delta, \gamma)$ must also be constant over time, or some or all of them vary through time in lockstep so as to maintain the constancy of \bar{L} .

A. Time-Varying Loss Probabilities

Specifically, if we acknowledge that risk varies over time and circumstances so that σ should be indexed by t , then it is clear from (4) that at least one of the remaining three parameters (μ, δ, γ) must also be time-varying to keep \bar{L} fixed. For example, assume for simplicity that μ and δ are fixed; this implies that γ must vary in tandem with σ for a fixed \bar{L} . As σ_t increases, the probability that the fixed leverage constraint \bar{L} will prevent losses of δ will decline and vice versa as σ_t decreases. In fact, under these assumptions, we can write γ_t as an explicit function of σ_t :

$$\gamma_t = 1 - \Phi\left(\frac{\delta + \mu\bar{L}}{\sigma_t\bar{L}}\right) = 1 - \Phi\left(\frac{\kappa}{\bar{L}} + SR_t\right) \quad , \quad SR_t \equiv \mu/\sigma_t \quad (7)$$

For sudden changes in volatility, (7) shows that the probability of exceeding the loss threshold of δ or κ changes just as suddenly.

B. An Empirical Illustration with the S&P 500 Index

To illustrate the dynamics of leverage limits summarized in (7), suppose we apply this framework to an entity that holds the S&P 500 index as its only asset, and we wish to impose a loss threshold δ of 10% of its capital with a loss probability γ of 1%. To be conservative, we assume $\gamma = 0$ and a historical volatility level of about 15%. Table 5 shows that under these assumptions and the additional assumption of normality of the distribution, 4.5 is the required leverage limit. Given this limit, we can construct a time series of γ_t using (7) and the parameters just specified.

Figure 3 displays this relation using daily S&P 500 price-index returns (not including dividends) and a rolling 125-day-window estimator of σ_t . It shows that despite the leverage constraint, there are a number of occasions where the probability of breaching the loss threshold of 10% far exceeds the 1% level because of volatility. Not surprisingly, these are periods when the 125-day rolling-window volatility estimator exceeds the long-run volatility of 15% for the S&P 500. These dynamics illustrate the potential problems with a

fixed leverage constraint when the risk of the constrained institution's assets can vary significantly over time.

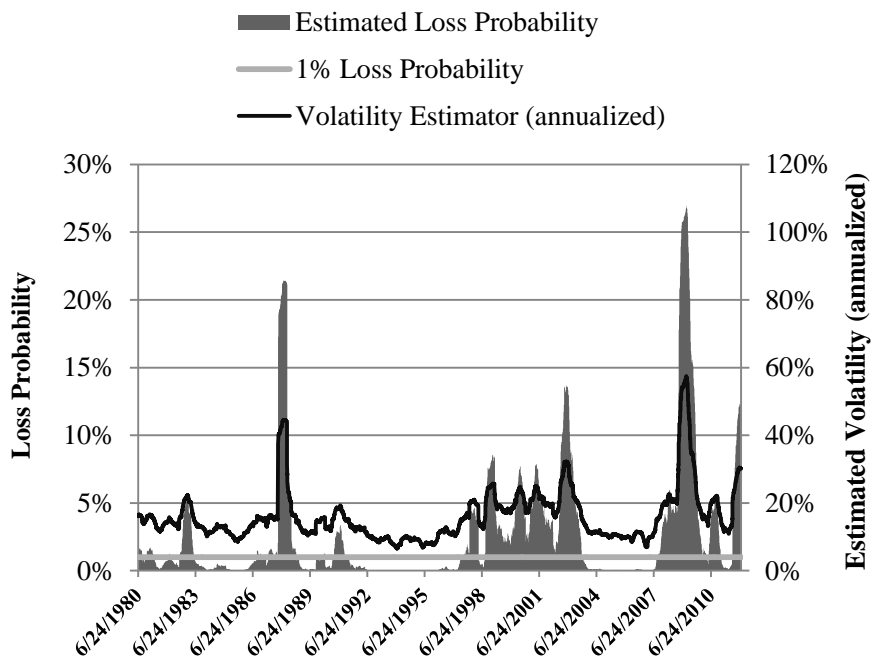


Figure 3. Daily estimated probabilities (gray bar graph) of a 10% one-day loss for a portfolio invested in the S&P 500 with a leverage constraint of $\bar{L} = 4.5$ under a normal distribution with a 125-day rolling-window volatility estimator (blue line graph) using S&P 500 returns excluding dividends, from June 24, 1980 to January 20, 2012. The leverage constraint of 4.5 is set to yield a loss probability of 1% under normality, indicated by the red line, and assuming a volatility of 15%.

There are at least two approaches to addressing the potential instabilities of portfolio parameters: model the specific interactions among the component securities, and model the time variation in the portfolio parameters. An example of the former approach is the specification of linear factor models that can be estimated security-by-security, and an example of the latter approach is the specification of a GARCH model for σ that includes measures of market distress like the VIX index as explanatory variables.⁶⁹

69. See JOHN Y. CAMPBELL ET AL, *THE ECONOMETRICS OF FINANCIAL MARKETS* 479–97 (1997) (describing various models of changing volatility over time).

C. Correlation and Netting

There is an important subtlety that is obscured by our focus on the *portfolio* return R_t , which is the implicit “netting” of risks within the portfolio. By modeling portfolio returns instead of the returns of the component securities, we are tacitly assuming that the parameters μ and σ are fixed constants and sufficient statistics for determining the statistical behavior of the portfolio. If, however, the correlations among the individual securities change over time and during adverse market conditions—as they did in August 1998 and August 2007—then μ and σ may be time varying and highly market dependent. For example, during August 2007, the volatilities of many equity market-neutral managers spiked dramatically, implying that the correlations of previously uncorrelated positions also spiked.

One way to capture this effect is to assume that a given institution’s portfolio consists of n securities with identical variances and equal pairwise correlation ρ to each other. Therefore, the covariance matrix Σ is given by:⁷⁰

$$\Sigma = \begin{bmatrix} \sigma^2 & \rho\sigma^2 & \cdots & \rho\sigma^2 \\ \rho\sigma^2 & \sigma^2 & \cdots & \rho\sigma^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho\sigma^2 & \rho\sigma^2 & \cdots & \sigma^2 \end{bmatrix}, \quad -\frac{1}{n-1} < \rho < 1 \quad (8)$$

Then for a given portfolio $\boldsymbol{\omega}$ of these assets,⁷¹ the variance σ_p^2 of the portfolio is simply:⁷²

70. The element in the i -th row and j -th column of the matrix Σ represents the covariance between the i -th asset and the j -th asset. The diagonal elements of the matrix are thus the variances of the n assets, and these are all equal to the common value σ^2 , by assumption. Each off-diagonal element is the covariance between two distinct assets and is equal to $\rho\sigma^2$, which is the product of the common correlation value between any pair of distinct assets, denoted ρ , and the product of the standard deviations of the two distinct assets, both of which are σ .

The upper bound of 1 for the possible values of ρ is imposed because correlation must always be less than or equal to 1. The lower bound of $-1/(n-1)$ is also necessary because a covariance matrix must be positive definite, and if ρ is below this bound, Σ will fail to meet this requirement. To see why this is so, note that positive definiteness is equivalent to the requirement that all eigenvalues of Σ are positive. To determine the sign of the eigenvalues, note that Σ can be written as $\Sigma = \sigma^2(1-\rho)\mathbf{I}_n + \sigma^2\rho\mathbf{1}\mathbf{1}'$, where \mathbf{I}_n is the $n \times n$ identity matrix and $\mathbf{1}$ is the n -dimensional column vector of all ones. Inspection shows that the eigenvectors of this matrix are $\mathbf{1}/\sqrt{n}$ and $(n-1)$ vectors orthogonal to $\mathbf{1}$. These last $(n-1)$ eigenvectors all have eigenvalue $\sigma^2(1-\rho)$, which is positive. The first eigenvector, however, has eigenvalue $\sigma^2(1+(n-1)\rho)$, which is positive exactly when $\rho > -1/(n-1)$.

71. We represent a portfolio with the n -dimensional vector $\boldsymbol{\omega}$, the i -th element of which is the fraction of the portfolio invested in the i -th asset. We assume that all weights are non-negative, so that no short positions are permitted, and we denote this non-negativity restriction as $\boldsymbol{\omega} \geq 0$.

72. The equality $\sigma_p^2 = \boldsymbol{\omega}\Sigma\boldsymbol{\omega}$ is the definition of the variance of the portfolio. The further results follow from the identity $\Sigma = \sigma^2(1-\rho)\mathbf{I}_n + \sigma^2\rho\mathbf{1}\mathbf{1}'$.

$$\begin{aligned}\sigma_\rho^2 &= \boldsymbol{\omega}' \boldsymbol{\Sigma} \boldsymbol{\omega} = \sigma^2 (\boldsymbol{\omega}' \boldsymbol{\omega} (1 - \rho) + \rho) \\ &= \sigma^2 (H(1 - \rho) + \rho) \quad , \quad H \equiv \boldsymbol{\omega}' \boldsymbol{\omega} \quad , \quad \boldsymbol{\omega} \geq 0\end{aligned}\quad (9)$$

where H is known as the ‘‘Herfindahl Index’’ and is a measure of the degree of concentration of a collection of non-negative percentages that sum to 100%.⁷³ Substituting (9) in (4) then yields the leverage constraint as a function of an entity’s asset correlations and Herfindahl index:

$$\bar{L} = \frac{\delta}{\sigma_\rho \Phi^{-1}(1 - \gamma) - \mu} = \frac{\delta}{\sigma [(1 - \rho)H + \rho] \Phi^{-1}(1 - \gamma) - \mu} \quad (10)$$

Because H is between 0 and 1, the leverage limit is strictly decreasing in ρ —as the correlation among individual assets rises, the leverage limit must be tighter to ensure a loss probability of γ . If the correlation varies over time and changes suddenly, as it has done on several occasions during the recent past, either the leverage constraint must change in tandem, or the loss probability will change if the other parameters are fixed.

To model sudden changes in correlation, consider the ‘‘phase-locking’’ two-factor model of the asset returns described by Professor Lo:⁷⁴

$$R_{it} = \alpha_i + \beta_i A_t + I_i Z_t + \varepsilon_{it} \quad (11)$$

Here R_{it} represents the return of asset i at time t , and α_i is a fixed component of this asset’s return. The quantities A_t , I_i , Z_t , and ε_{it} are variable across time and independent of each other, and ε_{it} is different for each asset i , with values of ε_{it} for different assets also independent of each other. In addition, each of these quantities is independently and identically distributed (IID) across time, and A_t , Z_t , and ε_{it} have the following expected values and variances:

$$\begin{aligned}E[A_t] &= \mu_\lambda \quad , \quad \text{Var}[A_t] = \sigma_\lambda^2 \\ E[Z_t] &= \mathbf{0} \quad , \quad \text{Var}[Z_t] = \sigma_z^2 \\ E[\varepsilon_{it}] &= \mathbf{0} \quad , \quad \text{Var}[\varepsilon_{it}] = \sigma_{\varepsilon_i}^2\end{aligned}\quad (12)$$

73. The Herfindahl Index is a measure of concentration defined as the sum of the squares of the weight of each factor in the portfolio, which exactly equal $\boldsymbol{\omega}' \boldsymbol{\omega}$. It is also used in the antitrust context to determine the concentration of market participants in an industry. See generally David S. Weinstock, *Using the Herfindahl Index to Measure Concentration*, 27 ANTITRUST BULL. 285 (1982).

74. ANDREW W. LO, HEDGE FUNDS: AN ANALYTIC PERSPECTIVE 18–22 (2008).

The quantity I_t is the phase-locking event indicator and is defined by:

$$I_t = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } p_0 = 1 - p \end{cases} \quad (13)$$

According to (11), expected returns are the sum of three components: the asset's alpha, α_i , a "market" component, A_t , to which each asset has its own individual sensitivity, β_i , and a phase-locking component that is identical across all assets at all times, taking only one of two possible values, either 0 (with probability p) or Z_t (with probability $1 - p$). If we assume that p is small, say 0.001, then most of the time the expected return of asset i is determined by $\alpha + \beta_i A_t$, but every once in a while an additional term Z_t appears. If the volatility σ_z of Z_t is much larger than the volatilities of the market factor, A_t , and the idiosyncratic risk, ε_{it} , then the common factor Z_t will dominate the expected returns of all assets when $I_t = 1$, i.e., there is phase-locking behavior.

More formally, consider the *conditional* correlation coefficient of two assets i and j , defined as the ratio of the conditional covariance divided by the square root of the product of the conditional variances, conditioned on $I_t = 0$:

$$\text{Corr}[R_{it}, R_{jt} | I_t = 0] = \frac{\beta_i \beta_j \sigma_\lambda^2}{\sqrt{\beta_i^2 \sigma_\lambda^2 + \sigma_{\varepsilon_i}^2} \sqrt{\beta_j^2 \sigma_\lambda^2 + \sigma_{\varepsilon_j}^2}} \quad (14)$$

$$\approx 0 \quad \text{for } \beta_i \approx \beta_j \approx 0 \quad (15)$$

where we assume $\beta_i \approx \beta_j \approx 0$ for illustrative purposes. Now consider the conditional correlation, conditioned on $I_t = 1$:

$$\text{Corr}[R_{it}, R_{jt} | I_t = 1] = \frac{\beta_i \beta_j \sigma_\lambda^2 + \sigma_z^2}{\sqrt{\beta_i^2 \sigma_\lambda^2 + \sigma_z^2 + \sigma_{\varepsilon_i}^2} \sqrt{\beta_j^2 \sigma_\lambda^2 + \sigma_z^2 + \sigma_{\varepsilon_j}^2}} \quad (16)$$

$$\approx \frac{1}{\sqrt{1 + \sigma_{\varepsilon_i}^2 / \sigma_z^2} \sqrt{1 + \sigma_{\varepsilon_j}^2 / \sigma_z^2}} \quad \text{for } \beta_i \approx \beta_j \approx 0 \quad (17)$$

If σ_z^2 is large relative to $\sigma_{\varepsilon_i}^2$ and $\sigma_{\varepsilon_j}^2$, i.e., if the variability of the catastrophe component dominates the variability of the residuals of both assets—a plausible condition that follows from the very definition of a catastrophe—then (17) will be approximately equal to 1! When phase-locking occurs, the correlation between i and j —close to 0 during normal times—can become arbitrarily close to 1.

An insidious feature of (11) is the fact that it implies a very small value for the *unconditional* correlation, which is the quantity most readily estimated

and the most commonly used in risk reports, Value-at-Risk calculations, and portfolio decisions. To see why, recall that the unconditional correlation coefficient is simply the unconditional covariance divided by the product of the square roots of the unconditional variances:

$$\text{Corr}[R_{it}, R_{jt}] \equiv \frac{\text{Cov}[R_{it}, R_{jt}]}{\sqrt{\text{Var}[R_{it}]\text{Var}[R_{jt}]}} \quad (18)$$

$$\text{Cov}[R_{it}, R_{jt}] = \beta_i \beta_j \sigma_\lambda^2 + \text{Var}[I_t Z_t] = \beta_i \beta_j \sigma_\lambda^2 + p \sigma_z^2 \quad (19)$$

$$\text{Var}[R_{it}] = \beta_i^2 \sigma_\lambda^2 + \text{Var}[I_t Z_t] + \sigma_{\varepsilon_i}^2 = \beta_i^2 \sigma_\lambda^2 + p \sigma_z^2 + \sigma_{\varepsilon_i}^2 \quad (20)$$

Combining these expressions yields the unconditional correlation coefficient under (11):

$$\text{Corr}[R_{it}, R_{jt}] = \frac{\beta_i \beta_j \sigma_\lambda^2 + p \sigma_z^2}{\sqrt{\beta_i^2 \sigma_\lambda^2 + p \sigma_z^2 + \sigma_{\varepsilon_i}^2} \sqrt{\beta_j^2 \sigma_\lambda^2 + p \sigma_z^2 + \sigma_{\varepsilon_j}^2}} \quad (21)$$

$$\approx \frac{p}{\sqrt{p + \sigma_{\varepsilon_i}^2 / \sigma_z^2} \sqrt{p + \sigma_{\varepsilon_j}^2 / \sigma_z^2}} \quad \text{for } \beta_i \approx \beta_j \approx 0 \quad (22)$$

If we let $p = 0.001$ and assume that the variability of the phase-locking component is 10 times the variability of the residuals ε_i and ε_j , this implies an unconditional correlation of:

$$\text{Corr}[R_{it}, R_{jt}] \approx \frac{p}{\sqrt{p + 0.1} \sqrt{p + 0.1}} = 0.001 / 0.101 = 0.0099$$

or less than 1%. As the variance σ_z^2 of the phase-locking component increases, the unconditional correlation (22) also increases so that eventually the existence of Z_t will have an impact. However, to achieve an unconditional correlation coefficient of, say, 10%, σ_z^2 would have to be about 100 times larger than $\sigma_{\varepsilon_i}^2$. Without the benefit of an explicit risk model such as (11), it is virtually impossible to detect the existence of a phase-locking component from standard correlation coefficients.

V. Qualifications and Extensions

The mathematical framework we have proposed must be qualified in at least three respects, and these qualifications suggest important extensions to be pursued in future research. The first involves incorporating liquidity into our analysis, the second has to do with the tradeoff between efficacy and

complexity, and the third is the distinction between microprudential and macroprudential policies. We discuss each of these issues in more detail in the following sections.

A. Liquidity

Implicit in all leverage constraints is the recognition that rapidly deteriorating market conditions can make it difficult if not impossible to reduce risk without suffering extreme losses, hence the need for minimum capital requirements. After all, if financial assets were equally liquid during good times and bad times, institutions could simply reduce their leverage as needed rather than being required to hold a minimum amount of capital in reserve. The failure of portfolio insurance to protect investors from the stock market crash of October 19, 1987, clearly illustrates the practical limitations of such an approach.⁷⁵ But if liquidity is so central to the motivation for leverage constraints, a more direct approach to measuring and managing liquidity may be preferred.

The challenge, of course, is how to measure liquidity, a concept that is reminiscent of Justice Potter Stewart's description of pornography: difficult to define but "I know it when I see it."⁷⁶ The difficulty lies in the fact that liquidity is multidimensional—an asset is said to be liquid if it can be bought or sold quickly in large quantities without significantly affecting its prevailing market price.⁷⁷ For assets traded on organized exchanges, a simple and effective measure of liquidity is an asset's bid/offer spread, measured as a percentage of the asset's mid-spread price.⁷⁸ For privately placed assets such as limited partnerships for which investment returns are computed only monthly or quarterly, liquidity can be measured by the statistical persistence of those returns through time, with more persistent returns corresponding to less liquidity.⁷⁹ Alternatively, more explicit theoretical models of liquidity may be constructed from which empirical predictions can be derived.⁸⁰ With explicit measures of liquidity in hand, dynamic leverage constraints can adapt to changes in both volatility and liquidity, reducing the possibility of

75. See Mark Carlson, Bd. of Governors of the Fed. Reserve, *A Brief History of the 1987 Stock Market Crash with a Discussion of the Federal Reserve Response* 15–16 (Fin. & Econ. Discussion Series No. 2007-13, 2006), available at <http://www.federalreserve.gov/pubs/feds/2007/200713/200713pap.pdf> (discussing the extent to which portfolio insurance may have contributed to the 1987 crash).

76. *Jacobellis v. Ohio*, 378 U.S. 184, 197 (1964) (Stewart, J., concurring).

77. CAMPBELL ET AL., *supra* note 69, at 99–100.

78. See LO, *supra* note 74, at 98–99 (identifying the bid/ask spread as a percentage of mid-spread price as a measure of liquidity).

79. See Getmansky et al., *supra* note 47, at 535 (observing that serial correlation, which measures persistence, "is a proxy for illiquidity").

80. See, e.g., Brunnermeier & Pedersen, *supra* note 44, at 2201–02 (providing a model featuring a "unified explanation for the main empirical features of market liquidity" based on the links between funding and market liquidity).

unanticipated losses. In several contexts, incorporating liquidity can significantly change the nature of the leverage constraint. For example, because illiquid investments often have higher estimated Sharpe ratios than their more liquid counterparts due to serial correlation in their monthly returns,⁸¹ this would permit greater leverage *ceteris paribus* if leverage constraint (6) were applied naively. Therefore, without some form of liquidity constraint, leverage constraints may cause regulated entities to favor higher Sharpe-ratio strategies, inadvertently increasing their illiquidity exposure and potential losses in the event of a market dislocation.

However, taking liquidity into account will mean dynamic leverage constraints that now depend on two variables instead of one, which increases the operational complexity for both regulated entities and their regulators. We turn to this issue next.

B. *Efficacy vs. Complexity*

Our theoretical and empirical analysis implies that dynamic leverage constraints are needed to yield fixed loss probabilities when risk varies over time. However, this result must be qualified by the fact that implementing dynamic leverage constraints is operationally more complex and demanding. An illustration of the practical challenges involved is provided by Figure 2, presented earlier; when asked to identify the meaning and source of these equations, even seasoned banking professionals do not always realize it is the supervisory formula proposed by the Bank of International Settlements for determining whether a bank is sufficiently well capitalized with respect to its securitized debt holdings, and is taken from Paragraph 624 of the Basel II Accord. This framework will soon be replaced by the even more complex Basel III Accord in which capital requirements have been raised, but nothing has been done to address the issue of complexity.⁸²

A consequence of this complexity is the difficulty in distinguishing policy failures from other causes of financial instability. A striking case in point is the ongoing misunderstanding and debate surrounding the role of the 2004 SEC rule change to its “net capital rule” (15c3-1) in the financial crisis of 2007–2009.⁸³ A former SEC official alleged that the immense losses suffered by large financial firms in the recent financial collapse were due in part to the SEC’s failure to apply traditional regulatory constraints on leverage.⁸⁴ Several

81. See generally Lo., *supra* note 60.

82. Andrew W. Lo, *Complexity, Concentration, and Contagion: A Comment*, 58 J. MONETARY ECON. 471, 472–73 (2011).

83. We thank Jacob Goldfield for bringing this example to our attention.

84. Lee A. Pickard, *SEC’s Old Capital Approach Was Tried—And True*, AM. BANKER, Oct. 8, 2008, at 10. In particular, Mr. Pickard argued that before the rule change,

the broker–dealer was limited in the amount of debt it could incur, to about 12 times its net capital, though for various reasons broker–dealers operated at significantly lower ratios. . . . If, however, Bear Stearns and other large broker–dealers had been subject to the typical haircuts on their securities positions, an aggregate indebtedness restriction,

newspapers published articles based on this claim, including the *New York Times* on October 3, 2008:

In loosening the capital rules, which are supposed to provide a buffer in turbulent times, the agency also decided to rely on the firms' own computer models for determining the riskiness of investments, essentially outsourcing the job of monitoring risk to the banks themselves.

Over the following months and years, each of the firms would take advantage of the looser rules. At Bear Stearns, the leverage ratio—a measurement of how much the firm was borrowing compared to its total assets—rose sharply, to 33 to 1. In other words, for every dollar in equity, it had \$33 of debt. The ratios at the other firms also rose significantly.⁸⁵

This rapid increase in leverage provided a compelling explanation for events related to the Financial Crisis of 2007–2009, and Pickard suggested a regulatory fix: “The SEC should reexamine its net capital rule and consider whether the traditional standards should be reapplied to all broker–dealers.”⁸⁶

The problem with this remedy is that the premise is false.⁸⁷ As Dr. Erik Sirri—the SEC’s director of the Division of Markets and Trading—explained

and other provisions for determining required net capital under the traditional standards, they would not have been able to incur their high debt leverage without substantially increasing their capital base.

Id.

85. Stephen Labaton, *Agency’s ’04 Rule Let Banks Pile Up New Debt, and Risk*, N.Y. TIMES, Oct. 2, 2008, at A1, *available at* http://www.nytimes.com/2008/10/03/business/03sec.html?pagewanted=1&_r=1.

86. Pickard, *supra* note 84.

87. The provisions of SEC Rule 15c3-1 are complex and contain more than a simple leverage test. *See* 17 C.F.R. § 240.15c3-1 (2011). A leverage test that does appear in the rule specifies a 15-to-1 ratio, with a 12-to-1 “early warning” obligation. *Id.* at § 240.15c3-1(a)(1)(ii). This component of the rule only applies to unsecured debt, however, and it did not apply to large broker–dealers, who were instead subject under the rule to net capital requirements based on amounts owed to them by their customers, i.e., a customer-receivable or “aggregate debit item” test. *Id.* This type of test requires a broker–dealer to maintain net capital equal to at least 2% of such receivable amounts, and it was by virtue of this rule that the five large investment banks were able to maintain higher leverage ratios in the 1990s than after the 2004 rule change. *Id.* Similarly, the broker–dealer subsidiaries of these investment banks, which were the entities actually subject to the net capital rule, had long achieved leverage ratios far in excess of 15-to-1. Eric R. Sirri, Dir., Div. of Trading and Mkts. U.S. Sec. & Exch. Comm’n, *Securities Markets and Regulatory Reform* (April 9, 2009), *available at* <http://www.sec.gov/news/speech/2009/spch040909ers.htm>. The historical leverage ratios of the investment banks were readily available to the public in their financial reports, and the facts regarding the true nature of the SEC net capital rule were also available in the public domain. *Id.*

So what was this rule change about, if not about changing leverage restrictions? It was meant to apply only to the five largest U.S. investment banks which were at a competitive disadvantage in conducting business in Europe because they did not satisfy certain European regulatory requirements dictated by the Basel Accord. *Id.* By subjecting themselves to *broader* regulatory supervision—becoming designated “Consolidated Supervised Entities” or CSEs—these U.S. firms would be on a more equal footing with comparable European firms. *Id.* As Sirri explains:

on April 9, 2009, “First and most importantly, the Commission did not undo any leverage restrictions in 2004.”⁸⁸ Moreover, the stunning jump in leverage from 12-to-1 to 33-to-1 reported by the press was also incorrect.⁸⁹ However, a number of prominent academics and policymakers repeated this misinterpretation of the rule change,⁹⁰ and despite several published articles attempting to set the record straight,⁹¹ the misunderstanding still persists in certain quarters.⁹²

Thus the Commission effectively added an additional layer of supervision at the holding company where none had existed previously. While certain changes were made in 2004 to the net capital rule to conform more closely with the methods of computing capital adequacy that would be applied at the holding company, the changes were unrelated to the ‘12-to-1’ restriction. . . . Thus, the Commission did not eliminate or relax any requirements at the holding company level because previously there had been no requirements. In fact, the Commission increased its supervisory access to the CSE investment bank holding companies.

Id. Now with respect to the net capital rule, Sirri explains that it had nothing to do with leverage constraints: “The net capital rule requires a broker-dealer to undertake two calculations: (1) a computation of the minimum amount of net capital the broker-dealer must maintain; and (2) a computation of the actual amount of net capital held by the broker-dealer. The ‘12-to-1’ restriction is part of the first computation and it was not changed by the 2004 amendments. The greatest changes effected by the 2004 amendments were to the second computation of actual net capital.” *Id.* We thank Bob Lockner for decoding the intricacies of the SEC net capital rule.

88. Sirri, *supra* note 87. Sirri cites several documented and verifiable facts to support this surprising conclusion. *Id.* This correction was reiterated by another SEC official. Letter from Michael Macchiaroli, Assoc. Dir., Div. of Mkts. and Trading, Sec. & Exch. Comm’n, to the General Accountability Office (July 17, 2009), reproduced in GEN. ACCOUNTABILITY OFFICE, FINANCIAL MARKETS REGULATION: FINANCIAL CRISIS HIGHLIGHTS NEED TO IMPROVE OVERSIGHT OF LEVERAGE AT FINANCIAL INSTITUTIONS AND ACROSS SYSTEM 117 (2009) [hereinafter GAO REPORT].

89. According to the GAO Report: “In our prior work on Long-Term Capital Management (a hedge fund), we analyzed the assets-to-equity ratios of four of the five broker-dealer holding companies that later became CSEs and found that three had ratios equal to or greater than 28-to-1 at fiscal year-end 1998, which was higher than their ratios at fiscal year-end 2006 before the crisis began” GAO REPORT, *supra* note 88, at 40. In footnote 68 of that report, the GAO observes that its 1999 report GAO/GGD-00-3 on Long-Term Capital Management “did not present the assets-to-equity ratio for Bear Stearns, but its ratio also was above 28 to 1 in 1998.” *Id.* at 40 n.68.

90. See, e.g., Alan S. Blinder, *Six Errors on the Path to the Financial Crisis*, N.Y. TIMES, Jan. 24, 2009, available at <http://www.nytimes.com/2009/01/25/business/economy/25view.html> (listing leverage ratios of 33-to-1 as one of the critical errors that led to the economic crisis); John Coffee, *Analyzing the Credit Crisis: Was the SEC Missing in Action?*, N.Y. L.J., Dec. 2008, available at www.law.com/jsp/law/article.jsp?id=1202426495544 (arguing that investment banks’ increased leverage ratios left them vulnerable to volatility in the market); CARMEN M. REINHART & KENNETH S. ROGOFF, THIS TIME IS DIFFERENT: EIGHT CENTURIES OF FINANCIAL FOLLY 213-14 (2009) (stating that allowing investment banks to triple their leverage ratios contributed to the economic crisis); Joseph Stiglitz, *Capitalist Fools: Five Key Mistakes that Led Us to Collapse*, in THE GREAT HANGOVER: 21 TALES OF THE NEW RECESSION FROM THE PAGES OF VANITY FAIR, at 145, 148 (Graydon Carter ed., 2010) (citing the increase in leverage ratios from 12-to-1 to 30-to-1 as one of the factors that contributed to the economic collapse); STIGLITZ, *supra* note 9, at 163 (2010) (stating that the SEC stood idly by as investment banks’ leverage ratios rose as high as 40-to-1).

91. See, e.g., Sirri, *supra* note 87 (arguing that media reports about institutions’ leverage ratios and how the rule change contributed to these ratios were incorrect and stating that the rule change actually made no changes to institutions’ leverage ratios); Bethany McLean & Joe Nocera, ALL THE DEVILS ARE HERE: THE HIDDEN HISTORY OF THE FINANCIAL CRISIS 244 (2010) (remarking that the

These considerations suggest that there is a trade-off between the efficacy of dynamic leverage constraints and the complexity they create for all stakeholders. Such trade-offs may not be easily quantifiable, but they exist nonetheless and must be kept in mind when proposing rule changes involving leverage constraints. Perhaps technological advances in risk estimation, systems integration, and trade execution will allow institutions to comply with ever more complex regulations. But even with such advances, complexity is not costless and must be carefully weighed against its potential benefits.

C. *Microprudential vs. Macroprudential Policies*

Prior to the Dodd-Frank Act of 2010, the vast majority of rules governing leverage has been focused on microprudential regulation, i.e., regulation designed to limit the risks to individual institutions.⁹³ However, the financial crisis of 2007–2009 has underscored the need for macroprudential regulation, which is designed to limit the risk to the entire financial system. The two types of regulation are obviously linked: microprudential rules such as minimum capital requirements for individual institutions may also reduce systemic risk. However, the purview of macroprudential regulation is far broader, and includes the impact of linkages and cross-effects of all financial institutions, as well as feedback loops as institutions respond to changing economic and regulatory environments.

The framework we propose has been developed from a microprudential perspective; however, it may have useful implications for macroprudential policy challenges when extended to the multivariate case. In particular, macroprudential regulation is meant to reduce the probability of systemic shocks, and simultaneous losses among the most systemically important financial institutions is one important example. The same analysis of Section III may be applied to the joint probability of simultaneous losses:

$$\text{Prob}(\pi_{1t} < -\delta_1 K_{1t}, \pi_{2t} < -\delta_2 K_{2t}, \dots, \pi_{nt} < -\delta_n K_{nt})$$

blame placed on the rule change was “part of the lore of the financial crisis” and declaring that the blame actually rested elsewhere); Andrew W. Lo & Mark T. Mueller, *WARNING: Physics Envy May Be Hazardous To Your Wealth!*, 8 J. INV. MGMT. 53–64 (2010) (addressing the alleged involvement of the SEC rule change in the increase of leverage ratios and concluding it was simply an easy scapegoat but not an actual cause of the issue); Lo, *supra* note 4, at 151–78 (debunking the popular argument that the SEC rule change caused investment banks to become vulnerable and contributed to the crisis).

92. *EconTalk: Simon Johnson on the Financial Crisis*, LIBR. OF ECON. & LIBERTY (Nov. 28, 2011), http://www.econtalk.org/archives/2011/11/simon_johnson_o.html.

93. *Macroprudential Approach to Financial Regulation 1* (J. of Econ. Perspectives, Working Paper, 2010), available at <http://web.williams.edu/Economics/seminars/steinJEP.pdf> (“Many observers have argued that a weakness of the existing framework is that it is largely microprudential.”).

While the analytics will likely be considerably more complex due to the interrelationships among the n institutions, the conceptual framework that emerges is straightforward. In particular, systemic risk arises from the fact that this joint probability distribution is not simply the product of the individual institution's distributions due to statistical dependence among the institutions' profits and losses, hence the evaluation of this joint probability will require us to model the dependencies explicitly.⁹⁴ Moreover, a fully dynamic version of this joint distribution of losses can capture feedback effects of many financial institutions shifting their leverage in response to market conditions, ultimately yielding a general equilibrium model of systemic risk. The computational demands for evaluating such a system may be daunting, but even a highly aggregated version may yield useful insights from a macroprudential policy perspective.

VI. Conclusion

Regulatory constraints on leverage are generally fixed limits that do not vary over time or with market conditions. From a legal and rulemaking perspective this may be optimal, but from an economic perspective it can be disastrous. Because financial investments can exhibit such dynamic properties from time to time, static constraints may have dramatically different consequences across market regimes. With the added complexity of financial innovation and regulatory arbitrage by the most sophisticated financial institutions, static financial regulations are woefully inadequate in meeting their stated objectives.

Using a loss-probability-based framework that connects leverage constraints with four other key parameters of a regulated entity's situation, we can integrate the disparate considerations involved in formulating these financial regulations. Our illustrative empirical examples show substantial variation in the estimated risk of the S&P 500 Index, implying equally substantial variation in the implied loss probability of an investment in this important asset under a fixed leverage limit. Time-varying expected returns and correlations can greatly amplify these instabilities and sharp spikes in the likelihood of losses.

These results suggest an urgent need for lawmakers, regulators, practitioners, and economists to collaborate to design more effective regulation—regulation that is both clear and adaptive. Without a fundamental change in the very nature of financial regulation, the process of regulatory reform will always be three steps behind the institutions that need supervision.

94. For a list of potential systemic risk measures, see generally Dimitrios Bisias et al., *A Survey of Systemic Risk Analytics* (U.S. Dep't of Treasury, Office of Fin. Research, Working Paper No. 0001, 2012), *available at* http://www.treasury.gov/initiatives/wsr/ofr/Documents/OFRwp0001_BisiasFloodLoValavanis_ASurveyOfSystemicRiskAnalytics.pdf.