VISCOUS ATTENUATION OF ACOUSTIC WAVES IN SUSPENSIONS

by

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ABSTRACT

A model for attenuation of acoustic waves in suspensions is proposed which includes an energy loss due to viscous fluid flow around spherical particles. The expression for the complex wavenumber is developed by considering the partial pressures acting on the solid and fluid phases of the suspension. This is shown to be equivalent to the results of the Biot theory for porous media in the limiting case where the frame moduli vanish. Unlike earlier applications of the limiting case Biot theory, however, a value for the attenuation coefficient is developed from the Stokes flow drag force on a sphere instead of attempting to apply a permeability value to a suspension. If the fluid and solid particle velocities have harmonic time dependence with angular frequency ω, the attenuation in this model is proportional to ω^2 at low frequencies and approaches a constant value at high frequencies. The predicted attenuation is very sensitive to the radius and density of the spherical particles. Accurate modeling of observed phase velocities from suspensions of spherical polystyrene particles in water and oil and successful inversion for kaolinite properties using attenuation and velocity data from kaolinite suspensions at 100 kHz show that this viscous dissipation model is a good representation of the effects controlling the propagation of acoustic waves in these suspensions. Attenuation predictions are also compared to amplitude ratio data from an oil-polystyrene suspension. The viscous effects are shown to be significant for only a limited range of solid concentration and frequency by the reduced accuracy of the model for attenuation in a kaolinite suspension at 1 MHz.

INTRODUCTION

Acoustic waves are attenuated by a variety of mechanisms in an inhomogeneous medium. The study of wave propagation in a fluid containing suspended solid particles is of in-

terest to both acoustic well logging, where the borehole is filled with a liquid with suspended clays, and ocean acoustic investigations. Full waveform acoustic modeling typically assumes a linear variation of attenuation with frequency with little or no theoretical basis (Burns, 1988). It would be of value to obtain a better understanding of this assumption in order to apply more rigorous values in synthesis of acoustic logs. "Geoacoustic models" for acoustic wave propagation through ocean bottom sediments must also include information on attenuation in order to properly interpret observations (Hamilton, 1980). The problem of attenuation in suspensions is directly relevant since the sediments of the sea floor can have porosities up to at least 80% to 90% (Akal, 1974; Hamilton, 1976).

Several theories for attenuation in suspensions have been formulated from scattering theory. One of the earliest applications of the scattering formulation for spherical particles was by Urick and Ament (1949), who used the zero and first-order scattering terms to estimate phase velocity and attenuation. The existence of shear waves in the scattering particles was incorporated into scattering theory by Faran, Jr. (1951). Ahuja (1972a) included the effect of particle viscosity in a study of suspensions and emulsions, while other workers have included thermal loss effects (Allegra and Hawley, 1972). Further studies of bulk properties of two-phase materials have analyzed the attenuation mechanisms and velocity dispersion predicted by scattering theories (for example, Kuster and Toksöz, 1974a,b; Hay and Burling, 1982; Lin and Raptis, 1983; and Hay and Mercer, 1985). These theories lead to solutions for the scattered waves in terms of Bessel or Hankel functions. The coefficients for the various wave types are determined by solution of complicated simultaneous equations, and therefore typically only the zero and first-order coefficients are examined to derive simplified expressions for attenuation. This leads to an attenuation coefficient which is linear in solid concentration, but examination of attenuation data for suspensions shows that this linearity is at best true only for very dilute concentrations of particles. In contrast, Davis (1979) showed that a theory including multiple scattering resulted in an attenuation coefficient with a second order correction term.

A different approach to the study of suspensions is to consider a representative volume of composite material and to examine the effective values of quantities such as density, compressibility and forces acting on the different components of a suspension. Urick (1947) attempted to estimate the acoustic phase velocity of suspensions of kaolinite by calculating effective bulk density and compressibility, and later (Urick, 1948) derived an attenuation coefficient equivalent to that predicted by zero and first-order scattering theory by looking at the Stokes drag force on a sphere. Ament (1953) estimated an effective density parameter for a suspension based on balancing the forces acting on solid and fluid components of a filter undergoing oscillatory motion. This was applied to suspensions by computing a filter permeability for the medium using Stokes drag force on spherical particles, and a linear approximation to attenuation was developed. Ahuja (1972b, 1973) employed a similar approach which included thermal

effects but also gave only a linear attenuation estimate.

More recently, the possible extension of the Biot (1956a,b) theory for porous media to suspensions by examining the limiting case where solid frame bulk and shear moduli vanish has been considered (Hovem, 1980a,b; Ogushwitz, 1985). The Biot theory includes attenuation due to flow of a viscous fluid through the pore structure of a solid matrix (Biot, 1956a,b; 1962). Although the modified Biot theory results are able to match data well, it is not obvious that this theory derived from a model containing a solid matrix framework can reasonably be assumed to apply to the suspension where a continuum consists of the liquid phase with solid particles as inclusions. In this paper, we consider a balance of the forces acting on the solid and fluid components instead of the modified Biot theory. Hovem (1980a, 1980b) and Ogushwitz (1985) both employ a permeability estimate based on concepts derived for porous media, but the present work instead includes attenuation due to Stokes flow drag on spherical particles. An analytic correction to the drag force compensating for the presence of multiple spheres (Hasimoto, 1959) is included in the derivation. This correction has not been included in previous theories utilizing the Stokes drag force approach. The resulting predictions are compared to laboratory measurements of velocity for spherical polystyrene particles, and the dispersion relationship is used to invert for the properties of kaolinite clays using velocity and attenuation data. The theory will apply to a limited range of concentrations and frequencies due to the assumption of Stokes flow and neglect of scattering and particle interaction, and the range suggested by the theoretical results is discussed along with some comparisons to other attenuation models.

MODEL FOR ATTENUATION IN SUSPENSIONS

Equations of Motion

The medium consists of two phases, a fluid continuum with suspended solid particles. The fluid is assumed to be a Newtonian viscous liquid, and the solid particles are taken to be spherical. An acoustic plane wave is assumed to propagate through the medium with a wavelength much greater than the dimension of the particles so that the fluid flow will behave as a simple Stokes flow with respect to the suspended particles. Finally, the distribution of the solid spheres is assumed to be statistically homogeneous so that the effective properties of the medium are isotropic. Then the same volume fraction of solids is found in any given volume and the same area fraction in any cross-section of the medium, and the suspended material will have the same effect on the plane wave in any given direction of propagation.

The equation describing motion of a simple fluid medium during propagation of an

acoustic wave is

$$-\nabla p = \rho_f \partial_t \mathbf{v}_f, \tag{1}$$

where p is pressure, ρ_f is fluid density and $\mathbf{v}_f$ is the velocity vector for the fluid particles. This expression results from neglecting inertial and viscous terms in the Navier-Stokes equations, which is valid for examination of pressure fluctuations due to acoustic waves (Clay and Medwin, 1977). The pressure will be altered by the introduction of solid spheres to a new value p^*. Considering the pressure gradient as a measure of the force applied to a unit volume of the composite material, when a volume fraction ϕ of solids is present a fraction ϕ of the pressure gradient will act on the solids in the suspension. Likewise, a fraction $1-\phi$ of the pressure gradient will act on the fluid component. The equations of motion for the fluid and solid particles are then

$$\begin{aligned} -(1-\phi)\nabla p^* &= (1-\phi)\rho_f \partial_t \mathbf{v}_f + \gamma(\mathbf{v}_f - \mathbf{v}_s) \\ -\phi \nabla p^* &= \phi \rho_s \partial_t \mathbf{v}_s - \gamma(\mathbf{v}_f - \mathbf{v}_s). \end{aligned} \tag{2}$$

The solid particle velocity is $\mathbf{v}_s$, and ρ_s is the density of the solids. The variable γ is a dissipation coefficient which is a function of ϕ and fluid dynamic viscosity η relating dissipation of energy to the relative motion of fluid and solid, a viscous coupling effect. An explicit form for this coefficient will be considered below.

The effective bulk modulus K^* of a two-phase medium is given by (see, for example, Kuster and Toksöz, 1974a)

$$(K^*)^{-1} = \phi K_s^{-1} + (1-\phi) K_f^{-1}, \tag{3}$$

where K_s and K_f are the solid and fluid bulk moduli, or incompressibilities. For a homogeneous fluid medium with bulk modulus K,

$$-\partial_t p = K \nabla \cdot \mathbf{v}_f, \tag{4}$$

but for the suspension a contribution to the pressure p^* will come from the motion of both fluid and solid particles. Both components will contribute to the pressure with the bulk modulus K^*, and considering the volume fractions of the materials, the effective pressure can be written

$$-\partial_t p^* = K^*[(1-\phi)\nabla \cdot \mathbf{v}_f + \phi \nabla \cdot \mathbf{v}_s]. \tag{5}$$

This can be used with equations (2) to fully describe the acoustic wave propagation in the composite medium.

The Biot theory for propagation of acoustic waves in porous media was originally obtained by considering the stress tensor in a solid framework and the pressure in

a pore-filling fluid (Biot, 1956a,b). Hovem (1980a,b) and Ogushwitz (1985) applied this theory to suspensions by allowing the bulk moduli describing the solid frame of the composite material to vanish. It is important to examine theoretically the consequences of the vanishing frame bulk moduli. This theoretical examination is both simple and enlightening. Begin with the equations for compressional wave propagation in the porous medium (Biot, 1962), with some notational change to correspond to the presentation in this paper:

$$\begin{aligned} \nabla^2(He - C\zeta) &= \partial_{tt}(\rho^* e - \rho_f \zeta) \\ \nabla^2(Ce - M\zeta) &= \partial_{tt}(-\rho_f e + m\zeta) - \tfrac{\eta}{B}\partial_t \zeta, \end{aligned} \tag{6}$$

where

$$\begin{aligned} e &= \nabla \cdot \mathbf{u}_s, \\ \zeta &= -(1-\phi)\nabla \cdot (\mathbf{u}_f - \mathbf{u}_s), \\ \rho^* &= \phi\rho_s + (1-\phi)\rho_f. \end{aligned} \tag{7}$$

The parameter B is permeability, $\mathbf{u}_s$ and $\mathbf{u}_f$ are solid and fluid displacement vectors, and H, C, and M are elastic coefficients relating stress and strain in the porous medium. The parameter m is related to mass coupling between solid and fluid (Biot, 1962) and to the tortuosity of the porous medium (Stoll, 1974), and is given by

$$m = \frac{a'\rho_f}{1-\phi}. \tag{8}$$

Here $a' \geq 1$, and increases with the tortuosity of the medium. Stoll (1974) showed that the elastic moduli can be expressed in terms of the fluid and solid bulk moduli and the solid frame bulk modulus K_b and shear modulus μ by

$$\begin{aligned} H &= \frac{(K_s - K_b)^2}{D - K_b} + K_b + \frac{4\mu}{3} \\ C &= \frac{K_s(K_s - K_b)}{D - K_b} \\ M &= \frac{K_s^2}{D - K_b} \\ D &= K_s[1 + (1-\phi)(\frac{K_s}{K_f} - 1)] = \frac{K_s^2}{K^*}. \end{aligned} \tag{9}$$

The first of equations (6) gives the divergence of the total stress tensor acting on the composite medium, while the second gives the divergence of the fluid pressure gradient only.

For a pore structure oriented parallel to the pressure gradient, $a' = 1$ in equation (8) (Stoll, 1974). It is reasonable to take $a' = 1$ for a suspension also, since, except for very near the spherical particles, the fluid motion will be entirely parallel to the acting pressure gradient. This has the same effect on the resulting equations of motion as neglecting mass coupling. When the frame bulk and shear moduli vanish, it is simple to show that $H = C = M = K^*$, which in turn causes the terms on the left hand sides of equations (6) to reduce to the p^* defined in equation (5). Setting the frame moduli to 0 and multiplying the first of equations (6) by -1 and the second by $-(1-\phi)$ gives after some algebra

$$-\nabla^2 p^* = \partial_{tt}(\phi\rho_s \nabla \cdot \mathbf{u}_s + (1-\phi)\rho_f \nabla \cdot \mathbf{u}_f) \tag{10}$$

$$-(1-\phi)\nabla^2 p^* = \partial_{tt}((1-\phi)\rho_f \nabla \cdot \mathbf{u}_f) + \gamma\partial_t(\nabla \cdot (\mathbf{u}_f - \mathbf{u}_s)). \tag{11}$$

Here the form of the dissipation coefficient γ has been temporarily ignored. Equation (10) is simply the sum of the divergence of equations (2), while equation (11) is simply the divergence of the first of equations (2). The total force per unit volume of the suspension is reflected in equation (10), and equation (11) indicates the force acting on the fluid component only.

This confirms that the approach of Hovem (1980a,b) and Ogushwitz (1985) is in fact reasonable. It should be noted that neglecting parameters describing a solid continuum is not a valid procedure in all cases. Kuster and Toksöz (1974a) note that in the scattering formulation it is not possible to derive relations for a fluid matrix with solid inclusions from those derived for the solid matrix by allowing the matrix rigidity to vanish. This is because the solid matrix case will have no relative motion between matrix and inclusions, while a fluid medium with suspended solid particles will allow for significant relative motion, particularly when the densities of the two components are significantly different. The reason it is possible to allow frame moduli to vanish in the Biot approach is because the original theory does in fact consider the relative motion of the solids and fluids as the primary viscous attenuation mechanism. Although it might seem that the porous medium is a case of a solid matrix with fluid inclusions, this is not true. In fact, both the fluid and solid components form continua which oscillate asynchronously during the passage of an acoustic wave. The suspension presents the same situation, except that it is not possible for a signal to propagate continuously through a solid component, and the resistance to fluid flow is a case of flow *around* obstacles instead of *through* a tortuous pore structure.

Dispersion Relationship

To develop a dispersion relationship from equations (2) and (5), consider a plane wave propagating in the x_3 direction of a Cartesian coordinate system with axes denoted by

x_1, x_2, and x_3. The equations reduce to

$$\begin{aligned}
(1-\phi)\partial_3 p^* &+ (1-\phi)\rho_f \partial_t v_f + \gamma(v_f - v_s) = 0 \\
\phi\partial_3 p^* &+ \phi\rho_s \partial_t v_s - \gamma(v_f - v_s) = 0 \\
\partial_t p^* &+ (1-\phi)K^* \partial_3 v_f + \phi K^* \partial_3 v_s = 0.
\end{aligned} \tag{12}$$

The components of solid and fluid velocities in the x_3 direction are represented by v_s and v_f respectively. Assuming a solution of the form $Ae^{i(\omega t + l x_3)}$ for each of the unknowns and substituting in equations (12) gives a matrix equation with a determinant that must equal 0 for a non-trivial solution:

$$\begin{vmatrix} il(1-\phi) & i\omega(1-\phi)\rho_f + \gamma & -\gamma \\ il\phi & -\gamma & i\omega\phi\rho_f + \gamma \\ i\omega & il(1-\phi)K^* & il\phi K^* \end{vmatrix} = 0. \tag{13}$$

The resulting dispersion relationship between wavenumber l and angular frequency ω is

$$\frac{l^2}{\omega^2} = \frac{\rho^*}{K^*} \frac{\phi(1-\phi)\frac{\rho_f \rho_s}{\rho^*} - i\frac{\gamma}{\omega}}{\phi(1-\phi)\rho' - i\frac{\gamma}{\omega}}, \tag{14}$$

$$\rho' = (1-\phi)\rho_s + \phi\rho_f. \tag{15}$$

The imaginary part of l from equation (14) gives the attenuation coefficient α in units of inverse length, and the phase velocity c is obtained from the real part $\Re\{l\}$:

$$\alpha = \Im\{l\}, \tag{16}$$

$$c = \frac{\omega}{\Re\{l\}}. \tag{17}$$

The dimensionless parameter Q, defined as the ratio of the maximum stored energy to the energy loss per cycle, is related to the attenuation α by (see, for example, Johnston et al., 1979)

$$Q^{-1} = \frac{2\alpha c}{\omega}. \tag{18}$$

Attenuation values cited below will be given in units of decibels per meter, which is related to attenuation in units of inverse length by 8.686α [1/m] $= \alpha$ [dB/m].

Expressions for the Dissipation Coefficient

To this point, no explicit form for the coefficient γ has been assumed. Biot (1962) showed that it is given by

$$\gamma = \eta R(1-\phi)^2, \tag{19}$$

where R is a measure of resistance to fluid flow. The factor of $(1-\phi)^2$ results from considerations of volume flow rates. In the original formulations, the value used for R was the reciprocal of permeability, B^{-1}. For applications to porous media, this is useful and valid, although the permeability B is at best difficult to predict theoretically and must in general be measured for a particular rock sample. Hovem (1980a,b) and Ogushwitz (1985) also used the permeability concept for suspension theory, estimating values from the Kozeny-Carman relationship for permeability in terms of particle radius r and concentration ϕ:

$$B = \left(\frac{r^2}{9k_0}\right)\frac{(1-\phi)^3}{\phi^2}, \tag{20}$$

where k_0 is a free parameter related to the local shape of the medium. This is usually assumed to be 5, which is the theoretical value for a circular tube pore. It has been shown that this equation is frequently observed to be inaccurate even for true porous media, and that *a posteriori* adjustment of parameters is necessary to match observations (Scheidegger, 1974).

Because of the ambiguity inherent in the Kozeny-Carman equation and its free parameter k_0, the drag coefficient γ in this paper is derived from Stokes flow drag force. For a single sphere this is (see, for example, Landau and Lifschitz, 1959)

$$\gamma_1 = 6\pi\eta r. \tag{21}$$

The application of the Stokes flow drag force to suspension dissipation is not new (for example, Urick, 1948; Ahuja, 1973) and it has also been applied in terms of the permeability concept (Ament, 1953). These previous applications have all attempted to account for the presence of multiple spheres simply by multiplying γ_1 by the number of spheres per unit volume, n. Some analytic approximations for the effects of the presence of more than one sphere exist. For a periodic array of spheres in a body centered packing, the drag force on an individual sphere to second order in ϕ is (Hasimoto, 1959)

$$\gamma_1 = 6\pi\eta k r, \tag{22}$$

$$k^{-1} = 1 - 1.791\phi^{1/3} + \phi - 0.329\phi^2. \tag{23}$$

Numerical studies indicate that this expansion is accurate to concentrations of about 20%, and is fairly close for values approaching 40% (Zick and Homsy, 1982). Both the analytic (Hasimoto, 1959) and numerical results (Zick and Homsy, 1982) show that the values for k are approximately the same as those shown above for simple cubic packings

and face centered packings. Solid particles in a suspension will certainly not have a periodic distribution in space, but on the average the distribution will at least be fairly uniform, and the deviation from the results for these packings should not be extreme. Equation (22) should be a valid approximation to the drag force in this case, and at least has a more rigorous theoretical basis than the Kozeny-Carman permeability for suspensions.

Multiplying the drag coefficient by the number of spheres per unit volume n, where

$$\phi = n\frac{4\pi r^3}{3}, \tag{24}$$

gives

$$\gamma = \frac{9\eta k\phi(1-\phi)^2}{2r^2}. \tag{25}$$

An equivalent permeability can be defined for the Stokes flow result of equation (25) for comparison to the predictions of the Kozeny-Carman relationship (20). The dissipation coefficient γ from Darcy's Law would be simply $\gamma = \frac{\eta(1-\phi)^2}{B}$. Noting this form for γ, the permeability implied by (25) is

$$B' = \frac{2r^2}{9k\phi}. \tag{26}$$

The results from the Kozeny-Carman equation (20) and from equation (26) are compared in Figure 1, where the values for $k_0 = 5$ and $k_0 = 10$ are presented. Noting that values from the Kozeny-Carman equation when $k_0 = 10$ are very close to the Stokes drag computations, it is not surprising that the Biot theory applications obtained best matches to observed data for this value of k_0 (Hovem, 1980a). This figure also shows that when solid concentration ϕ approaches 0.48, the results using the Hasimoto modification to drag force rapidly depart from the Kozeny-Carman predictions, indicating a strict upper limit on application of this approximation for the effects of an array of spheres on fluid flow.

Using the result for γ in equation (25), the dispersion relationship (14) becomes

$$\frac{l^2}{\omega^2} = \frac{\rho^*}{K^*}\frac{\dfrac{\rho_f\rho_s}{(1-\phi)\rho^*} - i\dfrac{9\eta k}{2r^2\omega}}{\dfrac{\rho'}{1-\phi} - i\dfrac{9\eta k}{2r^2\omega}}. \tag{27}$$

This equation should be a good approximation to attenuation and velocity for low concentrations. Due to the upper limit on the accuracy of the drag force function by Hasimoto (1959), the concentration should not be much greater than $\phi = 0.20$. A significant aspect of this result is that it involves no free parameters like the shape factor used in the Kozeny-Carman equation.

When $\phi = 0$, equation (27) reduces to the real wavenumber

$$l^2 = \left(\frac{\rho_f}{K_f}\right)\omega^2, \tag{28}$$

which therefore has no attenuation and a velocity $\alpha = (K_f/\rho_f)^{1/2}$, the velocity of acoustic waves in the pure fluid. This shows that equation (27) has the correct limit as the concentration of solids goes to zero.

Frequency Behavior of Attenuation

Assuming a harmonic time dependence, the inertial terms in both the equations for partial pressures in equation (2) vary as ω, while the viscous coupling terms are frequency independent. At low frequencies the inertial terms will be negligible, but at high frequencies the attenuation and velocity will be controlled by the inertial effects. Following the analysis of Schmitt (1986) in a summary of Biot theory applied to porous media, a characteristic frequency of the supension can be obtained by considering the case where the solids are motionless. Inertial terms in equation (2) can be neglected when

$$(1-\phi)\rho_f\omega \ll \gamma. \tag{29}$$

The characteristic frequency is then defined as that value for which the equality holds:

$$\omega_c = 2\pi f_c = \frac{\gamma}{(1-\phi)\rho_f}. \tag{30}$$

Substituting for γ yields

$$f_c = \frac{9\eta\phi(1-\phi)k}{4\pi r^2\rho_f}. \tag{31}$$

The value for k at $\phi = 0.05$ is 2.3 (Hasimoto, 1959), and for water with density 1.0 g/cm^3 and viscosity 1 cP, a 5% suspension of particles with radius 140 μm gives $f_c = 4.46$ hz. When the particle radius is 8 μm, the critical frequency increases to 1366 hz, a reasonable result since a larger radius makes forces related to volumes, the inertial forces, relatively more important than those related to surface area, viscous drag. When the particle is smaller, the viscous effects will dominate for a larger frequency range.

At low frequencies, the attenuation coefficient α is given approximately by

$$\alpha \approx \omega^2\sqrt{\frac{\rho^*}{k^*}}\frac{r^2}{9\eta k(1-\phi)}\left(\frac{\rho_f\rho_s}{\rho^*} - \rho'\right). \tag{32}$$

The Biot theory also predicts an attenuation proportional to ω^2 at low frequencies (Hovem, 1980). Given the above characteristic frequency analysis, this result is not

surprising since the viscous effects dominate the inertial terms at low frequencies, and viscous attenuation is proportional to ω^2.

The predicted attenuation at high frequencies for this model approaches

$$\alpha \approx \sqrt{\frac{\rho^*}{k^*}}\sqrt{\frac{\rho_s\rho_f}{\rho'\rho^*}}\frac{9\eta k}{4r^2}\left(\frac{1}{\rho'} - \frac{\rho^*}{\rho_s\rho_f}\right)(1-\phi). \tag{33}$$

In this case, the Biot theory differs, because a frequency correction term which varies as $\omega^{1/2}$ at high frequencies was applied to the fluid viscosity to account for the departure from Poiseuille flow (Biot, 1956b). However, the form of this correction utilized by both Hovem (1980) and Ogushwitz (1985) is only an analytical result for flow in a tube of circular cross-section. A frequency correction for the oscillatory motion of the spheres could be applied for the drag force on the spheres, but the results discussed below indicate that this is not necessary.

COMPARISONS TO LABORATORY DATA

Polystyrene Spheres

Kuster and Toksöz (1974b) conducted a series of laboratory experiments to test their theory developed from expressions for displacement fields caused by scattering obstacles (Kuster and Toksöz, 1974a). The data are well suited for theoretical tests, because the measurements were taken on suspensions of spherical particles of polystyrene in oil and water, and the bulk properties of all of these materials were known (Table 1). In addition, the solid material was chosen to have a density similar to that of the fluids to allow the suspension to be maintained longer without settling. Of the parameters listed in Table 1, the least well defined is the radius of the spheres. Kuster and Toksöz (1974b) determined that the grain size distribution was approximately Gaussian with a mean of 140 μm and standard deviation of 30 μm.

A detailed and significant comparison can be made to the ratios of measured velocities to pure fluid velocity, which were explicitly tabulated (Kuster, 1972). The data were obtained by measuring the travel times of pulses composed of many frequencies (Kuster and Toksöz, 1974b). Although the dispersion relation (27) requires a single frequency value, there is no difficulty in this case since the velocity is essentially constant with frequency for the two suspensions considered. This is because the densities of the fluids and polystyrene (see Table 1) are almost the same, and the relative motion which usually has a significant effect on velocity dispersion and attenuation is minimal here. Therefore the change in relative motion with frequency which will occur in a suspension composed of components of highly contrasting densities will be negligible, and the velocity will not vary with frequency. Data are presented for concentrations

greater than 0.50 by Kuster and Toksöz (1974b), but comparisons are only applied to velocities for $\phi < 0.40$ due to the limitation on the accuracy of the Hasimoto correction factor.

The predictions for the polystyrene suspension in water (WPS) are very accurate to $\phi \approx 0.30$, and then begin to increasingly overestimate the normalized velocity (Figure 2). The results for the oil-polystyrene suspension (OPS) were less satisfactory, and tend to systematically overestimate the normalized velocities by a small value (Figure 3). The estimated uncertainty in the velocities is 0.2%, and when errors in measured model parameters are considered, this estimate increases to 0.5% (Kuster and Toksöz, 1974b). The relative error in the theoretical predictions for OPS is less than 0.5% to concentrations of $\phi = 0.146$, but increases to 0.6% at $\phi = 0.198$ and 1.1% at $\phi = 0.30$.

Unfortunately, detailed attenuation data were not presented by Kuster and Toksöz (1974b). Instead, the results of the measurements are presented as the ratio of amplitude A_s of the signal in the suspension to the amplitude A_f of the signal in the pure fluid. This logarithm of this ratio has the form (Kuster and Toksöz , 1974b)

$$\ln\left|\frac{A_s}{A_f}\right| = x(\gamma_f - \gamma), \tag{34}$$

where γ_f is the attenuation coefficient of the fluid, γ is the attenuation coefficient produced by the dispersion relationship (27), and x is the separation of source and receiver. Because the offset x is not specified by Kuster and Toksöz (1974b) and the amplitude ratio data is presented with an arbitrary, unspecified shift of the ratio values, it is difficult to compare the attenuation data to theoretical predictions. However, if the theoretical attenuation values are multiplied by a factor of 100, and 39 is subtracted from the product, this scaled attenuation value is very similar to the observations for solid concentration $\phi = 0.05$ (Figure 4). The viscous attenuation model does reflect the increase in attenuation at lower frequencies, although the match is not perfect. Kuster and Toksöz (1974b) concluded that anelasticity of the polystyrene spheres made a significant contribution to the attenuation in the suspensions, and it is possible that the neglect of this factor does cause some of the error in the theoretical predictions.

Kaolinite Suspensions

Hampton (1967) presented a summary of a fairly large amount of data from laboratory measurements in sediments containing water. Included was a set of measurements of velocity and attenuation in suspensions of kaolinite clays of varying concentration at 100 kHz. There is a fair amount of scatter in the attenuation data, which Hampton (1967) attributes to varying concentrations. This might have been a consequence of flocculation or settling, or both. The measured attenuations increase to about 10%

concentration, when they level off and then begin to decrease, and thus it is inferred by Hampton (1967) that particle interactions begin to be significant at $\phi = 0.10$. Urick (1947, 1948) presented similar measurements of velocity and attenuation for a kaolinite suspension at a frequency of 1 MHz, and the attenuation and velocity display the same variations as a function of concentration as the 100 kHz data.

The availability of both velocity and attenuation data allows the parameters of the solids to be estimated by inversion, and a successful inversion yielding realistic parameter values helps to confirm that the theory models the data well. An indication of the need for both types of data is shown by Figure 5, where plots of the attenuation coefficient and velocity are given as functions of ρ_s and r for $\phi = 0.125$. Figure 5A shows that a fairly wide range of reasonable radius and density values will give the observed value for attenuation, but if the velocity predictions are considered as well (Figure 5B), a unique combination is determined. Although the predictions are comparatively weakly sensitive to solid bulk modulus, especially for attenuation, numerical study of the problem demonstrated the same sort of behavior in inversion for all three parameters. Inversions based only on attenuation data yielded very unrealistic results. See the Appendix for details of the damped least squares inversion procedure.

The parameter estimates for the 100 kHz data (see Tables 2 and 3) yielded a good fit to the data, especially the velocity data (Figure 6). The results of Ogushwitz (1985) are presented for comparison, and it appears that they match the data equally well, although the attenuation and velocity results may be slightly better and poorer, respectively, than those of the present investigation. A shape factor value $k_0 = 10$ was utilized for the Kozeny-Carman permeability estimates. This value gave results equivalent to the curves presented by Hovem (1980a) for $k_0 = 10$, but Ogushwitz (1985) obtained the same results with $k_0 = 5$. The source of the disagreement is unclear. A true estimate of the radius parameter is difficult to estimate, although Hampton (1967) presents a grain size cumulative distribution function by weight which gives a mode of approximately 1 μm. The effective radius value determined by inversion will reflect not only the volume distribution with respect to radius, but also the strong dependence of attenuation on radius. For clays, the problem is made even more difficult by the plate-like particle shape. In any case, the result used here, 1.13 μm is probably reasonable. The velocity ratio predictions match the observed decrease in velocity accurately. This decrease occurs in suspensions where solid and fluid densities differ significantly because the effective density of the medium increases more rapidly than does the effective bulk modulus (Kuster and Toksöz, 1974b).

The inversion result for solid density ρ_s is only about 10% less than the value used by Ogushwitz (1985), but the bulk modulus K_s result is quite different. An inversion for radius only using the same solid density and bulk modulus as Ogushwitz (1985) yielded $r = 0.952\mu$m, which is reasonable, but was unable to match the velocity data (Figure 7). It is apparent that in order for the dispersion relationship presented in

this paper to model the data, all three solid parameters must be allowed to vary in the inversion. The validity of the bulk modulus solution is difficult to determine. It was not possible to find a laboratory measurement of a bulk modulus for kaolinite, and values used by other workers for clays vary from 43.7 GPa (Ogushwitz, 1985) to 4.4 GPa (Wilkens et al., 1985). The result of the inversion, 12.5 GPa, is at least within this range.

The dependence of attenuation on radius r and particle density ρ_s at 1 MHZ is much the same as at 100 kHz, except that the maximum is shifted to a lower value of r (Figure 8). Likewise, the trends of the velocity predictions also shift to lower radius values. Observations indicate that velocity increases frequency for kaolinite suspensions (Hampton, 1967), and Figures 5 and 8 show that this effect is modeled by the dispersion relationship (27). The results of the inversion for the 1 MHz data were, however, much less satisfactory (Tables 2 and 4, Figure 9). Ogushwitz (1985) had an equal degree of difficulty matching observations using the Kozeny-Carman permeability approach (Figure 9). Urick (1948) cited a typical particle radius of about 0.5 μm, which is not too far from the inversion result, 0.415 μm. The value for ρ_s obtained by the inversion at this frequency, 2.38 g/cm^3, is almost the same as the result from 100 kHz, and the bulk modulus is also comparatively close to that obtained from the low frequency data. Some variation in parameters is expected, since two different clay samples were used in the experiments. Urick (1947) attempted to estimate the ratio of kaolinite compressibility to water compressibility on a single sample of kaolinite using velocity measurements, and the results ranged from 0.016 to 0.028, demonstrating the difficulty of this type of measurement.

The results of the inversions seem to be realistic and fairly consistent for two data sets, although the model predictions for the high frequency data are less satisfactory than for the low frequency measurements. Since the velocity data at 1 MHz is still very well predicted, the error in the model must be in the aspects related to attenuation. The inability of the model to predict the observations better than it does is likely due to an increased contribution of scattering to the attenuation.

DISCUSSION

Although the present model is one which should be restricted to fairly low concentrations and low frequencies in order to fulfill the assumptions regarding simple Stokes flow drag on particles and lack of particle interactions, it can predict some observations fairly well. It was shown above that the WPS normalized velocity calculations are accurate to concentrations of 40%, especially from 0 to 30%, and that attenuation and velocities in a kaolinite suspension could be predicted at 100 kHz. Although the data included an ambiguous scaling, at least some of the trends of attenuation in the OPS suspension were modeled. In contrast, the velocity in the OPS medium is con-

sistently overestimated by a small amount, and attenuation predictions for a kaolinite suspension at 1 MHz are poor. The results of calculations suggest that the maximum concentration for which the model is valid is about 30%.

The theory presented in this paper is closely related to some earlier developments. Urick (1947) attempted to model velocity in kaolinite suspensions by using effective density and bulk modulus for ideal mixtures, $c = \sqrt{\frac{K^*}{\rho^*}}$. It is simple to show that this results from setting $\mathbf{v}_f = \mathbf{v}_s$ in equations (2), thereby implying that there is no relative motion of the fluids and solids, and hence, no attenuation. The correction to this velocity expression which is present in the dispersion relationship in equation (27), although close to one in magnitude, is an important consequence of the behavior of a medium with a fluid matrix.

Although Ahuja (1973) did allow for the relative motion of spheres and fluid, the approximate expression for viscous attenuation was truncated at the first power of ϕ, and it is clear from the data that a linear dependence on ϕ is not satisfactory. In addition, there was no dependence of attenuation on the solid bulk modulus or density, and the lack of the Hasimoto (1959) correction term in the Stokes drag force expressions implies that the predicted resistance to fluid flow will have significant errors for concentrations over a few percent.

In order to match observed values of attenuation in the study of polystyrene spheres, Kuster and Toksöz (1974b) had to include significant contributions from viscous losses and anelasticity of grains as well as scattering. The anelasticity in fact accounted for the largest part of attenuation according to their analysis. Since this is not considered in the present model, the effect of particle anelasticity is probably not important in the kaolinite suspensions. This conclusion is supported by the observation of Urick (1947) that the elastic properties of water, specifically the high compressibility relative to kaolinites, tend to dominate the effective elastic properties of the suspension.

In the course of developing the dispersion relationship (27), mass coupling and frequency corrections for oscillatory motion were ignored. Biot (1956a) included mass coupling in the theory of acoustic propagation in porous media, and showed that there results a parameter relating the effects of the motion of the solids to induced forces in the fluid component. This accounts for a small change in the fluid and solid densities which cannot be theoretically predicted so that it introduces ambiguities in modeling. In any case, the present theory produces satisfactory results without consideration of the minor impact of the mass coupling.

A frequency correction could be introduced by using an expression for the drag

force on an oscillating sphere (Landau and Lifshitz, 1959):

$$\begin{aligned} F &= 6\pi\eta r\left(1+\tfrac{r}{\delta}\right)(v_f - v_s) + 3\pi r^2\sqrt{\tfrac{2\eta\rho_f}{\omega}}\left(1+\tfrac{2r}{9\delta}\right)\partial_t(v_f - v_s), \\ \delta &= \sqrt{\tfrac{2\eta}{\omega\rho_f}}. \end{aligned} \tag{35}$$

The depth of penetration of the disturbance created by the sphere in the fluid is given by δ. The second term in the drag force is a contribution to inertial forces, and only the first term contributes to dissipation. This dissipation term amounts to a scaling of the original steady-state Stokes flow drag force by a factor $1 + \frac{r}{\delta}$, and so the effect of using the frequency correction amounts to scaling the radius at each frequency of observation and does not change the form of the dissipation curve as a function of ϕ. In addition, the drag force expression is derived by assuming $\delta \ll r$. The depth of penetration at 100 kHz and 1 MHz in water is 1.8 μm and 0.56 μm respectively. In both cases, this is larger than the size of the particle in the kaolinite samples. Hence it would actually be invalid to apply this drag expression for the two sets of observations from kaolinite suspensions, and it is appropriate to use the simple Stokes flow result.

Because the 1 MHz kaolinite attenuation data could not be adequately modeled by any radius value, the equations of motion used to derive the dispersion relationship must not provide an accurate description of the attenuation at high frequency. The equations of motion (2) have inertial terms which vary as ω, given harmonic time dependence, while the viscous coupling terms are independent of ω. The model itself does therefore include the reduced significance of the viscous coupling. Scattering theory also predicts that the viscous effects diminish at high frequencies for spherical obstacles (Lin and Raptis, 1983). For the higher frequency range, other attenuation mechanisms such as scattering and thermal loss probably become dominant. The viscous dissipation model proposed here therefore should not be expected to fully account for the processes occurring during the propagation of an acoustic wave in a suspension, because it gives only an indication of the dominant effects occurring within a limited range of applicability. This is also true of the version of the theory applied by Hovem (1980a) and Ogushwitz (1985), since permeability still produces only viscous attenuation.

CONCLUSIONS

A theory for attenuation of acoustic wave propagation in suspensions was developed from a consideration of the partial pressures acting on the solid and fluid components of a suspension. It was shown that the resulting dispersion relationship is the same as that predicted by the Biot theory for acoustic wave propagation in porous media when the solid frame bulk modulus and rigidity vanish. However, instead of attempting

to estimate permeability values for a dilute suspension, dissipation of energy in the model is given by a Stokes flow drag on all spheres present in a given volume, with a modification factor to account for the effects due to the presence of more than one sphere. This approximate correction for the presence of multiple solid particles assumes a regular array of spheres, but in relatively dilute suspensions deviation from this assumed configuration should not be a significant problem. This form of dissipation allows an estimate of attenuation and velocity which has no arbitrary free parameters like previous applications of Biot theory to suspensions (Hovem, 1980a,b; Ogushwitz, 1985).

Results of application of this theory to suspensions of polystyrene spheres in water are quite accurate to concentrations of at least 30% for calculation of velocities, while the same calculations applied to polystyrene spherical particles suspended in oil are consistently slightly high. A damped least squares inversion for the properties of kaolinite yields a very good match to observed data for attenuation and velocity at a frequency of 100 kHz, but the results for 1 MHz attenuation data are poor, an indication of the reduced significance of viscous attenuation at higher frequencies. The results of the modeling of the acoustic properties of the different suspensions lead to the conclusion that the viscous effects are the dominant mechanism for a range of concentrations of solids to about 30% and for frequencies up to at least 100 kHz. Outside this range of concentration and frequency other effects become more significant, and a new theory which includes these effects must be applied.

ACKNOWLEGEMENTS

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APPENDIX

A damped least squares inversion was used for this paper. Because the functional relationship of attenuation and velocity to the various solid parameters in the dispersion relationship equation (27) is nonlinear, it was necessary to first linearize the inversion by assuming a starting model vector $\mathbf{m}_0$ which is close to the desired solution. The

components of this vector are

$$\mathbf{m}_0 = \begin{bmatrix} r \\ \rho_s \\ K_s \end{bmatrix}, \qquad (A-1)$$

where the various parameters are defined in the text. The data vector is modeled by

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{A}\boldsymbol{\Delta}\mathbf{m}, \qquad (A-2)$$

where $\mathbf{y}_0$ is the prediction of the starting model, each entry being an estimated attenuation or velocity. The matrix of first partial derivatives is $\mathbf{A}$, and the elements of $\mathbf{A}$ are defined by

$$A_{ij} = \frac{\partial y_i}{\partial m_j}. \qquad (A-3)$$

Values for the partial derivatives were estimated numerically. Each iteration of the inversion will give an estimate of the change in the model vector $\boldsymbol{\Delta}\mathbf{m}$. The damped least squares result for $\boldsymbol{\Delta}\mathbf{m}$ is obtained from the error in predictions $\boldsymbol{\Delta}\mathbf{y} = \mathbf{y} - \mathbf{y}_0$ by (Hatton et al., 1985)

$$\boldsymbol{\Delta}\mathbf{m} = (\mathbf{A}^{\mathbf{T}}\mathbf{A} + \epsilon^2\mathbf{I})^{-1}\mathbf{A}^{\mathbf{T}}\boldsymbol{\Delta}\mathbf{y}. \qquad (A-4)$$

The damping parameter ϵ^2 proved to be necessary only when the starting model was unusually far from the final result. Inversion results are assumed to be fairly unique, because different starting models yielded the same solution.

There were several practical problems unique to this particular inversion problem. Since no value of the acoustic velocity of the water used for suspension preparation was given by Hampton (1967) or Urick (1948), it was necessary to assume a value. This was calculated by using the same values for water density and bulk modulus as were applied by Ogushwitz (1985).

Hampton (1967) presented velocity ratio data at 50 and 200 kHz for each value of solid concentration. The velocity always increases with frequency, so the values for 100 kHz were obtained by linear interpolation. Any errors introduced by this procedure should be comparatively insignificant, considering that the scatter in the data is fairly large to begin with.

The velocity ratio data also presented a problem because the ratios were all less than one and attenuation values were 2 to 3 orders of magnitude larger, and so the rows of $\mathbf{A}$ corresponding to attenuation results were also several orders of magnitude larger than those corresponding to velocity ratios. This had the effect of causing the inversion to essentially ignore the velocity data. The problem was solved by rescaling the ratios by an arbitrary factor, typically about 1000, which caused the inversion to be equally sensitive to velocity and attenuation data.

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Parameter	Water	Oil	Polystyrene
Density (g/cm^3)	0.9982	0.8794	1.045
Incompressibility (GPa)	2.137	1.863	3.808
Viscosity (Poise)	0.01	1.8	

Table 1. Physical constants of materials used in model calculations for the water-polystyrene and oil-polystyrene suspensions (Kuster and Toksöz, 1974b).

Parameter	
Fluid density (g/cm^3)	1.00
Fluid incompressibility (GPa)	2.15
Fluid viscosity (Poise)	0.01

Table 2. Fluid parameters used in modeling attenuation and velocity in kaolinite suspensions. These are the values used by Ogushwitz (1985).

Parameter	Ogushwitz (1985)	Inversion results for Hampton data (Figure 6)
Particle radius (microns)	1.	1.13
Solid density (g/cm^3)	2.61	2.36
Solid incompressibility (GPa)	43.7	12.5

Table 3. Solid parameters used in modeling the Hampton (1967) data for kaolinite at 100 kHz. The solid parameters used in equation (27) were obtained by inversion, and the values used by Ogushwitz (1985) are given.

Parameter	Ogushwitz (1985)	Inversion results for Urick data (Figure 9)
Particle radius (microns)	0.5	0.415
Solid density (g/cm^3)	2.61	2.38
Solid incompressibility (GPa)	43.7	16.4

Table 4. Solid parameters used in modeling the Urick (1947, 1948) data for kaolinite at 1 MHz. The solid parameters used in equation (27) were obtained by inversion, and the values used by Ogushwitz (1985) are given.

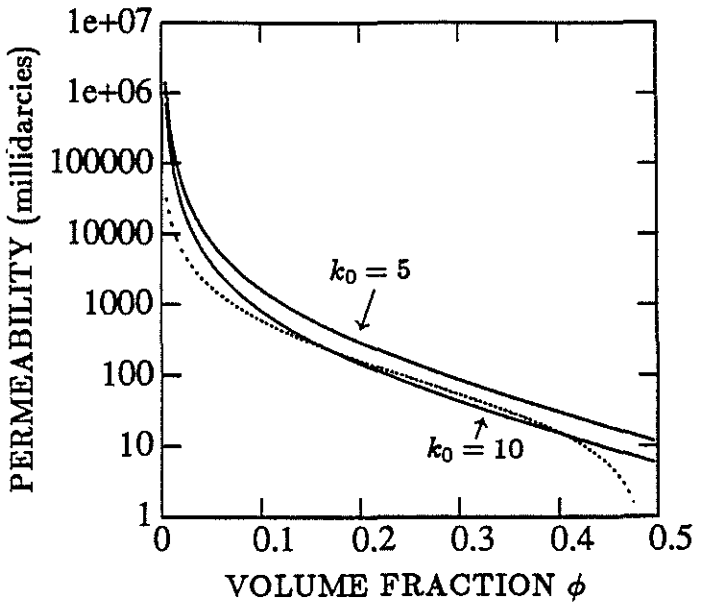

Figure 1: Predicted permeability versus solid volume fraction. The solid lines indicate values from the Kozeny-Carman equation with the indicated values of the parameter k_0, and the dashed line is the result from the Stokes drag force calculation.

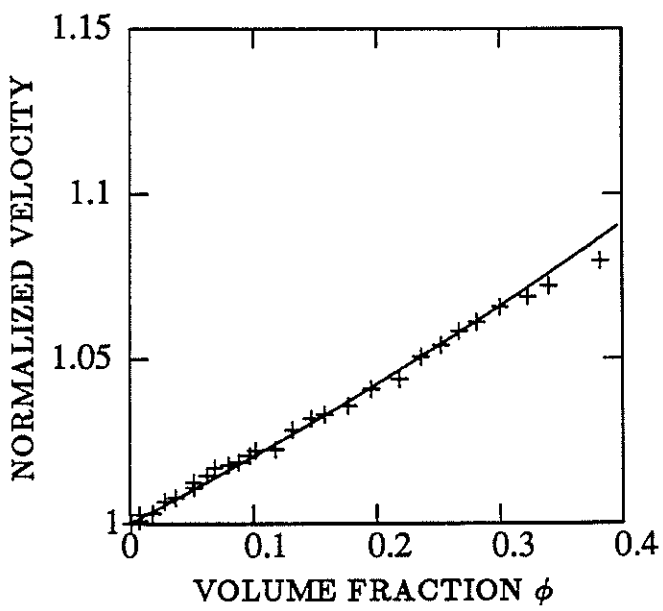

Figure 2: Water-polystyrene (WPS) suspension normalized velocities versus solid volume fraction. The points are laboratory data from Kuster (1972). The solid curve is the theoretical prediction of this paper. Model parameters are given in Table 1.

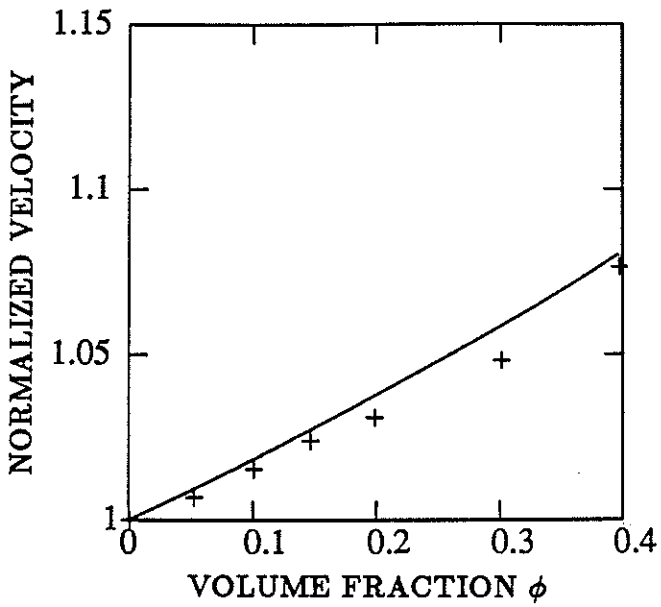

Figure 3: Oil-polystyrene (OPS) suspension normalized velocities versus solid volume fraction. The points are laboratory data from Kuster (1972). The solid curve is the viscous dissipation model prediction. Model parameters are given in Table 1.

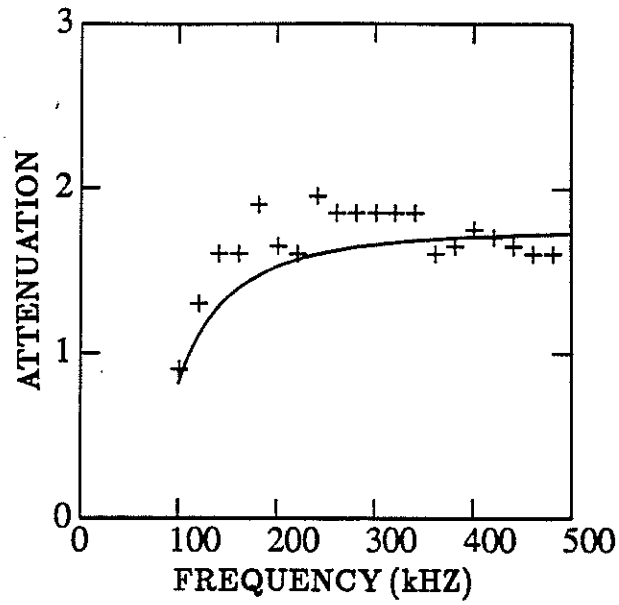

Figure 4: Oil-polystyrene (OPS) suspension attenuation versus frequency. The points are laboratory data in the form of amplitude ratios for concentration $\phi = 0.05$ from Kuster and Toksöz (1974b). The solid curve is the theoretical attenuation prediction of this paper calculated using the parameters in Table 1. The theoretical attenuation values α were scaled by $100\alpha - 39$ to produce this graph. This scaling was performed because the ratio data presented by Kuster and Toksöz (1974b) include an unspecified multiplicative factor and an arbitrary origin on the amplitude ratio, or attenuation, axis.

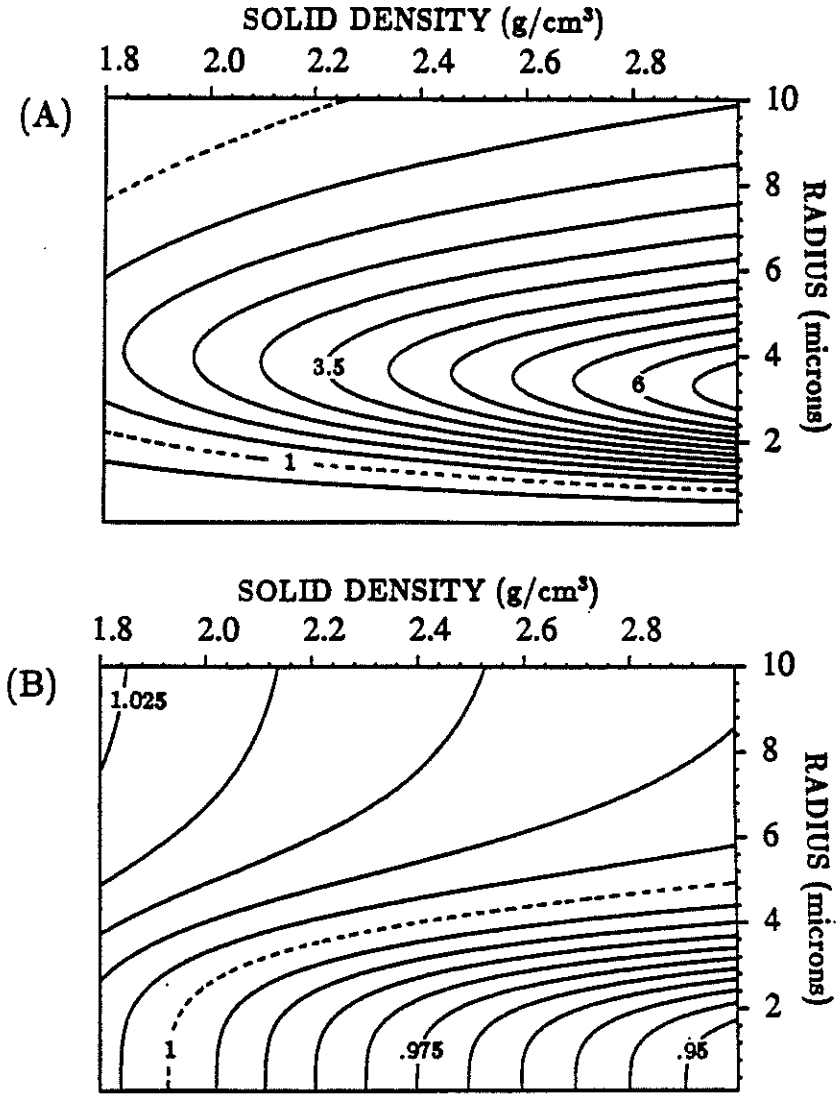

Figure 5: Dependence of attenuation and velocity on density (ρ_s) and radius (r) of solid inclusions at 100 kHz. The solid concentration for these calculations was $\phi = 0.125$, and other parameter values are given in Tables 2 and 3. (A) Ratio of predicted attenuation to the observed value, 18.7 dB/m Hampton (1967). The contour interval is 0.5. The contour corresponding to the observed value is dashed. (B) Ratio of predicted velocity to the observed value, 1466 m/s. This value is calculated from the normalized velocity ratio obtained by Hampton (1967) and the fluid parameters in Table 2. The contour interval is 0.005, and the contour corresponding to the observed normalized velocity is dashed.

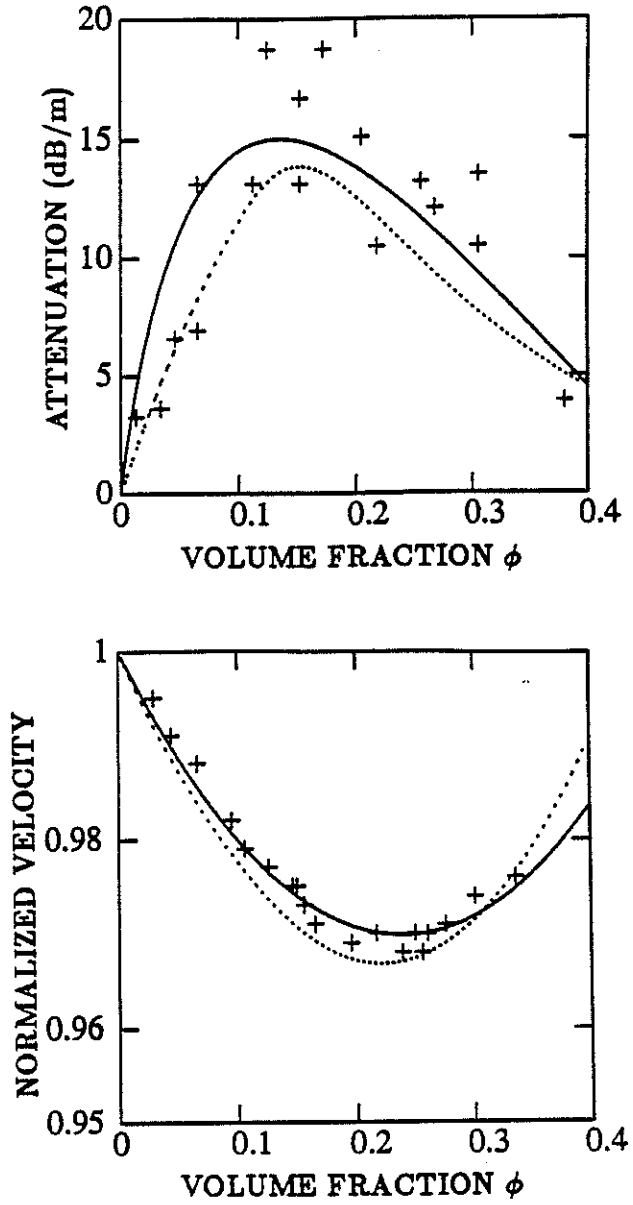

Figure 6: Attenuation and normalized velocity in a kaolinite suspension at 100 kHz. The solid line gives the predictions of the dispersion relation equation (27), and the dashed line is the prediction of the permeability approach of Hovem (1980a) and Ogushwitz (1985). An inversion based on the attenuation and velocity data yielded the solid parameters used for equation (27). The parameters used to calculate both curves are given in Tables 2 and 3, and the data are taken from Hampton (1967).

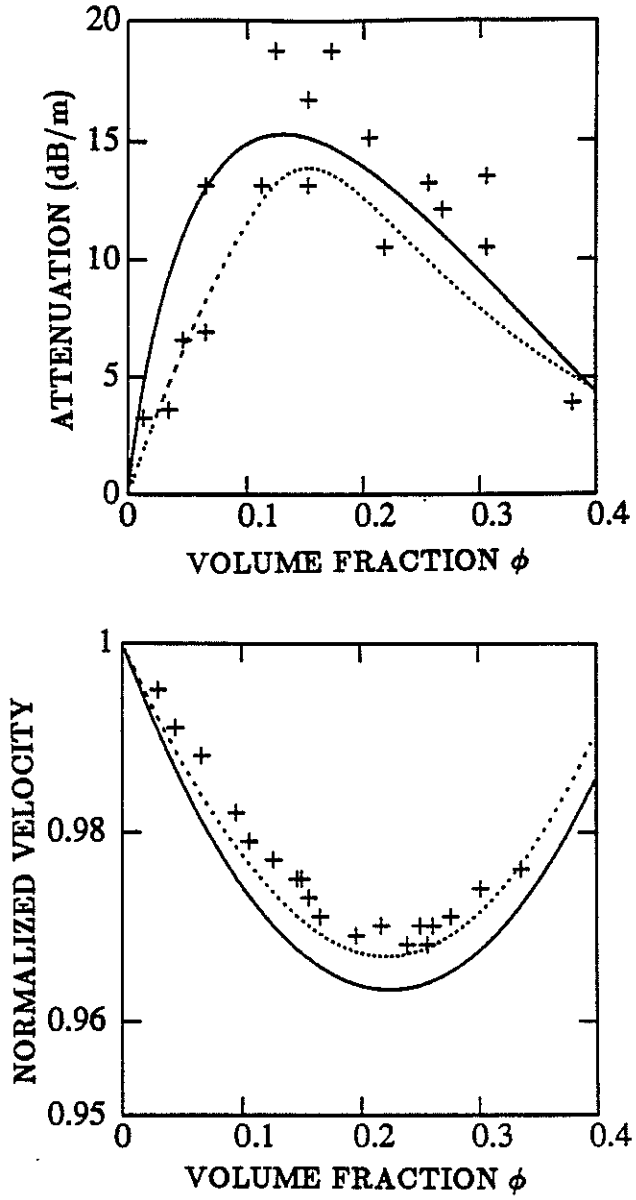

Figure 7: Attenuation and normalized velocity in a kaolinite suspension at 100 kHz where only the radius of solid particles is determined by inversion. The inversion result for radius r was 0.952 μm. The values for solid density ρ_s and solid incompressibility K_s were those used by Ogushiwitz (1985) (see Table 3), and fluid parameter values are given in Table 2. The solid line corresponds to the predictions of the dispersion relation equation (27), and the dashed line is the result of the permeabilty approach of Hovem (1980a) and Ogushwitz (1985).

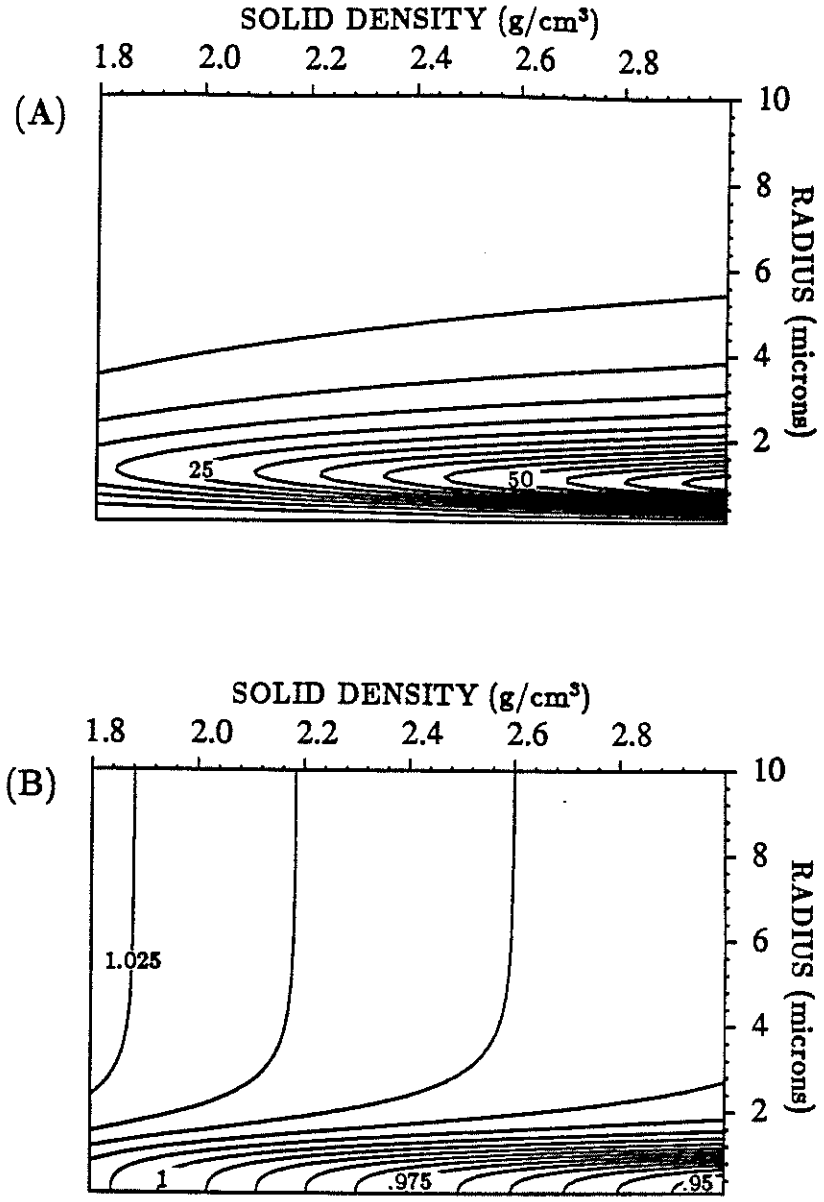

Figure 8: Dependence of attenuation and velocity on density (ρ_s) and radius (r) of solid inclusions at 1 MHz. The parameter values are the same as those used for Figure 5 and the results are presented as ratios to the same values as in Figure 5 so that the effect of increasing frequency is apparent. No observations were made by Urick (1947, 1948) at this value of ϕ. (A) Ratio of predicted attenuation to 18.7 dB/m. The contour interval is 5. (B) Ratio of predicted velocity to 1466 m/s. The contour interval is 0.005.

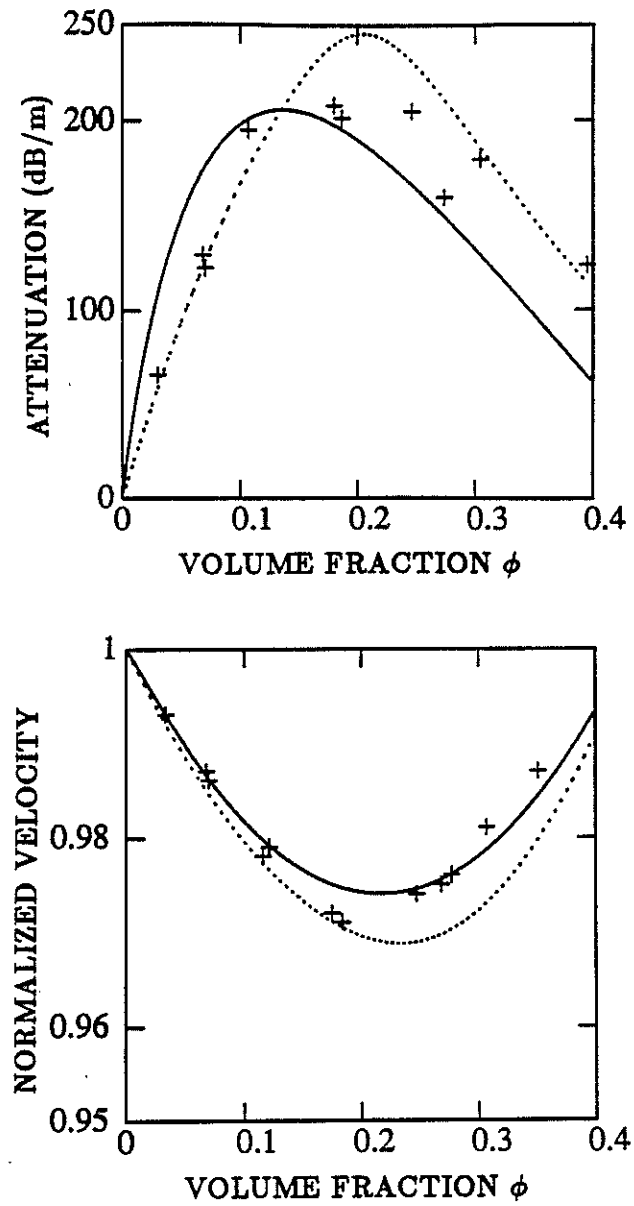

Figure 9: Attenuation and normalized velocity in a kaolinite suspension at 1 MHz. The solid line gives the predictions of the dispersion relation equation (27), and the dashed line is the prediction using the permeability approach of Hovem (1980a) and Ogushwitz (1985). The values used for the solid parameters used in equation (27) were obtained by inversion (Table 4). Fluid parameter values are given in Table 2, and the data values are taken from Urick (1947, 1948).