Unmanned Aerial Vehicle Route Planning On A Dynamically Changing Waypoint Based Map For Exploration Purposes.

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In the included work the Unmanned Aerial Vehicle (UAV) mission is represented by energy graphs motivated by the analysis in [1]. The problem of the shortest path routing is revisited when a dynamically changing environment is considered. It is assumed that information about the map is received while on flight due to events. In addition, UAVs are required, while on mission, to "*scout*" areas of interest which involves extracting as much intelligence as possible and traversing it in the most safe flyable means. Hence, the UAV should be capable of integrating knowledge from a variety of sources and re-plan its mission accordingly in order to fulfil objectives. Motivated by the previous, depending on the decision making process, the notion of a "*temporary*" optimum path can be of physical and functional sense. The problem is modeled as a multistage decision making process, where each stage is triggered by an event and is characterized by a current starting point, an area for reconnaissance purposes and a final destination. Hence, given the current availability between paths, the objective is to devise a policy that leads from an origin or current known location to a destination node while traversing the unknown region of interest with the minimal energy demand.

Index Terms:

UAV, Dynamically changing environment, Reconnaissance mission, Energy Graphs, shortest path, cover-sets, Traveling Salesman Problem.

1. Introduction

Shortest path problems are usually adapted to the problem's requirements (application domain) while incorporating constraints and special conditions for its solution. Depending on the problem, the classic static shortest path algorithm can be proved inadequate. Especially for cases where for a dynamically changing environment a shortest path is required to be computed real time and adapt to variations occurring on the system. For instance, consider the case of the optimal management of an emergency service. Determining all possible routes and choosing the optimum, prior to a mission, from a minimum cost sense is not sufficient for the solution of the problem. In fact, additional information should be accounted for the embedded decision process, the re-optimization should be performed in deterministic manner and a real-time computational schema should not be neglected from the design phase.

According to the application UAV missions can be divided in military or civilian served applications with different levels scenarios. The interested reader may refer to [2] for a thorough list of their classification. In this letter reconnaissance missions are investigated. Hence the problem is constrained to mandatory locations for reconnaissance, exploration or surveillance that the UAV should visit. Particularly for such missions the areas of interest can be totally unknown however priori information can be considered with respect to their location or environmental conditions affecting those.

Among many interesting works, few papers have been proposed for solving the dynamic shortest path problem. Among these some authors considered a quasi static network where discrete changes in the availability of paths are involved [3] and [4]. Other interesting works have proposed models to describe the effect of a time-variant weight alterations. In [5] and [6] a discrete function of time was used. Then in [7] the varying delays in inter-node communication networks were represented by a continuous function of time. The problem was solved utilizing a modified version of the Dijkstra's algorithm [8]. In [9] the authors examined the problem of routing in a dynamic network from the control theory perspective and various decision making procedures. For the special case of time varied blocked paths, the problem is referred in literature as the Canadian Traveler problem which was first proposed by Papadimitriou and Yannakakis in 1991 in [10]. Furthermore in [11] the approach is focused in a railway timetable system where changes are assumed to be discrete and periodical. The main difference in references previously stated and the proposed scenarios described further on is the fact that the "*events*" are occurring completely in a random manner, at a random time instant and of random amount. Consequently the sense of an optimum path can not be determined initially. Motivated by the previous and an emergency refueling scenario over a dynamically changing environment illustrated in work [12], depending on the decision making process, the notion of a "*temporary*" optimum path can be of physical and functional sense until mission objectives are fulfilled.

In section 2 preliminaries are included outlining the importance of the use of graph theory in the UAV context. In addition, the formulation of the dynamically changing energy cost matrix is stated. Then in 3 the re-optimisation of the routing problem is outlined and methodologies are stated for the reconnaissance mission. Those involve determination of mandatory nodes within the unknown area (Set covering problem) and their optimum tour to an origin through solving the Traveling Salesman Problem with genetic algorithms. Lastly a scenario is illustrating the whole analysis in section 4 and conclusions are stated in 5.

2. Preliminaries

2.1 Graph theory - Dynamically changing Energy cost matrix

Graph theory is mainly used in literature as a method for modeling complex networks with the edges associated to weights representing an oblique measurement in a space. For the UAV application discussed in this paper and motivated by the analysis in work [1], where the UAV energy route is employed through utilizing energy graphs the problem is extended to the one of an optimum route when a dynamically changing map is involved. The energy graph can be represented by an adjacency matrix which includes as elements the energy requirements, rather than just distances, for that specific UAV. In the case of a dynamically changing environment the adjacency matrix can be updated providing sensors, from lets say dispatch centers, send reports about events occurring. In addition it is valid to assume that in order for a UAV to travel at a constant speed, the propulsion thrust applied varies with time and is always opposed to the aerodynamic forces applied.

Adopting the terminology in works [1] and [12], the energy adjacency matrix Λ of order nxn for a UAV traveling at a constant speed u and impulse matrix $\mathbf{F}$ during time interval $\Delta_{i,j} = [t^0, t]$ for $\forall$ possible paths $\mathbf{\Lambda}[i, j]$, where all node-to-node paths are non-equal times $\underline{\Delta_{i,j}}$, is equal to $\mathbf{\Lambda} = [u.\int_{t_{i,j}^0}^{t_{i,j}} F_{i,j}(\tau) d\tau dt]$. Every element in Λ represent the energy demand to reach node j to i, for all $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,n$,respectively. The energy demand can be obtained like proposed in work [13]. In the particular the weights assigned were obtained from utilizing the acceleration profile of the UAV when a standard type of trajectory is followed. The trajectory illustrated was a Bezier-Bernstein piecewise polynomial formed by consecutive *Circle-Line-Circle* segments known as the Dubins path [14]. In the case where a dynamically changing environment is involved then that can alter by three different means blocked/unblocked edges, addition/elimination of nodes and their combinations. Hence provided that robust sensors can update both the connectivity matrix A_ℓ and the energy cost matrix W_ℓ at time index t_ℓ then for the general case the energy matrix is equal to $\mathbf{\Lambda}_{\mathbf{t}_\ell} = [W_{i,j}(t_\ell).A_{i,j}(t_\ell)]$. Hence for blocked paths the updated energy cost matrix is calculated by the dot product of $W(t_\ell) = \mathbf{W_0}.A(t_k)$, like illustrated in work [12], where $\mathbf{W_0}$ represents the energy demand for the complete graph case calculated priori to the mission. Contrary when new nodes are added the energy cost matrix has to be re-calculated $W(t_\ell)$. A graph can also be represented by the *distance adjacency matrix* $D(G)$ [12], which is going to be used in the included simulations. The distance adjacency matrix $\mathbf{D(G)} = [d_{ij}]$ is an $n \times n$ matrix which can be calculated using Floyd's algorithm [15]. Like mentioned above this paper is focused in reconnaissance purposes. The latter means to determine the adequate set of nodes to be visited (Covering sets) and to visit all of them at least once(Traveling Salesman Problem).

3. Formulation of a dynamically changing map and the re-optimization of an energy path subject to reconnaissance purposes

3.1 Dynamically changing map

Let a topological map be represented by a graph $G_k(V_k, E_k, C_k)$ for a time interval t_k, with $k \in N$ where V_k is a set of nodes of cardinality n, $E_k = \{(i,j) \vee i, j \in V_k\}$ the set of edges with cardinality of m and C_k a set of costs calculated for every $(i,j) \in E_k$. The problem of the shortest path (SP) can be defined as, given a source s_k, a reconnaissance area $S_{recon} = \{a_1^r, a_2^r, \ldots, a_m^r\}$ and a destination node z, where $s_k, a_m^r, z \in V_k$, find the path P_{r_k} at time t_k that yields the minimum value of the objective function:

$$J(P_{r_k}) = \begin{cases} \min \sum_{(i,j)\in E_k} c_k(i,j)x_k(i,j), & for x_s = s_k \; and x_f = a_o \in S_{recon} \\ \min \sum_{(i,j)\in E_k} c'_k(i,j)x'_k(i,j), & for x_s = a_o, x_f = a_w, a_o, a_w \in S_{recon} and length of the path equals m \\ \min \sum_{(i,j)\in E_k} c_k(i,j)x_k(i,j), & for x_s = a_w \in S_{recon} and x_f = z \end{cases} \quad (1)$$

the path $P_{r_k} \subseteq P_k$ where $P_k = \{(i_1,i_2),\ldots,(i_{p-1},i_p)\}$ is any simple path for $i_j \in V_k$, $(i_{j-1},i_j) \in E_k$ and $j = 2,\ldots,n$. In the objective function for $x_k(i,j) = 0$ the edge (i,j) does not belong to the set E_k, otherwise $x_k(i,j) = 1$. Let $P_{r_k} = \{(s_k, s_{k+1}),\ldots,(.,a_1^r),(a_1^r,.),\ldots,(.,a_m^r),\ldots,(\cdot,z)\}$ denote the path of optimum sense at time t_k. It should be noted that the optimum sequence within the reconnaissance area is obtained utilizing firstly equation (2) and thereafter (4). Then at time t_{k+1} the node s_{k+1} is reached with total cost c_{k+1} and a new event occurs. Hence the path P_{r_k} at time

t_k ceases to be optimum and a new solution $P_{r_{k+1}}$ for graph G_{k+1} should be found for initial starting conditions node s_{k+1} and terminal location the same, z. The previous procedure is repeated as soon as $s_{k+j} = z$. In addition the entry of the area of interest is also recalculated since the graph changed, providing that the vehicle has not entered or exited it yet. It should be noted that the solution is globally optimum at each time instant given local information. Apart from the three different scenarios mentioned previously, there is also the case of uncertainty or time-varied weights in each elementary path. The later has been addressed in [16] and many other interesting works. In the previous, the optimum energy route was shown to alter when uncertainty boundaries are assumed for the overall mission. The scenario involved a multi-sourced vehicle topology with a focus on the feasibility of an overall mission. In this letter blocked paths and addition of new nodes are investigated.

3.2 Cover sets for reconnaissance purposes and the Traveling Salesman Problem

The set covering problem is one of the most important discrete optimization problem since it serves as a model for real world problems. Such problems include facility location, scheduling, resource allocation, vehicle routing etc. Particularly, for the reconnaissance problem discussed it serves as a tool to chose the minimum set of nodes that need to be visited to extract the most information out of the area. Hence under the assumption that a graph $G(V,E)$ can be represented by its adjacency matrix A, then if one computes its transpose A^T and set all $a_{i,j} = 1$, for all $i = j = 1,2,\ldots,N$, then the problem of finding the minimum dominating set reduces to the one of choosing the least number of columns so that every row contains an entry of one under at least one of this set of columns. The later is referred in literature as the *Set Covering Problem* (SCP). Even though it seems a trivial problem it has been shown in Karp's work [17], in 1972, as one of the twenty-one NP-Complete problems included in his list. Mathematically speaking, given a set $R = \{r_1, r_2, \ldots, r_m\}$ and a family of sets $\Im = \{S_1, \ldots, S_n\}$, where $S_j \subset R$, any set $\Im' = \{S_{j1}, \ldots, S_{jk}\}$,where $\Im' \subset \Im$, such that $\bigcup_{i=1}^{k} S_{ji} = R$ is a set-covering of R, where S_{ji} are called covering sets. Hence the SCP attributes to the problem of finding the set-cover $\Im'$ of R that yields the minimum value of $\sum_{i=1}^{k} c_{ji}$ where $c_j \in R^+$ is the weight associated to $S_j \in \Im$ and $\Im' = \{S_{j1}, \ldots, S_{jk}\}$. In matrix form it results to a Mixed-Integer Linear Programming optimization. Thus the problem can be formulated as minimizing the objective function (2)

$$J(\Im') = \min \sum_{j=1}^{n} c_j x_j \tag{2}$$

subject to $\sum_{j=1}^{n} a_{ij} x_j \geq 1$, for $i = 1,2,\ldots,m$ and $x_j \in (0,1)$, for $j = 1,2,\ldots,n$. c_j is the cost of covering j column and x_j the decition variable satisfying equation 3. The first constraint is used to guarantee that each row is covered by at least one column and a_{ij} is a boolean element of an mxn matrix.

$$x_j = \begin{cases} 1, & if \;\; j \in S \\ 0, & otherwise \end{cases} \tag{3}$$

Like mentioned earlier, when the UAV has already entered the area that needs to be scouted the tasks arising are to extract as much information as possible while visiting locations of interest and to exit that area safeflyably. Thus provided that cover-sets were determined through the previous regime the task for the UAV bears similarity to the traveling salesman problem

(TSP). For the cover-set $\Im' = \{S_{j1},\ldots,S_{jk}\}$, not necessarily ordered, that forms a complete graph $G(V,E)$ whose edges are associated with arbitrary total costs $c_{ij} \in C$ of minimum sense to reach from one node to another, the problem is to find the optimum tour starting and ending at the origin node while visiting each node only and only once. In essence find the least cost Hamiltonian circuit using all nodes in the set.

The traveling salesman problem also falls under category of NP-complete problems. The computational time to determine its exact solution is exponentially increasing with respect to the number of nodes. Hence for large architectures, obtaining the optimum tour can be prohibitively expensive and heuristics should not be neglected to reduce computational costs. For our analysis a Genetic Algorithm (GA) is used rather than a recursive deterministic procedure in order to solve such problems with complex fitness landscape and large search space. Genetic algorithms fall under category of evolutionary algorithms that use crossover and mutation operators to solve optimization problems by means of *survival of the fittest*. For further details such as adequate representations of individuals, crossover, mutation operators, their significance and means of halting the involving genetic algorithm the interested reader may refer to [18] and [19] books. The fitness function used in the GA is equal to (4):

$$J_D = \sum_{i=1}^{N} \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} + \lambda(\pi_i - \pi_{i+1})^2, \quad \lambda \in Z^* \tag{4}$$

where x, y depict the localization of the node and the additional term denotes a penalty in J. In essence it can depict sections within the area for the UAV to try to avoid or cross. For instance, when a military application is concerned it can feature stealthy operations if the particular depicts a road section, a border or even patrols within that area.

4. Reconnaissance scenario

In this section a simulation example combining the formulation already stated is described. The problem to be solved is:

Let a topological map being represented by a graph $G(V,E)$. A UAV with finite reserves of fuel has to travel from a start node to a goal. Find the shortest path to reach the goal, while conserving energy demand, when the map is dynamically changing with respect to time. Additional constraints involve the UAV to safeflyable traverse through an area of interest where priori information is assumed with respect to its location within the map. In addition, locate the most adequate nodes to visit, at least once, within the reconnaissance area in order to extract the most information.

For the purpose of illustration the scenario is divided into three different phases. The first phase involves for the UAV to start its mission from a particular origin and enter the area of interest while conserving energy demand. By the second phase the UAV is receiving an event with respect to the environmental and topological conditions within the area in order to extract the most information and then safelyable exit it. Lastly the UAV has to reach its final destination (x_2) staring from the exit point. In the scenario outlined topological alterations are considered in phases.

<u>*Phase A :*</u>

At t_0 there is a priori topological map represented as a graph $G_0(V_0, E_0, W_0)$ and a UAV that has a mission to start from a particular source node s_k (x_{68}) and reach an entry point $x_{recon}^{enter} \in S_{recon}$, while conserving energy. The area included within the formed circle is the one of interest for the UAV to scout. For the particular time instant an optimum path is calculated P_{r_k} from the preflight planning to reach the particular goal while entering the less energy demanding node x_{recon}^{enter}.

Then at t_z, where t_z is a random time index where the UAV is reaching an intermediate waypoint, the UAV is receiving a report represented as a connectivity graph C_z. C_z depicts that the graph has changed to $G_z(V_z, E_z, W_z)$, where $V_0 = V_z$, $E_0 \neq E_z$ and $W_0 \neq W_z$. The task is to replan the mission from the node s_z, that the changes occurred, and find the new shortest path $P_{r_k}^z$ to reach the intermediate goal x_{recon}^{enter} with respect to energy demand. In addition the new entry point has to be redetermined.

The determination of the new energy cost matrix can be provided as proposed in section 3.1 and [13]. Due to reasons the problem is posed the Dijkstra's algorithm [8] is used for phase A and phase C, to determine the optimum path P_{r_k}, since for only 9 nodes it yields to perform faster than Floyd's algorithm as proposed in the work [20]. Hence provided that the necessary calculations are performed the mission initializes. The reconfiguration mechanism is checking every instant that the vehicle is reaching a node if changes occurred in the map. The control is taking place following the previous proposed formulation. Hence as soon as an event occurred the mission is replanned by determining the new shortest path from the energy perspective with sole purpose to reach the intermediate goal. Then at t_1, where the UAV has traveled up to node 19, an event is received that alters the connectivity matrix and energy cost matrix, respectively. In addition x_{recon}^{enter} changes to node 95 since its closer from energy perspective. Then a re-optimization takes place and the new optimum path to reach the reconnaissance area is determined. Thereafter the same procedure is performed as soon as the area of interest is reached. Results for Phase A are included in table 1 with total energy demand of 27 and resulting path sequence $\{68,19,65,37,73,20,73,40,27,81,7,48,84,95\}$. The entry point to the reconnaissance area is node 95. The black dotted line in figures 2 depict the nodes traversed until the occurrence of a new event for t_0, t_1, t_2, t_3 and t_4, respectively. In addition the connectivity (left) and energy (right) adjacency matrices can be seen in figure 1. All the parameters taken into account are all *SI* units.

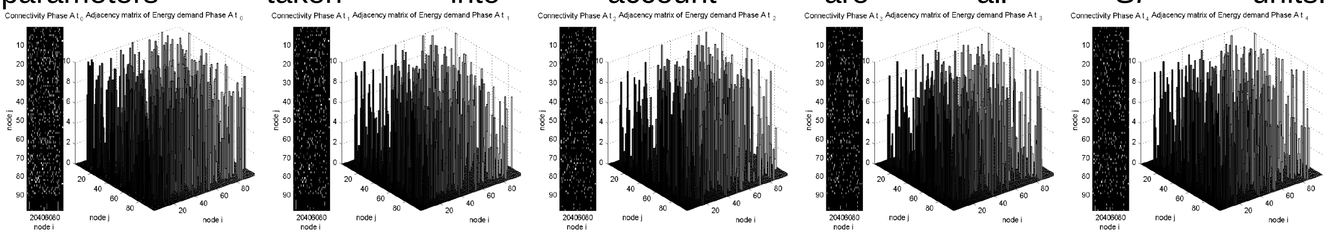

Figure 1: Connectivity matrices and Energy adjacency matrices at time instants t_0, t_1, t_2, t_3 and t_4.

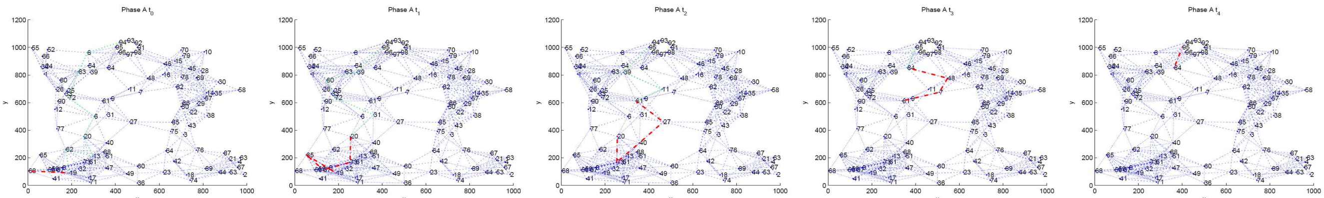

Figure 2: Phase A optimum routes for time indexes t_0, t_1, t_2, t_3 and t_4.

Phase B :

Like mentioned earlier as long as the UAV enters the unknown area of interest $x_{recon}^{enter} = x_{95}$ an event is sent, from lets say dispatch centres or operators, denoting topological and environmental conditions within that area. In other words new nodes are determined within the area and a complete graph is created. Hence the tasks of the UAV is to describe the energy requirements, find the set of those to visit, visit them in a sequence of optimum sense and safeflyable exit the area. In essence, provided that sensors are robust, all node to node energy weights are determined based on [13]. Then by utilizing cover-sets the set of nodes, that are mandatory to be visited in order to extract the most information from the environment, is determined. Afterwards through solving the traveling salesman problem the optimum sequence is obtained, and, lastly, a safeflyable route is determined to exit the reconnaissance area. Hence assuming that the complete graph created $G_{recon}(V_{recon}, E_{recon}, W_{recon})$ is the one depicted in figure 3 the Mixed-Integer Linear Programming illustrated in section 3.2 is solved the covering-set is obtained. $\Im'$ is depicted in figure 3 in red circles. It should be noted that the application domain that is involved is the energy perspective. However an optimum tour to start and finish at an origin while visiting all nodes in the set $\Im'$ at least once is still needed. The particular is solved utilizing genetic algorithms like formulated in section 3.2. The setup of the GA requires three basic steps such as the design of a chromosome representation, a way of mapping a genotype to a phenotype and the means of evaluating an individual. The representation usually chosen for ordering/sequencing problems, as the TSP, is an order based representation. For the particular, every individual is represented as permutations of the set to be used (16 in order) and is compared. In addition valid operators are also utilized in order to have valid permutations for the individuals. The evaluation and selection are the most costly steps. The former one depends on a fitness criterion equal to (4) and by the latter through a tournament selection where individuals directly compete member of the population. Then an arbitrary part from the first parent is chosen and is copied to the first child. The remaining genes that are not selected initially they are copied, starting right from the cut point of the copied part, using the order of genes from the second parent and wrapping around at the end of the chromosome. Afterwards the previous is repeated for the case where the parents roles are reversed. The mutation operator is based on swapping. The basic intuition behind the latter two steps is due to the fact that with crossover operator it is less probable to be trapped in local optima and because the cost function is a highly nonlinear *spiky* environment the mutation is also required. In addition an exhaustive tuning was carried out for the best compromise of parameters. However its beyond the scope of the particular analysis. The fitness of the TSP is depicted in figure 5 whereas the distance matrix used is depicted in 4. The resulting optimum tour in depicted in solid red in figure 3 and the exit point is node $x_{recon}^{exit} = x_{96}$. The total energy demand is 397 energy units. In addition a penalty term is involved in (4). Notice that it is only crossed twice.

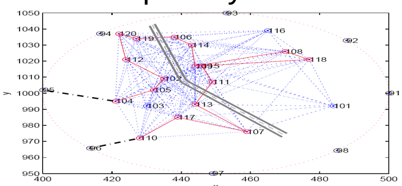

Figure 3: Reconnaissance area.

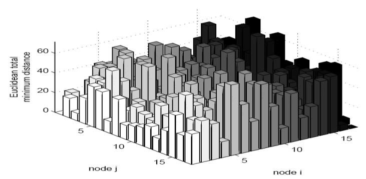

Figure 4: Distance matrix for the cover-set $\Im'$.

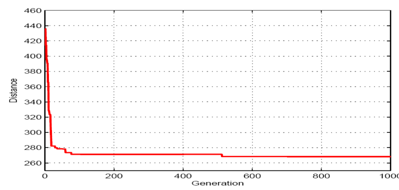

Figure 5: Best fitness of GA with respect to the generation index and the total distance of the sequence of nodes.

Phase C :

The last phase in the simulation involves the safeflyable route of the UAV to reach its final destination node x_2 from the exit point x_{96}. Generally it follows the same pattern as phase A, however the only differences are just the initialization and the terminal nodes. Hence utilizing the Dijkstra algorithm the optimum route with respect to energy is re-optimized whenever an event occurs. Figure 7 depict the succession of the mission until the final destination is reached. The updated adjacency matrices are depicted in 6, respectively. The results are summarised in table 1, where the overall sequence determined is $\{96,84,83,33,12,77,62,19,51,60,42,76,63,2\}$ with total energy demand equal to 42.

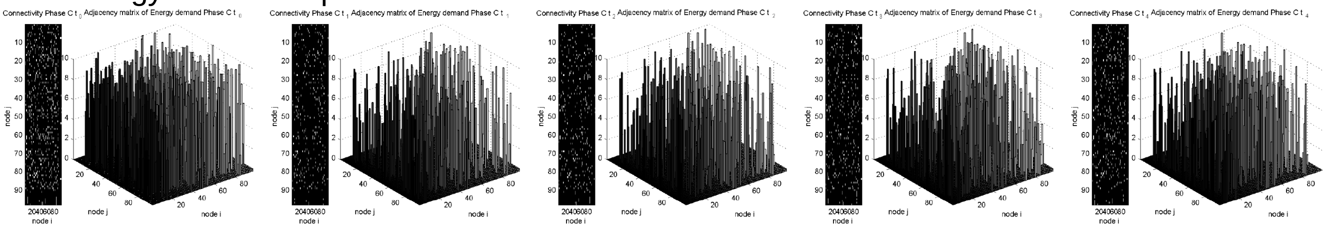

Figure 6: Connectivity matrices and Energy adjacency matrices at time instants t_0, t_1, t_2, t_3 and t_4.

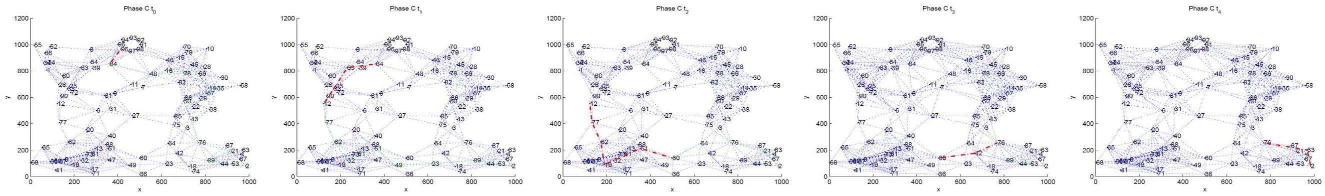

Figure 7: Phase C optimum routes for time indexes t_0, t_1, t_2, t_3 and t_4.

Event	$a_{i,j}(t_k)$	Path to reach goal	Total energy for goal	Spent energy	$r_{i,j}$	Intermediate goal	Total Energy demand
Phase A						Entry node	
t_0	68	68 **19** 62 13 20 6 72 83 8 94	21	1	$r_{68,94} \neq 0$	94	
t_1	19	19 65 37 73 **20** 31 72 80 39 95	18	7	$r_{19,95} \neq 0$	95	
t_2	20	20 73 40 27 **81** 11 39 95	12	6	$r_{20,95} \neq 0$	95	27
t_3	81	81 7 48 **84** 95	13	12	$r_{81,95} \neq 0$	95	
t_4	84	84 95	1	1	$r_{84,95} \neq 0$	95	
Phase B						Exit node	
t_{recon}	95	4 5 2 12 20 19 6 14 9 8 18 15 11 13 7 17	268.4	268.4	$r_{95,96} \neq 0$	96	397
Phase C						Exit node	
t_0	96	96 **84** 48 78 35 50 43 76 21 89 2	26	1	$r_{96,2} \neq 0$	96	
t_1	84	84 83 33 **12** 77 62 32 51 49 23 89 2	44	10	$r_{84,2} \neq 0$	96	
t_2	12	12 77 62 19 51 **60** 64 76 21 2	27	13	$r_{12,2} \neq 0$	96	42
t_3	60	60 42 **76** 89 2	14	6	$r_{60,2} \neq 0$	96	
t_4	76	76 63 2	12	12	$r_{76,2} \neq 0$	96	

Table 1: Concentrated table with results from the whole simulation when new edges appear or disconnected paths occur in Phase A and C, and the mission is subject to visit an area of interest, Phase B. From left to the right, event that occurs, number of node the event occurred ($a_{i,j}(t_k)$), optimum path to reach the goal with respect to energy, total energy demand of the path, spent energy until the next event occurs, whether the node of interest is reachable, refueling node and total energy demand for the overall mission.

5. Conclusion

In this work a methodology was illustrated for reconnaissance purposes of a UAV when a dynamically changing environment is considered, while conserving energy requirements. The graph theory tools proposed were shown to describe all key features of the scenario thus strengthening the use of energy graphs in the UAV context. In addition, through a simulation example the whole analysis was illustrated. The particular was divided in three phases where a UAV was employed to leave an origin, scout an unknown area and safeflyably reach its final destination, while conserving energy. The tasks involved within the reconnaissance area were to determine which nodes are mandatory to be visited in order to extract the most information (Set Covering Problem), safely visit them (TSP with GAs) and exit it.

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