ELEC1011 Communications and Control Coursework 2: Filters, Source Coding and Channel Coding

Introduction

This coursework contributes 5% of your mark for ELEC1011 Communications and Control. It is therefore worth half a credit and should take you up to 5 hours to complete it. Please do not spend any more time than this since I'm sure you have better things to be doing!

Besides discussing ideas with your classmates, you are required to work entirely on your own to produce a write-up, which only needs to include the figures, calculations and comments that are directly requested in the exercises below. Your write-up can contain a combination of hand-written work, computer print-outs and pages from this document. When you are finished, you need to print out a receipt from C-BASS
`https://handin.ecs.soton.ac.uk/handin/0910/ELEC1011/2/`
You then need to staple your receipt together with your write-up and submit it at the ECS reception before 4pm on 05/05/2010.

Here are some tips that will help you get a high mark in this coursework:

- Include your workings for each calculation in your write-up.

- Each figure in your write-up should have well labelled and accurately annotated axes, as well as a relevant title and/or legend that identifies which signals are shown.

- If you draw your figures by hand, make sure you use a ruler.

- You may like to use Matlab to help you complete the exercises and to produce the figures.

If you notice any mistakes in this document or have any queries about it, please email me at `rm@ecs.soton.ac.uk`

Have fun, Rob Maunder.

1   Filters

Figure 1 provides a schematic for an RLC circuit.

a) Show that the transfer function of the RLC circuit shown in Figure 1 is given by $H(j\omega) = 1/(1 + jX)$, where $X = (10^{-12}\omega^2 - 1)/10^{-4}\omega$.
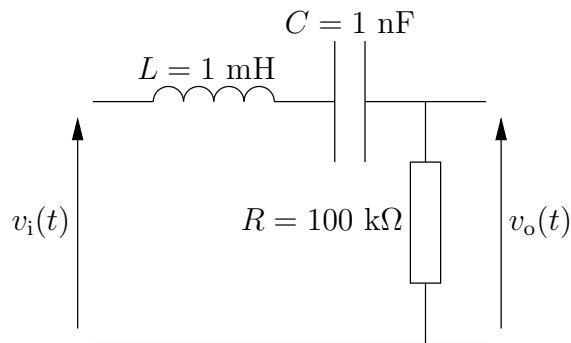
Figure 1: RLC circuit.

b) Show that the corresponding amplitude response is given by $|H(j\omega)| = 1/\sqrt{1 + X^2}$.

c) Show that the corresponding phase response is given by $\angle H(j\omega) = \tan^{-1}(-X)$.

d) Determine the amplitude and phase response at the angular frequencies $\omega \in [10^2, 10^3, 10^4, 10^5, 10^6, 10^7, 10^8, 10^9, 10^{10}]$.

e) Using these results, sketch a Bode plot for the RLC circuit shown in Figure 1.

f) Determine an expression for the output signal $v_\text{o}(t)$ in the case where the input signal is given by $v_\text{i}(t) = A\cos(2\pi f t + \phi)$, where $A = 3$ V, $f = 2$ kHz and $\phi = 45°$.

g) State the type of filter that is provided by the RLC circuit of Figure 1.

h) Derive an expression for the filter's 3 dB bandwidth in terms of R, L and C.

i) Use your expression or Bode plot to state the filter's 3 dB bandwidth.

2 Huffman Coding

Table 1 provides the probabilities of occurrence for the letters of the alphabet, when used in the English language. A Huffman codebook has been designed based on these probabilities of occurrence and the resultant codewords are also shown in Table 1.

a) Calculate the entropy H of the letters of the alphabet, when used in the English language.

b) Calculate the average codeword length L for the Huffman codebook provided in Table 1.

c) Calculate the Huffman coding efficiency R.

d) Determine the bit sequence that results from the Huffman coding of the letter sequence `HUFFMAN`.

Letter i	Probability of occurrence p_i	Huffman codeword $\mathbf{c}_i$
A	0.0856	1111
B	0.0139	010110
C	0.0279	01010
D	0.0378	11010
E	0.1304	100
F	0.0289	10100
G	0.0199	110111
H	0.0528	0100
I	0.0627	0111
J	0.0013	1101101110
K	0.0042	11011010
L	0.0339	10101
M	0.0249	00010
N	0.0707	1100
O	0.0797	1110
P	0.0199	00000
Q	0.0012	1101101101
R	0.0677	1011
S	0.0607	0110
T	0.1045	001
U	0.0249	00011
V	0.0092	1101100
W	0.0149	010111
X	0.0017	1101101111
Y	0.0199	00001
Z	0.0008	1101101100

Table 1: The letters of the alphabet, their probabilities of occurrence when used in the English language and the allocated Huffman codewords.

Figure 2: Binary tree for the Huffman codebook of Table 1.

e) Complete the binary tree shown in Figure 2 so that it describes the Huffman codebook of Table 1.

f) Decode the bit sequence `011011110111110000101101010000101010111`. You may find your binary tree useful for this.

3 Arithmetic Coding

A particular source generates the symbols [A, C, G, T] with the probabilities [0.4, 0.3, 0.2, 0.1], respectively. An arithmetic coding scheme is used to encode and decode these symbols. Figure 3 provides a diagram that may be used to assist the encoding and decoding of sequences comprising seven symbols.

a) Draw diagonal lines between the horizontal bars of Figure 3 to show the range that is expanded during the encoding of each symbol in the sequence GATTACA. Using at least six decimal places[1], label each of the vertical lines in Figure 3 with the values that separate the adjacent ranges. State the final range that results from the encoding of the symbol sequence GATTACA.

b) Draw a binary tree to determine the shortest bit sequence that represents a decimal value in the final range that you identified for the above symbol sequence.

c) Calculate to at least six decimal places, the decimal value that is represented by the arithmetic encoded bit sequence `11101101111111`.

d) Using a copy of Figure 3, determine the seven symbols that are represented by the above bit sequence.

e) Explain why arithmetic coding can potentially achieve a higher coding efficiency than Huffman coding.

4 Hamming Coding

The generator matrix $\mathbf{G}$ and the parity check matrix $\mathbf{H}$ of the $k/n = 4/7$ Hamming code are given by

$$
\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}
$$

The $n = 7$-bit encoded sequence $\mathbf{y} = [\; y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6 \quad y_7 \;]^T$ is given by $\mathbf{y} = \mathbf{Gx}$, where $\mathbf{x} = [\; x_1 \quad x_2 \quad x_3 \quad x_4 \;]^T$ is the $k = 4$-bit source sequence.

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[1]You can get Matlab to show more than four decimal places by using the `format long` command.

A	C	G	T

0 1

A	C	G	T

A	C	G	T

A	C	G	T

A	C	G	T

A	C	G	T

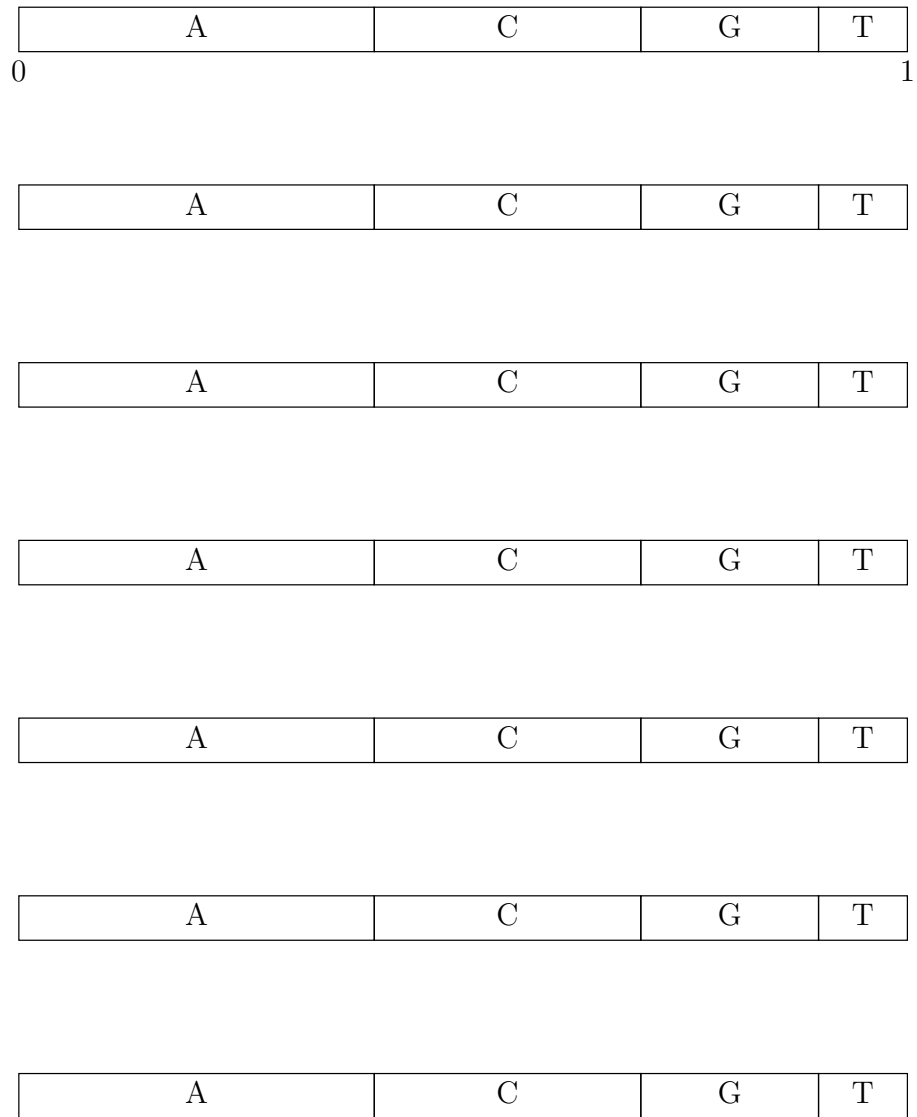| A	C	G	T

Figure 3: Diagram that may be used to assist the arithmetic encoding and decoding of sequences comprising seven symbols.

i	$\mathbf{x}_i$	$\mathbf{y}_i$
1	$\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$	$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$
2	$\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
3	$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
4	$\begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
5	$\begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
6	$\begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
7	$\begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
8	$\begin{bmatrix} 0 & 1 & 1 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
9	$\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
10	$\begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
11	$\begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
12	$\begin{bmatrix} 1 & 0 & 1 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
13	$\begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
14	$\begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
15	$\begin{bmatrix} 1 & 1 & 1 & 0 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$
16	$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$	$\begin{bmatrix} & & & & & & \end{bmatrix}^T$

Table 2: Hamming code mappings between the various permutations of the $k = 4$-bit source sequence $\mathbf{x}$ and the $n = 7$-bit encoded sequence $\mathbf{y}$.

j	i=1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	4														
3	3	3													
4	3	3	4												
5	3	3		4											
6	3	3	4	4	4										
7	4	4	3	3		3									
8	4	4	3	3	3	3	4								
9	3	3		4	4	4	4	7							
10	3	3	4	4	4	4	7	3	4						
11	4	4	3	3	3	7	4	4	3	3					
12	4	4	3		7	3	4	4		3	3				
13	4	4	3	7	3	3	4	4	3	3	4	4			
14	4	4	7	3	3	3		4	3	3	4	4	4		
15	3	7	4	4	4	4	3	3	4	4		3	3	3	
16	7	3	4	4	4	4	3	3	4	4	3	3	3	3	

$$d(\mathbf{y}_i, \mathbf{y}_j)$$

Table 3: Hamming distances between pairs of encoded sequences.

a) Using the generator matrix $\mathbf{G}$, express each of y_1, y_2, y_3, y_4, y_5, y_6 and y_7 as modulo-2 sums of x_1, x_2, x_3 and x_4.

b) Using your expressions, complete the table of mappings provided in Table 2.

c) In Table 3, fill in the 10 missing Hamming distances $d(\mathbf{y}_i, \mathbf{y}_j)$ between the encoded sequences $\mathbf{y}_i$ and $\mathbf{y}_j$ from Table 2.

d) Using Table 3, state the minimum Hamming distance $d_{\min} = \min_{i \neq j}[d(\mathbf{y}_i, \mathbf{y}_j)]$ between any pair of different encoded sequences in Table 2.

e) In the case where $\mathbf{y} = [\ 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0\ ]^T$ is transmitted but $\hat{\mathbf{y}} = [\ 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0\ ]^T$ is received, determine the permutation of $\mathbf{y}_i$ from Table 2 that has the minimum Hamming distance from $\hat{\mathbf{y}}$. State whether the transmission error can be corrected and/or detected.

f) In the case where $\mathbf{y} = [\ 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1\ ]^T$ is transmitted but $\hat{\mathbf{y}} = [\ 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1\ ]^T$ is received, determine the permutation of $\mathbf{y}_i$ from Table 2 that has the minimum Hamming distance from $\hat{\mathbf{y}}$. State whether the transmission errors can be corrected and/or detected.

g) In the case where $\mathbf{y} = [\ 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0\ ]^T$ is transmitted but $\hat{\mathbf{y}} = [\ 1 \quad 1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0\ ]^T$ is received, determine the permutation of $\mathbf{y}_i$ from Table 2 that has the minimum Hamming distance from $\hat{\mathbf{y}}$. State whether the transmission errors can be corrected and/or detected.

h) Explain how the Hamming code's ability to correct and/or detect these transmission errors can be predicted by considering the value of $d_{\min}$.

When the $n = 7$-bit encoded sequence $\hat{\mathbf{y}}$ is received, the $n - k = 3$-bit syndrome $\mathbf{s} = [\ s_1 \quad s_2 \quad s_3\ ]^T$ is given by $\mathbf{s} = \mathbf{H}\hat{\mathbf{y}}$.

i) In the case where the sequence $\hat{\mathbf{y}} = [\ 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0\ ]^T$ containing one bit error is received, determine the syndrome $\mathbf{s}$.

j) Explain how the erroneous bit in $\hat{\mathbf{y}} = [\ 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0\ ]^T$ can be identified using the syndrome $\mathbf{s}$.

k) Correct the erroneous bit in $\hat{\mathbf{y}} = [\ 1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0\ ]^T$ using the syndrome $\mathbf{s}$ and state the corresponding source sequence $\mathbf{x}$.