TFP Persistence and Monetary Policy

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PRELIMINARY AND INCOMPLETE

Abstract

In this paper, by using several statistical tools, we provide evidence of an increase in the persistence of the U.S. total factor productivity. In a forward looking model, agents' optimal behavior depends on the autocorrelation structure of the exogenous shocks. Since many monetary models are driven by exogenous TFP shocks, we study the interaction between monetary policy and TFP persistence. Considering a New-Keynesian model, first, we analytically derive the interaction between the TFP persistence, monetary policy parameters, and output gap and inflation. Second, we show that TFP persistence affects the optimal behavior of the monetary policy.

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1 Introduction

The rational expectation hypothesis is the cornerstone of the vast majority of recent macroeconomic models. This hypothesis implies that the optimal behavior of the agents depends on their predictions about future relevant state variables, given the forward-looking nature of these models. Therefore, a change in the autocorrelation structure of those state variables leads to different optimal choices and, as a consequence, a different equilibrium outcome. In this paper, we first document that the autocorrelation structure of the one of the main driving forces of up-to-date macroeconomic models, namely the total factor productivity (TFP henceforth), has changed throughout the last decades, and second, by considering a fairly standard New Keynesian model, we investigate the relationship between the TFP autocorrelation structure and monetary policy.

Carefully identifying the statistical properties of the stochastic processes that drive economic models is a key step to linking the theoretical model to the data. Intuitively, a successful model should predict an equilibrium path for macroeconomic variables that resembles their data counterpart; this ability depends also on the assumed specification of the exogenous processes driving the model. Eventually, these exogenous processes might be associated with an observable time series. For example, considering a neoclassical growth model driven by stochastic total factor productivity, Solow (1957) showed how to derive a time series for the empirical counterpart of TFP, the so-called Solow residuals. During the last two decades the large literature on Real Business Cycle models showed that models driven by the TFP, whose statistical process was calibrated using the Solow residuals, were able to match the properties of the economic cycle.

However, the parameters describing the process of TFP might change throughout the years. Many economists have intensively studied changes in the variance of the error terms of the exogenous processes. In fact, the literature on the Great Moderation, the term describing the reduction of the volatility of the macroeconomic variables after the early 1980s, has investigated whether a reduction of the magnitude of the shock hitting the driving forces of the economy was the main source of the moderation. This hypothesis, defined as the Good-Luck hypothesis, was endorsed by several authors[1], who documented a decline in the variance of the exogenous processes driving the models, in particular of the TFP. Their analysis uses rigorous statistical tools as well as the

[1] Stock and Watson (2002, 2003), Ahmed, Levin and Wilson (2004), Primiceri (2005), Galí and Gambetti (2009) and Liu, Waggoner and Zha,(2009).

estimation of rich macroeconomic models.

What are the consequences of a decline in the variance of the shock for the equilibrium of a rational expectations macroeconomic model? The policy functions that describe the relationship between control variables and state variables of the model depend on the set of the parameters of the model, including the variances of the shocks. However, a common procedure to solve macroeconomic models is to linearize the equilibrium conditions and to find a linear approximation of the true policy functions. In this case, however, a change in the variance of an exogenous shock does not alter the relationship between the control variables and the state variables, since the magnitude of the shock is only a scale-factor in a linearized equilibrium.

Whereas the interest in explaining the Great Moderation brought attention to the behavior of the variance of the shocks, there has been little focus on studying the dynamics of the autocorrelation structure of the shocks. In this paper we fill this gap by providing firm evidence of an increase in the persistence of the TFP. Our study of the evolution of the autocorrelation structure of the TFP is motivated by a recent study of Pancrazi (2011a) who shows that the volatility dynamics of macroeconomic variables changes with different sets of frequencies. In particular, the reduction of the volatility in the last three decades is particularly large at higher frequencies and much milder or even absent, at medium frequencies. This observation is at odds with the hypothesis that only a reduction of the magnitude of the shock drove the decline of the volatility of macroeconomic variables, since, in this case, their volatility should have decreased proportionally at all frequencies. The evidence brought by Pancrazi (2011a) is consistent with an increased persistence of macroeconomic variables. In fact, an increased persistence of a stochastic process implies a redistribution of its variance from higher to lower frequencies, which is consistent with the stylized facts mentioned above. Since it has been documented that the TFP is one of the main drivers of the economic cycle, we then investigate whether the increased persistence of economic variables is driven by a change in the autocorrelation structure of the TFP. By using a set of statistical tools, namely computing split sample statistics, rolling window estimates, recursive estimates, and by fitting a time-varying-parameters-stochastic-volatility model (TVP-SW, henceforth), we provide evidence that strongly supports an increase in the persistence of TFP. In particular, the statistical tools confirm that the TFP persistence has increased from values around 0.6 to values around 0.85 in the last few decades.

Unlike changes in the variance of its innovation, changes in the autocorrelation structure of an exogenous process have first-order effects on the equilibrium of a rational expectations model. Intuitively, a change in the persistence of an exogenous process affects the way agents compute expectations about the future state of the economy. This is a natural consequence of the forward-looking nature of the models. Moreover, the equilibrium outcome of the macroeconomic models is used by policy makers when deciding their policies. For example, in the optimal monetary literature, the policy parameters chosen by the monetary authority are those that minimize a given loss function, taking into account that the loss function depends on the equilibrium dynamics of the model, which depends on the parameters of the exogenous processes. It is therefore obvious that if the autocorrelation structure of the exogenous process affects the equilibrium, then it also affects the optimal policy decisions. Hence, in this paper we also thoroughly analyze the interaction between the autocorrelation structure of TFP, the monetary policy parameters, the equilibrium outcome of a model, and optimal monetary policy.

In particular, to gain some intuition, we first consider a simple monetary model, where the monetary authority solely chooses the nominal interest rate as a function of inflation. Since in this setting money is neutral, the monetary authority does not affect the equilibrium dynamics of real variables, but only that one of nominal variables, such as inflation. Nevertheless, this model is useful for understanding the interaction between monetary policy parameters and the persistence of TFP. This interaction is generated by the nature of the real interest rate, which in equilibrium is a function of the TFP persistence given the forward-looking nature of the model, by the assumed Taylor rule, which assumes the nominal interest rate to be a function of inflation, and by the Fisherian equation, which relates the nominal interest rate to the real interest rate and inflation. As a result, in equilibrium inflation is a non-linear function of the policy parameters and the TFP persistence. In this model, if the monetary authority responds more aggressively to inflation, the variance of inflation declines. However, the effectiveness of the monetary policy, measured by the change in inflation variance for a marginal change in the monetary policy parameters, is a non-linear function of the TFP persistence. We can easily pin down the value of the TFP persistence that maximizes this effectiveness for each value of the monetary policy parameter.

We then study a more realistic model in which money is non-neutral. In particular, we consider a fairly standard New Keynesian model, in which inflation dynamics is driven by the frictions on price settings and imperfect competition. The monetary policy is assumed to follow a standard Taylor rule. We focus on the equilibrium dynamics of two variables, output gap and inflation, since they are the two variables relevant for the welfare calculation. Considering a first-order approximation, we analytically derive the instantaneous response of these two variables to a technology shock. These responses are non-linear functions of the TFP persistence and monetary policy parameters. In particular, an increase in the response to inflation decreases the responses of both output gap and inflation to the shocks. This is the well-known Taylor principle: when the monetary authority responds more strongly to inflation, it guarantees that the real interest rate eventually rises with inflation. The increase in the real interest rate creates a counter-effect on inflation, since a higher real interest rate causes a fall in the output gap and in deviations of the marginal cost from the steady state. Moreover, we show that this effect depends on the TFP persistence. In particular, when the persistence of TFP is larger, its predictability increases, thus implying that the natural interest rate is closer to its steady state value. When the natural interest rate is stable around its steady state value, the output gap is less affected by a technology shock. However, the relationship between inflation response to a technology shock and TFP persistence is non-monotone. In fact, for the lower values of the monetary policy response to inflation, an increase in TFP persistence implies a larger effect of a technology shock on inflation, which, equivalently, implies a larger inflation variance. Given the tractability of the model, we can analytically pin down this interaction, as well as the relationship between TFP persistence and monetary policy effectiveness. We check the robustness of our results using a medium-scale dynamic stochastic general equilibrium model as in Smets and Wouters (2007) and show that our results carry over even when capital is added into our model.

A natural question to ask is whether the more aggressive monetary policy in the post 1980s documented in the literature (Clarida, Galí and Gertler (2000), Cogley and Sargent (2001,2005), and Boivin (2006)) is an optimal behavior given the increase in persistence of technology. To answer this question, we study the optimal monetary policy both in the basic New Keynesian model and in the slightly modified model, as in Giannoni (2010). The basic New Keynesian model is not the ideal setup to study optimal monetary policy, since the monetary authority does not face a trade-off between stabilizing inflation and output. However, by using this model we can investigate the welfare implications in case when the monetary authority does not internalize the increase in the TFP persistence. We find that, ceteris paribus (monetary policy parameters included), an increased persistence of TFP generates a larger welfare loss. However, by responding strongly to inflation, monetary authority mitigates the negative welfare effect of the increased persistence. Finally, we consider a model better suited for computing the optimal monetary policy, introducing cost-push shocks, as in Galí (2008). We conclude that the optimal monetary policy implies a stronger response to inflation, as well as output gap, as the persistence of technology rises. A drawback of this procedure is that for high values of the persistence, this method does not produce equilibrium since the determinacy condition is not satisfied for large values of the persistence, as also showed by Giannoni (2010).

The structure of the paper is as follows. In Section 2, we provide the evidence of the increased TFP persistence. Then we explore the relationship between monetary policy, TFP persistence and inflation dynamics using a simple monetary model in Section 3, and using a basic New Keynesian model in Section 4. In Section 5, we establish the link between the increased persistence of technology, monetary policy, and output gap and inflation dynamics. Finally, in Section 6 we study the optimal monetary policy decision as a function of TFP persistence. Section 7 concludes.

2 Total Factor Productivity Persistence

The overall volatility of macroeconomic variables declined after the early 1980s. While this phenomenon has been greatly investigated in the literature (see Giannone, Lenza and Reichlin (2008) for the summary of the literature), the behavior of the volatility at different frequencies was neglected. However, Pancrazi (2011a) investigates macroeconomic volatility dynamics of a large set of macroeconomic variables at different intervals of frequencies and shows that their higher-frequency volatility has dropped significantly, but the volatility at medium frequencies[2] has remained roughly constant. This redistribution of the variance towards lower frequencies can be interpreted as an increase in the persistence of macroeconomic variables. As an illustrative exercise, by using simple parametric models, we formally estimate an increase in the persistence of real per-capita output and consumption in the U.S. Output is measured as Gross Domestic Product, and consumption is measured as non-durable consumption and services. As in Pancrazi (2011b), we assume that the

[2] High frequencies are defined as variations from 6 to 32 quarters, whereas medium frequencies capture variations between 32 and 80 quarters.

stationary component of these variables, defined as the deviation from a non-linear trend[3], follows an AR(4) process. The rich autoregressive structure allows for a better characterization of the spectral density. We then estimate the persistence of the process by computing the largest root of the lag polynomial. Table 1 displays the point estimates of the largest root of the forth order lag polynomial and the standard deviation of its innovation, for output and consumption, before (Sample 1 from 1950:1 to 1982:4) and after (Sample 2 from 1983:1 to 2009:4) the early 1980s[4]. The estimates confirm increased persistence of the two variables during the last three decades and a reduction of the variance of the error terms.

A branch of the macroeconomic literature on time-varying volatility of the economic cycle has explained the reduction in the higher-frequency variance with a structural change of the magnitude of the shocks. For example, Stock and Watson (2002, 2003), Ahmed, Levin and Wilson (2004), Primiceri (2005), Galí and Gambetti (2009), Liu, Waggoner and Zha (2009) show that the standard deviation of the innovation of the shocks that drive macroeconomic dynamics shrank after the mid-80s, thus providing support for the Good luck hypothesis to explain Great Moderation. However, this hypothesis fails to explain the different dynamics of the volatility at different frequencies. In fact, a change of the magnitude of the innovation of a process implies a proportional shift of the spectral density at all frequencies, which is at odds with the stylized facts presented above. Therefore, in this section we investigate the dynamics of the autocorrelation structure of the shocks, namely the productivity shock, as a possible driver of this increased persistence of macroeconomic variables.

2.1 The Increased Persistence of Total Factor Productivity

Since the beginning of the real business cycle analysis, macroeconomists have recognized the importance of the TFP as one the main driving forces of the dynamics of macroeconomic variables (see for example Kydland and Prescott (1982, 1991), Long and Plosser (1983), Prescott(1986), King, Plosser and Rebelo (1988), Cogley and Nason (1995)). The contribution of technology to

[3] To eliminate a non-linear trend, we isolate the fluctuations with periodicity between 2 and 100 quarters.

[4] The choice of 1983 as the break date is in line with the estimate of the break in Stock and Watson (2002). In addition, Boivin and Giannoni (2006) use 1979:III as a break date, but they show that their results are robust to different break dates, such as 1984:I, a date consistent with some estimates of the date of change in the volatility of the U.S. economy.

capturing the movements and comovements among economic variables is large even when we allow for additional exogenous disturbances, as predicted by medium-scale DSGE models (Smets and Wouters (2007)). As a consequence, the literature on macroeconomic volatility dynamics has focused on exogenous disturbances, TFP in particular, and its role in explaining a change in the volatility dynamics. In fact, one branch of the literature suggests that the decrease of the variance of the technology shock accounts for a large fraction of the total decline of the volatility of macroeconomic variables (Stock and Watson (2002, 2003), Ahmed, Levin and Wilson (2004), Primiceri (2005), Galí and Gambetti (2009), Liu, Waggoner and Zha (2009)). Given these considerations, it is natural to use a similar approach to study whether a change of the autocorrelation structure of TFP can explain the additional dimensions of the volatility dynamics described above, namely the increased persistence of real variables and the uneven decline of the volatility at different frequencies.

In order to study whether the technology process has changed, we first construct the series of the TFP. Our definition of TFP accounts for a time-varying capacity utilization by constructing a measure of TFP as:

$$TFP_t = \left(\frac{Y_t}{L_t^{1-\alpha} \left(U_t K_t\right)^{\alpha}} \right). \tag{1}$$

This measure is consistent with the measure used in the medium-scale DSGE models, largely used in the recent macroeconomic literature. Greenwood, Hercowitz and Huffman (1988), Burnside and Eichenbaum (1996), Basu and Kimpball (1997), Altig, Christiano, Eichenbaum and Lindé (2005), among others, point out that accounting for a time-varying capacity utilization is important for obtaining a stronger propagation of the series in response to the shocks.

We set the labor share, $1 - \alpha$, equal to 0.64, which is the average value of the labor share series recovered from the Bureau of Labor Statistics (BLS). From the same source we recover annual data on capital services, K_t. We interpolate the capital services series to obtain quarterly series, assuming constant growth within the quarters of the same year. Non-farm business measures of hours, H_t, and output, Y_t, are also retrieved from the BLS. The series of capacity utilization, U_t, is retrieved from the Federal Reserve Board, and is based on the manufacturing data.

We document the increase in the persistence of TFP using several techniques: split sample statistics, rolling windows statistics, recursive estimate statistics and finally time-varying parameters estimation. The explanation of the techniques and the results regarding the persistence of TFP are described in detail below.

2.2 Split Sample Statistics

As an initial exercise we study the behavior of the persistence of TFP in two subsamples, before and during the Great Moderation. However, there is no particular reason to assume that a change in the autocorrelation structure of productivity took place in the early eighties, when many macroeconomists have dated a break in the variance of the shock. We use this assumption only for convenience and it will be relaxed in the following exercises. We assume that the stationary component of the total factor productivity, $\overline{TFP}_t$, obtained by eliminating a non-linear trend and displayed in Figure 1, follows a first order-autoregressive process, i.e.:

$$\overline{TFP}_t = \rho\overline{TFP}_{t-1} + \sigma_\varepsilon \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0,1). \tag{2}$$

Table 2 shows the estimates of ρ, the largest and unique root of the lag polynomial, and of σ_ε, the standard deviation of the innovation, in the two subsamples. The top panel of the table shows two important findings. First, the variance of the innovation of technology has largely decreased in the last thirty years. Second, the persistence of the TFP has increased instead. These results hold also in the case we consider a constant capacity utilization, i.e. $U_t = 1$.

In order to check the robustness of these results to different specifications of the statistical model, we then assume that the stationary component of the TFP follows a richer autoregressive process, an AR(4). The bottom panel of the Table 4 shows that, in this case, the increase of persistence is even more pronounced, and the decline in the variance of the error term is quantitatively comparable to the one estimated for the AR(1) process. As a result, the TFP is showed to exhibit dynamics similar to the one of the real variables: the increase in persistence shifts the volatility of TFP from higher to lower frequencies, thus implying an uneven reduction of the volatility across frequencies. These dynamics can be clearly visualized by plotting the log-spectrum of the TFP processes estimated before and after the early 1980s. Figure 2 displays the log-density resulting from the estimated AR(4) processes of TFP in the two subsamples. Recall that the variance attributable to a particular interval of frequencies corresponds to the area below the spectrum in that interval. We can notice that the higher frequency volatility of TFP declined in the second subsample. However, the reduction is smaller when lower frequencies are considered. This effect is driven by the higher persistence of the process in the second subsample, as visualized by the shift of density towards lower frequencies in the second subsample. In order to highlight the larger relative importance of the lower frequencies in explaining the volatility of TFP in the second subsample, Figure 3 plots the normalized-spectrum of the estimated AR(4) process for the TFP in the two subsamples. The area below the normalized spectrum in a given interval of frequencies is equivalent to the fraction of the variance attributable to those frequencies. It is evident that a portion of the total variance captured by the lower frequencies is much larger in the second than in the first subsample.

2.3 Rolling Windows Estimates

As mentioned above, there is no particular reason to date a possible increase of the persistence of TFP in the early eighties. Therefore, we now analyze the TFP persistence dynamics with no references to a particular date. Assuming that the stationary component $\overline{TFP}_t$ follows a first order autoregressive process as in (2), we can visualize the evolution of the persistence of TFP over time by constructing a rolling window estimates as follows:

$$\hat{\rho}_t = \hat{\rho}\left(\left\{\overline{TFP}\right\}_{j=t-k}^{t}\right) \quad \text{for } t = k+1, ..., T,$$

where $\hat{\rho}_t\ (x_t)$ indicates the point estimate of the first order autoregressive parameter for the time series x_t, k indicates the length of the window, T is the length of the time series, and $\{x_t\}_{t_1}^{t_2}$ represents the subset of observations of the time series x_t between the periods t_1 and t_2. Accordingly, $\hat{\rho}_t$ is the value of the persistence of TFP when k observations of the series $\overline{TFP}_t$ prior to time t are considered. Analogously, we compute the rolling windows estimate of the standard deviation of the innovations, as:

$$\hat{\sigma}_{\varepsilon,t} = \hat{\sigma}_{\varepsilon}\left(\left\{\overline{TFP}\right\}_{j=t-k}^{t}\right) \quad fort = k+1, ..., T,$$

where $\hat{\sigma}_{\varepsilon,t}\,(x_t)$ indicates the point estimate of the standard deviation of the error term when x_t follows a first order autoregressive process.

Figure 4 plots the rolling window estimates of $\hat{\rho}_t$ (solid line, left axes) and of $\hat{\sigma}_{\varepsilon,t}$ (dashed line, right axes). We observe that the persistence of TFP has gradually increased throughout the sample. On the other hand, the standard deviation of the innovations has declined, which is consistent with the change of the volatility dynamics described in the previous section. Interestingly, the increasing in persistence seems to match the timing of the declining of the variance of the shocks.

In order to assess whether our findings depend on the assumed statistical process, we compute similar rolling windows statistics by estimating sequences of forth-order autoregressive process. In this case, a measure of persistence is given by the largest root of the forth order lag polynomial. We still observe a noticeable increase in the TFP persistence. Therefore, this outcome is not an artifact of the assumed stochastic process.

2.4 Recursive Estimate Statistics

In the recursive least squares we repeatedly estimate the statistical model in (2), using a larger subset of the sample data of TFP for each repetition. For example, the first estimate $\hat{\rho}_1^{RE}$ is obtained by using the first k =16 observations of $\overline{TFP}_t$. Then the next observation is added to the data set and $k+1$ observations are used to compute the second estimate $\hat{\rho}_2^{RE}$. This process is repeated until all the T sample points have been used, yielding $T-k+1$ estimates of the $\hat{\rho}^{RE}$. Figure 5 plots the recursive estimate $\hat{\rho}_t^{RE}$. Since the number of observations used to obtain initial estimates $\hat{\rho}_t^{RE}$ is relatively small, the estimates might be imprecise, we cut the first twenty years of estimates and report, thus, the estimates starting from 1970, which is the starting date of the rolling-window statistics as well. Also this method suggests that the persistence of technology has increased in the second part of the sample.

Furthermore, at each step the last estimate of $\hat{\rho}^{RE}$ can be used to predict the next value of the dependent variable. The one-step-ahead forecast error resulting from this prediction, suitably scaled, is defined as a recursive residual. To test whether the value of the dependent variable at time t might have been generated from the model fitted to all the data up to that point, each error can be compared with its standard deviation from the full sample. In Figure 6 we plot the recursive residuals and standard errors together with the sample points whose probability value is at or below 15 percent. Residuals outside the standard error bands suggest instability in the parameters of the equation. Figure 7 shows that there are several periods in the middle of the sample in which it is likely that a break in the autoregressive parameter in (2) occurred.

Finally, we use a CUSUM of squares test (Brown, Durbin, and Evans, 1975). The expected value of this statistics under the hypothesis of parameter constancy is a straight line that goes from zero at $t = k$, to unity at $t = T$. A significant departure of the test statistics from its expected value is assessed by reference to a pair of parallel straight lines around the expected value.[5] Figure 8 displays the CUSUM of squares test against t and the pair of 5 percent critical lines. Since the CUSUM test moves outside the band approximately in the middle part of the sample, the diagnostic suggests the presence of a change in the autocorrelation structure of TFP.

2.5 Time-Varying Parameters Estimation

The statistical analysis presented in this section suggests a slow change in the persistence of TFP. It is then natural to estimate a time-varying parameter model for TFP. In addition, since Figures 4 and 5 suggest a decline of the variance of the error term in the regression, we include stochastic volatility in the model (TVP-SV) as well. In particular, we assume that the model is given by the following equations:

$$\begin{aligned}
\overline{TFP}_t &= \rho_t \overline{TFP}_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N\left(0, \sigma_t^2\right) \\
\rho_{t+1} &= \alpha_t + u_t \qquad u_t \sim N\left(0, \sigma_u^2\right) \\
\sigma_t^2 &= \gamma \exp\left(h_t\right) \\
h_{t+1} &= \phi h_t + \eta_t \qquad \eta_t \sim N\left(0, \sigma_\eta^2\right).
\end{aligned}$$

We follow Nakajima's (2011) Markov Chain Monte Carlo approach to estimate the parameters of the model. Appendix A reports the technical detail of the estimation procedure, as well as the outcome of the MCMC estimation.

Figure 9 and Figure 10 show that also a TVP-SV model estimate the increased persistence of TFP as well a decline of the variance of its innovations.

[5] See Brown, Durbin, and Evans (1975) or Johnston and DiNardo (1997, Table D.8)

3 A Simple Monetary Model and TFP persistence

The persistence of the exogenous shocks has a crucial role in defining the equilibrium dynamics of the macroeconomic variables in general equilibrium models. In fact, since these models are in general forward looking, the ability of the agents to forecast the future values of the exogenous variables affects their contemporaneous decisions. In general, the equilibrium dynamics of the model can be represented by the policy functions:

$$\begin{aligned} y_t &= g\left(x_t; \Theta, \Phi\right) \\ x_{t+1} &= h\left(x_t; \Theta, \Phi\right) \end{aligned}$$

where, y_t denotes the vector of control variables of the model, x_t denotes the vector of state variables, Θ is the set of structural parameters of the model, and Φ is the set of parameters describing the stochastic processes of the exogenous variables. It is evident that a change in the persistence of an exogenous process affects the equilibrium dynamics of the model.

Since the true policy functions $h\left(\cdot;\cdot\right)$ and $g\left(\cdot;\cdot\right)$ are usually hard to compute analytically, a linear approximation of the two functions is often a convenient way to represent the dynamics of the model. In this case, we have:

$$\begin{aligned} y_t &\simeq \tilde{g}\left(\Theta, \varrho\right) x_t \\ x_{t+1} &\simeq \tilde{h}\left(\Theta, \varrho\right) x_t \end{aligned}$$

where now $\tilde{g}$ and $\tilde{h}$ are reduced form parameters that depend both on the structural parameters of the model, Θ, and the set of parameters ϱ that describe the autocorrelation structure of the exogenous processes. It is important to notice that parameters that affect the variance of the exogenous processes but not their autocorrelation structure (for example the variance of the innovation of the process) do not have any impact on the equilibrium dynamics of the model. This is a trivial consequence of the first-order approximation. On the other hand, a change in the autoregressive component of the exogenous shocks, which is contained in the ϱ, alters the reduced form parameters $\tilde{g}$ and $\tilde{h}$, thus affecting the equilibrium path of the control variables.

In the previous section, we provided evidence of a change of the autoregressive coefficient of the total factor productivity, which is an important exogenous driving force of a large family of macroeconomic models, and, in particular, of monetary models. Since monetary authorities construct their policy based on the equilibrium dynamics of the economy, understanding the interaction between monetary tools and a change in the persistence of TFP is an important question to address. We will make use of standard monetary models to illustrate this relationship.

3.1 A Simple Monetary Model (Neutrality of Money)

In order to study the interaction between the persistence of total factor productivity and monetary policy, we first consider a very simple stylized model of classical monetary economy. Since the model is standard (see Galí, p.16) we present the formal equations in Appendix B and here we only describe its key features. The representative agent maximizes the lifetime utility function. The instantaneous utility function depends upon consumption and leisure. The agent can trade one-period nominally risk-less bonds. A representative firm produces output by employing labor. The productivity of labor evolves exogenously according to a first order autoregressive process. The model features perfect competition and fully flexible prices in all markets. In addition, the monetary authority follows an inflation-based interest rate rule. As a consequence of these assumptions, the real variables are determined independently of monetary policy. However, in this section we are interested in the dynamics of inflation, which will depend on the interaction between the monetary policy and the statistical properties of the technology shock.

The central bank adjusts the nominal interest rate, i_t, according to:

$$i_t = \rho + \phi_\pi \pi_t \tag{3}$$

where π_t denotes inflation, $\rho \equiv -\log(\beta)$ is the steady-state value of the real interest rate, with β being a discount factor, and with $\phi_\pi \geq 0$. Given the Fisherian equation:

$$i_t = E_t \pi_{t+1} + r_t,$$

where r_t is the real interest rate, E_t indicates the expectations operator conditional to the information available at time t, and assuming that $\phi_\pi > 1$, we can compute the stationary solution for inflation:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t \hat{r}_{t+k}, \tag{4}$$

where $\hat{r}_t = r_t - \rho$.

In equilibrium the real interest rate is given by:

$$r_t = \rho + \sigma\psi E_t \{\Delta a_{t+1}\} \tag{5}$$

where $\psi = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$, σ is the inverse of intertemporal elasticity of substitution, $1-\alpha$ is the labor share in the Cobb-Douglas production function, and φ is the inverse of the Frisch elasticity of labor supply. Assuming that the technology a_t evolves as a first order autoregressive stochastic process:

$$a_{t+1} = \rho_a a_t + \sigma_a \varepsilon_{t+1} \tag{6}$$

then, inflation dynamics in equilibrium is given by:

$$\pi_t = \delta_a a_t.$$

with

$$\delta_a = -\frac{\sigma\psi(1-\rho_a)}{\phi_\pi - \rho_a}. \tag{7}$$

It follows that the variance of inflation, σ_π^2, is given by:

$$\sigma_\pi^2 = \delta_a^2 \frac{\sigma_a^2}{(1-\rho_a^2)} \tag{8}$$

The two equations above display two important implications. First, as well known in monetary economics, the monetary policy can alter the volatility of inflation by increasing the monetary policy parameter ϕ_π. Second, as previously described, the autocorrelation structure of the exogenous process, driven by ρ_a, alters the equilibrium dynamics of inflation and its variance. In fact, the reduced form parameter δ_a, which measures the instantaneous effect of a technology shock on inflation, is a non-linear function of ρ_a. These two features imply that the effectiveness of the monetary authority in smoothing out the variance of inflation is a function of the persistence of technology, ρ_a. Since in the previous section we have showed that the persistence of the TFP has actually changed throughout the sample, it is interesting to study how the effectiveness of the monetary policy varies with $\rho_{a.}$

In order to graphically illustrate the connection between these two parameters, we first assign some values to the parameters of the model, using a standard calibration. In particular, following Galí (2008), we set $\beta = 0.99$, $\sigma = 1$, $\varphi = 1$, $\alpha = \frac{1}{3}$, $\sigma_a = 1$. Figure 11 and Figure 12 display this interaction. In Figure 11 the z-axes reports the instantaneous response of inflation to a technology shock, $\delta_{a.}$ The plot shows two relevant features. First, for any value of ρ_a, by responding more aggressively to inflation (higher ϕ_π) monetary policy can lower the effect of a technology shock on inflation. This, once again, is a well known result in monetary economics and comes directly from (7). Second, the magnitude of this effect drastically depends on the persistence of technology, ρ_a. For example, fixing ϕ_π close to 1.1, the lower value on the x-axis, a marginal change of the monetary policy parameter has the largest effect on δ_a when ρ_a takes values around 0.9 and smallest when ρ_a takes values at the extremes (0.99 and 0.5). This insight is confirmed when we plot the variance of inflation in Figure 12. The non-monotone shape of the surface is due to the interactions between the reduced form parameter δ_a and the unconditional variance of technology $\frac{\sigma_a}{(1-\rho_a^2)}$, when ρ_a varies. Interestingly, there is a value of the TFP persistence that maximizes the variance, when the monetary policy parameter is particularly low. Moreover, the stabilizing effect of a small change of ϕ_π largely varies in the space (ρ_a, ϕ_π). Therefore, by using a very simple model, we showed that the variance of inflation, and the effectiveness of a given change of the monetary policy parameters varies non-linearly with the autocorrelation structure of the exogenous shock.

We can analytically investigate the properties of the relationship between the instantaneous response of inflation to a technology shock, δ_a, the monetary policy parameter, ϕ_π, and the persistence of TFP, ρ_a, by studying how the variance of inflation and the effectiveness of the monetary policy, measured by the derivative of δ_a with respect to monetary policy parameter ϕ_π, varies with $\rho_{a.}$ Figure 12 shows that the relationship between variance of inflation, TFP persistence and monetary policy parameters is non-monotone for low values of ϕ_π. The tractability of this simple model allows us to compute analytically the locus of pairs (ρ_a, ϕ_π) that maximize the inflation variance. We can then express this relationship, as showed in the following Proposition.

Proposition 1 *Consider a monetary policy model characterized by the inflation dynamics in* (4) *, by the monetary policy rule in* (3) *, the equilibrium interest rate as in* (5) *, and by the stochastic* *process for the total factor productivity as in* (6) *. Then, the variance of inflation is non-monotone in* ρ_a *and the value of the monetary policy response to inflation* ϕ_π *that maximizes the variance of inflation in* (8) *is given by:*

$$\phi_\pi^* = 1 + \rho_a - \rho_a^2 \tag{9}$$

See Appendix C for the proof.

Next, we compute the level of ϕ_π for which the effectiveness of monetary policy is maximized. We define the effectiveness of the monetary policy as the effect of a marginal change in the monetary policy parameter ϕ_π on the instantaneous response of inflation to a technology shock, captured by δ_a. From (7), we obtain:

$$\frac{\partial \delta_a}{\partial \phi_\pi} = \frac{\sigma \psi (1 - \rho_a)}{(\phi_\pi - \rho_a)^2}. \tag{10}$$

Figure 13 plots how this effect varies with the current level of ϕ_π and the persistence of TFP, ρ_a. It is evident that a marginal increase in the monetary policy parameter has larger effect when ϕ_π is small (close to one) and when ρ_a assumes values around 0.9. When ρ_a is very close to unity, the monetary policy does not have much effect on the overall variance of inflation. The reason is that in this model inflation is a consequence of the departure of the real interest rate from its steady state value. When the persistence of TFP approaches one, the interest rate is always close to its steady state value and therefore the inflation is particularly small. As an obvious consequence, the monetary policy has no effect on the variance of inflation. Another interesting feature illustrated by the function in (7), is the non-linearity of the monetary policy effect on δ_a for different values of TFP persistence. This non-linearity is due to the term in the denominator $(\phi_\pi - \rho_a)$, which results from the assumed Taylor rule and the law of motion of the exogenous process. This term highlights the deep interaction between the autocorrelation structure of TFP and the effectiveness of monetary policy. Finally, an additional implication of Figure 13, is that the role of the persistence ρ_a on $\frac{\partial \delta_a}{\partial \phi_\pi}$ diminishes when ϕ_π is larger.

Using this illustrative model, it is trivial to derive the value of ϕ_π for which the effect of monetary policy on the response δ_a is maximized. To do that, we simply take the second derivative $\frac{\partial^2 \delta_a}{\partial \phi_\pi \partial \rho_a}$ and solve for ϕ_π. We obtain that the value of the persistence of technology, ϕ_π^{**} which maximizes the effect of the monetary policy is simply:

$$\phi_\pi^{**} = 2 - \rho_a.$$

Proposition 2 *Consider a monetary policy model characterized by the inflation dynamics in* (4) *, by the monetary policy rule in* (3) *, the equilibrium interest rate as in* (5) *, and by the stochastic process for the total factor productivity as in* (6) *. Then, the value of the monetary policy response to inflation* ϕ_π *for which the effect of a change in the monetary policy parameter on the instantaneous response of inflation to a technology shock,* $\frac{\partial \delta_a}{\partial \phi_\pi}$*, with* δ_a *defined in* (7) *, is maximized is given by:*

$$\phi_\pi^{**} = 2 - \rho_a. \tag{11}$$

See Appendix C for the proof.

In summary, this trivial model clearly illustrates the relationship between monetary policy parameters, technology persistence, and their effect on inflation. Next, we will study the same relationship in a model in which money is not anymore neutral as to allow for the real variables to depend on monetary policy parameters as well.

4 A New Keynesian Model and TFP Persistence

In the simple model presented above, the monetary policy can control only the volatility of inflation, since the neutrality of nominal variables implies that the real block of the model is independent from any monetary policy action. However, with fairly common assumptions, it is possible to set up an environment in which the monetary policy affects real variables as well. In this section we consider a fairly simple New Keynesian model as in Galí (2008). In this setting, the monetary authority can use its policy to affect both inflation and real variables, through the output gap. In what follows we explore how the interaction between monetary policy and TFP persistence affects inflation and output gap, which turn out to be the welfare-relevant variables.

4.1 Equilibrium

The model is characterized by two rigidities. First, the perfect competition assumption is abandoned by assuming that each firm produces a differentiated good and sets its price. Therefore, households must decide how to allocate its consumption expenditures among the differentiated goods in addition to making the usual consumption/savings and labor supply decision. Second, firms set their prices a lá Calvo (1983) and Yun (1996), i.e. in any given period, only a fraction of randomly picked firms is allowed to reset their prices. These assumptions imply that monetary variables are not neutral, since they affect the equilibrium path of real variables. As a consequence, we can also study how the interaction between monetary policy and TFP persistence affects the real block of the model. Since the model is fairly standard, we present only its equilibrium conditions. A complete representation of the model is provided in Appendix B. The non-policy block of the model is composed of the New Keynesian Phillips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{y}_t, \tag{12}$$

and the dynamic IS equation, given by

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right) + E_t\left(\tilde{y}_{t+1}\right). \tag{13}$$

Here, E_t denotes expectation conditional on the information at time t, π_t denotes the inflation rate at time t, i_t is the nominal interest rate at time t, r_t^n is the natural real interest rate, $\tilde{y}_t$ is the output gap defined as the deviation of output from its flexible-price counterpart, β is the discount factor, $\kappa = \lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ with $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\varepsilon)}$, σ is the inverse of intertemporal elasticity of substitution, $1-\alpha$ is the labor share in the production function, φ is the inverse of the Frisch elasticity of labor supply, θ is the price stickiness parameter, and ε is the elasticity of substitution among the differentiated goods. The dynamics of the model are governed by two exogenous processes. First, the level of technology, which we denote as a_t, follows a first order autoregressive, $AR(1)$, process[6]:

[6] The technology affects the logarithm of output: $y_t = a_t + (1-\alpha)\, n_t$, where n_t is the logarithm of hours worked.

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \text{ where } \varepsilon_t^a \sim N(0,1). \tag{14}$$

Second, the monetary policy shock, denoted as v_t, follows a similar first order autoregressive process:

$$v_t = \rho_v v_{t-1} + \sigma_v \varepsilon_t^v, \text{ where } \varepsilon_t^v \sim N(0,1). \tag{15}$$

The monetary policy shock is considered to be the exogenous component of the nominal interest rate rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t, \tag{16}$$

where i_t is the nominal interest rate at time t, and ρ is the household's discount rate, with $\rho = -\log(\beta)$.

Up to a first-order approximation, the output gap can be written as the following function of the two exogenous processes:

$$\tilde{y}_t = \Lambda_v\left(\phi_\pi, \phi_y, \rho_v, \Theta\right) v_t + \Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right) a_t, \tag{17}$$

where Λ_v and Λ_a are functions of the Taylor rule parameters $\left(\phi_\pi, \phi_y\right)$, the persistence parameters of the exogenous processes (ρ_a or ρ_v), and all the other structural parameters of the model gathered in the vector Θ. In particular, by using the methods of undetermined coefficients, we can compute the reduced form parameters $\Lambda_v\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)$ and $\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$:

$$\Lambda_v\left(\phi_\pi, \phi_y, \rho_v, \Theta\right) = -\frac{(1-\beta\rho_v)}{(1-\beta\rho_v)\left(\sigma(1-\rho_v)+\phi_y\right)+\kappa(\phi_\pi-\rho_v)} \tag{18}$$

$$\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right) = -\psi\frac{\sigma(1-\rho_a)(1-\beta\rho_a)}{(1-\beta\rho_a)\left(\sigma(1-\rho_a)+\phi_y\right)+\kappa(\phi_\pi-\rho_a)}, \tag{19}$$

where $\psi = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and κ is defined as above. Notice that these expressions imply that the relationship between the persistence of the exogenous shocks and the level of output gap is non-linear in the monetary policy parameters.

Assuming that ε_t^a and ε_t^v are independent, it is trivial to obtain the variance of output gap:

$$Var(y_t) = \left[\Lambda_v\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)\right]^2 \frac{\sigma_v^2}{1-\rho_v^2} + \left[\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)\right]^2 \frac{\sigma_a^2}{1-\rho_a^2}. \tag{20}$$

We can compute the equilibrium equation also for inflation, which is:

$$\pi_t = \Lambda_v^\pi \left(\phi_\pi, \phi_y, \rho_v, \Theta\right) v_t + \Lambda_a^\pi \left(\phi_\pi, \phi_y, \rho_a, \Theta\right) a_t \tag{21}$$

with

$$\Lambda_v^\pi \left(\phi_\pi, \phi_y, \rho_v, \Theta\right) = -\frac{\kappa}{(1-\beta\rho_v)\left(\sigma(1-\rho_v)+\phi_y\right)+\kappa(\phi_\pi-\rho_v)} \tag{22}$$

$$\Lambda_a^\pi \left(\phi_\pi, \phi_y, \rho_a, \Theta\right) = -\psi\left(\frac{\sigma(1-\rho_a)\kappa}{(1-\beta\rho_a)\left(\sigma(1-\rho_a)+\phi_y\right)+\kappa(\phi_\pi-\rho_a)}\right). \tag{23}$$

4.2 The Effects of Monetary Policy

The basic New Keynesian model has been a workhorse model for studying monetary policy. In fact, a lot of attention in the last decade has been devoted to understanding the stabilizing effects of the monetary authority on macroeconomic variables. When the monetary authority responds more strongly to inflation (higher ϕ_π), it guarantees that the real interest rate eventually rises with inflation. The increase in the real interest rate creates a counter-effect to inflation, since a higher real interest rate causes a fall in the output gap and in deviations of the marginal cost from the steady state. This is a well known intuition behind the Taylor Principle. Therefore, an increase in ϕ_π diminishes the exposure of output gap and inflation to monetary shocks, since they are smoothed out by the "lean-against-the-wind" strategy adopted by the monetary authority. This intuition explains why an increase of ϕ_π lowers both $\Lambda_v\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)$ and $\Lambda_v^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)$. In other words, a more aggressive monetary policy reduces the impact of monetary shocks both on inflation and on output gap.

However, an increase of ϕ_π has also a secondary effect, which has drawn much less attention in the literature. In fact, as displayed in equations (17) , and (21) ,the reduced form parameters $\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ and $\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ also depend on the monetary policy parameters. Therefore, a change in the monetary policy also leads to different responses of output gap and inflation to technology shocks. In particular, the effects of a change in the Taylor rule parameter ϕ_π on the reduced form parameters Λ_a and Λ_a^π are given by:

$$\frac{\partial \Lambda_a\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)}{\partial\ \phi_\pi} = \frac{\psi\kappa\sigma\left(1-\rho_a\right)\left(1-\beta\rho_a\right)}{\left[\left(1-\beta\rho_a\right)\left(\sigma\left(1-\rho_a\right)+\phi_y\right)+\kappa\left(\phi_\pi-\rho_a\right)\right]^2} \tag{24}$$

$$\frac{\partial \Lambda_a^\pi\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)}{\partial\ \phi_\pi} = \frac{\kappa^2\sigma\psi(1-\rho_a)}{\left[\left(1-\beta\rho_a\right)\left(\sigma\left(1-\rho_a\right)+\phi_y\right)+\kappa\left(\phi_\pi-\rho_a\right)\right]^2}. \tag{25}$$

Notice that both derivatives are positive. Since $\Lambda_a^\pi\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)$ and $\Lambda_a^\pi\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)$ are negative, a more aggressive monetary policy reduces the instantaneous response of inflation and output gap to a technology shock. This effect goes in the same direction as the Taylor-principle effects, which reduces the variance of inflation by eliminating both the technology-shock and the monetary-shock effect. Moreover, the effects of monetary policy on the instantaneous responses of output gap and inflation to a technology shock are also affected by the TFP persistence. We will explore this interaction in the next subsection.

4.3 TFP Persistence and Monetary Policy

In order to illustrate the relationship between monetary policy, technology persistence and instantaneous responses of inflation and output gap to a technology shock, we first use a fairly standard calibration of the New Keynesian model. We calibrate preference and technology parameters following Galí's baseline calibration: $\beta = 0.99$, $\sigma = 1$, $\alpha = 1/3$, $\varepsilon = 6$, and $\theta = 2/3$. We assume that the parameter of the Taylor rule with output gap is equal to $\phi_y = 0.125$. We then plot the values of the instantaneous responses $\Lambda_a\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)$ and $\Lambda_a^\pi\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)$, as a function of the monetary policy response to inflation, with $\phi_\pi \in [1.1, 2]$ and the persistence of TFP, with $\rho_a \in [0.5, 0.99]$.

Figure 14 plots the instantaneous response of output gap to a technology shock, $\Lambda_a\left(\phi_\pi,\phi_y,\rho_v,\Theta\right)$. Two effects are evident from the figure. First, fixing ρ_a, the instantaneous response of output gap is an increasing function of the Taylor rule parameter with inflation, ϕ_π. Similarly, fixing ϕ_π, the instantaneous response of output gap is an increasing function of the TFP persistence ρ_a. This result is general and does not depend on particular values of the structural parameters, but only on the conventional restrictions on their values, as proved in the following Proposition.

Proposition 3 *Consider the instantaneous response of output gap to a technology shock in the New Keynesian model presented above, as in* (3) *. Assume that* $\rho_a \in (-1, 1)$ *,* $\beta < 1$, $\phi_y > 0$, $\theta < 1$,

$\alpha < 1$, $\sigma > 0$, $\varepsilon > 0$, $\zeta > 0$, and $\phi_\pi > 1$. Then

$$\frac{\partial\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \phi_\pi} > 0 \tag{26}$$

and

$$\frac{\partial\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \rho_a} > 0 \tag{27}$$

for any structural parameter vector Θ.

See Appendix C for the proof. We now study the instantaneous response of inflation to a technology shocks, $\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)$, plotted in Figure 15. The figure shows that whereas an increase in the monetary policy parameters ϕ_π always decreases in absolute value the response of output gap due to the partial derivative in(26) being always negative, the effect of a change in the TFP persistence on $\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)$ is non-monotone, as it was for the output-gap response. In fact, when ϕ_π is particularly low, higher TFP persistence first increases the magnitude of the inflation-response to a technology shock and then it decreases it. This feature is important for the monetary policy authority: assume that the monetary authority measures welfare as a linear combination of inflation variance and output gap variance (as rationalized in the next session). An increase in the persistence of TFP has two opposite effect: it lowers the output gap variance (welfare improving) and it increases the inflation variance (welfare decreasing).

As in case of the simpler model, we can compute the value the monetary policy parameter ϕ_π for which the instantaneous response of inflation to a technology shock is maximized, as stated in the following Proposition.

Proposition 4 *Consider the instantaneous response of output gap to a technology shock in the New Keynesian model presented above, as in (23). Assume that the structural parameters satisfy the restriction of Proposition 3. Then there exists a value $\phi_\pi^{\pi*}$ that maximizes instantaneous response $\left|\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)\right|$. This value is:*

$$\phi_\pi^{\pi*} = \frac{\kappa + \beta\sigma\left(1-\rho_a\right)^2 - \left(1-\beta\right)\phi_y}{\kappa}$$

for any structural parameter vector Θ.

See Appendix C for the proof.

Figure 16 displays the TFP persistence $\rho_a^{\pi *}$ that maximizes the instantaneous impact $\Lambda_\alpha^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ as a function of $\phi_{\pi.}$

An additional feature of the model is that the effectiveness of monetary policy, defined as the effect of a marginal change in ϕ_π on the instantaneous response of a variable to a technology shock, also varies with ρ_a, as showed by equations (24), and (25). Figure 17 and Figure 18 display the proposed measure of effectiveness for output gap and inflation respectively. In particular, the figures show what is the effect of a marginal change of the monetary policy parameter ϕ_π on $\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ and $\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$. Hence, a large value in the z-axes means that the instantaneous responses are particularly sensitive to small changes in the monetary policy for the corresponding values of ϕ_π and ρ_a.

These figures show that effectiveness of the monetary policy is particularly sensitive to the persistence of technology when the inflation-Taylor rule coefficient is particularly low. However, this relationship is non-monotone, since there exist values of the monetary policy parameter that maximize the effectiveness for a given value of ρ_a. The next Proposition pins down these values.

Proposition 5 *Consider effectiveness of the monetary policy on the instantaneous responses of output gap and inflation to a technology shock, as defined in* (24) *and* (25). *Assume that the structural parameters satisfy the restrictions of Proposition 4. Then there exist values* $\phi_\pi^{eff(\tilde{y})}$ *and* $\phi_\pi^{eff(\pi)}$ *that maximize respectively the effectiveness* $\frac{\partial\, \Lambda_a\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)}{\partial\, \phi_\pi}$ *and* $\frac{\partial\, \Lambda_a^\pi\left(\phi_\pi,\phi_y,\rho_a,\Theta\right)}{\partial\, \phi_\pi}$. *For any structural parameter vector* Θ, *these values are:*

$$\phi_\pi^{eff(\tilde{y})} = \frac{\phi_y\left(1-\beta\right)\left(1-\beta\rho_a\right)-\kappa\left(2-\rho_a-\beta\rho_a\right)-\sigma\left(1-\rho_a\right)\left(1+\beta-3\beta\rho_a-\beta^2\rho_a\left(1-2\rho_a\right)\right)}{\kappa\left(\beta\left(2\rho_a-1\right)-1\right)} \tag{28}$$

$$\phi_\pi^{eff(\pi)} = \frac{\kappa\left(2-\rho_a\right)-\phi_y\left(1-\beta\left(2-\rho_a\right)\right)-\sigma\left(1-\rho_a\right)\left(2\beta\rho_a-1-2\beta\right)}{\kappa}. \tag{29}$$

The equations (28) and (29) can be used to derive the level of TFP persistence for which a marginal change in the monetary policy parameter, ϕ_π affects the output gap and inflation response to a technology shock the most, given the structural parameters. Figure 19 plots $\rho_a^{eff(\tilde{y})}$ (dashed line) and $\rho_a^{eff(\pi)}$ (solid line), for different values of ϕ_π.

5 Output Gap and Inflation Variance

In the previous section, we documented that the persistence of technology plays a key role in shaping the instantaneous response of output gap and inflation to the technology shock. Although it is crucial to understand the mechanism behind this relationship, we are ultimately interested in the behavior of the total variance of output gap and inflation since up to the second order, the objective function of monetary policy is a function of the two variances. Therefore, as can be seen from (17) and (21), in order to understand the behavior of the total variances of output gap and inflation we also need to consider instantaneous response of these two variables to the monetary policy shock (Λ_v and Λ_v^{π}). In this section, using a reasonable calibration, we quantify the effects of a change in ϕ_π on the total variance of output gap and inflation which will in turn help us quantify the effects on welfare.

Recall that the variance of output gap can be written as:

$$var\left(\tilde{y}_t\right) = \left[\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)\right]^2 \frac{\sigma_a^2}{1-\rho_a^2} + \left[\Lambda_v\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)\right]^2 \frac{\sigma_v^2}{1-\rho_v^2},$$

The role of ρ_a in shaping this expression is twofold: first, it affects the reduced form parameter Λ_a as extensively discusses in the previous section, and second, it affects the unconditional variance of the technology shock $\frac{\sigma_a^2}{1-\rho_a^2}$. Since we want to isolate only the first effect, we keep the unconditional variance of technology shock constant as ρ_a varies, by adjusting the variance of innovations σ_a^2. In addition, we keep the ratio between the unconditional variance of monetary shock and technology shock constant in order to eliminate the effect of the change in the relative importance of the two shocks. To do so, we adjust the variance of the innovation σ_v^2 as ρ_a varies. We calibrate the ratio between the unconditional variances of the two shocks using point estimates of the shock processes from Smets and Wouters (2007). In particular, we use the mean of the posterior distribution of ρ_a, σ_a, ρ_v and σ_v which are 0.95, 0.45, 0.15 and 0.24 respectively. [7] The rest of the structural parameters is calibrated as in the previous section.

Figure 20 displays the variance of output gap as a function of technology shock persistence ρ_a

[7] We are aware of the fact that Smets and Wouters (2007) use a richer model which allows for the fluctuations to be explained by more than these two shocks. However, had their model been estimated with only technology and monetary policy shock, the importance of the technology shock would be even higher which would be even more in line with our results.

and monetary policy parameter ϕ_π. In particular, the variance is given on the z-axis, while the values of ρ_a and ϕ_π are given on the x-axis and y-axis respectively. Notice that the shape of the surface is monotone and that it resembles the inverse of the shape of the instantaneous effect of technology shock on output gap, given by Λ_a. This is because the technology shock explains larger part of the total variance, and therefore the total variance inherits the behavior of Λ_a through Λ_a^2. In fact, variations in output gap will be the smallest for high values of ρ_a and high values of ϕ_π, which is in line with the intuition that monetary policy needs to increase ϕ_π in order to stabilize output gap.

Since we want to explore the total welfare in the economy and its dependence on changes in ϕ_π and ρ_a, we perform the same analysis for the case of inflation, as its variance is one of the components of the total welfare. From (21) it is trivial to obtain the variance of inflation:

$$var\left(\pi_t\right) = \left[\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)\right]^2 \frac{\sigma_a^2}{1-\rho_a^2} + \left[\Lambda_v^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)\right] \frac{\sigma_v^2}{1-\rho_v^2} \tag{30}$$

Figure 21 plots the variance of inflation as a function of technology shock persistence ρ_a and monetary policy parameter ϕ_π. Notice that the shape of the surface is rather different than that of the surface of the variance of output gap. In particular, variance of inflation exhibits highly non-monotone behavior. Again, as in the case of output gap, this was to be expected considering that the part of the variance due to the technology shock accounts for the most of the variance. Therefore, the total variance would inherit the properties of $\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ discussed in the previous section. There are two things worth noticing here. First, monetary policy stabilizes variance of inflation as it increases ϕ_π, which follows from the Taylor principle. However, more interestingly, the change of ρ_a largely influences the total variance of inflation. In particular, for low values of ϕ_π and values of technology persistence around 0.85 variance of inflation will be the highest. Therefore, given this value of ρ_a monetary authority would have to respond much stronger to inflation in order to reduce the variance. As can be seen from Figure 22, which plots the variance of inflation for specific values of ϕ_π (1.1, 1.2 and 1.5) and various values of technology persistence, the shape of the variance will be highly affected by the size of ϕ_π. In fact, for a low value of ϕ_π change in ρ_a will have highly significant effects on the variance of inflation.

5.1 Robustness Check: Medium-Scale DSGE Model

So far we have used a simple monetary model and a fairly simple New Keynesian model to convey the message of a nonlinear relationship among technology persistence, monetary policy response to inflation and variances of output gap and inflation. However, one might think that our results are specific to these models and do not carry over when more features are considered. To address these concerns, here we consider a medium-scale dynamic stochastic general equilibrium model as in Smets and Wouters (2007). The most distinctive additional feature is the introduction of capital and investment that are subject to convex adjustment costs. We also add habit persistence in consumption and the indexation of wages and prices.

We calibrate the model using the posterior mean of the estimates obtained by Smets and Wouters (2007, Tables 1A and 1B). Then we vary ρ_a and ϕ_π and for each combination of the two we calculate the variance of output gap and of inflation. Notice that the model of Smets and Wouters includes seven shocks. However, to make our exercises comparable we allow technology shock and the monetary policy shock to explain almost all the variations of output gap and inflation by lowering the variances of other five shocks. As before, we keep the unconditional variance of the technology shock constant as we vary the persistence. The results are shown in Figure 23. It is clear that the relationship remains highly nonlinear both in case of output gap and inflation. As in a simpler model, the inflation variance is much higher for the low values of ϕ_π and also highly dependent on the value of the technology persistence, as was the case in simpler models considered above. Therefore, we confirm that our results are not specific to simple models and are robust to the introduction of additional and rather standard features.

6 Optimal Monetary Policy and TFP persistence

6.1 No Trade-off Monetary Policy and TFP persistence

We documented that a change in technology persistence has different effects on the total variance of output and inflation: while the surface of the variance of output gap is monotone, the surface of the variance of inflation is rather non-monotone. This means that for different values of ϕ_π and ρ_a the effect of a change in monetary policy and technology persistence will have rather different implications on welfare. Therefore, it would be interesting to examine net effect on the total welfare, which is straightforward once we have the values of the total variance of output gap and inflation.

In particular, we assess the performance of a policy rule by using a welfare-based criterion, as in Rotemberg and Woodford (1999), which relies on a second-order approximation of the utility losses experienced by a representative consumer as a consequence of the deviations from the efficient allocation. The resulting welfare loss function, expressed in terms of equivalent permanent consumption decline, is given by:

$$WL = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\varepsilon}{\lambda} \pi_t^2 \right], \tag{31}$$

which leads to the following average welfare loss function per period:

$$AWL = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var\left(\tilde{y}_t\right) + \frac{\varepsilon}{\lambda} var\left(\pi_t\right) \right]. \tag{32}$$

The average welfare function is a linear combination of the variances of the output gap and inflation.

As in Taylor (1993), we consider the Taylor rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

where $\hat{y}_t = \log\left(\frac{Y_t}{Y}\right)$ is the log deviation of output from the steady state. We can rewrite this equation as

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

where $v_t = \phi_y \hat{y}_t^n$. In this scenario only the technology shock drives the dynamics of the model and v_t is an additional driving force of the nominal interest rate proportional to the deviations of natural output from the steady state.

Given this setting, we can compute the Average Welfare Loss that results from a change in the response of the monetary policy to inflation, ϕ_π. As we showed in the previous section, a change in monetary policy affects the volatility of both inflation and output gap. Moreover, since this effect depends on the persistence of the technology, we can study how changes in monetary policy affect the welfare loss for different values of ρ_a. Notice that in this setting there is no trade-off between output-gap and inflation stabilization: the optimal monetary policy trivially calls for an infinitely large response to inflation. However, in this section we explore the shape of the welfare loss function to understand its relationship with the TFP persistence. In the next sub-section we will study the optimal monetary policy in a setting with trade-off.

Figure 24 plots the average welfare loss as a function of ϕ_π and ρ_a. The figure shows some important results. First, an increase in the response to inflation improves the welfare of the agent. This is an intuitive finding since, as suggested by the Taylor principle, a larger ϕ_π stabilizes the total variance of output gap and inflation. Second, the persistence of the technology has a large impact on welfare, in particular when the monetary authority does not respond strongly to inflation. This is due to the fact that in this region the variance of inflation is high, which contributes to the high welfare loss. Notice that these effects are only driven by the changes in the reduced form parameters, Λ_a and Λ_v, since we are fixing the unconditional variance of a_t to be constant, as in the previous section. Therefore, for low values of ϕ_π , the welfare loss is directly related with the persistence of technology. Finally, as can be seen in Figure 25, when technology becomes more persistent, an increase in the response to inflation implies a larger change in welfare. In fact, for values of ρ_a around 0.9 a marginal increase in ϕ_π reduces welfare loss more significantly. However, when ϕ_π is close to 2, the welfare loss is the similar regardless the persistence of the technology. In conclusion, if the response of the monetary policy to inflation is too weak, an increase in persistence of the TFP brings a larger welfare loss, if the monetary policy does not update its parameters.

This analysis of the welfare loss function is illustrative, but it is silent about the optimal monetary policy. Without the presence of cost-push shocks, the monetary authority does not face any trade-off between stabilizing output gap variance and inflation variance. Therefore the optimal monetary policy, in this setting, suggests simply responding to inflation as strongly as possible. The optimal monetary policy in this setup is addressed in the next section.

6.2 Trade-off Monetary Policy and TFP persistence

The New Keynesian model presented above has two sources of inefficiency: first, the presence of market powers in the good market, and second, the presence of the price stickiness at the firm level. In order to isolate the distortive effect of the price adjustment setting, we can eliminate the first source of inefficiency by introducing an employment subsidy financed with a lump-sum tax. To eliminate the second distortion the markups should be identical across firms and goods at all time and equal to the frictionless markup on average. To achieve this outcome it is necessary to have a policy that stabilizes marginal costs to the "optimal level". In this case, no firm has an incentive to adjust her price, thus resulting in a zero-inflation scenario. Therefore, the price distortion disappears and the level of output equals its natural level, thus implying a zero output gap as well. Consequently, in the optimal case we have $\pi_t = 0$, $\tilde{y}_t = 0$, and $i_t = r_t^n$.

Therefore, to study the effect of an increased persistence of technology on the optimal monetary policy, we add cost-push shock in our model, as in Woodford (2003). We rationalize it by assuming that the elasticity of substitution among goods varies over time according to some stationary process ε_t. The associated desired mark-up is given by

$$\mu_t^n = \frac{\varepsilon_t}{\varepsilon_{t-1}}. \tag{33}$$

The resulting inflation equation (Galí, p.113) is then given by:

$$\pi_t = \beta E_t \left\{\pi_{t+1}\right\} + \kappa \left(y_t - y_t^n\right) + \lambda \left(\mu_t^n - \mu\right) \tag{34}$$

where y_t^n denotes the equilibrium level of output under flexible prices and a constant price markup μ. Defining $\tilde{y}_t = \left(y_t - \bar{y}_t^n\right)$ and $u_t = \lambda \left(\mu_t^n - \mu\right)$, we obtain

$$\pi_t = \beta E_t \left\{\pi_{t+1}\right\} + \kappa x_t + u_t \tag{35}$$

Therefore, the presence of cost-push shock modifies the New Keynesian Philips-Curve in (12), where u_t follows a first order autoregressive process:

$$u_t = \rho_u v_{u-1} + \sigma_u \varepsilon_t^u, \quad \text{where } \varepsilon_t^u \sim N\left(0, 1\right). \tag{36}$$

We proceed as in Giannoni (2010) to determine the optimal Taylor Rule under commitment. The monetary authority is assumed to commit to the rule (16), in which the parameters $\left(\phi_\pi, \phi_y\right)$ are chosen to minimize an expected loss function, described below, subject to equilibrium Philips-

Curve (12) and the Euler equation (13), and to the evolution of the exogenous shocks (14) and (36).The strategy is to first determine the optimal equilibrium consistent with the Taylor rule and second, to determine the policy coefficients that attain that equilibrium. The welfare function is assumed to depend on the present and future deviation of inflation, output gap, and nominal interest rate from their optimal level:

$$E\left(WL\right) = E\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^{t}\left[\pi_t^2 + \lambda_y\left(\tilde{y}_t - y^*\right) + \lambda_i\left(i_t - i^*\right)\right]\right\}. \tag{37}$$

We assume that the optimal level of the output gap y^* is zero, and that the optimal value of the nominal interest rate i^* is its steady state value. The expectation operator E is conditional on the state of the economy at the time the policy is evaluated, before the realization of the shocks in that period. The weights λ_y and λ_i are the weights associated to the stabilization of output gap and nominal interest rate. The welfare relevance of the nominal interest rate stabilization is motivated by reflecting welfare costs of transactions and an approximation to the zero lower bound.[8]

In this setting we can compute the optimal values of ϕ_π and ϕ_y as functions of the persistence of the technology ρ_a. It is important to notice that Giannoni (2010) points out the sensitivity of the determinacy region for this problem to the statistical properties of the exogenous processes. In particular, restricting to the case in which the Taylor rule parameters are positive, the policy rule (16) implies a determinate equilibrium if and only if

$$\phi_\pi + \frac{1-\beta}{\kappa}\phi_y > 1.$$

This relation is dependent on the persistence of exogenous shocks, since they affect the optimal monetary parameters. In order to study this relationship, we calibrate the model as described in section 3, assuming in addition that $\rho_u = 0.5$ and $\sigma_a = \sigma_u = 0.01$. We then compute the determinacy region as function of the persistence of the technology shock ρ_a and the persistence of the cost-push shock ρ_u. Figure 26 displays the determinacy region. The sensitivity of the determinacy of the optimal monetary policy problem, particularly to the persistence of the productivity, is evident; when its process becomes highly persistent, the problem displays indeterminacy. Never-

[8] See Giannoni (2010).

theless, in Figure 27 we plot the optimal policy parameters ϕ_π and ϕ_y as functions of the persistence of the technology ρ_a in the determinacy region. We observe that higher TFP persistence calls for a stronger response for both inflation targeting and output gap targeting. This result confirms our finding that a higher persistent technology implies a lower ability of the monetary policy to smooth the volatility of macroeconomic variables, thus leading to a need for stronger actions by the monetary authority to achieve stabilization. Notice that when the persistence of TFP is particularly large and close to the boundary of the determinacy region, the monetary policy is required to react very strongly to inflation.

In order to study the robustness of this result to different calibrations of the model that might result in a different determinacy region, we conduct the same analysis with different preference parameters. In particular, we assume a higher curvature of the utility function, by setting $\sigma = 6$. It turns out that this parameter plays a key role in shaping the determinacy region of the optimal problem. Figure 28 displays the determinacy region with this calibration: in this case, the indeterminacy is achieved for larger values of ρ_a with respect to the benchmark case in which $\sigma = 1$. However, as Figure 29 shows, the optimal monetary policy implications are quite similar to those from the previous calibration: an increase in the TFP persistence requires a stronger response of the monetary authority in order to offset the loss in its ability to affect the variance of macroeconomic variables.

7 Conclusion

In this paper we study the interaction between the TFP persistence and monetary policy. We first provide evidence of an increased persistence of the TFP, by using several statistical tools. In particular, we compute split-sample estimates, rolling-window estimates, recursive estimates, and we finally estimate a time-varying-parameters model augmented with stochastic volatility. These methods suggest that the autoregressive structure of the TFP process has likely changed, with an increased persistence from values around 0.6 to value around 0.85. A change in the autoregressive structure of the exogenous process has a first order effect on the equilibrium of forward-looking macroeconomic models. Since policy maker takes into account such equilibria when setting the optimal policy, it is important to understand how these equilibria are affected by the autocorrelation structure of the exogenous processes. We first consider a simple monetary model where money is neutral in order to show analytically that the variance of inflation is a non-monotone function of the TFP-persistence. The non-monotonicity is driven by the interaction of the Fisherian equation that defines the nominal interest rate, the Taylor rule which sets the nominal interest rate as a function of inflation, and the predictability of the real interest rate. We then analyze a standard New Keynesian model, featuring staggered prices and imperfect competition. In this setting money is not neutral, and, thus, the monetary policy affects real variables as well. We derive the relationship between TFP persistence, monetary policy, and both inflation and output gap dynamics, which are the two variables relevant for welfare. Finally, we analyze the optimal monetary policy as a function of the TFP persistence: ceteris paribus, an increase of the TFP persistence increases the welfare loss, thus calling for a stronger response to inflation.

References

[1] Ahmed, Shaghil, Andrew Levin and Beth Anne Wilson, "Recent U.S. Macroeconomic Stability: Good Policies, Good Practices, or Good Luck?," *The Review of Economics and Statistics* 86(3), (2004), pp. 824-832.

[2] Altig, David, Lawrence Christiano, Martin Eichenbaum and Jesper Lindé, "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," NBER Working Paper 11034, Cambridge, MA, (2005).

[3] Basu, Susanto and Miles S. Kimball, "Cyclical Productivity with Unobserved Input Variation," NBER Working Papers 5915, 1995.

[4] Boivin, Jean, "Has U.S. Monetary Policy Changed? Evidence from Drifting Coefficients and Real-Time Data," *Journal of Money, Credit and Banking* 38(5), (2006), pp. 1149-1173.

[5] Boivin, Jean, and Marc P. Giannoni, "Has Monetary Policy Become More Effective," *The Review of Economics and Statistics* 88, (2006), pp. 445-462.

[6] Brown, R. L. , J. Durbin and J. M. Evans "Techniques for Testing the Constancy of Regression Relationships over Time," *Journal of the Royal Statistical Society. Series B (Methodological)*37, (1975), pp. 149-192

[7] Burnside, Craig and Martin Eichenbaum, "Factor-Hoarding and the Propagation of Business-Cycle Shocks," *American Economic Review 86(5)*, (1996), pp. 1154-74.

[8] Calvo, Guillermo A., "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics* 12, (1983), pp. 383-98.

[9] Clarida, Richard, Jordi Galí, and Mark Gertler, "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory," *Quarterly Journal of Economics* 115:1, (2000), pp. 147–180.

[10] Cogley, Timothy and James M. Nason "Output Dynamics in Real-Business-Cycle Models," *American Economic Review 85(3)*, (1995), pp. 492-511.

[11] Cogley, Timothy, and Thomas J. Sargent, "Evolving Post-World War II U.S. Inflation Dynamics," *NBER Macroeconomics Annual* 16, (2001), pp. 332–373.

[12] Cogley, Timothy, and Thomas J. Sargent, "Drifts and Volatilities: Monetary Policies and Outcomes in the Post-WWII US," *Review of Economic Dynamics* 8, (2005), pp. 262–302.

[13] Galí, Jordi, *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework,* Princeton University Press, (2008).

[14] Galí, Jordi, and Luca Gambetti, "On the Sources of the Great Moderation," *American Economic Journal: Macroeconomics* 1, (2009), pp. 26–57.

[15] Giannone, Domenico, Michele Lenza, and Letizia Reichlin, "Explaining the great moderation: it is not the shocks," *Journal of the European Economic Association* 6, (2008), pp. 621–633.

[16] Giannoni, Marc P., "Optimal Interest-Rate Rules in a Forward-Looking Model, and Inflation Stabilization versus Price-Level Stabilization," NBER Working Paper no. 15986, May 2010.

[17] Greenwood, Jeremy, Zvi Hercowitz and Gregory W. Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review* 78(3), (1988), pp. 402-17.

[18] Johnston, John and John DiNardo *Econometric Methods*, McGraw-Hill, (1996).

[19] Kim, Chang-Jin, and Charles R. Nelson, "Has the U.S. economy become more stable? A Bayesian approach based on a Markov switching model of business cycle," *The Review of Economics and Statistics* 81(4), (1999), pp.608-616.

[20] King, Robert G., Charles I. Plosser and Sergio T. Rebelo, "Production, growth and business cycles : II. New directions," *Journal of Monetary Economics* 21(2-3), (1988), pp. 309-341.

[21] Kydlannd, Finn E. and Edward C. Prescott, "Time to Build and Aggregate Fluctuations," *Econometrica* 50, (1982), pp. 1345-1370.

[22] Kydlannd, Finn E. and Edward C. Prescott, "The Econometrics of the General Equilibrium Approach to Business Cycles," *Scandinavian Journal of Econometrics* 93, (1991), pp. 161-178.

[23] Liu, Zheng, Daniel Waggoner and Tao Zha, "Asymmetric Expectation Effects of Regime Shifts in Monetary Policy," *Review of Economic Dynamics* 12(2), (2009), pp. 284-303.

[24] Long, John B, Jr and Charles I. Plosser, "Real Business Cycles," *Journal of Political Economy* 91(1), (1983), pp. 39-69.

[25] Nakajima, Jouchi "Time-Varying Parameter VAR Model with Stochastic Volatility: An Overview of Methodology and Empirical Applications," IMES Discussion Paper Series 11-E-09, (2011)

[26] Pancrazi, Roberto, "Spectral Covariance Instability Test: An Application to the Great Moderation," Mimeo, (2011a).

[27] Pancrazi, Roberto, "How Beneficial was the Great Moderation After All?," Mimeo, (2011b).

[28] Prescott, Edward C., "Theory ahead of business cycle measurement," *Quarterly Review*, Federal Reserve Bank of Minneapolis, (1986), pp. 9-22

[29] Primiceri, Giorgio, "Time varying structural vector autoregressions and monetary policy," *Review of Economic Studies* 72, (2005), pp. 821–852.

[30] Rotemberg, Julio J. and Michael Woodford, "Interest Rate Rules in an Estimated Sticky Price Model," in John B. Taylor (ed.), Monetary Policy Rules, University of Chicago Press, (1999).

[31] Smets, Frank and Rafael Wouters, "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review* 97(3), (2007), pp. 586-606.

[32] Solow, Robert M., "Technical Change and the Aggregate Production Function," *Review of Economics and Statistics* 39, pp. 312-320.

[33] Stock, James, and Mark W. Watson, "Has the Business Cycle Changed and Why?," *NBER Macroeconomics Annual* 2002 (2002), pp. 159-218.

[34] Stock, James, and Mark W. Watson, "Has the Business Cycle Changed? Evidence and Explanations," *Monetary Policy and Uncertainty*, Federal Reserve Bank of Kansas City (2003), pp.9-56.

[35] Taylor, John B., "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy* 39, (1993), pp. 195-214.

[36] Woodford, Michael, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, (2003).

[37] Yun, Tack, "Nominal price rigidity, money supply endogeneity, and business cycles," *Journal of Monetary Economics* 37, (1996), pp. 345-370.

8 TABLES

Table 1: Properties of the Laws of Motion of Output and Consumption

	Sample 1: 1950:1- 1982:4		Sample 2: 1983:1-2009:4	
	Largest Root	Std. Dev. Innovations	Largest Root	Std. Dev. Innovations
Output	0.75	1.00	0.85	0.49
Consumption	0.80	0.52	0.91	0.29

Note: The process of the stationary component of consumption and output is assumed to follow an $AR(4)$ process. Standard deviations are in percent

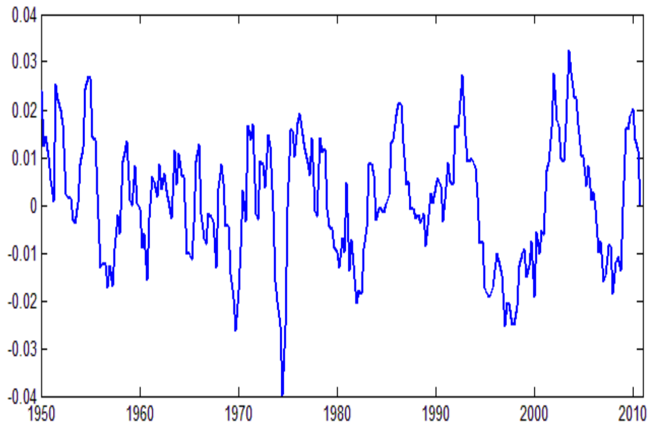
Table 2: Properties of the Laws of Motion of Total Factor Productivity

	Sample 1: 1950:1- 1982:4		Sample 2: 1983:1-2009:4	
	Largest Root	Std. Dev. Innovations	Largest Root	Std. Dev. Innovations
		AR(1)		
Varying utilization	0.74 [0.06]	0.78 [0.05]	0.95 [0.04]	0.61 [0.04]
Constant utilization	0.83 [0.05]	1.03 [0.06]	0.91 [0.04]	0.63 [0.04]
		AR(4)		
Varying utilization	0.60	0.77	0.84	0.60
Constant utilization	0.63	1.00	0.92	0.59

Note: The largest root of the lag polynomial is a measure of the persistence of the process. In case of an AR(1) process, the largest root corresponds to the parameters ρ. Standard deviations are in percent

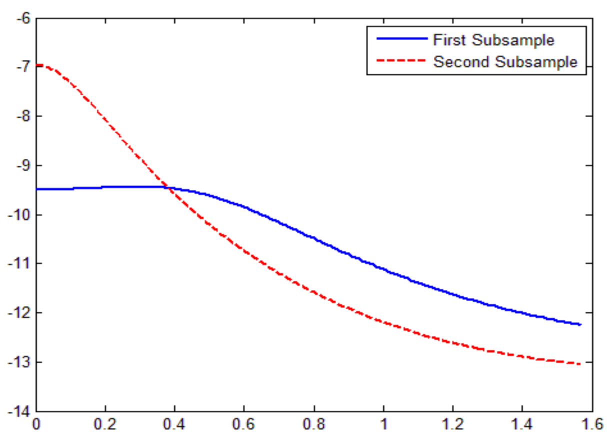
9 Figures

Figure 1: Total Factor Productivity



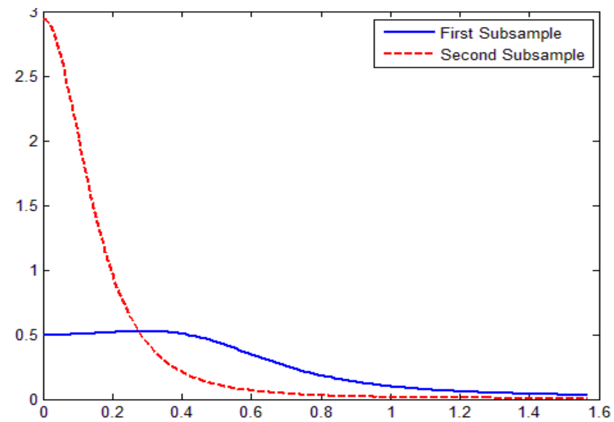
Note: The total factor productivity time series are computed accounting time-varying capacity utilization The sample includes quarterly observations from 1950:1 to 20010:4. The labor share is equal to 0.64.

Figure 2: Log-Spectrum of the AR(4) process of Total Factor Productivity



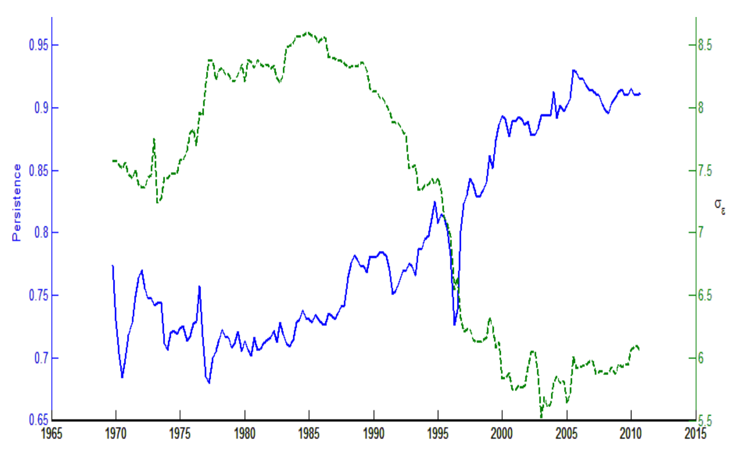
Note: The figure shows the log-spectral density of AR(4) processes of total factor productivity with time-varying capacity utilization within the frequencies 0 and $\frac{\pi}{2}$. The solid line is the log-spectrum of the process estimated in the first subsample (1950:1-1982:4), the dashed line is the log-spectrum of the process estimated in the second subsample (1983:1-20010:4).

Figure 3: Normalized Spectrum of the AR(4) process of Total Factor Productivity



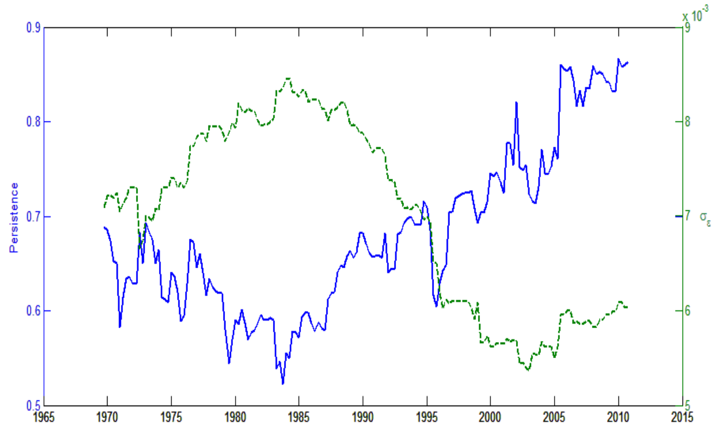
Note: The figure shows the normalized spectral density of AR(4) processes of total factor productivity with time-varying capacity utilization within the frequencies 0 and $\frac{\pi}{2}$. The solid line is the normalized spectrum of the process estimated in the first subsample (1950:1-1982:4), the dashed line is the normalized spectrum of the process estimated in the second subsample (1983:1-2010:4).

Figure 4: Rolling window estimate for the largest root and standard deviation of innovations of tfp: AR(1) model



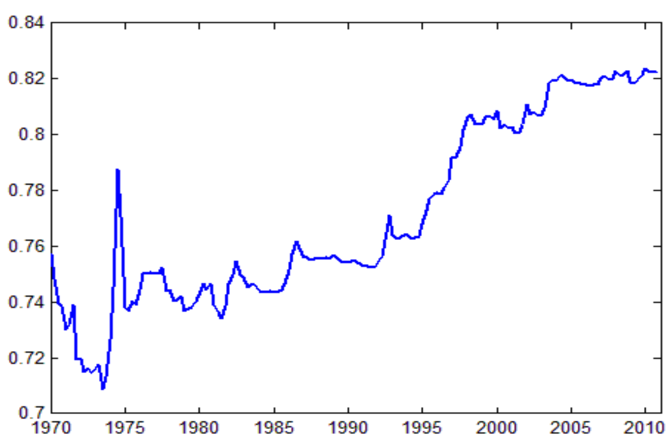
Note: The figure shows the rolling window estimates of the persistence of TFP (solid line) and the standard deviation of its error term (dashed line) when assuming an $AR(1)$ structure. The window has length of 80 quarters.

Figure 5: Rolling window estimate for the largest root and standard deviation of innovations of tfp: AR(4) model



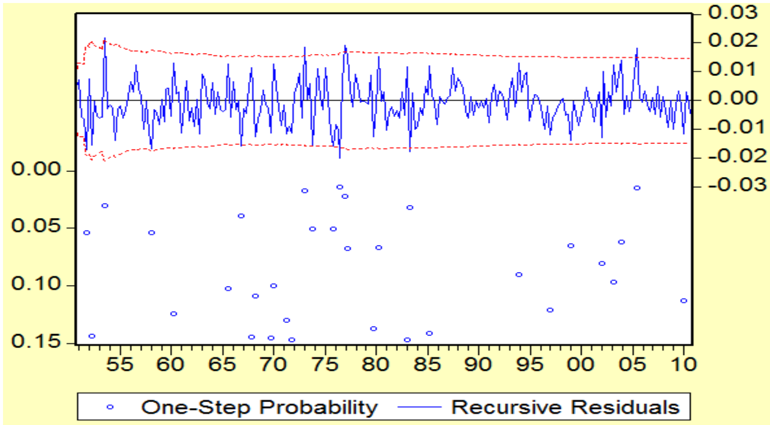
Note: The figure shows the rolling window estimates of the persistence of TFP (solid line) and the standard deviation of its error term (dashed line) when assuming an $AR(1)$ structure. The window has length of 80 quarters.

Figure 6: Recursive estimate for the autoregressive parameter in a AR(1) model for tfp



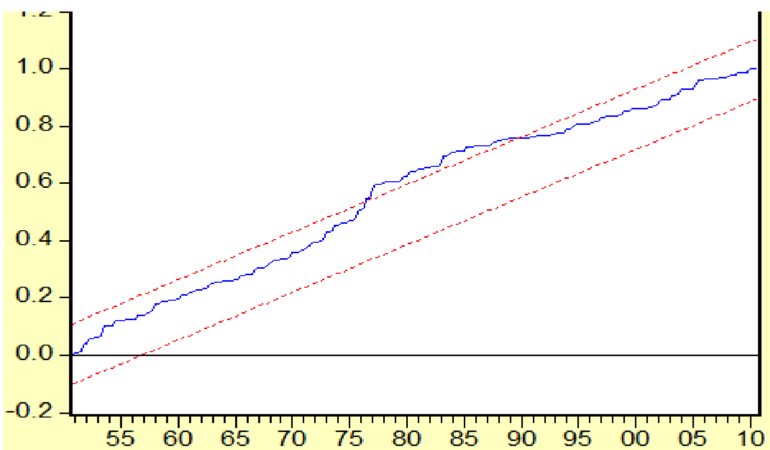
Note: The figure shows the recursive estimate of the persistence of TFP and the standard deviation of its error term when assuming an $AR(1)$ structure

Figure 7: Recursive Residuals for the ar parameter in AR(1) model for tfp



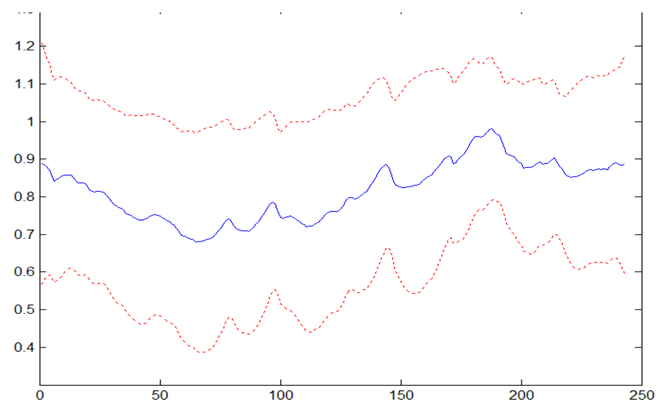
Note: The figure shows the recursive residuals (solid line) for fitting an AR(1) model for TFP, their standard errors bands (dashed line) together, and the sample point (circle) whose probability values is below 15 percent. Residuals outside the standard error bands suggest instability in the parameters of the equation

Figure 8: CUMSUM Squared statistics for the ar parameter in AR(1) model for tfp



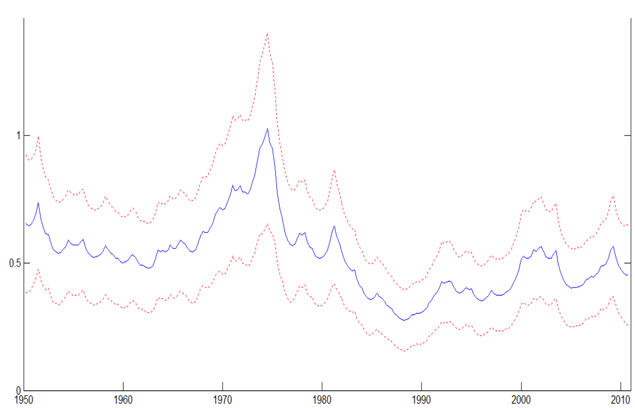
Note: The figure shows the CUSUM of squares test statistic (solid line), and the pair of 5 percent critical lines. Since the CUSUM test moves outside the band approximately in the middle part of the sample, the diagnostic suggests the presence of a change in the autocorrelation structure of TFP

Figure 9: Posterior mean of the persistence of a TVP-SV model



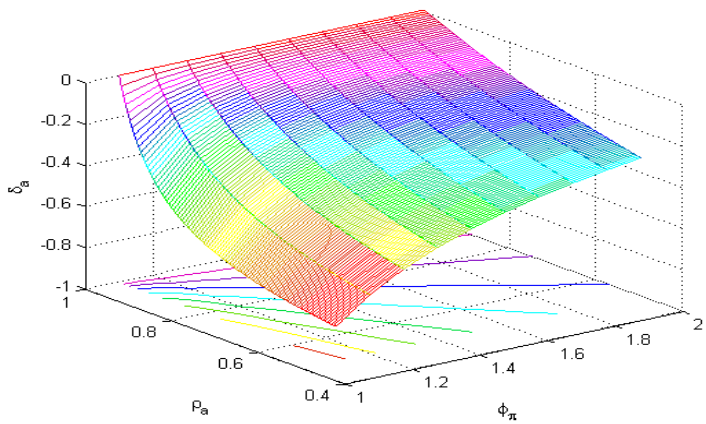
Note: The figure shows the estimated posterior mean (solid line) of the autoregressive parameter ρ_t of a Time-Varying-Parameters-Stochastic-Volatility model. The model is estimated using a Markov Chain-Monte Carlo procedure with one million repetitions The 2.5 and 97.5 percentile of the posterior distribution are also plotted (dashed line).

Figure 10: Estimates of the persistence of a TVP-SV model



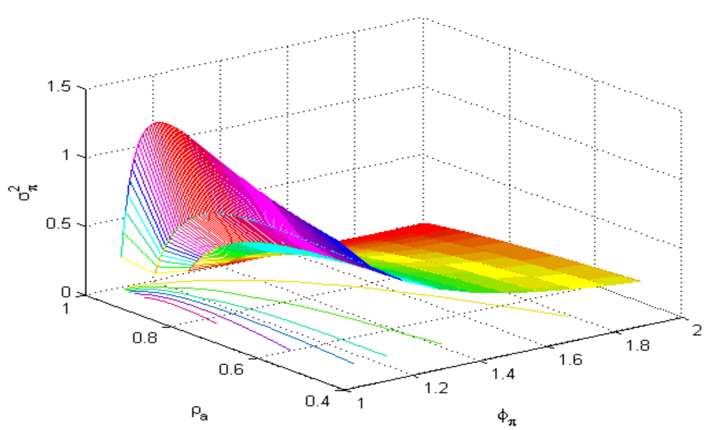
Note: The figure shows the estimated posterior mean (solid line) of the variance of the innovation σ_t^2 parameter of a Time-Varying-Parameter-Stochastic-Volatility model. The model is estimated using a Markov Chain-Monte Carlo procedure with one million repetitions. The 2.5 and 97.5 percentile of the posterior distribution are also plotted (dashed line).

Figure 11: Instantaneous response of inflation to a technology shock as a function of the monetary policy parameter and the persistence of technology in the simple monetary model



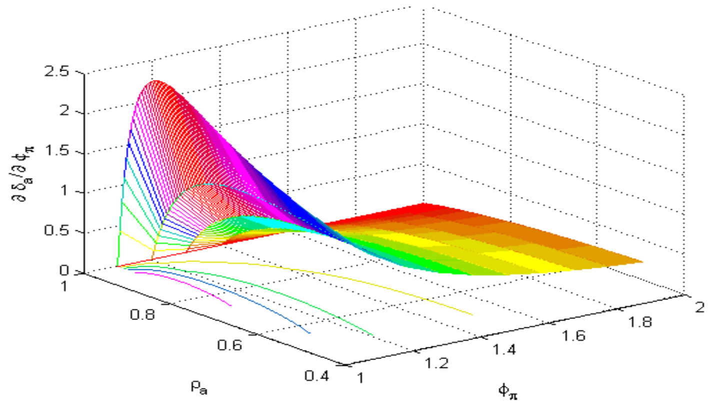
Note: The figure shows the instantaneous response of inflation to a technology shock, δ_a, as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$.The model considered is the simple monetary model.

Figure 12: Variance of inflation as a function of the monetary policy parameter and the persistence of technology in the simple monetary model



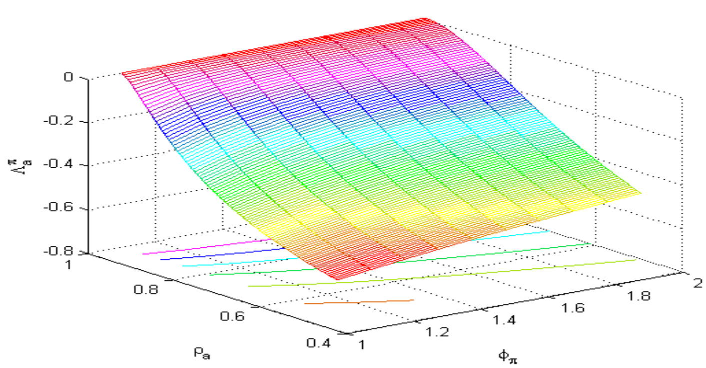
Note: The figure shows the instantaneous response of inflation to a technology shock, δ_a, as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$. The model considered is the simple monetary model.

Figure 13: Effectiveness of monetary policy on the instantaneous response of inflation to a technology shock in the simple monetary model



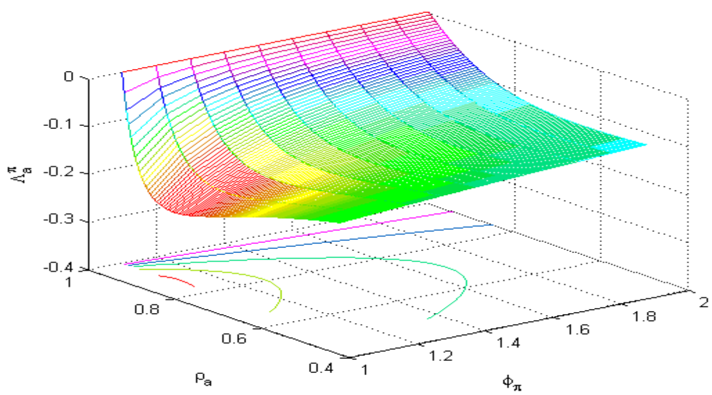
Note: The figure shows the effectiveness of the monetary policy on the instantaneous response of inflation to a technology shock as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$. The model considered is the simple monetary model. This effectiveness is defined as the change in the response δ_a to a marginal change in the monetary policy parameter ϕ_π, i.e. $\frac{\partial \delta_a}{\partial \phi_\pi}$.

Figure 14: Instantaneous response of output gap to a technology shock in the New Keynesian model



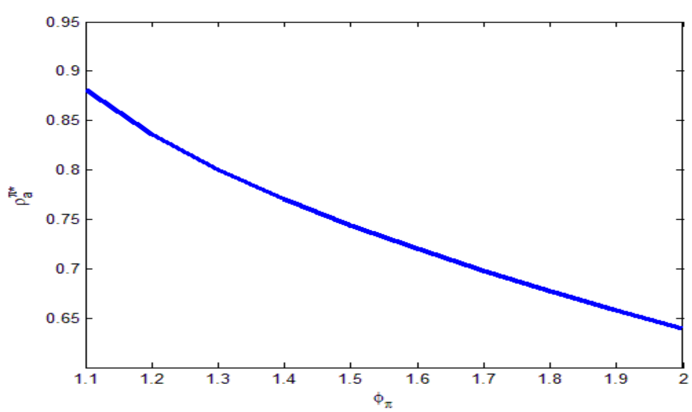
Note: The figure shows the instantaneous response of output-gap to a technology shock, $\Lambda_a(\cdot)$, as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$. The model considered is the New Keynesian model.

Figure 15: Instantaneous response of inflation to a technology shock in the New Keynesian model



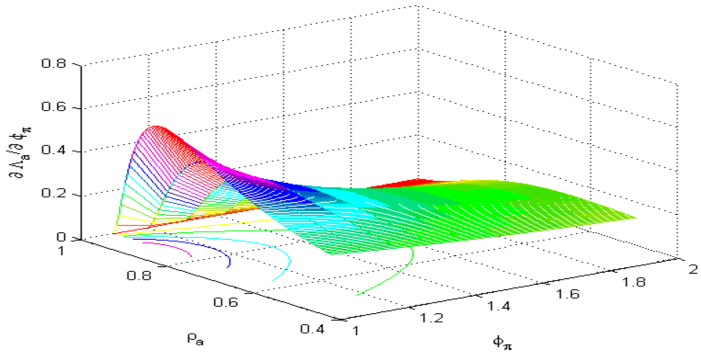
Note: The figure shows the instantaneous response of inflation to a technology shock, $\Lambda_a^\pi(\cdot)$, as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$. The model considered is the New Keynesian model.

Figure 16: Value of the TFP persistence that maximizes the instantaneous response of inflation to a technology shock parameter in the New Keynesian model



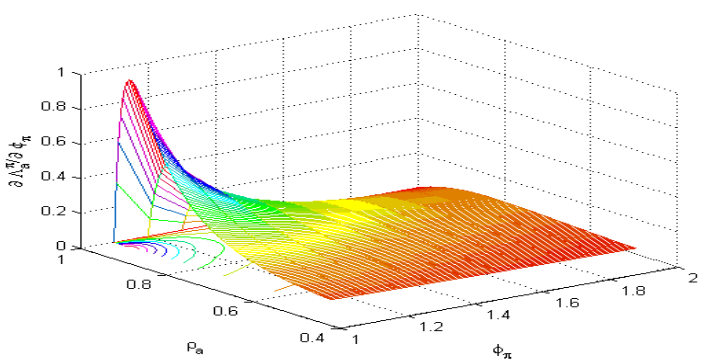
Note: The figure shows the value of the TFP persistence $\rho_a^{\pi*}$ that maximizes the the instantaneous response of output-gap to a technology shock, $\Lambda_a^\pi(\cdot)$, as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$. The model considered is the New Keynesian model.

Figure 17: Effectiveness of monetary policy on the instantaneous response of output gap to a technology shock in the New Keynesian model



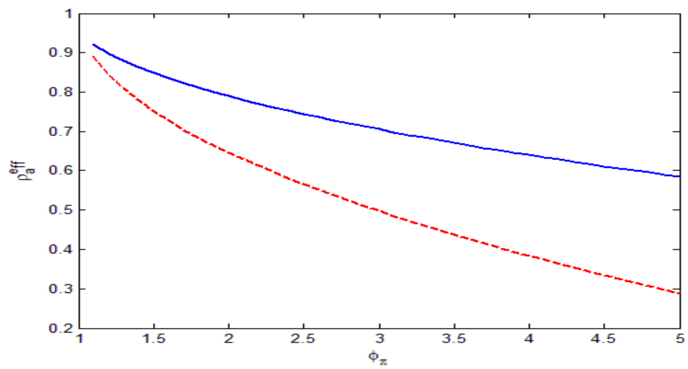
Note: The figure shows the effectiveness of the monetary policy on the instantaneous response of output-gap to a technology shock as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$. The model considered is the New Keynesian model. This effectiveness is defined as the change in the response $\Lambda_a(\cdot)$ to a marginal change in the monetary policy parameter ϕ_π, i.e. $\frac{\partial \Lambda_a(\cdot)}{\partial \phi_\pi}$.

Figure 18: Effectiveness of monetary policy on the instantaneous response of inflation to a technology shock in the New Keynesian model



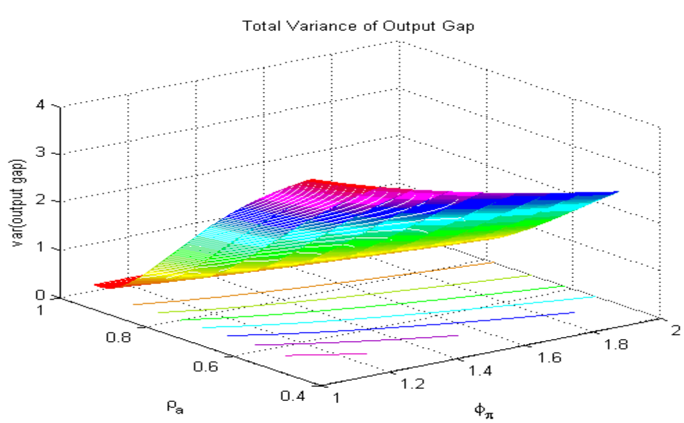
Note: The figure shows the effectiveness of the monetary policy on the instantaneous response of inflation to a technology shock as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$. The model considered is the New Keynesian model. This effectiveness is defined as the change in the response $\Lambda_a^\pi(\cdot)$ to a marginal change in the monetary policy parameter ϕ_π, i.e. $\frac{\partial \Lambda_a^\pi(\cdot)}{\partial \phi_\pi}$.

Figure 19: Value of the TFP persistence that maximizes the effectiveness of monetary policy on the instantaneous response of output gap and inflation to a technology shock in the New Keynesian model



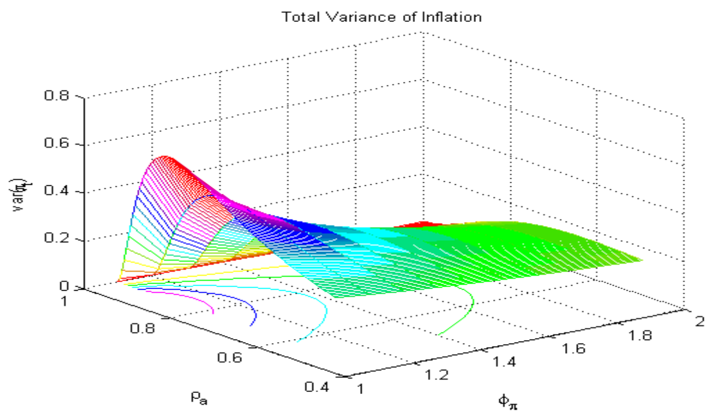
Note: The figure shows the value of the TFP persistence $\rho_a^{eff(\tilde{y})}$ (dashed line) and $\rho_a^{eff(\pi)}$ (solid line) that maximizes the instantaneous response of output-gap and inflation respectively to a technology shock, $\Lambda_a^{\pi}(\cdot)$, as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$. The model considered is the New Keynesian model.

Figure 20: Variance of Output Gap as a Function of the Monetary Policy Response to Inflation and the Persistence of Technology



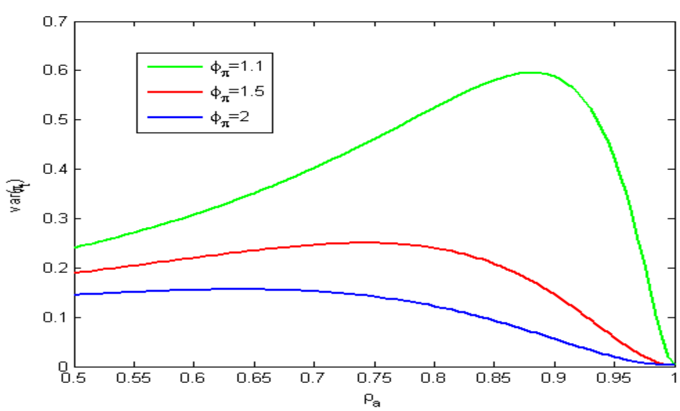
Note: The figure shows a variance of output gap as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$.

Figure 21: Variance of Inflation as a Function of the Monetary Policy Response to Inflation and the Persistence of Technology



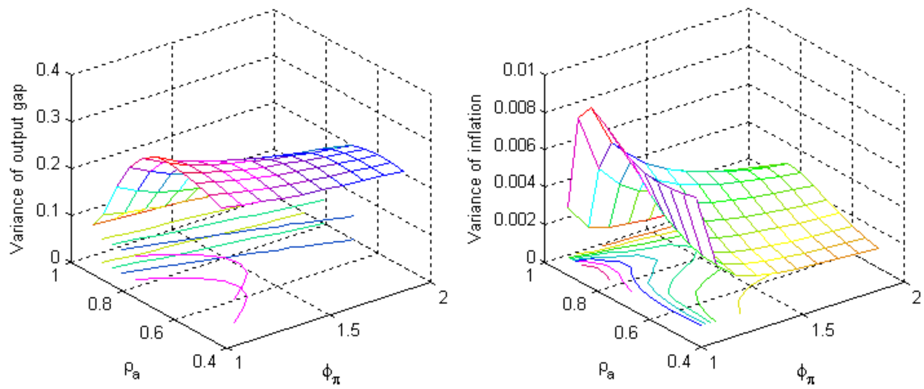
Note: The figure shows a variance of inflation as a function of monetary policy parameter ϕ_{π}, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$.

Figure 22: Variance of Inflation as a Function of Technology Persistence for Different Values of Monetary Policy Response to Inflation



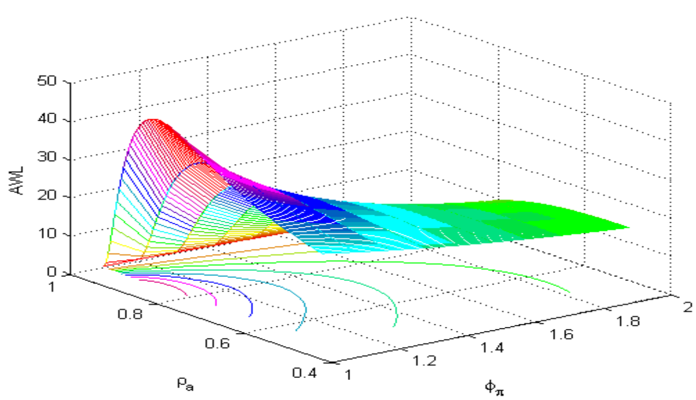
Note: The figure shows a variance of inflation as a function of technology persistence parameter ρ_a, which takes values $[0.5, 1]$, for three different values of monetary policy parameter ϕ_{π} : $1.1, 1.2$ and 2.

Figure 23: Variance of Output Gap (left panel) and Inflation (right panel) as a Function of Technology Persistence for Different Values of Monetary Policy Response to Inflation



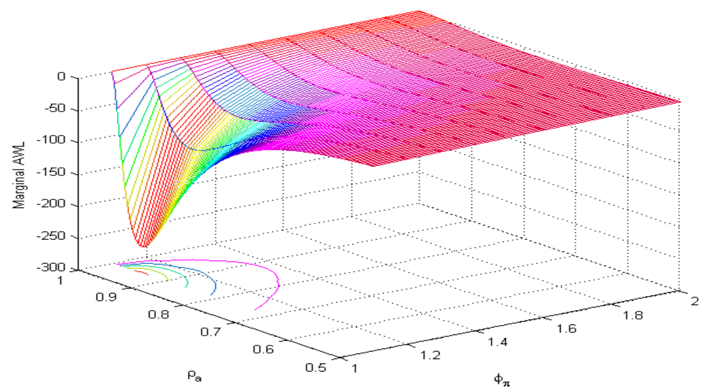
Note: The figure shows a variance of output gap (left panel) and of inflation (right panel) in a medium-scale DSGE model, as a function of technology persistence parameter ρ_a, which takes values $[0.5, 1]$, monetary policy parameter ϕ_π, which takes values $[1.1, 2]$.

Figure 24: Average Welfare Loss as a Function of the Monetary Policy Response to Inflation and the Persistence of Technology



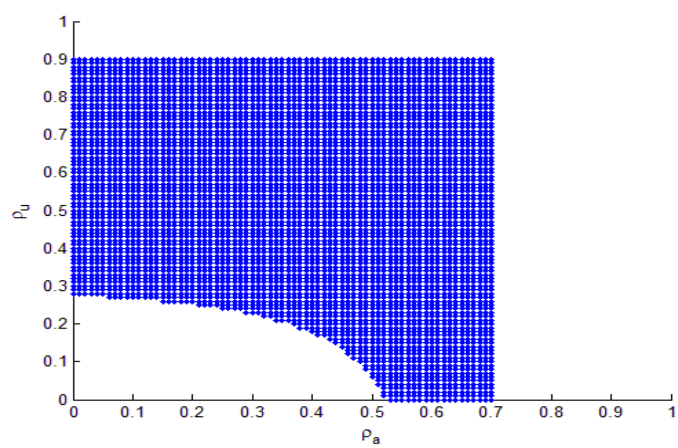
Note: The figure shows average welfare loss as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$.

Figure 25: Effect of a Marginal Change in Monetary Policy Response to Inflation on the Average Welfare Loss as a Function of the Monetary Policy Response to Inflation and the Persistence of Technology



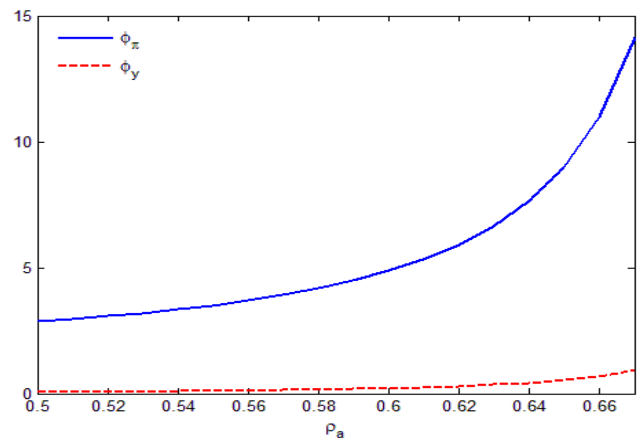
Note: The figure shows the effect of a marginal change in monetary policy parameter $\phi_\pi$on the average welfare loss (given by the derivative of the average welfare loss with respect to ϕ_π) as a function of monetary policy parameter ϕ_π, which takes values $[1.1, 2]$, and the persistence of technology ρ_a, which takes values $[0.5, 1]$

Figure 26: Indeterminacy Region for the optimal monetary policy with $\sigma = 1$.



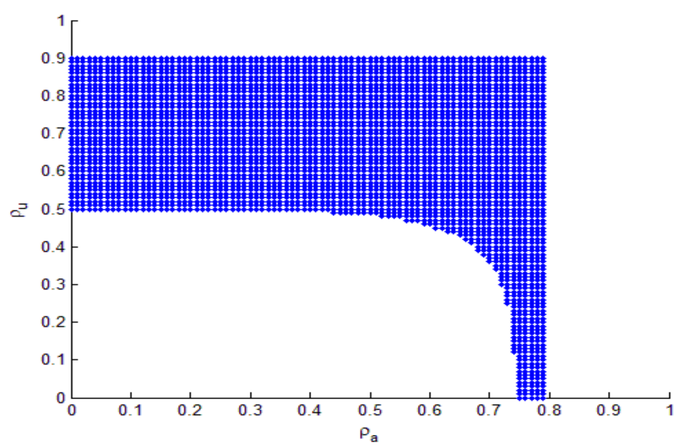
Note: The figure shows the indeterminacy region of the optimal policy problem assuming that $\sigma = 1$, as a function of the persistence of the technology, ρ_a, and the persistence of the cost-push shock, ρ_u.The dots represents a combination of the (ρ_a, ρ_u) that lead to a determinate equilibrium

Figure 27: optimal monetary parameters as function of the persistence of technology with $\sigma = 1$.



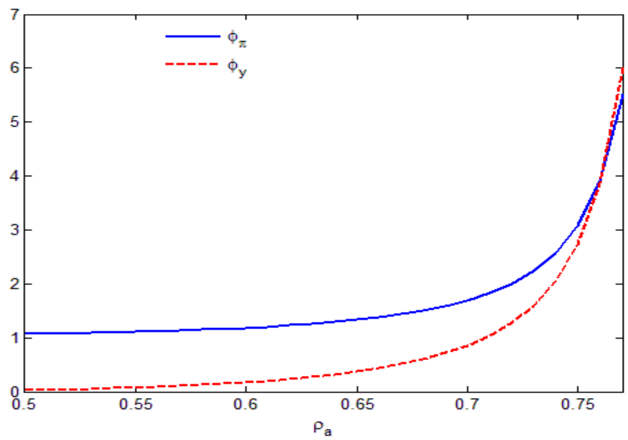
Note: The figure shows the optimal monetary policy parameters ϕ_π (solid line) and ϕ_y (dashed line) as a function of the persistence of the technology ρ_a, when $\sigma = 1$. The optimal parameters are computed only inside the determinacy region, assuming $\rho_u = 0.5$.

Figure 28: Indeterminacy Region for the optimal monetary policy with $\sigma = 6$.



Note: The figure shows the indeterminacy region of the optimal policy problem assuming that $\sigma = 6$, as a function of the persistence of the technology, ρ_a, and the persistence of the cost-push shock, ρ_u.The dots represents a combination of the (ρ_a, ρ_u) by which there is a determinate equilibrium.

Figure 29: optimal monetary parameters as function of the persistence of technology with $\sigma = 6$.



Note: The figure shows the optimal monetary policy parameters ϕ_π (solid line) and ϕ_y (dashed line) as a function of the persistence of the technology ρ_a, when $\sigma = 6$. The optimal parameters are computed only inside the determinacy region, assuming $\rho_u = 0.5$.

10 APPENDICES

APPENDIX A

ESTIMATION PRIORS AND RESULTS

TBW

APPENDIX B

This appendix describes two models used in the analysis: a simple monetary model and a basic New Keynesian model. We do not describe the model of Smets and Wouters (2007) and leave it to the reader.

A Simple Monetary Model

Here we summarize a simple model of a classical monetary economy, as in Gali (2008, pages 16 -19).

Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

where C_t is the quantity consumed and N_t denotes hours worked, P_t is the price of the consumption good, W_t is the nominal wage, B_t is the quantity of one-period, nominally riskless bonds purchased at time t which pays one unit of money at maturity $t+1$, and its price is Q_t, and T_t are nominal lump-sum taxes. The non-Ponzi condition $\lim_{T\to\infty} E_t \{B_T\} \geq 0$ for all t. Considering the utility function of the form $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$The first order conditions of this problem are

$$\frac{W_t}{C_t} = C_t^{\sigma} N_t^{\varphi} \tag{38}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{39}$$

Each period a representative firm takes prices and wages as given and maximizes profits

$$P_t Y_t - W_t N_t$$

subject to

$$Y_t = A_t N_t^{1-\alpha}$$

where A_t is the level of technology which evolves exogenously according to some stochastic process.

This maximization problem yields a standard optimality condition

$$\frac{W_t}{P_t} = (1-\alpha)\, A_t N_t^{-\alpha}$$

which tells that firm hires labor up to the point where its marginal product equals the real wage.

This model abstracts from aggregate demand components like investment, government purchases or net exports. Therefore, the goods market clearing condition

$$Y_t = C_t$$

states that all output must be consumed.

As described in the main text, the Central Bank adjusts the nominal interest rate, i_t, according to:

$$i_t = \rho + \phi_\pi \pi_t$$

where π_t denotes inflation, $\rho \equiv -\log(\beta)$ is the steady-state value of the real interest rate, with β being a discount factor, and with $\phi_\pi \geq 0$.

The Basic New Keynesian Model

This model departs from a simple monetary model described above in two directions: imperfect competition in the goods market is introduced and prices are assumed to be sticky (Gali (2008, pages 41-50)).

Households maximize the same utility function as in the simple model, except that now

$$C_t \equiv \left(\int_0^1 C_t\,(i)^{1-\frac{1}{\varepsilon}}\, di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $C_t\,(i)$ represents the quantity of good i consumed by the household in period t. Since we assume continuum of goods on the interval $[0,1]$ the period budget constraint will be given by

$$\int_0^1 P_t\,(i)\, C_t\,(i)\, di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

where $P_t\,(i)$ is the price of good i in period t, while the other variables are as defined above.

In addition to choosing consumption, savings and labor household also chooses how to optimally allocate its consumption expenditure across different goods. That is, household maximizes C_t subject to a given expenditure level

$$\int_0^1 P_t(i) C_t(i) \, di \equiv Z_t$$

which leads to the set of demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

with $P_t \equiv \left[P_t(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}}$ being an aggregate price index. Conditional on this behavior $P_t C_t = \int_0^1 P_t(i) C_t(i) \, di$ which implies that the budget constraint will be identical to the one in a simple model, and therefore the first order conditions on consumption/savings and labor (38) and (39) do not change.

There is a continuum of firms indexed by $i \in [0, 1]$, and each of them produces a differentiated good, using the identical technology given by

$$Y_t(i) = A_t N_t(i)^{1-\alpha}.$$

Each firm may reset its price with probability $1 - \theta$, and with probability θ it keeps its price unchanged. Therefore, if we denote with $S(t) \subset [0, 1]$ the set of firms not reoptimizing their posted price at period t, then using the definition of the aggregate price level and the fact that all the firms that get to reoptimize will choose the same price P_t^*, we can write

$$\begin{aligned} P_t &= \left[\int_{S(t)} P_{t-1}(i)^{1-\varepsilon} \, di + (1-\theta) \left(P_t^* \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \\ &= \left[\theta \left(P_{t-1} \right)^{1-\varepsilon} + (1-\theta) \left(P_t^* \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \end{aligned}$$

The firms that are allowed to change price will choose price P_t^* by maximizing the present discounted value of the profits generated while that price is effective

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k} Y_{t+k|t} \right) \right\}$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t-1}} \right)^{-\varepsilon} C_{t+k}$$

for $k = 0, 1, 2, \ldots$ where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$ is the stochastic discount factor for nominal payoffs, $\Psi(\cdot)$ is the cost function, and $Y_{t+k|t}$ is the output in period $t+k$ for the firm that reset its price at period t. The first-order condition associated with this problem is then

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \frac{\varepsilon}{\varepsilon - 1} \Psi'_{t+k} Y_{t+k|t} \right) \right\} = 0.$$

Finally, market clearing in the goods market implies

$$Y_t(i) = C_t(i)$$

for all $i \in [0,1]$ and all t. Defining output as $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ it is straightforward to obtain

$$Y_t = C_t.$$

A monetary policy is characterized by the interest-rate rule as described in the main text.

APPENDIX C

Proof. Proposition 1. Differentiating (5) with respect to ρ_a, we have:

$$\frac{\partial \sigma_\pi^2}{\partial \rho_a} = \frac{2\psi^2 \sigma^2 \sigma_a^2}{(1-\rho_a)^2 (\phi_\pi - \rho_a)^3} \left(+\rho_a^2 - \rho_a + \phi_\pi - 1\right)$$

Equating the expression above to 0 and solving for ϕ_π, we obtain

$$\phi_\pi^* = 1 + \rho_a - \rho_a^2$$

Finally, computing the second derivative:

$$\frac{\partial^2 \sigma_\pi^2}{\partial \rho_a \partial \phi_\pi} = \frac{2\sigma^2 \sigma_a^2 \psi^2 \left(-3 + 2\phi_\pi - 2\rho_a + 3\rho_a^2\right)}{(1+\rho_a)^2 (\phi_\pi - \rho_a)^4}$$

and evaluating at the optima ρ_a^*, we have:

$$\frac{\partial^2 \sigma_\pi^2}{\partial \rho_a \partial \phi_\pi} (\phi_\pi^*) = \frac{2\sigma^2 \sigma_a^2 \psi^2}{(\rho_a - 1)^3 (\phi_\pi - \rho_a)^4}.$$

The last expression is negative since $|\rho_a| < 1$, which assures that (9) is a maximum. ∎

Proof. Proposition 2. Taking the derivative of (10) with respect to ρ_a, we obtain

$$\frac{\partial^2 \delta_a}{\partial \phi_\pi \partial \rho_a} = -\frac{\sigma\psi \left(-2 + \phi_\pi + \rho_a\right)}{(\phi_\pi - \rho_a)^3}$$

Equating this expression with zero and solving for ϕ_π, we obtain (11). Finally, computing the third-order derivative:

$$\frac{\partial^3 \delta_a}{(\partial \phi_\pi)^2 \partial \rho_a} = 2\sigma\psi \frac{(-3 + \phi_\pi + 2\rho_a)}{(\phi_\pi - \rho_a)^4}$$

and evaluating it at the optimum ϕ_π^{**}, we have:

$$\frac{\partial^3 \delta_a}{(\partial \phi_\pi)^2 \partial \rho_a} (\phi_\pi^{**}) = \frac{\sigma\psi}{8(\rho_a - 1)^3} < 0.$$

Since $|\rho_a| < 1$, the third order derivative is negative, and ρ_a^* is then a maximum of $\frac{\partial \delta_a}{\partial \phi_\pi}$. ∎

Proof. Proposition 3. The inequality (26) comes directly from differentiating $\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ with respect to ϕ_π, as in (24) . Since by assumption $\rho_a \in (-1,1)$ and $\beta < 1$, this partial derivative is always positive.

Differentiating $\Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ with respect to ρ_a we have:

$$\frac{\partial\ \Lambda_\alpha\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \rho_a} = \psi\sigma \frac{\phi_y\left(1-\beta\rho_a\right)^2 + \kappa\left[-1+\beta\rho_a^2+\phi_\pi\left(1+\beta-2\beta\rho_a\right)\right]}{\left[\left(1-\beta\rho_a\right)\left(\sigma\left(1-\rho_a\right)+\phi_y\right)+\kappa\left(\phi_\pi-\rho_a\right)\right]^2}$$

The denominator is obviously always positive. The first term in the numerator is also positive since ϕ_y and β and ρ_a are less than unity. Then, since $\kappa = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\varepsilon)}\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$ is also positive, we need to prove that

$$-1+\beta\rho_a^2+\phi_\pi\left(1+\beta-2\beta\rho_a\right) > 0.$$

Provided that $\left(1+\beta-2\beta\rho_a\right) > 0$, then, since $\phi_\pi > 1$ we have:

$$-1+\beta\rho_a^2+\phi_\pi\left(1+\beta-2\beta\rho_a\right) > -1+\beta\rho_a^2+1+\beta-2\beta\rho_a = \beta\left(\rho_a-1\right)^2 > 0$$

where the last inequality comes from the restriction on β and ρ_a.

Finally, we need to prove that $\left(1+\beta-2\beta\rho_a\right) > 0$. If $\rho_a < 0$ the expression is trivially satisfied. If $\rho_a > 0$, rearranging the terms we obtain:

$$1+\beta-2\beta\rho_a > 0 \Longleftrightarrow \rho_a < \frac{1+\beta}{2\beta} < 1,$$

where the last equality depends on β being less then unity. Since $\rho_a \in (-1,1)$, the inequality is always satisfied. ■

Proof. Proposition 4. Differentiating (23) with respect to ρ_a we have:

$$\frac{\partial\ \Lambda_\alpha^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \rho_a} = \kappa\psi\sigma \frac{\phi_y-\beta\phi_y+\kappa\left(\phi_\pi-1\right)-\beta\sigma\left(\rho_a-1\right)^2}{\left[\left(1-\beta\rho_a\right)\left(\sigma\left(1-\rho_a\right)+\phi_y\right)+\kappa\left(\phi_\pi-\rho_a\right)\right]^2}.$$

The solution of the expression above equated to zero is:

$$\phi_\pi^{\pi*} = \frac{\kappa+\beta\sigma\left(1-\rho_a\right)^2-\left(1-\beta\right)\phi_y}{\kappa}.$$

To prove that the $\phi_\pi^{\pi*}$ is the maximum of $\left|\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)\right|$, we need to show that the second derivative $\frac{\partial^2\ \Lambda_\alpha^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \rho_a \partial \phi_\pi}$ evaluated at $\phi_\pi^{\pi*}$ is positive (since $\Lambda_\alpha^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)$ is negative). We have:

$$\frac{\partial^2\ \Lambda_\alpha^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \rho_a \partial \phi_\pi}\left(\phi_\pi^{\pi*}\right) = -\frac{\kappa^2 \psi \sigma \begin{pmatrix} \kappa^2 + \kappa\left(\rho_a - 2\right) + \phi_y\left(1 - \kappa + \beta\left(\kappa + \rho_a - 2\right)\right) \\ -\sigma\left(\rho_a - 1\right)\left(2\beta\rho_a - 1 - \beta\right) \end{pmatrix}}{\begin{pmatrix} \kappa^2 - \kappa\rho_a + \phi_\pi\left(1 + \left(\beta - 1\right)\kappa - \beta\rho_a\right) \\ +\sigma\left(\rho_a - 1\right)\left(2\beta\rho_a - 1 - \beta\right) \end{pmatrix}^3}$$

It is not possible to sign the second derivative at the optimum analytically. Numerical computation shows that the second derivative condition is satisfied for any value in the restricted parameter space. ■

Proof. Proposition 5. First let us analyze the effectiveness of monetary policy on inflation. The derivative of $\frac{\partial\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial\ \phi_\pi}$ with respect to ρ_a is:

$$\frac{\partial^2\ \Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial\ \phi_\pi \partial \rho_a} = \kappa^2 \psi \sigma \frac{\left[\phi_y\left(1 + \beta\left(\rho - 2\right)\right) + \kappa\left(\phi_\pi + \rho_a - 2\right) - \sigma\left(\rho_a - 1\right)\left(3\beta\rho_a - 2\beta - 1\right)\right]}{\left[\left(1 - \beta\rho_a\right)\left(\sigma\left(1 - \rho_a\right) + \phi_y\right) + \kappa\left(\phi_\pi - \rho_a\right)\right]^3}$$

By setting $\frac{\partial^2\ \Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial\ \phi_\pi \partial \rho_a} = 0$, solving for ϕ_π,we obtain (29).

Analogously, consider the effectiveness of monetary policy on output gap. The derivative of $\frac{\partial^2\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial\ \phi_\pi \partial \rho_a}$ with respect to ρ_a is:

$$\frac{\partial^2\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial\ \phi_\pi \partial \rho_a} = \kappa \psi \sigma \frac{\begin{bmatrix} \left(\beta - 1\right)\left(\beta\rho_a - 1\right)\phi_y + \kappa\left(\rho_a - 2 + \beta\rho_a + \phi_\pi\left(1 + \beta - 2\beta\rho_a\right)\right) \\ +\sigma\left(\rho_a - 1\right)\left(1 + \beta - 3\beta\rho_a + \beta^2\rho_a\left(-1 + 2\rho_a\right)\right) \end{bmatrix}}{\left[\left(1 - \beta\rho_a\right)\left(\sigma\left(1 - \rho_a\right) + \phi_y\right) + \kappa\left(\phi_\pi - \rho_a\right)\right]^3}.$$

By setting $\frac{\partial^2\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial\ \phi_\pi \partial \rho_a} = 0$, solving for ρ_a, and considering the only real solution, we obtain (28). To prove the solution is effectively a maximum, we compute the third order derivatives $\frac{\partial^3\ \Lambda_a\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial^2\ \phi_\pi \partial \rho_a}$ and $\frac{\partial^3\ \Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right)}{\partial^2\ \phi_\pi \partial^3 \rho_a}$, we evaluate it at the optimum, and observe that it is negative in the restricted parameter space. Since it is not possible to sign this third-derivative analytically, given the large interaction of many structural parameters, we study it numerically. ■