DETECTION OF SUBSURFACE FRACTURES AND PERMEABLE ZONES BY THE ANALYSIS OF TUBE WAVES

W.B. Beydoun, C.H. Cheng and M.N. Toksöz

Earth Resources Laboratory
Department of Earth and Planetary Sciences
Massachusetts Institute of Technology
Cambridge, MA 02139, U.S.A.

ABSTRACT

In Vertical Seismic Profiling tube waves are generated by compressional waves impinging on subsurface fractures or permeable zones. The amplitude of tube waves is dependent upon the formation permeability, the length of the fracture, and on the source frequency. The generation of tube waves is formulated theoretically and the relative effects of these parameters are studied individually. Field examples are shown for open fractures in granite. From tube wave amplitudes normalized to P-wave amplitudes, calculated permeabilities are on the order of 400 millidarcys.

INTRODUCTION

In field experiments with compressional wave Vertical Seismic Profiling (VSP) surveys, it has been observed that tube waves are generated at fractures intersecting the borehole (Huang and Hunter, 1981a,b). Examples of tube waves originating at fractures and propagating up and down the borehole are shown in a VSP section in Figure 1. A possible mechanism for the generation of tube waves is that the incident P-waves compress the fracture and inject a fluid pulse into the borehole. Figure 2 schematically illustrates this efficient mechanism for the generation of tube waves.

The fluid pulse mechanism for the generation of tube waves is formulated mathematically. The geometrical model is that of a fracture intersecting a borehole normal to the borehole axis, with a compressional plane wave impinging normally on the fracture. First, the volume of fluid ejected from the fracture into the borehole is calculated. Then, the tube wave amplitudes generated in the borehole fluid and in the formation are evaluated. The in-situ fracture permeability can then be estimated from the ratio of the tube wave amplitude to the P-wave amplitude measured in the formation or in the fluid using this model.

THEORY OF FRACTURE COMPRESSION AND FLUID FLOW

Consider the specific example of a parallel-walled, fluid-filled fracture with low aspect ratio (width to length ratio $<< 10^{-3}$). The fracture is penetrated normally by an uncased borehole. The fluid in the fracture is in hydrostatic

equilibrium with the fluid in the borehole when there is no perturbation. A P-wave impinges on the fracture at normal incidence. The wavelength of the P-wave is much greater than the fracture width $L(t)$. For very small strains, $L(t)$ is assumed to oscillate about the static shape L_0 as:

$$L(t) = L_0 + \zeta_0 \cos(\omega t) \tag{1}$$

where $2\zeta_0$ is the maximum fracture displacement (Figure 3) and $\zeta_0 \ll L_0$.

To simplify the calculations of the fluid injection from the fracture into the borehole, the following assumptions are taken: (1) One dimensional laminar flow (low Reynold's number) within the fracture, (2) the fluid compressibility is small, (3) the fluid injected into the borehole does not significantly perturb the borehole pressure P_0, (4) low frequency approximation with frequency dependence and (5) the fracture permeability, K, does not vary with time.

Two-dimensional Model

The two-dimensional problem, where the fracture is infinitely long in the y direction, will be solved first. The fluid flow rate in the presence of a pressure gradient $\partial p(x,t)/\partial x$ is related to the fracture width $L(t)$, the fluid viscosity μ, and the fracture permeability K by Darcy's law:

$$q(x,t) = -\frac{KL(t)}{\mu}\ \partial p(x,t)/\partial x \tag{2}$$

The volume of fluid ejected from the fracture into the borehole during the one-half cycle of the incident wave (see Appendix) is:

$$\Delta V_{2D}(K) = -\zeta_0 \left[\frac{K}{\pi\gamma\mu}\right]^{\frac{1}{2}} F(\omega, \zeta_0/L_0) \tag{3}$$

where:

$$F(\omega, \zeta_0/L_0) \approx 2\omega \int_0^{T/2} (T/2-t)^{\frac{1}{2}} \sin(\omega t)\ dt \quad \text{for } \zeta_0 \ll L_0.$$

The effective length of the fracture, d, is defined as the radial distance from the borehole wall to a point at which the pressure gradient (see Appendix, equation A.4) falls to $1/e$ times the pressure gradient at the borehole wall over a time interval of $T/2$. The effective fracture length is given by the expression:

$$d(K) = \left[\frac{2K\ T}{\mu\gamma}\right]^{\frac{1}{2}} \tag{4}$$

Extension to Three Dimensions

The volume obtained in (3) is two dimensional. The actual borehole geometry involves a three-dimensional configuration which can be approximated by extrapolation of the two-dimensional solution assuming the geometry is axisymmetric. This geometrical extrapolation is assumed to be independent of the small fluid compressibility and of the low frequency of

excitation.

The two-dimensional stationary problem consists of an incompressible viscous fluid flow, in the ξ direction (oriented away from the borehole), enclosed between two parallel planes separated by a distance L. The boundary conditions for pressure are: at $\xi=R$, $p=P_0$ and at $\xi=R+d$, $p=P_f$ ($P_f > P_0$). The pressure distribution is:

$$p_{2D} = (\xi-R)\frac{(P_f - P_0)}{d} + P_0 \tag{5}$$

The 3D axisymmetric problem involves two discs separated by a distance L with radial fluid flow into the center. The outer radius is $r=R+d$ and inner radius is $r=R$. The boundary conditions are: at $r=R$, $p=P_0$ and at $r=R+d$, $p=P_f$. The stationary solution for the pressure distribution is:

$$p_{3D} = (P_f - P_0)\frac{ln(r/R)}{ln(\frac{(R+d)}{R})} + P_0 \tag{6}$$

The rate of fluid flow for these two problems is

$$q_{2D} = \frac{KL}{\mu} \frac{dp_{2D}}{d\xi} \tag{7a}$$

$$q_{3D} = \frac{KL}{\mu} 2\pi r \frac{dp_{3D}}{dr} \tag{7b}$$

or explicitly

$$q_{2D} = \frac{KL}{\mu} \frac{P_f-P_0}{d} \tag{8a}$$

$$q_{3D} = \frac{KL}{\mu} \frac{(P_f-P_0)2\pi}{ln(\frac{R+d}{R})} \tag{8b}$$

Use of Navier-Stokes equation and the continuity equation (Landau and Lifchitz, 1971) with the assumptions stated in this paper indicate that the permeability of two-disc radial fluid flow is the same as the permeability of two parallel planes fluid flow. Combining the two results of (8) leads to

$$q_{3D} = 2\pi R \chi q_{2D} \tag{9}$$

where

$$\chi(K) = \frac{d}{R \, ln(\frac{R+d(K)}{R})}$$

The χ component is defined as the geometrical factor; d is the effective length defined previously in (4) and R is the borehole radius. For any given time interval the equation relating the two- and three-dimensional volumes is similar to equation (9).

$$\Delta V(K) = 2 \pi R \chi \ \Delta V_{2D}(K) \qquad (10)$$

where ΔV is the volume ejected from the circular fracture into the borehole during a time interval of $T/2$.

TUBE WAVE GENERATION

Tube waves are low frequency Stoneley waves (White, 1965; Cheng and Toksöz, 1981). These guided waves reach their largest amplitudes at the solid-fluid boundary and decay approximately exponentially away from it. The fluid pulse ΔV forced from the fracture into the borehole by an incident compressional wave generates tube waves which propagate up and down the borehole.

To determine the relationship between the ejected fluid volume and the tube waves generated, the tube wave volumetric strain in the fluid (ε_{ii}) and seismic potentials are used. Let us define $z = 0$ as the plane of symmetry of the fracture. Further, the tube wave and the P-wave are assumed to have the same frequency. The integrated tube wave volumetric strain in the borehole fluid (at $z=0$) in the time period $T/2$ can be equated to the fluid volume ejected from the fracture, $\Delta V(K)$, over the same time period. In axisymmetric cylindrical coordinates $\Delta V(K)$ can be expressed as

$$\Delta V(K) = 2 \pi c \int_0^{T/2} \int_0^R \varepsilon_{ii}(r,t,z=0) \, r dr \, dt \qquad (11)$$

The volumetric strain and amplitude of the tube wave can be determined using the seismic potential for the tube wave (Cheng and Toksöz, 1981). The tube wave pressure amplitude in the borehole fluid p^T and its displacement amplitude in the formation u_z^T are described by:

$$p^T = \rho_f \ \omega^2 \ C \ I_0(nr) \qquad r \leq R^- \qquad (12)$$

$$u_z^T = A[kK_0(lr) + mGK_0(mr)] \qquad r \geq R^+ \qquad (13)$$

where

$$G = \frac{2l\beta^2 \ K_1(lr)}{k \ (c^2-2\beta^2) \ K_1(mr)} \qquad (14)$$

$$A = \frac{C \ n \ I_1(nr) \ (2\beta^2-c^2)}{l \ c^2 \ K_1(lr)} \qquad (15)$$

The factor C is calculated from equation (11), and is given by

$$C(K) = \frac{(1 - c^2/\alpha_f^2)^{1/2}}{4\pi R \ (c^2/\alpha_f^2 - 2) \ I_1(nr)} \Delta V(K) \tag{16}$$

The amplitudes of the up and down going tube waves will be the same because of symmetry.

The fracture permeability can be evaluated by considering the inverse problem. The *in situ* fracture permeability can be determined from the tube wave amplitude normalized to the direct P-wave amplitude in the fluid (pressure ratio) or in the formation (displacement ratio). The ratio of fracture displacement to incident P-wave displacement, ζ_0/u_z^α, is a function of the fracture shape (aspect ratio), the stiffness of the formation, and the fluid compressibility. For a thin fracture with large surface area and small fluid compressibility, $\zeta_0 \approx u_z^\alpha$, and when there is no fluid in the fracture, $\zeta_0 \approx 2 u_z^\alpha$ (ex: a free surface). In following calculations ζ_0 will be assumed to be equal to u_z^α.

At low frequencies ($\lambda >> R$), the P-wave pressure in the borehole fluid can be written in terms of displacement in the formation (White, 1965):

$$p^\alpha / |\dot{u}_z^\alpha| = \frac{\rho_f \ c^2 \alpha (1 - 2\beta^2/\alpha^2)}{\beta^2 (1 - c^2/\alpha^2)} \tag{17}$$

where $|\dot{u}_z^\alpha| = \omega u_z^\alpha$.

Using $\zeta_0 = u_z^\alpha$ and equations (12) to (17), the ratios of fracture induced tube wave amplitudes to incident P-wave amplitudes can be determined. The pressure ratio in the borehole (measured by a hydrophone) is:

$$p^T / p^\alpha = \ C(K) \ \frac{\omega\beta^2 \ (1 - c^2/\alpha^2) \ I_0(nR)}{\zeta_0 c^2 \alpha \ (1 - 2\beta^2/\alpha^2)} \tag{18}$$

The vertical component of displacement ratio at the wall of the borehole (measured by an anchored borehole geophone) is:

$$u_z^T / u_z^\alpha = \ \left[k K_0(lR) + mG K_0(mR) \right] A / \zeta_0 \tag{19}$$

The factor A is related to $C(K)$ by equation (15) and $C(K)$ is related to the volume of fluid ejected in $T/2$ by equation (16). The fluid volume is related to the fracture permeability by combining equations (10) and (3). Equation (18) or (19) can be solved to determine the fracture permeability K. Normally, given the formation and fluid properties, displacement or pressure ratios can be determined as a function of frequency with permeability, K, as a parameter. These values can then be compared with observations to determine K.

A simple relation exists between the fracture permeability (K) and the fracture hydraulic conductivity (or coefficient of permeability) (K_c):

$$K_c = K \rho_f \ g \ / \ \mu \ , \tag{20}$$

where g is the gravitational acceleration. The SI units of K are m^2, and the common unit is the darcy (1 $d = 10^{-12} \ m^2$). The SI units of K_c are m/sec, and

the common unit is $10^{-3} \, cm/sec$, which is sometimes called the darcy; to avoid any confusion this nomenclature shall not be used. In these units the following relation exists: $1d \approx 10^{-3}$ cm/sec.

RESULTS AND DISCUSSION

In order to demonstrate the different efficiencies of tube wave generation, we considered three separate formations: a granite, representing a typical crystalline rock; a "hard" sediment that would represent relatively dense carbonate and hard sandstones; and a "sediment" to represent the more typical sedimentary rocks such as sandstones and shales. The properties of these formations and other physical parameters used in this study are listed in Table 1. The tube wave phase velocity c is calculated by solving the period equation (Cheng and Toksöz, 1981) for the given formation and borehole parameters as a function of frequency.

Pressure and Displacement Amplitude Ratios

The ratios of tube wave amplitudes to P- wave amplitudes are shown in Figures 4, 5 and 6. The first comparison made is that of the ratios of pressure amplitudes inside the borehole fluid. This is useful for interpreting the hydrophone data. The second comparison is that of the ratios of displacements that may be measured by a borehole seismometer locked to the borehole wall. Only the vertical (z) component of the displacement is considered. The tube wave particle motion at the borehole interface is highly elliptical and it is important to specify the component of displacement under consideration. Figure 4 shows the frequency dependence of the tube wave amplitude for the two formations considered. Owing to the much larger amplitudes of tube waves in the fluid it is preferable to use hydrophone data as opposed to geophone data to detect highly permeable zones. The borehole essentially acts as an amplifier. The tube wave pressure amplitudes in the fluid decrease with increasing frequency. The displacement ratios in the formation increase with frequency. The pressure ratios for granite, "hard" and "sediment" models are shown in Figures 5a, b and c, for different values of permeability. Higher fracture permeability yields higher tube wave amplitudes. For a given fracture permeability the "hard" formation gives a higher value of the tube wave pressure amplitude in the fluid, and a lower value of the tube wave displacement amplitude in the formation than the "sediment" (Figure 6).

Non-horizontal Fractures

For the two-dimensional case in which the fracture does not perpendicularly intersect the borehole, the normal displacement of the fracture wall, ζ_f, can be expressed in terms of the inclination, ϑ, of the fracture with respect to a plane normal to the borehole. The fluid volume ejected is found by replacing ζ_0 by $\zeta_f = \zeta_0 \cos(\vartheta)$, where $\zeta_0 = u_z^\alpha$. Equation (16) becomes:

$$C = f(K) \cos(\vartheta) \tag{21}$$

where

$$f(K) = \frac{\zeta_0 (1-c^2/\alpha_f^2)^{\frac{1}{2}}}{2(2-c^2/\alpha_f^2)\, I_1(nR)} \left(\frac{K}{\pi\mu\gamma}\right)^{\frac{1}{2}} F(\omega, \zeta_f/L_0)\, \chi(K)$$

The factor C is proportional to the tube wave amplitudes (equations 12 and 13). Therefore C is determined directly from the data. $f(K)$ is a monotonically increasing function of the permeability K. The fracture inclination, ϑ, in our problem is assumed to be zero; therefore K is calculated by the following equation:

$$C = f(K) \tag{22}$$

Given the tube wave amplitude (i.e., C), this model ($\vartheta=0$) will yield a lower bound of the *in situ* fracture permeability of a tilted fracture (equation (21), $\vartheta\neq0$).

Field Example

Field examples of tube wave generation in a borehole that intersects open fractures in granite are shown in Figures 1 and 7 (Huang and Hunter, 1981b). These examples are pressure measurements in the borehole fluid. Comparison of data from the televiewer, core logs and the tube waves show a good correlation between tube wave generation and open fractures. For open fractures at several depths, the tube wave to P- wave amplitude ratios were measured. These are listed in Table 2. The borehole radius is about 3.8 cm. The generated tube wave amplitude is dependent on the borehole radius (roughly inversely proportional to the borehole cross-sectional area). A granite model is calculated with a borehole radius of 3.8 cm. The pressure ratios as a function of permeability and frequency are shown in Figure 8. The mean frequency of tube waves (and P- waves) is about 200 Hz. Using the observed amplitude ratios the permeability for each fracture zone was determined using Figure 8. They are listed in Table 2. and range from 18 to 67 millidarcys. These values are consistent with other permeability calculations in fractured granite as compiled by Brace (1980).

Conclusions

Tube waves can be used to detect open fractures intersecting a borehole and to determine an equivalent fracture permeability using tube wave to P- wave amplitude ratios in the borehole fluid (pressure ratio). It should be noted that the tube- to P- wave pressure ratio in the borehole fluid is approximately 3 orders of magnitude greater than the displacement ratio in the formation. For this reason it is preferable to use hydrophone data, as compared to wall-locked borehole geophone data, to locate these permeable zones. It is important to mention that a number of assumptions were made in this study and complex "equivalency" was established when going from the 2-dimensional cartesian geometry to a circular crack model. Additional theoretical work is now in progress to solve the 3-dimensional problem directly.

Table 1. Physical parameters used in this study

Formation	$\alpha(m/s)$	$\beta(m/s)$	$\rho(kg/m^3)$
granite	5500	3300	2700
"hard"	4500	2500	2300
"sediment"	3000	1200	2100
fluid	1500	0	1200
fluid viscosity	μ	$= 10^{-3}$	Poiseuilles
fluid incompressibility	γ^{-1}	$= 2.10^9$	Pa
borehole radius	R	$= 0.1$	m

Table 2.

Calculated fracture permeablity for a granite model from observed p^T/p^α and for a frequency of 200 Hz				
Depth (m)	80	130	392	460
p^T/p^α	4.5	3	2	3.5
K (md)	67	35	18	45

NOMENCLATURE

c : tube wave phase velocity

d : effective length of fracture

f : P-wave or tube wave frequency

$I_i(z)$, $K_i(z)$: modified Bessel function of the i-th order

K : fracture permeability (1 darcy = $10^{-12} m^2$)

K_c : fracture hydraulic conductivity ($10^{-3} cm/sec$)

k : ω/c, tube wave vertical wavenumber

L(t) : fracture width

L_0 : static fracture width

l : $k(1-c^2/\alpha^2)^{1/2}$, tube wave radial wavenumber from P-wave contribution

m : $k(1-c^2/\beta^2)^{1/2}$, tube wave radial wavenumber from S-wave contribution

n : $k(1-c^2/\alpha_f^2)^{1/2}$, tube wave radial wavenumber from fluid P-wave contribution

p(x,t) : fluid pressure distribution in the fracture

P_0 : static borehole pressure at a given depth

p^T : generated tube wave pressure amplitude in borehole fluid in the vicinity of the fracture

p^α	: pressure amplitude in the borehole fluid of the first P-wave arrival, in the vicinity of the fracture
q(x,t)	: rate of fluid flow in the fracture
Q(t)	: source function
R	: borehole radius
u_z^α	: vertical displacement amplitude in the formation of the first P-wave arrival, in the vicinity of the fracture
u_z^T	: generated tube wave vertical displacement amplitude in the formation, in the vicinity of the fracture
ΔV_{2D}	: two-dimensional volume of fluid ejected by the fracture in T/2
ΔV	: three-dimensional extrapolation of ΔV_{2D}
T	: 1/f, P-wave or tube wave period
α	: compressional (P-) wave velocity of the formation
α_f	: compressional (P-) wave velocity of the fluid
β	: shear (S-) wave velocity of the formation
γ	: fluid compressibility
ε_{ii}	: tube wave volumetric strain in the fluid
$\zeta(t)$	: normal displacement of fracture wall
ζ_0	: maximum $\zeta(t)$
μ	: dynamic fluid viscosity
ρ	: formation density
ρ_f	: fluid density
χ	: geometrical factor
ω	: $2\pi f$, P-wave or tube wave angular frequency

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APPENDIX

The net change of fluid dq in a volume element $L(t)dx$ (Figure 3.) is due to the volume of fluid ejected by the fracture closure and the compressibility of the fluid (no mass is generated or lost in the element). During a time increment dt, this total change is:

$$dq\ dt = \frac{dL(t)}{dt}\ dxdt + L(t)\gamma \frac{\partial p(x,t)}{\partial t}\ dxdt \qquad (A.1)$$

where γ is the fluid compressibility and $dL(t)/dt = -\omega\zeta_0 \sin(\omega t)$, the velocity of the fracture wall. The net storage given by (A.1) must equal the net volume of fluid $\{\partial q(x,t)/\partial x\}$ dxdt flowing into the differential volume, giving

$$\frac{\partial}{\partial x}\left[\frac{K}{\mu} L(t)\ \frac{\partial p(x,t)}{\partial x}\right] = L(t)\gamma\ \frac{\partial p(x,t)}{\partial t} + \frac{dL(t)}{dt}$$

Setting $a^2 = K/\mu\gamma$ and Q(t) $= -\omega\zeta_0 \sin(\omega t)/(\gamma L(t))$, the following equation is

obtained:

$$a^2 \frac{\partial^2 p(x,t)}{\partial x^2} - \frac{\partial p(x,t)}{\partial t} = Q(t) \qquad (A.2)$$

with the boundary conditions for pressure

$$\begin{array}{lll} p(x,t=0) & = P_0 & x \geq 0 \\ p(x=0,t) & = P_0 & t \geq 0 \\ \partial p(x,t)/\partial x & = 0 & x = \infty \end{array}$$

This last condition states that there is no fluid flow in the fracture far $(x \gg d)$ from the borehole intersection.

Equation (A.2) is a one-dimensional inhomogeneous diffusion equation. The heat conduction analogy corresponds to a semi-infinite half-space $(x \geq 0)$ having a^2 as thermal diffusivity and a time varying heat source $Q(t)$. The solution to this partial differential equation (A.2) is

$$p(x,t) = P_0 - \int_0^t Q(t-\tau) \, erf\left[\frac{x}{2a \ \tau^{\frac{1}{2}}}\right] d\tau$$

where $erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt$ is the error function.

Note that

$$\lim_{x \to \infty} p(x,t) = P_0 - \gamma^{-1} \ln\left[\frac{L(t)}{L_0 + \zeta_0}\right]$$

The pressure gradient follows from (2)

$$\frac{\partial p(x,t)}{\partial x} = - \frac{1}{a\sqrt{\pi}} \int_0^t Q(t-\tau) \, \exp(\frac{-x^2}{4a^2 \ \tau}) \ \tau^{-1/2} \ d\tau \qquad (A.4)$$

The rate at which fluid flows is given by equation (2). By calculating the volume ejected from the fracture for the maximum fracture displacement $2\zeta_0$, we can obtain the maximum volume ejected in a finite amount of time. This maximum volume occurs during a time interval of $t = T/2$. Therefore the volume of fluid forced from the fracture into the borehole in $T/2$ is:

$$\Delta V_{2D} = \int_0^{T/2} q(x=0,t) \ dt$$

or explicitly,

$$\Delta V_{2D}(K) = - \zeta_0 \left[\frac{K}{\pi \gamma/\mu}\right]^{\frac{1}{2}} F(\omega, \zeta_0/L_0) \qquad (A.5)$$

where

$$F(\omega, \zeta_0/L_0) = \omega \int_0^{T/2} \int_0^t \frac{1 - (\zeta_0/L_0)\cos(\omega t)}{1 - (\zeta_0/L_0)\cos(\omega\tau)} \ \frac{\sin(\omega\tau)}{(t-\tau)^{\frac{1}{2}}} \ d\tau \ dt$$

Computation shows that for increasing frequency, $F(\omega, \zeta_0/L_0)$ decreases. As the frequency increases less fluid is ejected into the borehole. For $\zeta_0 \ll L_0$, an asymptotic expression for F can be found by interchanging the order of integration in the (τ, t) plane:

$$F(\omega, 0) = 2\omega \int_0^{T/2} (T/2 - t)^{\frac{1}{2}} \sin(\omega t) \, dt$$

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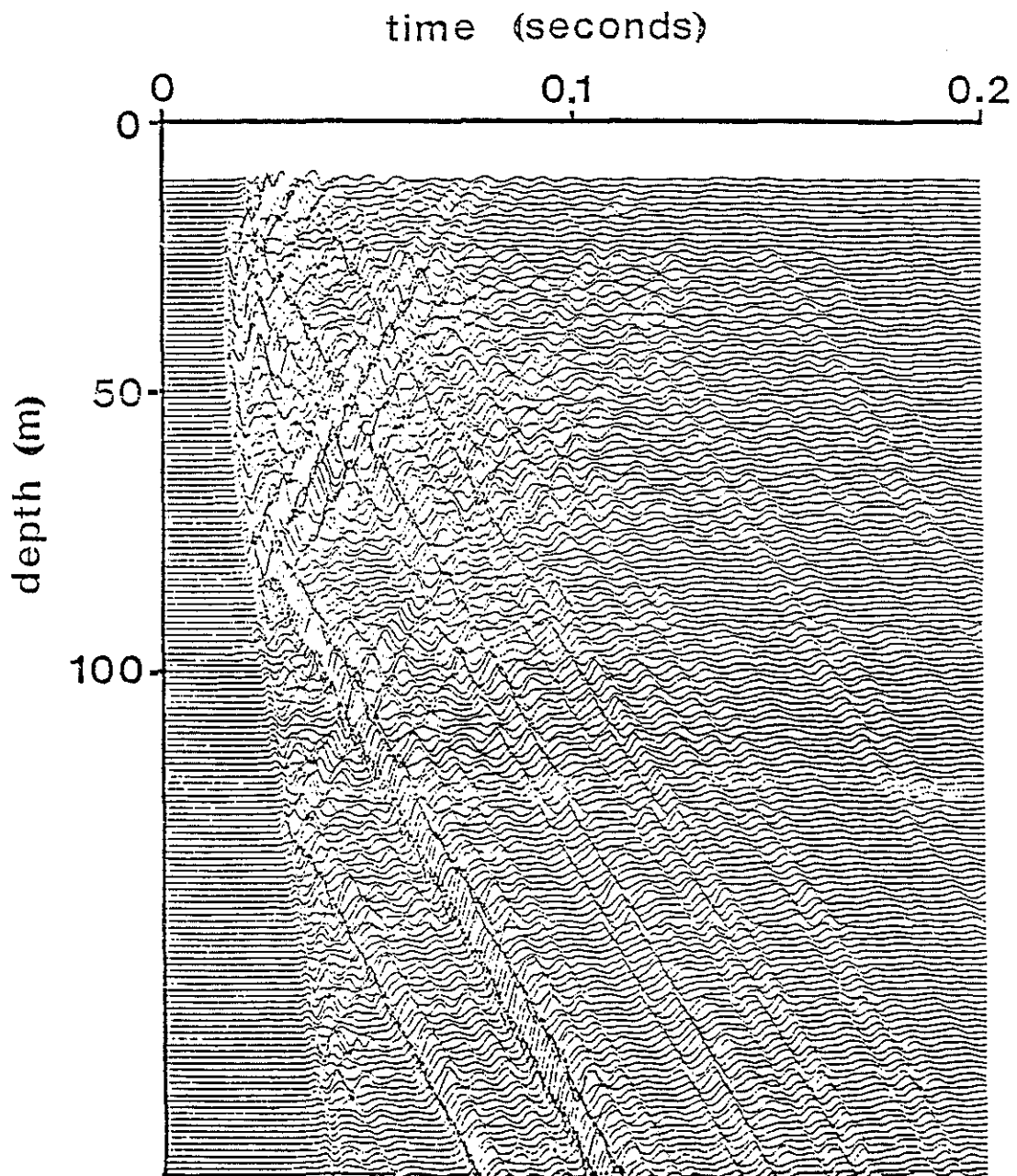
Figure 1. Example of fracture generated tube wave in a VSP (after Huang and Hunter, 1981b).
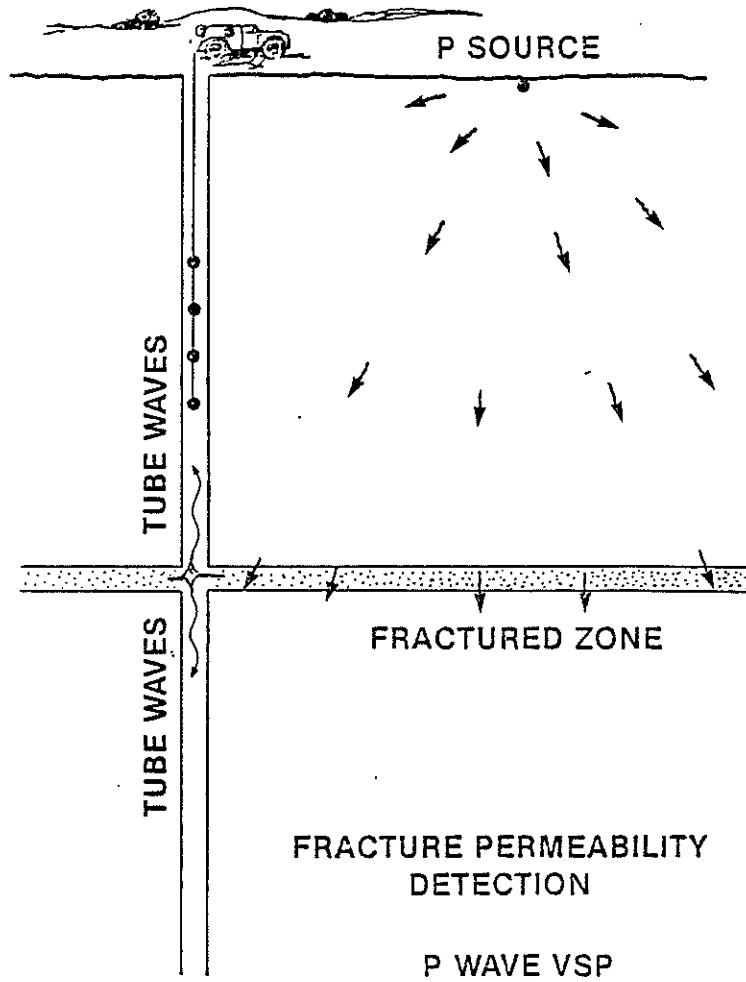
Figure 2. Schematic diagram of tube wave generated from P-waves impinging on a subsurface fracture zone.
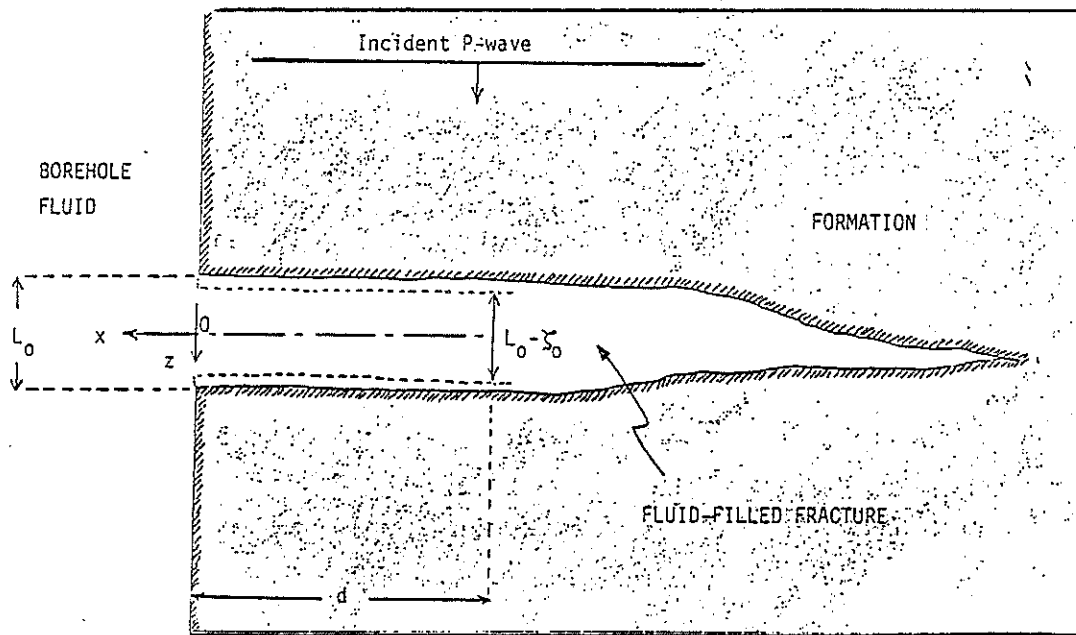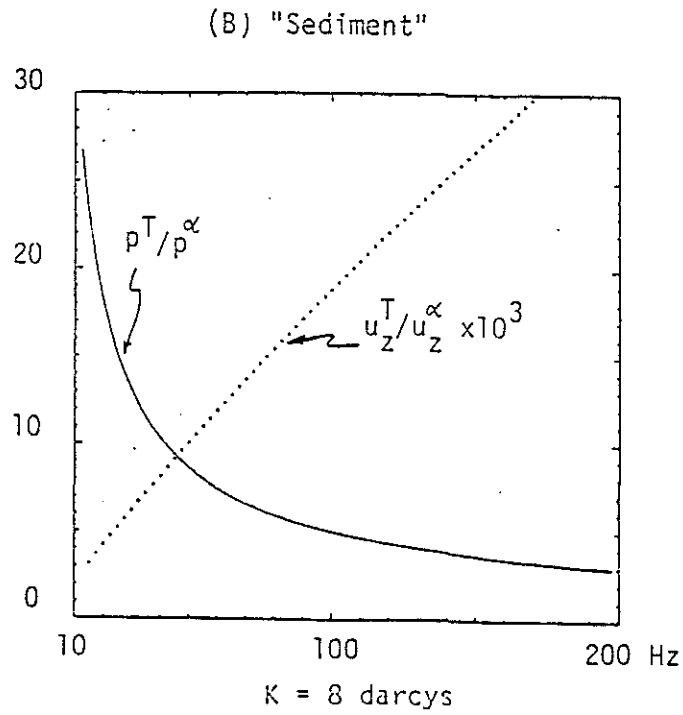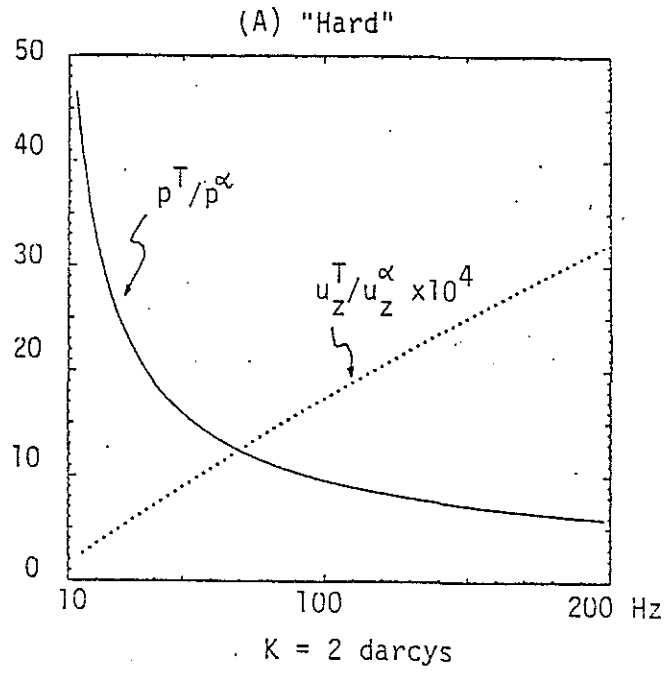
Figure 3. Fracture model used in this study.

Figure 4. Ratios of tube to P- wave pressures and displacements in two different formations as functions of frequency.
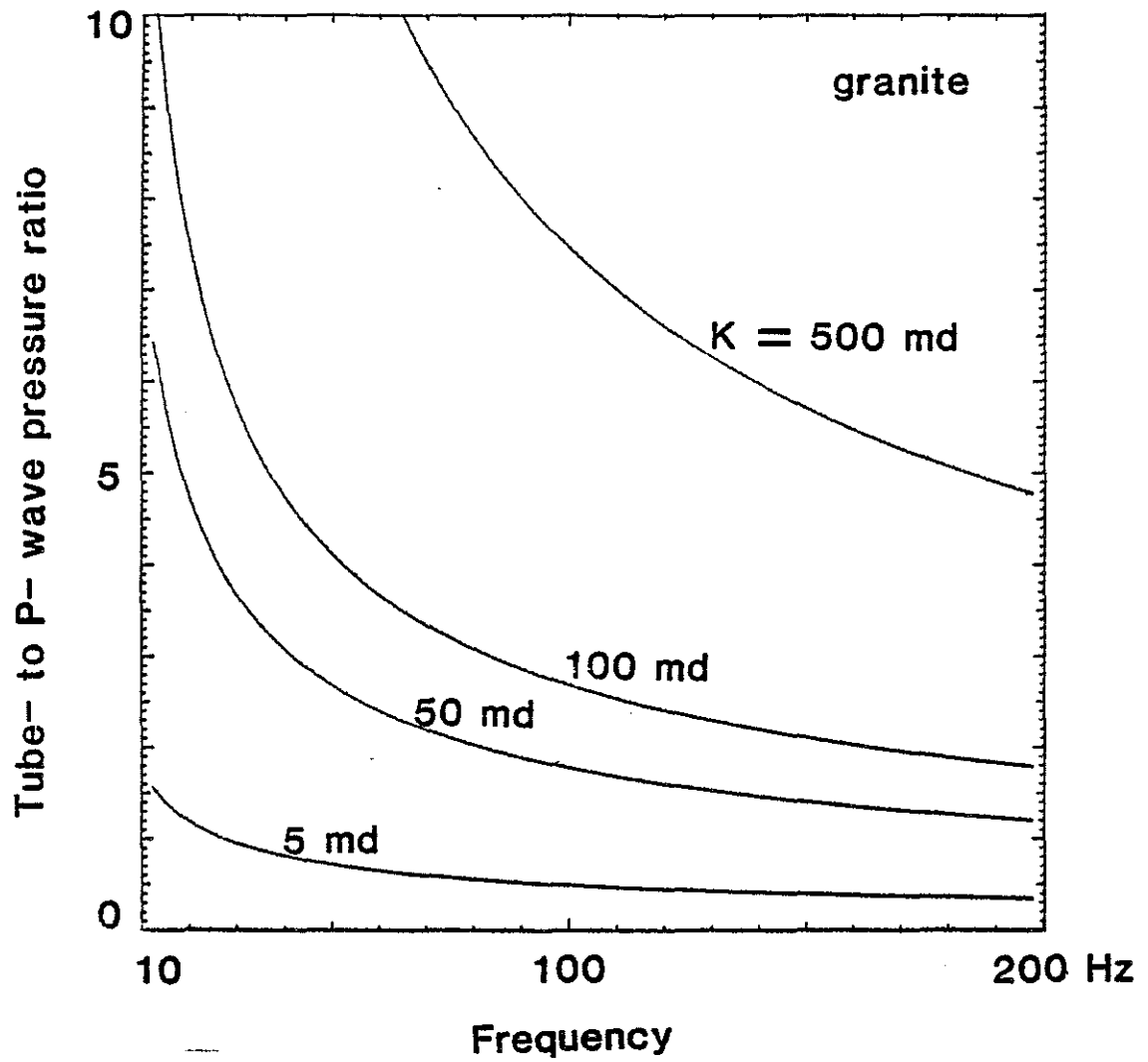
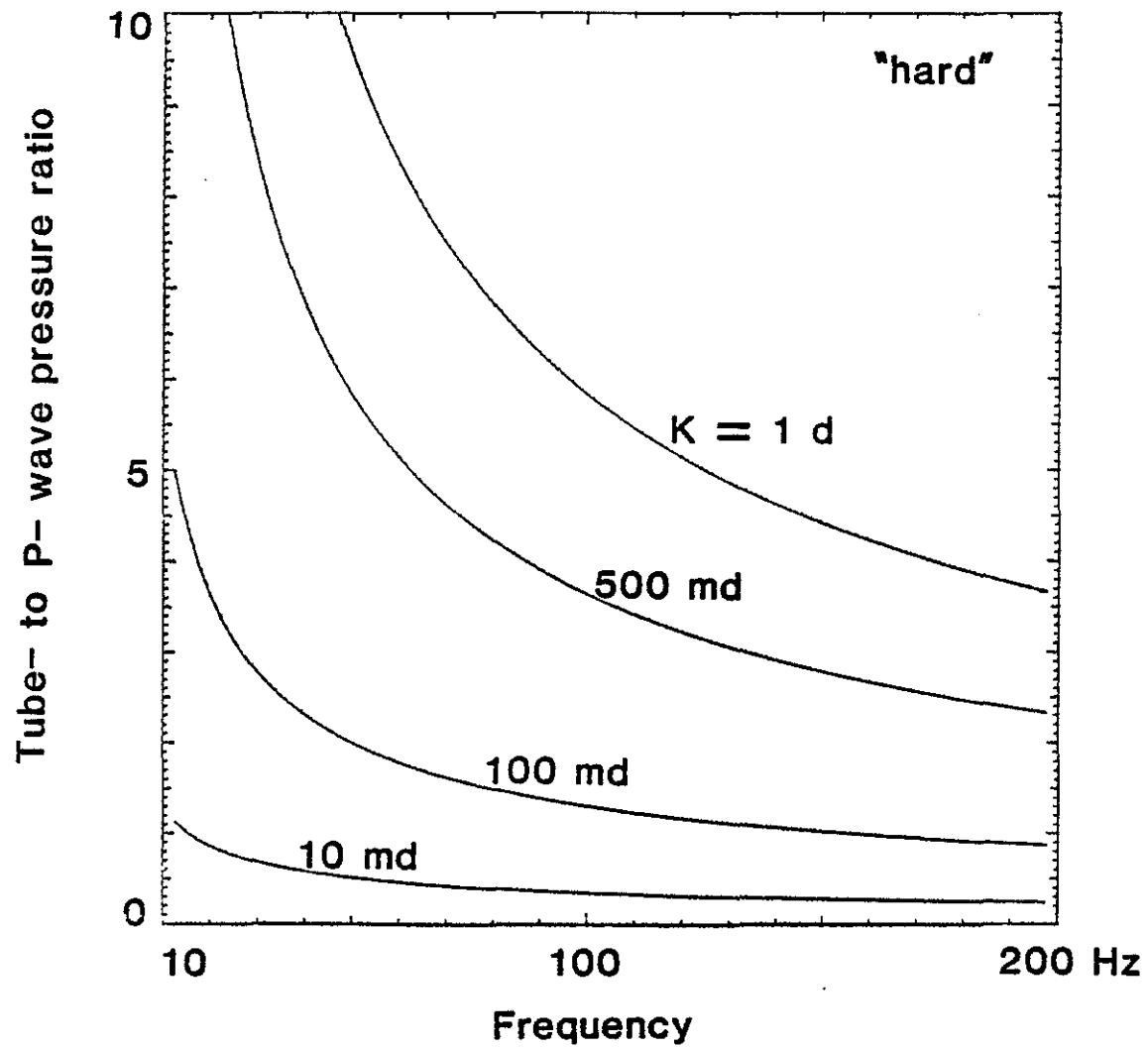Figure 5a. Tube to P- wave pressure ratios as functions of frequency and permeability in a granite.

Figure 5b. Tube to P- wave pressure ratios as functions of frequency and permeability in a "hard" formation.
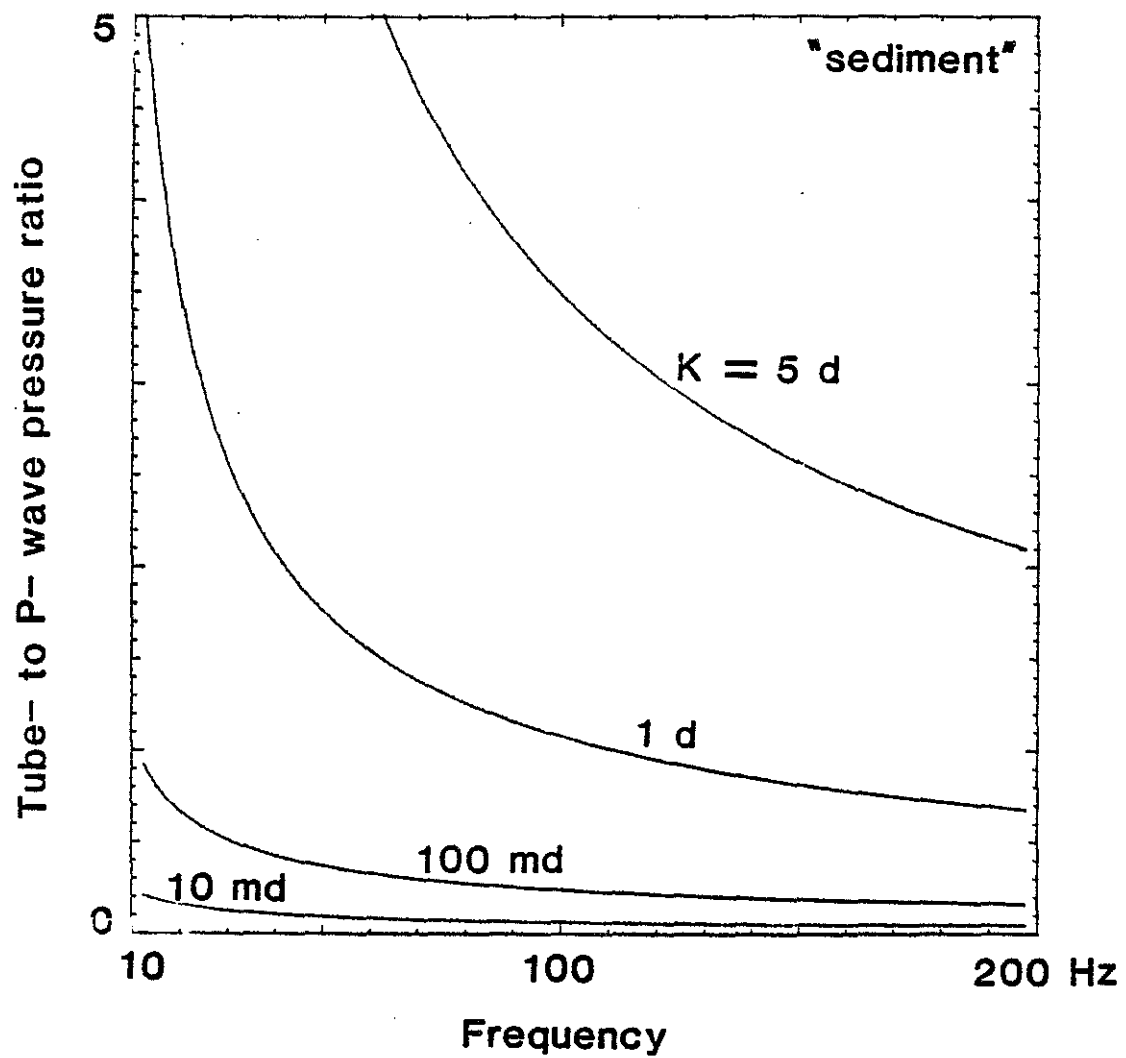
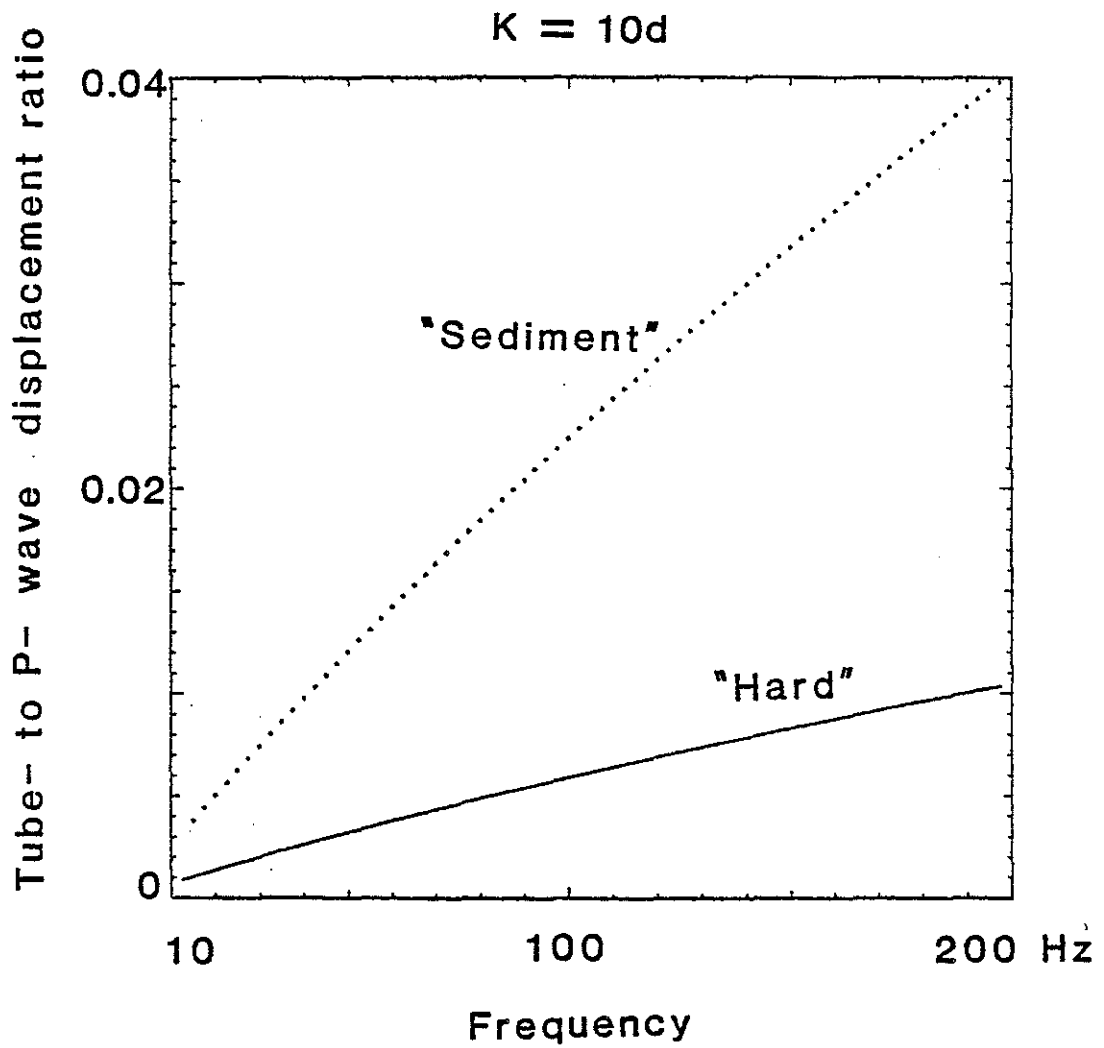Figure 5c. Tube to P- wave pressure ratios as functions of frequency and permeability in a "sediment".

Figure 6. Tube to P- wave vertical displacement ratios in two different formations as a function of frequency.
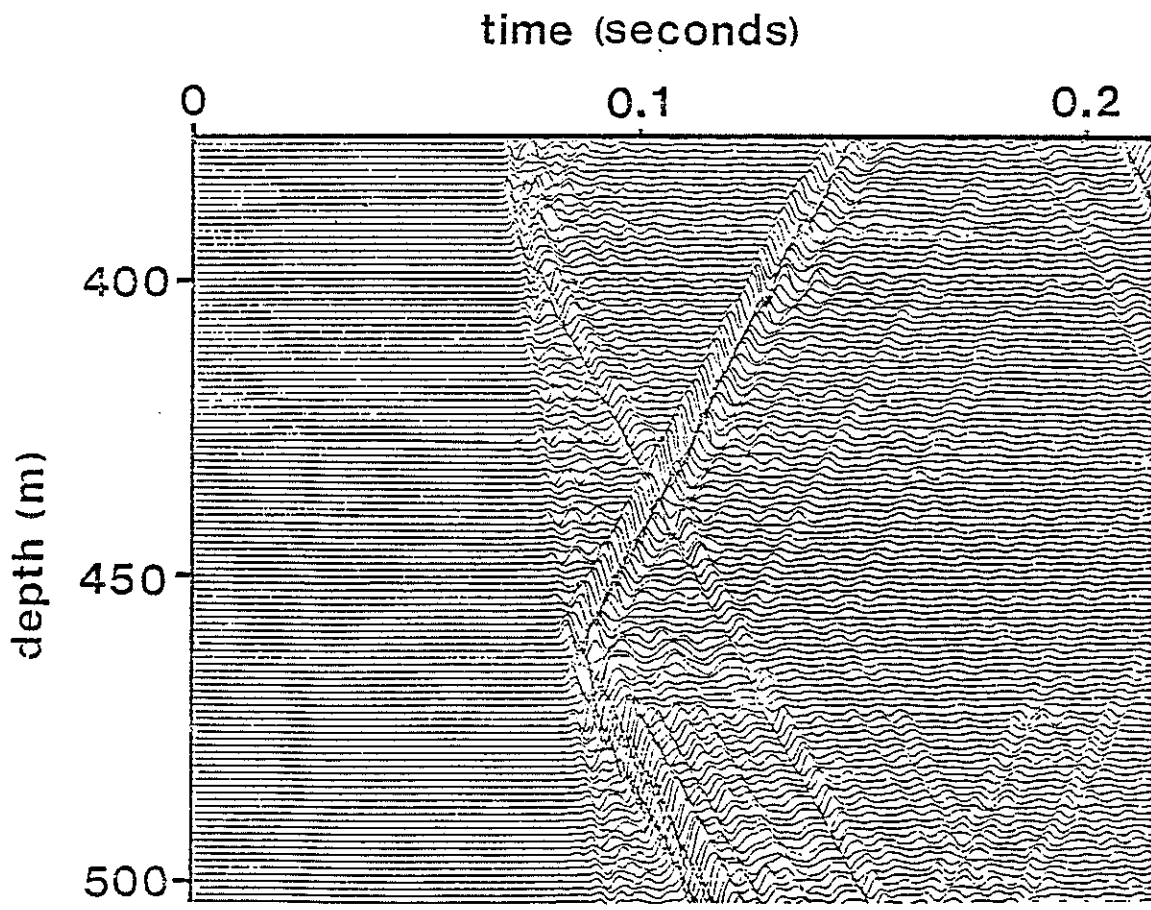
Figure 7. Further examples of tube waves generated by fractures in a VSP (after Huang and Hunter, 1981b).
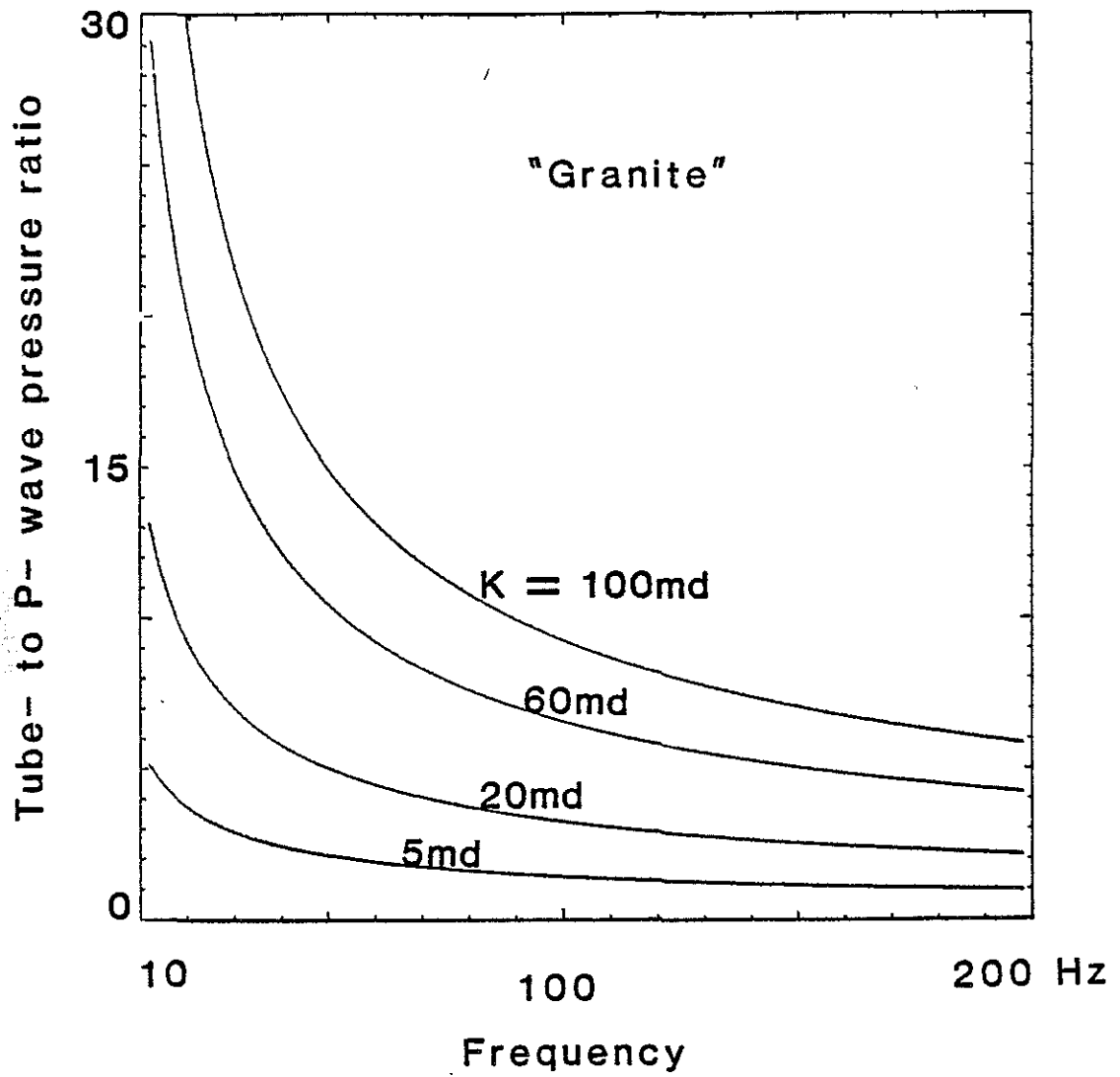
Figure 8. Tube to P- wave pressure ratios as functions of frequency and permeability in a granite, with a borehole radius of 3.8 cm.