## Exam Question

## Topic: LaplaceODE

Find the solution of the differential equation

$$
\frac{d^{2} y}{d x^{2}}+\frac{d y}{d x}=f(x)
$$

where

$$
f(x)= \begin{cases}0 & \text { if } \quad x<3 \\ 1 & \text { if } x \geq 3\end{cases}
$$

and where $\frac{d y}{d x}=0$ when $x=0$ and $y(0)=1$.

## Solution

Using the Heaviside function we can write $f(x)=H(x-3)$.
Transforming the differential equation gives

$$
\begin{aligned}
p^{2} \bar{y}-p+p \bar{y}-1 & =\frac{\mathrm{e}^{-3 p}}{p} \\
p(p+1) \bar{y} & =\frac{\mathrm{e}^{-3 p}}{p}+(p+1) \\
\bar{y} & =\frac{\mathrm{e}^{-3 p}}{p^{2}(p+1)}+\frac{1}{p} \\
& =\mathrm{e}^{-3 p}\left[\frac{1}{p+1}-\frac{1}{p}+\frac{1}{p^{2}}\right]+\frac{1}{p} \\
\text { So } y(x) & =\left[\mathrm{e}^{-(x-3)}-1+(x-3)\right] H(x-3)+1
\end{aligned}
$$

