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ABSTRACT: This paper presents an axiomatic characterization of difference-form group contests, that is, contests fought among groups and where their probability of victory depends on the difference of their effective efforts. This axiomatization rests on the property of Absolute Consistency, stating that the difference in winning probabilities between two contenders in the grand contest must be the same as when they engage in smaller contests. This property overcomes some of the drawbacks of the widely-used ratio-form contest success functions. Our characterization shows that the criticisms commonly-held against difference-form contests success functions, such as lack of scale invariance, are unfounded. Finally, we extend our axiomatization to relative-difference contests where winning probabilities depend on the difference of contenders effective efforts relative to total aggregate effort.

JEL Codes: D31, D63, D72, D74.

Keywords: Contests, groups, contest success function, axioms

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1 Introduction

Despite the relevance and ubiquity of contests in the real world, contest theory has been often criticized for its great reliance on a particular construct: The Contest Success Function (Hirshleifer, 1989). This function is a mapping from the efforts made by contenders into their probability of attaining victory or, alternatively, their share of the contested prize. Critics argue that the CSF is too reduced form, too much of a black-box. For instance, the widely-used Tullock CSF (Tullock, 1967; 1980) under which winning probabilities or shares depend on relative efforts might seem sensible. But there is no apparent reason for this functional form to govern most types of contests, ranging from interstate wars to sport competitions.[1] Because of this, the predictions of contest theory could be seen as too reliant on very specific functional forms rather than on sound economic principles.

This view is somewhat unfair for two reasons: Firstly, because there are other areas of Economics where very specific functional forms are often assumed. Secondly, because there is an active and fruitful strand of the contest literature which in the last few years has provided a variety of foundations to the most frequently employed CSFs.[2] This literature has even attempted to estimate these functions econometrically.[3] As a result of these efforts, economists have now at their disposal a growing menu of well-founded CSFs to chose from. The next natural question is which type of CSF is better suited to each specific application. A systematic study of the properties of each family of CSFs can contribute to that aim.

One family of contests assumes that winning probabilities depend on the difference of contenders' efforts. These *difference-form* contests were introduced by Hirshleifer (1989; 1991) and explored later by Baik (1998) and Che and Gale (2000) for the case of bilateral contests. Difference-form CSFs have been shown to emerge naturally in a number of settings. Gersbach and Haller (2009) show that a linear difference-form CSF is the result of intra-household bargaining when partners must decide how much time to devote to themselves or to their partner. Corchón and Dahm (2010) microfound a difference-form CSF as the result of a game where contenders are uncertain about the type of the external decider; by interpreting the CSF as a share, they also show that the difference-form coincides with the claim-egalitarian bargaining solution. Corchón and Dahm (2011) obtain the difference-form

[1] For excellent surveys of the contest literature see Corchon (2007) and Konrad (2009).

[2] These characterizations fall into four main categories: Axiomatic, stochastic, optimally-designed and microfounded (Jia, Skaperdas and Vaidya, 2013).

[3] For a detailed discussion of the econometric issues involved in the estimation of CSFs see Jia and Skaperdas (2011) and Jia et al. (2013).

as the result of a problem where the contest designer is unable to commit to a specific CSF once contenders have already exerted their efforts. Skaperdas and Vaydia (2012) show that the difference-form CSF can be derived in a Bayesian framework in which contenders produce evidence stochastically in order to persuade an audience of the correctness of their respective views. Finally, Polishchuk and Tonis (2013) show that a logarithmic difference-form CSF results from using a mechanism design approach when contestants have private information over their valuation of victory. In summary, it is fair to conclude that difference-form contests are by now well micro-founded. However, little is known about their actual properties and about how they differ from the properties of the more often used ratio-form CSFs, where winning probabilities are a function of the ratio of contenders' effective efforts.

The present paper offers the first axiomatic characterization of the family of difference-form CSFs. All the existing axiomatizations have limited themselves to CSFs of the ratio-form. The key property in those characterizations is the Relative Consistency axiom. This axiom states that the success of a group in a smaller contest should be equal to the ratio of the contenders' winning probabilities in the big contest. We replace this axiom by an Absolute Consistency axiom imposing that the difference in winning probabilities of two contenders in a smaller contest should be the same as when they are engaged in the grand contest. We show that Absolute Consistency can overcome some of the problems presented by the ratio-form CSFs. Our Theorem 1 shows that Absolute Consistency, together with a number of reasonable axioms already employed in the literature, characterize a generalized version of the linear difference-form CSF introduced by Che and Gale (2000). This family of CSFs also encompass as particular cases the ones micro-founded in the aforementioned literature as well as the ones employed in Levine and Smith (1995), Rohner (2006), Besley and Persson (2008, 2009) and Gartzke and Rohner (2011).

With our axiomatization, we help to clarify the properties that characterize the families of CSFs studied in the literature. Contrary to the received wisdom, we show that the difference-form CSF can be scale invariant, i.e. homogeneous of degree zero (Theorem 2), and that the difference between the winning probabilities of two contenders diminishes when their efforts increase whilst keeping constant the difference between them (Theorem 3). These misconceptions are due to the common assumption of linear impacts, which we dispense with, and to the common usage of the term "difference-form CSF" to refer to the logistic functional introduced by Hirshleifer (1989; 1991), under which winning probabilities are proportional to contenders' exponential efforts.

In the last part of the paper, we show that a generalization of the Relative

Consistency axiom can characterize the family of Relative-Difference CSFs introduced recently by Beviá and Corchón (2014a). In this family of CSFs, winning probabilities depend on the difference of contenders' effective efforts relative to total aggregate effective effort. This functional form can overcome the criticisms against the difference-form CSF mentioned above. Although our results demonstrate that these criticisms are unfounded, the axiomatization of the Relative-Difference CSFs shows that this family of functions presents their own advantages, advantages which make them a worthy addition to the toolkit of researchers working in contest theory.

This paper contributes to the axiomatic work on CSF pioneered by Skaperdas (1996) and Clark and Riis (1998). Later, Münster (2009), which we follow closely, extended this characterization from individual to group contests. Arbatskaya and Mialon (2009) and Rai and Sarin (2009) axiomatized multi-investment contests, whilst Blavatskyy (2010) did the same for contests with ties. More recently, Hwang (2012) axiomatized the family of CSF with constant elasticity of augmentation, which encompasses the logistic and the ratio forms as particular cases. Vesperoni (2013) and Lu and Wang (2014) axiomatized contests producing a ranking of players instead of a sole winner. Lu and Wang (2014) characterized success functions for contests with strict rankings of players, whereas Vesperoni (2013) axiomatized an alternative success function for rankings of any type. Finally, Bozbay and Vesperoni (2014) characterized a CSF for conflicts embedded in network architectures. Let us add that in our axiomatization we make strong connections with the income inequality literature, and in particular with the concept of absolute inequality introduced by Kolm (1976a,b) and with the family of compromise indexes of inequality formalized by Blackorby and Donaldson (1980). The literature on inequality measurement offers valuable insights on the properties of functional forms which we explicitly employ at several points of the text. In this same spirit, Chakravarty and Maharaj (2014) have recently characterized a new family of individual contests success functions which satisfy properties akin to the intermediate inequality and ordinal consistency axioms employed in the income distribution literature.

2 Axiomatization

In order to be as general as possible, we consider a society divided in $K \geq 2$ disjoint groups formed by a number $n_k \geq 1$ of individuals each.[4] Denote the

[4]Individual contests are a particular case of the ones studied here when groups are formed by just one individual. All our results, except those in Section 4 which deal with the aggregation of individual efforts within groups, thus apply to individual contests as

set of groups by $\mathbb{K}$. These K groups are in competition. They are engaged in a contest which can only have one winner. Members of the contender groups can expend non-negative effort in order to help their group to win the contest. Depending on the specific type of contest, these efforts can be money, time, physical effort or weapons. Denote by $\mathbf{x}_k \equiv \{x_{1k}, ..., x_{n_k k}\}$ the vector of efforts in group k and by $\mathbf{x}$ the vector $(\mathbf{x}_1, ..., \mathbf{x}_K)$. For convenience we will denote by $\mathbf{x}_{-k}$ the vector of efforts in groups other than k.

Efforts determine the winning probability of each group according to a Contest Success Function (CSF) $p_k : \mathbb{R}_+^n \rightarrow \mathbb{R}_+$. The function $p_k(\mathbf{x})$ can also be thought of as the share of the prize associated to victory that group k obtains. For most of the paper, we will favor the former interpretation.

Let us now state the axioms that we would like to impose on our CSF.

2.1 Two basic axioms: Let us first present two axioms introduced by Skaperdas (1996) in his axiomatization of CSFs for individual contests, later generalized by Münster (2009) to group contests. These axioms are rather natural and should thus apply to the class of difference-form group contests we study in this paper.

Axiom 1 (Probability) $\sum_{k=1}^{K} p_k(\mathbf{x}) = 1$ *and* $p_k(\mathbf{x}) \geq 0$ *for any* $\mathbf{x}$ *and all* $k \in \mathbb{K}$.

Axiom 2 (Monotonicity) *Consider two generic vectors* $\mathbf{x}_k$ *and* $\mathbf{x}'_k$ *such that* $\mathbf{x}'_k > \mathbf{x}_k$. *Then,*

(i) $p_k(\mathbf{x}'_k, \mathbf{x}_{-k}) \geq p_k(\mathbf{x}_k, \mathbf{x}_{-k})$, *with strict inequality whenever* $p_k(\mathbf{x}_k, \mathbf{x}_{-k}) \in (0, 1)$.

(ii) $p_l(\mathbf{x}'_k, \mathbf{x}_{-k}) \leq p_l(\mathbf{x}_k, \mathbf{x}_{-k})$ *for all* $l \neq k$ *and* $l \in \mathbb{K}$.

The axiom of Probability just states that the CSF generates a probability distribution over the set of groups. The Monotonicity axiom implies that group winning probabilities are increasing in the effort of their members and weakly decreasing in the effort of outsiders. Note that this axiom is weaker than the Monotonicity axiom employed in Münster (2009) and than the analogous one in Rai and Sarin (2009).

2.2 Subcontest axioms: The next two axioms relate to contests played among a generic non-empty subset $S \subseteq \mathbb{K}$ of groups. We refer to this contest among groups in S as a *subcontest*. Let us denote by $p_k^S(\mathbf{x})$ the winning

well.

probability of group k in the subcontest S. In particular, denote by $p_k^{\{k,l\}}(\mathbf{x})$ the winning probability of group k in the bilateral contest against group l. Finally, denote by $\mathbf{x}_S$ and $\mathbf{x}_{-S}$ the vector of efforts in the groups inside and outside S respectively.

Axiom 3 (Independence) *$p_k^S(\mathbf{x})$ does not depend on $\mathbf{x}_{-S}$; or $p_k^S(\mathbf{x})$ can be written as $p_k^S(\mathbf{x}_S)$.*

Independence implies that the efforts made by contenders outside a subcontest should not matter to its result. As discussed by Skaperdas (1996) and Clark and Riis (1998), this property relates to the axiom of Independence of Irrelevant Alternatives in probabilistic individual choice. Thus, it is a reasonable property in contests where nature determines the winner. Independence also implies that there are no spillovers across subcontests or that spillovers affect all contenders in S equally.[5]

The next axiom is essential to the characterizations of the ratio-form CSFs. Skaperdas (1996) and Münster (2009) who call it Consistency:

Axiom 4 (Relative Consistency) *For any vector $\mathbf{x}$, any two groups $k, l \in S$ and any subcontest $S \subseteq \mathbb{K}$ it must be that*

$$\frac{p_k^S(\mathbf{x})}{p_l^S(\mathbf{x})} = \frac{p_k(\mathbf{x})}{p_l(\mathbf{x})}. \tag{1}$$

We discuss this axiom below. We rename it as Relative Consistency in order to avoid confusion with the next axiom, which is crucial in our characterization of the family of difference-form CSFs.

Axiom 5 (Absolute Consistency) *For any vector $\mathbf{x}$ and any two groups $k, l \in S$ and any subcontest $S \subseteq \mathbb{K}$ it must be that*

$$p_k^S(\mathbf{x}) - p_l^S(\mathbf{x}) = p_k(\mathbf{x}) - p_l(\mathbf{x}). \tag{2}$$

Let us now devote some time to compare the implications of these two axioms. Relative Consistency has been invoked as a natural assumption. It states that the relative success of two groups should be identical across subcontests. However, it necessarily bounds winning probabilities away from zero. Suppose that a contender k has zero winning probability in the big contest, whereas group l has a winning probability ε arbitrarily close to zero. Then group k must have a zero winning probability in any subcontest S, including the subcontest against the similarly weak group l. Hence, Relative

[5] We thank Luis Corchon for pointing this out.

Consistency does not apply in contests where contenders with positive efforts can lose with certainty.

The Absolute Consistency axiom allows contenders to have zero winning probability in a natural way. Following the example above, group k would enjoy a winning probability of $\frac{1-\varepsilon}{2}$ when fighting against the also weak group l. Nonetheless, the Absolute Consistency axiom has its own limitations. Suppose that l is instead a very strong contender but that the grand contest involves such a large number of identically strong groups so l's winning probability is just ε. In that case, Absolute Consistency again implies that the weak group k should enjoy a winning probability of $\frac{1-\varepsilon}{2}$ in the pairwise contest between k and l. Absolute Consistency seem thus well-suited to model contests with so severe frictions that differences in winning probabilities across contenders can seldom become large.

One last word on the comparison of these two axioms. It is interesting to note that there exists an approximate equivalence between them. Assuming that all probabilities are strictly positive, taking logs in (1) yields

$$\ln p_k^S(\mathbf{x}) - \ln p_l^S(\mathbf{x}) = \ln p_k(\mathbf{x}) - \ln p_l(\mathbf{x}).$$

Knowing that the first-order Taylor approximation of $\ln z$ around one is $z-1$, and applying this to all terms in the expression above, one can obtain precisely expression (2). Therefore, for relatively high winning probabilities, the two axioms are approximately equivalent.

2.3 The main theorem: We are now in the position to state our main axiomatization theorem. But before that, and for completeness, let us state the theorem that characterizes the family of ratio-form CSFs.

Theorem 0 (Münster, 2009) *The CSF $p_k(x)$ is continuous and satisfies axioms A1-A4 if and only if it can be written as*

$$p_k(\mathbf{x}) = \frac{h_k(\mathbf{x}_k)}{\sum_{l \in \mathbb{K}} h_l(\mathbf{x}_l)} \qquad \textit{for any } \mathbf{x},$$

where $h_k(\mathbf{x}_k) : R_+^{n_k} \to R_+$ is a non-negative and strongly increasing function for each $k \in \mathbb{K}$.

The function $h_k(\mathbf{x}_k)$ is commonly known as the *impact function.* It aggregates members' efforts into a measure of their group influence in the contest. Alternatively, it can be seen as the function determining how effective players' efforts are. In this case, this function must be strongly increasing, which implies that $h_k(\mathbf{x}_k') > h_k(\mathbf{x}_k)$ whenever $\mathbf{x}_k' > \mathbf{x}_k$.

Let us now characterize the family of the difference-form CSFs. They emerge from replacing the Relative Consistency axiom by the Absolute Consistency axiom in the previous Theorem.

Theorem 1 *The CSF $p_k(x)$ is continuous and satisfies axioms A1-A3 and A5 if and only if it can be written as*

$$p_k(\mathbf{x}) = \max\left\{\min\left\{\frac{1}{K} + h_k(\mathbf{x}_k) - \frac{1}{K}\sum_{l\in\mathbb{K}} h_l(\mathbf{x}_l), 1\right\}, 0\right\} \quad \textit{for any } \mathbf{x}, \quad (3)$$

where all $h_k(\mathbf{x}_k)$ are continuous and weakly increasing functions, i.e. $h_k(\mathbf{x}'_k) \geq h_k(\mathbf{x}_k)$ whenever $\mathbf{x}'_k > \mathbf{x}_k$.

Proof. Recall that $p_k^{\{k,m\}}(\mathbf{x})$ denotes the winning probability of group k in the pairwise contest against group m. Then, we can rewrite (2) as

$$p_k^{\{k,m\}}(\mathbf{x}_k, \mathbf{x}_m) - p_m^{\{k,m\}}(\mathbf{x}_k, \mathbf{x}_m) = p_k(\mathbf{x}) - p_m(\mathbf{x}).$$

Note that by A3 we can write $p_k^{\{k,m\}}(\mathbf{x})$ as $p_k^{\{k,m\}}(\mathbf{x}_k, \mathbf{x}_m)$. Now consider the pairwise subcontest $\{l, m\}$. Employing (2) again we get

$$p_l^{\{l,m\}}(\mathbf{x}_l, \mathbf{x}_m) - p_m^{\{l,m\}}(\mathbf{x}_l, \mathbf{x}_m) = p_l(\mathbf{x}) - p_m(\mathbf{x}).$$

After substracting these two expressions and noting that by the probability axiom $p_m^{\{k,m\}}(\mathbf{x}_k, \mathbf{x}_m) = 1 - p_k^{\{k,m\}}(\mathbf{x}_k, \mathbf{x}_m)$ and $p_m^{\{l,m\}}(\mathbf{x}_l, \mathbf{x}_m) = 1 - p_l^{\{l,m\}}(\mathbf{x}_l, \mathbf{x}_m)$ we can rewrite

$$p_k(\mathbf{x}) - p_l(\mathbf{x}) = 2[p_k^{\{k,m\}}(\mathbf{x}_k, \mathbf{x}_m) - p_l^{\{l,m\}}(\mathbf{x}_l, \mathbf{x}_m)]. \quad (4)$$

Since this holds for any vector $\mathbf{x}$ and the left hand side of the expression does not depend on $\mathbf{x}_m$, hence the right hand side cannot depend on $\mathbf{x}_m$ either. Therefore, we can rewrite the right hand side as the difference of two functions

$$p_k(\mathbf{x}) - p_l(\mathbf{x}) = h_k(\mathbf{x}_k) - h_l(\mathbf{x}_l). \quad (5)$$

Note that $h_k(\mathbf{x}_k)$ and $h_l(\mathbf{x}_l)$ are continuous functions given that $p_k(\mathbf{x})$ and $p_l(\mathbf{x})$ are continuous too. Adding up across all groups in $\mathbb{K}$, we get

$$Kp_k(\mathbf{x}) - \sum_{l\in\mathbb{K}} p_l(\mathbf{x}) = Kh_k(\mathbf{x}_k) - \sum_{l\in\mathbb{K}} h_l(\mathbf{x}_l). \quad (6)$$

By the Probability axiom $\sum_{l\in\mathbb{K}} p_l(\mathbf{x}) = 1$, so this expression in addition to the bounds imposed by the Probability axiom boils down to the expression stated in the text of the Theorem.

We must finally prove that each function $h_k(\mathbf{x}_k)$ is weakly increasing. Consider a pair of vectors $\mathbf{x}'$ and $\mathbf{x}$ such that $\mathbf{x}' = (\mathbf{x}_1, ..., \mathbf{x}'_k, ..., \mathbf{x}_K)$ where $\mathbf{x}'_k > \mathbf{x}_k$. That is, vector $\mathbf{x}'$ is identical to vector $\mathbf{x}$ except for group k. By the Monotonicity axiom it must be that $p_k(\mathbf{x}) \leq p_k(\mathbf{x}')$ and $p_l(\mathbf{x}) \geq p_l(\mathbf{x}')$. Combining this with the expression (5) implies that

$$\begin{aligned} h_k(\mathbf{x}_k) - h_l(\mathbf{x}_l) &= p_k(\mathbf{x}) - p_l(\mathbf{x}) \\ &\leq p_k(\mathbf{x}') - p_l(\mathbf{x}') = h_k(\mathbf{x}'_k) - h_l(\mathbf{x}_l), \end{aligned}$$

thus proving that $h_k(\mathbf{x}_k)$ is weakly increasing. This finalizes the proof. ■

The difference-form group CSF in (3) relates the success of a group to the difference between its impact and the average impact of all the groups involved in the contest. If the impact of the group is above (below) the average impact, its winning probability must be above (below) the winning probability that the group would be awarded under a fair lottery.

For the case of individual contests and linear and identical impact functions, i.e. $h_k(x_k) = sx_k$, the form in (3) boils down to

$$p_k(\mathbf{x}) = \max\left\{\min\left\{\frac{1}{K} + \frac{s}{K}\sum_{l=1}^{K}(x_k - x_l), 1\right\}, 0\right\},$$

which is a generalization to K-players contests of the difference-form introduced by Che and Gale (2000) and later employed by Rohner (2006), Besley and Persson (2008, 2009) and Gartzke and Rohner (2011).

3 Invariance

3.1 Scale invariance

In this section, we study two other properties employed in previous axiomatic characterizations of CSFs. These properties impose the invariance of winning probabilities to certain changes in the profile of contestants' efforts. The first one, and most-commonly used, is homogeneity of degree zero, which here we will refer to as scale invariance.

Axiom 6 (Scale Invariance) *For all $\lambda > 0$ and all $k \in \mathbb{K}$,*

$$p_k(\lambda \mathbf{x}) = p_k(\mathbf{x}).$$

This axiom states that winning probabilities must remain constant to equiproportional changes in all contenders' efforts. Scale invariance thus implies that units of measurement should not matter. This is a desirable property when efforts are measured in money or military units (battalions, regiments, etc). It is a property which is also satisfied by the indices of relative inequality introduced by Atkinson (1970).

Münster (2009) proved that if a CSF satisfies axioms A1-A4 and A6, then the impact function of all groups must be homogeneous of the same degree. Let us now characterize the family of scale invariant difference-form CSFs.

Theorem 2 *If a CSF satisfies axioms A1-A3, A5 and A6, then it is of the form (3) and the impact functions $h_k(\mathbf{x}_k)$ are homothetic functions satisfying*

$$h_k(\mathbf{x}_k) = \alpha_k + \beta \ln g(\mathbf{x}_k), \tag{7}$$

where α_k and $\beta > 0$ are parameters and $g(\mathbf{x}_k)$ is homogeneous of degree one.

Proof. By Theorem 1 and A6, and for any two groups $k, m \in \mathbb{K}$ their impact functions satisfy

$$h_k(\lambda \mathbf{x}_k) - h_k(\mathbf{x}_k) = h_m(\lambda \mathbf{x}_m) - h_m(\mathbf{x}_m) = \frac{1}{K}[\sum_{l \in \mathbb{K}} h_l(\lambda \mathbf{x}_l) - \sum_{l \in \mathbb{K}} h_l(\mathbf{x}_l)].$$

Hence, because the first two terms of the above expression do not depend on x_k or x_m they must depend only on λ. Denote by $\mathbf{1} = (1, ..., 1)$ the vector of appropriate length whose components are all equal to one. Then it must hold true that

$$h_k(\lambda \mathbf{x}_k) - h_k(\mathbf{x}_k) = h_k(\lambda \cdot \mathbf{1}) - h_k(\mathbf{1}).$$

Now add and substract $h_k(\mathbf{1})$ to the left hand side of this expression and denote $H(\mathbf{x}_k) = h_k(\mathbf{x}_k) - h_k(\mathbf{1})$. It can then be rewritten as

$$H(\lambda \mathbf{x}_k) = H(\lambda \mathbf{1}) + H(\mathbf{x}_k).$$

If $\mathbf{x}_k = t \cdot \mathbf{1}$ for $t > 0$ then

$$H(\lambda t \mathbf{1}) = H(\lambda \mathbf{1}) + H(t \mathbf{1}).$$

Define now $G(\lambda) = H(\lambda \mathbf{1})$. This is a function of just one variable and it is weakly increasing and continuous since by Theorem 1 we know already that $h_k(\mathbf{x}_k)$ must be weakly increasing and continuous. We can then rewrite

$$G(\lambda t) = G(\lambda) + G(t).$$

This is one of the Cauchy functional equations whose only solution is given by $G(z) = \ln z^\beta$ where $\beta > 0$ is a constant (Aczél, 1969). Now, this implies that

$$H(\lambda\mathbf{1}) = G(\lambda) = \ln \lambda^\beta,$$

and by the same token that

$$H(\lambda\mathbf{x}_k) = H(\mathbf{x}_k) + \ln \lambda^\beta.$$

Given our definition of the function H, this implies that

$$h_k(\lambda\mathbf{x}_k) = h_k(\mathbf{x}_k) + \ln \lambda^\beta.$$

By A6, it must be that β is identical for all impact functions. Now define $F(\mathbf{x}_k) = \exp\{h_k(\mathbf{x}_k)\}$. It is clear that $F(\mathbf{x}_k)$ is an homogeneous function of degree β. This is a function of one variable, which in turn must be a multiple of a power function, i.e. $F(s) = as^\beta$ (Münster, 2009; p 355). The argument of this function can be expressed as a function $g_k : \mathbb{R}^n_+ \to \mathbb{R}_+$, so we can rewrite

$$F(\mathbf{x}_k) = a_k(g_k(\mathbf{x}_k))^\beta.$$

This function $g_k(\mathbf{x}_k)$ must be homogeneous of degree one since

$$F(\lambda\mathbf{x}_k) = \lambda^\beta F(\mathbf{x}_k) \to g_k(\lambda\mathbf{x}_k) = \lambda g(\mathbf{x}_k).$$

Finally, tracing back our steps

$$h_k(\mathbf{x}_k) = \ln F(\mathbf{x}_k) = \ln a_k + \beta \ln g(\mathbf{x}_k) = \alpha_k + \beta \ln g(\mathbf{x}_k).$$

■

The difference-form CSF has been often criticized because it seemed to violate scale invariance (Skaperdas, 1996; Hirshleifer, 2000; Alcalde and Dahm, 2007, p. 103; Corchón, 2007, p. 74). Theorem 2 proves that such criticism is ungrounded. If the impact function is of the form (7), winning probabilities under the family of difference-form CSFs in (3) are invariant to equiproportional changes in contenders' efforts. To the best of our knowledge, this family of scale invariant difference-form CSFs has only been studied in Polishchuk and Tonis (2013). They microfound the CSF of the form

$$p_k(\mathbf{x}) = \max\left\{\min\left\{\frac{1}{K} + \ln g(x_k) - \frac{1}{K}\sum_{l=1}^{K}\ln g(x_l), 1\right\}, 0\right\},$$

by using a mechanism design approach when contenders are individual who have private information over their valuation of victory. For the case of group contests, (7) is satisfied by the function $h_k(\mathbf{x}_k) = \ln(\sum_{i=1}^{n_k} x_{ik})^\beta$, which we study in more detail in Section 4.

3.2 Translation invariance

If a CSF is defined as a function of the difference between contenders' efforts, another natural invariance property is the following: Winning probabilities should remain constant when the effort of all contenders increase by the same amount. This is equivalent to the following property. Let us denote by $\mathbf{1} = (1, ..., 1)$ the vector whose components are all equal to one.

Axiom 7 (Translation Invariance) *For all $\lambda > 0$ and all $k \in \mathbb{K}$,*

$$p_k(\mathbf{x}+\lambda\cdot\mathbf{1}) = p_k(\mathbf{x}).$$

Skaperdas (1996) and Münster (2009) used this property as an alternative to homogeneity of degree zero in their axiomatization of the ratio-form CSFs. Actually, translation invariance can be traced back to the income distribution literature, and in particular to the concept of absolute inequality introduced by Kolm (1976a,b). Absolute inequality states that the level of inequality in a distribution should not vary when the income of every individual increases by the same fixed amount. Hence, any measure of absolute income inequality must be translation invariant.

However, note that the Translation Invariance axiom builds in an implicit bias against big groups. Adding a constant λ to the effort of each member means that the total group effort increases by λn_k. Bigger groups increase more their effort than smaller groups. Despite of this, Translation Invariance requires that winning probabilities should remain invariant. In order to correct this bias, we introduce the following axiom:

Axiom 8 (Group Translation Invariance) *For all $\lambda > 0$ and all $k \in \mathbb{K}$,*

$$p_k(\mathbf{x}_1+\frac{\lambda}{n_1}\cdot\mathbf{1}, .., \mathbf{x}_K+\frac{\lambda}{n_K}\cdot\mathbf{1}) = p_k(\mathbf{x}).$$

This property implies that if the total group effort increases equally across all groups by the same positive amount λ (by increasing each member effort by a fix amount $\frac{\lambda}{n_k}$), winning probabilities should not change. Group Translation Invariance thus levels the playfield: It eliminates the bias against big groups implicitly built in the standard Translation Invariance property, a bias which has been so far overlooked by the literature.[6]

Before stating our next theorem, consider the following definition:

[6] Such bias is not built in the Scale Invariance property: When each member's effort increases in the same proportion, the total effort of all groups increases in that same proportion.

Definition *The impact function $h_k(x_k)$ is said to be translatable if*

$$h_k(\mathbf{x}_k + \lambda \cdot \mathbf{1}) = h_k(\mathbf{x}_k) + \beta_k \lambda \qquad \textit{where } \beta_k, \lambda > 0.$$

We will refer to the scalar β_k as the degree of (linear) translatability of the impact function. Translatability is analogous to linear homogeneity when a fixed amount is added to the arguments of a function. We again borrow this concept from the income distribution literature; it is a building block in the analysis of absolute inequality (Kolm, 1976a, 1976b; Blackorby and Donaldson, 1980).

We are finally ready to state our theorem characterizing the family of translation invariant difference-form CSFs.

Theorem 3 *If a CSF satisfies axioms A1-A3, A5 and A7, then it is of the form (3) and the impact functions $h_k(\mathbf{x}_k)$ must be translatable of the same degree $\beta > 0$. If A7 is replaced by A8, then the impact functions $h_k(\mathbf{x}_k)$ must be translatable of degree βn_k.*

Proof. Combining Theorem 1 with the Translation Invariance axiom,

$$p_k^{\{k,l\}}(\mathbf{x}_k + \lambda \cdot \mathbf{1}, \mathbf{x}_l + \boldsymbol{\lambda} \cdot \mathbf{1}) = p_k^{\{k,l\}}(\mathbf{x}_k, \mathbf{x}_l) \rightarrow h_k(\mathbf{x}_k + \lambda \cdot \mathbf{1}) - h_k(\mathbf{x}_k) = h_l(\mathbf{x}_l + \lambda \cdot \mathbf{1}) - h_l(\mathbf{x}_l).$$

Since this holds for any $k, l \in \mathbb{K}$ and for any $\mathbf{x}_l$, the right hand side cannot depend on $\mathbf{x}_l$ but on λ so the expression can be rewritten as

$$h_k(\mathbf{x}_k + \lambda \cdot \mathbf{1}) = h_k(\mathbf{x}_k) + \phi(\lambda),$$

where $\phi(\cdot)$ is a continuous function because it is the difference of two continuous function. This expression holds for any λ so

$$\begin{aligned} h_k(\mathbf{x}_k + (\lambda + \mu) \cdot \mathbf{1}) &= h_k(\mathbf{x}_k + \lambda \cdot \mathbf{1}) + \phi(\mu) = h_k(\mathbf{x}_k) + \phi(\lambda) + \phi(\mu) \\ &\rightarrow \phi(\lambda + \mu) = \phi(\lambda) + \phi(\mu). \end{aligned}$$

This is just the Cauchy functional equation whose only solution is of the form $\phi(\lambda) = \beta\lambda$ where $\beta > 0$ is an arbitrary real number.

The proof when A7 is replaced by Group translation invariance runs along the same lines. It must be that

$$h_k(\mathbf{x}_k + \frac{\lambda}{n_k} \cdot \mathbf{1}) = h_k(\mathbf{x}_k) + \psi_k(\frac{\lambda}{n_k}),$$

where $\psi_k(\cdot)$ is also continuous because it is the difference of two continuous function. Note that for this expression to hold, it must be also that

$$\psi_k(\frac{\lambda}{n_k}) = \psi_l(\frac{\lambda}{n_l}) \qquad \forall k, l \in \mathbb{K}. \tag{8}$$

Because this holds for any λ then one can write

$$\begin{aligned} h_k(\mathbf{x}_k+\frac{\lambda+\mu}{n_k}\cdot\mathbf{1}) &= h_k(\mathbf{x}_k+\frac{\lambda}{n_k}\cdot\mathbf{1})+\psi_k(\frac{\mu}{n_k}) = h_k(\mathbf{x}_k)+\psi_k(\frac{\lambda}{n_k})+\psi_k(\frac{\mu}{n_k}) \\ &\rightarrow \psi_k(\frac{\lambda+\mu}{n_k}) = \psi_k(\frac{\lambda}{n_k})+\psi_k(\frac{\mu}{n_k}). \end{aligned}$$

By induction, it is easy to see that this property implies that $\psi_k(\lambda) = n_k\psi_k(\frac{\lambda}{n_k})$. Hence, $\psi_k(\lambda+\mu) = \psi_k(\lambda)+\psi_k(\mu)$. This is just the familiar Cauchy functional equation whose solution is $\psi_k(\lambda) = \beta_k\lambda$. This together with (8) implies that

$$\frac{\beta_k}{\beta_l} = \frac{n_k}{n_l} \qquad \forall k, l \in S,$$

so $\beta_k = \beta n_k$ for all $k \in \mathbb{K}$. This completes the proof. ■

For an example of a translation invariant difference-form CSF, consider the following impact function which we employ in a companion paper (Cubel and Sanchez-Pages, 2014):

$$h_k(\mathbf{x}_k) = \ln(\frac{1}{n_k}\sum_{i=1}^{n_k} e^{-\gamma_k x_{ik}})^{-\frac{\beta_k}{\gamma_k}} \qquad \text{for } \gamma_k \geq 0,\ \beta_k > 0.$$

This is the log of a CES function where efforts are exponential. The parameter γ_k measures the degree of complementarity of members' efforts. This function is linear when $\gamma_k = 0$. It violates A2 when $\gamma_k \rightarrow \infty$ as it converges to the weakest-link technology (Hirshleifer, 1983).

One remark is in order at this point: In his Theorem 3, Münster (2009) characterizes the class of ratio-form CSF which are also translation invariant. He shows that for individual contests, this class boils down to the logistic CSF introduced by Hirshleifer (1989; 1991). In the literature, the logistic form is often referred to as a difference-form CSF. We see this as a misnomer. As our axiomatization makes clear, this form does not satisfy the Absolute Consistency axiom. Hence, in order to be precise and rigorous, we believe that the logistic form should remain classified as an element in the family of translation invariant ratio-form CSFs.

4 Aggregation

So far, none of the properties we have posit on CSFs is specific to group contests. A distinctive feature of confrontations among groups is that members' efforts must be aggregated in some form. This is modelled through the impact function. Further assumptions on the aggregation of efforts are thus needed in order to obtain sharper characterizations. Consider for instance the following axiom introduced by Münster (2009).

Axiom 9 (Summation) *For any $k \in \mathbb{K}$ consider two effort vectors $\mathbf{x}_k$ and $\mathbf{x}'_k$ such that $\sum_{i=1}^{n_k} x_{ik} = \sum_{i=1}^{n_k} x'_{ik}$. Then it must be that*

$$p_k(\mathbf{x}_k, \mathbf{x}_{-k}) = p_k(\mathbf{x}'_k, \mathbf{x}_{-k}).$$

This axiom implies that winning probabilities should remain invariant to changes in the distribution of efforts within groups which leave total group effort unchanged. In the context of lobbying or rent-seeking, when efforts are monetary, such assumption seems to be granted. Underlying this axiom is the assumption that efforts within groups are perfect substitutes, so the marginal productivity of individual effort does not depend on the effort made by other group members.

Let us now apply this property to our characterization of the family of difference-form CSFs.

Proposition 1 *If a CSF satisfies axioms A1-A3, A5, A6 and A9, then it is of the form (3) and the impact functions $h_k(\mathbf{x}_k)$ satisfy*

$$h_k(\mathbf{x}_k) = \alpha_k + \beta \ln(\sum_{i=1}^{n_k} x_{ik}), \tag{9}$$

where $\alpha_k, \beta > 0$ are parameters.

Proof. Given Theorem 2, we only need to prove that $g(\mathbf{x}_k) = \sum_{i=1}^{n_k} x_{ik}$. By A9, we know that the impact function can be expressed just as a function of the total sum of efforts in the group. Hence, it must be that

$$h_k(\mathbf{x}_k) = h_k(\frac{1}{n_k}\sum_{i=1}^{n_k} x_{ik}, ..., \frac{1}{n_k}\sum_{i=1}^{n_k} x_{ik}).$$

This together with expression (7), implies that it is possible to write $g(\mathbf{x}_k) = \phi(\sum_{i=1}^{n_k} x_{ik})$. Since $\phi(\sum_{i=1}^{n_k} x_{ik})$ must be homogeneous of degree one and it is a function of one variable it must be a multiple of a power function. Hence,

$$\phi(\sum_{i=1}^{n_k} x_{ik}) = a\sum_{i=1}^{n_k} x_{ik},$$

which leads to a function of the form (9) . ■

The addition of Summation to our set of axioms yields a tighter characterization of the impact function. Proposition 1 highlights again that the difference-form CSFs can be homogeneous of degree zero when the function mapping members' efforts into group impact is logarithmic. This result can thus respond to a criticism often made against this family of CSFs and which was originally raised in Hirshleifer (2000)[7]: If the difference between the efforts of two contenders is kept fixed, the weaker side should be more likely to win as the efforts of the contenders increase; formally, $p_i^{\{i.j\}}(x_i, x_i + c)$ should be increasing in x_i, where $c > 0$. This is equivalent to a positive elasticity of augmentation (Hwang, 2012). This property is not satisfied by difference-form CSFs with linear impact functions, such as the logistic form or the linear CSF in Che and Gale (2000). This is because a linear mapping from effort into impact implies that the CSF is translation invariant which in turn implies a zero elasticity of augmentation.[8] Such feature seems indeed unreasonable in many circumstances. It is easy to see that if the difference-form group CSF satisfies Summation, i.e. the impact functions are as in (9), the weaker group has a higher winning probability as the total efforts of the two groups increase whilst keeping the difference between the two constant.[9]

Let us now turn our attention to the case of translation invariant CSFs:

Proposition 2 *If a CSF satisfies axioms A1-A3, A5, A7 and A9, then it is of the form (3) and the impact functions $h_k(\mathbf{x}_k)$ satisfy*

$$h_k(\mathbf{x}_k) = \alpha_k + \frac{\beta_k}{n_k} \sum_{i=1}^{n_k} x_{ik}, \tag{10}$$

where $\alpha_k, \beta_k > 0$ are parameters, and $\beta_k = \beta$ for all k. If A7 is replaced by A8, then $\beta_k = \beta n_k$.

[7] "It might be thought a fatal objection against the difference form of the CSF that a force balance of 1,000 soldiers versus 999 implies the same outcome (in terms of relative success) as 3 soldiers versus 2! [...] Any reasonable provision for randomness would imply a higher likelihood of the weaker side winning the 1,000:999 comparison than in the 3:2 comparison." (Hirshleifer, 2000, p 779)

[8] Translation Invariance implies $p_i^{\{i.j\}}(x_i, x_i + c) = p_i^{\{i.j\}}(x_i + t, x_i + t + c)$. Hence, $\frac{\partial p_i^{\{i.j\}}(x_i, x_i+c)}{\partial x_i} = 0$.

[9] Scale Invariance plus Summation imply that for $\lambda > 1$

$$p_1(\sum_{i=1}^{n_1} x_{i1}, c + \sum_{i=1}^{n_1} x_{i1}) = p_1(\sum_{i=1}^{n_1} \lambda x_{i1}, \lambda c + \sum_{i=1}^{n_1} \lambda x_{i1}) \leq p_1(\sum_{i=1}^{n_1} \lambda x_{i1}, c + \sum_{i=1}^{n_1} \lambda x_{i1}),$$

where the last inequality follows from the monotonicity axiom.

Proof. Again, it is possible to define the impact function as a function of $\sum_{i=1}^{n_k} x_{ik}$ but in this case, by Theorem 2, with the property that

$$\phi_k(\sum_{i=1}^{n_k} x_{ik} + n_k\lambda) = \phi_k(\sum_{i=1}^{n_k} x_{ik}) + \beta\lambda,$$

in case A7 is invoked. Define again $H_k(t) = \exp\{\phi_k(t)\}$ so then$H_k(\sum_{i=1}^{n_k} x_{ik} + n_k\lambda) = \exp\{\phi_k(\sum_{i=1}^{n_k} x_{ik}) + \beta\lambda\} = \exp\{\beta\lambda\}H_k(\sum_{i=1}^{n_k} x_{ik})$. and

$$\phi_k(\sum_{i=1}^{n_k} x_{ik} + \lambda) = \phi_k(\sum_{i=1}^{n_k} x_{ik}) + \lambda,$$

if A8 is employed instead. ■

Summation plus Translation Invariance imply that impact functions must be linear. This has an additional implication. Given that the form (3) is already separable, the equilibrium of any simultaneous-move contest with a linear difference-form CSF must be in dominant strategies under 1) risk neutrality or 2) when $p_k(\mathbf{x})$ is interpreted as a winning probability. This is because the marginal benefit of effort does not depend in these cases on neither the effort of other group members or the effort of outsiders. It is thus natural that Beviá and Corchón (2014b) have been able to microfound this type of CSFs by means of dominant strategy implementation. Dominance solvability does not apply when $p_k(\mathbf{x})$ is instead a share, and utilities are non-linear as, for instance, in Levine and Smith (1995).[10]

One undesirable consequence of the Summation axiom is that the resulting CSFs can admit biases. Take for instance the linear impact in (10) and plug it in the difference-form (3). That yields

$$p_k(\mathbf{x}) = \max\left\{\min\left\{\frac{1}{K} + \alpha_k - \frac{1}{K}\sum_{l\in\mathbb{K}}\alpha_l + \beta[\bar{x}_k - \frac{1}{K}\sum_{l\in\mathbb{K}}\bar{x}_l], 1\right\}, 0\right\},$$

where $\bar{x}_k$ denotes the average effort in group k. Note that any group with an above-average α_k enjoys a head-start in the contest. The reason why the CSF admits this type of bias is because the Summation axiom remains silent on the relative success of different groups with the same total effort. One possibility is to modify this property in order to account for this.

Axiom 10 (Total Effort) *For any two groups $k, l \in \mathbb{K}$ such that $\sum_{i=1}^{n_k} x_{ik} = \sum_{i=1}^{n_l} x_{il}$ it must be that*

$$p_k(\mathbf{x}_k, \mathbf{x}_{-k}) = p_l(\mathbf{x}_l, \mathbf{x}_{-l}).$$

[10] We thank Alberto Vesperoni for pointing this out.

The axiom is a stronger version of Summation. It is actually a combination of Summation and the Between-Group Anonymity axiom of Münster (2009). It requires that two groups with the same total effort must have the same winning probability regardless of their size. Again, this property can make sense when efforts are monetary units, but not when efforts represent time or when group size matters: for instance, the impact of a group of 100 people demonstrating for 10 hours may not be the same as the impact of a group of 1000 people demonstrating for an hour.

The following Proposition shows that when Total Effort replaces Summation, the bias described above vanishes.

Proposition 3 *If A9 is replaced by A10, then the impact functions characterized in Propositions 1 and 2 must satisfy* $\alpha_k = \alpha$ *for all* $k \in \mathbb{K}$.

Proof. It suffices to show that when A10 holds impact functions, whatever their functional form, should be identical across groups. To see this note that

$$h_k(\mathbf{x}_k) = h_k(\frac{\sum_{i=1}^{n_k} x_{ik}}{n_k}, ..., \frac{\sum_{i=1}^{n_k} x_{ik}}{n_k}),$$

because A10 also applies to changes in the distribution of efforts within groups which maintain total effort constant. Hence, for any vector $\mathbf{x}_k$ it is possible to write the impact of the group as a function of the total effort, i.e. $h_k(\mathbf{x}_k) = \phi_k(\sum_{i=1}^{n_k} x_{ik})$. Similarly for group l, that is, $h_l(\mathbf{x}_k) = \phi_l(\sum_{i=1}^{n_l} x_{il})$. From this it is clear to see that ϕ_k and ϕ_l are identical functions since by A10 they yield the same value whenever they are applied to the same argument. Hence, impact functions (9) and (10) must not differ across groups and $\alpha_k = \alpha$ for all $k \in \mathbb{K}$. ■

Total Effort eliminates biases in favor of certain groups. Denoting by X_k the sum of efforts within group k, a particularly interesting CSF satisfying the Total Effort axiom together with Scale Invariance is

$$p_k(\mathbf{x}) = \max\left\{\min\left\{\frac{1}{K} + \beta \ln \frac{X_k}{G_X}, 1\right\}, 0\right\},$$

where $G_X = (\prod_{l=1}^{K} X_l)^{\frac{1}{K}}$ is the geometric mean of groups' total efforts.

5 Relative difference functions

In the previous sections, we have seen that the criticisms often leveled against difference-form CSFs rest critically on the implicit assumption of a linear relationship between efforts and impact. Within that framework, some authors have proposed new difference-form CSFs which can overcome these problems. Alcalde and Dahm (2007) and Beviá and Corchón (2014a) introduced new functional forms where the difference in efforts is divided by some aggregate. This is again reminiscent of the income inequality literature, and in particular, of the family of compromise indices, which result from dividing an absolute index by the per capita income of the distribution and are, therefore, scale-invariant (Blackorby and Donaldson, 1980).[11]

Let us now characterize axiomatically the relative-difference contest success function (RDCSF) introduced in Beviá and Corchón (2014a). Before proceeding, let us mention that their family of CSFs admits three parameters so here we just characterize one subset of them which, as it turns out, emerges from a natural generalization of the Relative Consistency axiom.

Axiom 11 (Affine Relative Consistency) *For any vector* $\mathbf{x}$*, any subcontest* $S \subseteq \mathbb{K}$ *and any two groups* $k, l \in S$ *it must be that*

$$\frac{\delta + p_k^S(\mathbf{x})}{\delta + p_l^S(\mathbf{x})} = \frac{\delta + p_k(\mathbf{x})}{\delta + p_l(\mathbf{x})}, \tag{11}$$

where $\delta \in [0, 1]$,

Note that when $\delta = 0$ this axiom coincides with the Relative Consistency axiom. We bound the parameter δ to be in the interval $[0, 1]$ for the following reason. Taking logs on both sides of (11) yields

$$\ln(\delta + p_k^S(\mathbf{x})) - \ln(\delta + p_l^S(\mathbf{x})) = \ln(\delta + p_k(\mathbf{x})) - (\delta + \ln p_l(\mathbf{x})),$$

which after repeatedly applying the first-order Taylor approximation around δ yields again (2). Therefore, Affine Relative Consistency and Absolute Consistency become equivalent properties when winning probabilities are close to δ. Because of that, zero and one are the natural limits to the value of this parameter.

We are now in the position to state a new Theorem characterizing a RDCSF belonging to the family introduced in Beviá and Corchón (2014a).[12]

[11] The equivalence between inequality indexes and CSFs requires a normalization first. Note that when all impacts are identical, winning probabilities are all equal to $\frac{1}{K}$ whereas any inequality index should be equal to zero when all incomes are identical . Now, taking this into account, observe that substracting $\frac{1}{K}$ from the difference-form in (3) and dividing by the per capita impact $\frac{1}{K}\sum_{l\in\mathbb{K}} h_l(\mathbf{x}_l)$ yields the RDCSF in (12) minus $\frac{1}{K}$ when $\delta = \frac{K-1}{K}$.

[12] Specifically, for the case $s = 1$ and $\beta = (\delta + \frac{1}{K})(K-1)$. See their equation (15).

Theorem 4 *The CSF $p_k(x)$ is continuous and satisfies axioms A1-A3 and A11 if and only if it can be written as*

$$p_k(\mathbf{x}) = \max\left\{\min\left\{\frac{1}{K} + (\delta K + 1)\frac{h_k(\mathbf{x}_k) - \frac{1}{K}\sum_{l\in\mathbb{K}} h_l(\mathbf{x}_l)}{\sum_{l\in\mathbb{K}} h_l(\mathbf{x}_l)}, 1\right\}, 0\right\} \textit{ for any } \mathbf{x} \text{ s.t. } \mathbf{x} \neq 0, \tag{12}$$

where $\delta \in (0, 1]$ and the impact functions $h_k(\mathbf{x}_k)$ are continuous and weakly increasing functions.

Proof. Take three contender groups k, l and m. By A11 we know that

$$\begin{aligned}
\frac{\delta + p_k(\mathbf{x})}{\delta + p_m(\mathbf{x})} &= \frac{\delta + p_k^{k,m}(\mathbf{x})}{\delta + p_m^{k,m}(\mathbf{x})} = \frac{\delta + p_k^{k,m}(\mathbf{x}_k, \mathbf{x}_m)}{1 + \delta - p_k^{k,m}(\mathbf{x}_k, \mathbf{x}_m)} \\
\frac{\delta + p_l(\mathbf{x})}{\delta + p_m(\mathbf{x})} &= \frac{\delta + p_l^{l,m}(\mathbf{x})}{\delta + p_m^{l,m}(\mathbf{x})} = \frac{\delta + p_l^{l,m}(\mathbf{x}_l, \mathbf{x}_m)}{1 + \delta - p_l^{l,m}(\mathbf{x}_l, \mathbf{x}_m)},
\end{aligned}$$

where the last equalities hold because of A3. Dividing these expressions yields

$$\frac{\delta + p_k(\mathbf{x})}{\delta + p_l(\mathbf{x})} = \frac{\frac{\delta + p_k^{k,m}(\mathbf{x}_k, \mathbf{x}_m)}{1 + \delta - p_k^{k,m}(\mathbf{x}_l, \mathbf{x}_m)}}{\frac{\delta + p_l^{l,m}(\mathbf{x}_l, \mathbf{x}_m)}{1 + \delta - p_l^{l,m}(\mathbf{x}_l, \mathbf{x}_m)}}.$$

Now fix $\mathbf{x}_m = m \neq 0$. It is thus possible to write

$$h_k(x_k) = \frac{\delta + p_k^{k,m}(\mathbf{x}_k, m)}{1 + \delta - p_k^{k,m}(\mathbf{x}_l, m)}, \tag{13}$$

so then

$$\frac{\delta + p_k(\mathbf{x})}{\delta + p_l(\mathbf{x})} = \frac{h_k(\mathbf{x}_k)}{h_l(\mathbf{x}_l)}.$$

Adding up across all $l \in \mathbb{K}$ and then solving for $p_k(\mathbf{x})$ yields

$$p_k(\mathbf{x}) = (\delta K + 1)\frac{h_k(\mathbf{x}_k)}{\sum_{l\in\mathbb{K}} h_l(\mathbf{x}_l)} - \delta, \tag{14}$$

which, after some rewriting and after applying the necessary bounds due to A1, becomes expression (12). We finally need to prove that the impact functions $h_k(x_k)$ are strongly increasing. We leave out the case $\delta = 0$ because then A11 becomes A4 and that case is covered by Theorem 1 in Münster

(2009). For the case $\delta \in (0,1]$ take two vectors $\mathbf{x}'_k$ and $\mathbf{x}_k$. By A2 it must be that $p_k^{k,m}(\mathbf{x}_k, m) < p_k^{k,m}(\mathbf{x}'_k, m)$ when $p_k^{k,m}(\mathbf{x}_k, m) \in (0,1)$. Combining this with the expression (13) implies that $h_k(\mathbf{x}'_k) > h_k(\mathbf{x}_k)$. Inequalities are weak if $p_k^{k,m}(\mathbf{x}_k, m) \in \{0,1\}$. Hence, $h_k(\mathbf{x}_k)$ is a weakly increasing function. This finalizes the proof. ■

This Theorem shows that the family of RDCSF is characterized by the axiom of Affine Relative Consistency. Given that Relative Consistency is a particular case of this property for $\delta = 0$, this implies that the ratio-form CSFs is a particular case of the RDCSF.

Let us now characterize the RDCSF satisfying the two invariance properties considered in Section 3.

Proposition 4 *If a CSF satisfies axioms A1-A3, A6 and A11, then it is of the form (12) and all the impact functions $h_k(\mathbf{x}_k)$ are homogeneous of degree $\beta > 0$. If A6 is replaced by A7 then all the impact functions satisfy*

$$h_k(\mathbf{x}_k + \lambda \mathbf{1}) = e^{\beta\lambda} h_k(\mathbf{x}_k), \tag{15}$$

for all $\lambda > 0$ and where $\alpha_k, \beta > 0$ are parameters.

Proof. A6 implies that $p_k(\mathbf{x}) = p_k(\lambda\mathbf{x})$. Recall that (*12*) can be written as in (14). Assume for the time being that $\mathbf{x}_k \neq 0$. It is then possible to write

$$\frac{\delta + p_k(\lambda\mathbf{x})}{\delta + p_k(\mathbf{x})} = \frac{h_k(\lambda\mathbf{x}_k)}{h_k(\mathbf{x}_k)} = \frac{h_k(\lambda\mathbf{1})}{h_k(\mathbf{1})} = \frac{\delta + p_k(\lambda\mathbf{1})}{\delta + p_k(\mathbf{1})}.$$

From here the proof can just continue identical to the one for Theorem 2 in Münster (2009) in order to show that $h_k(\lambda\mathbf{x}_k) = \lambda^\beta h_k(\mathbf{x}_k)$. For the case where $\mathbf{x}_k = 0$ we need to be a bit more careful. Assume that for some group m it holds that $\mathbf{x}_m \neq 0$. Then it must be that

$$\begin{aligned} p_k(\mathbf{0}, \mathbf{x}_{-k}) &= \max\left\{\min\left\{(\delta K + 1)\frac{h_k(\mathbf{0})}{h_k(\mathbf{0}) + \sum\limits_{l\neq k, l\in\mathbb{K}} h_l(\mathbf{x}_l)} - \delta, 1\right\}, 0\right\} \\ &= \max\left\{\min\left\{(\delta K + 1)\frac{h_k(\mathbf{0})}{h_k(\mathbf{0}) + \lambda^\beta \sum\limits_{l\neq k, l\in\mathbb{K}} h_l(\mathbf{x}_l)} - \delta, 1\right\}, 0\right\} \\ &= p_k(\mathbf{0}, \lambda\mathbf{x}_{-k}). \end{aligned}$$

If $p_k(\mathbf{0}, \mathbf{x}_{-k}) \in (0,1)$ then it must be that $h_k(\mathbf{0}) = 0$ because $\sum\limits_{l\neq k, l\in\mathbb{K}} h_l(\mathbf{x}_l) > 0$. If $p_k(\mathbf{0}, \mathbf{x}_{-k}) = 0$ then A6 implies that $p_k(\mathbf{0}, \lambda\mathbf{x}_{-k}) = 0$ so for any $\lambda > 0$ it must be that

$$\frac{\delta K+1-\delta}{\delta}\frac{h_k(\mathbf{0})}{\sum\limits_{l\neq k,l\in\mathbb{K}} h_l(\mathbf{x}_l)}\leq \min\{\lambda^{\beta},1\}.$$

If $h_k(\mathbf{0})>0$ then the left hand side of the above expression is strictly positive so it is possible to find a value of λ arbitrarily close to zero which contradicts the above expression. Hence it must be that $h_k(\mathbf{0})=0$. A similar argument can be used if $p_k(\mathbf{1},\mathbf{x}_{-k})=0$.

Regarding the second part of the Proposition, using A7 is possible to write

$$\frac{\delta+p_k(\lambda\mathbf{1}+\mathbf{x})}{\delta+p_k(\mathbf{x})}=\frac{h_k(\lambda\mathbf{1}+\mathbf{x})}{h_k(\mathbf{x})}=\frac{h_k(\lambda\mathbf{1})}{h_k(\mathbf{0})}=\frac{\delta+p_k(\lambda\mathbf{1})}{\delta+p_k(\mathbf{0})}.$$

From here, the proof continues identically to the one for Theorem 3 in Münster (2009) only if $h_k(\mathbf{0})>0$. Let us show that this is the case. Assume instead that $h_k(\mathbf{0})=0$. Applying A7 implies that for any $\lambda>0$

$$\begin{aligned}
p_k(\mathbf{0},\mathbf{x}_{-k}) &= 0\\
&= \max\left\{\min\left\{(\delta K+1)\frac{h_k(\lambda\mathbf{1})}{h_k(\lambda\mathbf{1})+\sum\limits_{l\neq k,l\in\mathbb{K}} h_l(\mathbf{x}_l+\lambda\mathbf{1})}-\delta,1\right\},0\right\}\\
&= p_k(\lambda\mathbf{1},\mathbf{x}_{-k}+\lambda\mathbf{1}).
\end{aligned}$$

For this expression to hold it must be that for any $\lambda>0$

$$h_k(\lambda\mathbf{1})\leq\frac{\delta}{\delta K+1-\delta}\sum_{l\neq k,l\in\mathbb{K}} h_l(\mathbf{x}_l+\lambda\mathbf{1}).$$

Because the impact functions are increasing, the summation at right hand side of the above expression is bounded from below by $h_m(\mathbf{x}_m)>0$. Because impact functions are also continuous, then there exists a value of λ arbitrarily close to zero which leads to a contradiction. ■

As we have reiterated, the main criticisms made against the difference-form CSF characterized in (3) can be overcome with the appropriate choice of the impact function. It is thus not necessary to resort to the RDCSF in order to restore scale invariance. Notwithstanding, this family of CSFs has its own advantages. Let us discuss two of them. First, the RDCSF is not subject to the "zero probability problem" suffered by the CSFs satisfying the Relative Consistency axiom. Simple inspection of (11) shows that if $p_k(\mathbf{x})=0$ the axiom does not force group k to have zero winning probability in all subcontests in which k can participate. Secondly, RDCSF is less rigid than the family of CSFs satisfying Absolute Consistency. Take again our example in Section 2.2, where group k is very weak so $p_k(\mathbf{x}) = 0$ whereas group l is very strong but $p_l(\mathbf{x}) = \varepsilon$ because there is a large number of equally mighty groups in the grand contest. In this case, Affine Relative Consistency implies that in the pairwise contest between k and l, the weak group k has a winning probability of $p_k^{\{k,l\}}(\mathbf{x}) = \frac{\delta(1-\varepsilon)}{2\delta+\varepsilon}$, which ranges between zero and $\frac{1-\varepsilon}{2+\varepsilon}$, which is always smaller that the value that $p_k^{\{k,l\}}(\mathbf{x})$ would take if Absolute Consistency were imposed instead. Therefore, changes in winning probabilities across subcontests seems more reasonable when CSFs satisfy Affine Relative Consistency than when they satisfy Absolute Consistency.

6 Conclusion

In this paper, we have offered the first systematic study of group contests where winning probabilities depend on the difference between their effective efforts. Our axiomatic characterization encompassed both absolute and relative difference-form CSFs. This axiomatization employed some tools from the inequality measurement literature.

We aimed to provide a complete cartography of this family of CSFs. We demonstrated that, contrary to what has been argued in the literature, difference-form CSFs can be homogeneous of degree zero, and that they do not force differences in winning probabilities to remain invariant when absolute differences in raw efforts remain constant. In addition, we were the first to flag up that the Translation Invariance property builds in an implicit bias against big groups which should be corrected. In this process, we argued that the logistic function (Hirshleifer, 1989, 1991), although often referred to as a difference-form function, does not actually belong to this family. For us, this label should be reserved only to CSFs satisfying the Absolute Consistency axiom, which the logistic form does *not* satisfy.

Finally, our axiomatization of relative-difference CSFs showed that this family of functions presents its own distinctive advantages. One of these advantages, which this family shares with the family of Absolute Difference CSFs, is that having a zero winning probability in the grand contest does not bound to zero the probability of winning smaller contests. This might be adequate in Political Economy applications where a party may have no chances in a general election but a large probability of winning a local one.

Difference-form CSFs have not been employed in the contest literature as often as other functional forms. We hope that, by clarifying its properties, our axiomatization will persuade researchers in the area to include this family of CSFs in their toolkit. Of course, our characterization is normative and leaves out strategic interactions. Che and Gale (2000) showed that their linear difference-form CSF often leads to mixed-strategy equilibria and that any equilibrium in pure-strategies involves that at most one contender is active. One possible next step would be to check whether the equilibria of contests under the generalized difference-form CSF axiomatized here still presents such features. In addition, this form implies the separability on contenders' efforts, leading to dominant strategy equilibria when impacts are linear. We explore these issues in a companion paper (Cubel and Sanchez-Pages, 2014).

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