Optimal Parameter Inference Method for Effective Design of Synchronous Reluctance Machines

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***Abstract*— This paper presents a method for evaluating, both qualitatively and quantitatively, the effects of specific rotor design parameters on the performance of a synchronous reluctance machine. The method uses multi-factor experimental design, with Analysis of Variance (ANOVA), and Finite Element Analysis (FEA) to determine the optimal rotor design parameter according to a specific objective. Using this method, two factors - rotor flux barrier pitch angle and barrier width - are selected at simultaneously varied levels for assessment with the aim of analyzing the response variables, which are, the average torque and torque ripple. Results from the investigation show that the influence of the rotor flux barrier pitch angle on the torque ripple is more statistically significant than the influence of the barrier width. However, the effect of the barrier width on the average torque is more significant.**

***Keywords*— Analysis of Variance, Average Torque, Finite Element Analysis, Rotor Design, Synchronous Reluctance machine, Torque Ripple**

I. Introduction

Synchronous reluctance machines (SynRMs) have high torque densities, fault-tolerant capabilities, high efficiencies, low rotor inertias and relatively uncomplicated controllability in comparison to induction machines [1, 2, 3, 4]. Despite several advantages, one of the common problems with SynRMs is the high content of torque ripple [4]. This is due to the interaction between spatial harmonics of the electrical loading and the rotor anisotropy which causes a high torque ripple that is intolerable in most of applications [5, 6, 7].

Therefore, much emphasis has been placed on mitigating torque ripple in SynRMs. It is reported in [8] that skewing of the rotor by a stator-slot pitch can reduce the torque ripple caused by slot harmonics, but also decreases the average torque by ± 2 %. In [9] it has been shown that a reduction in torque ripple can be achieved by means of a suitable choice of number of flux-barriers with respect to the number stator slots per pole pair. Torque ripple reduction for SynRMs using asymmetric flux barrier has been reported in [3]. In [10], asymmetric flux barrier angles and a flipped rotor structure have been presented as an approach of torque ripple reduction without loss in the average torque.

As mentioned in above literature, there is a vast number of different rotor geometries that are possible, specifically due to the variations in flux barriers that are available. Modifications to the number, arrangement and dimensions of the flux barriers yield changes in the performance of the machine. This makes the design process quite expensive as optimization is usually required to determine a suitable geometry for best performance. This paper presents a method for evaluating, both qualitatively and quantitatively, the effects of specific rotor design parameters on the performance of a SynRM. Multifactor ANOVA is used together with FEA to assist with analysis and selection of optimal parameter/s during the design process. The proposed method is intended to be used in conjunction with optimization to enhance FEA-based design.

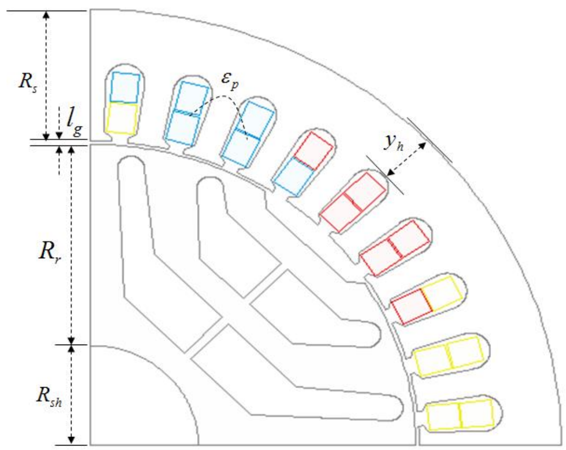
II. Machines Specifications and Design Variables

The specifications of a traditional 5.5 kW, three-phase, 50-Hz, induction machine are used to design and model the SynRM. Table I gives the general specifications and design variables of the machine.

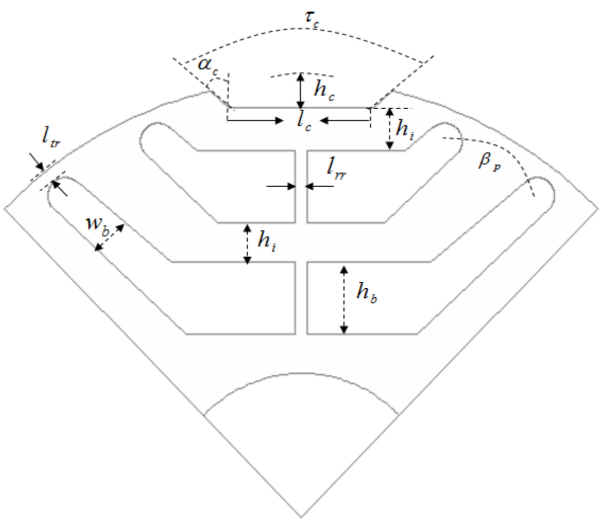
TABLE I: Design Specifications and Design Variables

Description	Values
Stator slot Pitch ε_p	10º*mech*
Airgap length l_g	0.88 *mm*
Barrier height h_b	12 *mm*
Barrier width w_b	7, 8, 9, & 10 (*mm*)
Barrier Pitch β_p	10º, 12.5º, 15º and 10º *mech*
Cut-off angle α_c	40º *mech*
Cut-off pitch τ_c	25º *mech*
Cut-off height h_c	4.8 *mm*
Iron width h_i	6.5 mm
Stack length	160.00 *mm*
Number of barriers per pole	2
Number of pole pairs	2
Number of stator slots	36
Rotor radius R_r	48.80 *mm*
Stator radius R_s	31.62 *mm*
Shaft radius R_{sh}	24.00 *mm*
Yoke height y_h	12.87 *mm*
Barrier end radius b_{er}	2.80 mm
Radial rib length l_{rr}	2 *mm*
Tangential rib length l_{tr}	2 *mm*

It is widely known that in SynRMs the increase of the reluctance on the *q*-axis reduces the magnetizing inductance and further increases the saliency ratio and the inductances difference. The air to iron ratio on the *q*-axis should be taken into consideration for SynRM optimal performance. In this paper the SynRM has two flux barriers and three iron segments per pole. The air to iron ratio is 1.487. Fig.1(a), and Fig.1(b) show the cross-sections of main machines dimensions and rotor design specifications respectively.



(a)



(b)

Fig.1. (a) main machine dimensions, (b) rotor design specifications

The pole pitch to airgap length ratio is 188; such a ratio is acceptable. The airgap length is kept as low as possible, in order to increase the torque, because a greater ratio provides greater saliency [12].

The rotor segments are interconnected to each other using radial bridges in the q-axis and tangential bridges near the airgap as shown in Fig.1. The tangential bridges hold the whole structure of the lamination together, while the radial bridges support the structure of the rotor and they are much needed especially when the SynRM is used for high speed applications. These magnetic bridges are the weakest parts in SynRM rotor structure, since centrifugal forces are concentrated locally in the bridges. In addition, to be vulnerable to mechanical forces, the bridges will be saturated by the q-axis MMF during normal operation, and directly causes a torque reduction [13].

The radial bridge width l_{rr} is dependent on the mechanical limit, and the effect of radial bridges width on the saliency and inductances difference of basic SynRMs are well presented [14]. The magnetic bridges that are chosen should be thick enough to withstand the mechanical stress caused by centrifugal forces and torque. On the other hand, thinner tangential and radial bridges will result in a better electromagnetic performance. A thinner tangential bridge decreases the leakage flux as well as increases the airgap flux density due to an increase in saturation of the thinner bridges.

III. Background

The use of multi-objective optimisation algorithms together with Finite Element Method (FEM) can be time consuming, computationally expensive and does not present qualitative information regarding the influence of the input parameters on the performance variable being studied. In other words, such optimisation only yields numerical values for the design parameters. These results still need to be analysed and evaluated afterwards to assess the overall design, and to perceptively stipulate tolerances required for the manufacturing process. ANOVA is typically utilised in the experimental design but can offer analytical benefits to the design process. Multifactor ANOVA is used here to investigate effects of the flux barrier width and pitch angle parameters on the output torque and its ripple contents of a SynRM.

IV. Finite Element Analysis and ANOVA Design

A. *Finite Element Analysis*

Fig.2. shows the magnetic flux density distributions for the SynRM with βp = 15º mech, when the d-axis current i_d =8 A and q-axis current i_q = 8 A.

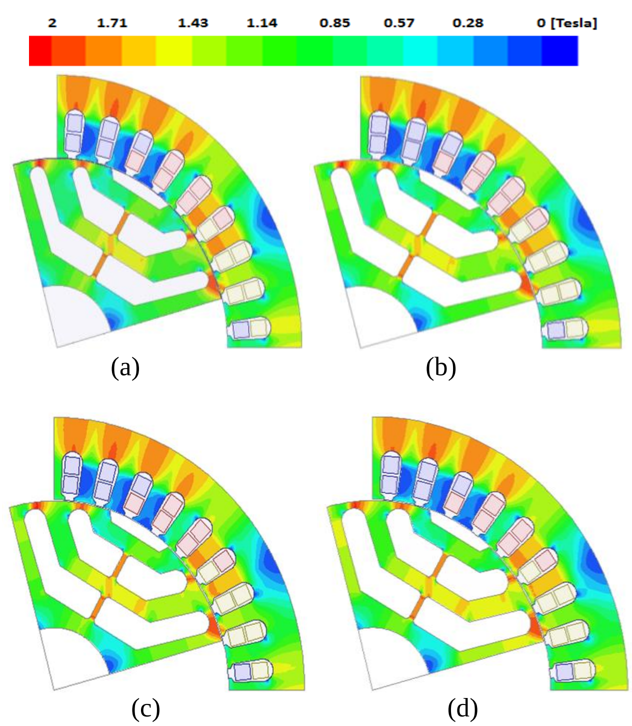


Fig. 2. Flux density distribution for the SynRM with β_p of 15º mech, (a) $\boldsymbol{w_b}$ =7 mm, (b) $\boldsymbol{w_b}$ = 8 mm, (c) $\boldsymbol{w_b}$ = 9 mm, (d) $\boldsymbol{w_b}$ =10mm

The FEA is carried out at constant speed and frequency of 1500-rpm and 50-Hz respectively. The three-phase double layer lap windings are chorded by one slot and excited by 3-phase sinusoidal currents. It should be noted that saturation and stator slotting are not disregarded in the FEA results. The models were run at current space phasor angle of 45° *elect.*

Observing from Fig.2, it is clear that the radial and tangential bridges are highly saturated in all four cases. The distribution of the flux density in the *d-axis* rotor iron parts varies with change in the barrier width. The FEA results indicate that by increasing the barrier width, the flux density in the *d-axis* rotor iron parts increases as well.

B. Analysis of Variance Design

There are numerous design configurations that are possible through adjustment of geometrical parameters of the rotor. The number of flux barriers and dimensions thereof influence the performance of the machine. The parameters selected for this study are the flux barrier width (w_b) and the pitch angle (β_p) The response variables of interest in the presented study are the torque and its ripple contents. Analysis of Variance (ANOVA) is used in experimental design to determine what factors affect a response variable. In the presented methodology, ANOVA is used to determine the significance of specific geometrical factors on each of the response variables of interest. Additionally, this method enables the pitch angle and barrier width. The following sets of hypotheses – i.e. null and alternative - are being testing with the use of a multifactor model where multiple factors may be varied simultaneously in order to analyse interaction effects. The multifactor model presented here consists of two factors – i.e. barrier presented ANOVA. The three sets of hypotheses are given as:

1. H_{01} - Means for each level of barrier width are equal.

   H_{A1} - Means for each level of barrier width race are different.
2. H_{02} - Means for each level of barrier pitch angle are equal.

   H_{A2} - Means for each level of barrier pitch angle are different.
3. H_{03} - There is no significant interaction between the barrier width and pitch angle.

   H_{A3} - There is significant interaction between the barrier width and pitch angle.

The third set of hypotheses is tested through replication of observations. The levels of each of the factors are firstly selected. Four levels are selected for w_b and β_p. The first (w_b) is varied as a function of the barrier vertical height (h_b) in order to maintain the iron to insulation ratio in both *q*-and *d-axis* within an acceptable margin. The second (β_p) is varied as a function of the stator slot pitch (α_s) as the torque ripple production in SynRMs is also due mainly to slot harmonics caused by the stator slotting. The variation Δw_b is from 7 *mm* to 10 *mm* with 1 *mm* increments, and elsewhere, the variation $\Delta\beta_p$ is from 10° to 17.5° at 2.5° *mech* increments. Δw_b and $\Delta\beta_p$ are expressed as in (1) and (2) respectively.

$$\Delta w_b = \frac{10h_b}{8} \leq w_b \geq \frac{7h_b}{8} \tag{1}$$

$$\Delta\beta_p = \alpha_s \leq \beta_p \geq \frac{\alpha_s}{4} \tag{2}$$

Using these parameters, 16 separate (4 barrier widths by 4 barrier pitch angles) FE models are constructed. For replication purposes, a subroutine was implemented to randomly vary the excitation by ±5% in order to mimic the randomness that is inherently achieved under experimental conditions. Each of the 16 models is simulated with a fixed current value to conduct ANOVA without replication. The models are then simulated, with randomly generated currents, to conduct ANOVA with replication. A balanced design is used resulting in 16 responses for the case without replication and 64 responses for the case with replication. The overview of the parameter evaluation methodology is well illustrated in Fig.3.

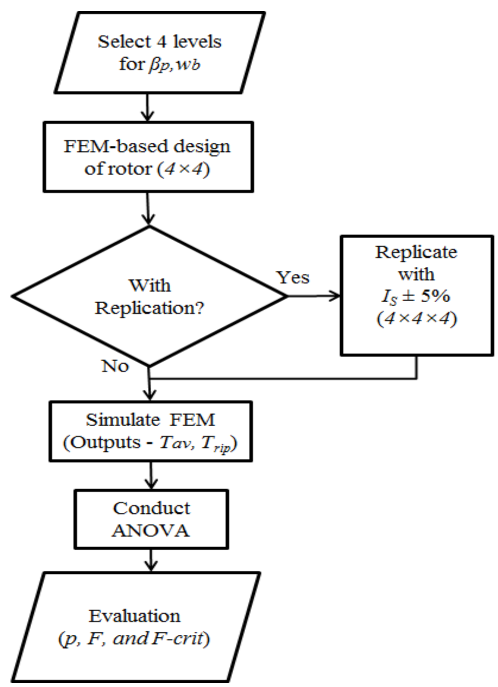


Fig.3: Overview of parameter evaluation methodology

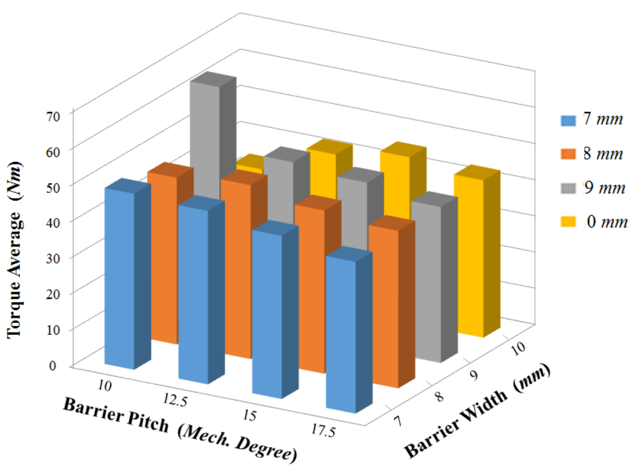
In a two-way ANOVA model, a particular combination of levels is called a treatment or cell [11]. In this case, there are 16 cells (4 by 4 levels). The two-way ANOVA model is given by

$$Y_{ijk} = \mu + \beta_{pi} + W_{bj} + \left(\beta_p w_b\right)_{ij} + \varepsilon_{ijk} \tag{4}$$

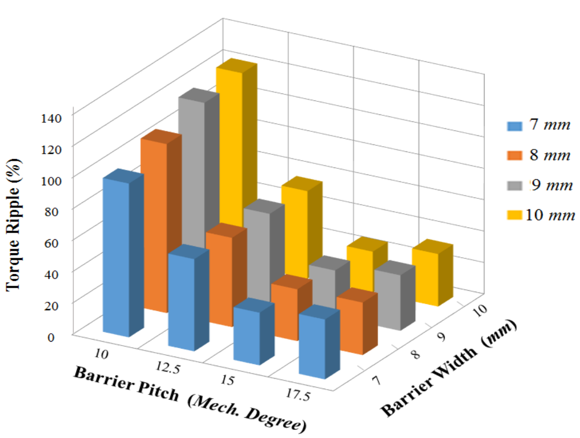
Where i = 1,.., 4 levels of factor β_p, j = 1,.., 4 levels of factor w_b, Y_{ijk} is the k^{th} response at level combination ij, μ is the grand mean, k = 1,..,4 observations per cell, β_p and w_b terms represent the main effects of each of the factors, $(\beta_p \quad w_b)_{ij}$ represents the interaction effect, and ε_{ijk} is the error. It should be noted that the interaction effects term does not feature in an additive model – i.e. without replication.

V. Results

The parameter evaluation methodology is carried out twice for this investigation. The first iteration through the methodology does not utilise replication of observations for each treatment and therefore the ANOVA does not account the interaction effects between the factors. The simulation results for the average torque and torque ripple responses for each treatment are given in Fig. 4 (a) and Fig. 4 (b), respectively.



(a)



(b)

Fig. 4: Responses for each treatment of w_b and β_p, (a) Average torque, (b) Torque ripple

The results indicate that the flux barrier width and pitch angle have a more noticeable effect on the torque ripple in comparison to the average torque. Furthermore, there is no apparent pattern to the behaviour of the responses relative to the treatment. Thus, the two-factor ANOVA is used here in order to obtain more information about the effects of these factors on the response variables. Tables II and III give summaries of the main results obtained from the ANOVA tests performed without replication for the average torque and torque ripple respectively, where *SS* is the sum of squares, *df* is the degrees of freedom, *MS* is the mean squares, *F* is the ratio of between- and within-group variance, *p-value* is the probability of obtaining an *F-value* (or more extreme) under the null hypothesis, and *F-crit* is the Critical value of *F*-distribution.

TABLE II: Average Torque as Response Variable-Without Replication

Source of Variation	ANOVA Model Parameters					
	SS	***df***	***MS***	***F***	***p-value***	***F-crit***
Rows	146	3	48.75	1.35	0.31	3.86
Columns	59	3	19.64	0.54	0.67	3.86
Error	326	9	36.22			
Total	531	15				

TABLE III: Torque Ripple as Response Variable –Without Replication

Source of Variation	ANOVA Model Parameters					
	SS	***df***	***MS***	***F***	***p-value***	***F-crit***
Rows	22.81	3	7.27	0.17	0.92	3.86
Columns	16584	3	5528	127	1E-7	3.86
Error	390	9	43.44			
Total	16997	15				

A significance level of 0.05, corresponding to a 95% confidence interval, was used for the ANOVA tests. The *p*-value is below the significance level for the (β_p) factor in Table III and below the significance level for the (w_b) factor in Table II. This implies that the null hypothesis (columns) may be rejected for the torque ripple and the null hypothesis (rows) may be rejected for the average torque. Simply put, the *p*-values indicate that the effects of the variation in barrier pitch angle are statistically significant on the torque ripple, and the effects of the barrier width are statistically significant on the torque.

Additionally, these *p*-values also indicate that the opposite is not true – i.e. effects of modifying the barrier pitch angle and barrier width are not statistically significant on the average torque and torque ripple responses respectively. The resulting *F*-statistics principally confirm these results however the *F*-crit value for the average torque response is relatively larger for both factors. The second iteration of the parameter evaluation methodology is carried out with replication of observations for each treatment in order to study the interaction effects between factors. As previously mentioned, each treatment is repeated by randomly varying the current excitation. Results for the two-factor ANOVA test performed with replication are given in Tables IV and V for the torque and torque ripple respectively.

TABLE IV: AVERAGE TORQUE AS RESPONSE VARIABLE-WITH REPLICATION

Source of Variation	ANOVA Model Parameters					
	SS	***df***	***MS***	***F***	***p-value***	***F-crit***
Rows	611	3	204	106	4E-21	2.8
Columns	202	3	67.42	34.93	3E-12	28
Error	1317	9	146	75.81	3E-25	2
Interaction	92.65	48	1.93			
Total	2223	63				

TABLE V: TORQUE RIPPLE AS RESPONSE VARIABLE-WITH REPLICATION

Source of Variation	ANOVA Model Parameters					
	SS	***df***	***MS***	***F***	***p-value***	***F-crit***
Rows	99.53	3	38.18	48.92	1E-14	2.8
Columns	6.5E4	3	2.2E4	3.2EA	3E-79	2.8
Error	1685	9	187	276	3E-38	2
Interaction	32.55	48	0.68			
Total	6.7E4	63				

These results follow a similar pattern to the first iteration, however the *F*-statistics and *p*-values indicate that the effects of both factors are statistically significant in the case of both response variables. The smaller *p*-values and larger *F*-statistics indicate greater statistical significance of the interaction effect. Furthermore, the interaction effects between factors are highly significant for the torque ripple. This means that the relationship between these two factors cannot be assumed as independent when analysing or optimising the torque ripple. Fig. 5 and Fig. 6 show the predominant 6th and 18th torque harmonic orders respectively. The difference in torque harmonic magnitudes is very significant for the 18th components, which are caused by the slotting effects. It is observed that the magnitudes of the 18th torque harmonics drop with the significance level for the (β_p) factor. The significance level for the (w_b) factor has less impact on the 18th harmonic order, but could be significant on the 6th torque harmonics, mainly caused by saturation of the rotor iron in the *d*-axis.

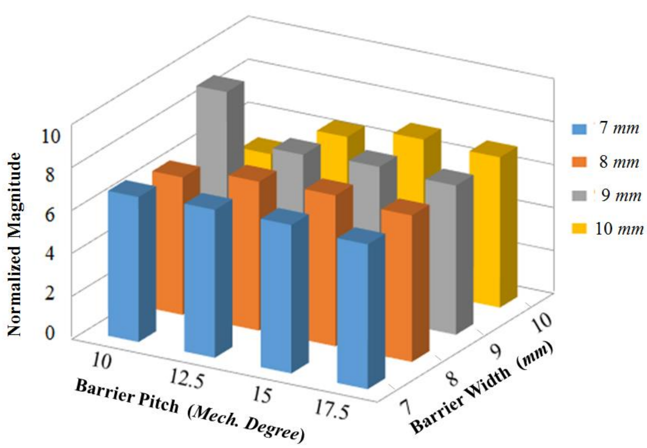
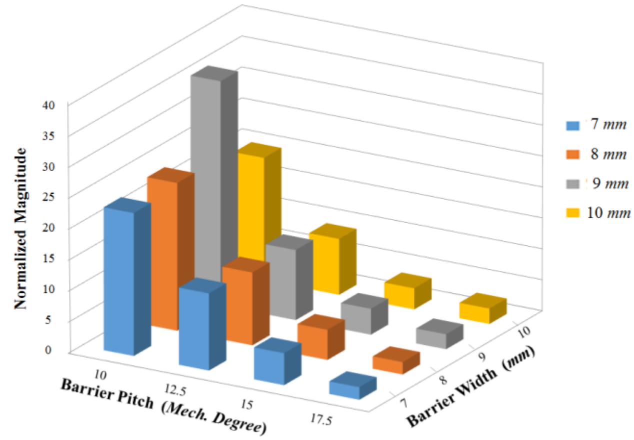


Fig. 5: The 6th torque harmonic components



(b)

Fig. 6: The 18th torque harmonic components

VI. CONCLUSION

The ANOVA-FEA based methodology presents some key qualitative findings regarding the rotor parameters of the synchronous reluctance machine. Although there is merit in contemporary optimisation methods used during the design process – this only provides quantitative results. For practical purposes, a qualitative method is required to better understand the interaction effects of parameters. The presented methodology is intended to assist with optimisation by informing decisions regarding the practical trade-offs during the design process. Results from this specific investigation shows that the interaction effects of the barrier pitch angle and width are statistically significant for both the average torque and torque ripple responses. Furthermore, the torque ripple is affected more significantly by variations in the flux barrier pitch angle, while the average torque is more significantly affected by the variations in the barrier width.

Furthermore, the results evidenced that the magnitudes of the 18th torque harmonics caused by slotting effects drop with the significance level for the barrier pitch factor. The significance level for the barrier width factor has less impact on the 18th harmonic order, but could be significant on the 6th torque harmonics, mainly caused by saturation of the rotor iron in the *d*-axis.

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