PERMEABILITY ESTIMATION FROM VELOCITY ANISOTROPY IN FRACTURED ROCK

by

Richard L. Gibson, Jr. and M. Nafi Toksöz

Earth Resources Laboratory
Department of Earth, Atmospheric, and Planetary Sciences
Massachusetts Institute of Technology
Cambridge, MA 02139

ABSTRACT

Cracks in a rock mass subjected to a uniaxial stress will be preferentially closed depending on the angle between the fracture normal and the direction of the applied stress. If the prestress fracture distribution is isotropic, the effective elastic properties of such a material are then transversely isotropic due to the preferred alignment of the cracks. Velocity measurements in multiple directions are used to invert for the probability density function describing orientations of crack normals in such a rock. We suggest a means of using the results on fracture distribution from the velocity inversion to estimate the anisotropic permeability of the fracture system. This approach yields a prediction of permeability as a function of the angle from the uniaxial stress direction.

INTRODUCTION

A common goal of seismic field experiments is to estimate rock properties such as permeability from the information contained in the seismic waveforms. Fractured media provide a particularly interesting example of a permeable medium, since a material containing an aligned system of cracks will be effectively anisotropic for elastic wavelengths much greater than the crack dimensions (Hudson, 1980,1981; Crampin, 1984). While a particular rock may have a randomly oriented distribution of cracks, application of a uniaxial stress will preferentially close fractures depending on orientation with respect to the stress axis (Walsh, 1965; Nur, 1971). It has been suggested that the prevailing tectonic stress regimes in the earth frequently include a maximum compressive stress which is horizontal, resulting in such an alignment of vertically oriented cracks (Crampin, 1981). A uniaxial stress is easily produced in laboratory experiments as well (Nur and Simmons, 1969).

Analysis of the elastic anisotropy produced by crack alignment can be used to investigate fracture properties. Sayers (1988a,b) suggested a means of inverting for the orientations of crack normals using these velocity measurements. This method involves an expansion of the fracture orientation distribution function in terms of harmonics related to the system of Euler angles describing the orientations. The coefficients in the expansion are subsequently related to perturbations in elastic moduli predicted by the Hudson (1981) theory for the properties of a cracked medium, and an inversion was performed based on an approximate expression for elastic wave velocity derived from a variational approach (Sayers, 1988a,b).

In this paper, we apply a similar inversion for crack orientations but do not use an approximate expression for the velocities. Instead, a nonlinear inversion is performed by linearizing the problem about an initial estimate of crack density and a parameter describing the distribution of crack aspect ratios. The resulting estimate of crack orientations and the distribution of aspect ratios with respect to direction is used to predict permeability as a function of direction with respect to the uniaxial stress axis. The method is applied to ultrasonic velocity data for Barre granite (Nur and Simmons, 1969) and the implications of the results for permeability prediction are discussed.

THEORY

Inversion for Crack Orientations

The rock medium is assumed to contain an isotropic distribution of cracks in the unstressed state so that the effective elastic parameters of the material are also isotropic in this case. When a uniaxial stress is applied to such a material, some of the cracks will close depending on the angle of the crack normal with respect to the stress axis (Walsh, 1965) (Figure 1). This angle γ_0 is given by

$$\cos\gamma_0 = \sqrt{\frac{\alpha E_0}{\sigma}} \quad , \tag{1}$$

where α is the crack aspect ratio, E_0 is the Young's modulus of the uncracked material, and σ is the applied uniaxial stress. The initially isotropic material will become anisotropic after application of the stress with rotational symmetry about the stress axis (Nur, 1971). The effective elastic properties of the stressed, cracked material will then have a transversely isotropic symmetry.

The effective elastic moduli of the medium can be estimated by averaging the elastic constants of the fractured material over a crack orientation distribution function $N(\theta, \psi, \phi)$, where θ, ψ, and ϕ are Euler angles of rotation specified in Figure 2. These angles define the set of rotations necessary to obtain the orientation of the crack Cartesian coordinate system x, y, z for each crack with respect to the composite medium reference Cartesian coordinate system denoted by X, Y, Z. We specify the initial orientation of the fracture prior to rotation such that the crack normal (parallel to z) is parallel to Z, and the other two axes x and y are therefore in the plane of the fracture. Note that for a circular crack, only θ and ψ are necessary to fully specify crack orientations, and ϕ can freely range from 0 to 2π. The crack orientation distribution function $N(\theta, \psi, \phi)$ is defined so that integration over the full domain is one:

$$\int_0^{2\pi} \int_0^{2\pi} \int_0^{\pi} N(\theta, \psi, \phi)\, d\theta\, d\psi\, d\phi = 1 \quad . \tag{2}$$

This function can be expanded in generalized spherical harmonics (Roe, 1965)

$$\sum_{l=0}^{\infty} \sum_{m=-l}^{l} \sum_{n=-l}^{l} W_{lmn} Z_{lmn}(\zeta) e^{-im\psi} e^{-in\phi} \quad . \tag{3}$$

Here $\zeta = \cos\theta$. The derivation of the generalized Legendre functions $Z(\zeta)$ and some of their properties are described by Roe (1965). Each coefficient W_{lmn} in the expansion of the orientation distribution function $N(\zeta, \psi, \phi)$ is obtained by integrations of the following form:

$$W_{lmn} = \frac{1}{4\pi^2} \int_0^{2\pi} \int_0^{2\pi} \int_{-1}^{1} N(\zeta, \psi, \phi) Z_{lmn}(\zeta) e^{im\psi} e^{in\phi}\, d\zeta\, d\psi\, d\phi \quad . \tag{4}$$

With this expansion, the orientation distribution function can be decomposed into harmonic components.

If a polycrystalline aggregate were considered, an estimate of the elastic properties of the aggregate could be obtained by simply averaging the elastic constants of the individual crystals with respect to the orientation distribution. This method, the Voigt approach, is known to yield an upper bound on the elastic constants (Hearmon, 1961). The same procedure can be applied to the fractured medium by averaging the effective elastic constants of fractured material over all sets of fracture orientations in the rock (Sayers, 1988a). The averaged constants can be written (Morris, 1969)

$$\begin{aligned} \overline{c}_{ijkl} &= c_{mnpq} \int_0^{2\pi} \int_0^{2\pi} \int_{-1}^{1} T_{ijklmnpq}(\zeta, \psi, \phi)\, d\zeta\, d\psi\, d\phi \\ &= c_{mnpq} \overline{T}_{ijklmnpq} \quad , \\ T_{ijklmnpq} &= \frac{\partial x_i}{\partial X_m} \frac{\partial x_j}{\partial X_n} \frac{\partial x_k}{\partial X_p} \frac{\partial x_l}{\partial X_q} \quad . \end{aligned} \tag{5}$$

The Einstein summation convention is applied. The matrix $\overline{T}_{ijklmnpq}$ essentially defines an average rotation of the elastic constants of the individual components c_{mnpq}. Morris (1969) has calculated a table of values for the matrix elements $\overline{T}_{ijklmnpq}$ in terms of the coefficients of the expansion of the distribution function up to order $l = m = n = 4$ for composites of materials with orthorhombic symmetry which can also be applied to material with hexagonal symmetry. The orthogonality properties of the harmonics cause terms for indices greater than 4 to disappear since the fourth order elastic tensor c_{mnpq} will only have coefficients for $l = m = n = 4$. The Morris (1969) table can easily be used in Eq. (5) to find the overall properties.

The theory of Hudson (1980,1981) for the stiffness constants of a fractured medium can be used to obtain values for c_{ijkl} to use on the right hand side of Eq. (5). This theory provides an expression for the effective elastic tensor c_{ijkl} of a homogeneous medium containing a single set of parallel penny shaped cracks with dimensions much less than a wavelength. This expression is in terms of a first order correction c^1_{ijkl} to the elastic tensor of the unfractured material c^0_{ijkl}:

$$c_{ijkl} = c^0_{ijkl} + \epsilon\, c^1_{ijkl} \quad . \tag{6}$$

Here ϵ is the crack density defined by $\epsilon = n\alpha^3$, n is the number of cracks per unit volume, and a is the crack radius. Hudson (1980) also derived a second order term which results in values of the stiffnesses which are quadratic functions of the concentration of cracks, and hence the second order theory actually predicts increasing velocities for very large crack concentrations. In order to match the observed data discussed below, the second order correction was therefore not applied, and only the first order term was involved in the inversion.

If we apply a stress along the z−axis, the only nonzero coefficients in the expansion of the resulting crack distribution will be W_{000}, W_{200}, and W_{400} due to the symmetry around the z−axis and the circular symmetry of the cracks. For purposes of the inversion, we follow Nur (1965) and Sayers (1988b) and take as a model for the crack aspect ratio distribution in the unstressed state a simple linear function

$$N(\alpha) = N_0(1 - \frac{\alpha}{\alpha_m}), \qquad 0 < \alpha < \alpha_m \quad . \tag{7}$$

The parameter α_m sets the maximum aspect ratio present in the rock sample and is given by $\alpha_m = \sigma_0/E_0$, where σ_0 is the hydrostatic pressure required to close all cracks. To serve as a density function, Eq. (7) is normalized by the total number of cracks present n_σ:

$$n_\sigma = N_0 \left[\frac{\alpha_m}{2} + \frac{\sigma^2}{E_0^2}\frac{1}{10\alpha_m} - \frac{\sigma}{3E_0} \right] \quad . \tag{8}$$

Given this distribution of cracks, the crack orientation distribution function after application of a uniaxial stress can be obtained using the closure model given by Eq. (1). At any given angle θ from the stress axis, all fractures with aspect ratio $\alpha > \sigma \cos\theta / E_0$ are open. The resulting coefficients in the expansion of the orientation distribution function are:

$$W_{000} = \frac{1}{8\pi^2} \; ,$$

$$\begin{aligned} W_{200} &= -\frac{1}{5n'_\sigma 2\pi^2}\sqrt{\frac{5}{2}}\frac{\sigma}{E_0}\left(\frac{1}{3}+\frac{1}{7}\frac{\sigma}{E_0\alpha_m}\right) , \\ W_{400} &= \frac{1}{315}\frac{1}{n'_\sigma \pi^2}\sqrt{\frac{9}{2}}\frac{\sigma^2}{E_0^2\alpha_m} , \\ n'_\sigma &= \frac{n_\sigma}{N_0} . \end{aligned} \tag{9}$$

One important aspect of this particular distribution model is that the expansion up to terms $l = 4$ is exact, and there is therefore no truncation error from termination of the series. If, however, only a single crack aspect ratio were considered, the post-stress distribution of cracks resulting from the closure model governed by Eq. (1) would be a box car function with respect to the θ (or η) variable, and strong Gibbs phenomena effects would result since accurate representation of this discontinuous function will require a large number of terms in the expansion. Truncation of the expansion series in this case would yield unrealistic results due to strong oscillations of the predicted distribution function.

Given the values of the elastic constants resulting from the averaging process, velocities can be computed for the stressed, cracked material. The quasi-compressional wave phase velocity v_{qp}, vertically polarized quasi-shear wave velocity v_{qSV}, and horizontally polarized quasi-shear wave velocity v_{qSH} in a general transversely isotropic medium are given by (Musgrave, 1970)

$$\rho v_{qp}^2 = C_{44}+\frac{1}{2}\left\{h\cos^2\beta + a\sin^2\beta + \left[(h\cos^2\beta + a\sin^2\beta)^2 - 4(ah-d^2)\cos^2\beta\sin^2\beta\right]^{1/2}\right\} \tag{10}$$

$$\rho v_{qSV}^2 = C_{44}+\frac{1}{2}\left\{h\cos^2\beta + a\sin^2\beta - \left[(h\cos^2\beta + a\sin^2\beta)^2 - 4(ah-d^2)\cos^2\beta\sin^2\beta\right]^{1/2}\right\} \tag{11}$$

$$\rho v_{qSH}^2 = C_{44}\cos^2\beta + C_{66}\sin^2\beta \tag{12}$$

$$\begin{aligned} a &= C_{11} - C_{44} \\ h &= C_{33} - C_{44} \\ d &= C_{13} + C_{44} . \end{aligned}$$

Here β is the angle measured from the symmetry axis, in this case the z−axis. This expression uses the averaged elastic constants to predict the phase velocity value in a given direction.

For a given uniaxial stress σ and intrinsic Young's modulus E_0, the only unknown parameters necessary to compute velocity from Eq. (10) are crack density ϵ and maximum crack size α_m. Therefore, these are the natural quantities to determine through inversion procedures. Since Eq. (10) is a nonlinear function of ϵ and α_m (through the dependence of the elastic constants on the orientation function), an inversion is performed by linearizing the problem about an initial estimate of model parameters:

$$\mathrm{d} \cong \mathrm{G}\mathrm{m}_0 + \mathrm{A}\Delta\mathrm{m} \ . \tag{13}$$

Here d is the data vector containing observed velocity values, G is the forward model operator yielding velocity predictions for a given set of model parameters in starting model vector m_0, A is a matrix of partial derivatives, and $\Delta\mathrm{m}$ is a perturbation to the starting estimate of model values. The partial derivatives are somewhat complicated algebraically, but can be computed analytically with no approximations. We then perform an iterative least squares inversion for α_m and ϵ, which allows an estimate of the crack normal orientation distribution.

This inversion procedure is similar to that proposed by Sayers (1988a,b), but there are several significant differences. For example, Sayers (1988b) considers a stress applied along the x−axis, which results in a more complicated expansion of the crack orientation distribution function since the orientation is in that case a function of angle ψ as well as θ. The approach described in this paper uses the exact expression for phase velocity, while Sayers (1988b, see also 1986) uses an approximate expression derived from a variational method.

Permeability Prediction

The crack orientation distribution function resulting from the inversion can be used to predict permeability values. The permeability of a single fracture is simply

$$k_1 = \frac{L_0^3}{12} \ . \tag{14}$$

This cubic law permeability results from the analysis of flow through a single parallel plane walled fracture (Snow, 1965), and gives the flow rate per unit length along the fracture. Conventional permeability values are defined from Darcy's law relative to flow across a unit surface element area. To make this conversion, consider as a model a block volume containing a set of cracks which extend through the length of the block (Figure 3). The permeability of the volume relative to the surface area of the block is obtained by simply adding the contribution of each fracture, which amounts to multiplying Eq. (14) by the number of cracks in the volume shown in Figure 3. The number of cracks of interest is the number with normals perpendicular to the direction in which permeability is to be estimated, which requires a knowledge of the crack normal distribution function $P(\chi, \eta)$, $\chi = \cos\delta$ (Figure 4). Due to the circular symmetry of the cracks, this function is equal to $2\pi N(\zeta, \psi, \phi)$ so that $\chi = \zeta$ and $\eta = \psi$. Remembering that $\epsilon = na^3$, a set of cracks of a given aspect ratio yields a permeability k_α:

$$k_\alpha = \frac{\epsilon\alpha^3}{12} \ , \tag{15}$$

where the product in the numerator gives nL_0^3. Since the model considers a unit volume (Figure 3), the value α in Eq. (15) is length squared, where the length unit will be the same as that of the unit volume under consideration.

Integrating over the range of crack aspect ratios for cracks still open in a given direction, from $\alpha_{cl} = \cos^2\gamma_0\sigma/E_0$ to α_m, gives permeability as a function of angle θ measured from the stress axis:

$$k(\theta) = \frac{N(\theta,\psi)\epsilon}{48(\alpha_m - \frac{\sigma}{E_0}\cos^2\theta)}\left[\alpha_m^4 - (\frac{\sigma}{E_0}\cos^2\theta)^4\right] \quad . \tag{16}$$

While this is a rather simple approach to permeability estimation, it is related to other studies of fluid flow through crystalline rock. Bernabé (1986) examines in detail the applicability of the equivalent channel approach to permeability modeling for several granites and concludes that it is a valid approach. In our case, we in essence develop an equivalent channel for each direction of interest based on the distribution of crack aspect ratios in that direction. This allows the model to include the effects of crack closure with orientation. The principal effects which are neglected are the influence of asperities in diminishing the mean hydraulic radius of the cracks and the tortuosity created by the asperities and the interconnection of cracks. The simplest way to include these effects of the equivalent channel approach is to simply normalize the permeability predictions by some constant so that the values are of the correct order of magnitude. This can be done easily if permeability for one direction is known or if the permeability of the unstressed rock is known.

Another possible approach is to assume that only fractures with aspect ratio smaller than some cut off value are involved in fluid flow, the remainder being isolated fractures. This will clearly greatly decrease the calculated permeabilities due to the width cubed dependence of fracture permeability. However, this will yield vanishing permeabilities for directions where only fractures with aspect ratio greater than the cut off are open, which is not likely to be true for real rocks.

APPLICATION TO ULTRASONIC DATA

The inversion procedure together with the permeability model Eq. (16) provides a method for predicting permeability values given observations of elastic wave velocities which could be obtained from either laboratory samples or field data. Nur and Simmons (1969) made velocity measurements on samples of Barre granite as a function of direction for several magnitudes of applied uniaxial stress. Measurements were presented for both quasi-compressional wave signals and quasi-transverse waves, SV and SH. The Barre granite sample used by Nur and Simmons (1969) was dry, so the Hudson (1981) formulation for dry cracks is appropriate.

Inversion results for the qP data are presented for uniaxial stresses of 0, 10, 20, 30 and 40 MPa in Table I. Corresponding quasi-compressional wave velocity predictions and observations are compared in Figure 5. The theory is able to match the data well, with a fit essentially the same as that obtained by Sayers (1988). The inversion results become less accurate for the higher applied stresses, which suggests that the simple physical model of crack closure becomes less adequate at these values of stress. Only two data points are available for inversion at the stress value of 40 MPa, but some experimentation using data from lower stress values using only the velocities parallel and perpendicular to the applied stress axis yielded results very similar to those using all available data. An important aspect of the approach is that for the no stress case, the inversion is unable to obtain an estimate of α_m, since the forward model is insensitive to the aspect ratio in this case. Therefore, a value of 4.4×10^{-4} was used for permeability predictions based on the trends indicated for the results of other stress values. The trends in crack density shown by the inversion results in Table 1 are reasonable. As stress increases, more cracks will close reducing the overall crack density, as occurs for these results. The increase in α_m with stress is more difficult to explain, however, as is the sudden decrease at 40 MPa. It may reflect changes in aspect ratio with stress which are not included in the present physical model.

Results for the qSH and qSV data are presented in Tables II and III, respectively. Data and velocity predictions are shown in Figures 6 and 7. While the qSH data are similar to those in Figure 5 for the quasi-compressional waves, the qSV results are relatively poor. A transversely isotropic medium always has equal qSV velocities parallel and perpendicular to the symmetry axis (see Eq. (11)), but it is clear from the data in Figure 6 that this condition is not quite true for these observations. It is possible that the distribution of the Barre granite fracture system has some slight anisotropy which would cause the stressed system to have some overall symmetry other than transversely isotropic. A more likely cause of these results is that the principal source of SV velocity variation is preferred grain orientation in the granite. Lo et al. (1986) clearly demonstrate such a residual anisotropy after crack closure in measurements of velocity in Chelmsford granite. If the residual anisotropy is the cause of most of the velocity variation for the SV data, the inversion results are not significant for inference of crack orientation since the forward model involved in the inversion includes only anisotropy due to cracks.

The effects of this residual anisotropy seem to be evident to a smaller degree at high pressures for the quasi-compressional and qSH wave data also (Figures 5 and 7). Since the total velocity anisotropy is greater for the quasi-compressional and qSH data, however, the fractures have more effect on observed velocities and the inversion results are more significant for these cases. The values of crack density ϵ obtained from the two quasi-shear wave data sets are almost identical, but the quasi-compressional wave data consistently yielded a somewhat lower estimate of crack density. The results for all data sets are also essentially the same as those obtained by Sayers (1988b). The cause of the difference in results for quasi-compressional and quasi-shear wave data is difficult to explain but may be related to the difference in interaction with crack surfaces of dilatational and shear strains.

The inversion results for α_m from qP and qSH data are also comparable. Both sets of results show a trend of increasing α_m with increasing stress, except for the highest applied stress value of 40 MPa. Although the qSV results for α_m are quite different, it was clear from the behavior of the inversion process that this is a poorly constrained parameter for the qSV data.

Permeability predictions using the qP and qSH results are shown in Figures 8 and 9 respectively. The curves for 10 MPa were normalized to match a permeability measurement for Barre granite under 10 MPa hydrostatic pressure (Bernabé, 1986). This normalization assumes that the permeability in the direction perpendicular to the applied uniaxial stress shows the same behavior as does isotropic permeability in the hydrostatic case. The other permeability curves were normalized to have the same permeability in the stress direction. Zoback and Byerlee (1975) show that there is in fact a decrease in permeability in this direction, but this effect is not too large, on the order of about 50 nD over a stress range of 160 MPa. The two sets of predictions from qP and qSV inversion results compare well, especially at 20 and 30 MPa.

While measurements of anisotropic permeability in the uniaxial stress case are not available to confirm these predictions, a simple check can be made by comparison with permeability measurements as a function of hydrostatic pressure. The permeability values perpendicular to the stress axis from both the qP and qSH results are compared to the measurements by Bernabe (1986) for Barre granite in Figure 10. The match is not perfect, but the values at least show approximately the correct degree of change with pressure. Other examples of permeability measurements reported in the literature for granites show a wide variation both in the absolute value of permeabilities under pressure and in the magnitude of change in isotropic permeability with increasing pressure (e.g., Brace et al. (1968) and Bernabe (1986)).

DISCUSSION AND CONCLUSIONS

The results of the velocity inversion suggest that the physical model for the fracture behavior under uniaxial stress is capable of describing most of the effects of the cracks on the elastic properties of the medium and that the model is able to match observations of velocity in the Barre granite. The aspect ratio of the dry fractures does not affect the elastic wave velocities in the Hudson formulation. Only the density of cracks ϵ is important in this case, and so the inversion results suggest that we are modeling this aspect of the system fairly well. On the other hand, permeability critically depends on the aspect ratio due to the cubic dependence on crack width in Eq. (14). The permeability predictions are highly sensitive to this parameter, and thus permeability values are predicted to show fairly rapid change as a function of direction (Figures 8 and 9).

It is important that we have only included a fracture contribution to permeability in this model. If the medium under consideration has a significant amount of a different type of interconnected pores and tubular fluid conduits, such as is the case in some sandstones, the effects of the crack closure must be added to the permeability due to other porosity types. Since the equidimensional pores of a sandstone will only be minimally affected by the applied stress, the effects of the cracks may not be so important for overall permeability values. This crack model is most important for low porosity rocks such as fractured limestones or granites and other crystalline and metamorphic rocks which would be essentially impermeable except for the cracks.

The present version of this theory of permeability has a limitation in that we do not include any change in aspect ratio for the cracks which remain open. In actuality, the aspect ratio of the open cracks will decrease as the uniaxial stress is applied (Toksöz et al., 1976). This effect will tend to decrease the permeability values in directions away from the stress axis.

The change in aspect ratio with stress will affect only the permeability predictions as long as the cracks are dry. If, however, the cracks are assumed to be filled with a fluid, the aspect ratio also affects the elastic constant values. The forward modeling of velocities would then have to include the variation of aspect ratio with direction in order to compute the velocity values. However, a relatively small amount of gas mixed with the fluid will still cause the effective properties of the medium to be essentially those of a gas, since the effective bulk modulus k^* of a two phase medium is given by

$$(k^*)^{-1} = Vk_f^{-1} + (1-V)k_g^{-1} \quad , \tag{17}$$

where k_f and k_g are fluid and gas bulk moduli, respectively, and V is the volume fraction of fluid (Kuster and Toksöz, 1974). The large compressibility of the gas will tend to dominate the overall properties of the crack filling material, and it will tend to behave as though the cracks are filled with a gas. As long as the shear modulus and bulk modulus of the crack filling material are small, the aspect ratio of the cracks has little impact on the elastic constants in the Hudson (1981) approach, and the present approach will be sufficient.

This approach should at least provide a means of obtaining an initial estimate of permeabilities for use in modeling of fluid flow in subsurface fractured media. Potential areas of application include both hydrological studies and petroleum reservoir modeling. Perhaps the most important aspect of the theory is that it represents an attempt to extend knowledge of the permeability of a subsurface feature to regions beyond the borehole using seismic data.

ACKNOWLEDGEMENTS

This material has been supported by the Full Waveform Acoustic Logging Consortium at M.I.T., the Department of Energy under grant DE-FG02-86ER13636, and a National Science Foundation Graduate Fellowship (RLG).

REFERENCES

Bernabé, Y., Pore volume and transport properties changes during pressure cycling of several crystalline rocks, *Mech. Mater.*, *5*, 235–249, 1986.

Brace, W.F., J.B. Walsh and W.T. Frangos, Permeability of granite under high pressure, *J. Geophys. Res*, *73*, 2225–2236, 1968.

Crampin, S., A review of the effects of anisotropic and cracked elastic-media., *Wave Motion*, *3*, 343-391, 1981.

Crampin, S., Effective anisotropic elastic constants for wave propagation through cracked solids, *Geophys. J. Roy. Astr. Soc.*, *76*, 135–145, 1984.

Hearmon, R.F.S., *An Introduction to Applied Anisotropic Elasticity*, Oxford, New York, 1961.

Hudson, J.A., Overall properties of a cracked solid, *Math. Proc. Camb. Phil. Soc.*, *88*, 371–384, 1980.

Hudson, J.A., Wave speeds and attenuation of elastic waves in material containing cracks, *Geophys. J. Roy. astr. Soc.*, *64*, 133–150, 1981.

Kuster, G.T. and M.N. Toksöz, Velocity and attenuation of seismic waves in two-phase media: Part I. Theoretical formulations, *Geophysics*, *39*, 607–618, 1974.

Lo, T.W., K.B. Coyner and M.N. Toksöz, Experimental determination of elastic anisotropy of Berea sandstone, Chicopee shale, and Chelmsford granite, *Geophys.*, *51*, 164–171.

Morris, P.R., Averaging fourth-rank tensors with weight functions, *J. Appl. Phys.*, *40*, 447–448, 1969.

Musgrave, M.P.J., *Crystal Acoustics*, Holden-Day, San Francisco, 1970.

Nur, A., Effects of stress on velocity anisotropy in rocks with cracks, *J. Geophys. Res.*, *76*, 2022–2034, 1971.

Nur, A. and G. Simmons, Stress-induced velocity anisotropy in rock: an experimental study, *J. Geophys. Res.*, *27*, 6667–6674, 1969.

Roe, R.J., Description of crystallite orientation in polycrystalline materials. III. General solution to pole figure inversion, *J. Appl. Phys.*, *36*, 2024–2031, 1965.

Sayers, C.M., Angular dependent ultrasonic wave velocities in aggregates of hexagonal crystals, *Ultrasonics*, *24*, 289–291, 1986.

Sayers, C.M., Inversion of ultrasonic wave velocity measurements to obtain the microcrack orientation distribution function in rocks, *Ultrasonics*, *26*, 73–77, 1988a.

Sayers, C.M., Stress-induced ultrasonic wave velocity anisotropy in fractured rock, *Ultrasonics*, *26*, 311–317, 1988b.

Snow, D.T., A parallel plate model of fractured permeable media, Ph.D. thesis, University of California, Berkeley, 1965.

Toksöz, M.N., C.H. Cheng, and A. Timur, Velocities of seismic waves in porous rocks, *Geophysics*, *41*, 621–645, 1976.

Walsh, J.B., The effects of cracks on the uniaxial elastic compression of rocks, *J. Geophys. Res.*, *70*, 399–411, 1965.

Zoback, M.D. and J.D. Byerlee, The effect of microcrack dilatancy on the permeability of Westerly granite, *J. Geophys. Res.*, *80*, 752–755.

Uniaxial stress (MPa)	ϵ	α_m
0	0.275	
10	0.255	4.82×10^{-4}
20	0.236	5.64×10^{-4}
30	0.221	7.16×10^{-4}
30	0.217	6.72×10^{-4}

Table I. Inversion results using quasi-compressional wave velocity measurements of Nur and Simmons (1969). A value of α_m for the 0 MPa uniaxial stress is not included, since the inversion cannot determine information on aspect ratio in this case.

Uniaxial stress (MPa)	ϵ	α_m
0	0.315	
10	0.283	4.98×10^{-4}
20	0.255	5.94×10^{-4}
30	0.234	6.78×10^{-4}
30	0.217	5.61×10^{-4}

Table II. Inversion results using qSH velocity measurements of Nur and Simmons (1969). As in the quasi-compressional case, the inversion cannot determine information on aspect ratio at 0 MPa.

Uniaxial stress (MPa)	ϵ	α_m
0	0.314	
10	0.282	
20	0.256	1.103×10^{-4}
20	0.230	1.708×10^{-4}

Table III. Inversion results using qSV velocity measurements of Nur and Simmons (1969). Values of α_m for the 0 MPa and 10 MPa uniaxial stress are not included, since the inversion cannot determine information on aspect ratio for low pressure qSV data.

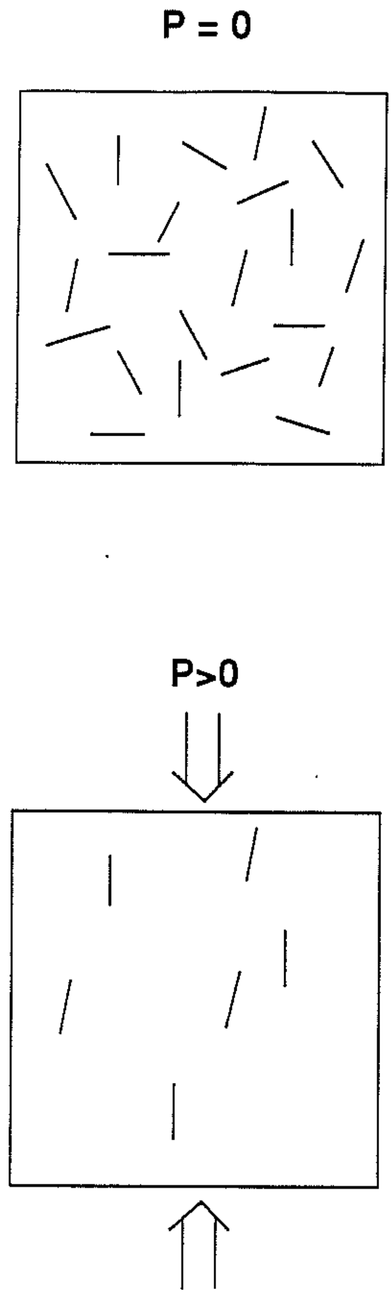

Figure 1: Schematic diagram illustrating the behavior of a randomly fractured medium under an applied uniaxial stress. The upper figure shows a possible random crack system with no stress applied. The lower portion shows the same system after application of the uniaxial stress, where cracks have closed depending on their orientation with respect to the stress.

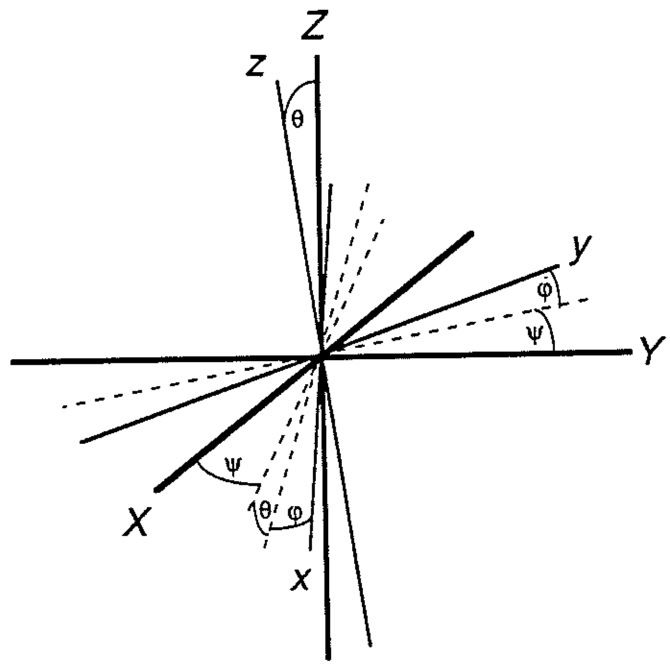

Figure 2: Euler angles of rotation θ, ψ, ϕ describing orientation of a given crack coordinate system x, y, z (fine lines) with respect to the composite medium coordinate system X, Y, Z (heavy lines). Dashed lines indicate intermediate orientations of the crack coordinate system. The set of rotations is defined as follows: 1) rotate by ψ about Z (the same as z initially). 2) rotate by θ about the new y-axis. 3) rotate by ϕ about the new z-axis. Since the cracks are assumed to have circular symmetry, the first two rotations actually uniquely specify the orientation of a single crack.

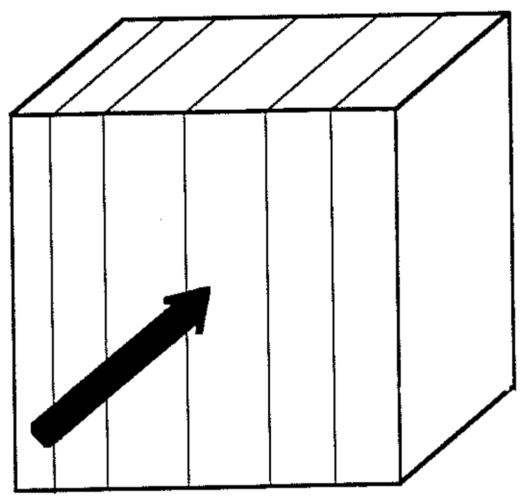

Figure 3: Schematic block model showing the hypothetical distribution of cracks used to estimate permeability. The arrow indicates the direction of fluid flow and pressure gradient.

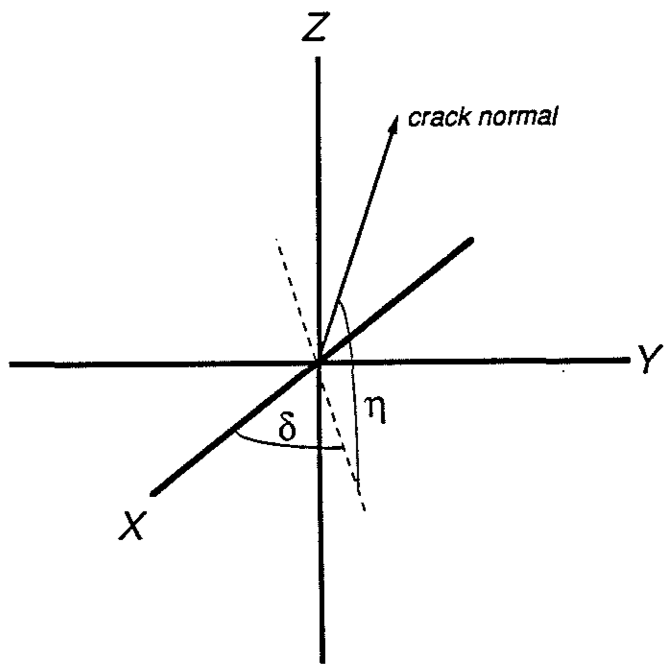

Figure 4: The angles δ and η necessary to specify the orientation of a crack normal.

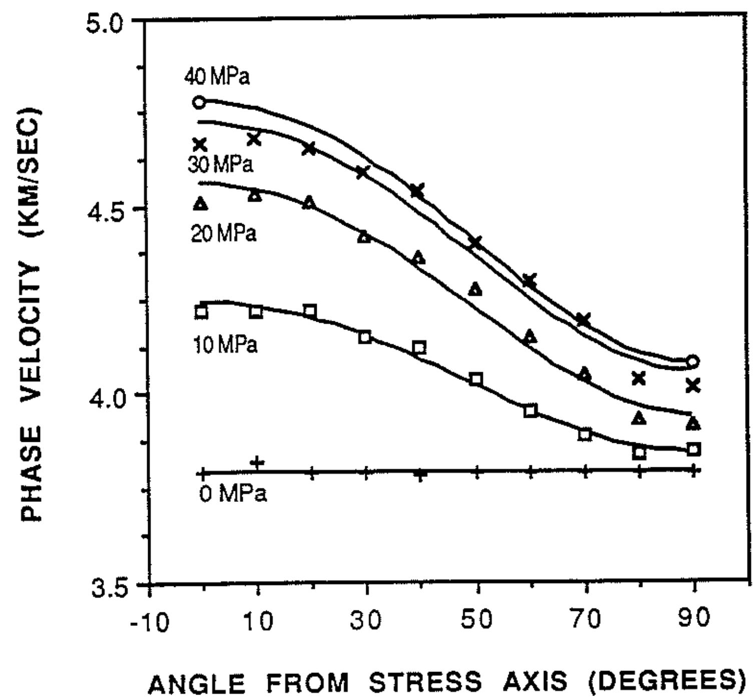

Figure 5: Results of inversion for crack density ϵ and maximum crack size α_m using Barre granite quasi-compressional wave velocity data. The points are data collected by Nur and Simmons (1969), and the lines indicate the inversion results. The value of the applied uniaxial stress is indicated for each curve. Note that only two data points were available at 40 MPa.

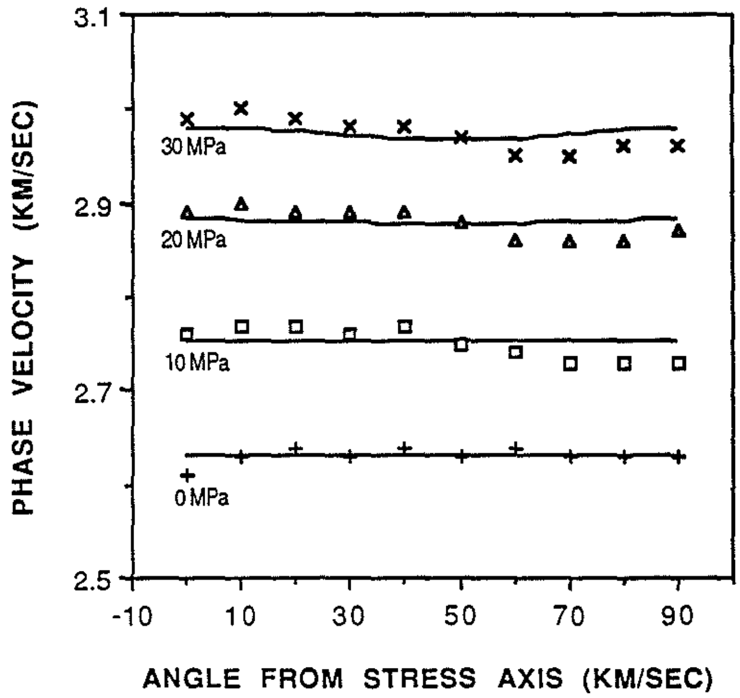

Figure 6: Results of inversion for crack density ϵ and maximum crack size α_m using Barre granite qSV velocity data. The points are data collected by Nur and Simmons (1969), and the lines indicate the inversion results. The value of the applied uniaxial stress is indicated for each curve.

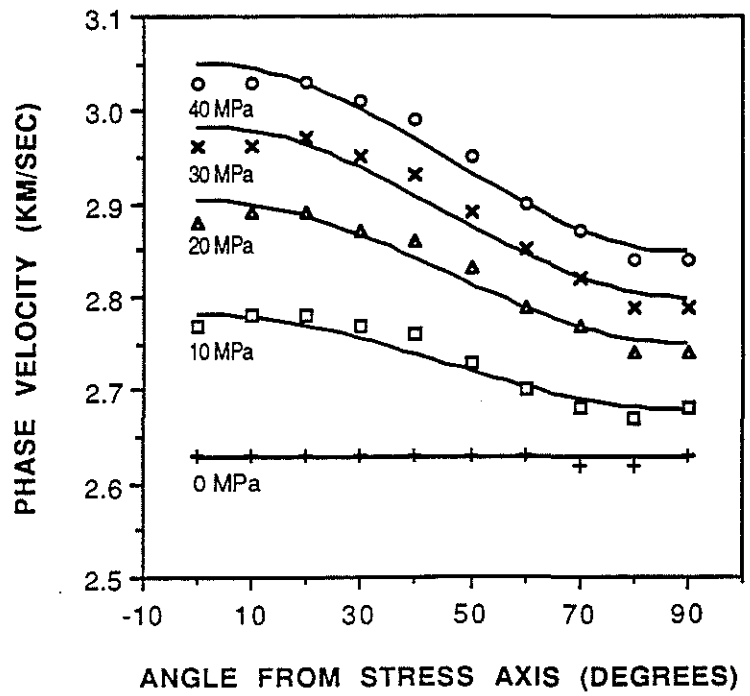

Figure 7: Results of inversion for crack density ϵ and maximum crack size α_m using Barre granite qSH velocity data. The points are data collected by Nur and Simmons (1969), and the lines indicate the inversion results. The value of the applied uniaxial stress is indicated for each curve.

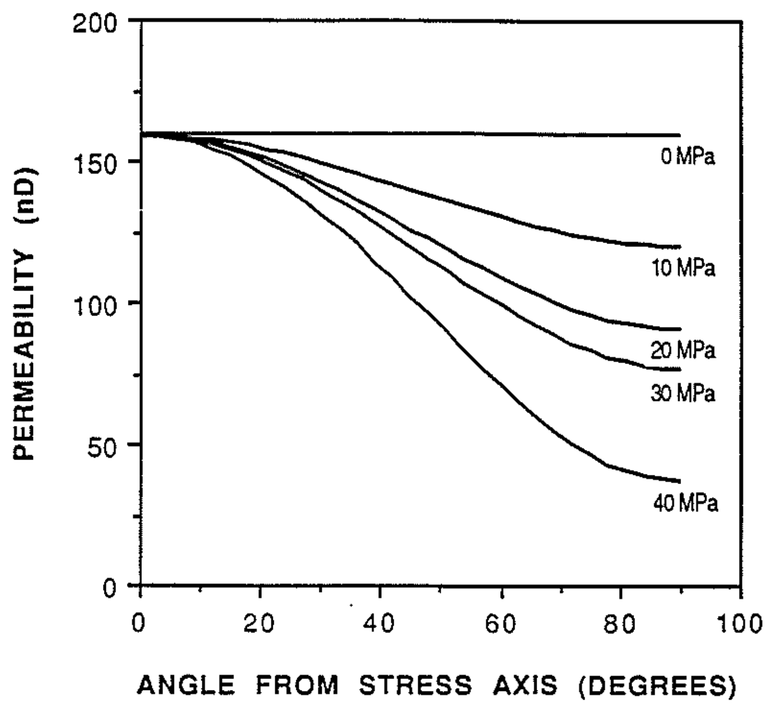

Figure 8: Permeability predictions as a function angle from the applied uniaxial stress axis. The predictions use the results from the inversion of qP data. The uniaxial stress is indicated for each curve.

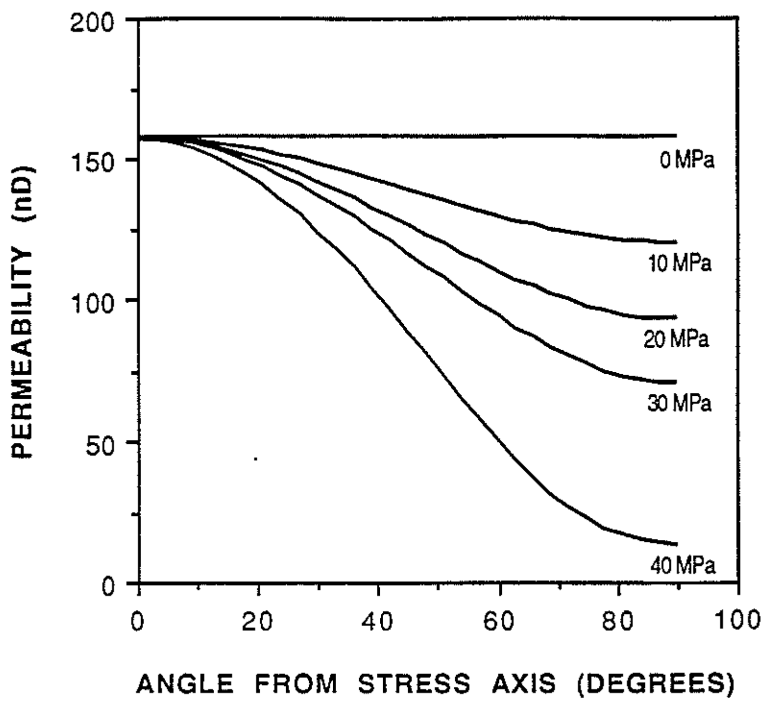

Figure 9: Permeability predictions as a function angle from the applied uniaxial stress axis. The predictions use the results from the inversion of qSH data. The uniaxial stress is indicated for each curve.

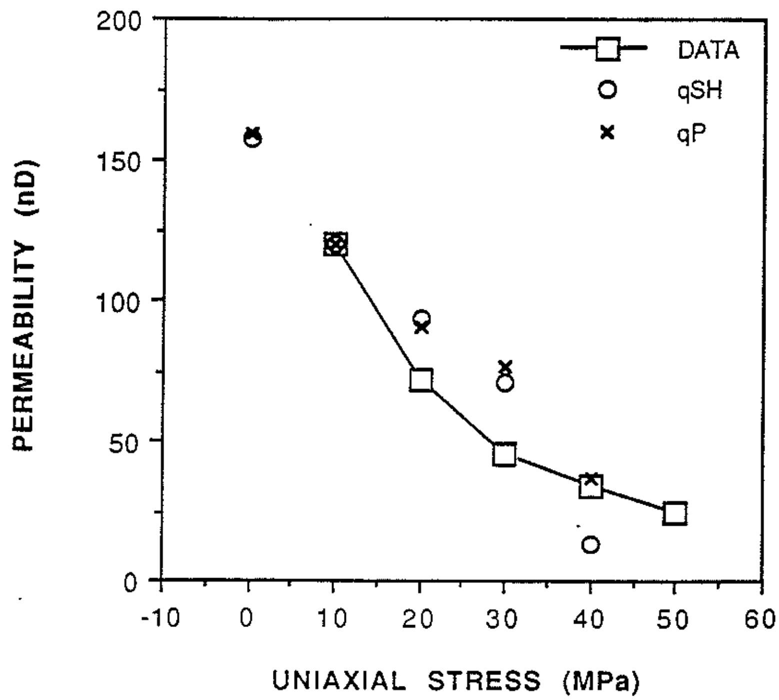

Figure 10: Comparison of hydrostatic permeability measurements with theoretical permeabilities perpendicular to the stress axis. The data, measurements on Barre granite (Bernabé, 1986), are indicated by the line and the points are calculated from Eq. (16). Since the permeabilities are normalized to have the same value at 10 MPa, the data and theoretical points overlap at this value of stress. The velocity data types used to calculate the permeability predictions are indicated in the figure.