Estimating Phase Velocity and Attenuation of Guided Waves in Acoustic Logging Data

by

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ABSTRACT

Phase velocity and attenuation of guided waves have been estimated from multireceiver, full waveform, acoustic logging data using the extended Prony's method. Since a formation affects velocity and attenuation, estimating these quantities is important in evaluating the formation properties. The estimation is performed using an array processing technique which requires two steps: (1) the traces for all receivers are transformed into the frequency domain, and (2) for each frequency the extended Prony's method is used to determine the presence of a guided wave propagating past the array of receivers. The guided wave properties estimated by the Prony's method include amplitude, attenuation, and phase change which is related to phase velocity. An important assumption in this array processing technique is that the formation, borehole fluid, and tool are homogeneous along the receiving array. For synthetic data, the phase velocities and attenuation of the tube wave and two modes of the pseudo-Rayleigh wave are accurately estimated over many frequencies, with the exception that the low amplitude of the second mode causes its attenuation estimate to be somewhat less accurate. For laboratory data, very good estimates of the phase velocities of the tube wave and three modes of the pseudo-Rayleigh wave are obtained. Since the materials used in the laboratory experiment had very large quality factors, the attenuation could not be estimated. For field data, the dispersion of the tube wave and the velocity of the pseudo-Rayleigh wave at its cutoff are very close to those predicted by another, independent method. Accurate attenuation estimates could not be made because the data are noisy and consist of only eight traces.

INTRODUCTION

In acoustic logging, guided waves are generated which propagate parallel to the axis of the borehole, and the properties of the formation partly affect the characteristics of these waves. Because some acoustic logging tools now have eight or more receivers, array processing methods can be used to calculate the phase velocity and attenuation of these waves. This information is crucial when estimating formation properties like permeability, which affects the tube wave velocity and attenuation (Burns and Cheng; 1986, Williams et al., 1984; Staal and Robinson, 1977); anisotropy, which affects the guided wave dispersion (White and Tongtaow, 1981); and S-wave velocity, which cannot be directly measured in slow formations but can be estimated from the tube wave velocity (Stevens and Day, 1986; Chen and Willen, 1984; Cheng and Toksöz, 1983).

The first array processing of acoustic logging data was done by Schoenberg et al. (1981), who tried to estimate the frequency-wavenumber spectrum. By using fast Fourier transforms in conjunction with the maximum-likelihood method, they were able to identify the tube and pseudo-Rayleigh waves in the two-dimensional spectrum, but these estimates had poor resolution. To overcome this problem, Parks et al. (1983) and McClellan (1986) used the extended Prony's method to obtain a high resolution spectrum from which accurate dispersion curves for the guided waves were calculated. Lang et al. (1985) implemented a variation of the Prony's method to make it more robust, but their approach is not appropriate when trying to estimate the attenuation. Other methods of array processing like high resolution slant stacking and semblance can be used to obtain the velocities of the refracted P and S waves and tube wave (Block et al., 1986; Hsu and Baggeroer, 1986) but not highly dispersive waves like the pseudo-Rayleigh wave.

The purpose of this paper is to demonstrate the applicability of the extended Prony's method for estimating phase velocity and attenuation of the borehole-guided waves. Although determining wave attenuation is an inherent feature of this method, a modification which gives a more robust estimate has been developed and is presented here. The appropriateness of the Prony's method to modelling the guided waves is discussed in mathematical terms and then demonstrated by estimating phase velocity and attenuation from synthetic, laboratory, and field data. The accuracy of the results is verified by comparing them to either theoretical predictions or results from the slant stack method.

METHOD

Array processing of acoustic logging data using the extended Prony's method requires two steps. First is the application of the fast Fourier transform over time. Second is the estimation of each wave properties using the extended Prony's method.

This method is based upon the assumption that the Fourier transform of the trace at receiver n may be represented by a finite sum of damped exponentials:

$$s(n,\omega_i) = \sum_{l=1}^{p} A_l(\omega_i) e^{(-\alpha_l(\omega_i)+jk_l(\omega_i))(n-1)\Delta z + j\theta_l(\omega_i)} \quad . \tag{1}$$

This expression is valid for $1 \leq n \leq N$, where N is the total number of receivers. The number, p, is called the model order and must be less than or equal to $N/2$. The spacing between the receivers is Δz. The meaning of the other variables will be clear if only one term in the summation for the last receiver is examined:

$$Ae^{j\theta} e^{(-\alpha+jk)z} \quad . \tag{2}$$

The frequency, ω_i, and index, l, have been omitted for convenience, and z is the length of the array. As the guided wave passes the first receiver in the array, its amplitude is A, and phase θ. As the wave passes the last receiver, its amplitude has diminished by $e^{-\alpha z}$, and its phase changed by kz. The attenuation coefficient, which will be called just attenuation, is α, and the phase velocity is determined via the relation $c = \omega/k$. The sum of p terms like that in equation 2 gives the value of the Fourier transform of the last trace at frequency ω_i.

This approach to the estimation of the exponential parameters is similar to that used for typical geophysical inverse problems in which an underlying model is assumed, and the parameters, which describe the behavior of that model, are altered until they accurately predict the observed data. For this array processing application, the model is a sum of p damped exponentials each of which is described by its amplitude, phase, attenuation, and wavenumber (Equation 1). These parameters are selected to make the sum of the exponential terms closely match the data, $s(\omega_i, n)$. How these parameters are estimated is discussed in the Appendix.

An important assumption underlying this representation of the guided waves is that the formation, borehole fluid, and tool are homogeneous along the length of the array. This method would not be suitable when, for example, the receiving array straddles a large fracture or a wash-out.

In practice, the attenuation is calculated using the amplitude of the wave because the estimation of this parameter is much more robust than that of α. This calculation begins by estimating the amplitude, A, using all of the traces in the array. Next one or more traces are deleted, and the amplitude is estimated again giving A'. Finally, the attenuation is calculated via the relation (Aki and Richards, 1980):

$$\frac{A'}{A} = e^{-\alpha z'} \tag{3}$$

where z' is the distance over which the traces were deleted.

An important issue is the appropriateness of the damped exponential model to array processing of acoustic logging data. To address this issue, consider the frequency-wavenumber Fourier transform for the pressure at the center of the borehole (Tsang and Rader, 1979):

$$s(z,t) = \int_{-\infty}^{\infty} d\omega X(\omega) e^{j\omega t} \int_{-\infty}^{\infty} dk B(k,\omega) e^{-jkz} \tag{4}$$

where $X(\omega)$ is the spectrum for a point source, k the wavenumber in the z direction, and $B(k,\omega)$ the frequency wavenumber response of the borehole and formation. After performing a Fourier transform over time, this equation becomes:

$$s(z,\omega) = X(\omega) \int_{-\infty}^{\infty} dk B(k,\omega) e^{-jkz} \quad . \tag{5}$$

The straightforward evaluation of this integral would require integration along the real wavenumber axis. An alternative approach involves contour integration in the complex wavenumber plane. The contributions to $s(z,\omega)$ will come from the poles which are enclosed by the contour and the branch-line integrals (Peterson, 1974). For this approach the integral may be written:

$$s(z,\omega) = 2\pi j \sum_{l} X(\omega) \left[\text{ Residue of } B(k,\omega) e^{jkz} \text{ at pole } k_l \right] - \int_{bl} \quad . \tag{6}$$

The notation, $\int_{bl}$, indicates the contributions which come from the branch-line integrals. The discrete form of this equation is obtained by replacing z by $\Delta z(n-1)$ and ω by ω_i:

$$s(n,\omega_i) = \sum_{l} \left[2\pi j X(\omega_i) \left(\text{ Residue of } B(k,\omega_i) \text{ at pole } k_l \right) \right] e^{jk_l(n-1)\Delta z} - \int_{bl} \quad . \tag{7}$$

Comparing this expression with equation 1 shows that (1) the terms within the brackets involving the source spectrum and the residue may be identified with $A_l(\omega_i) e^{j\theta_l(\omega_i)}$ and (2) the pole, k_l, with $\alpha_l(\omega_i) + jk_l(\omega_i)$. These poles correspond to the guided waves, and so the damped exponential model will represent these waves well. Lang et al. (1985) considered the applicability of an undamped sinusoidal model which is not appropriate when trying to estimate attenuation. The contributions from the branch-line integrals are generally not well modeled by the extended Prony's method.

RESULTS

Array processing using the extended Prony's method was applied to synthetic, laboratory, and field data to estimate phase velocity and attenuation over a broad range of frequencies. The results are shown in the figures as solid lines and are compared to theoretical predictions which are the dotted lines. The tube wave is labelled "T", the first mode of the pseudo-Rayleigh wave "R1", the second mode "R2", etc.

The synthetic data, which were calculated for a hard formation, consist of twelve traces (Figure 1). The model order (Equation 1) was selected to be six, half the number of traces. The estimated dispersion curves for the tube and pseudo-Rayleigh waves are compared to the theoretical curves in Figure 2. With only twelve traces, the best estimates for the Prony parameters occur either when only one wave is present or when one wave has a very large amplitude relative to the other waves. Otherwise, the estimates for the velocity and attenuation are noisy and must be edited. For this reason, the dispersion curves for the different waves do not overlap. The estimated attenuation curves are compared to the theoretical predictions in Figure 3. The estimated and predicted curves for the tube wave and first mode of the pseudo-Rayleigh wave are reasonably similar. However, the estimated attenuation curve for the second mode is noisy because this wave amplitude is relatively low. The estimates of phase velocity and especially the attenuation would improve with more traces.

The laboratory data, which consist of twenty-two traces (Figure 4), were collected in a water-filled borehole through an aluminum block. The estimated dispersion curves, which were calculated using a model order (Equation 1) of ten, are shown in Figure 5. The curves are confined to three frequency bands which result from the frequency characteristics of the source. The theoretical dispersion curves were computed by assuming that the aluminum and water are perfectly elastic (which is reasonable because they have very large quality factors) and using a tabulated value for the aluminum S-wave velocity. Despite these approximations the agreement between the calculated and theoretical curves is very good. The guided waves in this laboratory experiment were virtually unattenuated, and consequently this parameter could not be estimated.

The field data (Figure 6), which consist of eight traces, were collected in a limestone formation, in which the invaded zone is believed to have significant mechanical damage (Blackway, 1987, personal communication). The estimated dispersion curves, which were calculated using a model order of four, are shown in Figure 7. Despite the poor quality of the field data and the small number of traces, apparently accurate dispersion curves were obtained. The dispersion of the tube wave and the velocity of the pseudo-Rayleigh wave near its cutoff are very close to those predicted by an independent method. Good attenuation estimates could not be made because the data are noisy and the array has few receivers.

DISCUSSION

The phase velocity estimates are very robust. Accurate dispersion curves are obtained even when the data are noisy and the array has few receivers. The estimates can be improved by isolating a particular guided wave using a window in the time domain and then determining its Prony parameters. This process is done for each of the guided waves. Experience with several data sets suggests that the minimum number of receivers necessary for good estimates is about eight.

The accuracy of the attenuation estimates is primarily influenced by noise which includes mismatched receivers and formation heterogeneity. The best method of ameliorating the noise effects is to increase the number of receivers making more data available for determining the Prony parameters. The disadvantage of this solution is that the receiving array will lengthen, increasing the likelihood that the formation heterogeneity will degrade the estimates.

CONCLUSIONS

Array processing using the extended Prony's method may be used to estimate the guided wave phase velocity and attenuation over a broad frequency range. The phase velocity estimates are robust, and accurate dispersion curves were obtained for all data sets even for the short array with noisy traces. Although attenuation estimates are less robust, moderately good results were obtained from the synthetic data. Extracting useful attenuation estimates from field data will probably require array data with many receivers and little noise.

ACKNOWLEDGEMENTS

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APPENDIX

The explanation of the extended Prony's method will follow that given by Kay and Marple (1981) and will focus on estimating the parameters for a one-dimensional sequence. Frequency and damping for this sequence are analogous to wavenumber and the negative of the attenuation coefficient for the array data.

A set of N complex data samples, $x(1), ..., x(N)$ is assumed to be approximated by a sum of p complex exponentials:

$$\hat{x}(n) = \sum_{k=1}^{p} A_k e^{(\alpha_k + j2\pi f_k)(n-1)T + j\theta_k} \qquad (A-1)$$

For the extended Prony's method, the number of data samples must be greater than or equal to $2p$. The sampling interval is T, the amplitude A_k, the damping factor α_k, the frequency f_k, and the phase θ_k. A more concise form for this equation would be

$$\hat{x}(n) = \sum_{k=1}^{p} h_k z_k^{n-1} \qquad (A-2)$$

where the complex constants are defined as

$$h_k = A_k e^{j\theta_k} \qquad (A-3)$$

and

$$z_k = e^{(\alpha_k + j2\pi f_k)T} \quad . \qquad (A-4)$$

Finding the parameters which minimize the squared error,

$$\rho = \sum_{n=1}^{N} |x(n) - \hat{x}(n)|^2 \quad . \qquad (A-5)$$

is a difficult nonlinear problem.

An alternative approach to finding the parameters, h_k and z_k, is based upon the Prony's method for which equation A-2 is regarded as the solution to some homogeneous, linear, constant-coefficient, difference equation. To find this equation, a polynomial, $\phi(z)$, is defined such that the p exponents, z_k, are its roots:

$$\phi(z) = \prod_{k=1}^{p} (z - z_k) \quad . \qquad (A-6)$$

After performing the multiplication, $\phi(z)$ may be expressed as a summation:

$$\phi(z) = \sum_{m=0}^{p} a(m) z^{p-m} \quad . \qquad (A-7)$$

where $a(m)$ are complex coefficients with $a(0) = 1$. Equation A-2 has its index shifted by $-m$, is multiplied by $a(m)$, and is summed over the index m to give:

$$\begin{aligned}\sum_{m=0}^{p} a(m)\hat{x}(n-m) &= \sum_{i=0}^{p} h_i \sum_{m=0}^{p} a(m) z_i^{n-m-1} \\ &= \sum_{i=0}^{p} h_i z_i^{n-p} \sum_{m=0}^{p} a(m) z_i^{p-m-1} \\ &= 0 \end{aligned} \qquad \text{(A-8)}$$

which is valid for $p+1 \leq n \leq 2p$. To see that the right hand side is zero, notice $\phi(z)$ is zero when it is evaluated at any of its roots. The previous expression may be written as a recursive difference equation:

$$\hat{x}(n) = -\sum_{m=1}^{p} a(m)\hat{x}(n-m) \qquad (\mathrm{A}-9)$$

The difference between the actual data, $x(n)$, and the approximation, $\hat{x}(n)$, is $e(n)$:

$$x(n) = \hat{x}(n) + e(n) \qquad (\mathrm{A}-10)$$

which is defined for $1 \leq n \leq N$. Substituting equation A-9 yields

$$x(n) = -\sum_{m=1}^{p} a(m)x(n-m) + \sum_{m=0}^{p} a(m)e(n-m) \quad . \qquad (\mathrm{A}-11)$$

The last term is defined as the sum:

$$\epsilon(n) = \sum_{m=0}^{p} a(m)e(n-m) \qquad (\mathrm{A}-12)$$

for $p+1 \leq n \leq N$, which will then give

$$x(n) = -\sum_{m=1}^{p} a(m)x(n-m) + \epsilon(n) \quad . \qquad (\mathrm{A}-13)$$

This equation expresses $x(n)$ as an autoregressive sequence. The term $\epsilon(n)$ is the difference between $x(n)$ and its linear prediction based upon p past data samples. The terms $a(m)$ are linear prediction parameters and are computed by the covariance method which minimizes the squared error $\sum_{n=p+1}^{N} |\epsilon(n)|^2$.

The terms $a(m)$ are used to find the roots of the equation A-7 (i.e. $\phi(z) = 0$). The exponential approximation becomes linear in the remaining unknowns, $h(1), ..., h(p)$ which may be expressed in matrix form as

$$\mathbf{Zh} = \mathbf{x} \qquad (\mathrm{A}-14)$$

where

$$\mathbf{Z} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_p \\ \vdots & \vdots & & \vdots \\ z_1^{N-1} & z_2^{N-1} & \cdots & z_p^{N-1} \end{pmatrix}, \ \mathbf{h} = \begin{pmatrix} h_1 \\ h_2 \\ \cdots \\ h_p \end{pmatrix}, \text{ and } \mathbf{x} = \begin{pmatrix} x(1) \\ x(2) \\ \cdots \\ x(N) \end{pmatrix} \quad . \qquad (A-15)$$

The method of least squares is used to solve this equation.

The use of the extended Prony's method requires three steps. First, the linear prediction parameters are computed using the covariance method (Equation A-13). Second, these parameters are used to characterize the polynomial, $\phi(z)$, (Equation A-7), and the p roots of this polynomial yield the exponents, z_k. Third, the complex constants, h_k, are found by the method of least squares (Equation A-14).

In the presence of significant noise, the extended Prony's method does not perform well, and the damping parameters are often estimated larger than they really are. Using a model order, p, larger than the actual number of damped sinusoids helps to ameliorate this problem.

Using the complex constants, z_i and h_i, the damping, frequency, amplitude, and phase may be determined from these relationships:

$$\alpha_i = \ln |z_i| \quad , \qquad (A-16)$$

$$f_i = \arctan \frac{\left(\frac{\mathrm{Im}(z_i)}{\mathrm{Re}(z_i)}\right)}{2\pi T} \quad , \qquad (A-17)$$

$$A_i = |h_i| \quad , \text{ and} \qquad (A-18)$$

$$\theta_i = \arctan \frac{\mathrm{Im}(h_i)}{\mathrm{Re}(h_i)} \quad . \qquad (A-19)$$

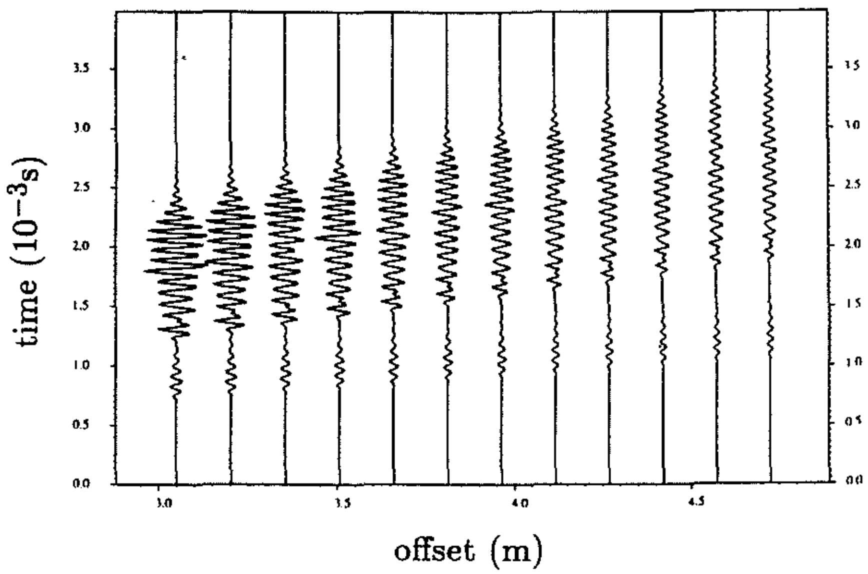

Figure 1: Twelve traces of synthetic, open hole, acoustic logging data (after Tubman, 1984).

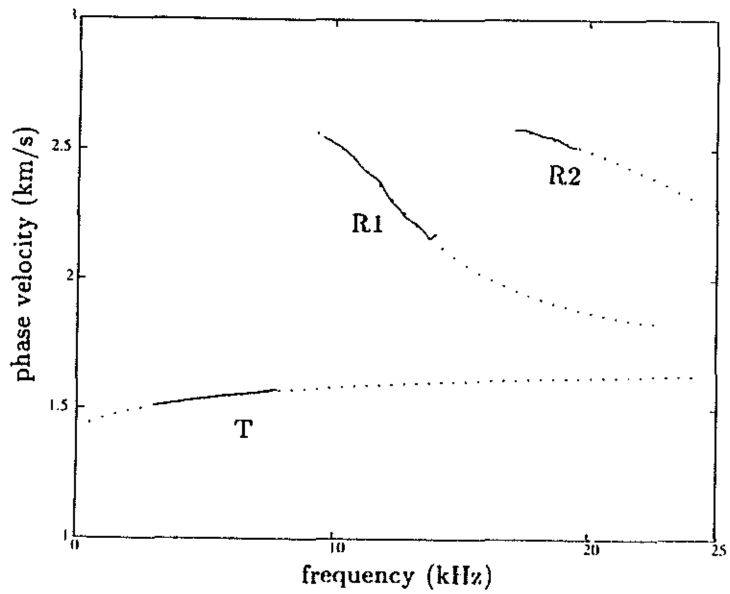

Figure 2: Velocity dispersion curves calculated by the Prony's method from the synthetic data shown in Figure 1 and the theoretically predicted dispersion curves.

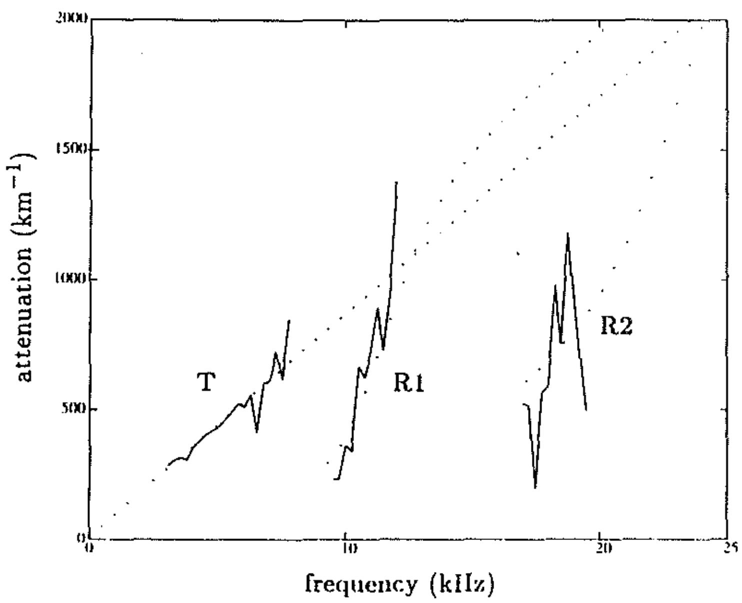

Figure 3: Attenuation curves calculated by the Prony's method from the synthetic data shown in Figure 1 and the theoretically predicted attenuation curves.

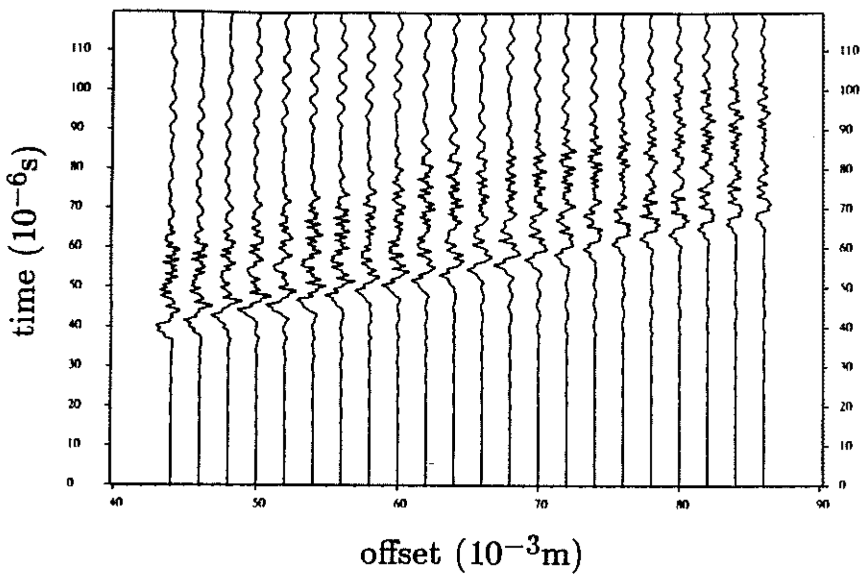

Figure 4: Twenty-two traces of laboratory, acoustic logging data collected in a water-filled borehole through an aluminum block.

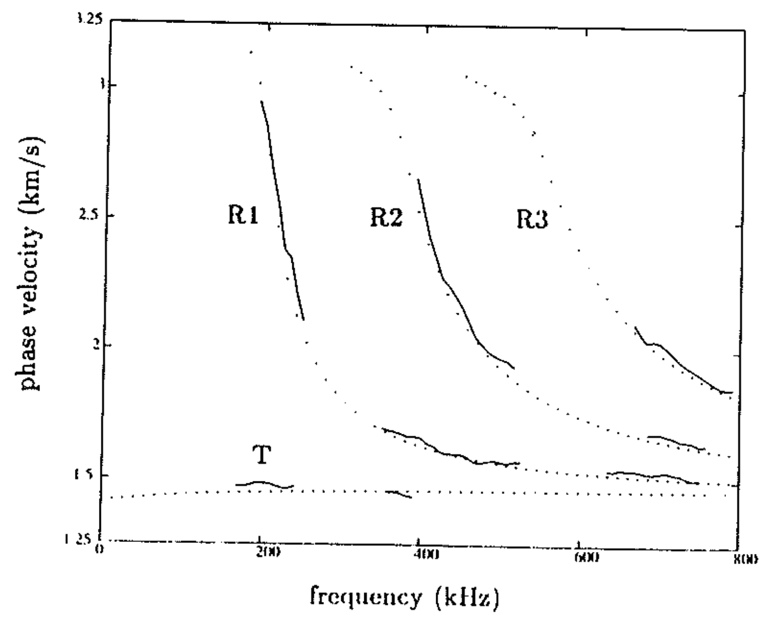

Figure 5: Velocity dispersion curves calculated by the Prony's method from the laboratory data shown in Figure 4 and the theoretically predicted dispersion curves.

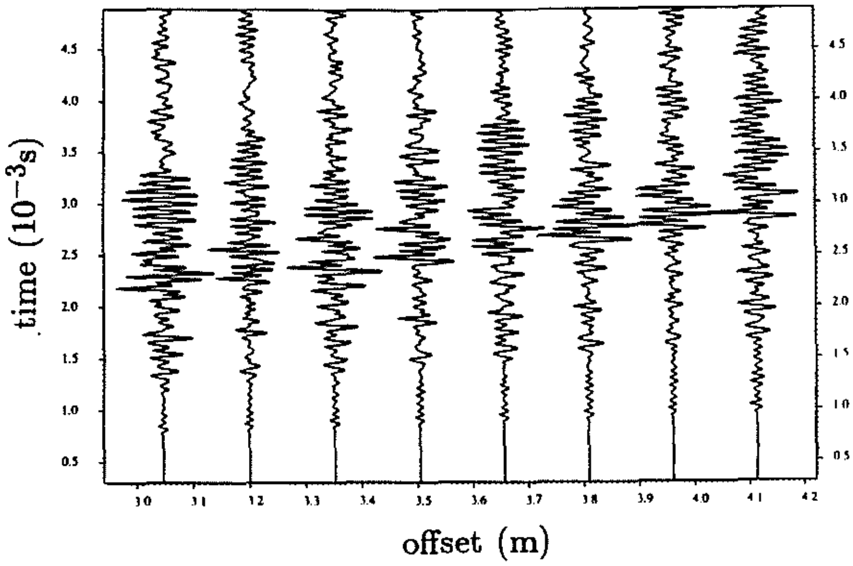

Figure 6: Eight traces of field, acoustic logging data collected in an open borehole in a limestone formation.

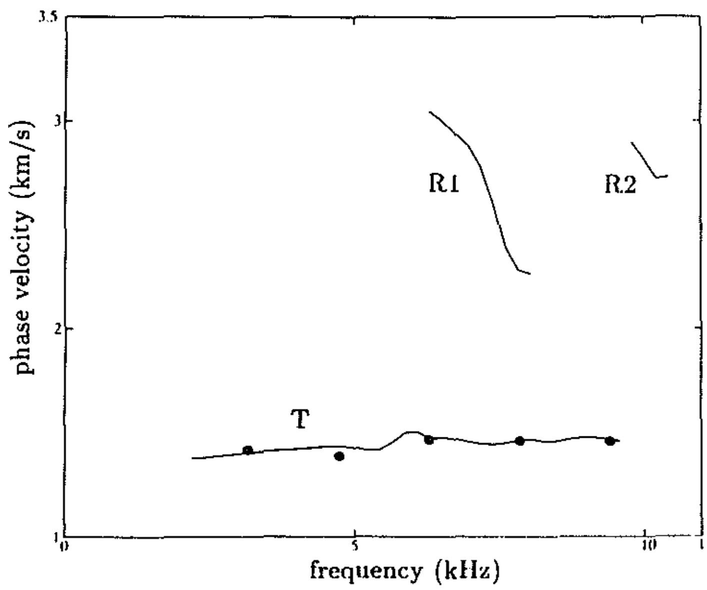

Figure 7: Velocity dispersion curves calculated by the Prony's method from the field data shown in Figure 6. Using the high resolution, slant stack method, tube wave velocities at several frequencies were estimated. These are indicated by the dots. Also, the refracted S-wave velocity was determined to be about 3.0 km/s.