QUESTION

An oil company is considering an offshore drilling venture. A preliminary survey indicates that there may be large (L) or small (S) deposits with probabilities $\mathrm{P}(L) = 0.1$ and $\mathrm{P}(S) = 0.9$. Exploration costs £1 000 000. If large deposits are found, the company makes a profit of £16 000 000, less the exploration cost. If small deposits are discovered, the project will be abandoned.

A team of experts can be employed for £500 000 to perform a survey. They would find that either large deposits are likely (P), large deposits are unlikely (N), or there is inconclusive evidence (I). The previous record of experts is shown in the following table.

	Prediction		
Actual result	P	N	I
L	60%	20%	20%
S	10%	80%	10%

(a) Determine the probabilities of the predictions $\mathrm{P}(P)$, $\mathrm{P}(N)$ and $\mathrm{P}(I)$.

(b) Determine the posterior probabilities for each possible prediction.

(c) Use a decision tree to find what course of action the company should follow.

ANSWER

The table gives

P(P—L)=0.6 P(N—L)=0.2 P(I—L)=0.2

P(P—S)=0.1 P(N—S)=0.8 P(I—S)=0.1

(a)

$$\begin{aligned} P(P) &= P(P|L)P(L) + P(P|S)P(S) = (0.6 \times 0.1) + (0.1 \times 0.9) = 0.15 \\ P(N) &= P(N|L)P(L) + P(N|S)P(S) = (0.2 \times 0.1) + (0.8 \times 0.9) = 0.74 \\ P(I) &= P(I|L)P(L) + P(I|S)P(S) = (0.2 \times 0.1) + (0.1 \times 0.9) = 0.11 \end{aligned}$$

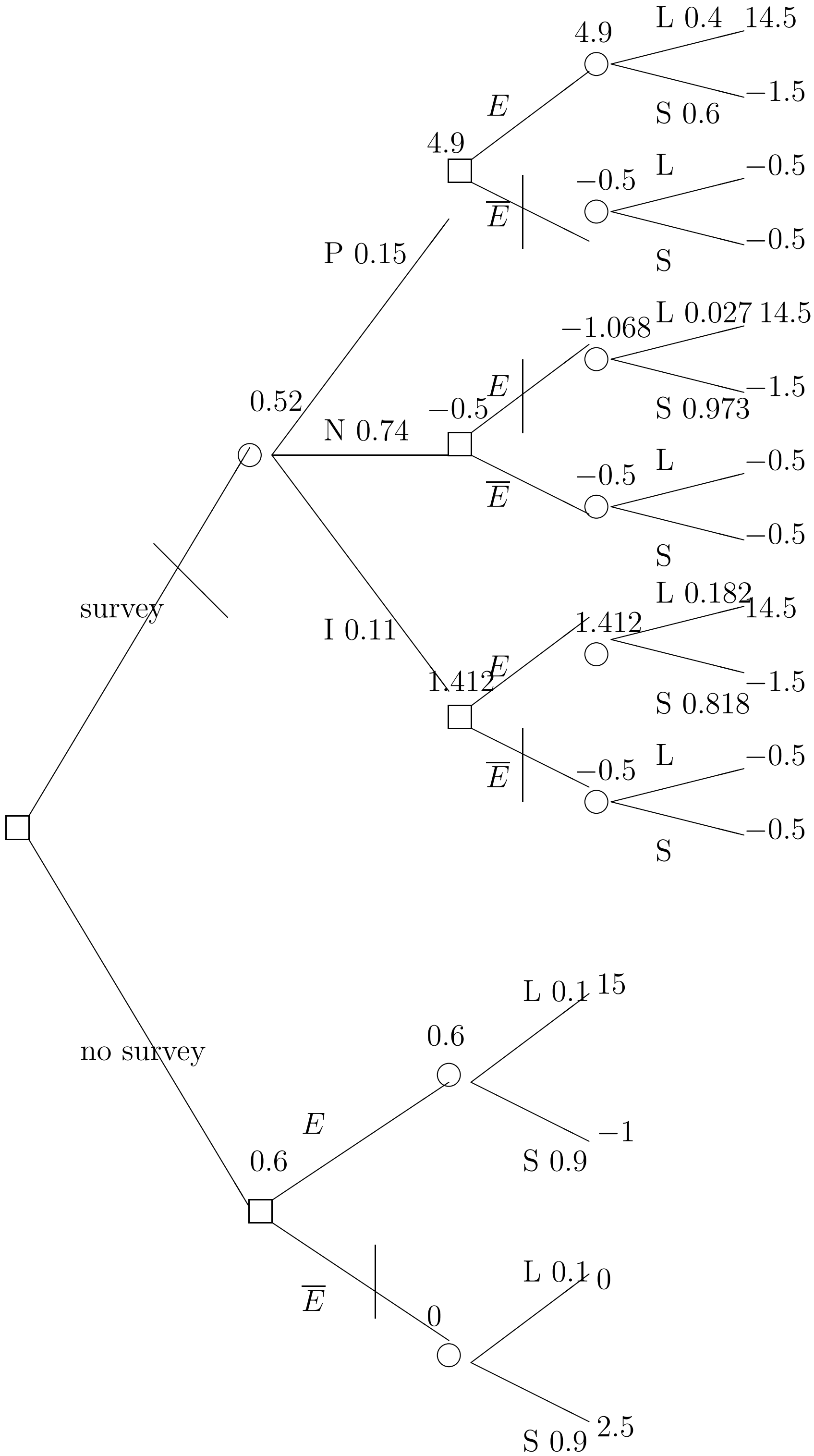
(b)

$$\begin{aligned} P(L|P) &= \frac{P(P \cap L)}{P(P)} = \frac{P(P|L)P(L)}{P(P)} = 0.4 \\ P(S|P) &= \frac{P(P \cap S)}{P(P)} = \frac{P(P|S)P(S)}{P(P)} = 0.6 \\ P(L|N) &= \frac{P(N \cap L)}{P(N)} + \frac{P(N|L)P(L)}{P(N)} = 0.027 \\ P(S|N) &= \frac{P(N \cap S)}{P(N)} = \frac{P(N|S)P(S)}{P(N)} = 0.973 \\ P(L|I) &+ \frac{P(I \cap L)}{P(I)} = \frac{P(I|L)P(L)}{P(I)} = 0.182 \\ P(S|I) &= \frac{P(I \cap S)}{P(I)} = \frac{P(I|S)P(S)}{P(I)} = 0.818 \end{aligned}$$

Returns (without survey) are:

	Action	
	E	$\overline{E}$
L	15	0
S	-1	0

E=explore, $\overline{E}$=do not explore



$P(L) = 0.1,\ P(S) = 0.9$ are prior probabilities.

From the table:

$P(P|L) = 0.6,\ P(N|L) = 0.2\ P(I|L) = 0.2,$
$P(P|S) = 0.1,\ P(N|S) = 0.8,\ P(I|S) = 0.1$