

Well-to-Well Log Correlation Using Knowledge-Based Systems and Dynamic Depth Warping

by

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ABSTRACT

We present a novel system for well-to-well log correlation using knowledge-based systems and dynamic depth warping techniques. This approach overcomes a major drawback inherent in previous methods, namely the difficulty in correlating missing or discontinuous rock units.

The system has three components: (1) A Dynamic Programming algorithm to correlate the logs and to find the minimum-cost or "best" match; (2) A set of "rules" to guide the correlation; (3) A data base that contains the logs and other relevant geologic and seismic information. The Dynamic Programming algorithm calculates the cost of correlating each point in the first well with each of the points in the second well. The resulting matrix of dissimilarity contains cost information about every possible operation which matches the well logs. The cost of matching the two wells is measured by the difference in the log values. The dynamic programming approach allows correlation across geologic structures, thinning beds, and missing or discontinuous units. A path finding algorithm then traces through the matrix to define a function which maps the first well onto the second. The minimum cost path is the optimal correlation between the wells.

The system's database contains the well logs themselves and other relevant data including information about the geologic setting, seismic ties, interpreted lithologies, and dipmeter information. Rules operating on the data affect the dynamic programming and

path finding algorithms in several ways: (1) Seismic ties or marker beds define a point in the warping path, thereby removing calculations over large portions of the search space; (2) Dipmeter results and knowledge of geologic structure further constrain the path to certain global areas and save calculation time; (3) The system assigns weights to different logs based on log quality and sensitivity; (4) Knowledge of the paleoenvironment allows the program to choose a set of rules (model) which accounts for changes in sediment type or thickness within a field. For example, when the program is operating in a deltaic environment, it will correlate the shales before attempting to correlate the sands.

We demonstrate the method with synthetic examples in which the program successfully correlates across geologic structures and pinch-outs. We also applied the program to field examples from two widely separated oil provinces. In both cases, the automated correlation agreed very well with correlations provided by geologic experts.

INTRODUCTION

One of the primary uses of wireline logs is well-to-well correlation. Since the earth's geologic record has been modified by tectonics and erosion, correlating even closely spaced boreholes can be a complicated problem, requiring some rules that are not easily programmed on a computer. As well data is continually increasing in volume, automated correlation methods become more attractive to reduce the burden on the geologist and allow consideration of more possible matches. Attempts at using automated correlation methods date back to Testerman (1962). More recently, computer algorithms have been proposed to work with Fourier transformed logs, performing the correlation in the (spatial) frequency domain (Rudman and Lankston, 1973; Robinson, 1978). Although these methods work well in some cases, they do not account for nonlinear correlations. They determine a depth offset and a stretch factor which most closely match the test well into a reference, but the offset and stretch are constant for the section analyzed. Perhaps the greatest pitfall of these methods, therefore, is their inability to handle correlation across missing or discontinuous rock units. This is such a common occurrence in geological sequences that it must be handled by any automated technique that is to be applied in a variety of areas.

We describe a new automated computer program for well-to-well correlation, using ideas developed in computer science about expert systems. Expert systems are computer programs that attempt to emulate the behavior of a human expert in a problem-solving task (Startzman and Kuo, 1986). They are best at employing heuristic rules, complementing conventional programming's use of mathematical functions. The program (named COREX) has three components: (1) A dynamic programming algorithm to correlate the logs and find the minimum-cost match; (2) A data base containing the logs, and other geological and geophysical data relevant to the correlation problem; (3)

A flexible set of rules and geologic models, which when applied to the data, serve to constrain the correlation within certain meaningful limits.

In the next section we describe the dynamic programming method used to perform the correlation algorithm. Multiple logs are used from each well, and the resulting matching costs calculated. We account for insertions and deletions of rock units by considering nonlinear matches. Next we describe the rules used and their organization in the knowledge base. The knowledge base in essence serves to make some correlations more attractive and others less attractive, based on a model of the geologic setting. Then we include a section on how we implemented the methods. Finally we demonstrate the methods with examples in which the program successfully correlates across different geologic structures and pinch-outs. We also applied the program to field examples from two widely separated hydrocarbon provinces.

DYNAMIC DEPTH WARPING: THE CORRELATION ALGORITHM

The mathematical algorithm used to determine the match between wells must be able to model the geologic process and must allow for deletions and insertions of new rock units in a sequence. For this purpose we use a method called dynamic depth warping. This approach has been used in speech processing where problems similar to those in geologic correlation occur. When comparing test and reference words, mismatches may result from differences in the length of the word, as well as local variations when one portion of the word is sped up relative to another. A successful approach to the speech processing problem is the dynamic time warping method described by Anderson and Gaby (1983). The dynamic depth warping we developed benefits from the studies in speech.

Dynamic Warping

As an example of dynamic warping, consider the transformation of the test word "MILLER" into the reference word "HILLIER". There are many possible ways to make the transformation by changing, inserting, or deleting individual characters, but each of these edit operations has an implied cost. Several possible matchings are shown below:

	Operation	Description	Edit Cost
MILLER : T(M)	<i>d</i>	Delete the character	1
HILLIER : R(N)	<i>cX</i>	Change the character to X	1
d cM cI - cL - - D=4	<i>iX</i>	Insert character to the right of this character.	1
	-	No Change	0
MILLER : T(M)			
HILLIER : R(N)			
cM d cI - cL - - D=4			
MILLER : T(M)			
HILLIER : R(N)			
cM - - - d - - D=2			

(After Anderson and Gaby, 1983)

Figure 1 summarizes the cost information for every possible match of the test and reference words. This display, called a dissimilarity matrix, is calculated recursively by the dynamic warping algorithm. Finding the optimal correlation is now readily done by tracing the minimum cost path through the dissimilarity matrix. This path is called the warping function; it shows the least costly correlation at each point in the sequence (Delcoigne and Hansen, 1975; Sankoff et al., 1983). Further details of dynamic warping are also provided in the first appendix.

The Cost Function for Dynamic Depth Warping

An assumption in the dynamic warping approach is that at any point in the match all of the relevant differences between the objects being matched can be summarized by a single measure of pairwise dissimilarity (Gordon and Reyment, 1979). In correlating well logs, the cost of matching corresponding points is set equal to the absolute value of the difference in log values at any point. For example, the cost of matching a gamma ray log value of 100 in the test well to a gamma ray log value of 100 in the reference well is zero, while matching the same point in the test well to a gamma ray value of 20 in the reference well has a cost of 80. In practice any log sensitive to lithologic change can be used, and all logs are normalized to a scale of 0 - 100. For a pair of logs (such as gamma ray) we define the cost function as a difference metric $d(n, m)$ of matching the n th point in Well A with the m th point in Well B as $|A(n) - B(m)|$. Figure 2 shows the correlation of two well log sequences by the dynamic depth warping method. The dissimilarity matrix shows running costs for all possible matches; the warping path follows the least costly path through the matrix.

An important feature of the dynamic depth warping cost function is it can be extended to correlate multiple logs from each well. This allows more reliable results for two reasons. First, redundant information sometimes carried by multiple logs may diminish random errors of measurement. Second, some lithologic changes are not equally manifested on all logs. For multiple logs, the cost function is defined as the square root of the sum of the squared differences in corresponding log values. If we are matching two wells A and B, each with $i=1, \dots, k$ logs, then the local cost is given by:

$$d(n, m) = \frac{\sqrt{\sum_{i=1}^k |A_i(n) - B_i(m)|^2 W(k)}}{k} \quad (1)$$

where $d(n, m)$ is the cost of matching depth point n in well A with depth point m in well B, and $W(k)$ is a weighting coefficient for the k -th log. Weightings can be used to adjust the confidence level assigned to a log based on the log's quality or on local knowledge. Weighting coefficients in COREX are determined by querying the user or by inferences from information stored in the data base.

KNOWLEDGE-BASED SYSTEM: INTERACTION OF RULES AND DATA

In well-to-well correlation of logs, one would generally incorporate available geological and geophysical information into the process. The "Expert System" described in this paper incorporates the information (complementary data and knowledge) into the correlation process. The expert system consists of a knowledge base and a data base. The knowledge base is an executable section of the program containing a number of conditional statements which may effect the numerical calculations in various ways. There are presently about thirty-five rules implemented in the knowledge base. The data base is flexible and need only contain the well logs themselves. It may also contain other relevant data such as seismic ties, lithologies (from log interpretation or from mud logging), interpreted dipmeter results, and local geologic information (e.g., a regional marker bed).

Before discussing how we incorporate geologic knowledge, we show first how some common geologic situations are manifested in the warping paths described in the last section. Figure 3 shows four schematic structures and the corresponding warping path for each case. Notice in each case that deletion of a section in Well 1 relative to Well 2 corresponds to purely horizontal motion in the warping path, while a deletion in Well 2 corresponds to vertical motion in the warping path. A diagonal motion corresponds to simple stretching of 1 relative to 2. A 45 degree diagonal corresponds to a perfect match between the wells, and a curved path results from non-linear stretching between sections. In Figure 3a, for example, correlation across a normal fault with a throw of

200 meters would result in a simple shift of Well 1 relative to Well 2. To accommodate this shift, the correlation will effectively delete all of the section in Well 1 that is not in Well 2. Thus, the warping path proceeds along the edge of the matrix perpendicular to the section being deleted, until it reaches the point where the wells begin to match. At this point the path is a 45 degree diagonal, proceeding to the lower edge of the matrix. In Figure 3b, when a growth fault separates the two wells, the amount of stretch and shift is no longer constant, but instead increases as a function of depth. In this case the warping path will be a curved line, concave toward the upthrown block. Figures 3c and 3d show respectively the warping paths for insertion or deletion (pinchouts or lenses) and for the flank of a salt diapir.

Notice from Figure 3 that a wide variety of structural situations can be accounted for by only a few different restrictions on the warping path—namely, horizontal or vertical motion, diagonal motion with some slope, and curved paths restricted to one half of the global space. Thus, it becomes possible to program very general structural rules into the dynamic depth warping algorithm. Figure 4a shows the correlations using synthetic logs generated to represent two examples of the geologic structures shown in Figure 3. Correlations were performed on wells separated by a normal fault, and on wells with sandstone pinchouts. We deliberately added random noise to the synthetic logs. Figure 4b shows the warping paths that resulted from the match. As the figure shows, the paths are very similar to those discussed in Figure 3. Departures of the synthetic examples from the theoretical paths can be accounted for by noise present in the logs. By removing the noise from the logs, we can effectively smooth the warping function to the straight lines shown in Figure 3. Noise reduction is discussed later in the section on distance metric rules.

We can thus impose restrictions on the warping path using a priori knowledge of the local geology. To do this, the knowledge base interacts with the dynamic programming through a set of rules. These rules fall into three categories: (1) Lithologic or Depositional Environment Rules; (2) Geologic Structure Rules; and (3) Local Distance Rules, discussed in turn below.

Lithologic Rules

In correlation problems, one proceeds by first matching the most prominent units such as thick-continuous beds or highly distinct marker beds. This has the advantage of breaking the large problem down into several smaller ones. The COREX program approaches the problem in a similar way by initially performing a coarse matching of the lithologies present in each well. For this, the knowledge base stores information about common depositional models, and how correlation strategies should change based on the particular environment. An example of a lithologic rule might state:

If we are correlating in a meandering stream environment,
Then shales will be more continuous units than sands,
and we should correlate shales first, and then the sands.

Another factor the program considers is the strike of the correlation line relative to the paleoshoreline direction, if known. In this way the program can account for changes in thickness and lithology that may occur down a paleoslope, or changes in the lateral bed continuity as we change the orientation with respect to the shoreline. For example, one rule states:

If we are correlating in a continental shelf environment,
and the strike of the wells is perpendicular to the shoreline,
Then allow shaly-sands to correlate with sandy-shales down the slope,
and let thin units correlate with thick units of the same lithology.

The results of this initial lithologic match are a series of tie points (Figure 5). In dynamic depth warping, this is equivalent to forcing the warping path through one point in the dissimilarity matrix, thus eliminating large sections of the global area as possible paths. Next, if any seismic interpretation is stored in the data base, the program creates tie points based on these. (Because there is some uncertainty in converting seismic information into depth, tie points can be specified inexactly as tie 'regions'.) The correlation thus breaks down into a number of smaller correlations, and at the same time eliminates the need to consider a large number of correlations which are no longer possible.

Structural Rules

Once the initial tie points are determined, the problem is further limited by applying rules that relate the depth warping algorithm to geologic structures. As Figure 3 showed, certain geologic features can define the warping path. These structures can either be input by the user or inferred from the seismic and or dipmeter information. For example, if we expect insertions and deletions of rock units, then we could weight our path finding algorithm to favor horizontal and vertical motions in the warping path. In a second example, we might want to limit the amount of shift that is allowed between the wells. Then a structural rule would state:

If we are in a deep basin,
 and the dip between the wells is nearly flat,
Then impose a maximum shift of $d \sin \delta$ between the wells
 (where d is the horizontal well spacing and δ the dip angle.

The program handles such a shift constraint by automatically assigning very high matching costs to points separated by a depth greater than the maximum shift. This shows up in the dissimilarity matrix as a possible match band whose width is equal to the range distance (Figure 6).

Distance Metric Rules

Rules also impose knowledge on the dynamic depth warping algorithm through the local cost calculations. Equation 1 showed that a family of weighting coefficients can be used to alter the influence of a particular log on the correlation. For example, if one log is not diagnostic in a particular formation, or is determined to be too noisy, it can be weighted accordingly. If the program expects to correlate down a paleoslope where shaliness would increase away from the shoreline, then a sandy-shale can be made to correlate with a shaly-sand of the same dimension. Before the matching process begins, the COREX program runs through rules concerning the lithologies and the noise analysis of the logs to adjust the local distance measure between points. Figure 7 shows how the correlation can be improved by modifying the local distance measure because of noisy logs. Here is a sample rule that alters the local distance measure between two points:

If the caliper log shows a large positive deflection,
Then we expect the density and neutron log responses
 to be altered by a washout,
 and their weighting coefficients should be reduced.

A complete list of the rules presently implemented in COREX is given in Appendix B.

IMPLEMENTATION OF THE METHOD

This section describes how the expert system interacts with the dynamic programming algorithm to solve a correlation problem. As described in the previous section, the knowledge base contains information about common depositional models, simple geologic structures, and rules which allow this knowledge to affect the data. The database

of the program contains the digitized well logs, and information about seismic ties, structural dip, and lithology. The program must run through the rules in the knowledge base, and by combining the data with geologic knowledge, translate this information into meaningful constraints on the depth warping algorithm. The end result is a constrained least-cost match which the knowledge base has forced to be geologically meaningful.

The program attempts to emulate a human expert by breaking the problem into smaller, more manageable parts. It accomplishes this by first performing a large-scale match of the lithologies present in each well. Gross lithologies come from drilling records or from well logging data. With a knowledge of the depositional environment, the program assigns continuity to specific lithologies. Information supplied by the user at the start of a session determines an appropriate geologic model. In particular, the program wants to know the horizontal distance between the wells, the depositional environment, and the strike of the correlation relative to the paleoshoreline.

For example, in a fluvial meandering stream environment, shale units will be more continuous (in lateral directions) than sands, and thick shales will be more continuous than thin ones. Using these rules, and the others outlined in Appendix B, the system assigns a similarity measure or matching cost for matching lithologic units in two different wells. An ordinal rank such as 'good' might describe the match between two thick evaporite units in a shallow shelf environment. Once these local costs have been determined, the program uses a simple string matching algorithm as described in Appendix B to match corresponding lithologies. The result of this initial match is a series of tie points limiting the search space for the optimal warping path.

Once the program performs the initial match, it has a series of tie points which separate the correlation into a number of smaller problems, represented in the global search space as a number of rectangular regions connected at the corners (see Figure 5). Next the program runs through rules concerning the geologic structure of the particular section. Seismic and dipmeter information further reduce the search space. For example, if the structural dip between the wells is known, and corresponding sections to correlate are of roughly the same thickness, then a maximum shift constraint can be imposed. Another rule may force the warping path to travel only in the lower half of the global area, or to travel along edges of the area, as seen in Figure 3. After these rules are applied, the dynamic depth warping algorithm is ready for the final correlation with all of the imposed constraints. At this point the program has reduced significantly the amount of calculation from the original, unconstrained problem, and enhanced greatly the chances of a geologically meaningful result.

The final correlation is performed on a point-to-point scale using dynamic programming. During the calculations, distance metric rules are sometimes fired by the program, since these rules are generally applicable over sub-sections of the interval. The caliper rule changes the weighting coefficients of density and neutron logs over sections

of washed out hole. Other log quality curves can be used to alter the weightings of other logs. The amount of noise present in any log can be estimated by statistical variance methods, where noise amplitude would be expressed in terms of standard deviations from the norm. By assigning some noise "threshold" to the logs, the program can decide which deflections are likely due to noise and which ones are likely due to formation boundaries. As mentioned before, the use of multiple logs further enhances this resolution. This local distance measure may also be adjusted by noting that some logs are not as sensitive as others to bed boundaries, and their influence on the correlation can be weighted accordingly.

The depth warping algorithm calculates all possible matching costs between each point in the first well, and all other points in the second well. Dynamic programming then recursively fills the global cost matrix, each point in the matrix representing the total cost of matching the two sets of logs to that point. The total cost at the end of the warping is a measure of the quality of the match (analogous to the peak in the cross correlation function). If two curves are identical, then the total cost to match them will be zero. If one or both of the curves is noisy, then the matching cost will be very high.

When the calculation is complete, the program uses a separate routine to trace back the minimum distance path through the matrix. The program can then draw tie lines between all points which correspond in the match. We should point out that not all points will be matched, but only those which required a minimum matching cost in the original calculation. In other words, points may be connected by the warping path, but unless the matching cost between them was above some value (the noise threshold) no tie lines will be drawn. This helps to prevent the program from correlating noise, as opposed to geologic "signals".

One of the advantages of knowledge-based programming is that the program provides commentary on how it reached a conclusion. Although not implemented in our system, a sample output for a typical correlation is as follows:

Correlated in a barrier beach environment, well separation was 2 kilometers, strike of correlation line was parallel with the paleoshoreline, structural dip was 2 degrees, a maximum shift constraint of 40 meters was applied, the density log was eliminated due to excessive noise (interval 2000-2050 m), and final correlation proceeded with 4 logs.

FIELD EXAMPLES

Field Example from West Africa

Figures 8–11 show an example of correlation using log data from two offshore wells in the Western margin of Africa. In this study, Well 1 contains 250 depth points and spans 250 feet (150 m) of section, and Well 2 contains 225 depth points and spans 225 feet (135 m) of section. Each well had five logs available for correlation—gamma ray, sonic, density, neutron, and resistivity. The depositional environment is a small deep marine basin along the continental shelf of the passive East Atlantic margin. Figure 8 shows the results of the initial lithologic match between the two wells. A section of sandy-shales in Well 1 correlates with a section of shaly-sands in Well 2. This is allowed by the program since the environment is a shelf and the strike of the well correlation line is roughly perpendicular to the shoreline. In this case, it is likely for a sand sequence to increase in shaliness away from shore, and hence the matching cost between the units was reduced.

Figures 9–11 show the resulting correlation, the final warping path, and the constrained global cost matrix for the West African wells. Also displayed are the rules 'fired' by the program in the correlation process. Figure 9 shows that the program does very well matching the particular sand and shale units across the wells, even though the section contains both thinning and thickening units. We can also see from Figure 9 the advantage of using multiple logs in the correlation. Looking at the gamma ray log, it appears that the program is making meaningless correlations over the depth range of 8300 – 8400 feet. Looking at the density log correlation, however, we see that changes in rock properties are occurring, even though they are not reflected in the gamma ray log. This evidence supports the use of multiple logs in the correlation.

In Figure 10 we can see how information about the geologic structure between the wells is represented in the warping path. First, referring back to Figure 3, we can see the superposition of different geologic factors on the warping path. Over the entire depth section, the beginning and ending regions match quite well in depth, which accounts for the general 45 degree diagonal trend in the warping path. Notice, however, that in a number of sections non-linear expansions and contractions occur between Wells 1 and 2, and that this imposes a curved section on the warping path. Most notably, the strongly radioactive zone near 8250 feet is expanded in Well 2 relative to Well 1, which shows up as vertical motion in the path. At 8500 feet, however, we have expansion of a sand body from Well 2 relative to Well 1, which shows up as horizontal motion in the path. Finally, the entire shale section at 8550–8600 in Well 1, which is expanded to the section 8825–8925 in Well 2, shows up as a significant curve in the warping path. Referring back to Figure 3, we see the features displayed in parts (a) and (c) of that

figure, normal faulting and pinching out.

Figure 11 shows the global cost matrix that resulted from the constraints imposed by the initial match. Maximum shift constraints were also imposed by the system for each section in the initial match. The original, unconstrained correlation required over 168,000 distance calculations, and took about 7 minutes of computer time. The final problem, after initial matching and shift constraints, required only 20,500 calculations, and took only 127 seconds.

Field Example from Thrace Basin, Turkey

The dynamic depth warping algorithm is efficient because it can detect complex patterns in many wells simultaneously. As a test of the fine scale matching ability of the system, we used a pair of wells located in the Thrace Basin of Turkey. These wells are in a trough filled by a turbidite sand shale sequence and capped by reefal limestones. Little character is displayed in the logs (Figure 12). Lithology changes show up as only small deflections and are very difficult to observe. Obvious, however, are the top of the formation coming in early in each well, and a more subtle area of volcanic tuffs, considered to be reliable time surfaces in each well. Both of these sections are shown in the lithology log on either side of the wells (Figure 12).

No initial lithologic match was performed. The program imposed a maximum shift constraint of 80 meters based on seismic results. The results of the correlation are shown in Figures 12-14, along with the warping path and the global cost matrix. As Figure 12 shows, the top of the sandy-shale sequence is clearly reflected in the correlation. Close scrutiny of the correlation also reveals three individual volcanic beds which are traced from well to well. Another feature seen from the correlation is a general thickening of corresponding sections with depth from Well 1 to Well 2, consistent with the well locations in the trough. This shows up in the warping path (Figure 13) as a diagonal line through the matrix, with a non-linear trend toward the bottom sections. Looking at the warping path, we see two of the basic kinds of motions superimposed on the path. First, the depth difference between the logs shows up as horizontal motion at the bottom portion of the path. Stretching of Well 1 relative to Well 2 shows up as a diagonal line through the matrix, as we saw in Figure 3. The non-linear increase in bed thickness with depth that is reflected by the tie lines, however, curves the warping path from the upper left corner to the bottom of the match. Once again, the separate geological features show up as basic motions in the warping path. A comparison with Figure 3 will show the similarity with parts (b) and (d) of that figure, listric faulting and folding.

CONCLUSIONS

We presented a new approach to well-to-well correlation which combines dynamic programming and expert systems techniques. Combining these two methods overcomes some of the fundamental problems that have hindered automated correlation in the past. The dynamic depth warping technique has two advantages over older techniques. First, it permits correlations across missing or discontinuous units. Second, because it can be calculated recursively, it is extremely efficient and runs faster than even the spectral methods. Multiple logs from each well are used for correlation, and we have seen evidence that this can produce superior results.

The ability to apply rules from a geologic knowledge base to the matching algorithm provides another set of advantages. Correlation strategies must change from basin to basin and even from field to field. COREX bases its choice of correlation strategy on encoded information about depositional models and the geometry of the wells in a given environment. This information allows the program to make decisions about the continuity of specific rock units, and how this continuity can change within a depositional model. The program performs a coarse initial lithologic match, and uses this match to constrain the dynamic depth warping. Other rules consider dipmeter and seismic information, and respond to changes in log quality.

We applied the program to field examples from two widely separated hydrocarbon provinces. In both cases, the automated correlation agreed very well with correlations provided by geologic experts.

APPENDIX A: Dynamic Waveform Matching

An Example With String Matching

The following is a detailed description of dynamic programming for dynamic waveform matching. For further information the reader is referred to Sankoff and Kruskal (1983), Myers (1980), or Anderson and Gaby (1983). To illustrate the principles behind dynamic waveform matching, we will consider the simple problem of matching some reference word R with characters $R(n)$, $1 > n > N$ with a test word T with characters $T(m)$, $1 > m > M$ (Anderson and Gaby, 1983). The word matching will be performed by editing some of the letters of the input test word $T(m)$ until it matches the reference word $R(n)$. For alphabetical characters, these edit operations are defined as:

Operation	Description	Edit Cost
<i>d</i>	Delete the character	1
<i>cX</i>	Change the character to X	1
<i>iX</i>	Insert character to the right of this character.	1
-	No Change	0

Associated with each edit operation is a cost, listed on the right. We will assume for simplicity that each operation has a cost of 1 except for the "no change" option, which has no cost. For example, below are several possible edit sequences that transform the word HILLIER into the word MILLER, and the total edit cost of each operation:

MILLER : T(M)
HILLIER : R(N)

d cM cI - cL - - D=4

MILLER : T(M)
HILLIER : R(N)

cM d cI - cL - - D=4

MILLER : T(M)
HILLIER : R(N)

cM - - - d - - D=2

From the example we see that of the three possible edit sequences, one is less costly than the others. This match represents the "minimum distance" edit sequence, where here the "distance" that we are concerned with is the total cost of matching the words. Since there are at most $N' = \max(N, M)$ edit operations required to transform T into R and there are 4 possible edit operations, there are $4^{N'}$ possible edit sequences. Dynamic Programming¹ is used to efficiently explore this space of possible solutions and determine the minimum distance (or least costly) match.

Let $D(n, m)$ be the minimum distance (or cost) required to match the first n characters of R to the first m characters of T. $D(n, m)$ will depend at any point in the match on the choices of edit operations used so far in the match as follows:

¹Dynamic Programming was first introduced by Bellman in 1962 as a method of optimizing by linear programming.

Edit	$d(n, m)$
-	$D(n-1, m-1)$
c	$D(n-1, m-1) + 1$
i	$D(n, m-1) + 1$
d	$D(n-1, m) + 1$

The creed of dynamic programming is to minimize the total cost of the match by always choosing the minimum distance operation (cheapest local cost) at each point in the matching process. In other words:

The total cost of matching two words up to the letters $T(m)$ and $R(n)$, $[D(n, m)]$, is equal to the total cost up to the previous move $[D(n-1, m-1)$ or $D(n-1, m)$ or $D(n, m-1)]$, plus the cost of the next move [Either 1 or 0].

Thus, as the matching process continues, our total matching cost increases every time we perform an operation besides a perfect match. Thus our cost function is a monotonically increasing function. For each costly operation (i.e., an insertion, deletion, or no change), we will choose the least costly one. Therefore, our matching costs are continuously increasing, but increasing by a locally minimal amount.

We can now recursively define the running total cost of the match $D(n, m)$ in terms of the previous total cost as:

$$D(n, m) = \min \left\{ \begin{array}{l} D(n-1, m-1) + d(n, m), \\ D(n, m-1) + 1, \\ D(n-1, m) + 1 \end{array} \right\} \quad (\text{A-1})$$

with the boundary condition $D(n, m) = 0$ whenever $n = 0$ or $m = 0$, and

$$d(n, m) = \min \left\{ \begin{array}{l} 0 \text{ if } R(n) = T(m), \\ 1 \text{ otherwise.} \end{array} \right. \quad (\text{A-2})$$

where $d(n, m)$ is the local cost and is referred to as the "distance metric".

Figure 1 shows a graphical representation of this matching process, with the distances computed for the MILLER and HILLIER example. Referring to Figure 1, the running total matching costs $D(n, m)$ are computed on an N by M grid, starting from the point $D(1, 1)$ and proceeding to the point $D(N, M)$ column by column. The minimum total edit cost at the end of the match [the point $D(N, M)$] is $D(6, 7) = 2$, as was shown above. The path drawn from $D(N, M)$ to $D(1, 1)$ for which the value of $D(n, m)$ is monotonically decreasing corresponds to the optimum edit sequence.

Figure 1 shows how the individual moves through the grid correspond to the edit operations described above. Horizontal motions in the grid correspond to deleting portions of the reference pattern, and always have a cost associated with them. Likewise, vertical motions correspond to deletion of the test pattern with the associated costs. Motions along a diagonal can either result in no cost, if the corresponding features match ($R(n) = T(m)$), or a cost of 1, if they are different.

Since $D(n, m)$ (the running total cost) depends only on $d(n, m)$ (the cheapest next move) and the $D(n, m)$ values to its left (the total matching cost so far), the path is locally constrained to follow one of the three paths shown in Figure 1. Since these motions correspond to the edit operations, the matching problem becomes equivalent to a pathfinding problem. If we can find the path through the grid that minimizes the total distance $D(N, M)$, we have found our optimal match. Notice that this path is exactly the warping function $W(n)$ described earlier. Dynamic programming can efficiently determine the optimal path whenever the dynamic programming principle applies:

Whenever the path from a starting point S [$D(1,1)$ here] to an intermediate point I does not influence the optimum choice of paths for traveling from I to a goal point G [$D(N, M)$ here], then the minimum distance from S to G is the sum of the minimum distance from S to I and the distance from I to G

Dynamic programming is efficient because it ignores all paths from S to I other than the minimum distance one. This feature will prove vital when we wish to reduce some of the grid calculations in matchings that involve hundreds or thousands of points.

Dynamic Depth Warping

Now that we have outlined the properties for dynamic matching of patterns that consist of alphabetical characters, we would like to generalize this to the matching of discrete curves. Figure 2 shows an example of a warping path that matches two discrete curves. Each point in the curve has a value associated with it, which for well logs will be the readings from the tool responses. These values now take the place of the alphabetical characters in the word matching problem. Each point in the grid will have a value associated with it that represents the total cost of the match to that point. As in the word matching problem, the program must trace its way back through the grid, touching the points which represent the now monotonically decreasing minimum distance path. As the program traces through the matrix, it saves the discrete local motions in memory, and then uses these motions to draw the corresponding tie lines and reproduce the warping path. Note that in other applications the warping function is interpolated as a continuous curve, whereas in the dynamic depth warping algorithm the path is

always saved as discrete moves. In this way, the user can retrieve information about the correlation from point to point in the global path.

APPENDIX B: Rules in the Geologic Knowledge Base

The following section contains a list of all the rules presently implemented in the COREX knowledge base. The rules do not appear as they do in LISP code, but instead as their English language translations. The rules are divided into three categories:

1. **Lithologic Rules:** Rules fired as COREX performs the initial match of lithologies between the wells. These have three sub-classes: general lithologic rules, scale rules, and shoreline rules.
2. **Structural Rules:** Rules that translate structural information from seismic and dipmeter into dynamic programming constraints.
3. **Distance Metric Rules:** A miscellaneous category of general correlation rules which concentrate mostly on modifying the local distance metric.

Lithologic Rules

If there is no other information about the depositional environment,
Then shales are the most continuous units, and sands are the next most continuous, and limestones the next.

If there is no other information about the depositional environment,
Then "thick" units are more continuous than "thin" units.

If a lithology is designated as a marker bed,
Then it will be the most continuous unit in the area, and it will be assigned a "continuity" ranking of 1, and be assigned a matching cost of zero.

If a unit is known to cover a large geographical area, and it is present in both of the wells,
Then it will be ranked high in continuity.

If two units are the equal in lithology and thickness,
Then their matching distance is proportional to their ranking of continuity in the environment.

If two units match in lithology, but not in thickness,
Then their matching distance is proportional to .7 times their continuity.

If two units do not match in thickness or lithology,
Then their matching distance is the maximum of 100.

If we are in a braided stream environment,
Then thick-sands are the most continuous units, and set the thin-shale matching distance to 100.

Scale Rules

If a particular unit is less than an arbitrary thickness,
Then it is designated a "thin" bed.

If a particular unit is greater than an arbitrary thickness,
Then it is designated a "thick" bed.

If we are in a braided stream environment, and the section is greater than an arbitrary thickness,
Then thick-shales cap vertical sequences, and thick-shales should be marker beds.

If we are in a meandering stream environment, and the vertical section is greater than an arbitrary thickness,
Then we should expect insertions and deletions of units.

Shoreline Rules

If correlation is in a braided stream, meandering stream, beach, delta, slope, or shelf environment,
Then ask the user for shoreline trends and invoke the shoreline rules.

If we are in a meandering stream environment, and the strike of correlation is perpendicular to shoreline, and the well spacing is less than 10 km,
Then make sands the most continuous units.

If the well spacing is greater than 10 km, and the paleostrike is parallel,
Then assign sand-sand matching distance to 100.

If we are in a beach environment, and the strike of the wells is parallel,
Then sands are the most continuous units. Else if the well spacing is greater than 2 km, Then sands are the least continuous units.

If we are in a reef environment, and the strike of the wells is parallel to shoreline,
Then limestones are the most continuous units. Else if the strike is perpendicular and the well spacing is greater than 10 km, Then limestones are the least continuous units.

If we are in a continental shelf environment, and the strike of the wells is perpendicular,
Then allow sandy-shales to correlate with shaly-sands of the same dimension, and allow conglomerates to correlate with sands of the same dimension.

If we are in a continental slope environment, and the strike of the wells is perpendicular,
Then allow thin units to correlate with thick units of the same lithology down the slope, and allow sandy-shales to correlate with shaly-sands down the slope.

If we are in a deltaic environment, and the strike of the wells is parallel, and the well spacing is less than 15 km,
Then sands are the most continuous units. Else if the well spacing is greater than 20 km, Then set sand-sand matching distance to 100.

If we are in a deltaic environment, and the strike of the wells is perpendicular to shore,
Then allow thin units to correlate with thick units of the same lithology down the slope, and allow sandy-shales to correlate with shaly-sands down the slope.

If we are in a deltaic environment, and there is coal in both wells,
Then designate coal as a marker bed.

Structural Rules

If the section in well 1 is at least 1.5 times as thick as the section in well 2,
Then limit the warping path to the lower section of the global path.

If two regions are forced to match by the interpreter,
Then force the warping path to go through the region.

If two points are tied from the lithologic match,
Then impose a tie point with an error on each side equal to twice the minimum resolv-

able bed thickness.

If there is a normal fault between the two wells, and it has an offset of x meters,
Then start the warping (x - an arbitrary number) meters in the downthrown well.

If there is structural dip between the wells of δ degrees, and the well separation is d meters,
Then start the warping path ($d \sin \delta$) meters in the down-dip well.

If we expect insertions and deletions of rock units,
Then use a weighted type I continuity constraint.

Distance Metric Rules

If the two points are separated by a distance greater than the maximum allowable shift,
Then assign a maximum matching cost of 100.

If the matching cost is below the threshold of the smoothing operator,
Then assign the points a matching cost of zero.

If log A is determined to be beyond the noise threshold,
Then weight it by zero in the local distance measure.

If the caliper log at depth D shows a spike,
Then assign a very high matching distance at that depth.

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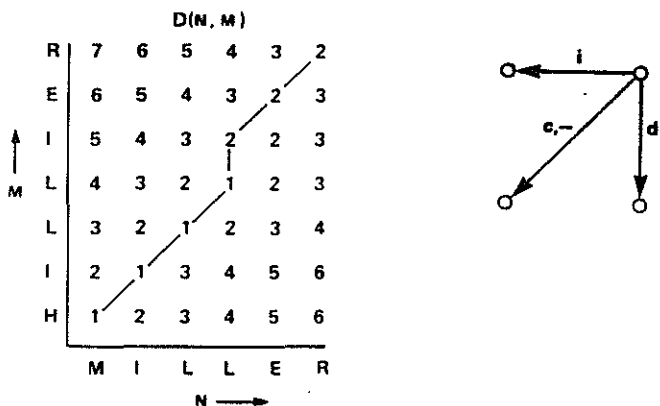


Figure 1: String matching as a path finding problem. Matching two character sets is equivalent to finding the minimum-cost path through a matrix. (a) Each value in the grid represents the total cost of matching the words to that point. At any point in the match, the warping path is restricted to one of the three motions shown in (b). Each of the three possible motions - horizontal, vertical, or diagonal - corresponds to the three possible edit operations - insertion, deletion, or substitution. (After Anderson and Gaby, 1983)

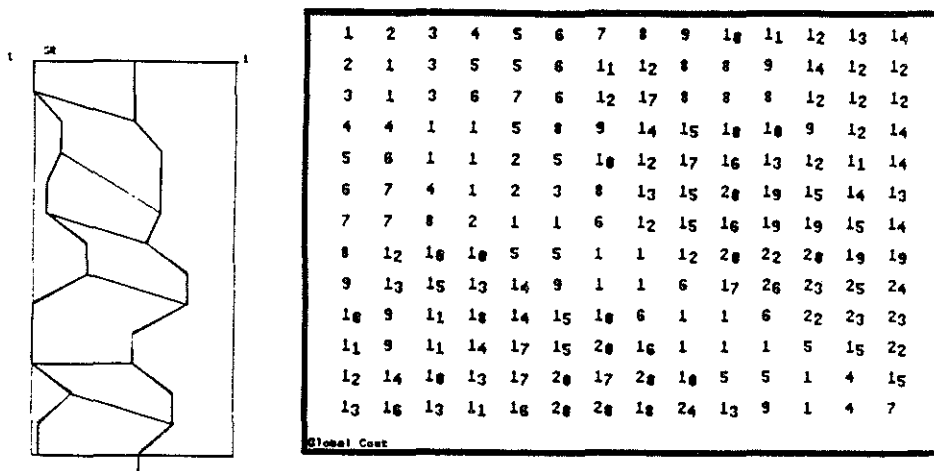


Figure 2: Discrete warping as path finding. The same dynamic programming principles apply as in the string matching problem, but the characters are replaced by discrete values from a digitized well log. The same path restrictions apply, representing insertion, deletion, and substitution, as shown in Figure 1.

SOME GEOLOGIC SITUATIONS AND THEIR CORRESPONDING WARPING PATHS

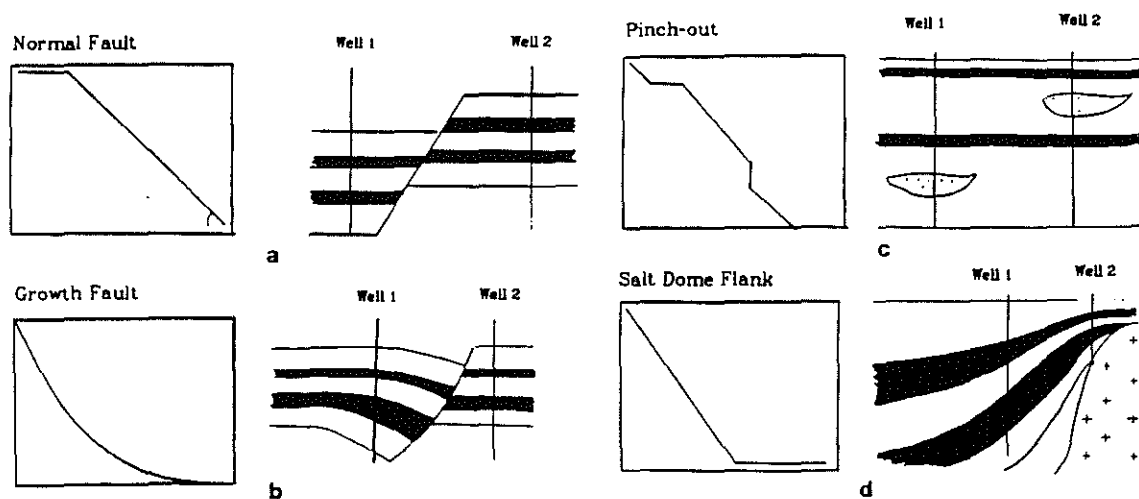


Figure 3: Geological expression in the warping path. Some common geological settings and the corresponding depth warping paths that would result from a correct correlation across the feature. (a) Simple offset, resulting from a normal fault. (b) Non-linear stretching with depth, from a growth fault. (c) Pinch-outs, from isolated sand lenses. (d) Linear stretching, from flanks of a salt dome.

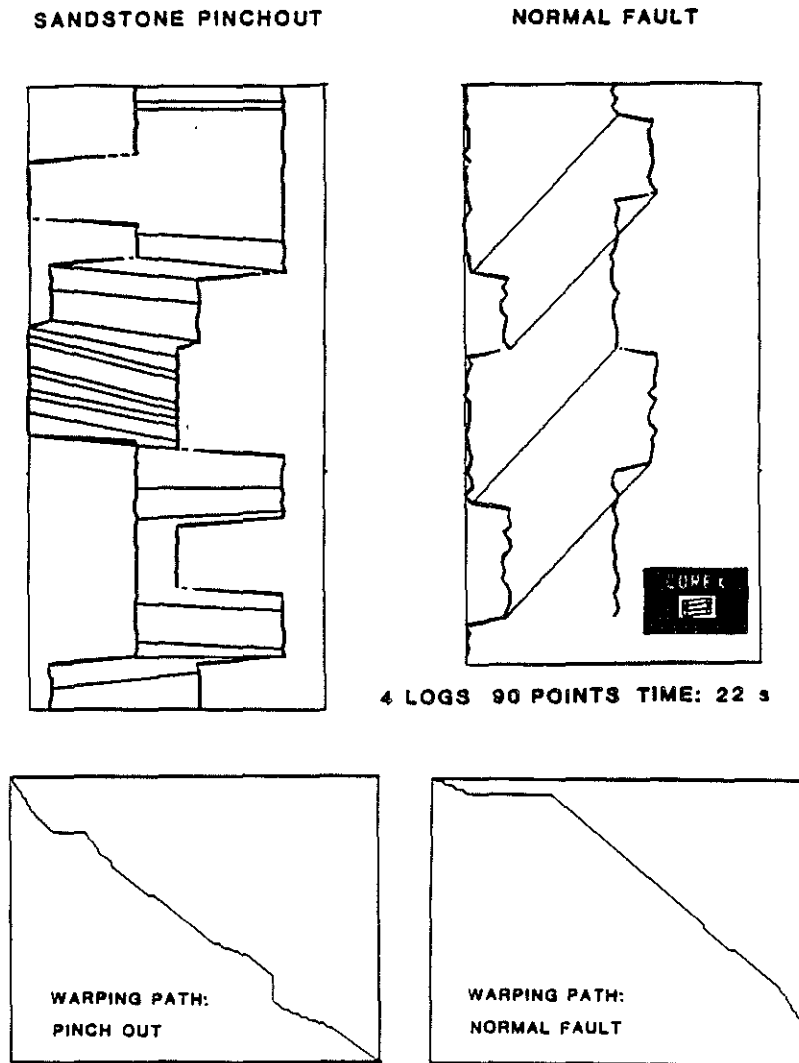


Figure 4: Results of COREX correlation on synthetic examples of geologic structure. The upper right shows correlations across a normal fault with simple depth shift between corresponding points, and the upper left shows correlation where sandstone units pinch-out between wells, and must be deleted or inserted from one well for a correct correlation. The correct warping paths are shown beneath each correlation. Note the correspondence to the theoretical diagrams of Figure 3, in the presence of noise which we deliberately added to the synthetic logs.

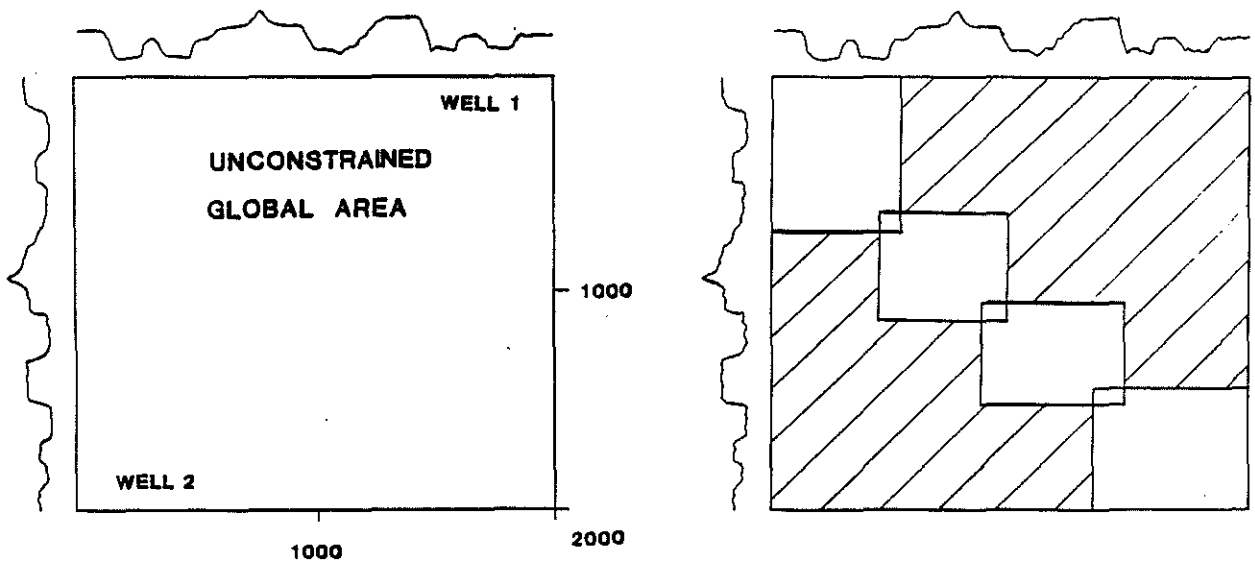


Figure 5: Output from the COREX program after the initial match between two wells shows how tie points input from the knowledge base effectively limit the scope of the warping calculations.

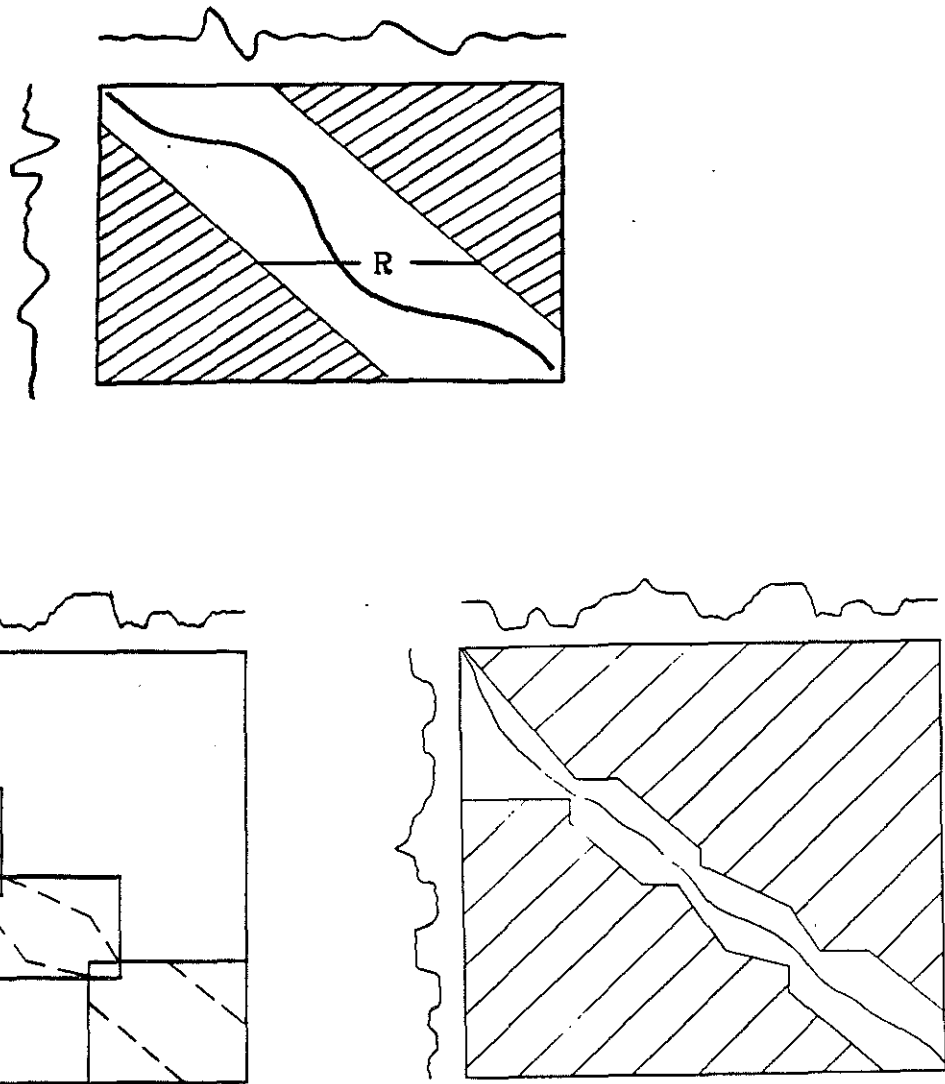


Figure 6: Limiting the global area with maximum shift constraints. Two points separated by a distance greater than the maximum shift (R) are automatically assigned high matching costs, eliminating them as possible points for the warping path. The lower half of the figure shows the result of superimposing range constraints with the tie point constraints of Figure 5.

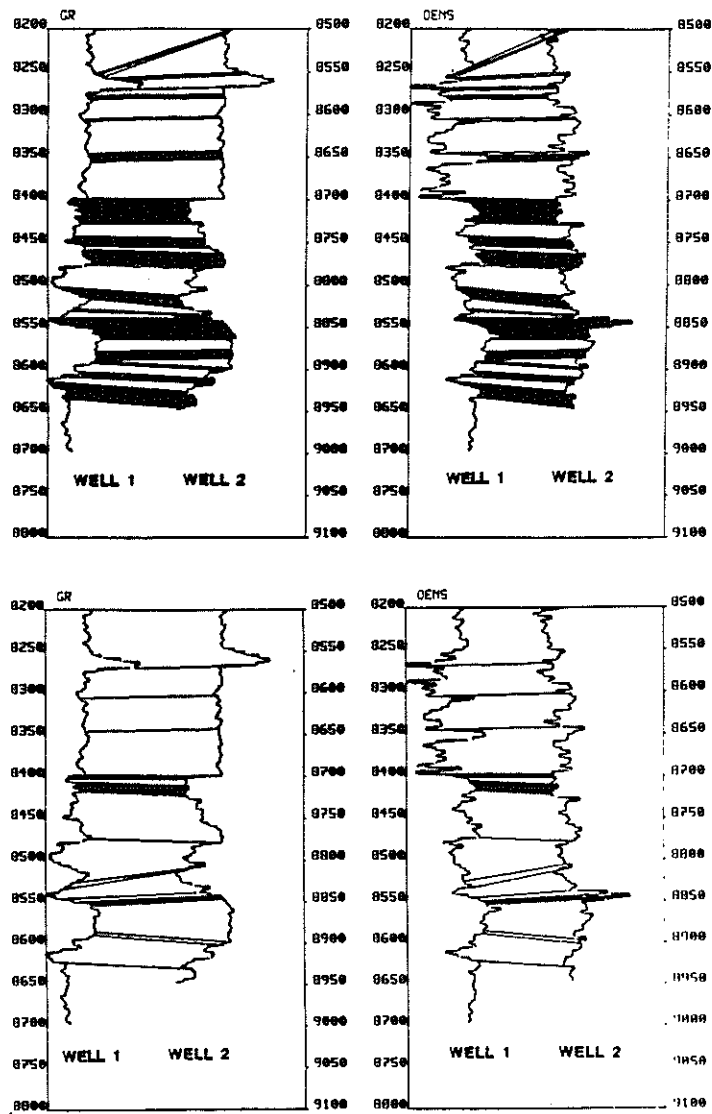
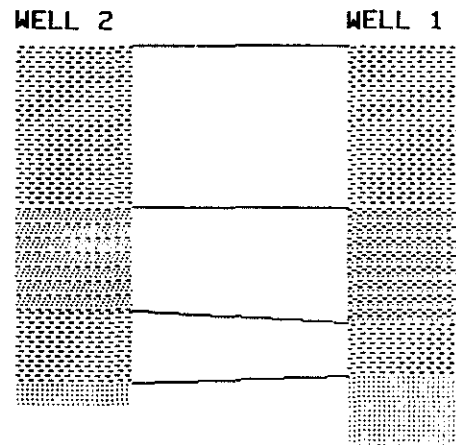


Figure 7: Improved correlation from noise analysis. In part (a), correlation proceeded using all logs giving poor results. After noise analysis, the density and neutron logs were omitted from the calculation, resulting in the improved correlation shown in part (b).

8	100	200	300	400
100	5	105	112	212
200	105	85	185	212
300	119	185	95	195
400	219	219	195	110



WARPING FUNCTION FOR WELLS:

Well 1 Well 2

- SAND
- SHALE
- SHALY-SAND
- SANDY-SHALE

Figure 8: Results of the initial lithologic match for the West African wells. Notice in the correlation and also in the cost matrix that a sandy-shale section is correlated with a shaly-sand section. This was allowed because the orientation of the wells was perpendicular to the shoreline, and shaliness is expected to increase down the paleoslope.

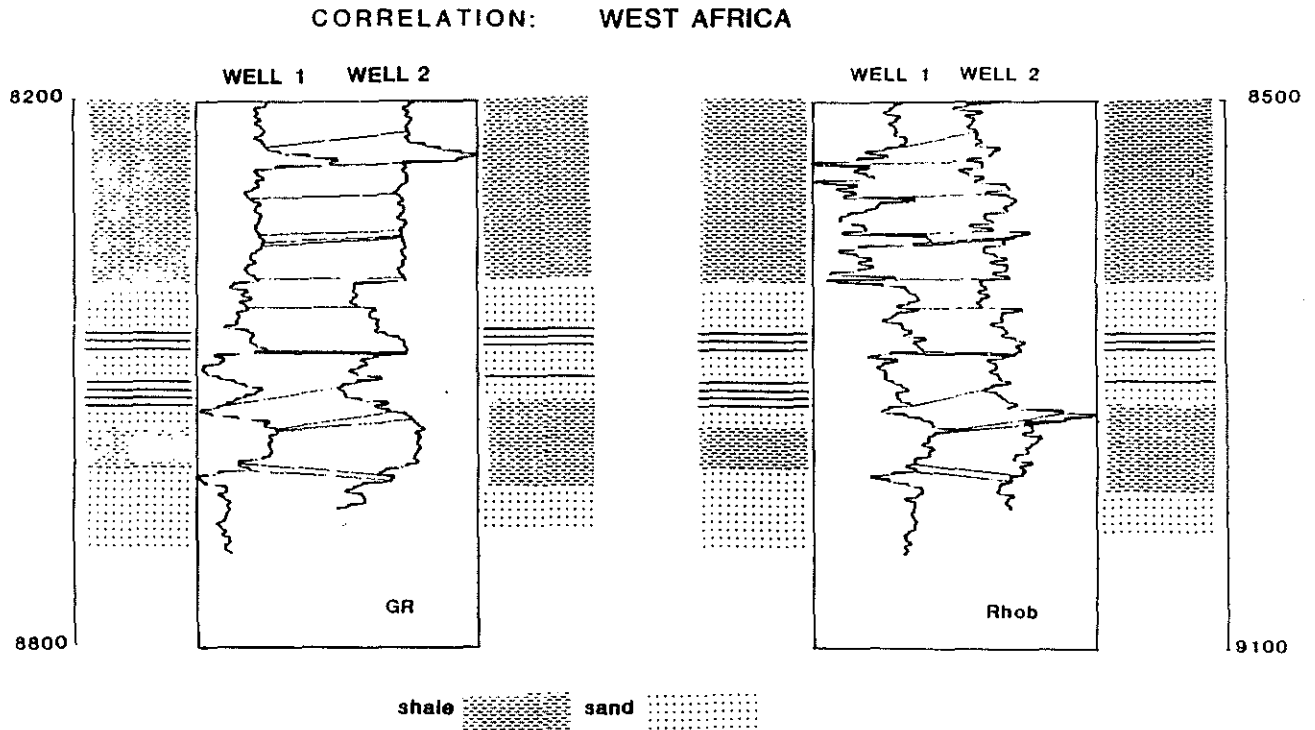


Figure 9: Final correlation for West African wells displaying gamma ray and bulk density logs, with lithology inversion displayed next to the logs. Note the successful matching of the lithologic boundaries, and the expansion and contraction of various sands and shales in the lower section of the wells. The advantage of a multiple-log correlation shows in the display of the various logs. There appears to be correlation of meaningless features in the thick upper shale section of the gamma ray log, but the density log reveals that lithology changes are taking place, even though they are not reflected in the gamma ray log.

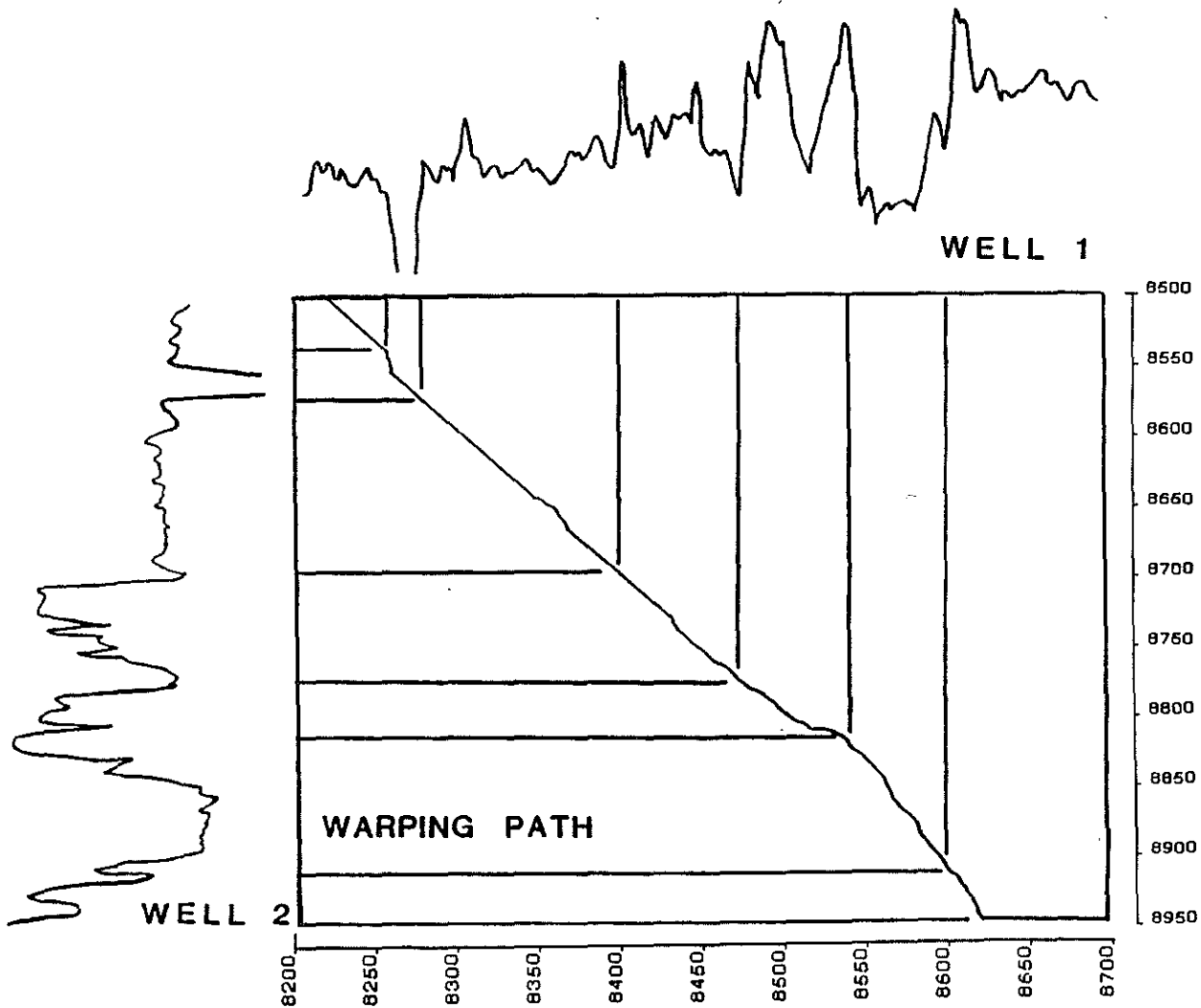


Figure 10: The warping path which matched the West African wells. Note the reflection of the geologic features in the warping path: 1) Slight expansion of the highly radioactive zone (around 8260 feet) from well 1 relative to well 2 shows up as a vertical line in the path; (2) Expansion of a sandy-zone at 8525 from well 2 relative to well 1 shows up as a horizontal section in the warping path; (3) The superposition of a curved section onto a rough 45 degree diagonal, representing the fact that the sections correlate well with depth at the beginning and end points (a diagonal line), but that non-linear stretching is required in a few areas (the curved sections). Note the similarity with parts (a) and (c) of Figure 3.

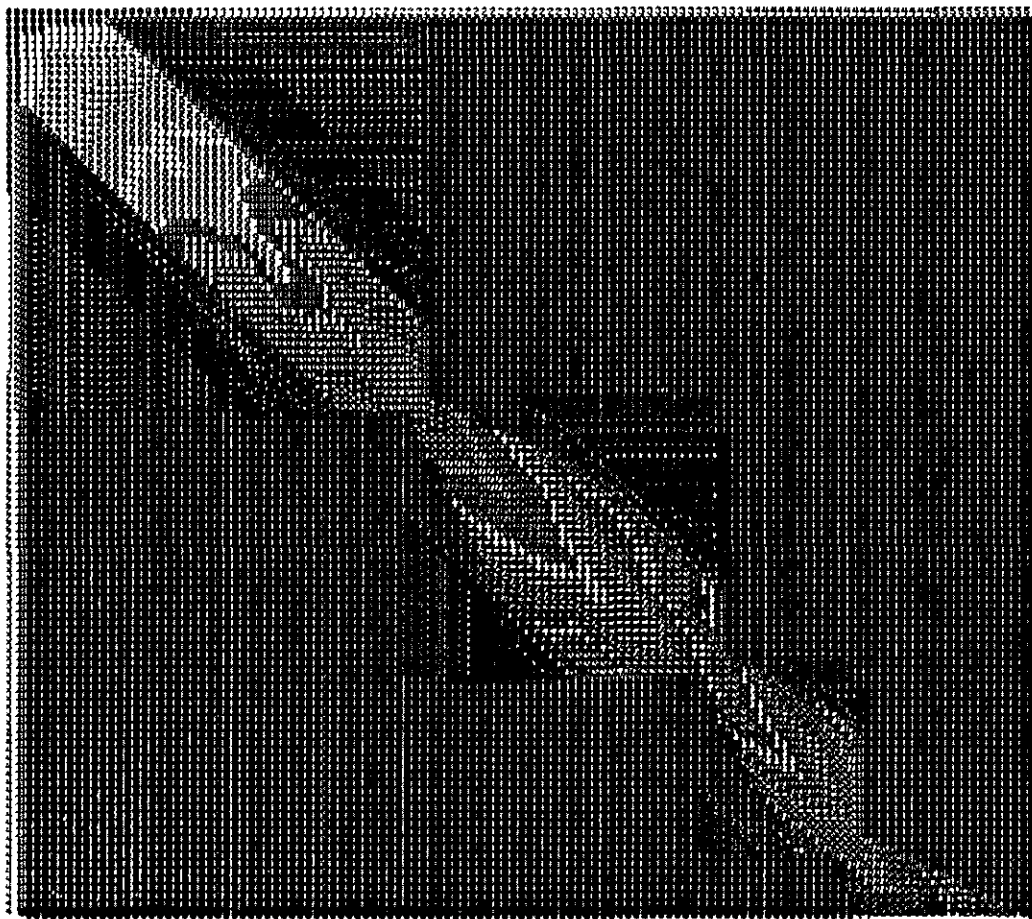


Figure 11: The global cost matrix that resulted from the correlation of the West African wells. Darker regions mean higher cost values. Black areas represent portions of the global area that were eliminated by tie lines and maximum shift constraints. Notice the “propagation” of the lower cost values through the diagonal of the matrix, and how this corresponds with the warping path of Figure 10.

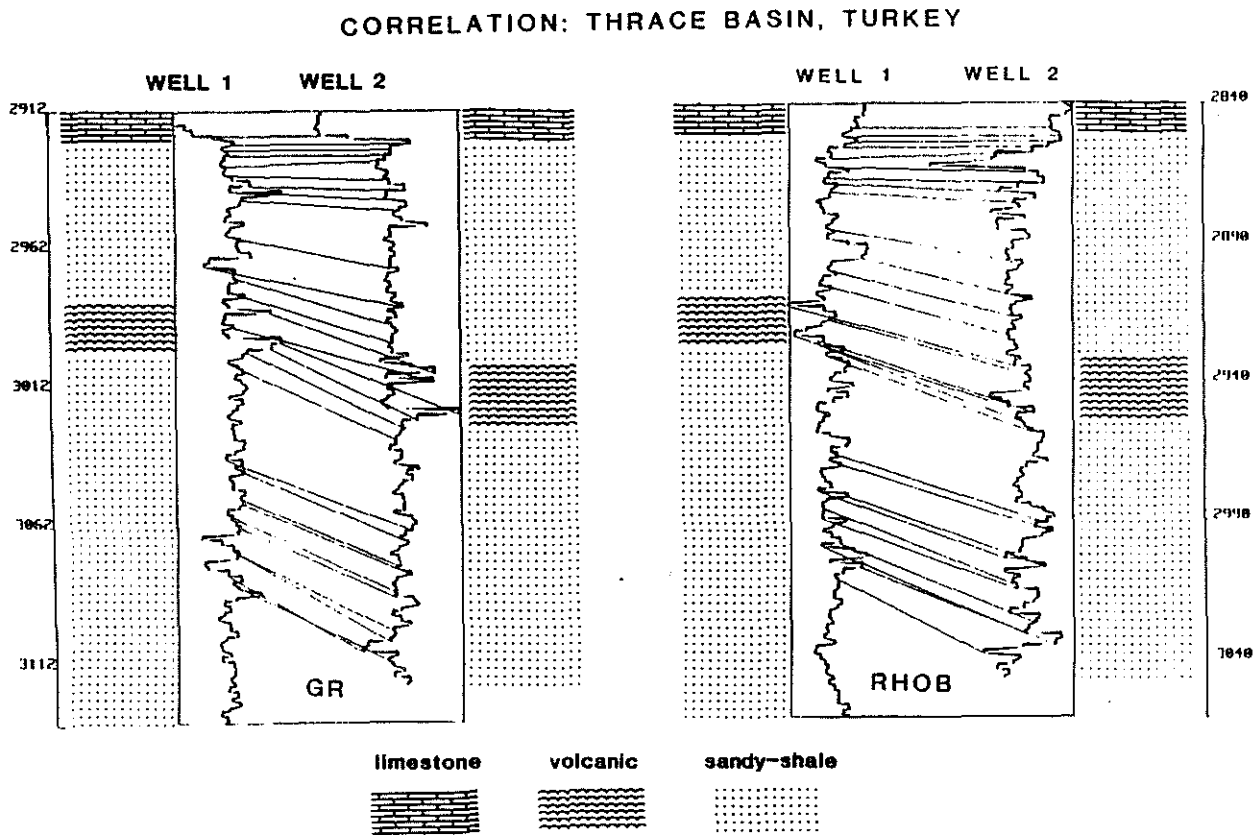


Figure 12: Final correlation from wells in the Thrace Basin, Turkey. No lithologic correlation was performed. Notice the successful match of the formation onset and the section of volcanic tuffs in each well. The tie lines display some non-linear stretching of well 1 relative to well 2, which may represent a thickening of section down the correlation line.

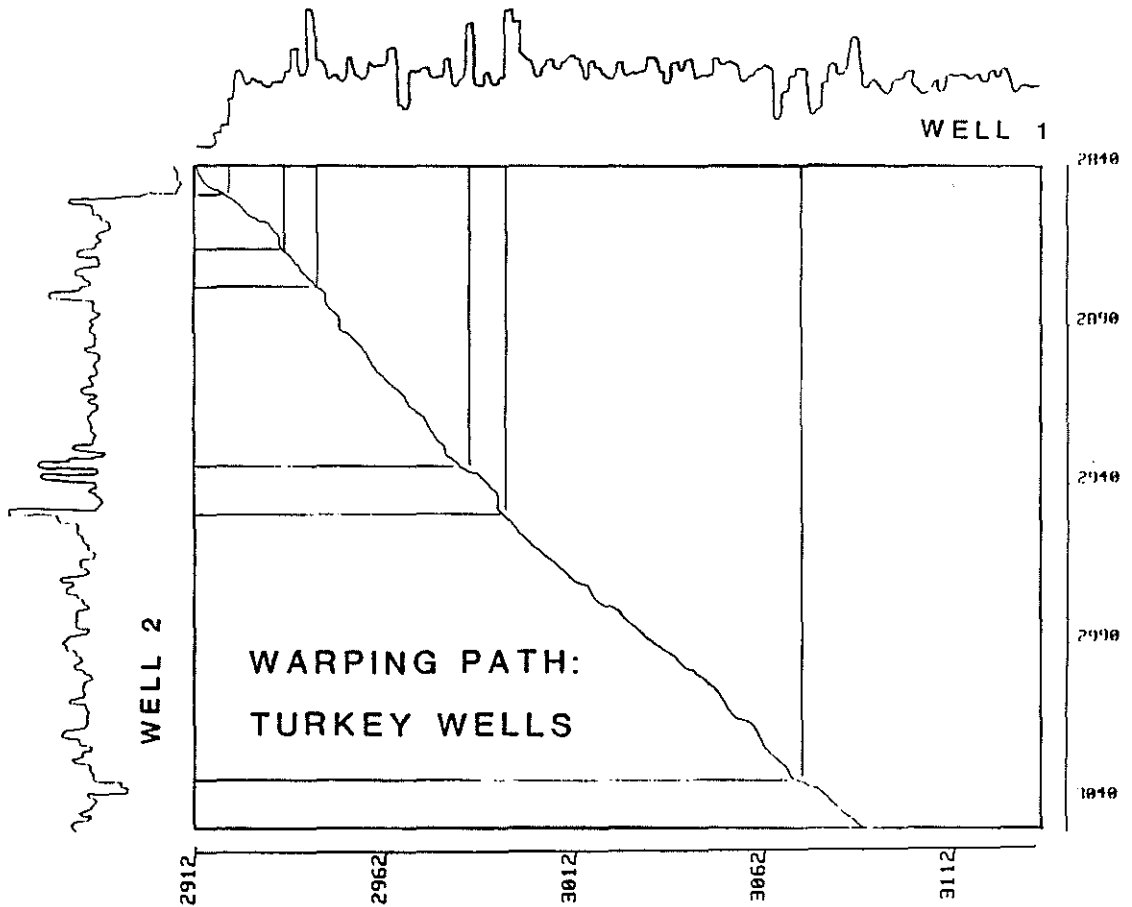


Figure 13: Warping path which matched the wells in the Thrace Basin. Notice the superposition of the different geometric properties on the warping path, and how these correspond to the theoretical paths discussed in Figure 3. (1) A diagonal path toward the bottom of the matrix represents the constant stretching of section in well 1 relative to well 2; and (2) a curved portion is superimposed on this general trend to account for the non-linear stretching with depth that occurs from well 1 to well 2, as is shown in the tie lines of Figure 12. Note the similarity with parts (b) and (d) of Figure 3.

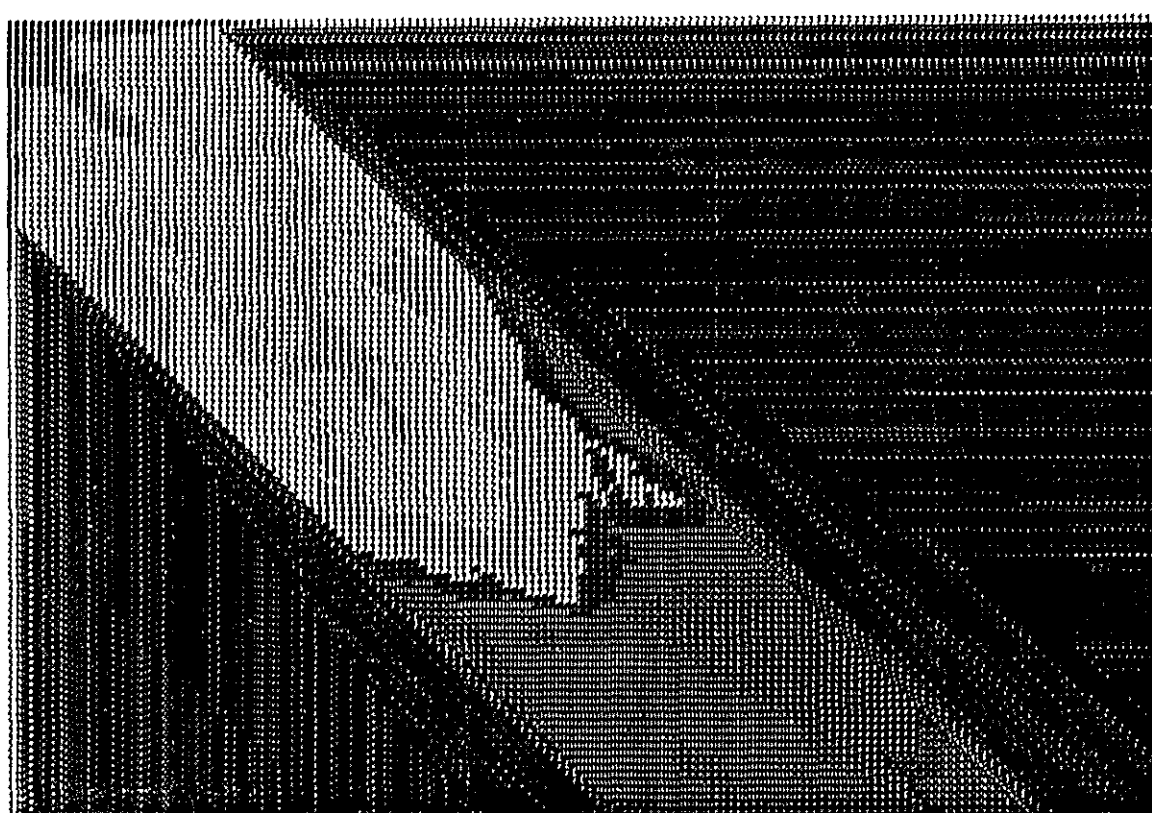


Figure 14: Global cost matrix for Thrace Basin correlation. Darker areas represent high matching costs. Black areas are sections that were eliminated from calculation by a maximum shift constraint. Compare the minimum distance areas with the warping path seen in Figure 13.