# An Access Optimization Approach for FFH-OCDMA System's Fiber Bragg Gratings Encoder 

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#### Abstract

This paper suggests an adaptive 2-D Optical CDMA coding system based on one-coincidence frequency hopping (OCFH) code combined with an optical orthogonal code (OOC) in the format OCFH/OOC, suitable for the fast frequency hopping optical code division multiple access (FFH-OCDMA) channel, encoded by the Bragg gratings encoder with an aim to optimize the access network in terms of number of users and transmitted power. As wavelength hopping (WH) code, the OCFH code is herein adapted to the constraints of the encoder: the Bragg gratings chain put on the optical fiber. As the time spreading (TS) code, the code length bounds of the OOC and its new extended sequence derived from the introduction of run length constraints are given. The multiple access interference (MAI) of the resulting combined sequence is evaluated based on the number of coincidences and a performance comparison is made between the new congruence class of $(9,3,1)$-OOC obtained and the previous well-known congruence class of $(7,3,1)$-OOC.

Index Terms-Bragg gratings, FFH-OCDMA, multiaccess communication, one-coincidence frequency-hopping code, spread spectrum communication


## I. Introduction

With its bandwidth of about 15 THz , the optical fiber communication can reach a transmission rate of the order of $\mathrm{Tb} / \mathrm{s}$ that is difficult to match by any other medium. Bringing that transmission rate to the access network, via fiber to the home (FTTH) and fiber to the curb (FTTC), will allow the backbone to run at its full capacity. However, the backbone is characterized by at least one interconnection and therefore an end-user access problem.

Many techniques have been developed to address the issue of users' access. From TDMA, through WDMA and up to CDMA, the aim is the same: share resources among multiple users. Since the first two techniques are the particular cases of the latter, this paper focuses on CDMA in an optical network. In general, optical CDMA (OCDMA) is one of the promising technologies that is able of implementing optical access networks, optical signal multiplexing and switching in backbone networks with higher speed and efficient data transmission.

[^0]The superposition of signals at the network access categorizes OCDMA into coherent or incoherent (uses unipolar codes). Incoherent OCDMA in terms of encoding/decoding is categorized by time spreading codes, spectral amplitude codes (bit " 1 " in the sequence when the frequency is present at the required time interval) and wavelength hopping/time spreading codes (WH/TS). The latter scheme is a technique that allows multiple access by allocating a code sequence to a transmitterreceiver pair, thus a sequence that is achieved in the time and frequency domains that identifies the channel.

If the WH/TS is suitable for an FFH-OCDMA, the sequence must be designed or chosen with the required robustness to resist impairments caused by the increased number of users at a rate of $\mathrm{Tb} / \mathrm{s}$. To achieve such a sequence, the encoder/decoder must be able to tune as fast as possible at different frequencies and to introduce required delays to avoid inter-pulse overlapping. The Bragg gratings is therefore that encoder which yields a 2-D OCDMA coding: one-coincidence frequency hopping code/optical orthogonal code (OCFHC/OOC) that can match the required transmission speed and most importantly, the amount of users in terms of cardinality, and the acceptable power. This paper optimizes both resources, that is to maximize the number of users while minimizing the required power.

The cardinality issue was raised by Yin and Richardson [1] where it was found that the optimal cardinality for such a scheme is given by $\leq \frac{(Q(Q-1))}{L}$ [2] for OCFH, where $Q$ is the number of frequencies available and $L$ is the OCFH sequence length, while OOC was optimal if $\left\lfloor\frac{n-1}{w(w-1)}\right\rfloor[3]$ according to Johnson's bounds, with $n$ being the OOC sequence length and $w$ its Hamming weight. According to [1], this has resulted in an OCFH code constructed in a Galois field $G F\left(p^{k}\right)$ where $p$ is a prime number, with cardinality per OOC given by $|N|=$ $p^{2 k}\left\lfloor\frac{n-1}{w(w-1)}\right\rfloor$. But the problem is that with the Bragg gratings encoder, when a frequency is used, it is no longer available for the sequence. This results in a considerable reduction of the cardinality, which this paper addresses by adapting the results of [1] to the constraints of the medium.

Thereafter, increasing the number of users raises the issue
of increased multiple access interference (MAI) and power reduction. This paper suggests an optimization of the access network, leading to a trade-off between maximizing resources available while minimizing the MAI.

This paper is organized as follows: we start in Section II by identifying OCDMA and the encoder model, and in Section III outlining the constraints of the medium and consequently the ones over the codes which motivates our code selection. In Section IV, we present the one-coincidence code construction as it is proposed, but adapt it to the constraints of the medium, while Section V introduces the run-length constraints in the OOC construction. Section VI gives the performance analysis in terms of cardinality and signal over MAI noise ratio.

## II. Optical Code Division Multiple Access

In optical code division multiple access (OCDMA) the sequence called code can be achieved in time, frequency or both domains. The superposition of the resulting signals classifies the sequence as coherent or incoherent [4].

## A. Incoherent OCDMA

Because of the simplicity and the cost of the scheme, only the incoherent sequences are outlined through the following different techniques [5]:

- Direct sequence CDMA (DS-CDMA) is a time domain encoding technique that uses direct detection in its incoherent scheme. The direct detection means amplitude modulation, hence the so-called unipolar code. In this list, we can outline the optical orthogonal code (OOC) and the prime sequences (PS) which are the main family of DS codes. The codes are obtained by splitting the incoming signal and each line is delayed by an integer multiple of the time interval to avoid overlapping. This arrangement is the code sequence.
- Frequency encoding CDMA (FE-CDMA) [6], as its name implies, is intended to achieve a sequence in the frequency domain. To achieve a comb which is the code sequence, there is a need of equipment capable of manipulating the signal spectrum. This is made possible by the Bragg gratings.


## B. System Model

The system is a Bragg gratings encoder/decoder which is made up of gratings [7], a chain of frequencies is delayed


Fig. 1. Bragg gratings encoder
by a time interval, manipulated by the selector reflecting out the frequency required (Fig. 1). The signal reflected depends entirely on the transmission rate and the distance between two gratings. For that distance close to zero the propagating pulses overlap, the resulting pulses are one-like and the signal is 1bit time axis (chip). The aim being to achieve a sequence in time and frequency, the encoder must be able to select a frequency among a set of tunable frequencies well-spaced in time. The major challenge with such a system is to be able to tune frequencies as fast as the required speed of the network. A data of rate $R_{D}=\frac{1}{T_{d}}$ is tuned at a subset of frequencies taken from the set of $Q$ frequencies, $\left\{\lambda_{1}, \lambda_{2}, \ldots, \lambda_{Q}\right\}$. The propagation of the resulting signals is such that at most one frequency is present in a train of pulses, each of duration $\frac{T_{d}}{Q}$, yielding a transmission rate of $R_{T}=\frac{1}{(Q-1) T_{c}}$, where $T_{c}$ is the chip duration related to the refraction index $n_{0}$ of the fiber and the distance between two gratings $L_{g}=\frac{c T_{c}}{2 n_{0}}$, with $c$ the speed of light [8]. The transmission rate is therefore the speed of the system hopping from one frequency to another. One such fast frequency tuner is the Bragg gratings encoder/decoder.

## III. FFH-OCDMA System Constraints

## A. Transmission Rate

Studies conducted by Chen et al. [9] showed that when the transmission rate increases, the source emitted power decreases, hence the need for the amplified spontaneous emission (ASE) technique to avoid interference by keeping the signal level high.

Also, the number of subscribers is proportional to the number of frequencies available in the chain and the data rate is inversely proportional to the same spreading length, as a consequence, the larger the number of users, the higher the multiple access interference (MAI) becomes, hence the trend to keep the number of users small. Furthermore, the fixed number of gratings reduces the system to constant Hamming weight code. To overcome these limits, a 2-D solution has been proposed in [1], in the format of wavelength hopping/time sequences (WH/TS).
This paper follows the same suggestion but with some particularities in the time sequence that comply with the distance between gratings. The time spacing between successive reflections from the frequencies of the Bragg gratings chain, as mentioned above, results in the same number of chips that propagate along the fiber. If that time spacing is small, one can experience interference and when it is larger than the transmission rate, we can experience overlapping processes (a data chip will still be in the encoding process while another chip's processing is starting). This requires a construction method that manipulates the sequence with the aim to achieve a particular pattern that fits the constraints of space between gratings within the bounds of the code length. In this paper we are therefore suggesting an extension of the code length with the introduction of run-length constraints as solution to minimize the MAI while maximizing the number of users taking into account the constraints imposed by the optical fiber and the encoder over the frequency and time sequences.

## B. Code Constraints

The code selection is of utmost importance with regard to the physical constraints of the medium as the performance of the system relies on it.

In the encoder (Bragg gratings chain), a frequency cannot be reflected more than once. The number of frequencies available depends on the tuning capability of the Bragg grating. The code length is the number of Bragg gratings in the chain, which also determines the transmission rate; and the longer the codeword the better the performance. Hence the following code characteristics in terms of frequency are of interest:

- A frequency is present in a code word only once. Once reflected by a grating, the related frequency becomes unavailable.
- Since the system can only reflect one frequency at a time, we need a frequency hopping code sequence with nonrepetitive frequencies.
- The code cannot be circular since the Bragg gratings selector is not able to respect the required modulo structure.
Furthermore, because of this limit on the WH's flexibility, the optimization of the cardinality can only be achieved through the TS part of the WH/TS scheme. The first idea with regard to the cardinality is to extend the code length. However, this extension can only be possible within the bounds of the code length if some properties that we will outline later in this paper have been satisfied. Therefore, the code to choose must be flexible enough for such an operation, extension that allows the introduction of run length constraints which are also given herein. The resulting extension of the code length has a drawback: the increase in number of users, resulting in an increase of MAI in the superposed signal.


## C. Superposition Condition of Received Signals

Assume pulse signals and that the receiver with an adaptive filter outputs the $i$-th user's information, since the detector has to adapt to the optical fiber constraints and the access network philosophy. According to the latter, all users have to receive and send asynchronously. Hence, good detection is required. A sequence from the $i$-th user is

$$
s_{i}=\sum_{f=1}^{Q} \sum_{n=1}^{N} c_{i}(f, n) p\left(t-n T_{c}\right)
$$

where

$$
c_{i}(f, n)= \begin{cases}1, & \text { if } f \text { is present at } n T_{c} \\ 0, & \text { otherwise }\end{cases}
$$

and $p(t)$ is the impulse shape of duration $T_{c}=\frac{T_{d}}{N}$. The received signal is such that $r(t)=\sum_{k=1}^{K} d_{k} s_{k}\left(t-\tau_{k}\right)$ where $d_{k}$ is the user $k$ transmitted information value and $\tau_{k}$ is the
delay experienced. The adaptive detector's filter picks up the $i$-th user signal and outputs:

$$
\begin{aligned}
y(t) & =\int_{0}^{T_{c}} s_{i}(t) r(t) d t \\
& =\int_{0}^{T_{c}} \sum_{k=1}^{K-i} d_{k} s\left(t-\tau_{k}\right) s_{i}(t) d t \\
& =\int_{0}^{T_{c}} d_{i} s_{i}(t)^{2}+\int_{0}^{T_{c}} \sum_{k=1}^{K-i} d_{k} s_{i}(t) s_{k}\left(t-\tau_{k}\right)
\end{aligned}
$$

The first term of the last equality is the actual information while the last term represents the MAI noise which contains the correlation property. The code to choose will then have to nullify or at least reduce that value.

Taking into account the conditions and constraints mentioned above, together with the importance of cardinality, the 2-D OCFHC/OOC is the sequence that is selected.

## IV. One-Coincidence Frequency Hopping Code

As already stated, we need a trade-off between the MAI and the cardinality: a code where the codewords are of nonrepetitive frequencies to combat MAI and the code length is smaller than the number of frequencies available. The onecoincidence code is then chosen and its construction is done as in [10], but adapted to the constraints of the medium.

## A. Definition

A one-coincidence code is a sequence of frequencies represented by $(0,1)$ with " 0 " if no frequency is present and " 1 " if there is frequency, satisfying the following Hamming correlation condition [8]:

$$
H_{X Y}(\tau)=\sum_{k=0}^{L-1} h\left(x_{k} y_{k+\tau}\right)= \begin{cases}1, & x_{k}=y_{k} \\ 0, & \text { otherwise }\end{cases}
$$

where $X=\left(x_{0}, x_{1}, \ldots, x_{L-1}\right)$ and $Y=\left(y_{0}, y_{1}, \ldots, y_{L-1}\right)$ are the related sequences and the subscript $k+\tau$ being taken modulo $L$. If this condition is at its maximal value, we denote the one-coincidence frequency hopping code as $\left(L, Q, H_{\max }\right)$ OCFHC, with $L$ the code length, $Q$ the frequencies available and Hamming correlation condition $H$ at its maximum (which for the present case is at most one).

## B. Construction of One-Coincidence Frequency Hopping Code

This construction follows the one-coincidence code steps as in [1], [2] and [11]. The $i$-th sequence of frequency hopping can be modeled as a vector of length $L, s_{i}=$ $\left.s_{j}(0), s_{j}(1), \ldots, s_{j}(L-1)\right)$ where $s_{j}(l)$ denotes the $l$-th chip, $0 \leq l \leq L-1$, is selected from one frequency generator of $Q$ frequencies available. That is, a frequency $f_{j}$ is varied accordingly to achieve the frequencies $f_{j}=\left(f_{j}+\lambda_{0}, f_{j}+\right.$ $\left.\lambda_{1}, \ldots, f_{j}+\lambda_{Q-1}\right)$, which yields the generator of sequences $G=\left\{g_{0}, g_{1}, \ldots, g_{N-1}\right\}$.

Let $A$ be that set of frequencies obtained from the Bragg gratings $A=\left\{f_{0}, f_{1}, \ldots, f_{Q-1}\right\}$. We can obtain a frequency

TABLE I
Some OCFHC/OOC CONSTRUCTED

| Group 1 | Group 2 | $\cdots$ | Group 7 |
| :---: | :---: | :---: | :---: |
| $\lambda_{1} \lambda_{2} 0 \lambda_{4} 000$ | $\lambda_{0} \lambda_{3} 0 \lambda_{5} 000$ | $\cdots$ | $\lambda_{4} \lambda_{7} 0 \lambda_{1} 000$ |
| $\lambda_{3} \lambda_{6} 0 \lambda_{7} 000$ | $\lambda_{2} \lambda_{7} 0 \lambda_{6} 000$ | $\cdots$ | $\lambda_{6} \lambda_{3} 0 \lambda_{2} 000$ |
| $\lambda_{5} \lambda_{1} 0 \lambda_{2} 000$ | $\lambda_{4} \lambda_{0} 0 \lambda_{3} 000$ | $\cdots$ | $\lambda_{0} \lambda_{4} 0 \lambda_{7} 000$ |
| $\lambda_{4} \lambda_{3} 0 \lambda_{6} 000$ | $\lambda_{5} \lambda_{2} 0 \lambda_{7} 000$ | $\cdots$ | $\lambda_{1} \lambda_{6} 0 \lambda_{3} 000$ |
| $\lambda_{7} \lambda_{5} 0 \lambda_{1} 000$ | $\lambda_{6} \lambda_{4} 0 \lambda_{0} 000$ | $\cdots$ | $\lambda_{2} \lambda_{0} 0 \lambda_{4} 000$ |
| $\lambda_{2} \lambda_{4} 0 \lambda_{3} 000$ | $\lambda_{3} \lambda_{5} 0 \lambda_{2} 000$ | $\cdots$ | $\lambda_{7} \lambda_{1} 0 \lambda_{6} 000$ |
| $\lambda_{6} \lambda_{7} 0 \lambda_{5} 000$ | $\lambda_{7} \lambda_{6} 0 \lambda_{4} 000$ | $\cdots$ | $\lambda_{3} \lambda_{2} 0 \lambda_{0} 000$ |

hopping code, $C$, of code length $L$ and cardinality $N$, with the number of frequency sequences is $C=\left\{s_{0}, s_{1}, \ldots, s_{N-1}\right\}$. If the Hamming condition above is satisfied, i.e. each frequency is present at most once in each code word, the code becomes an OCFHC with $N \leq \frac{Q(Q-1)}{L}$ codewords [2].

Based on $\operatorname{GF}\left(p^{k}\right)$, an OCFHC code with length $L=p^{k}-1$ can be constructed with an optimum of $N=p^{k}$ codewords with the elements of the field expressed as power of the primitive $a$. The generator $G=\left\{a^{0}, a^{1}, \ldots, a^{l-1}\right\}$ is used and the sequence $s_{i}=\left(s_{i}(0), s_{i}(1), \ldots, s_{i}(L-1)\right)$ is found by adding (modulo $p$ ) to each element of $G$, the $p$-ary element of the field $G F\left(p^{k}\right)$. That is, $s_{i}(j)=\left(a^{j}+j\right)(\bmod p)$ with $j$ the element of the field.

Example 1 Construct an OCFHC over $\operatorname{GF}\left(2^{3}\right)$ combined with a $(7,3,1)$-OOC 1101000. Each element of the generator $G=\{001,010,100,011,110,111,101\}$ from the polynomial $X^{2}+X+1$ is added to the binary elements of the field $\{000,001,010,100,011,110,111,101\}$ to form a sequence of seven frequencies. When combined with the OOC time sequence 1101000 for instance, the resulting time-frequency sequences are exactly the ones given in [1].

However, considering that the frequencies generated by the Bragg gratings encoder can only be present at most once in the sequence, the $w$-set shifts are reduced to $L$, which means we can only get $p^{k}-1$ sequences. The resulting sequences are shown in Table I, with a considerable reduction of cardinality. Since the gratings are fixed on the fiber, the constant Hamming weight requirement as stated above, this number of users can only be improved by adapting the time spreading sequence whereby the OOC is applied.

## V. Adaptive $(n, w, 1)$-OOC

Let $(n, w, 1)$-OOC denote the optical orthogonal code of length $n$, Hamming weight $w$ and correlation index one. The adaptive operation results in extending the code length. The extension, based on the difference family $n \cong 1 \bmod (w(w-$ 1)) [12] and the optimal OOC of [13], comes from the need to improve the cardinality of the code while taking into account the required power.

## A. Code Length Bounds

1) Code Length Upper Bound: The upper bound of the code length is derived from the cardinality of the code. The cardinality of an $(n, w, 1)$-OOC is given by Johnson's bound $C \leq\left\lfloor\frac{n-1}{w(w-1)}\right\rfloor$ and equality means that the OOC is optimal.

Consider $l$ as the largest integer resulting from the ratio $\frac{n-1}{w(w-1)}$, one can write $n_{\max } \geq w(w-1) l+n^{\prime}$, where $1 \leq n^{\prime} \leq w(w-1) \epsilon: 0<\epsilon<1$, therefore $1 \leq n^{\prime}<w(w-1)$.
2) Code Length Lower Bound: The lower bound is generated by the neighbor positions of bits " 1 " taken at their minimum values as follows:

- Define the $i$-th and the $k$-th neighbor of the $j$-th bit " 1 " by $1 \leq i \neq k \leq w-1$.
- If for any $i \neq k$ neighbor of any $j$-th bit " 1 " position, the neighbor differences are such that $\delta_{k, j}=\delta_{k, j-1}+1$ and $\delta_{i j}=\delta_{i, j-1}+1$; and if furthermore the maximum run of bits " 0 " after the last bit " 1 " in the sequence $r(\bmod n)$ is the minimum neighbor difference available with respect to the same arithmetic progression $\left(\delta_{i j}=\delta_{i, j-1}+1\right)$, the first neighbors $\delta_{1, j}$ containing the entire information of the sequence, then the minimum code length is given by: $n_{\text {min }}=\sum_{j=1}^{w-1} \delta_{1, j}+r(\bmod n)$.
Hence the following proposition.
Proposition 1 For any starter block $B_{i}$ of the family $F=$ $\left\{B_{1}, B_{2}, \ldots, B_{l}\right\}$ of an $(n, w, 1)-O O C$, if $\delta_{k, J}=\delta_{k, j}+1$, for any $i \neq k$, then the code length $n$ is bounded as $\sum_{j=1}^{w-1} \delta_{1, j}+r$ $\bmod (n) \leq n \leq w(w-1) l+n^{\prime}$, where $n^{\prime}$ is such that $1 \leq n^{\prime}<$ $w(w-1) l, l$ is the number of blocks called codewords, and the lower bound is necessarily $\sum_{j=1}^{w-1} \delta_{1, j}+r(\bmod n) \cong 1$ $\bmod (w(w-1))$.

From the above proposition, one can extend the code length accordingly based on the run-length constraints of bits " 0 " and the consideration of having full orbit blocks of [12].

## B. Zero Run-Length Constraints

The extension of the code length with the requirement of having a difference leave $L=\emptyset$, is only possible within the bounds of the code length. The sequence resulting from these bounds and requirements can be obtained with the introduction of the minimum run-length of bit " 0 " $r_{0}$ and the maximum run length of bits " 0 " $R_{0}$. The pair $\left(r_{0}, R_{0}\right)$ is called the bits " 0 " run-length constraints or asymmetric run-length constraints. The run-length constraints can be given in a certain pattern that defines the congruence class as $n \cong 3(\bmod (w(w-1)))$ in the following proposition:
Proposition 2 For any block $B$ of an $(n, w, 1)$-OOC, if the sequence of any codeword satisfies the constraints $r_{0}=1$ and $R_{0}$, an arithmetic sequence defined such that $R_{0 i}=R_{0 i-1}+2$ where $R_{0 i}$ is the value of $R_{0}$ for $i=w$, with the first term being $R_{0 i}=3$ when $w=3$, then $\left(r_{0}, R_{0}\right)$ represents the minimum and maximum run-length of bits " 0 ", $B$ is unique and the difference leave $L=\Phi$ yields $n \cong 3 \bmod (w)(w-$ 1)).

Example 2 The (7, 3, 1)-OOC sequence, 1101000, becomes the $(9,3,1)$-OOC sequence, 101001000 , by imposing $r_{0}=1$ and $R_{0}=3$. The neighbor differences and their complementaries for $(7,3,1)$ are $(1,6),(2,5),(3,4)$ with the first neighbors being $\{1,2\}$ to yield 1101000 above. Introducing the run-length constraints yields $n=9$. The run length are 1

TABLE II
Comparison of Different Cardinalities

| Construction | Cardinality |
| :---: | :---: |
| Yin and Richardson | $p^{3 k}-p^{2 k}$ |
| $n \cong 1 \bmod (w(w-1))$ | $\left(p^{3 k}-p^{2 k}\right)-p^{2 k}+p^{k}$ |
| $n \cong 3 \bmod (w(w-1))$ | $p^{3 k}-p^{k}$ |

and 3 hence the neighbor differences and their complementaries are $(1,8),(2,7),(3,6),(4,5)$. The neighbor differences $(1,8),(4,5)$ are excluded pairs, then $(2,7),(3,6)$ remains with first neighbors $\{2,3\}$. Hence, the above $(9,3,1)$-OOC sequence 101001000 , which corresponds to the block $B=$ $\{0,2,5\}$ and it is unique.

## VI. Performance Evaluation

## A. Cardinality Issue

With the construction of Yin and Richardson [1] adapted to the optical Bragg gratings encoder (Table I), we can quickly observe that the cardinality has decreased from $N=$ $p^{2 k}\left(\frac{n-1}{w(w-1)}\right)$ to $N=p^{k}\left(p^{k}-1\right)\left(\frac{n-1}{w(w-1)}\right)$. To overcome this reduction in cardinality, we chose the introduction of zero run-length constraints with the minimum of $r_{0}=1$, and maximum of $R_{0}=R_{0 i}=R_{0 i-1}+2$ for the congruence class $n \cong 1 \bmod (w(w-1))$ to improve the cardinality as shown in Table II.

## B. Bit Error Rate

We base this performance analysis in evaluating the MAI impact on the number of coincidence in a multiple user scenario. For an $(n, w, 1)$-OOC, the number of coincidence a chip of user 1 can experience is $w$ with a probability of $\frac{w}{n}$ and the resulting pulse has the probability of appearance during the specific time interval of $\frac{1}{2}$. Hence, the probability that a chip experiences coincidence is $p=\frac{w^{2}}{2 n}$. The bit error rate is then according to [13] given by

$$
B E R=\binom{N-1}{i} p^{i}(1-p)^{(N-1-i)} .
$$

One can realize that the above probability can be approximated using the central limit theorem as the number of users becomes large. Accordingly, for a given $N$ users, the bit error rate above can be reduced to

$$
\sqrt{N p q} B E R=\frac{1}{\sqrt{2 \pi}} \exp \left(\frac{x^{2}}{2}\right) .
$$

This yields the final bit error rate of

$$
B E R=\frac{1}{\sqrt{2 \pi N p q}} \operatorname{erfc}\left(\sqrt{\frac{N q}{p}}\right)
$$

Figs. 2, 3 and 4 show the resulting bit error rate evaluated using the number of users with MAI as sole noise and the gap between signal-to-noise ratio of $(7,3,1)$-OOC and the one of $(9,3,1)$-OOC.

## VII. Results Analysis

From the results above, one can observe that the increase in number of users can be better handled by the introduction of run-length constraints, as in our code construction, at least in terms of multiple access interference. It is true that we lost 0.04 in terms of code rate, but we can see a positive code gain of about 0.08 dB in terms of MAI signal-to-noise ratio, and most importantly, an improved system cardinality which we wanted to achieve as shown in Table II. In fact, the extension of the code length from $(7,3,1)$-OOC to $(9,3,1)$ OOC through the introduction of " 0 " run-length constraints has reduced the multiple access interference effects. This is shown in Figs. 2, 3 and 4 where one can observe that the MAI of $(7,3,1)$-OOC has a bit of an edge in terms of BER on $(9,3,1)$-OOC for small number of users, but its BER becomes


Fig. 2. Multiple access interference probability error


Fig. 3. BER vs. SNR for $N=20$


Fig. 4. Coding gain for $(7,3,1)$ and $(9,3,1)$ OOC
worse particularly as the number of users increase towards the expected value where a BER in the order of $10^{-9}$ is no longer achievable.

## VIII. Conclusion

We have shown in this paper that for a better performance of the combination OCFHC/OOC in an OCDMA network, one solution can be the introduction of " 0 " run-length constraints on the TS pattern, which is chosen to be the OOC. The latter allows the extension of the code length within its bounds which consequently shows a significant improvement in the cardinality of an FFH-OCDMA system. We have also shown that, despite the increase of users in the network, the sequence exhibits a better signal-to-noise ratio in terms of MAI caused by the number of coincidences, particularly when the number
of users tends to its mean value, which is the average number of users expected in the network.

## REFERENCES

[1] H. Yin and D. J. Richardson, Optical Code Division Multiple Access Communication Networks: Theory and Applications, Tsinghua University Press, Beijing, 2008.
[2] L. Bin, "One-coincidence sequences with specified distance between adjacent symbols for frequency-hopping multiple access," IEEE Trans. Commun., vol. 45, no. 4, pp. 408-410, Apr. 1997.
[3] Y. Chang and Y. Miao, "Construction for optimal optical orthogonal codes," Discrete Math., vol. 261, no. 1-3 pp. 127-139, Jan. 2003.
[4] P. R. Prucnal (ed.), Optical Code Division Multiple Access: Fundamentals and Application, CRC Press, Taylor and Francis Group, 2005.
[5] D. J. G. Mestdagh, "Multiple access techniques for fiber-optic networks," Opt. Fiber Technol., vol. 2, no. 1, pp. 7-54, Jan. 1996.
[6] A. Iocco, H. G. Limberger and R. P. Salathe, "Bragg grating fast tunable filter," Electron. Lett., vol. 33, no. 25, pp. 2147-2148, Dec. 1997.
[7] K. Kiasaleh, "Fiber optic frequency hopping multiple access communication system," IEEE Photon. Technol. Lett., vol. 3, no. 2, pp. 173-175, Feb. 1991.
[8] H. Fathallah, L. A. Rusch and S. LaRochelle, "Optical frequency-hop multiple access communications system," IEEE Int. Conf. Commun., vol. 3, Atlanta, USA, Jun. 7-11, 1998, pp. 1269-1273.
[9] L. R. Chen, S. D. Benjamin, P. W. E. Smith and J. E. Sipe, "Ultrashort pulse reflection from fiber gratings: A numerical investigation," $J$. Lightwave Technol., vol. 15, no. 8, pp. 1503-1512, Aug. 1997.
[10] C. D. Frank and M. B. Pursley, "On the statistical dependence of hits in frequency-hop multiple access," IEEE Trans. Commun., vol. 38, no. 9, pp. 1483-1494, Sep. 1995.
[11] S. Shurong, H. Yin, Z. Wang and A. Xu, "A new family of 2-D optical orthogonal codes and analysis of its performance in optical CDMA access networks," J. Lightwave Technol., vol. 24, no. 4, pp. 1646-1653, Apr. 2006.
[12] R. M. Wilson, "Cyclotomy and Difference Families in Elementary Abelian Groups," Journal of Number Theory, vol. 4, pp. 17-47, 1972.
[13] H. Chung and P. V. Kumar, "Optical orthogonal codes-New bounds and an optimal construction," IEEE Trans. Inform. Theory, vol. 36, no. 4, pp. 866-873, Jul. 1990.
[14] M. J. Colbourn and C. J. Colbourn, "On cyclic block designs," Math. Rep. of Canadian Academy of Sci., vol. 2 pp. 95-98, 1980.
[15] C. Malik and S. Tripathi, "Performance evaluation and comparison of optical CDMA networks," Int. J. Electron. Commun. Technol., vol. 2, no. 1, pp. 55-59, Mar. 2011.


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