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UK: Managing Editor
International Journal of Innovative Technology and Creative Engineering
1a park lane,
Cranford
London
TW59WA
UK
E-Mail: editor@ijitce.co.uk
Phone: +44-773-043-0249

USA: Editor
International Journal of Innovative Technology and Creative Engineering
Dr. Arumugam
Department of Chemistry
University of Georgia
GA-30602, USA.
Phone: 001-706-206-0812
Fax:001-706-542-2626

India: Editor
International Journal of Innovative Technology & Creative Engineering
Dr. Arthanariee. A. M
Finance Tracking Center India
17/14 Ganapathy Nagar
2nd Street
Ekkattuthangal
Chennai -600032
Mobile: 91-7598208700

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From Editor's Desk

Dear Researcher,

Greetings!

Research article in this issue discusses about Covert Kidnapping Alert and New Algebraic Graph.

Let us review research around the world this month; Soft biometrics is the new way to monitor people. CAMERAS are strewn around our environment, catching glimpses of our faces everywhere we go, yet even the best facial recognition technology still has a hard time picking us out of the crowd. The agency announced a contest on 8 November, challenging teams of the country's top researchers to revolutionise how machines recognise people. Those entering the competition already know that conventional facial recognition won't cut it. There are other applications too. Airport security could be streamlined to allow passengers to walk freely from check-in to the gate, their movements monitored and identities verified automatically by cameras.

Who needs wires? An idea for sending power over long distances via lasers and balloons could help provide emergency power where it is needed. Stephen Blank of the New York Institute of Technology wants to use aerostats, military-grade balloons, to send hundreds of kilowatts of power over several hundred kilometres. A laser would be sent up to the aerostat through a fibre-optic cable, then beamed through the air to a distant aerostat where the high-energy light is converted into electricity, which streams back down to earth via a tether. The ultimate goal is space-based solar power, beamed to Earth via lasers from orbit. This research is at its most advanced in Japan, says Reza Zekavat of Michigan Technological University. A $21 billion Japanese project aims to put 1 gigawatt of solar generation capacity in space within the next 30 years.

Eye-tracker lets you drag and drop files with a glance. A system called EyeDrop uses a head-mounted eye tracker that simultaneously records your field of view so it knows where you are looking on the screen. Gazing at an object – a photo, say – and then pressing a key, selects that object. It can then be moved from the screen to a tablet or smartphone just by glancing at the second device, as long as the two are connected wirelessly."The beauty of using gaze to support this is that our eyes naturally focus on content that we want to acquire," says Jayson Turner, who developed the system with colleagues at Lancaster University, UK.Christian Holz, a researcher in human-computer interaction at Yahoo Labs in Sunnyvale, California, says the system is a nice take on getting round this fundamental problem of using gaze-tracking to interact. "EyeDrop solves this in a slick way by combining it with input on the touch devices we carry with us most of the time anyway and using touch input as a clutching mechanism," he says. "This now allows users to seamlessly interact across devices far and close in a very natural manner."

It has been an absolute pleasure to present you articles that you wish to read. We look forward to many more new technologies related research articles from you and your friends. We are anxiously awaiting the rich and thorough research papers that have been prepared by our authors for the next issue.

Thanks,
Editorial Team
IJITCE

Editorial Members

Review Board Members

Contents

A Covert Kidnapping Alert and Location Identifier (CKALI)

Matthews V.O[1], Osafehinti S[2], Adetiba E[1], Dike Ike[1],John S.N[1].

1. **Covenant University, Electrical/Information Engineering Department, Ota,Nigeria**
2. **M2M Technologies, Nigeria**

***Abstract*- Kidnapping is the taking away or transportation of a person against that person's will, usually to hold the person in false imprisonment and confinement without legal authority. This paper reports a complete research work in kidnapping emergency alert situations. The authors were able to program a microcontroller incorporating a trigger activated GSM/GPS module to report via the GSM communication network (using SMS messaging) to a monitoring center, giving the exact position of the point where the kidnapping took place, and the route taken by the kidnappers and the victim. All the components are miniaturized using advanced very large scale integration techniques, and embedded in a belt which can be worn as a regular clothing accessory. This device (CKALI) will allow fast response and rescue of kidnapped victims; thereby saving lives and money. The paper reports its experimental results, and gives appropriate conclusions and recommendations.**

Keywords - Emergency, GPS/GSM, Kidnapping Alert, microcontroller, SMS, CKALI.

I INTRODUCTION

Kidnapping is a global menace, which is more prevalent in countries like Mexico, Brazil, Colombia, Russia and most recently Nigeria. Findings revealed that kidnappers rake in billions of dollars yearly in Brazil. In Mexico too, it is a serious business as gangs encourage their young members to practice on pets and domestic animals so as to master the game before going for human beings [1]. The rate of kidnapping in Nigeria has risen considerably in the last decade, with Nigeria now accounting for 25% of global kidnappings. Kidnapping in Nigeria has become very prominent and according to a 1999 statistics, Nigeria was fifth in ranking to Columbia, Russia, Mexico and Brazil for ransom kidnappings and was tagged the “global capital of kidnapping” in a statement at the African Reinsurance Forum [2]. When a kidnap occurs, the reaction of security agencies now becomes a battle between life and death. Today, innovations in wireless communications have tilted the odds in favour of success than failure. Before, security agencies had nothing to rely on other than raw courage. Now the world of wireless communication has led to an entirely new way of minimizing the death rate due to kidnapping.

GPS tracking systems are used to track anyone and anything these days [3], [4], [5]. Technology has rapidly advanced in the past few years and it has become very easy for the average person to use a tracking system [4]. If you have a vehicle, then you may want to place a GPS tracking system in your car, this way if your car ever gets stolen, it can easily be located at the shortest possible time.

Kidnapping Alert System is quite a novel research area. A good example of a kidnapping alert system is the KINGXIN watch tracker made in China; it uses a GPS+GSM+GPRS wireless communication network positioning system to communicate real time location information of a kidnap victim. Most GPS trackers today are worn in hidden clothing, pockets, necklaces, armbands, or watches, where they can easily be found. One inventor has already patented an ingestible GPS tracking device [6]. Additionally, a company in Mexico, XEGA, produces implantable tracking devices to help combat kidnappings [7]. But these devices are RFID tags and would become useless if separated from an external GPS tracker [8].

The approach adopted for our research work on the belt tracking device (CKALI) is based on the provision and proper translation of location information in terms of latitude and longitude using the GPS/GPRS module connected with a microcontroller and a battery, embedded in the belt, and having the ability to effectively communicate with the Google Earth Software installed on a computer system via the GSM network, where the location information in latitude and longitude terms is decoded into actual places on the map. Another feature is that an SMS can be triggered by pressing the miniature panic button located on the belt. We adopted the use of SMS messaging because in the third world (Nigeria in particular), the attention being paid to emergency calls is grossly inadequate and this has wrecked a lot of havoc on lives and properties [9].

The Real-time GPS/GPRS/SMS belt tracker (CKALI) which is a novel device, can be worn at all times, just as a normal clothing accessory and would serve as a means of protection against kidnapping, and a kidnap victim can be located in the shortest possible time.

II MATERIALS AND METHODS

In this research work, we made use of Machine to Machine (M2M) technologies and GSM/GPS module which is a device that operates mostly under M2M platform. This is a devices that can operate over a network without human interference [9]. The system has two main parts. The first part is the tracking device which is attached to the belt. It comprises of a GPS module, a microcontroller and a GSM module. The second part of the system consists of a receiver device which we built; it receives the GPRMC messages and sends it to the monitoring workstation PC connected to the internet and running Google Earth software and Franson GpsGate client software. Franson GpsGate Software is an important tool utilized in the design of this system. It is a web-based GPS tracking software with real time view, advanced system alerts and reporting. Some of its functions include using JavaScript to gather GPS position; normalizing the GPS connection; GPS simulation and logging; sharing one GPS to several applications using virtual communication ports; connecting a GPS to Google Earth; sending GPS data over HTTP to a personal server; multiplexing and splitting NMEA streams. The version used in this research work is Franson GpsGate v2.6.0.402. This software installed on the monitoring system acquires the GPS coordinates reaching the GPS module interfaced with the receiver, logs it continually and connects dynamically with Google Earth to provide a trace of the path of the tracked kidnap victim. The core tracking functionality of the system is done by the Google Earth software. Google Earth is a virtual globe, map and geographical information program that maps the earth by the superimposition of images obtained from the satellite imagery, and is able to show all kinds of images overlaid on the surface of the earth, and is also a Web Map Service Client.

A. Belt Tracker Architecture and Signal Flow

The circuit diagram of CKALI and the receiver device used to receive the GPS coordinates on the monitoring system is as shown in fig.1. If an individual is kidnapped and he/she presses the trigger on the belt, the information flow is as shown in fig. 2.

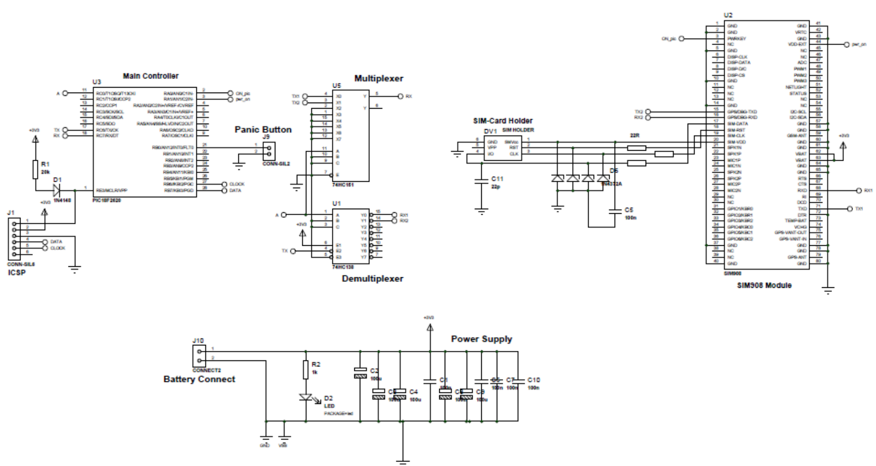


Fig. 1 (a) CKALI Circuit Design Layout

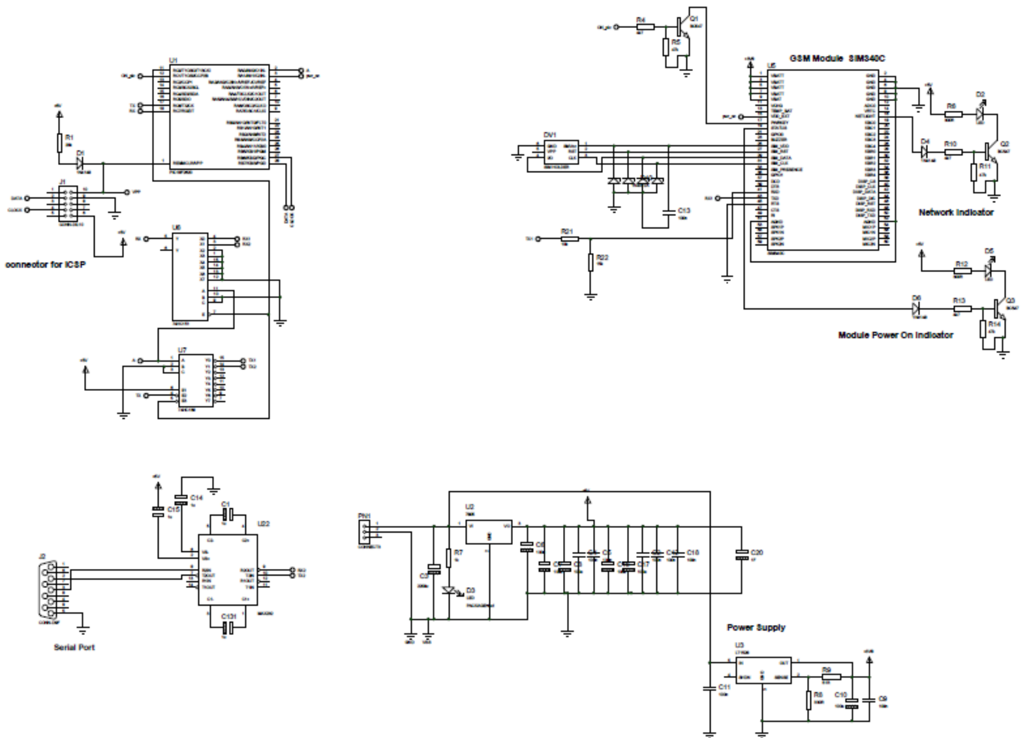


Fig. 1 (b) GPS coordinates Receiver Circuit Design Layout

The GPS/GSM module works with an installed trigger on the belt. This module is constantly being tracked by a constellation of satellites located in six orbital planes at a height of 20,200km and circle the earth every 12 hours, each plane is inclined at 55 degrees to the earth's equator and contains 4 satellites each [4]. These satellites enable the GPS receivers embedded in the belt tracker when activated by the trigger, to pinpoint the exact location, in terms of longitude and latitude, of the kidnapped victim.
In case of a kidnapping, the victim presses the trigger at the bottom of the leather enclosure of the belt, this trigger sends an activating signal to the GPS/GSM module, and the GPS module retrieves the location information in real time from the satellites in the form of longitude and latitude readings. The microcontroller processes the GPS information and extracts the desired values, this information is then sent to the monitoring computer using the GSM module (modem) by SMS which changes every 40 seconds. The pre-configured phone which is supposed to alert the security personnel for tracking is sent an SMS message "Help! I have been abducted, please start tracking". On the internet; connected monitoring computer running the Google Earth software the SMS is translated into GPS location information and real-time tracking is done, and as the kidnap victim moves, the route travelled is displayed. The Franson GpsGate Client software running in the monitoring computer acquires the GPS coordinates reaching the GSM module interfaced with the receiver, logs it continually and connects dynamically with Google Earth to provide a trace of the route of the tracked kidnapped victim. This information can then be used by security operatives to safely rescue the kidnapped victim.

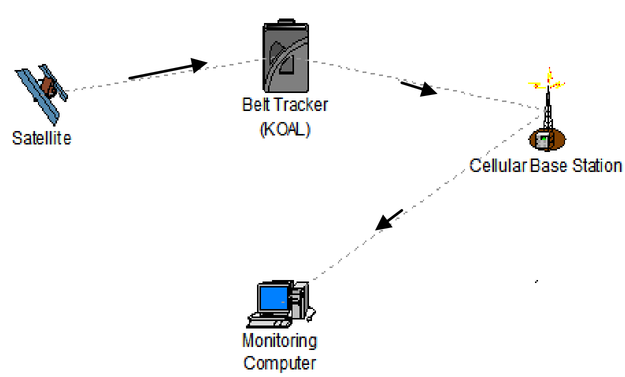


Fig.2. CKALI Signal Flow

B. CKALI Experimental Setup

CKALI was tested by setting up a system to simulate a kidnapping scenario. In order to do this, a kidnapping scenario was aranged, the belt tracker was worn at each instant by different persons scattered across a geographical area (around Ogun State in Nigeria). Each person was told to press the trigger on the belt and then to drive around with the device for twelve minutes, while we did the monitoring of the persons location and route from the monitoring computer system.

III RESULT

For each individual that wore the belt in the various locations, when the trigger on the belt was pressed, the GPS module retrieved the location information from satellites in the form of longitude and latitude real time readings. This GPS information was then processed and sent by the microcontroller as an SMS through the GSM network using the GSM module to our alert pre-configured phone and the GSM modem of our monitoring system. We were alerted by the SMS reaching the phone, the SMS message was as follows:

"HELP!, I HAVE BEEN ABDUCTED, PLEASE START TRACKING "

The GSM modem on our monitoring system also received the SMS which included the GPS coordinates of the individual wearing the belt. A total of 18 messages was sent by the belt tracker device during the 12 minutes duration, this is due to the fact that the belt tracker sends a new SMS every 40 seconds. The Longitude and Latitude readings were accurate and reflected the exact location of the individual wearing the belt, and we were also able to trace the route of the individual. The GPS coordinates as received in the monitoring system is shown in fig.3, and the route taken by the individual wearing the belt tracker as seen on the monitoring system is shown in fig.4. It was observed that the delivery of the SMS messages was network dependent, thus a reliable GSM network connection is needed to improve the effectiveness of the device, and thus greatly enhance the reliability of the platform for combating dangerous kidnap situations.

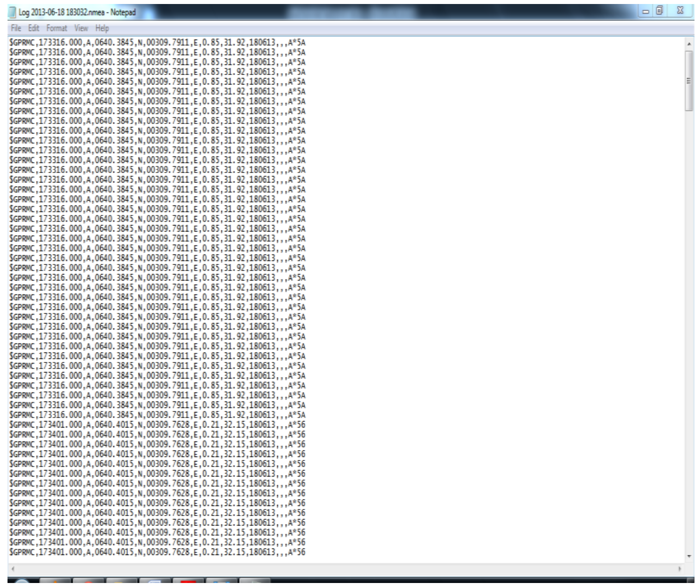


Fig 3. GPS Coordinates Received on the Monitoring System

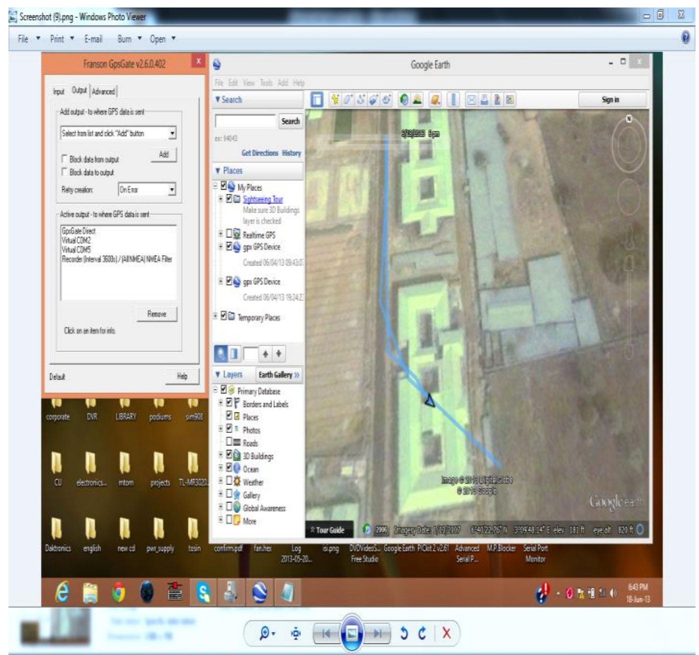
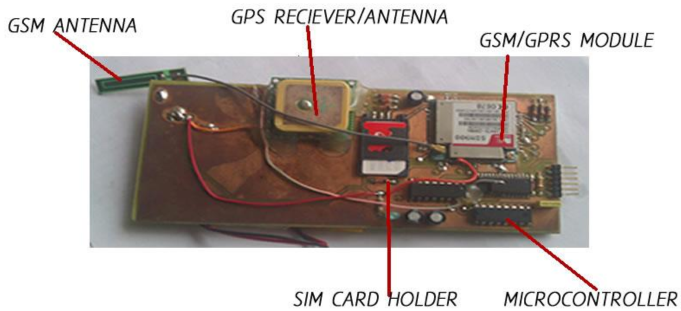
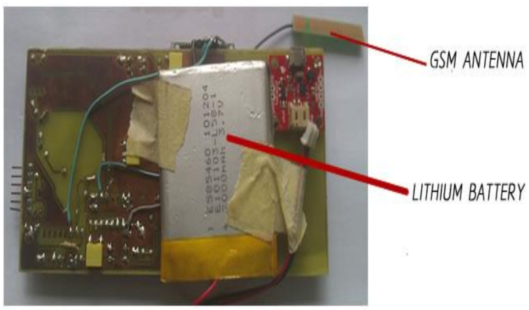


Fig. 4: Google Earth and Franson GpsGate Snapshot Showing the Live Location Route of the tracked kidnap victim

The various pictures taken while CKALI prototype was being built in the workshop and the final prototype are shown in fig.5.





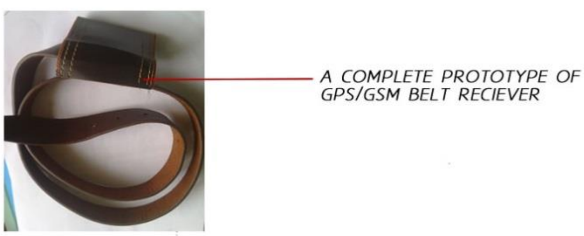


Fig.5 Prototype

IV. ACKNOWLEDGMENT

We wish to thank Mr. Ayoola Akindele for assisting us in the field test of the device, also our thanks go to M2M Technologies Nigeria Ltd for their assistance during the experiment.

V CONCLUSION

In order to develop CKALI, three main players were identified i.e. the belt incorporating a machine-to-machine (M2M) device (GPS/GSM module with a trigger activator), the mobile operator and the security monitoring organizations, where all players are joined by the geographic information network.In this research work, we have designed a platform for prompt rescue of a kidnap victim and developed a prototype and tested it. We identified the possible drawbacks and dealt with them. Therefore, the platform operates effectively and efficiently.

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Matrix Graph: A New Algebraic Graph

Dr.R.Stella Maragatham [#1], S.Peninal Dhanamani [*2]

[#] Department of Mathematics, Queen Mary's College, University of Madras,
Chennai – 04 India.
[1] rstellamar@yahoo.com

* Department of Mathematics, Siva Institute of Frontier Technology-Technical Campus,
Affiliated to Anna University,
Thiruvallur-601 103
[2] penijohn@yahoo.co.in

Abstract— In the past Four decades, the study of graph theory has grown beyond leaps and bounds. In one direction, more and more new finer concepts, for example, labelling, dominations and many such graph theoretic phenomenon have been developed. On the other hand, new planar graphs have been constructed using algebraic concepts such as groups, characters and linear transformations of vector spaces. A few examples are the Brattili diagrams [1], the Relative character graph [3], [4], [6], [7].

Keywords: Matrix Graph, Algebraic Graph, Matrix Ring, Lie Algebra, Dominations.

1. INTRODUCTION

This is the first of a series of papers of our attempt to construct yet another new finite, simple, planar graph. (These materials could form a part of the second author's Ph.D thesis, under the supervision of the first author). In this paper, we introduce a new graph called a 'matrix graph' and study some of its properties. Further deep concepts such as connectivity, tree problem, complements, dominations, etc., will be taken up in the subsequent papers.

2. Basic Concepts from Matrix Theory

We assume that all the matrix entries are complex numbers.

2.1 Definition: Let A be an $n \times n$ matrix. A complex number λ is an eigen value of A if λ is a root of the characteristic polynomial $|A - \lambda I|$
(I be an $n \times n$ identity matrix). The following basic facts may be recalled.

(i) A has n eigen values, not necessarily distinct.
(ii) A is invertible if and only if 0 is not an eigen value.
(iii) If A is invertible and $\lambda_1, \lambda_2 \dots \lambda_n$ are its eigen values, then ${}^1/_{\lambda_1}, {}^1/_{\lambda_2}, \dots, {}^1/_{\lambda_n}$ are the eigen values of A^{-1} and $\lambda_1^r, \lambda_2^r, \dots, \lambda_n^r$ are the eigen values of A^r for any positive integer r.
(iv) A satisfies the expansion of the polynomial expression $|A - \lambda I|$ in λ (Cayley-Hamilton theorem).

It is important to note that even if the entries of A are reals, the eigen values need not be real. For example, consider the matrix $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

The Characteristic polynomial is $\det\left\{\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right\} = \lambda^2 + 1$.

The roots are i, -i.

3. The Matrix Graph

Let $S = \{A_1, A_2, \dots, A_p\}$ be a collection of $n \times n$ matrices over $\mathbb{C}$. We define a graph $\Gamma_s = (V, E)$ where the vertex set V is the p matrices in S and two distinct matrices A_i and A_j of S are adjacent if and only if, A_i and A_j share a common eigen value.

If S is fixed, we sometimes write Γ for Γ_s, clearly Γ is a finite, simple, undirected planar graph.

3.1 Proposition: Let $S = \{A_1, A_2, \dots, A_p\}$ where each A_i is either an upper or lower triangular matrix, with diagonal entries$(d_{i1}, d_{i2}, \dots d_{in})$ with $d_{ik} \neq d_{jl}$ for $i \neq j$. Then Γ_s is the null graph.

Proof: Clearly the eigen values of A_i are $d_{i1}, d_{i2}, \dots d_{in}$ and those of A_j are $d_{j1}, d_{j2}, \dots d_{jn}$. By assumption $d_{ik} \neq d_{jl}, i \neq j$.

After deleting the $d_s{}'$ with the same second entries, we see that the remaining entries corresponding to A_i and $A_j (i \neq j)$ are all distinct. Hence A_i and A_j have no common eigen value, which means that the corresponding graph Γ_s has no edges.

The corresponding discussion for complete graphs will be taken up a little later. We shall first go for a criterion for adjacency of A_i and A_j.

But first we recall the following well-known result, whose proof is easy, but not trivial.

3.2 Proposition: Let A and B be two $n \times n$ matrices having a common eigen value α. Let $P_A(\lambda)$ and $P_B(\lambda)$ be the characteristic polynomials of A and B respectively. Then

$$\det P_A(B) = P_B(A) = 0.$$

Proof: Write $P_A(\lambda) = (\lambda - \alpha_1)(\lambda - \alpha_2) \dots (\lambda - \alpha_n)$ where $\alpha_1, \alpha_2 \dots \alpha_n$ are the eigen values of A. Let $\alpha_1 = \alpha$. Substituting B for λ in the above, we get, $P_A(B) = (B - \alpha_1 I)(B - \alpha_2 I) \dots (B - \alpha_n I)$.

Then

$det P_A(B) = det(B - \alpha_1).det(B - \alpha_2 I) \dots . det(B - \alpha_n I)$.

Since $\alpha = \alpha_i$ is an eigen value of B as well, $det(B - \alpha_1 I) = 0$. Hence $det_A(B) = 0$. Arguing similarly inter-changing A_i and A_j we get $det_B(A) = 0$.

3.3 Corollary: A_i and $A_j$are adjacent in the previous notation, if and only if $det_{Ai}(A_j) = det_{Aj}(A_i) = 0$.

3.4. Proposition: Let $\{b_1, b_2, \dots, b_p\}$ be distinct positive integers. Define $n \times n$ matrices $S = \{A_{\sigma_1}, A_{\sigma_2}, \dots, A_{\sigma_p}\}$ as follows:

Take $X = \{1, 2, \dots p\}$ and let $\sigma_1, \sigma_2, \dots \sigma_p$ be distinct permutations of X.

Put $A_{\sigma_i} = diag(\sigma_i(1), \sigma_i(2), \dots \sigma_i(p))$. Then the graph Γ_s is complete.

Proof: $A_{\sigma_1}, A_{\sigma_2}, \dots, A_{\sigma_p}$ are all distinct $p \times p$ matrices, but all of them have the same set of eigen values $\{\sigma_i(1), \sigma_i(2), \dots \sigma_i(p)\}$. Hence Γ_s is clearly complete.

A few more basic results stated below will be useful. For details we refer to [5].

I. Any $A \in M_n(\mathbb{C})$ (the full matrix ring of $n \times n$ matrices over $\mathbb{C}$) is similar to an upper triangular matrix.

II. A and B are unitarily similar if there exists a unitary matrix P (ie., $\bar{P}P^T = I$) such that $P^{-1}AP = B$. Then any $A \in M_n(\mathbb{C})$ is unitarily similar to an upper triangular matrix.

III. If rank A = 1, then trace A (= sum of diagonal entries) is an eigen value of A.

Since similar matrices of the same order have the same eigen values, our construction of Γ revolves around (similarity of) matrices similar to certain known forms.

3.5 Proposition: Let $S = \{A_1, A_2, \dots, A_p\}$ where rank $A_i = 1$ for each i. Then in Γ_s, A_i is adjacent to A_j if and only if trace $A_i =$ trace A_j

Proof: From III, A_i and A_j have one common eigen value (=trace). Hence they are adjacent,

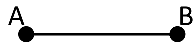
3.6 Example:

A typical example from Lie Algebras.

The simplest Lie Algebra is the usual $L = sl_2(\mathbb{C})$ the Lie algebra of matrices of trace 0.

Take, $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. Then A, B and C form a basis of L (as a vector space).

We can take S = {A, B}. (rank A = rank B = 1). Then clearly the graph Γ_s is



Next we can take $L = sl_3(\mathbb{C})$ of dimension 8 and look for a corresponding construction.

In fact, we can generalize this to the following.

3.7 Proposition. The set S of all submatrices of the basic matrices of the Lie algebra $L = s_{l+1}^{l}(\mathbb{C})$ of rank l forms a complete matrix graph.

Proof: Imitating the steps in 3.6 we see that the elements of S clearly satisfy all the required conditions: rank1 and trace 0. Hence any two vertices of S are adjacent proving the completeness of Γ_s.

We can go to matrices of the group $gl_n(\mathbb{C})$ (where trace 0 restriction is not there).

It may turn out that the resulting graphs are not complete.

3.8 Corollary: Let $S = \{A_1, A_2, \dots, A_p\}$ where each A_i is $E_{mn}(m \neq n)$ an elementary matrix then Γ_s is complete.

Proof: Trivial

3.9 Proposition:

Let $S = \{A_1, A_2, \dots, A_p\}$ where each A_i is a $n \times n$ permutation matrix. Then Γ_s is complete.

Proof: Each A_i has 1 at only one row and only one column and all other entries are 0's. If X denotes the column vector $(1,1,\dots,1)^T$ then it is easily seen that $A_i(1,1,\dots,1)^T = (1,1,\dots,1)^T$ for all i; ie., $A_iX = X$ showing that 1 is an eigen value of each A_i and hence Γ_s is complete.

3.10 Remark 1: Monomial matrices are those having a non zero entry in a unique column and row. Our question is whether the above proposition goes through for monomial matrices also. For simplicity first take $n = p = 2$.

Let $A = \begin{pmatrix} 0 & a_1 \\ a_2 & 0 \end{pmatrix}$, $B = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix}$

$|A - \lambda I| = \lambda^2 - a_1a_2$, giving $\lambda = \pm\sqrt{a_1a_2}$. $|B - \lambda I| = \lambda^2 - \lambda(b_1 + b_2) + b_1b_2$, which gives $\lambda = (b_1, b_2)$. Required adjancency condition is

$b_1 = \pm\sqrt{a_1a_2}\,, b_2 = \mp\sqrt{a_1a_2}$.

Remark 2: The graph Γ_s can be complete without any of the above conditions. For instance, the matrices A_1, A_2, A_3 given below do not have any of the conditions said above.

$$A_1 = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}, A_2 = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix}.$$

The eigen values of A_1, A_2 and A_3 are the following triplets in the same order:

(1, 1, 3), (2, 2, 3) and (1, 2, 4). Obviously Γ is complete.

It seems that the question of characterizing the completeness of Γ_s, for a given S, is not easy! In this context we have the following interesting algorithm for completeness.

4. An Algorithm

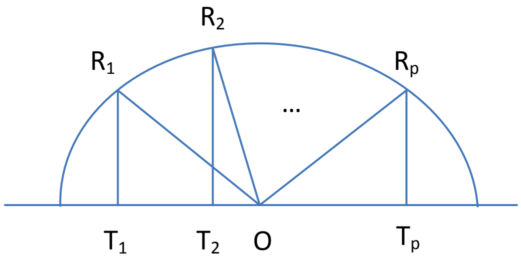
4.1 Let $S = \{A_1, A_2, \dots, A_p\}$ be a set of $n \times n$ symmetric matrices. Diagonalize each A_i to get the corresponding diagonal matrix

$D_i = diag(\alpha_{i1}, \alpha_{i2}, \dots \alpha_{in})$, $i = 1,2,\dots,p$ so that α_{ik}'s are precisely the eigen values of D_i for every i. Then,

I. If $\alpha_{ik} = \alpha_{jl}$, for every pair i, j and for some k, l then Γ_s is complete.

II. If for any given A_i there exists α_{ik} such that $\alpha_{ik} = \alpha_{jl}, i \neq j$ such that for some k and l then Γ_s is a connected graph.

We shall discuss the above in a very special case.



Consider the matrices $S = \{A_1, A_2, \dots, A_p\}$ given by $A_i = \begin{pmatrix} a_i & c_i \\ c_i & -a_i \end{pmatrix}$ where $a_i = |OT_i|$ and $C_i = |R_iT_j|$ in the above semi-circle of radius r. Then clearly, for each i,

$a_1^2 + c_1^2 = a_2^2 + c_2^2 = \cdots = a_p^2 + c_p^2 = r^2$.

Hence the eigen values of A_i are $\pm r$.This proves that the graph Γ_s is complete.

Proof is obvious.

Another interesting variation is the following. Continuing our earlier notation,

III. If a_i's and c_i's are chosen so that

$a_i^2 + c_i^2 \neq a_j^2 + c_j^2$ for any pair (i,j) then it is easily seen that the corresponding graph is the null graph. For a proof, one can easily verify that the eigen values of A_i are $\pm\sqrt{a_i^2 + c_i^2}$.

5. Unitarily similar matrices

First recall that an $n \times n$ matrix A over $\mathbb{C}$ is unitarily similar to an upper triangular matrix T. This means that there exists a unitary $n \times n$ matrix P (ie., $P\bar{P}^T = I$) and an upper triangular matrix T such that satisfying $\bar{P}^TAP = T$. Here

(P and) T need not be unique but A and T do have the same eigen values.

5.1 Definition: Let X be the collection of all matrices T satisfying the above conditions. Let S be a finite subset of X. We then get the corresponding matrix graph Γ_s. We denote this by $\Gamma_{s,A}$ and call this the **unitary matrix graph** of A. Once again Γ_s is complete.

We obtain an interesting result to get a tree.

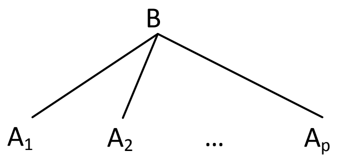
5.2 Proposition: Let $S = \{B, A_1, A_2, \dots A_p\}$ be $n \times n$ matrices such that

i) no two A_i's have the same eigen value.

ii) B has 1 as an eigen value

iii) $A_iB = BA_i$ for all i. Then Γ_s is a tree, in fact a star.

Proof: By the choice of eigen values $\{A_i\}$, no two $A_i's$ are adjacent. By a basic result from matrix theory (see [5]) eigen values of A_iB are products of eigen value of A_i and B Since 1 is an eigen value of B, if a_i is an eigen value of A_i, then a_i is an eigen value of A_iB too. Hence B and A_i are adjacent for each i. Hence Γ_s is a tree, a star.



6. Some applications.

6.1. Definition: An $n \times n$ matrix $A = (a_{ij})$ is **stochastic** if $a_{ij} \geq 0$ for all i, j and $\sum_{i=1}^{n} a_j$
for all j. It is an interesting fact that if A is stochastic, then 1 is an eigen value of A. We have the following easy

6.2 Proposition: If S is a set of p stochastic $n \times n$ matrices, Γ_s is complete.

Proof: Each A in S has 1 as an eigen value and hence any two matrices in S are adjacent.
In this way we can associate probability theory with graph theory.

6.3 Liapunov Matrix Graphs.

If A, B, C are $n \times n$ matrices over $\mathbb{C}$, an equation of the form $AX + XB = C$ is known as the **Sylvester matrix equation**.

6.4 Proposition. The Sylvester matrix equation $AX + XB = C$ has a unique solution if and only if $Spec\ A \cap -Spec\ B = \varphi$. (Spec A denotes the spectrum of A, ie, the set of all eigen values of A). For a proof we refer to [5].

A special case is the **Liapunov matrix equation** $\bar{A}^T X + XA = -I$. (*)
(It is used in a stability criterion for the linear differential equation $\frac{dt}{dx} = Ax$.) We say that the matrix A is **stable** if all its eigen values lie in the left hand half plane.

A well known result in stability theory says that A is stable if and only if there exists a positive definite solution X to the equation $\bar{A}^T X + XA = -I$. It follows that A is stable if and only if there exists a positive definite (hermitian) matrix X which is a solution of $\bar{A}^T X + XA = -I$.

6.5 Definition: Let S be a finite set of stable matrices. The **stability matrix graph** is defined as the graph Γ_s of matrix graph obtained by the (unique) solution matrices X_A for each A in S. Note that condition (*) is automatically satisfied by A and $\bar{A}^T$.

The advantage of the matrix X_A is that it is hermitian and positive definite. When n and p are large, computation part will be easier.

6.6. The Rayleigh Matrix Graph.

Let A be an $n \times n$ hermitian matrix. Let v be a nonzero vector in $\mathbb{R}^n$. The Reyleigh quotient for A at the vector v is the real number $\rho(v) = \frac{\bar{v}^T A v}{\bar{v}^T v}$. (Since A is Hermitian, $\rho(v)$ must be real.) If A is real symmetric then $\rho(v) = \frac{v^T A v}{v^T v}$.

6.7 Theorem: Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigen values of an $n \times n$ hermitian (or real symmetric) matrix A written so that $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Then for each $v \neq 0, \lambda_1 \leq \rho(v) \leq \lambda_n$. It also follows that if v_i is an eigen vector for λ_i then $\rho(v_i) = \lambda_i$ for each i.

In particular $\lambda_n = \rho(v_n) = Max\{v, v \neq 0\}$ and $\lambda_1 = \rho(v_1) = Min\{v, v \neq 0\}$ .The **Rayleigh matrix graph** $\Gamma_{s,A}$, is defined as follows:
Let A be a hermitian (a real symmetric) matrix such that $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Choose nonzero vectors, $v_1, v_2, \ldots, v_p$ such that
$\rho(v_1) < \rho(v_2) < \cdots < \rho(v_p).\text{Then } \rho(v_i) = \lambda_i$
for each i (by the above theorem).
Let $\| v \| = (x_1^2 + \cdots + x_n^2)^{1/2}$
$\big(v = (x_1, x_2, \ldots, x_n)\big)$ be the Euclidean norm on $\mathbb{R}^n$.

Let $S = (w_1, w_2, \ldots, w_p)$ be 'p' n-vectors such that $\| w_1 \| < \| w_2 \| < \cdots < \| w_p \|$. The vectors in S form the vertices of our new graph called the Rayleigh matrix graph of A relative to S and with two vectors w_r and w_s are adjacent $(r < s)$ if and only if there exist a pair (i, j) such that $\lambda_i \leq \| w_r \| \text{ and } \| w_s \| \leq \lambda_j$. This graph could be a tool in approximation and perturbation theory.

7.Neighbourhood Matrix Graph.

We now generalize our matrix graph using little bit of analysis.

7.1 Take S as before. Then Γ_s absolute graph is the graph having adjacency if and only if A_i and A_j have at least one common value in absolute value.

7.2.Definition: Let $\delta \geq 0$ be a real number.
Let $S = \{A_1, A_2, \ldots, A_p\}$ be 'p' $n \times n$ complex matrices. Then the δ-neighbourhood matrix graph $\Gamma_{\delta,S}$ of S relative to δ is the graph (V,E) where V, as before denotes the matrices $\{A_i\}$ and two A_i, A_j $(i \neq j)$ are adjacent if and only if $A_i$has one eigen value α_i and $A_j$has one eigen value α_j such that $|\alpha_i - \alpha_j| \leq \delta$.This definitely generalizes absolute matrix graphs by taking $\delta = 0$.

Absolute graph and neighbourhood graphs vastly generalized and lead us into real time application situations. First, due to matlab techniques,we can actually locate eigen values of a given (real) $n \times n$ matrix A . Then one can approximate eigen values of A by means of another matrix B all whose entries are 'close' enough to those of A in a neighbourhood sense, ie., by a famous 'continuity theorem' [5] which says that if entries of A and the corresponding entries of B are 'arbitrarily close', then the eigen values of A and B are 'sufficiently close'. Finally, coupling this with the 'Greshgorin discs' $D_i \left(= Z \in \mathbb{C} / |z - a_{ii}| \leq \sum_{\substack{i=1 \\ j \neq 1}}^{n} |a_{ij}| \right)$ one can further reach deeper results.

CONCLUSION

In conclusion, we say that the construction of the matrix graph of a finite collection of matrices of the same order is new and we hope that we can continue to analyse further deep graph theoretic aspects such as connectivity, domination, etc., of this graph. Also neighbourhood matrix graphs could vastly be seen in the application context, using matlab techniques, including QR-algorithms.

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