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Web Science and Web Technology
„The Small World Problem"

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Overview

Topics

- Definition of the Small World Problem
- Results from a social experiment
- The importance of „weak ties"

Course Organization

Attendance:

* Home assignment 1: 5%
* Home assignment 2: 5%
* Home assignment 3: 5%
* Home assignment 4: 5%
* Home assignment 5: 5%

Prerequisite for obtaining these points: attending 9 out of the following 11 classes (week 2-12, sign the list of attendees)
In other words: you can miss up to two classes, for more information see website

* Home assignment 6: 25%
* Final Exam: 50%

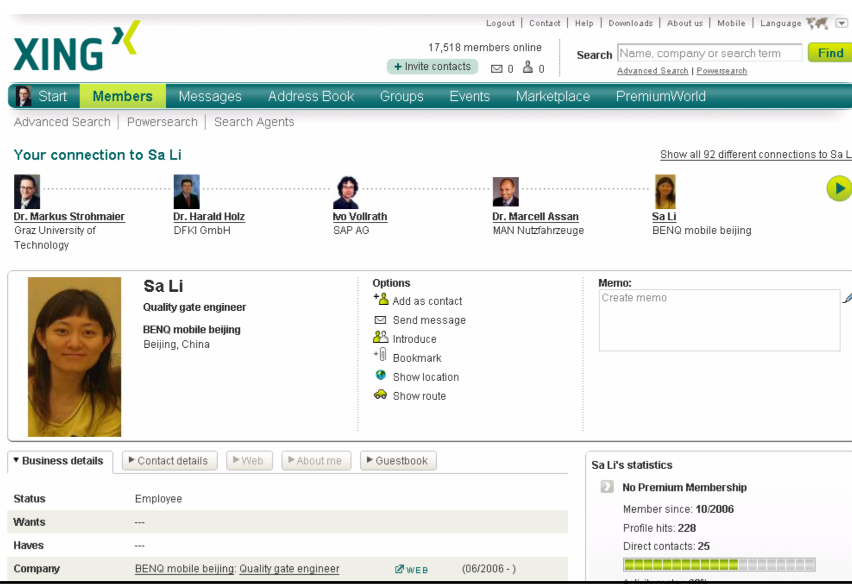
No prerequisites

Communication:

– Your question might be of interest to other students!
– Therefore, before sending an e-mail to the instructor or the teaching assistants, please consider posting it to the course newsgroup *tu-graz.lv.web-science*. The course team reads the newsgroup frequently and will try to answer your question as soon as possible.

No "Nachklausur"

Do I know somebody in …?



The Bacon Number

http://www.imdb.com/name/nm0000102/

The Kevin Bacon Game

The oracle of Bacon

www.oracleofbacon.org

The Bacon Number
[Watts 2002]

TABLE 3.1 DISTRUBUTION OF ACTORS ACCORDING TO BACON NUMBER

BACON NUMBER	NUMBER OF ACTORS	CUMULATIVE TOTAL NUMBER OF ACTORS
0	1	1
1	1,550	1,551
2	121,661	123,212
3	310,365	433,577
4	71,516	504,733
5	5,314	510,047
6	652	510,699
7	90	510,789
8	38	510,827
9	1	510,828
10	1	510,829

The Erdös Number

Who was Erdös?
http://www.oakland.edu/enp/

A famous mathematician, 1913-1996

Erdös posed and solved problems in number theory and other areas and founded the field of discrete mathematics.

- 511 co-authors (Erdös number 1)
- ~ 1500 Publications

The Erdös Number

The Erdös Number:
Through how many research collaboration links is an arbitrary scientist connected to Paul Erdös?

What is a research collaboration link?
Per definition: Co-authorship on a scientific paper ->
Convenient: Amenable to computational analysis

What is my Erdös Number?
➔ **5**

me -> S. Easterbrook -> A. Finkelstein -> D. Gabbay -> S. Shelah -> P. Erdös

Stanley Milgram

- A social psychologist
- Yale and Harvard University
- Study on the Small World Problem, beyond well defined communities and relations (such as actors, scientists, …)
- Controversial: The Obedience Study
- What we will discuss today: *„An Experimental Study of the Small World Problem"*

1933-1984

Introduction

The simplest way of formulating the small-world problem is: **Starting with any two people in the world, what is the likelihood that they will know each other?**

A somewhat more sophisticated formulation, however, takes account of the fact that while person X and Z may not know each other directly, they may share a mutual acquaintance - that is, a person who knows both of them. One can then think of an acquaintance chain with X knowing Y and Y knowing Z. Moreover, one can imagine circumstances in which X is linked to Z not by a single link, but by a series of links, X-A-B-C-D…Y-Z. That is to say, person X knows person A who in turn knows person B, who knows C… who knows Y, who knows Z.

[Milgram 1967, according to]http://www.ils.unc.edu/dpr/port/socialnetworking/theory_paper.html#2]

An Experimental Study of the Small World Problem [Travers and Milgram 1969]

A Social Network Experiment tailored towards

- Demonstrating
- Defining
- And measuring

Inter-connectedness in a large society (USA)

A test of the modern idea of “six degrees of separation”

Which states that: every person on earth is connected to any other person through a chain of acquaintances not longer than 6

Experiment

Goal

- Define a single target person and a group of starting persons
- Generate an acquaintance chain from each starter to the target

Experimental Set Up

- Each starter receives a document
- was asked to begin moving it by mail toward the target
- Information about the target: name, address, occupation, company, college, year of graduation, wife's name and hometown
- Information about relationship (*friend/acquaintance*) [Granovetter 1973]

Constraints

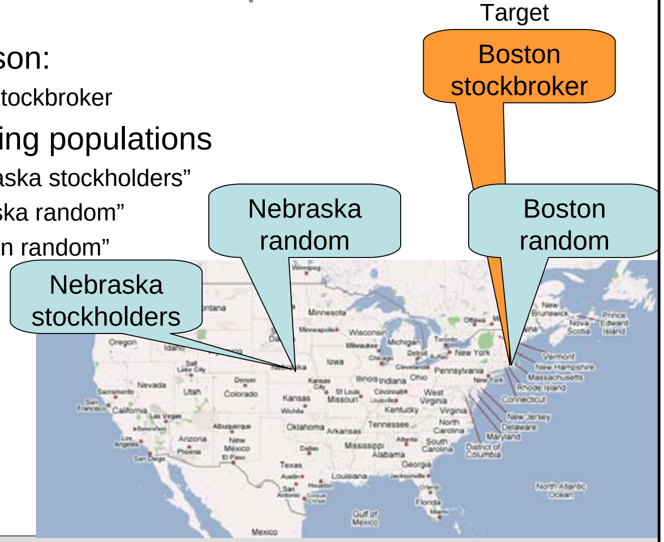
- starter group was only allowed to send the document to people they know and
- was urged to choose the next recipient in a way as to advance the progress of the document toward the target

Questions

- How many of the starters would be able to establish contact with the target?
- How many intermediaries would be required to link starters with the target?
- What form would the distribution of chain lengths take?

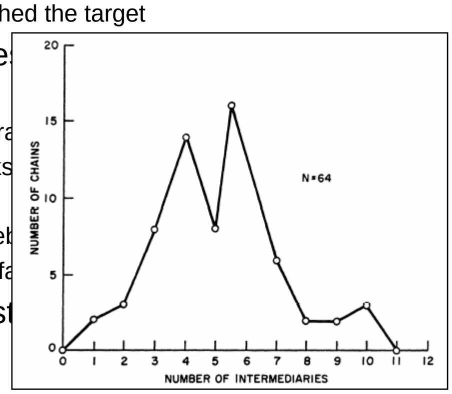
Set Up

- Target person:
  - A Boston stockbroker
- Three starting populations
  - 100 “Nebraska stockholders”
  - 96 “Nebraska random”
  - 100 “Boston random”



Results I

- How many of the starters would be able to establish contact with the target?
  - 64 (or 29%) out of 296 reached the target
- How many intermediarie starters with the target?
  - Well, that depends: the overa
  - Through hometown: 6.1 links
  - Through business: 4.6 links
  - Boston group faster than Neb
  - Nebraska stakeholders not fa
- What form would the dist take?



Results II

- Incomplete chains

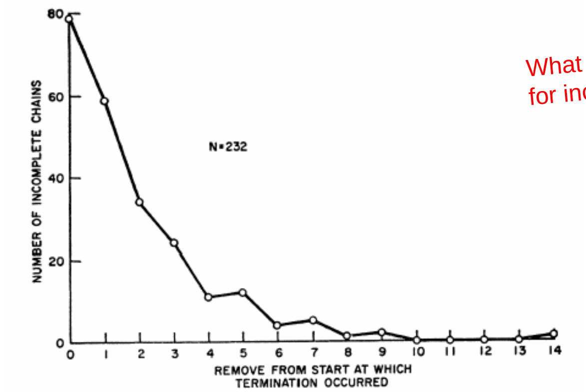


FIGURE 2

Lengths of Incomplete Chains

What reasons can you think of for incomplete chains?

Results III

- Common paths
- Also see:
  Gladwell's "Law of the few"

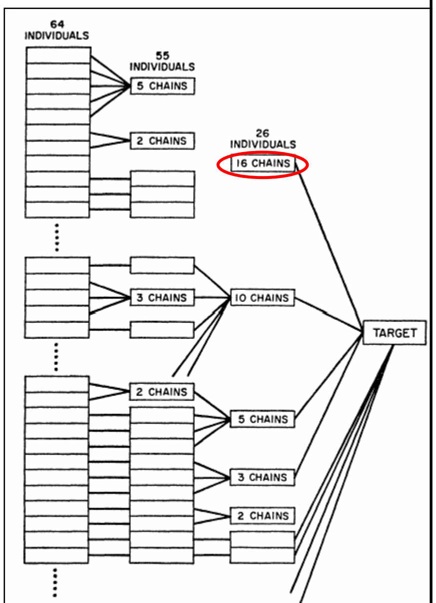


FIGURE 3

Common Paths Appear as Chains Converge on the Target

6 degrees of separation

- So is there an upper bound of six degrees of separation in social networks?

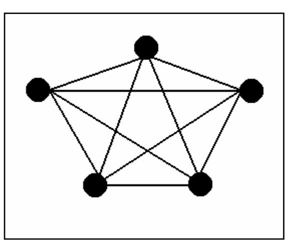
What kind of problems do you see with the results of this study?

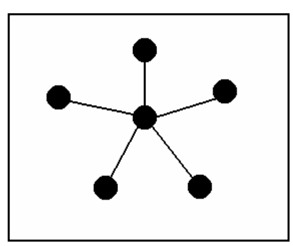
  - Extremely hard to test
  - In Milgram's study, ~2/3 of the chains didn't reach the target
  - 1/3 random, 1/3 blue chip owners, 1/3 from Boston
  - Danger of loops (mitigated in Milgram's study through chain records)
  - Target had a "high social status" [Kleinfeld 2000]

Small Worlds

http://www.infosci.cornell.edu/courses/info204/2007sp/

- Every pair of nodes in a graph is connected by a path with an extremely small number of steps (low diameter)
- Two principle ways of encountering small worlds
  - Dense networks
  - sparse networks with well-placed connectors

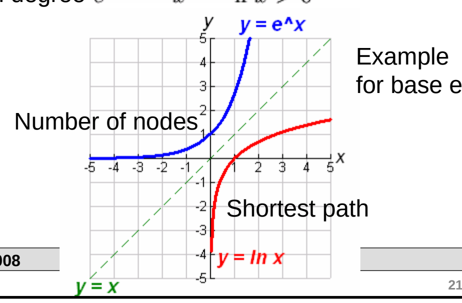




Small Worlds
[Newman 2003]

- The small-world effect exists, if
  - „The number of vertices within a distance r of a typical central vertex grows exponentially with r (the larger it get, the faster it grows) $x(t) = x_0 e^{kt}$

  In other words:
  - Networks are said to show the small-world effect if the value of l (avg. shortest distance) scales logarithmically or slower with network size for fixed mean degree $e^{\ln(x)} = x \quad \text{if } x > 0$



Contemporary Software

- Where does the small-world phenomenon come into play in contemporary software, in organizations, ..?
- Xing, LinkedIn, Myspace, Facebook, FOAF, …
- Business Processes, Information and Knowledge Flow

How do Small World Networks form?

Preferential Attachment [Barabasi 1999]

„The rich getting richer"

Preferential Attachment refers to the high probability of a new vertex to connect to a vertex that already has a large number of connections

Example:

1. a new website linking to more established ones
2. a new individual linking to well-known individuals in a social network

Preferential Attachment Example

Which node has the highest probability of being linked by a new node in a network that exhibits traits of preferential attachment?

Example

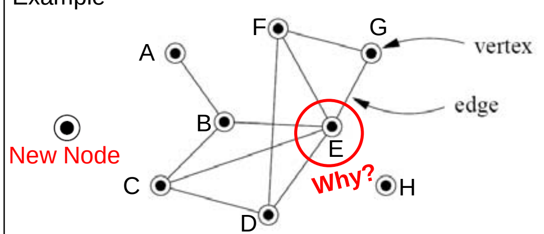


FIG. 1 A small example network with eight vertices and ten edges. [Newman 2003]

Assortative Mixing (or Homophily) [Newman 2003]

Assortative Mixing refers to selective linking of nodes to other nodes who share some common property

- E.g. degree correlation
  high degree nodes in a network associate preferentially with other high-degree nodes
- E.g. social networks
  nodes of a certain type tend to associate with the same type of nodes (e.g. by race)

Assortative Mixing (or Homophily) [Newman 2003]

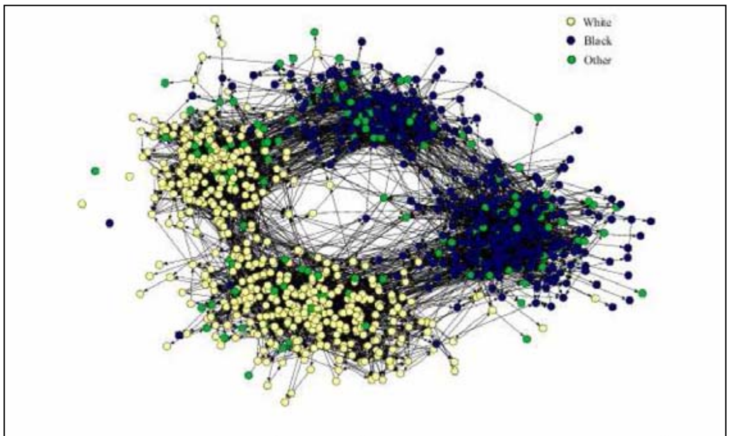


FIG. 8 Friendship network of children in a US school. Friendships are determined by asking the participants, and hence are directed, since A may say that B is their friend but not *vice versa*. Vertices are color coded according to race, as marked, and the split from left to right in the figure is clearly primarily along lines of race. The split from top to bottom is between middle school and high school, i.e., between younger and older children. Picture courtesy of James Moody.

Disassortativity
[Newman 2003]

Disassortativity refers to selective linking of nodes to other nodes who are different in some property

- E.g. the web
  low degree nodes tend to associate with high degree nodes

Network Resilience
[Newman 2003]

The resilience of networks with respect to vertex removal and network connectivity.

If vertices are removed from a network, the typical length of paths between pairs of vertices will increase – vertex pairs will be disconnected.

Examples:

1. Deletion of a hub
2. Deletion of a leaf node element

The web is highly resilient against random failure of vertices, but highly vulnerable to deliberate attack on its highest-degree vertices

Network Resilience
[Newman 2003]

Delete the node with the highest degree, what happens to the network?

Deleting which nodes introduces a new component?

Example

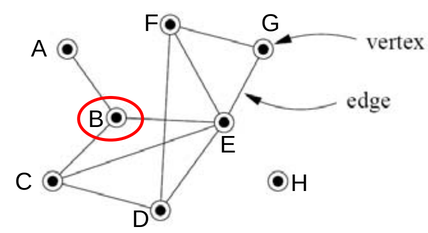


FIG. 1 A small example network with eight vertices and ten edges.

[Newman 2003]

Connectivity: a function of whether a graph remains connected when nodes and/or lines are deleted. [Wassermann 1994]

Network Resilience
[Newman 2003]

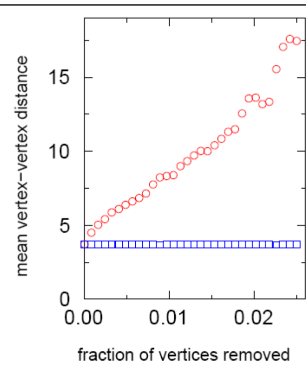


FIG. 7 Mean vertex–vertex distance on a graph representation of the Internet at the autonomous system level, as vertices are removed one by one. If vertices are removed in random order (squares), distance increases only very slightly, but if they are removed in order of their degrees, starting with the highest degree vertices (circles), then distance increases sharply. After Albert *et al.* [15].

Removal of high degree nodes first

Removal of random nodes

Connectivity of the Web [Newman 2003, Broder et al 2000]

What does it need to destroy the connectivity of the web?

According to Broder et al 2000, you need to remove all vertices with a degree greater than five.

Because of the highly skewed degree distribution of the web, the fraction of vertices with degree greater than five is only a small fraction of all vertices.

But …

Isn't all of this an over simplification of the world of social systems?

- Ties/relationships vary in intensity
- People who have strong ties tend to share a similiar set of acquaintances
- Ties change over time
- Nodes (people) have different characteristics, and they are *actors*
- …

The Strength of Weak Ties [Granovetter 1973]

The strength of an interpersonal tie is a

- (probably linear) combination of the amount of time
- The emotional intensity
- The intimacy
- The reciprocal services which characterize the tie

Mark Granovetter,

Stanford University

Can you give examples of strong / weak ties?

The Strength of Weak Ties and Mutual Acquaintances [Granovetter 1973]

Consider:

Two arbitrarily selected individuals A and B and

The set S = C,D,E of all persons with ties to either or both of them

Hypothesis:

The stronger the tie between A and B, the larger the proportion of individuals in S to whom they will both be tied.

Theoretical corroboration:

Stronger ties involve larger time commitments – probability of B meeting with some friend of A (who B does not know yet) is increased

The stronger a tie connecting two individuals, the more similar they are

The Strength of Weak Ties
[Granovetter 1973]

The forbidden triad

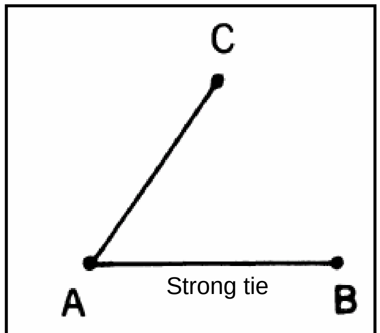


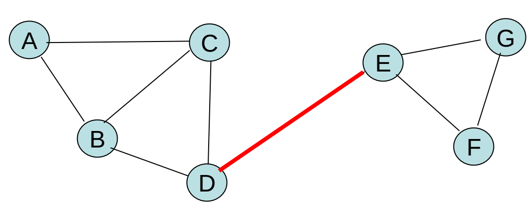
FIG. 1.—Forbidden triad

Why is it called the forbidden triad?

Bridges
[Granovetter 1973]

A bridge is a line in a network which provides **the only path** between two points.

In social networks, a bridge between A and B provides the only route along which information or influence can flow from any contact of A to any contact of B



Which edge represents a bridge? Why?

Bridges and Strong Ties [Granovetter 1973]

Example:

1. Imagine the strong tie between A and B
2. Imagine the strong tie between B and C
3. Then, the forbidden triad **implies** that a tie **exists** between C and B (it forbids that a tie between C and B does not exist)
1. From that follows, that A-B is not a bridge (because there is another path A-B that goes through C)

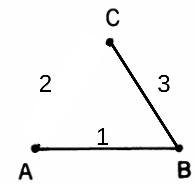


FIG. 1.—Forbidden triad

Why is this interesting?

⇨Strong ties can be a bridge ONLY IF neither party to it has any other strong ties

⇨Highly unlikely in a social network of any size

⇨Weak ties suffer no such restriction, though they are not automatically bridges

⇨But, all bridges are weak ties

In Reality …. [Granovetter 1973]

it probably happens only rarely, that a specific tie provides the only path between two points

Local bridges: the shortest path between its two points (other than itself)

- Bridges are efficient paths
- Alternatives are more costly
- Local bridges of degree n
- A local bridge is more significant as its degree increases

What's the degree of a bridge in an absolute sense?

Alternative

Alternative

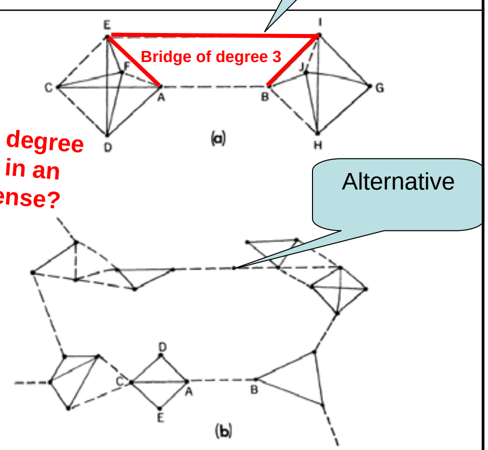


FIG. 2.—Local bridges. *a*, Degree 3; *b*, Degree 13. ——— = strong tie; – – – = weak tie.

In Reality …

Strong ties can represent *local* bridges BUT
They are weak (i.e. they have a low degree)

Why?

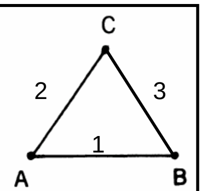


Fig. 1.—Forbidden triad

What's the degree of the local bridge A-B?

Implications of Weak Ties [Granovetter 1973]

- Those weak ties, that are local bridges, create more, and shorter paths.
- The removal of the average weak tie would do more damage to transmission probabilities than would that of the average strong one
- **Paradox**: While *weak ties* have been denounced as generative of alienation, *strong ties*, breeding local cohesion, lead to overall fragmentation

What are sources of weak ties/bridges?

Can you identify some implications for social networks on the web / for search in these networks?

How does this relate to Milgram's experiment?

Completion rates in Milgram's experiment were reported higher for acquaintance than friend relationships [Granovetter 1973]

Implications of Weak Ties [Granovetter 1973]

- Example: Spread of information/rumors in social networks
  - Studies have shown that people rarely act on mass-media information unless it is also transmitted through personal ties [Granovetter 2003, p 1274]
  - Information/rumors moving through strong ties is much more likely to be limited to a few cliques than that going via weak ones, bridges will not be crossed

How does information spread through weak ties?

Next Week

We will have a look at

Network theory and terminology including (excerpt)

- Degree
- Degree distributions
- Clustering Co-efficients
- Random networks
- Scale Free networks
- And others

Any questions?

See you next week!