

### Exam Question

#### Topic: Arc Length

Calculate the length of the curve given by

$$x = t^2, y = 2t, (0 \leq t \leq 1).$$

Give your answer both in exact form and also as an approximation rounded to four decimal places, using your calculator.

### Solution

$$L = \int_0^1 \sqrt{x'(t)^2 + y'(t)^2} dt = \int_0^1 \sqrt{4t^2 + 4} dt = 2 \int_0^1 \sqrt{t^2 + 1} dt$$

Let  $t = \sinh u$ ;  $dt = \cosh u du$ .

$$\begin{aligned} L &= \int_0^{\sinh^{-1} 1} 2 \cosh^2 u du \\ &= \int_0^{\sinh^{-1} 1} (1 + \cosh 2u) du \\ &= \left[ u + \frac{\sinh 2u}{2} \right]_0^{\sinh^{-1} 1} \\ &= [u + \sinh u \cosh u]_0^{\sinh^{-1} 1} \\ &= \sinh^{-1} 1 + 1 \cdot \cosh (\sinh^{-1} 1) \\ &= \sinh^{-1} 1 + \sqrt{1 + \sinh^2 (\sinh^{-1} 1)} \\ &= \sinh^{-1} 1 + \sqrt{2} = 2.2956 \text{ (4 d.p.)} \end{aligned}$$