Critical behaviour of thermopower and conductivity at the metal-insulator transition in high-mobility Si-MOSFETs

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Abstract

This paper reports thermopower and conductivity measurements through the metal-insulator transition for 2-dimensional electron gases in high mobility Si-MOSFETs. At low temperatures both thermopower and conductivity show critical behaviour as a function of electron density. When approaching the critical density from the metallic side the diffusion thermopower appears to diverge and the conductivity vanishes. On the insulating side the thermopower shows an upturn with decreasing temperature. These features have much in common with those expected for an Anderson transition.

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I. INTRODUCTION

Scaling theory of non-interacting, disordered, electron gases predicts that no metal-insulator transition (MIT) occurs in 2 dimensions[1–3] as temperature $T \to 0$. Nevertheless, what appears to be an MIT has been observed (at finite, though low T), first in $n-$Si-MOSFETs[4] and more recently in many other 2-dimensional (2D) hole and electron gases.[5] In the particular case of Si-MOSFETs, the transition is most clearly visible in high-mobility samples, roughly $\mu \geq 1\,\mathrm{m}^2/\mathrm{V\,s}$. As the electron density, n, is varied, there is a particular value, n_0, above or below which the resistivity ρ shows metallic or insulating temperature dependence respectively. For the present purposes we will use as a working definition that negative $d\rho/dT$ indicates an 'insulator', and positive $d\rho/dT$ at the lowest temperatures we can reach corresponds to a 'metal' (possible deviations from this definition and the consequences will be mentioned later). At $n > n_0$ and not too close to n_0, metallic behaviour is visible over a wide range of T, roughly $T < 0.5T_F$ where T_F is the Fermi temperature. The decrease of ρ in the metallic state for high mobility Si-MOSFETs samples is typically a few orders of magnitude larger than can be accounted for by electron-phonon scattering. There is no accepted explanation of this behaviour as yet. On the insulating side ρ is found to increase exponentially as T is reduced, typically showing good agreement with variable range hopping models.

Most previous work on these systems has focused on ρ, though measurements of the compressibility[6] have also appeared recently. The present paper presents experimental data on thermopower, S, and conductivity, $\sigma = 1/\rho$, at temperatures down to 0.3 K. When we extract the diffusion thermopower, S^d, we find that it diverges when approaching n_0. At n_0 there is an abrupt change in behaviour of the thermopower with lower densities showing an upturn in S as T is decreased. We also attribute this to S^d.

In addition we find that σ exhibits critical behaviour around n_0 which appears to be largely consistent with the behaviour of S^d. Earlier, a scaling behaviour was described[4] for the temperature dependence of $\rho(T)$ over a temperature range $\sim (0.05-0.3)T_F$. In contrast,

the present experiments are concerned with the limiting, low temperature behaviour of σ. As already mentioned, ρ decreases with temperature when $n > n_0$ but it saturates at low temperatures. (We are unable to say what happens below 0.3 K and return to this point later). Our low temperature saturation values of σ, say σ_0, show a power-law critical behaviour as a function of $(n/n_0 - 1)$. Similar critical behaviour of σ around n_0 has previously been reported in 2D p-type GaAs[7] and lower mobility Si-MOSFETs.[8] The observed characteristics of S^d and σ_0 are strongly reminiscent of those expected for an Anderson MIT in 3D[9–12] but such a transition should not occur in 2D.

We have reported some of our preliminary results, particularly on the thermopower, in conference form.[13] The present paper extends the thermopwer results to a wider range of temperatures and densities, and extends the data into regions of high sample resistivities which were inaccessible to us earlier. It is interesting to note that, unlike resistivity, S is measured with no current flow in the system and hence no self heating occur. We also present the results of systematic studies on ρ on a number of samples; these were primarily taken to augment our understanding of the thermopower, but they are also interesting in their own right.

II. SAMPLE AND EXPERIMENTS

The main sample used for the present ρ and S measurements (Sample 1) is the same as that described in a previous paper[14] and the general techniques used to measure S can also be found there. This sample has $n_0 = 1.01 \times 10^{15}\,\mathrm{m}^{-2}$ (as defined as above) and a peak mobility $\mu = 1.82\,\mathrm{m}^2/\mathrm{Vs}$ at $T = 0.3\,\mathrm{K}$. S and ρ have been measured as a function of T, down to about 0.3 K, at many different values of n. We have also analyzed independent $\rho(T, n)$ data for two other samples over the same range of T, Sample 2 from the same wafer with $n_0 = 0.96 \times 10^{15}\,\mathrm{m}^{-2}$ and with peak $\mu = 1.96\,\mathrm{m}^2/\mathrm{Vs}$ (again at $T = 0.3\,\mathrm{K}$) and Sample 3^{15} with $n_0 = 0.96 \times 10^{15}\,\mathrm{m}^{-2}$ and peak $\mu = 3.6\,\mathrm{m}^2/\mathrm{Vs}$. All connections to the 2D gas had isolation resistance $> 50\,\mathrm{G}\Omega$ and all leads into the cryostat were well shielded and filtered

against rf interference. For the sample leads, commercial, bulkhead, rf rejection filters were used covering a range 1 MHz - 10 GHz, and these were augmented by extra, series inductors to extend the range down to about 50 kHz.

Measurements were made at dc. With the sample in the metallic state ($n > n_0$), a Keithley 182 digital voltmeter usually gave the best compromise of input bias current, input impedance and noise. As previously,[14] we obtained spurious voltages from the sample even with no temperature gradient or current present. We traced this to the input bias current of the voltmeter. With any amplifier, the input bias current combined with the source impedance produces spurious (offset) voltages which must be separated from the true signals. Spurious voltages have been noticed previously by many people and probably have a number of causes, but we verified the consistency of the explanation in the present case by measuring the 2-terminal resistance of the sample over a wide range of conditions, as well as measuring the input bias current of the amplifier. The bias current of the input amplifier of our Keithley 182 was 15 pA for sources up to about 1 MΩ but it increased to 30 pA at higher source impedances.

When the system was metallic the offset voltages were always rather small and not usually too T dependent, especially at lower temperatures where the thermoelectric signals are also small, so they were not a serious problem. For example, at $n = 2.1 \times 10^{15}\mathrm{m}^{-2}$, the sample is metallic and has a resistance of $\sim 20\,\mathrm{k}\Omega$ at low temperatures. This gives an offset voltage of $\sim 30\,\mathrm{nV}$. Offsets were taken into account by taking the thermoelectric signal to be the difference between the measured voltages with the temperature gradient present and not present, ensuring that the mean sample temperature, and thus resistance, was the same in both cases.

On the other hand, when the sample was in the insulating state its resistance was much higher and strongly $T-$dependent. In this case the offset voltage rapidly overwhelmed the thermoelectric signal as the temperature was lowered. To reduce this problem, an amplifier with input bias current < 1 pA and input impedance $> 10^{12}\Omega$ was used. With some averaging it had a resolution of $0.1\,\mu$V for source impedances of less than a few hundred kΩ rising to

about 1 μV at 10-20 MΩ. However, when used with the sample there was always an apparent offset current of $\sim$ 4 pA independent of sample voltage, i.e., offset signals were produced corresponding to a dc current through the sample of this magnitude. We were unable to determine the origin of this current. It was unaffected by changing the rf filters in the leads, or by rerouting some of the wiring, particularly that carrying the gate voltage. When the offset signals become large, one would like to have some check that the method of extracting the signal used above was reliable. Fortunately, it was found that if the sample was grounded at different contacts, the offset current always flowed in a direction towards ground. Thus by measuring the thermoelectric signals with the source or drain grounded in turn, the offset voltage reversed sign but the thermoelectric voltage did not. This gave two independent measurements and is the origin of the error bars on our data at low temperatures with the sample in the insulating state (see Fig. (4) later). The width of the error bars typically increases very rapidly as T is lowered because of the strong variation of ρ.

Finally we provide some information on the temperature difference ΔT across the sample at various temperatures so that the signal voltages $\Delta V = S\Delta T$ may be estimated under various conditions. The electric field in the sample may also be estimated given the length was 2.5 mm. Between 0.7 and 7 K, $\Delta T \approx 100$ mK. Below 0.7 K ΔT dropped steadily until by 0.4 K it was about 30 mK and at 0.34 K about 15 mK.

III. RESULTS AND DISCUSSION

In the metallic region n, as obtained from the Hall constant, was found to be a linear function of gate voltage V_g, and it is believed to follow approximately the same dependence in the insulating region,[16] at least close to n_0. On different cooldowns from room temperature there were small shifts in n at fixed V_g of up to $0.04 \times 10^{15}\,\mathrm{m}^{-2}$, but if the sample was held at 77 K between cooldowns, as was done for most of these measurements, the density and other properties were very reproducible. In general the absolute uncertainty in n should be no worse than $\pm 0.1 \times 10^{15}\,\mathrm{m}^{-2}$ in the region of n_0 and 2% at high n; the reason for the

larger uncertainty near n_0 is the fact that the Hall resistivity shows non-linearities of up to about 10% there.

Because our analysis of thermopower makes extensive use of ρ data, examples of the temperature dependence of ρ on both the metallic and insulating side are shown in Figs. 1 and 2. They are very similar to those seen in previous work.[4,17]

Well into the insulating regime (roughly $n \leq 0.65 \times 10^{15}\mathrm{m}^{-2}$) ρ can be well represented by an equation developed for variable range hopping (VRH) through localized states in the presence of a Coulomb gap at the Fermi energy E_F (referred to as Efros-Shklovskii VRH),

$$\rho = \rho_c \exp\left(T_{ES}/T\right)^{1/2}, \tag{1}$$

where ρ_c and T_{ES} are constants that depend on n but not T. Examples are given in Fig. 1 for Sample 1. The values that we obtain for the constants are similar with those determined by others.[17] T_{ES} varies from $\sim 35\,\mathrm{K}$ at $n = 0.650 \times 10^{15}\mathrm{m}^{-2}$ to $\sim 90\,\mathrm{K}$ at $0.48 \times 10^{15}\mathrm{m}^{-2}$. If the exponent is left as a variable we find an average value of 0.50 ± 0.05 over this same range of n. We find poorer fits assuming Mott VRH (no Coulomb gap at E_F) which corresponds to a power law of $1/3$ in the above expression.

For $n \geq 0.70 \times 10^{15}\mathrm{m}^{-2}$, a simple activated behaviour

$$\rho = \rho_c \exp\left(T_A/T\right) \tag{2}$$

provides a very good fit to the data below about 1 K. Thus these results suggest a transition from simple activated behaviour to VRH as n decreases.

In the metallic region ($n > n_0$) we have fitted our data to the equation[15]

$$\rho = \rho_0 + \rho_1 \exp\left[-(T_0/T)^q\right] \tag{3}$$

where ρ_0, ρ_1, T_0 and q are constants that depend on n but not T. Examples are shown in Fig. 2 for Sample 2. For $n \geq 1.5 \times 10^{15}\mathrm{m}^{-2}$, excellent fits were obtained by fixing $q = 0.5$. For lower n, the values of q (if this is left as a variable) increased and approached 2.0 at $n \approx 1.0 \times 10^{15}\mathrm{m}^{-2}$. The model curves fit the data successfully over the range 0.3 K (our

lowest T) to about $0.3\,T_F$. ($T_F = 7.3n\,\mathrm{K}$ with n in $10^{15}\,\mathrm{m}^{-2}$ units). Above this region the curves usually exhibit a maximum.[18,19] T_0 also provides a natural scale for the data and, since T_0 decreases as n_0 is approached (Fig. 2 indicates T_0 for the lowest densities), a lowering of the upper temperature limit is also consistent with this. The reduced range of fitting at low n means that, ρ_1, T_0 and q are not well known here but, even so, ρ_0 retains a relatively low error.

Figure 3 shows the results on $\sigma_0 = 1/\rho_0$ as a function of n.[20] For all three samples σ_0 follow the critical behaviour

$$\sigma_0 = \sigma_m + \sigma_s \left(\frac{n}{n_0} - 1 \right)^{\nu}. \tag{4}$$

The solid lines are the best fits with the following parameters, with σ in units of e^2/h (the data points at the nominal values of n_0 are $\sigma(0.3\mathrm{K})$ and were not used in the fitting). For sample 2, $\sigma_m = 0.0 \pm 0.3$, $\sigma_s = 13.6 \pm 0.7$ and $\nu = 0.94 \pm 0.03$. A good fit was not possible over the whole range of n and so was restricted to $n < 3.0 \times 10^{15}\,\mathrm{m}^{-2}$ (a range of $(n/n_0 - 1)$ of about 0.03 to 3). Sample 1 has an identical behaviour within experimental error. The higher-mobility Sample 3 also follows the same equation with $\sigma_m = 0.36 \pm 0.15$, $\sigma_s = 34 \pm 5$ and $\nu = 1.39 \pm 0.05$. We have a smaller range of n for this sample, but the fit still covers a range $(n/n_0 - 1) \approx 0.02 - 0.4$. These results suggest ν increases with peak mobility but clearly more data on a variety of samples are required. The values of σ_m for Sample 1 and 2 are consistent with zero within experimental uncertainty. The fits are good over the whole range except for the last points near $n = n_0$ which do not fall on the curves. This is not necessarily unexpected because a value of $\sigma = 0$ is not possible at a finite temperature due to thermal broadening of the electronic distribution. Small errors in n_0 will have a similar effect. For Sample 3, σ_m may be finite. However, if n_0 is allowed to decrease from 0.96 to about $0.93 \times 10^{15}\,\mathrm{m}^{-2}$, a fit which is essentially indistinguishable over the range of the data can also be obtained with $\sigma_m = 0.0 \pm 0.2$, $\sigma_s = 32 \pm 5$ and $\nu = 1.48 \pm 0.05$; both fits are actually shown on Fig. 3. A small discrepancy in n_0 could easily arise from the identification of the critical density for the MIT with that density, n_0, where $d\rho/dT$ changes

sign, a procedure which has no firm physical foundation;[18] in addition there are uncertainties and possibly non-linearities in the determination of n from gate voltage values as mentioned above.[16]

The critical behaviour described by Eq. (4) with $\sigma_m = 0$ is formally the same as that expected for a (continuous) Anderson transition with a 'mobility edge' at n_0, whereas a finite σ_m would correspond to a (discontinuous) Mott-Anderson transition; neither transition should arise in a non-interacting 2D gas.[1,2] Our ρ data in the insulating region are also consistent with this scenario in the sense that we observe activated behaviour for n below n_0. Similar critical behaviour, usually with σ_m consistent with zero, has been seen in many 3D systems, typically with values[12] of ν in the range $0.5 - 1.3$. There are also two previously reported cases related to 2D. Hanein *et al.*[7] have made a similar analysis to the one above for a 2D hole gas in GaAs and found a linear relation between σ_0 and n, but with a finite σ_m. Feng *et al.*[8] also found critical behaviour in σ_0 in a low mobility Si-MOSFET though we note that their data all show negative $d\rho(T)/dT$, even in the range which they identify with 'metallic' behaviour. They found $\sigma(T)$ *increased* with T and fitted a T^2 dependence. This behaviour was attributed to local momemts and is so different from that found here that the two cases may not be closely related. Recently[21] a model based on percolation of non-interacting electrons through local quantum point contacts also led to zero-temperature conductivity consistent with our observations.

Note that the data presented in Fig. 3 were obtained from extrapolating σ to $T = 0$ by a procedure which focusses only on the 'strong' exponential, $T-$dependence of Eq. (3). It ignores any other weaker dependences[20] that may be present below $T = 0.3$K, including those due to weak localization and screening.

We now turn to the thermopower data. A selection of data on S as a function of T is shown in Fig. 4. From our previous work[14] we know that the diffusion thermopower, S^d, is almost zero at $n = 8.5 \times 10^{15}\,\mathrm{m}^{-2}$ and one sees only phonon drag, S^g which varies approximately as T^6 at the lowest temperatures. The fact that S^d is very low at high n was predicted by Karavolas and Butcher[22] who attributed it to a particular combination of

scattering mechanisms. Indeed, when $n \gg n_0$ the behaviour of both S^d and S^g appears to be in generally good agreement with theory.[14]

The T^6 dependence of S^g obtains only in the Bloch limit, i.e., when $q \ll 2k_F$ where q is the average phonon wave number and k_F the Fermi wave number. At $n = 8.5 \times 10^{15}\,\mathrm{m}^{-2}$ the condition $q \ll 2k_F$ is satisfied below about 1.4 K. Above this temperature one sees a gradual decrease in the exponent to roughly $S^g \propto T^3$ by 4 K, but no simple power law is expected in this region.

As n decreases, S begins to show two distinct regions with different T dependences. At $T < 1\,\mathrm{K}$, S has a much weaker, approximately linear, T-dependence indicative of S^d becoming dominant. For $n < n_0$, this low-T dependence, which is characteristic of ordinary metals, is replaced by an upturn in S; we will return to this feature later.

Concentrating first on the metallic region, we analysed the data over the range 0.3-1.5 K to obtain S^d by assuming

$$S = S^d + S^g = \alpha T + \beta T^s \tag{5}$$

where α $(= S^d/T)$ and β are constants that depend on n. For the 2 highest densities we obtained good straight lines by plotting S/T as a function of T^{s-1} with $s = 6$ as expected. At lower n, proportionately lower temperatures are required to reach the Bloch limit and so we no longer expect to see $S^g \propto T^6$ in our temperature range. Hence we simply used s as a variable to give the best fit. For $n = 1.9 - 5 \times 10^{15}\,\mathrm{m}^{-2}$ we found $s = 4$, and for $n = 0.97 - 1.5 \times 10^{15}\,\mathrm{m}^{-2}$, $s = 5$. That s seems to increase again at the lowest n is unexpected, but more data are required before this is taken to be an experimental fact. We have no reason to believe that S^g is not well-behaved in this temperature, density and resistivity range, but we must caution that our understanding of S^g in these circumstances is not well founded as yet.[13,23] Given this it is important to note that the value of α depends only weakly on the choice of s. Indeed, providing S^g has a much stronger T dependence than linear, then simply taking the measured values of S/T at 0.3 K to be equal to α gives values in good agreement with those obtained using Eq. (5).

Fig. 5 shows that α as a function of n appears to diverge as $n \to n_0$. One would expect such a divergence if E_F approaches a band edge or a gap in the DOS giving $S^d \propto 1/n$. Das Sarma *et al.*[24] have suggested that a similar mechanism, carrier freezeout, is responsible for the MIT in 2DEGs. However, the present results are inconsistent with these explanations because Hall data, both our own and those in Ref. 16, show that in the vicinity of n_0 the mobile carrier density is not approaching zero but equals n within a few percent; also any temperature dependence of n at fixed V_g is very small in the range $0.3 - 4.2\,\mathrm{K}$. Given that theory successfully predicted[22] S^d to be zero at high n as mentioned above, it is possible that the present results have an explanation in terms of the specific scattering processes important at low n, but there is no theory available for this situation at present.

However, Eq. (4) also implies a divergence of S^d. With the assumption of a constant density of states (DOS), Eq. (4) is consistent with

$$\sigma(E_F) = \sigma_m + \sigma_s \left(\frac{E_F}{E_c} - 1 \right)^{\nu} . \tag{6}$$

Again, with $\sigma_m = 0$ this is formally equivalent to an Anderson transition with E_c being the mobility edge. Taking $\sigma_m = 0$, the use of the Mott relation $S^d = -(\pi^2 k_B^2 T/3e)(\partial \ln \sigma/\partial E)_{E_F}$ with Eq. (6) then gives[10,12]

$$S^d = -\frac{\nu \pi^2 k_B^2 T}{3e(E_F - E_c)} . \tag{7}$$

The use of the Mott relation, and hence this result, is valid only if $(E_F - E_c)/k_B T \gg 1$; in the opposite limit S^d tends to a constant[11,12] ($\sim 228\,\mu\mathrm{V/K}$ in 3D). The saturation is a direct consequence of the fact that the contribution to the conductivity of electrons below E_c is zero in this model. The relevant integral[10,12] for S^d has a weighting factor $\sigma(E)(E - E_F)$ which, under normal conditions, leads to S^d being a measure of the derivative of $\sigma(E)$ with respect to energy, and hence to the Mott relation. This is consistent with Eq. (7). However, when $E_F = E_c$ only electrons above E_c contribute to the integral, which leads to a constant value of S^d. Numerical calculations[12] show that the approximation of Eq. (7) gives a magnitude roughly a factor of 2 too large when $(E_F - E_c)/k_B T \approx 2$ (i.e. because of the approach to

saturation). For our sample at $T = 0.4\,\mathrm{K}$ this corresponds to $(n - n_0)/n_0 = \Delta$ (say) ≈ 0.11, using the ideal DOS, g_0, with an effective mass of $0.19\,m_0$. If the present model was indeed appropriate, it suggests that we should see saturation of S^d as T increases for the samples with $n = 0.97$ and $1.06 \times 10^{15}\,\mathrm{m}^{-2}$ (for higher n any saturation would be masked by the rapidly increasing contribution from S^g). We are unable to see any obvious evidence for saturation at these densities and Eq. (5) still seems to be valid there. However, we should recall that S^g is not yet understood[23] at low n and high ρ and there remains the possibility that an unexpected behaviour of S^g could mask important features in S^d.

Proceeding with the model, we have fitted Eq. (7) to the measured α, allowing for the possibility of saturation as n_0 is approached. To simulate a saturation we add Δ in the denominator (but allow it to be a variable when determining the best fit to the data) and, rewriting Eq. (7) in terms of n, we have

$$\alpha = S^d/T = -K/\sqrt{\Delta^2 + (\frac{n}{n_0} - 1)^2} \tag{8}$$

where $K = \nu\pi^2 k_B^2/(3eE_c)$ is a constant expected to be about $32\,\mu\mathrm{V/K}^2$ for Sample 1, again using g_0. If σ_m is finite in Eq. (6), then the Mott relation suggests that it will contribute to the denominator of Eq. (7), also softening the divergence at $n = n_0$. However, the experimental σ_m is so small that this is probably negligible compared to the finite-T effect considered here.

The best fit of the data to Eq. (8) is shown by the solid line in Fig. 5 and uses $\Delta = 0.15 \pm 0.01$ and $K = (9.5 \pm 1.5)\,\mu\mathrm{V/K}^2$. Only data with $n < 4 \times 10^{15}\,\mathrm{m}^{-2}$ are fitted, a range consistent with that used for σ_0. Δ is consistent with that expected from the argument above, but K is too small by a factor of about 3. As with σ, the fit can be improved if n_0 is decreased, perhaps by $0.05 \times 10^{15}\,\mathrm{m}^{-2}$, in which case Δ decreases and may even become zero.

In spite of the relatively good correspondence between the model and the data, we emphasize that we should be cautious in necessarily concluding that the model is fundamentally correct. We are comparing our results on a 2D system with a theoretical model of an Ander-

son MIT believed to be appropriate only to non-interacting electrons in 3D. At this point it is difficult to know whether the situation in 2D might be significantly changed by including interactions along with the strong disorder; this is a complex and ongoing theoretical problem (e.g. see Refs. 2,3 and references therein). Some progress has been made on calculating S^d with the inclusion of weak interactions and disorder.[25] We are, however, unable to explain the observed strong n dependence in terms of the calculations which predict universal values. The calculated corrections are logarithmic in T. These would be the equivalent of similar corrections in ρ and, if they exist, would require much lower temperatures than we have available and would be difficult to detect. On the other hand, these facts mean that the analysis of α, ρ_0, and their relationship as zero-temperature quantities, is a self-consistent procedure which has a semi-classical meaning. Nevertheless, whether or not σ goes sharply to zero near n_0 as would be required in the Anderson model, the observed change in behaviour of S^d as n is varied around n_0 points to a fundamental change in transport taking place in this region.

We should finally mention that we can also represent the data reasonably well over the same range using the simple expression $\alpha = -56/n^{2.5}\,\mu\mathrm{V/K^2}$, with n in units of $10^{15}\,\mathrm{m^{-2}}$, but this has no obvious physical explanation; in particular, it does not have the form that we would expect for S^d approaching a band edge at $n = 0$, i.e. $\alpha \propto 1/n$.

Our data in the insulating regime also show a behaviour qualitatively consistent with activation across an energy or mobility gap. In such cases we expect an upturn of S^d at low temperatures according to

$$S^d = -\frac{k_B}{e}\left(A + \frac{E_c - E_F}{k_B T}\right) \tag{9}$$

where A is a constant of order unity. In the particular case of the Anderson model[9,12,26] Ak_B/e corresponds to the saturation value when $E_c = E_F$ noted above in our discussion of the metallic region. When n is close to, but less than, n_0, we see activated behaviour in ρ (see Eq. (2)) which is in accord with this. We do not see a high temperature saturation of S as Eq. (9) predicts, but this is not surprisingly because it will be masked by other

contributions to S, particularly S^g (see also below).

There might also be another contribution to S^d from VRH. Demishev *et al.*[27] have recently demonstrated the existence of such a component in S^d using a 3D GaSb sample. The T dependence of our ρ data and other previously published data[17] well into the insulating region are consistent with Efros-Shklovskii VRH. (When two or more conduction mechanisms are present, the appropriate S^d are weighted by their contributions to σ). For this mechanism one expects S^d to be a constant given by[28,29] $S^d = -(k_B/e)(k_B T_{ES}/C)(\partial \ln g(E)/\partial E)_{E_F}$ where T_{ES} is obtained from the temperature dependence of ρ in Eq. (1), $g(E)$ is the background DOS, i.e, on which the Coulomb gap is superimposed, and C a dimensionless constant ≈ 6. If we take $(\partial \ln g(E)/\partial E)_{E_F} \sim 1/E_F$ (implying that E_F may be in the tail of the DOS) and again using g_0 to estimate E_F, we find that the calculated S^d are typically a factor of two smaller than the values of S at the observed minima. The argument is not significantly changed if Mott VRH is assumed.[28,29] In this case $S^d \propto T^{1/3}$ but the magnitudes calculated for S^d are similar. The references give the relevant details.

In the insulating regime, one expects a diverging localization or correlation length as the critical point is approached. Given this, one might also question whether the thermopower results are in the linear region close to the critical point. However, on this side of the transition we are never very close to the critical density. The closest point is at about $n_0 - n \approx 0.5 \times 10^{14}\mathrm{m}^{-2}$. The correlation length estimated in Ref. [30] under these conditions is about $20\,\mu$m. Assuming a magnitude of the disorder potential of order 1 K gives a threshold electric field for non-linear effects of order 80 mV/m whereas the thermoelectric electric field is about 1 mV/m over the range of 0.7-7 K. Thus non-linear effects do not appear to be a problem for the present data.

To put the present results in perspective, as far as we are aware the only previous work which attempted to follow S^d into the region of 2D electron localization was that of Burns and Chaikin[31] on thin films of Pd and PdAu. They found an upturn of S^d in the strong localization region but no divergence at higher conductivities. The authors attributed their results to the opening of a Mott-Hubbard gap. In 3D, Lauinger and Baumann[32] observed

critical behaviour of σ and a divergence of S^d for metallic AuSb films, but the magnitude of the latter was 2 orders of magnitude smaller than seen here. Other experiments on bulk SiP[33] and NbSi[34] saw no divergence of S^d on the metallic side.

We close this section by making a few comments about S at higher T. S^g appears to be dominant in this region because the strong T dependence of S (roughly T^2 to T^3) is inconsistent with any other mechanism. As we have already indicated, little is known about the behaviour of S^g where n is low and ρ is high,[13,23] and these are just the conditions that pertain around n_0. Thus, although S^g should be present on the metallic side, its precise form is not known with any certainty.

When conduction is dominated by VRH, we expect $S^g = 0$ because drag is based on the conservation of crystal momentum for electron-phonon scattering which will not hold for transitions between localized electron states.[29,35] Thus, S^g on the insulator side should only exist when carrier excitation to delocalized states occurs, though we emphasize that there are no calculations appropriate to these conditions. Our ρ data are consistent with activated behaviour for n just below n_0, but deeper into the insulating state VRH becomes dominant and so we would expect to see a strong diminution of S^g in this region. Figure 6 shows experimental data on S as a function of n at fixed temperatures of 2 K, 3 K and 4 K. We see that S rises as n decreases but crosses n_0 smoothly, i.e., we no longer see divergent behaviour of S at n_0 as we did for S^d. Indeed there is no feature that indicates that anything unusual occurs at n_0. This behaviour is consistent with activated conduction below n_0. However, we continue to see a rise in S, and presumably S^g, to the lowest densities, which is not expected from our argument above. This can only be understood if significant activated conduction is also present at all densities. In this light, the fact that our ρ data at lowest n in Fig. 2, (and the data of others[17]) appear to follow the Efros-Shklovskii VRH model so well over a limited range of temperatures appears to be somewhat coincidental.

IV. SUMMARY AND CONCLUSIONS

Generally speaking, the main focus of attention with regard to the MIT in 2D systems has been whether the metallic behaviour is the result of a transition to a new state induced by the strong disorder and possibly electron-electron interactions, or whether it is the result of a conventional physical mechanism that has yet to be unambiguously identified.

On the metallic side, our analysis has mainly concentrated on data at low T, essentially below the region of rapid T dependence of ρ. Our main result is that the critical behaviour of σ_0 and S^d of Si-MOSFETs in this region are surprisingly consistent with equations that are formally equivalent to those describing a 3D Anderson MIT. On the insulating side, the behaviour of ρ and S^d over a wider range of T are also consistent with such a scenario. However, not all features that we see can be understood in this way. In particular, although the Anderson model exhibits scaling behaviour in both $\sigma(T)$ and $S^d(T)$,[36] it does not appear to be able to explain the large increase of ρ with T in the metallic region observed in Si-MOSFETs.

An Anderson transition can arise purely from disorder in 3D but general scaling results predict that the equivalent transition should not occur in 2D. Still, given that many features of the data do mimic an Anderson transition, a key question in the present work would then be whether a 'mobility edge' actually exists, in particular at low or zero temperature. If we look for more conventional explanations of the data, a point to bear in mind is that $k_F l \sim 0.3$ at n_0, where l is the electron mean free path estimated from the conductivity, which implies very strong disorder. Under these conditions it seems plausible that a sufficiently rapid drop in ρ with decreasing n could be mistaken for a mobility edge, but it might actually be caused by a smooth, though very rapid, transition from weak to strong (exponential) localization of the carriers in keeping with the scaling model. If this is the explanation, then it remains to be shown how this rapid drop can mimic a critical behaviour over a relatively wide range of n.

In our analysis, the apparent divergence of S^d as n_0 is approached from above is basically

a reflection of the rapid drop in σ_0, a result that might remain valid even if no Anderson transition occurred. An unambiguous indicator of a mobility edge would be saturation of S^d as a function of temperature. Our data near n_0 show no visible indication of this, but the presence of phonon drag is a complicating factor here and might mask any saturation. Clearly it would be an advantage to suppress phonon drag and reveal S^d over a wider range of T, but this is difficult to do in 2D. In principle, it could be done by the use of very thin substrates. We do not see any features in the overall behaviour of S which appear to correlate with the strong T dependence of ρ in the metallic region.

Finally, an important point that must be stressed is that to reliably identify any observed critical behaviour with a MIT requires data in the zero T limit. Although our analysis of σ_0 and S^d is based on an extrapolation to zero T, the actual data extend only to 0.3K. If the strong drop of ρ is caused by conventional physics, then we are examining the low temperature limit of this mechanism, but not necessarily the true low temperature limit of the system. This is true of practically all the experimental data published so far.

In spite of these reservations, it is clear that the observed critical behaviour in both σ_0 and S^d indicates an unknown but interesting physics of strongly disordered and interacting systems. The results open up a new window on these systems that further constrains any theoretical model proposed to explain the MIT, whether such a transition be a quantum property or simply the result of a conventional physical mechanism.

V. ACKNOWLEDGEMENTS

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[20] In fitting the data to Eq. (3) and thus in assigning $\rho(T \to 0)$ to be equal to ρ_0, we ignored any other weaker temperature dependences that might exist. In particular, we ignored any T dependence due to quantum corrections to the conductivity (see e.g., Pudalov et al. Phys. Rev. B **60**, 2154 (1999) and Ref. 19) which, in the high density range, develop for temperatures $T < T_{WL} = 0.007 T_F$. Since $T_F(n_0) \approx 7\,\mathrm{K}$ in the vicinity of n_0, T_{WL} is below our lowest accessible temperature. On the other hand, there are no calculations of the quantum correction for the case of low density and strong Coulomb interaction. We also ignored a possible quasi-linear T-dependence due to temperature dependent screening effects[37] which could be pronounced over the range $T = (0.01 - 0.05)T_F$. However, the latter dependence is expected to vanish[37] below $T < \hbar/k_B\tau_q \sim T_F/\sigma_0$, where τ_q is the quantum lifetime and σ_0 is in units of e^2/h. Because this temperature is always higher than 0.3 K, any dependence from this cause should not affect our estimates of ρ_0. Thus, the procedure used to derive ρ_0 gives us an estimate for the zero T-limit valid for the exponentially strong $T-$dependence, i.e. that part which obeys temperature scaling[4,5] and which is of primary interest in these systems.

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FIGURES

FIG. 1. Examples of resistivity ρ at various densities (in units of $10^{15}\mathrm{m}^{-2}$) as a function of temperature T for Sample 1 in the insulating region. The data are plotted in the form $\log\rho$ as a function of $T^{-1/2}$ to show that Eq. (1) (Efros-Shklovskii VRH) gives a good representation at low n (0.52 and 0.61 on the figure). At higher n (0.70, 0.79 and 0.88) the data are well described by Eq. (2) below 1 K, corresponding to simple activated behaviour.

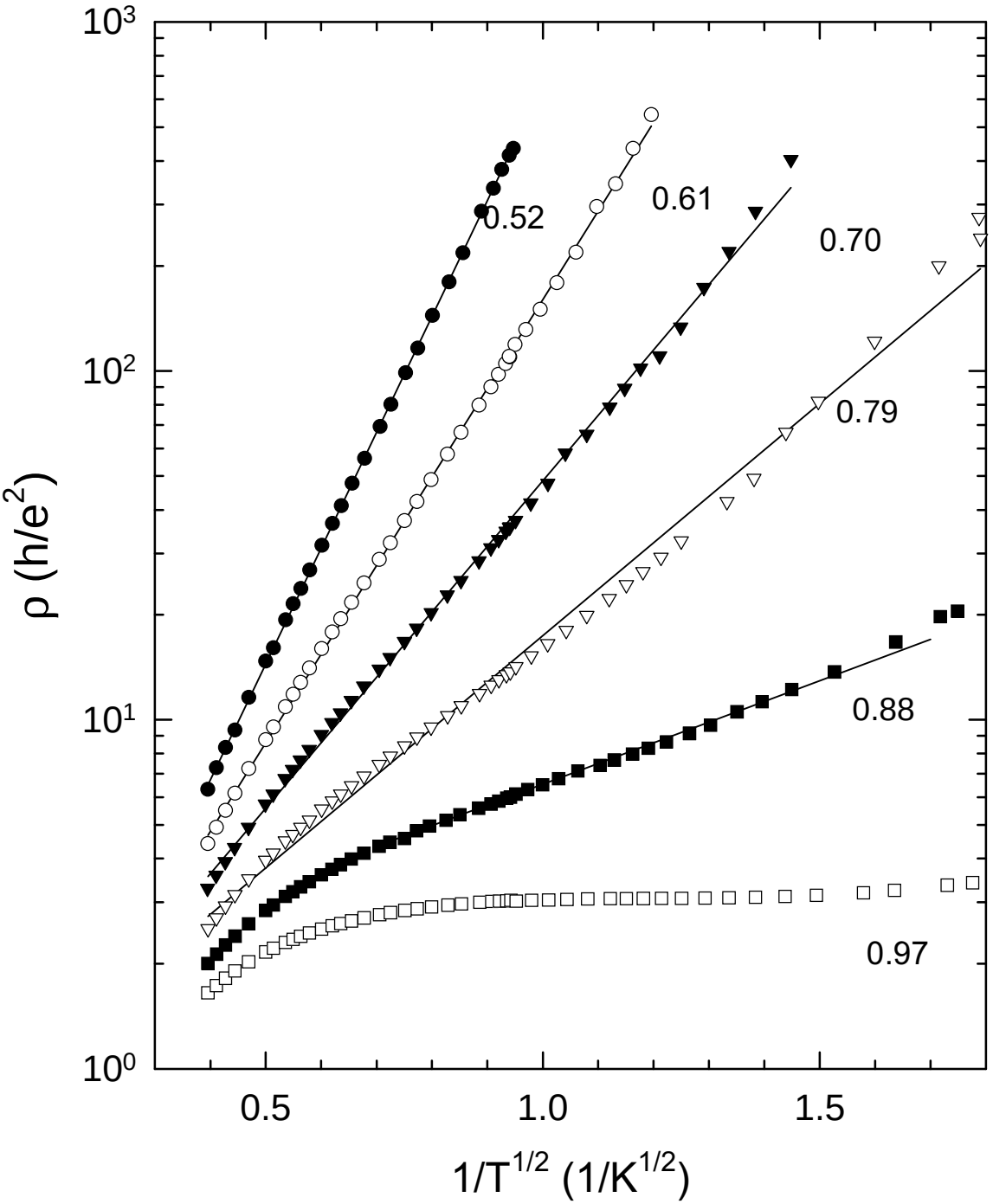
FIG. 2. Resistivity ρ at various fixed densities n (in units of $10^{15}\mathrm{m}^{-2}$) as a function of temperature T for Sample 2 in the metallic region. The solid lines are fits to Eq. (3) using only the points represented by open symbols. The closed symbols for the data near n_0 are points not used in the fitting (see text). The short vertical lines give the value of T_0 for each density. The data at $n = 0.96$ are nominally at n_0.

FIG. 3. The main panel shows the density dependence of the conductivity σ_0 in the $T \to 0$ limit of Sample 2. The inset shows the same data for Sample 3. In both cases circles correspond to data obtained using Eq. (3). The lowest points designated by triangles are simply the measured σ(T=0.3K). The dashed vertical lines are the values of n_0 used in obtaining the fitted curves. In the case of Sample 3, two fitted curves are shown (but are almost indistiguinshable over the data range) corresponding to the two values of n_0.

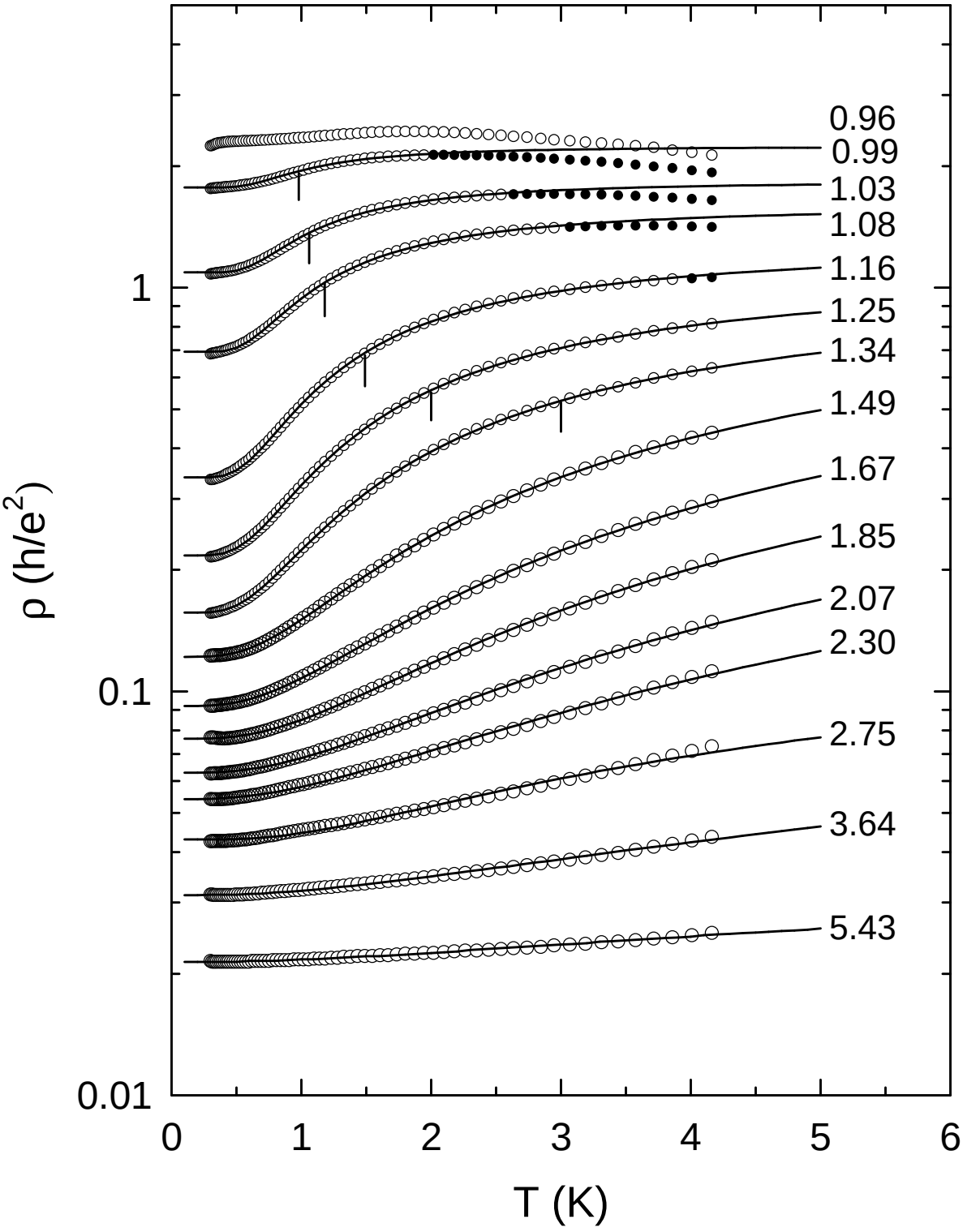
FIG. 4. The thermopower S for Sample 1 at various fixed electron densities n (in units of $10^{15}\,\mathrm{m}^{-2}$) as a function of temperature.

FIG. 5. Density dependence of $\alpha = S^d/T$ for Sample 1. The closed symbols are obtained using Eq. (5) and the open symbols are simply the measured S/T at at 0.3 K (the values of α from the two methods at $n > 3 \times 10^{15}\,\mathrm{m}^{-2}$ are indistinguishable). The points at $n = 0.97 \times 10^{15}\,\mathrm{m}^{-2}$ are just below n_0 but k_BT broadening should make these indistinguishable from n_0. The line is the best fit to Eq. (8) for $n < 4 \times 10^{15}\,\mathrm{m}^{-2}$.

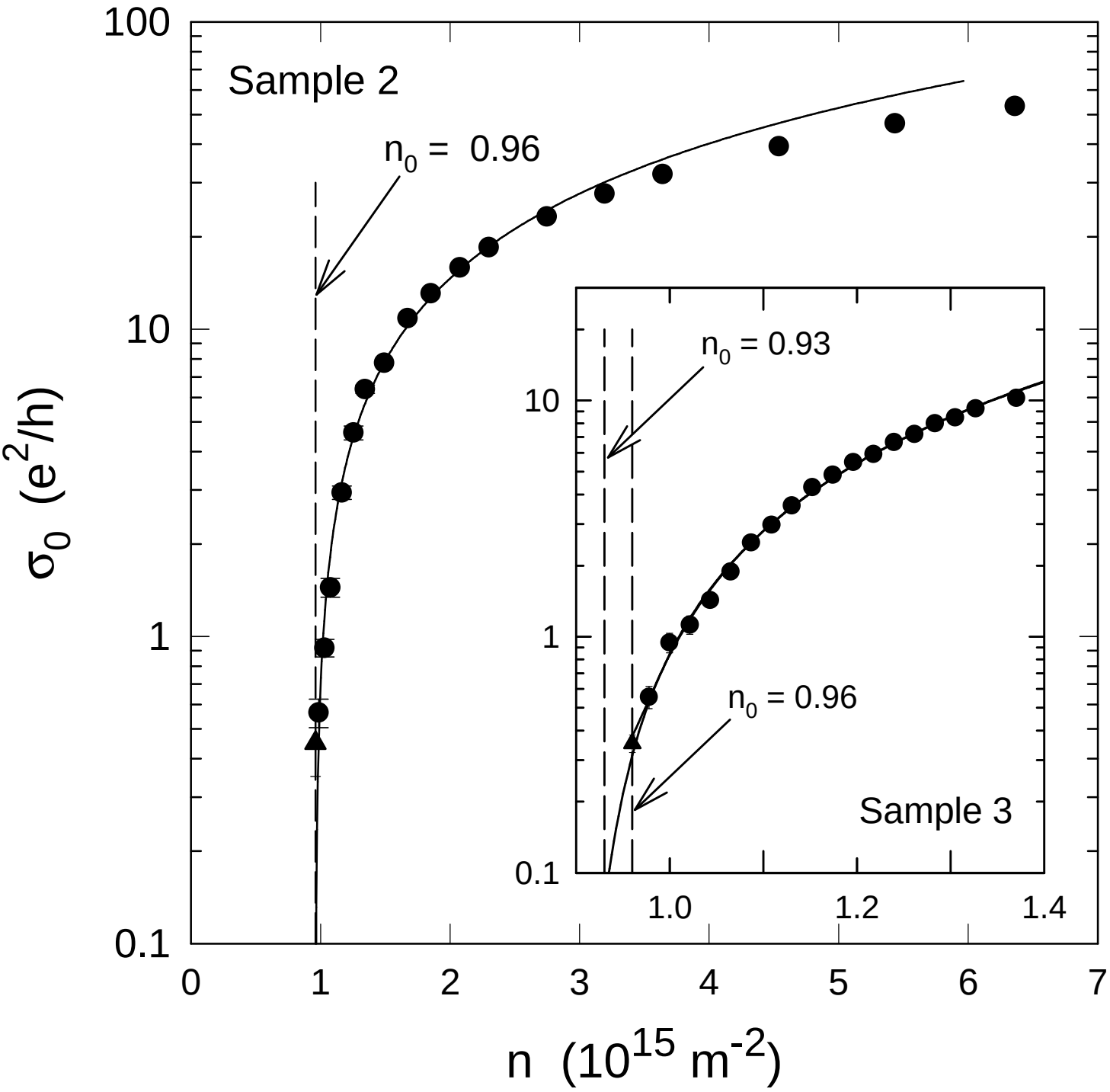
FIG. 6. Thermopower at fixed temperatures of 2.0 K, 3.0 K and 4.0 K as a function of density.



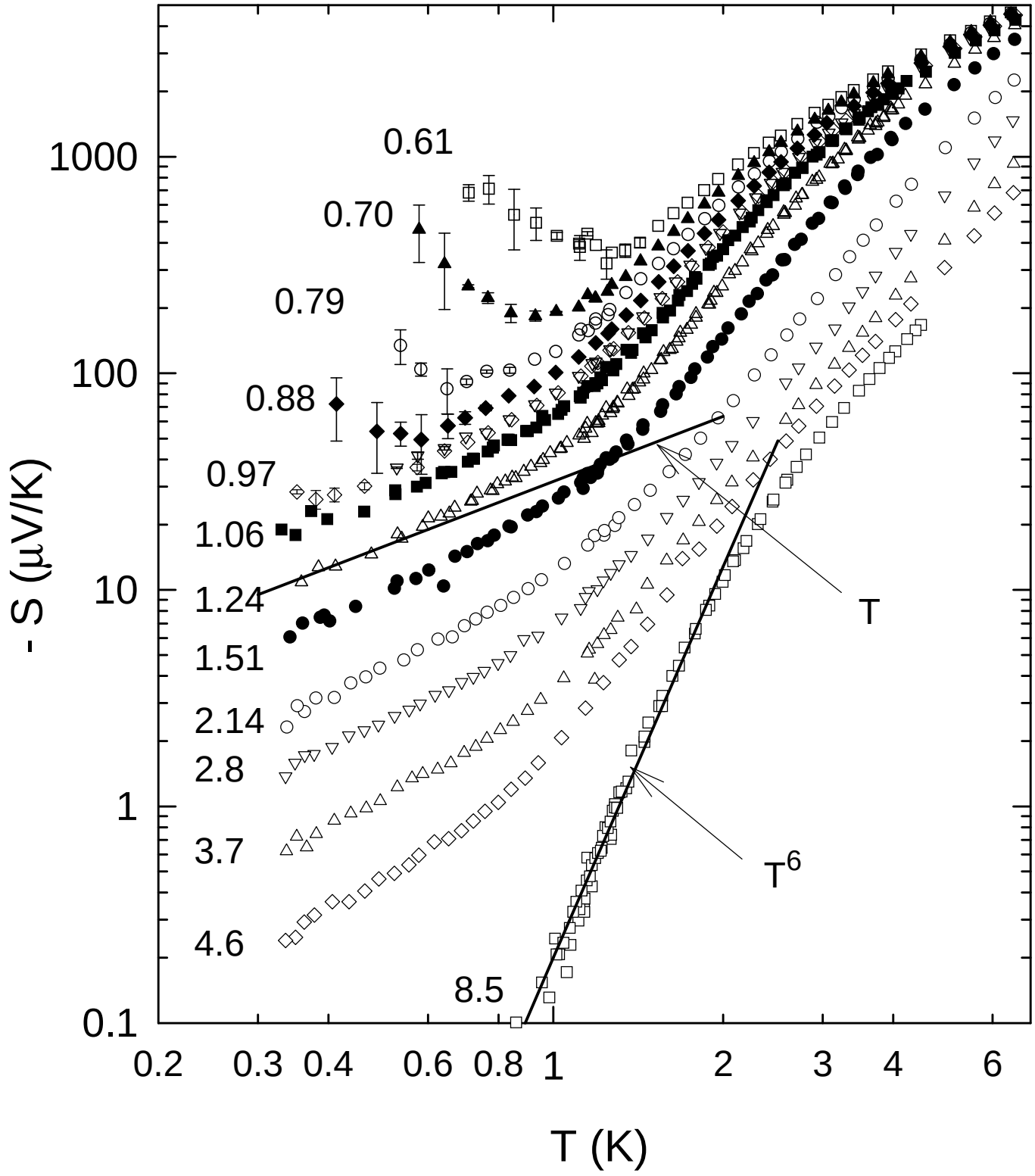
Fletcher et al Fig. 1



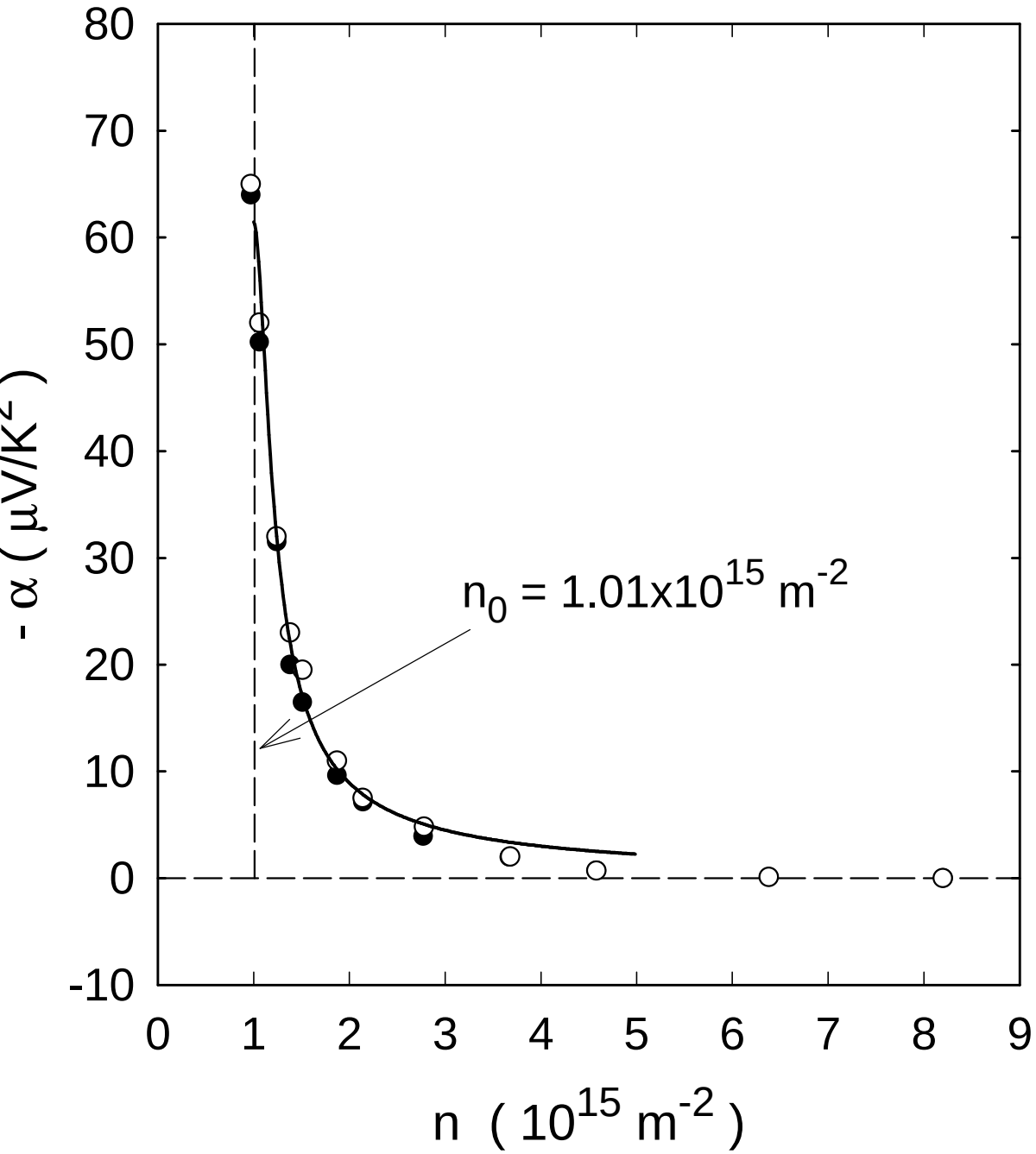
Fletcher et al Fig. 2



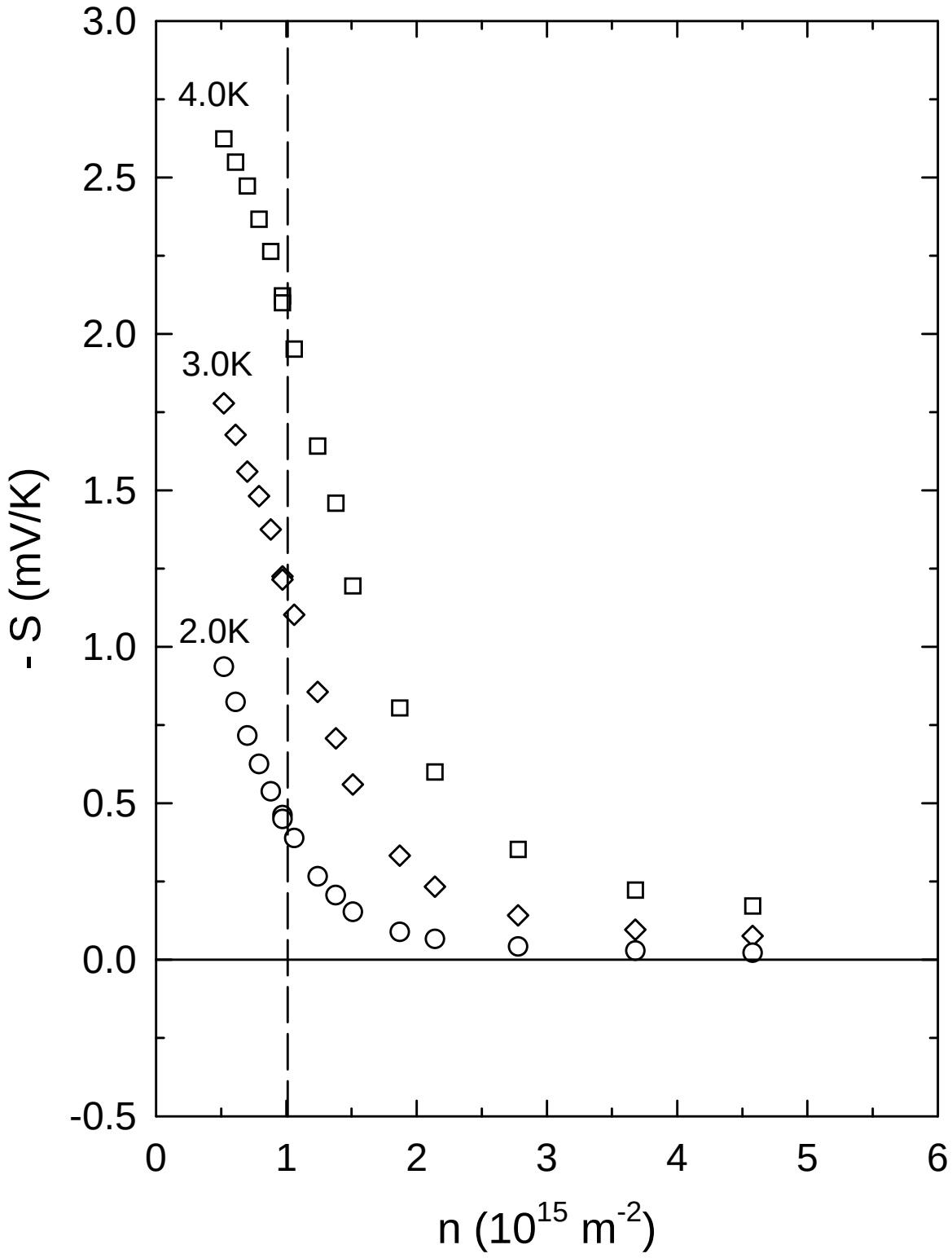
Fletcher et al Fig. 3



Fletcher et al. Fig.4



Fletcher et al. Fig. 5



Fletcher et al Fig. 6