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WORKING PAPER

COG-FPOM: Adapted Fuzzy Pay-Off Method for Real Options Valuation – Application in the Abandonment Decision of Petroleum Producing Fields

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COG-FPOM: Adapted Fuzzy Pay-Off Method for Real Options Valuation – Application in the Abandonment Decision of Petroleum Producing Fields

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Abstract

This paper presents the COG-FPOM, a model based on the Fuzzy Pay-Off Method (FPOM). The FPOM is a scenario-based real option valuation method that uses fuzzy numbers as possibility distributions. The paper shows an unexpected result generated by the original FPOM, in which the real option would have a negative value. It further analyses its reasons and suggests a way to overcome it, by using the center of gravity (COG) instead of the possibilistic mean to summarize a fuzzy number. The overall work is an ongoing project that aims to apply the presented model to support the abandonment decision of petroleum producing fields – an initial attempt together with its preliminary outcomes are shown here. Although not concluded, the COG-FPOM and its pilot results indicate a good potential for the sequence of the project.

1. Introduction

Many petroleum producing fields located at mature sedimentary basins around the world are approaching the end of their lifetimes. Every time oil prices fall, companies face the question of whether to stop the production or not. Lower oil prices mean lower revenue and consequently lower results for the field, both present and forecasted. In principle, producers are supposed to abandon the field as soon as its results are negative, that is when the revenues from the field are less than the costs of producing its oil. However, as Jafarizadeh and Bratvold (2012) highlight, given the uncertain nature of both revenue and cost elements, the decision maker has to continuously evaluate the expected values of either continuing the production or abandoning the field. Since the abandonment decision is usually irreversible, the company might miss future profits in case of a potential scenario improvement.

In the oilfield abandonment case, the corporate methods for supporting the decision are based on future cash flow projections. However, these traditional valuation methods typically use a single static most likely yearly value to generate the decision suggestion. In this way, less likely possibilities (potentially with high impact) are not considered in the analysis. In order to deal with the uncertainty – and the flexibilities this can offer to the decision makers –, the real option analysis shows up as an important valuation tool. Following Carlsson and Fullér (2011), the decision rule, derived from option pricing theory, is that producers should only abandon the field now if the value of this action is high enough to compensate for giving up the value of the option to wait.

The commonly used models for computing the real option value are based on the methods that have been used to value financial options: differential equation-based, especially Black-Scholes option pricing formula; lattice-based, especially the binomial option valuation method; and simulation-based methods. Most of these models are complex and are based on the assumption that they can accurately model the underlying markets. As Collan, Fullér, and Mezei (2009) point out, this assumption may be acceptable for some financial securities, like stocks and currencies, which are quite efficiently traded.

As opposed to the aforementioned methods, Favato, Cottingham, and Isachenkova (2015) defend the use of scenario planning in real option valuation. In their argument, what decision-makers need is a flexible valuation tool that is easy to understand and which can be lightly re-executed any time after the first decision is made – for example, when new information become available. There are two main methods that follow that premise: the Datar-Mathews method (DMM) (Mathews, Datar, & Johnson, 2007) and the Fuzzy Pay-Off Method (FPOM) (Collan, Fullér, & Mezei, 2009). They both use forecasted scenarios for the cash flows to derive a distribution of net present value (NPV) for the project. While DMM uses simulation to generate a probability distribution and its associated probabilistic expected value, FPOM utilises the possibilistic expected value out of a fuzzy number, which can also be seen as a possibility distribution (Zadeh, 1978). Favato, Cottingham, and Isachenkova (2015) show that, all else equal, the application of FPOM is feasible and useful without the necessity to engage in high-level and daunting mathematics.

The objective of this paper is to present a model based on the FPOM to compute the value of an oilfield with abandonment real option and support the correspondent decision of interrupting or continuing its production. Section 2 presents an overview of the oilfield abandonment process, focusing on the analytical tools used to support this decision. Section 3 discusses real option valuation, from the traditional methods all the way to FPOM – it also gives an overview of fuzzy sets. The developed COG-FPOM model is presented in more details in section 4, focusing on suggested improvements when compared to original FPOM. In section 5 an application of the model is shown together with an analysis of its results. Section 6 finalizes the paper with conclusions and suggestions for future works.

2. Petroleum Field Abandonment Decision

Petroleum exploration and production (E&P) is an activity that encompasses several phases, the last of which is the oilfield abandonment. As stated by Parente et al. (2006), this last stage highlights a difference from E&P to many others industries: the projects typically present an additional third period of cash flow – after the investment and production phases. This abandonment cash flow refers to all decommissioning expenses, which are costly and involves regulatory and environmental considerations (Osmundsen & Tveterås, 2003). Additionally, the companies should also account for the potential value of selling or reusing production equipment and thus, as emphasized by Jafarizadeh and Bratvold (2012), the abandonment cash flow can be either negative or positive.

The abandonment decision draws special attention when the production rate of a field approaches an economic limit below of which continuing its production would yield a net loss. In principle, producers are supposed to abandon the field as soon as its results are negative, that is when the revenues from the field are less than the costs of producing its oil. However, the timing of abandoning is a tough decision because the uncertainty of the future increases the difficulty of *ex-ante* analysis (Taleb, 2007). Therefore, only the course of time will tell if it the right call was made – both if the decision was to abandon or not.

Considering that the decommissioning is irreversible, the abandonment decision prunes all alternative development options and may prevent future profits, which might be possible under improving conditions. On the other hand, as explained by Carlsson and Fullér (2011), the company may have a difficult time with stakeholders if they continue producing from an oilfield in conditions which cut into its profitability.

Following Dias (2015), the traditional method to support the oilfield abandonment decision is to build yearly operational cash flow projections and suggest to produce until the year which has the last positive estimate, abandoning in the following year. As a way to consider the estimated abandonment cash flow into the analysis – especially when it is negative – some companies calculate the benefit of postponing this expense and producing even after the projection is negative. This approach, which may be seen as an opportunity cost analysis, basically accounts for the fact that investing the abandonment value – instead of spending it – will yield some profit that may compensate the expected operational loss.

The aforementioned approaches have an important issue in common: they rely on one single most likely estimate of cash flow items, for example: future production rate, petroleum price and abandonment cost. However, although the subsurface models used nowadays are very sophisticated, the future production rate remains uncertain (Bickel & Bratvold, 2008); several models try to represent the petroleum market behaviour (Dias, 2004), but, as a commodity, its prices are unpredictable (Jafarizadeh & Bratvold, 2012); the abandonment costs are highly uncertain, mainly because of the industry's lack of experience (Parente et al., 2006).

Since our forecasting ability is limited, Bickel and Bratvold (2008) suggest that the focus should be on making good decisions, instead of reducing/removing uncertainty. In this way, less likely possibilities may carry important information regarding the decision (Taleb, 2007), and therefore should be considered. Moreover, the static most likely value used in the traditional methods consider that the decision depends only on the information available now, ignoring the additional information that might be revealed in the future.

The options offered by this flexibility can be modelled by decision trees. However, Jafarizadeh and Bratvold (2012) observe that the optimization that occurs at each downstream node changes the expected future cash flow of the project, which changes its risk characteristics and prevent the achievement of a correct result. The uncertain petroleum prices, complex cash flows structures, and interrelated decisions transform the timing of oilfield abandonment into a complex real option (Jafarizadeh & Bratvold, 2012), discussed in the next section.

3. Real Options Valuation

Real options (RO) valuation is a methodology that highlights the value of managerial flexibility to respond optimally to the uncertainty. By observing that corporate investments opportunities can be viewed as financial call options on real assets, Myers coined the term "real options" (Myers, 1977 apud Dias, 2004). Collan, Fullér, and Mezei (2009) point out that RO may be seen both as a qualitative method, like a mental model to analyze options for operational and strategic decision-making, and as a quantitative method, like a tool to perform numerical analysis for valuation purposes.

It comes as no surprise that the first efforts to valuate RO were to use financial options pricing techniques. The Black-Scholes (BS) model (Black & Scholes, 1973), which derives a partial differential equation and uses its solution to predict the price of the option, is the base of many RO valuation methods. The binomial option pricing model originally uses a lattice-based model to provide a discrete-time approximation to the continuous process used in BS (Cox, Rox, & Rubinstein, 1979). These two methods assume that the underlying asset prices follow respectively a geometric Brownian motion and its discrete analog, but other stochastic processes were used in subsequently derived models. Dias (2004) made an extensive survey of stochastic processes used to model oil prices, including properties like mean-reversion and jumps. Simulation-based methods, also known as Monte Carlo techniques may also be used to calculate the value of an option combining multiple sources of uncertainty in complex RO models (Dias, 2017). One of the earliest simulation models for option valuation was presented by Boyle (1977). Simulation overcomes the "curse of dimensionality", and allows the use of different probability distributions, especially in problems without analytical solution.

According to Favato, Cottingham, and Isachenkova (2015), RO research took the direction of searching for more sophisticated statistical models, increasing the complexity of calculus instead of focusing on management relevance. In the same direction, Mathews, Datar, and Johnson (2007) argue that the field of RO has been slow to develop because of the complexity of the techniques and the difficulty of fitting them to the realities of corporate strategic decision-making.

Another criticism of the aforementioned "realistic" models is about the assumption that they can adequately mimic the underlying markets. As Collan, Fullér, and Mezei (2009) point out, this assumption may hold for some financial securities – like stocks and currencies, which are quite efficiently traded –, but may not hold for real investments that do not have existing markets or whose markets don't exhibit even weak market efficiency. Jafarizadeh and Bratvold (2012) state that being a commodity, oil prices are simply impossible to be correctly forecasted. An additional observation is that the traditional methods require the uncertainty to be typically of the parametric type, not considering structural or procedural uncertainty (Collan, Haahtela, & Kyläheiko, 2016).

In favour of blending scenarios into RO valuation, Favato, Cottingham, and Isachenkova (2015) say that companies should not be restricted to single forecasts, which are like predictions; instead, scenarios should be seen as speculative descriptions of possible outcomes for the future, widening the chances of capturing potential opportunities and threats. By encouraging managers to envision future states of the world, scenario planning is a strategic management tool primarily used for qualitative analysis. If combined with RO, however, scenario planning may contribute to powerful quantitative assessments. In this way, decision-makers can work with a flexible valuation tool that is easy to understand and which can be lightly re-executed any time after the first decision is made – for example, when new information become available. This approach also allows for using separate risk adjusted discount rates for different cash flow items – like operational revenues, operational costs and capital investment – thus better representing the different types and levels of uncertainty within a project.

There are two main scenario-based methods for RO valuation: the probability-based Datar-Mathews method (DMM), introduced by Mathews, Datar, and Johnson (2007), and the possibility-based Fuzzy Pay-Off Method (FPOM), introduced by Collan, Fullér, and Mezei (2009). They both use (usually three) forecasted scenarios for the cash flows to derive a distribution of NPVs. Mathews, Datar, and Johnson (2007) state that this look and feel of an extended NPV analysis, with easy modelling in a spreadsheet, gives more transparency to the method and helps its understanding and adoption by practitioners.

The DMM uses Monte Carlo simulation to generate a probability distribution of NPVs – in this way it can also be seen as a simulation-based model. A risk-adjusted pay-off distribution is then derived, considering flexibilities and rational decision-making – negative outcomes would not proceed and are valued as zero. At that time, it's possible to calculate the value of the project considering the options, and consequently the RO value. The authors show that their results converge to the results from the analytical BS method if the same assumptions are used.

When choosing a RO valuation type, it is important to identify the sort of uncertainties that characterise the available information – Collan, Haahtela, & Kyläheiko (2016) discuss the usability of some RO valuation types under different types of uncertainty: parametric, structural and procedural. In this way, the different RO valuation methods are not competitors to each other, as there are more adequate models for each real world decision-making situation.

Kozlova, Collan, and Luukka, (2016) say that DMM is usable when there is enough information available for the construction of a credible model to underlie a simulation. However, when the information available is imprecise, incomplete, non-stochastic and include normative judgement of managers, a probability-based RO valuation method may not be suitable (Luukka & Collan, 2015). An alternative to model this kind of uncertainty is the use of fuzzy sets, which are a basis for a possibility theory (Zadeh, 1978). The next subsection gives an overview of fuzzy sets, showing its benefits for investment analysis models and finally presenting the FPOM in more details.

3.1. Fuzzy Sets and the Fuzzy Pay-Off Method

Zadeh (1965) introduced fuzzy sets to mathematically represent imprecise and vague information and to provide formalized tools for dealing with these non-statistical uncertainties intrinsic to human language and perception – for example, in cash flow projections. Extending the classical crisp sets, to which an element may either belong or not, a fuzzy set assigns a real number between zero (complete non-membership) and one (complete membership) to each element of its universe of discourse – values in between represent a gradation of belonging. This flexibility may be helpful in making explicit the imprecision with which experts and modellers estimate parameter values used in models (Collan, Haahtela, & Kyläheiko, 2016).

Let X be a nonempty classical set, known as the universe of discourse. A fuzzy set A of X is characterized by its membership function:

$$\mu_A : X \to [0,1] \quad (1)$$

The fuzzy set A is called normal if there is at least one element $x \in X$ with membership as close to the unity as desired: $\sup_{x \in X} \mu_A(x) = 1$. Otherwise it is called subnormal. The support of A is a crisp subset of X whose elements all have non-zero membership degrees in A: $supp(A) = \{x \in X \mid \mu_A(x) > 0\}$. The core of A is a crisp subset of X whose elements all have full membership degrees in A: $C(A) = \{x \in X \mid \mu_A(x) = 1\}$.

The support and the core may be seen as the largest and the smallest classical sets characterizing A, but sometimes it may be of interest to represent the fuzzy set by another crisp set between them. For $\alpha \in [0,1]$, an α-level set (or α-cut) of A is defined by:

$$[A]^\alpha = \begin{cases} \{x \in X \mid A(x) \geq \alpha\}, & if\ \alpha > 0, \\ cl\big(supp(A)\big), & if\ \alpha = 0, \end{cases} \quad (2)$$

where $cl\big(supp(A)\big)$ denotes the closure of the support of A. A fuzzy set A of X is called convex if $[A]^\alpha$ is a convex subset of X for all α (if $X = \mathbb{R}$, A is convex if $[A]^\alpha$ is a connected set, that is an interval, for all α). A fuzzy number A of X is a fuzzy set of the real line ($X = \mathbb{R}$) with a normal, convex and upper semi-continuous membership function of bounded support.

Triangular fuzzy numbers are commonly used in problem modelling and may be seen as representing the statement "*x is approximately equal to a*". A triangular fuzzy number A with peak (or centre) a, left width $\alpha > 0$ and right width $\beta > 0$ may be referenced as $A = (a, \alpha, \beta)$ and has a membership function of the form:

$$\mu_A(x) = \begin{cases} 1 - \dfrac{a - x}{\alpha}, & if\ a - \alpha \leq x \leq a \\ 1 - \dfrac{x - a}{\beta}, & if\ a \leq x \leq a + \beta \\ 0, & otherwise \end{cases} \tag{3}$$

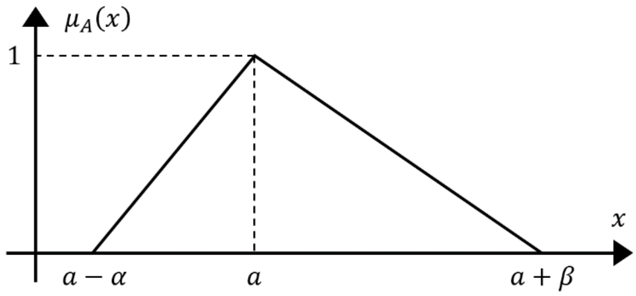


Figure 3.1 – Triangular fuzzy number

As explained by Zadeh (1978), fuzzy numbers may be seen as possibility distributions. To notice the difference in the interpretation, consider a fuzzy number $young$, for which the numerical age $x = 28$ has a grade of membership $\mu_{young}(28) = 0.7$. The usual way of seeing this is that 0.7 depicts the degree of compatibility of 28 with the concept labeled young (fuzzy restriction). The other interpretation is that 0.7 represents the degree of possibility that somebody is 28 given the proposition that this person is young (possibility distribution).

In general, a variable may be associated both with a possibility distribution and a probability distribution, with the weak connection between the two expressed as the possibility/probability consistency principle (Zadeh, 1978). Carlsson and Fullér (2011) state that probability distributions can be interpreted as carriers of incomplete information, whereas possibility distributions can be interpreted as carriers of imprecise information. Kuchta (2000) argues that probability theory is much less flexible than fuzzy sets theory because it has several assumptions about their distributions and operations that are seldom fulfilled in investment decisions cases.

Collan, Haahtela, and Kyläheiko (2016) made a survey regarding fuzzy numbers utilization in RO valuation. They show the use of fuzzy numbers together with differential equation-based models, lattice-based models and decision tree approaches. These fuzzy versions of RO analysis methods are generally usable under the same types of uncertainty as the underlying original methods with crisp numbers.

The Fuzzy Pay-Off Method (FPOM) uses NPVs of (usually three) cash flow scenarios to create a (usually triangular) pay-off fuzzy number – or possibility distribution. In this way, that fuzzy number illustrates the degree to which a particular NPV estimate belongs to the set of possible NPVs. Much like DMM does to its (probability) pay-off distribution in order to include the RO flexibility within a project, FPOM maps the negative NPV values of its (possibility) pay-off distribution into zero, reflecting the right of not proceeding with the project if a negative outcome is expected. Figure 3.2 illustrates this procedure, showing the "original" and the "modified" distributions.

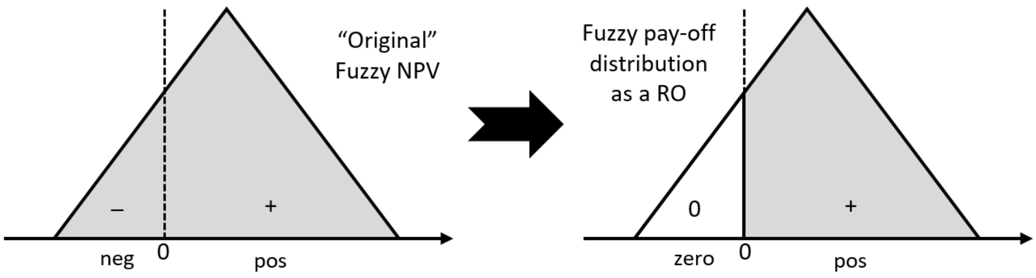


Figure 3.2 – FPOM's creation of pay-off distribution as a RO, based on Collan, Haahtela, & Kyläheiko (2016)

With the purpose of obtaining the value of the project with RO, it is necessary to calculate a most likely value of this modified distribution. In FPOM, this is done by calculating the possibilistic mean of the positive side of the distribution – according to the definition by Carlsson and Fullér (2001) – and multiplying it by the fraction of the positive area of the distribution over its whole area. It is important to notice that this operation is effectively valuing all negative outcomes as zero. As defined by Collan, Fullér, and Mezei (2009):

$$ROV = E(A_+) \times \frac{\int_0^\infty A(x)dx}{\int_{-\infty}^\infty A(x)dx} \quad (4)$$

where A stands for the fuzzy pay-off distribution; $E(A_+)$ denotes the possibilistic mean value of the positive side of A; $\int_0^\infty A(x)dx$ computes the area below the positive part of A and $\int_{-\infty}^\infty A(x)dx$ computes the area below the whole fuzzy pay-off distribution.

The FPOM for RO valuation has been used for analysis of research and development projects (Collan & Luukka, 2013), patents (Collan & Heikkilä, 2011), (Collan & Kyläheiko, 2013) investments into information systems (Collan, Björk, & Kyläheiko, 2014), corporate acquisitions (Collan & Kinnunen, 2011), and large industrial investments (Collan, 2011). As argued by Collan, Haahtela, and Kyläheiko (2016) the method's input can range from hunches to detailed qualitative historical data-based information, which means that it can be useful not only under parametric, but also structural and procedural uncertainty. The price for this flexibility is that the output is not a precise RO valuation, but directions to be followed – which is in line with Bickel and Bratvold (2008) reasoning. Moreover, it is possible to argue that "it is better to be vaguely right than exactly wrong" (Read, 1898, p. 351). Finally, Favato, Cottingham, and Isachenkova (2015) show that, all else equal, the application of FPOM is feasible and useful without the necessity to engage in high-level and daunting mathematics.

4. Proposed COG-FPOM Model

The original FPOM was proposed for the case in which the options are either to invest in a development project or not. Therefore, the alternative pay-off has a value of zero, reflecting the right of not proceeding with the project if a negative outcome is expected (see Figure 3.2). In the oilfield abandonment case, the alternative options are slightly different: the company can either continue or stop the petroleum production. As explained in section 2. , the abandonment pay-off (APO) may be negative or positive, meaning it is not necessarily (almost never) zero. In this way, the first adaptation in the original FPOM is required: the model should reflect the right of not continuing with the production in those cases where the expected outcomes are below the expected APO – and not below zero.

In order to cope with this necessity, this model translates the forecasts, so that the expected APO may be thought of as being zero. To keep the coherence, it is necessary to also subtract the expected APO value from the three estimates (pessimistic, base and optimistic) that characterize the triangular fuzzy NPV of the production (step 1). After performing the FPOM calculations (step 2), it is essential to add the APO to the obtained value of the field with the RO, so that the result is meaningful (step 3). Figure 4.1 illustrates this procedure for a negative expected APO and negative base case, without loss of generality.

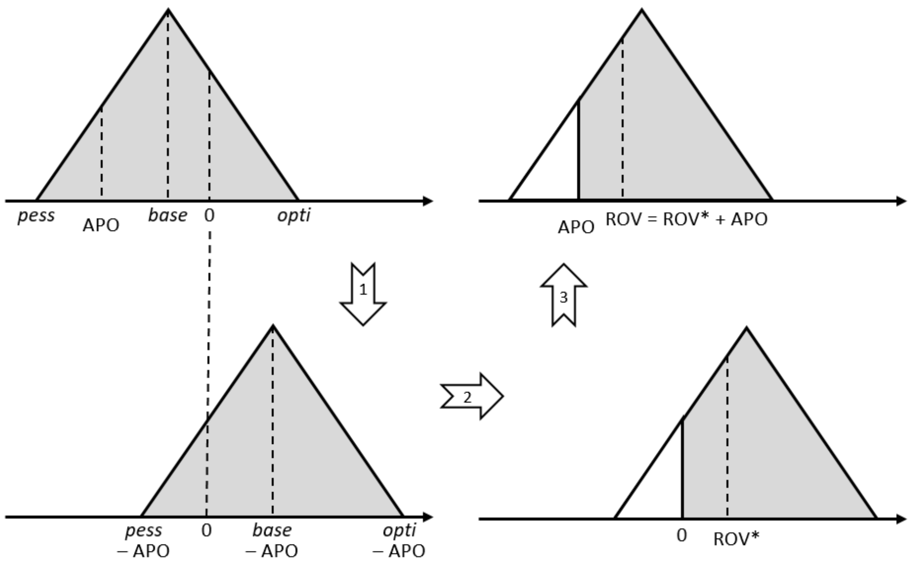
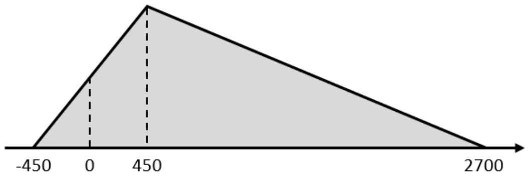


Figure 4.1 – First adaptation: procedure to use FPOM with non-zero alternative

As remarked in section 3. , RO should add value to the company, either by upgrading profit opportunities or by mitigating downside risks. Ceteris paribus, a project with RO is worth more than the same project without RO – in the limit when the option is worthless, the values should be equal (Amram & Kulatilaka, 1999). It happens that the original FPOM might not always follow this premise. Considering that the value of a project without RO is calculated by the possibilistic mean of the whole fuzzy number – counting on expected negative outcomes –, there are cases in which the project without RO results in a higher value than the same project with RO. The following example in Figure 4.2 shows one such case.

NPV Estimate	Value (MM $)
Pessimistic	-450
Base	450
Optimistic	2700



$$E(A_+) = 693.8$$

$$\int_0^{\infty} A(x)dx = 1462.5$$

$$\int_{-\infty}^{\infty} A(x)dx = 1575$$

$$ROV = 644.2 \quad < \quad E(A) = 675$$

Figure 4.2 – Example of unexpected result using FPOM: the value of the option would be negative

The reason for this surprising result seems to be related to the method used for generating a crisp representative value out of the fuzzy number – original FPOM uses the possibilistic expected value, as defined by Carlsson and Fullér (2001). With the purpose of avoiding these unexpected situations, another modification is now suggested to the FPOM. Instead of using the possibilistic mean to obtain the single representative value from a fuzzy number, this model uses the center of gravity (COG), the most popular defuzzification technique and widely utilized in actual applications (Bai & Wang, 2006). This method is similar to the formula for calculating the center of gravity in physics, and gives name to the proposed model: COG-FPOM. The weighted average of the membership function is computed to be the most representative value of the fuzzy number:

$$COG(A) = \frac{\int_{-\infty}^{\infty} x\, \mu_A(x)dx}{\int_{-\infty}^{\infty} \mu_A(x)dx} \quad (5)$$

The calculation of the COG of the positive side of the fuzzy pay-off distribution (see Figure 3.2) depends on where the zero pay-off is located within the fuzzy number. In order to have analytical solutions – which can be readily incorporated in spreadsheet software – equation (5) was solved for the four possible locations that the zero may be in relation to a triangular fuzzy number $A = (a, \alpha, \beta)$. It is worth mentioning that the COG-FPOM can easily be derived to accommodate four scenarios and trapezoidal fuzzy numbers.

→ *Case 1:* $0 < a - \alpha$

In this situation, depicted in Figure 4.3, the whole fuzzy number is above zero, and the COG is calculated for the entire triangle. The result for this case is also used to calculate the value of the project without the option.

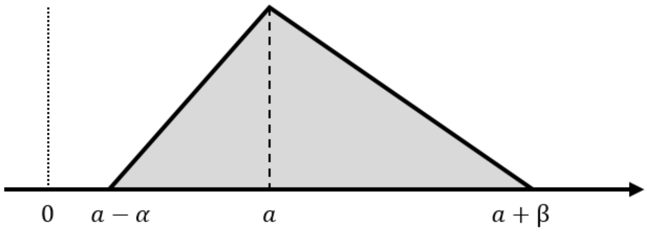


Figure 4.3 – Fuzzy pay-off distribution as RO with $0 < a - \alpha$

$$COG(A_+) = \frac{\int_{a-\alpha}^{a} x\left(1 - \frac{a-x}{\alpha}\right)dx + \int_{a}^{a+\beta} x\left(1 - \frac{x-a}{\beta}\right)dx}{\int_{a-\alpha}^{a}\left(1 - \frac{a-x}{\alpha}\right)dx + \int_{a}^{a+\beta}\left(1 - \frac{x-a}{\beta}\right)dx} = \frac{3a - \alpha + \beta}{3}$$

→ *Case 2:* $a - \alpha < 0 < a$

This situation is depicted in Figure 4.4, and the equation is developed thereafter.

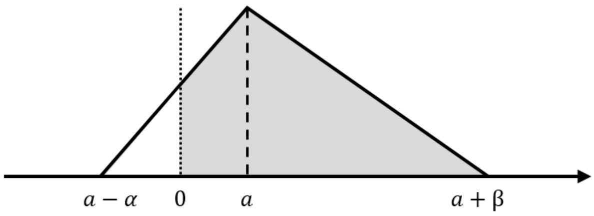


Figure 4.4 – Fuzzy pay-off distribution as RO with $a - \alpha < 0 < a$

$$COG(A_+) = \frac{\int_{0}^{a} x\left(1 - \frac{a-x}{\alpha}\right)dx + \int_{a}^{a+\beta} x\left(1 - \frac{x-a}{\beta}\right)dx}{\int_{0}^{a}\left(1 - \frac{a-x}{\alpha}\right)dx + \int_{a}^{a+\beta}\left(1 - \frac{x-a}{\beta}\right)dx} = \frac{\alpha(a+\beta)^3 - a^3(\alpha+\beta)}{3[\alpha(a+\beta)^2 - a^2(\alpha+\beta)]}$$

→ *Case 3:* $a < 0 < a + \beta$

This situation is depicted in Figure 4.5, and the equation is developed thereafter.

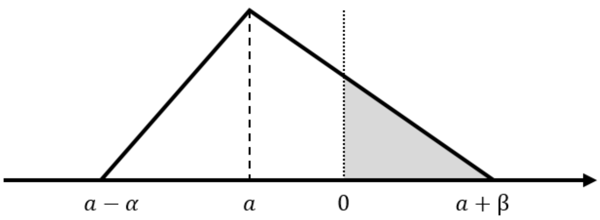


Figure 4.5 – Fuzzy pay-off distribution as RO with $a < 0 < a + \beta$

$$COG(A_+) = \frac{\int_{0}^{a+\beta} x\left(1 - \frac{x-a}{\beta}\right)dx}{\int_{0}^{a+\beta}\left(1 - \frac{x-a}{\beta}\right)dx} = \frac{a+\beta}{3}$$

→ *Case 4:* $a + \beta < 0$

In this situation, depicted in Figure 4.6, the whole fuzzy number is below zero. It follows that: $COG(A_+) = 0$.

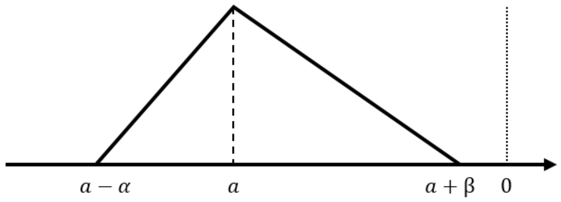
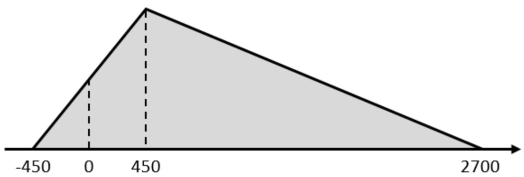


Figure 4.6 – Fuzzy pay-off distribution as RO with $a + \beta < 0$

The use of COG-FPOM seems to overcome the problem shown in Figure 4.2, as illustrated by the calculation of the same example in Figure 4.7.

NPV Estimate	Value (MM $)
Pessimistic	-450
Base	450
Optimistic	2700



$$COG(A_+) = 980.8$$
$$\int_0^{\infty} A(x)dx = 1462.5$$
$$\int_{-\infty}^{\infty} A(x)dx = 1575$$
$$ROV = 910.7 \;\; > \;\; COG(A) = 900$$

Figure 4.7 – The use of COG instead of possibilistic mean in FPOM

Therefore, in this model the value of a project with RO using the COG-FPOM described above shall be calculated as:

$$ROV = COG(A_+) \times \frac{\int_0^{\infty} A(x)dx}{\int_{-\infty}^{\infty} A(x)dx} \tag{6}$$

As highlighted in section 2, the APO is uncertain itself. Consequently, the expected APO used in the procedure depicted in Figure 4.1 has to be adequately estimated. In the COG-FPOM, the expected APO is obtained as the COG of the triangular fuzzy number build from the pessimistic, base and optimistic projections for APO.

5. Application and Discussion

In order to apply the COG-FPOM to support the oilfield abandonment decision, the starting point is the estimation of variables. As highlighted in section 3. , it is common practice in companies to work with scenarios – often one pessimistic, one most likely and one optimistic –, which are carefully build and justified by strategy teams. However, in this initial application the projections were made by the author, as described below, and are not real data.

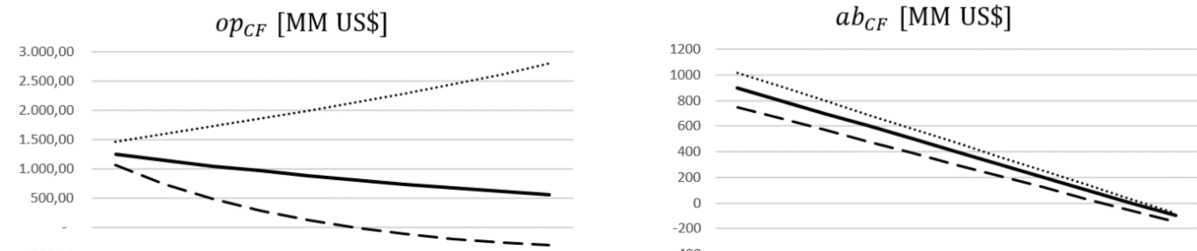
Based on Dias (2014), the oilfield cash flows estimated for each year can be described as follows:

$$\begin{cases} op_{CF} = prod \times (price - var_{cost}) - fixed_{cost} \\ ab_{CF} = res_{value} - ab_{cost} \end{cases}$$

where op_{CF} [MM US$] is the operating profit, $prod$ [MM un] is the petroleum production, $price$ [US$/un] is the price of petroleum, already considering the benchmark crude oil projection and the spread to the specific oilfield petroleum price, var_{cost} [US$/un] is the variable operating cost, $fixed_{cost}$ [MM US$] is the fixed operating cost, ab_{CF} [MM US$] is the abandonment cash flow, res_{value} [MM US$] is the residual value of the oilfield and ab_{cost} [MM US$] is the abandonment cost.

All the aforementioned variables are treated as uncertain and have their yearly values estimated/calculated for each of the three scenarios. In this application, for the sake of simplicity but without loss of generality, $prod$ is considered to start from an arbitrary initial value and follow an exponential decline – a well-known analytical technique for petroleum production forecasting. Furthermore, res_{value} is considered to fall linearly from an initial value to zero, reflecting the wear and tear of the facilities.

The calculated op_{CF} and ab_{CF} used in this example are shown in Figure 5.1. Obviously, it could be adapted to a producer's official projection of variables. Also, if any of the variables are not included in scenario planning, the analyst may use its most likely value in the model.



	1	2	3	4	5	6	7	8	9	10
pess	1.060	742,3	486,0	280,4	116,3	-13,63	-115,7	-195,1	-256,0	-301,9
base	1.243	1.143	1.049	962,0	880,9	805,7	735,9	671,2	611,2	555,6
opti	1.464	1.594	1.724	1.856	1.992	2.133	2.283	2.443	2.615	2.802

	1	2	3	4	5	6	7	8	9	10	11
pess	750	660	570	480	390	300	210	120	30	-60	-150
base	900	800	700	600	500	400	300	200	100	0	-100
opti	1020	910	800	690	580	470	360	250	140	30	-80

Figure 5.1 – Forecasts for application of COG-FPOM

From ab_{CF} estimates, it is possible to calculate the expected APO for each year using COG. It is important to notice that the expected APO has to be estimated up to one year after the final year of forecasted production – in this application, year 11. This is because at that point the company has no choice: the field has to be abandoned (the most common reasons are technical or contractual limits). It means that the expected payoff for year 11 is its expected APO.

In the beginning of the last year of forecasted production (year 10), the company would have to decide between stopping or keeping the production. Following the proposed model, decision-makers would behave rationally and seek the RO value related to this flexibility. The three op_{CF} estimates for year 10, together with the expected payoff for year 11 discounted to year 10, make it possible to build the corresponding fuzzy number. The expected APO of year 10 defines the threshold below which the projections should be valued as zero, making it possible to use one of the 4 cases derived from equation (6). This calculated RO value becomes the expected payoff for year 10 in case the company decides to produce that far. In a backwards process, it is possible to calculate the estimated value of the field with RO in the present.

The RO value is calculated from the difference between the value of the field with RO and the value of the field without RO. This last element can be calculated by using COGs of the triangular fuzzy numbers of each year, without disregarding its negative side – which is similar to ignoring the integral terms in equation (6). After discounting and summing the elements, the value of the field without the option is calculated, and therefore the RO value is achieved.

For the example of Figure 5.1, using a single discount rate of 10% per year, the RO calculated value was 4.16 MM US$. This result indicates that the possibility of being able to abandon increases the value of the field, as expected. It also shows numerically what is an estimated value of this increase. In the example, both values of the field were positive – meaning the company should decide to keep producing even if not considering the option. Nevertheless, in some specific circumstances the value of the field without RO may be negative while the value of the field with RO is positive. In those cases, the COG-FPOM would suggest to keep producing whereas the traditional methods would suggest to abandon.

6. Conclusions and Future Works

This working paper describes the COG-FPOM and its application to support the abandonment decision of a petroleum field. Its main contribution is to show a weakness of the traditional FPOM method and to suggest the COG-FPOM as a way to overcome it. This is an ongoing project, but the results show that the model has potential.

The model has a current limitation, which should be further approached: in reality, the decision is seldom to stop now or produce until the end. Beyond the RO value, the model should generate an expected abandonment year. This year is very important in practice because it will limit the estimations for the field, making it possible to calculate its real value. It also has impacts in reserves estimation, which influences other processes, like impairment tests and depletion rate of assets.

The test of the model with some sort of real data is also foreseen, in order to check if the results are really useful. Other improvements that might make the model more professional are related to the variables (include others, like the exchange rate) and to the structure of cash-flows (consider government take of specific countries, for example).

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