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Keywords: Conglomerates; Delegation; Diversi¯cation; Monitoring.
JEL classi¯cation: D23; D82; G20; G32; L22.

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1 Introduction

This paper is about the corporate structure, the organizational structure, and the ¯nancial structure of ¯rms, and how they relate to each other. We often model the ¯rm as producer of a single good. In reality, however, many ¯rms produce more than one single good and some ¯rms are indeed quite diversi¯ed. Why do then ¯rms diversify and is diversi¯cation good or bad from a social point of view? Merging di®erent types of activities inside one ¯rm could without any doubts be bene¯cial whenever there are economies of scope in production, but why would conglomerates, whose main activity is to undertake projects in unrelated lines of business, arise?

A standard result in corporate ¯nance is that, with perfect capital markets, conglomerates do not add any value. The theoretical explanation is that if the investors set of opportunities is not restricted, each single investor can replicate on his own the diversi¯ed portfolio of a conglomerate.[1] Furthermore, investors are able to diversify at lower costs than ¯rms do, as there are agency costs in a divisionalized structure that runs many di®erent lines of business as the conglomerate ¯rm does. This implies that there are no bene¯ts from this type of diversi¯cation, but ¯rms have better to focus their activity in order to exploit the gains from specialization. How come then that we observe conglomerates?

In the literature there are several explanations as to why ¯rms diversify, although there are no synergies. Many of these explanations are not consistent with the e±cient use of resources and some of them are not even consistent with pro¯t maximization. One explanation o®ered is for example that managers are empire builders, and therefore ¯rms with separation between ownership and control will be too diversi¯ed.[2]

However, when capital markets are imperfect, ¯rm diversi¯cation may add value. For instance, from the literature on ¯nancial intermediation we know that diversi¯cation is the mean through which intermediaries can provide liquidity and information services to investors.[3]

The aim of this paper is to explain the role of conglomerates as a solution to information problems in the capital market. We show that ¯rm diversi¯cation may be bene¯cial although it gives rise to agency costs. In a context where entrepreneurs have the ability to run projects and to improve the future cash °ow of projects by exercising e®ort, but have no capital to start the projects, valuable projects might not

[1] See Brealey and Myers (1991), Chap.33, pp.854-856 for a discussion of the principle of value additivity of mergers between ¯rms.

[2] See as a reference Jensen (1986) and Shleifer and Vishny (1989). Li and Li (1996) show in a model with empire building managers that diversi¯cation can be bene¯cial as it increases the e®ectiviness of debt as a bonding device.

[3] The standard references are Diamond and Dybvig (1983) and Diamond (1984). Also in the ¯nancial intermediation literature, however, there are arguments against diversi¯cation. One is that specialization in monitoring similar projects adds value. See for instance Hellwig (1998) for a discussion of the trade-o® between specialization and diversi¯cation. In Winton (1999) diversi¯cation can be bad when projects in di®erent sectors are subject to correlated shocks.

be funded. This rationing occurs as there is moral hazard between the entrepreneur, who runs the project, and external investors, because they do not observe the e®ort choice of the entrepreneur.

It has been shown that diversi¯cation by a borrower, debt ¯nanced, increases the borrower's incentive to exert e®ort.[4] The intuition is that debt makes the agent residual claimant in all states except in bankruptcy and the probability of bankruptcy decreases with diversi¯cation. As a matter of fact in some cases non-risky debt maximizes the incentive by the agent exercising the e®ort. However for a single entrepreneur running many di®erent projects might be increasingly costly due to overload. In other words if limited attention implies that the time spent on each single project a®ects the time available for the other projects, overload costs might become important. Thus a single entrepreneur might not be able to diversify as much as he would like, if he is going to run all the projects himself.

In this paper we show that delegating the running of projects to several managers can keep overload costs down, although it introduces agency costs, as there is need to monitor the e®orts of the managers. Hence, separation of ownership and control arises in response to overload costs. As a matter of fact, a wealthy entrepreneur, who does not need external ¯nance and thus does not diversify, but have capital enough to ¯nance a large number of projects, will indeed delegate the running of projects to managers because this allows to better exploit his managerial skill.

A self-¯nanced entrepreneur will only delegate the running of projects to managers when overload costs are large compared to agency costs. However, an entrepreneur that need external ¯nance has stronger incentives to delegate. First of all, the entrepreneur may be forced to diversify to be able to raise external ¯nance and delegation can be the key ingredient to be able to diversify. Secondly, we show that delegation reduces the moral hazard problem of external ¯nance. In the paper we show that under some circumstances, building a conglomerate is the only solution to overcome asymmetry of information in capital markets.[5]

Our paper thus suggests that we should not worry too much about the agency costs of conglomerates, as they come with the solution of the asymmetric information ¯nancing problem. What we should analyze are instead the ways to exploit the gains from diversi¯cation. Conglomerates are one possible way to exploit these gains when overload costs prove to be important.

Cross-subsidization within conglomerates has attracted lot of attention, and the

[4]See Cerasi and Daltung (2000) for a proof of this result in a context where an intermediary has to exerce an unobservable e®ort in monitoring several projects. This result hinges on the e®ort choice not being observable to investors. Boot and Schmeits (2000) show that, if there is a positive probability that the e®ort choice will be observable to investors, there are cases in which diversi¯cation reduces the e®ort level.

[5]The idea that diversi¯cation adds value only when there is separation of ownership and control is also in Markides and Williamson (1996). However they focus on economies of scope, while we concentrate on the bene¯ts of delegation for external ¯nance.

common opinion seems to be that there is misallocation of funds inside the conglomerate.[6] This paper shows that cross-subsidization among subsidiaries in a conglomerate actually is bene¯cial when it allows the ¯rm to make more credible promises to investors and thereby to reduce the moral hazard in external ¯nancing.

There is evidence that conglomerates trade, on average, at a discount relative to a portfolio of single-segment ¯rms in the same industries[7]. This has been taken as evidence that conglomerates are not pro¯table. However, this paper shows that conglomerates can be a solution to information problems in the capital market. Firms' head-quarters provide information services to investors, i.e. head-quarters monitor managers of the di®erent divisions within the same conglomerate on behalf of investors. Thus the return of the conglomerate's shares should actually be lower compared to the return on equity in ¯rms where the investors have to do the monitoring by themselves.

Our paper is related to the literature on multiple agents within the principal-agent literature.[8] In contrast to this literature, however, we have two moral hazard layers. The paper is in fact closely related to Quian (1994), where incentives are studied in order to ¯nd the optimal hierarchical structure in ¯rms with delegation. There are two aspects in common: the ¯rst is the idea that the larger the span of control of one's

case of self-¯nanced entrepreneurs as it helps to reduce the moral hazard between entrepreneurs and investors. Finally in section 5 we show that, when overload costs are too high for a diversi¯ed entrepreneur to be able to raise any external funding, delegating the task of running the projects to managers inside the ¯rm can make the ¯rm viable, thus explaining the rise of conglomerates.

2 The setup

Consider a one-period economy in which risk-neutral agents are endowed with di®erent amounts of capital. There are two types of investment technology in the economy. There is a safe constant-return technology which requires capital and returns y per unit of capital. There are also indivisible projects which require capital and some management skill. Each project requires an initial input of one unit of capital. An agent must run the project in order for it to return anything. There are two types of agent in the economy; entrepreneurs who have the ability to run projects, and investors, who do not have this ability. We assume that there are in¯nitely many projects in the economy, but that there is a limited number of entrepreneurs so that management skill is a scarce resource in the economy. Therefore the total endowment of capital exceeds the total number of projects seeking ¯nance.

Project cash °ows are assumed to be stochastic and independently distributed. The expected return of a speci¯c project depends on how much e®ort the entrepreneur puts into the project. Each project returns R in case of success and 0 in case of failure. The probability of success of the project, p; depends on the e®ort level, e; according to the following function:

$$p(e) = p_L + e\Delta p \ (= p_H - (1 - e)\Delta p);$$

where $\Delta p = p_H - p_L > 0$ and $e \in [0;1]$: By putting e®ort into the project the entrepreneur can increase the probability of success of the project from p_L. If the project is badly run, that is if the entrepreneur does not exert any e®ort, the probability of success is p_L, but if the entrepreneur puts enough e®ort into the project (e = 1) it will with certainty have the highest probability of success, p_H.

We assume that only properly run projects are worth ¯nancing, while without e®ort the net present value of the project is negative, that is:

Assumption 1 $p_L R < y$:

Assume that e®ort is costly to the entrepreneur and that the marginal cost of e®ort is increasing. Moreover, one entrepreneur can run several projects, but the e®ort cost is increasing more than proportionally with the number of projects, due to overload. This cost structure is captured by the following cost function

$$c(\underline{e}) = \frac{c_1}{2}\sum_{i=1} e_i^2 + \frac{c_2}{2}\sum_{i=1}\sum_{j\neq i} e_i e_j;$$

where $\underline{e} = \{e_1; ...; e_n\}$ is a vector of e®orts, n is the number of projects run by the entrepreneur, and c_1 and c_2 are positive parameters. A strictly positive value of c_2 implies that the marginal cost of running a particular project increases with the number of projects run by the entrepreneur. Furthermore, it implies that increasing the e®ort put into the running of one project does not only increase the marginal cost of running that speci¯c project, but also the marginal cost of running all the other projects. Hence, when $c_2 > 0$ there are overload costs.

We assume that:

Assumption 2 $¢pR > c_1$:

This implies that the social marginal return from e®ort exceeds the social marginal cost of e®ort for any e®ort level when the entrepreneur runs one project alone. We also assume that:

Assumption 3 $p_H R ¡ y ¡ \frac{c_1}{2} > 0$:

This assumption implies that the project is socially valuable.

Finally, we assume that e®ort levels are not observable to outsiders. An entrepreneur, however, can use his management skill to ¯nd out the true probability of success of a project run by another entrepreneur. We assume that if the entrepreneur spends m units of e®ort on monitoring a project, he will ¯nd out with probability m the expected return of the project. The e®ort cost of monitoring is assumed to be the same as the e®ort cost of running a project.[10] The key assumption, however, is that monitoring more than one project is increasingly costly for the same reason as running more than one project.

We would like to study the incentives of owners of ¯rms to diversify and delegate the running of projects to managers and how this incentive is a®ected by the ¯nancial structure of the ¯rm. Let us ¯rst consider, as a benchmark, the incentives of entrepreneurs who have enough inside capital to be able to run their projects without seeking external ¯nance.

3 The incentive to delegate in self-¯nanced ¯rms

As management skill is a scarce resource in the economy, entrepreneurs can earn rents on running projects. Therefore, an entrepreneur may want to run more than

[10] There are two justi¯cations for this. First, if there was to be a di®erence between the two costs, running a project would be presumably more costly than monitoring somebody else running the project. But since the paper wants to discuss the bene¯ts of delegating the running activity to managers, our assumption is conservative. Secondly, to monitor a project the agent must have had some experience in running some other projects himself to be able to understand the quality of a speci¯c project and make forecast on the cash °ow. Thus, monitoring a project embodies some initial learning cost which is likely to be larger than the monitoring cost.

one project although it involves overload costs. Since there is an unlimited number of projects in the economy, an entrepreneur can carry out as many projects as he wants. In this section we discuss the benchmark case of an entrepreneur who is going to ¯nance the projects himself. We will also discuss whether self-¯nanced entrepreneurs prefer to run the projects on their own or to delegate the task of running the projects to managers. Given the assumed cost structure, each single entrepreneur might ¯nd too costly to run several projects on his own. Thus he might ¯nd pro¯table to delegate the running of projects to managers in order to avoid overload costs. On the other hand however delegation involves agency costs as the manager faces a moral hazard problem due to the private cost of e®ort. Therefore the answer will depend upon the balance between overload costs and agency costs.

3.1 The self-¯nanced entrepreneur

We ¯rst de¯ne as a benchmark the optimal number of projects that a wealthy entrepreneur would like to undertake given that he is going to ¯nance and run the projects himself. From Assumption 3 it follows that the entrepreneur will at least run one project. Since there are many more projects than entrepreneurs, it could be optimal for the entrepreneur to run more than one project in spite of overload costs. The entrepreneur chooses the optimal number of projects in order to maximize

$$\sum_{i=1}^{n} p_i(e_i)R - ny - c(\underline{e});$$

where $\underline{e} = \{e_1,...,e_n\}$ is the vector of e®orts of running projects and $p_i(e_i) = p_H - (1 - e_i)\Delta p$. The optimal e®ort that the entrepreneur will put into project i, when running n projects, is given by the following ¯rst order condition (FOC):

$$\Delta pR - c_i'(\underline{e}) \geq 0. \qquad (1)$$

In the symmetric equilibrium[11], i.e. when $e_i = e$ for all i, the FOC becomes:

$$\Delta pR - ce \geq 0, \qquad (2)$$

where $c = c_1 + (n - 1)c_2$ is the marginal cost of an additional unit of e®ort. Running more than one project will eventually reduce the optimal e®ort into each project. Substituting the optimal symmetric e®ort, $e^*(n)$, into the total pro¯ts of the entrepreneur, we get the equilibrium pro¯t function:

$$\pi_n(e^*) = n\left[p(e^*)R - y - \frac{c}{2}(e^*)^2\right] = n\Delta p\mu_n(e^*), \qquad (3)$$

[11] We show in the Appendix that for $c_1 > c_2$ the only equilibrium for the choice of the e®orts is the symmetric one.

where $¼_n(e^¤) = \frac{¦_n(e^¤)}{n}$; the per-project pro¯t, is decreasing in the number of projects, directly, through n, and, indirectly, through the e®ort $e^¤$; which is decreasing in the number of projects whenever eq.(2) is satis¯ed with equality.

By applying the Envelope Theorem, we can compute the net gain from running an additional project for the entrepreneur's pro¯ts:[12]

$$\frac{d¦_n(e^¤)}{dn} = ¼_n(e^¤) ¡ \frac{nc_2}{2}(e^¤)^2: \tag{4}$$

The derivative in eq.(4) can be positive or negative for n = 1 depending on the size of overload costs. From Assumption 3 it follows that, for su±ciently small c_2, the entrepreneur may want to run more than one project. However, since $e^¤$ eventually decreases with n, it follows from Assumption 1 that there is a limit to the number of projects that the entrepreneur wants to run ($¼_n(0) = p_L R ¡ y$). Denote this number by $n^¤$: We have that $n^¤$ decreases with c_2 and that as c_2 approaches zero, $n^¤$ goes to in¯nity.

3.2 The incentive to delegate

When overload costs are large, is it better to delegate the task of running the projects to di®erent managers in order to limit the costs? The bene¯ts of delegation have to be counterbalanced by the costs, because delegation introduces agency costs.

Let's analyze the case of a wealthy entrepreneur, called the owner, undertaking n projects. The owner hires n entrepreneurs (without capital), called the managers, to run the projects. Hence, each manager runs only one project.[13]

Managers must be compensated for running projects. Assume that the owner promises to pay the same wage, w, to every manager. In order to maximize the incentives of the manager to exert e®ort, the manager should not get any return if the project fails or if the owner ¯nds out that he has shirked, that is if the owner ¯nds out that the manager has chosen an e®ort level, e, smaller than 1. Hence, the manager is promised a salary w, but the owner is allowed to ¯re him without paying anything if the project fails or if he ¯nds out that the manager has shirked.[14] Given this contract, manager i chooses his e®ort to maximize his utility

$$U_i = q_i w ¡ \frac{c_1}{2}(e_i)^2;$$

[12]Note that this equation holds true even when the FOC for the optimal e®ort choice is not binding, since in that case $de^¤(n)=dn = 0$:

[13]This hierarchical structure minimizes overload costs. We discuss further this assumption in the conclusions.

[14]We assume that the principal can ¯re the manager without veri¯ability of the e®orts. This assumption ¯ts with the fact that managers are given much more risky contracts than simple workers and thus there is no need for a good cause to ¯re a manager, while this is necessary in the case of a worker.

where $q_i = e_i p_H + (1 ¡ e_i)(1 ¡ m_i)p_L$ and m_i is the e®ort of the owner when monitoring the manager. If the manager chooses the e®ort level e_i, with probability e_i the project will have a high expected return and the manager will get his salary if the project succeeds. With probability $(1 ¡ e_i)$ the project will have a low expected return and the manager gets the salary when the project succeeds only if the owner does not ¯nd out the true expected return of the project. Whether or not the owner learns the expected return of the project depends on how much e®ort, m_i, he puts into monitoring this project. With probability $(1 ¡ m_i)$ the owner will not be able to verify that the expected return of the project is low, and the manager gets his salary if the project succeeds. The FOC for the manager's e®ort choice, that is his incentive compatibility (IC) constraint is:

$$(¢p + m_i p_L)w ¡ c_1 e_i = 0: \tag{5}$$

Hence, monitoring by the owner will increase the incentive of the manager to put e®ort into the project.[15] Whether the IC constraint of the manager is binding will depend on the salary. The pro¯t of the owner in the delegation case will always be lower if the IC constraint of the manager does not bind than if it binds, since a non-binding constraint means that the salary could be reduced without reducing the expected return of the project. We will consider the case in which the manager's IC constraint is binding.[16]

Since each manager is paid out of the return of his project, there is no cross-subsidization between projects. This means that the managers could either run independent ¯rms or be division managers within the same ¯rm; there is no di®erence between these two cases. Hence, the owner can either own n small ¯rms or one large ¯rm. In other words, the owner has no incentive to set up a conglomerate, although there are no costs from doing it. This, as we will show, is di®erent with respect to the case of an owner who needs to raise external ¯nance. In that case the owner will indeed have incentive to build a conglomerate, even under the assumption that he pays all managers the same salary.

Let us now examine the incentive of the owner to monitor the managers. We assume that the owner, once he has monitored the project, is in full control of the new manager hired to continue the project after the ¯rst has been ¯red. In order to simplify formulas we assume that this new manager is paid the same wage of all other managers.[17] The expected return of the owner running n projects can thus be

[15] This framework borrows from Aghion and Tirole (1997) the idea that the e®orts of the owner and the manager in°uence each others and that both e®orts are essential for the project. In our model however the two e®orts are complements, while in their model they are substitutes.

[16] The salary depends on the market for managers. The manager's IC constraint will bind if there are su±ciently many managers around for there not to be any competition for managers. Delegation could of course be more expensive if there is shortage of managers so that managers have some bargaining power.

[17] This assumption is actually making delegation more costly. One could just assume that this

written as:

$$\sum_{i=1}^{n} p_i^d(e_i; m_i)(R - w) - ny - c(\underline{m}); \tag{6}$$

where $\underline{m} = \{m_{1;:::}m_n\}$ is the vector of monitoring e®orts and $p_i^d(e_i; m_i) = p_H - (1 - e_i)(1 - m_i)¢p$ is the probability of success of project i, that depends upon the e®orts of both the manager and the owner.

For a given salary, w, the owner chooses the monitoring e®ort for each project to maximize pro¯ts in eq.(6). The FOCs are given by

$$(1 - e_i)¢p(R - w) - c_i^0(\underline{m}) \geq 0; \qquad i = 1; :::; n: \tag{7}$$

For delegation to be feasible, w must be positive. If $w = 0$, it follows from eq.(5) that $e_i = 0$ for all i and the FOC of the owner is equivalent to that in the non-delegation case, that is eq.(1). Hence, $w = 0$ can be thought of as the owner running the project by himself. For a larger salary the owner reduces his e®ort. The interesting case is when the FOCs of the owner are satis¯ed with equality. In this case, the owner, who delegates the project to a manager, exerts less e®ort compared to the owner running the project himself. Delegation is thus appealing for the owner only if he manages to reduce his e®ort, otherwise he would prefer to save on the salary w and run the project himself. If $e_i = 1$, the owner has no incentive to monitor the manager. In other words, when w is large enough, the manager runs the project with maximum e®ort, although the owner does not monitor him at all.

Let us de¯ne the symmetric equilibrium e®orts $\{e(n); m(n)\}$. Thus the system of FOCs becomes[18]:

$$(¢p + m¢p_L)w - c_1e = 0; \tag{8}$$

$$(1 - e)¢p(R - w) - cm = 0: \tag{9}$$

where $c \equiv c_1 + (n - 1)c_2$. Using Cramer's rule to solve for the equilibrium e®orts, we get:

$$m = \frac{¢p(R - w)(c_1 - ¢pw)}{H}; \tag{10}$$

$$e = \frac{w¢p(c + p_L(R - w))}{H}; \tag{11}$$

where $H \equiv c_1c + p_Lw¢p(R - w)$. In equilibrium the e®ort levels depend upon the salary; the e®ort of the owner decreases with the salary, while the e®ort of the manager increases with the salary. A higher salary means that the owner can rely more on the manager behaving, and therefore can reduce his monitoring e®ort.

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manager is paid just the alternative salary, but then it could look like we were biasing the result towards delegation.

[18] We focus on symmetric equilibria. It is easy to show that when $c_1 > c_2$, there is only a symmetric equilibrium.

Substituting the equilibrium e®orts, we derive the equilibrium pro¯ts of the owner who delegates the running of n projects to managers, in a self-¯nanced ¯rm:

$$\Pi_n^d(e,m) = n\left[p^d(e,m)(R - w) - y - \frac{c}{2}(m)^2\right] = n\,\pi_n^d(e,m); \qquad (12)$$

where $p^d(e,m) = p_H - (1-e)(1-m)\Delta p$ is the equilibrium probability of success:

3.2.1 The focused ¯rm (n=1)

We will ¯rst discuss the incentives to delegate by the owner of a focused ¯rm. When n = 1; the pro¯t of the owner is given by

$$p^d(e;m)(R - w) - y - \frac{c_1}{2}m^2;$$

where $p^d(e;m) = p_H - (1-e)(1-m)\Delta p$: The owner chooses his monitoring e®ort according to

$$(1-e)\Delta p(R - w) - c_1 m = 0; \qquad (13)$$

while the manager according to

$$(\Delta p + mp_L)w - c_1 e = 0: \qquad (14)$$

There are agency costs when delegating the running of the project to a manager. This can be illustrated as follows. Assume that the owner does not monitor the manager, the IC constraint of the manager then would be:

$$\Delta p w - c_1 e = 0:$$

Assume further that the owner pays the manager a salary which induces the same e®ort level as that of the owner, if he were running the project himself, that is

$$w = \frac{c_1}{\Delta p}e^{\alpha}(1):$$

From Assumption 2 it follows that $e^{\alpha}(1) = 1$: Substituting this into the pro¯t function, we get:

$$\pi_1^d(e^{\alpha};0) - \pi_1(e^{\alpha}) = -pw + \frac{c_1}{2}: \qquad (15)$$

Substituting w gives

$$-\frac{(p_L + \Delta p)}{\Delta p}c_1 + \frac{c_1}{2} < 0:$$

Due to agency costs, it is more costly for the owner to pay the manager in order to induce him to run the project, rather than running it himself. The owner can reduce the agency cost somewhat by monitoring the manager. We have thus the following result:

Proposition 1 The owner of a focused ¯rm does not want to delegate to a manager at a salary for which the manager chooses the same e®ort of the owner, when running the project himself.

Proof: De¯ne f$\hat{e}$; $\hat{m}$g the e®ort levels of the owner and the manager in equilibrium, that is the solution to the system of equations (14) and (13). We will show that the pro¯ts in delegation, $¼_1^d$; are lower than the pro¯ts without delegation, $¼_1$; for $\hat{e} > \hat{m}$: The di®erence between the pro¯ts can be written as:

$$¼_1^d(\hat{e};\hat{m}) - ¼_1(e^¤) = \left[(p_L + \hat{e}¢p + \hat{m}¢(1-\hat{e})¢p)(R-w) - y - \frac{c_1}{2}\hat{m}^2\right] - \left[(p_L + e^¤¢p)R - y - \frac{c_1}{2}(e^¤)^2\right]:$$

The right hand side (RHS) can be rewritten as

$$(p_L + \hat{e}¢p)(R-w) + \hat{m}\left[(1-\hat{e})¢p(R-w) - \frac{c_1}{2}\hat{m}\right] - \left[(p_L + e^¤¢p)R - \frac{c_1}{2}(e^¤)^2\right]:$$

From eq.(13) it follows that the second term is equal to $\frac{c_1}{2}\hat{m}^2$: Using this and rearranging terms gives

$$\hat{e}¢pR - (p_L + \hat{e}¢p)w + \frac{c_1}{2}\hat{m}^2 - e^¤(¢pR - \frac{c_1}{2}e^¤):$$

We will now show that

$$(p_L + \hat{e}¢p)w > c_1\hat{e}^2:$$

Hence,

$$w = \frac{c_1\hat{e}}{(¢p + \hat{m}p_L)};$$

and we should show that

$$(p_L + \hat{e}¢p)\frac{c_1\hat{e}}{(¢p + \hat{m}p_L)} > c_1\hat{e}^2:$$

By multiplying both sides by $\frac{¢p+\hat{m}p_L}{c_1\hat{e}}$ and subtracting $\hat{e}¢p$ we get

$$p_L > \hat{e}\hat{m}p_L$$

which indeed holds true, since $\hat{e}$ and $\hat{m}$ cannot be both equal to one simultaneously.

Hence we have that

$$¼_1^d(\hat{e};\hat{m}) - ¼_1(e^¤) < \hat{e}¢pR - c_1\hat{e}^2 + \frac{c_1}{2}\hat{m}^2 - e^¤(¢pR - \frac{c_1}{2}e^¤):$$

Rewriting the RHS, we have that

$$¼_1^d(\hat{e};\hat{m}) - ¼_1(e^¤) < \hat{e}\left(¢pR - \frac{c_1}{2}\hat{e}\right) - e^¤\left(¢pR - \frac{c_1}{2}e^¤\right) + \left[\frac{c_1}{2}\hat{m}^2 - \frac{c_1}{2}\hat{e}^2\right]: \quad (16)$$

We have that the ¯rst term of the RHS is non-positive, since e(¢pR ¡ $\frac{c_1}{2}$e) is increasing in e for all e < e¤and equal to zero for e = e¤, and indeed ê · e¤: Also the second term is negative if ê > m̂: Q.E.D

Hence, we can conclude that the owner of a focused ¯rm does not want to pay a manager for doing most of the job running the project. This is not surprising, since delegation introduces an agency cost for the owner. However, we can show that the bene¯ts of delegation are potentially larger in a ¯rm that undertakes many projects when the owner faces overload costs. As pointed out, delegation allows the owner to reduce the e®ort into each project and to substitute his own e®ort with the manager's e®ort. This may indeed reduce costs, since the manager does not face any overload costs. Let us therefore consider the case in which the ¯rm invests in several projects, that is when n > 1:

3.2.2 The unfocused ¯rm (n>1)

Let us discuss whether an owner of an unfocused ¯rm would like to delegate. We will ¯rst show that the incentive to delegate is larger in a unfocused ¯rm compared to a focused ¯rm. We will also show that there are cases in which the owner actually wants to delegate despite internal agency costs arising from the delegated structure. Then we will argue that a hierarchial structure allows the owner to increase the size of the ¯rm, therefore exploiting better his management skill.

Proposition 2 The owner of a unfocused ¯rm has larger incentives to delegate compared to the owner of a focused ¯rm.

Proof: Assume that the owner gives the managers a salary such that

$$w = \frac{c_1}{¢p} e^{¤}(n); \tag{17}$$

where e¤(n) is the e®ort level chosen by the owner when running the projects himself given in eq.(2). Assume that n is large enough for eq.(2) to be ful¯lled with equality. Then we have

$$e^{¤}(n) = \frac{¢pR}{c}:$$

If the owner does not monitor the managers, substituting the salary in eq.(17) into the IC constraint of the manager, eq.(5), implies that each manager will choose the e®ort level e¤. Then the di®erence in pro¯ts per-project can be written as

$$¼^{d}_{n}(e^{¤}; 0) ¡ ¼_{n}(e^{¤}) = ¡pw + \frac{c}{2}(e^{¤})^2:$$

Substituting w; gives

$$¼^{d}_{n}(e^{¤}; 0) ¡ ¼_{n}(e^{¤}) = ¡\frac{(p_L + ¢pe^{¤})}{¢p} c_1 e^{¤} + \frac{c}{2}(e^{¤})^2;$$

which is equal to

$$e^{\alpha}\left(\frac{p_L}{\phi p}c_1 - c_1 e^{\alpha} + \frac{1}{2}ce^{\alpha}\right): \tag{18}$$

The owner has incentive to delegate whenever the term within the parentheses is positive. Since, according to eq.(2), $ce^{\alpha} = \phi pR$ for any n su±ciently large for the FOC of the owner to be binding, and since e^{α} decreases with n; the term within the parentheses increases with n. Q.E.D.

Through delegation, the owner can save on overload costs. This bene¯t increases with the size of the ¯rm. Now, we will show that there are parameter values for which the owner indeed wants to delegate.

Let us discuss whether the owner would like to delegate, when the number of projects is the optimal number of projects, n^{α}; in the previous section. If he would like to delegate in that case, a fortiori he certainly would like to delegate if he were allowed to freely choose the number of projects. We postpone the discussion about the optimal number of projects in this case. Before we have assumed that the owner does not monitor at all the managers. However, the owner might want to monitor the managers, although monitoring involves overload costs, as he could reduce the salary.

Now we will show that there are conditions under which delegation dominates non-delegation.

Proposition 3 There are parameter values for which an entrepreneur would ¯nd optimal to delegate the running of projects to managers.

Proof: Consider the following numerical example:

$$R = 3; p_H = 0.7; p_L = 0.3; y = 1; c_1 = 0.25; c_2 = 0.05; n = 20:$$

Substituting the parameter values into the FOC of an owner who runs the projects himself given by eq.(2) gives, for a binding condition, $e^{\alpha} = 1$: This in turns implies a probability of success for each project $p^{\alpha} = 0.7$. The portfolio return is $p^{\alpha}R = 2.1$; larger than y, so the entrepreneur would prefer to run projects rather than investing in the safe alternative asset. Notice that $n^{\alpha} = 20$ is the optimal number of projects that maximizes pro¯ts in eq.(4). The maximum pro¯ts of the owner are then $\Pi_{20}(e^{\alpha}) = 10$:

However, if the owner delegates the running of projects he could do better. For instance, this is true when the salary is 0.25. Substituting the parameter values into eq.(10) and (11) for $n^{\alpha} = 20$; gives $\pi = 0.22034$ and $\theta = 0.74576$; which in turns implies a probability of success $p^{d} = 0.62071$: This means that the average portfolio return net of salaries, $p^{d}(R - w) = 1.6139$; is larger than y: Furthermore, if the owner delegates the running of the projects to 20 managers he can earn $\Pi^{d}_{20}(\theta; \pi) = 11.694 > 10$. Since the manager gets at least his reservation utility, that is $U_i(\theta) = .16308 > 0$; this solution is feasible and the owner would prefer it to running the $n^{\alpha} = 20$ projects on his own. Q.E.D.

This result depends upon the fact that e®ort costs are increasing in the number of projects and that delegating allows the owner to reduce overload costs. However, delegation introduces agency costs so that this result holds true whenever overload costs overcome internal agency costs.

Notice that in this speci¯c example, when n = 1 it turns out that the e®ort of the manager is larger than the monitoring e®ort of the owner, that is $\hat{e} = 0.87987 > \hat{m} = 0.49973$: In the previous subsection, we have shown that the owner would not like to delegate when running only one project. Still, the above Proposition holds true; thus the gains from delegation increase with the number of projects.

In the numerical example, the bene¯ts of delegation come through the savings on overload costs, since the owner monitors less compared to the e®ort he has to exercise when he runs the project himself. Delegation could also be bene¯cial as, under some conditions, it increases the overall probability of success of each project.

Proposition 4 When the entrepreneur, running the project himself, chooses an e®ort level smaller than 0.5, delegation increases the probability of success of the project.

Proof: When delegating, the probability of success is $p^d(e;m) = p_H - (1-e)(1-m)\Delta p$: Substituting from eq.(9), it follows that the equilibrium probability of success for each project can be written as:

$$\hat{p}^d(\hat{e};\hat{m}) = p_H - \frac{c}{\Delta p(R-w)}\hat{m}(1-\hat{m})\Delta p:$$

We have that $\hat{m} = e^{\alpha}$ for $w = 0$, that is when $w = 0$ the owner chooses the same e®ort level as if he were running the project himself. Since $\hat{m}$ decreases with the salary, and $m(1-m)$ reaches its maximum for $m = \frac{1}{2}$; it follows that $\hat{p}^d$ increases monotonically with the salary when the owner, running the projects himself, exercises an e®ort level smaller than 0:5. Q.E.D

Thus, when the owner runs several projects, putting little e®ort into each of them due to overload, he would have even stronger incentives to delegate. Moreover, the owner might want to ¯nance a larger number of projects compared to the non-delegation equilibrium. We have in fact that:

$$\frac{d\Pi_n^d(\hat{e};\hat{m})}{dn} = \frac{\partial}{\partial n}\Pi_n^d(\hat{e};\hat{m}) - \frac{nc_2}{2}\hat{m}^2 + n(1-\hat{m})\Delta p(R-w)\frac{d\hat{e}}{dn}: \qquad (19)$$

The sum of the ¯rst two terms in eq.(19) is larger than the derivative of the pro¯ts in the non-delegation case given by eq.(4), whenever the owner prefers to delegate, for a given number of projects. The last term in eq.(19), however, is negative. Increasing the number of projects reduces monitoring by the owner and therefore the e®ort of the manager. Hence we cannot generally conclude that the owner would like to ¯nance more projects when delegating. However, in the numerical example in Proposition 3

the owner would indeed like to do so. For instance if the owner hires one thousand managers, his pro¯ts will be equal to 450:65 compared to 11:694; the pro¯ts when running 20 projects. Actually, in this case the projects are so pro¯table that the owner could pay the managers a salary high enough for each of them to put maximum e®ort into the project, even if the owner is not monitoring at all, and still earn a positive pro¯t for each project. In this case the last two terms in eq.(19) are equal to zero so that the derivative is positive for any n:

To conclude this section, without external ¯nance, the bene¯ts of delegation are in the reduction of overload costs, due to the fact that the e®ort of the owner is lower than in the non-delegation case. In addition in some cases delegation increases the probability of success of the project. Hence, separation of ownership and control arises endogenously as optimal response to overload costs, although it involves agency costs.

However, when a ¯rm has to raise external ¯nance, we can show that delegation adds a further bene¯t in that it reduces the moral hazard between the owner and his ¯nanciers. Furthermore, we will show that externally ¯nanced ¯rms might have to diversify more in order to be able to attract outside capital, reinforcing thus the incentive to delegate as the number of projects increases. Hence, external ¯nancing might force the owner to give up some degree of control to managers.

4 The scope for diversi¯cation and delegation in externally ¯nanced ¯rms

Assume that entrepreneurs have no capital at all, while investors do, and that there are many investors. Entrepreneurs who do not have capital on their own, will try to use their essential management skill to raise funds from investors. However, since e®orts are non observable to investors, managerial e®ort is non contractible. Because of limited liability[19] and unobservability of e®orts, there is moral hazard between the entrepreneur and investors, although observability and veri¯ability of project returns. In other words, external ¯nance reduces the incentive of the entrepreneur, since part of the bene¯ts of increased e®ort accrues to investors.

In Cerasi and Daltung (2000) we have shown that diversi¯cation reduces the moral hazard problem when the ¯rm is debt ¯nanced. In the next section we will show that externally ¯nanced entrepreneurs might want to diversify more compared to self-¯nanced entrepreneurs. In the previous section we have shown that delegation may help increasing the overall performance of the entrepreneur's portfolio of projects by

[19]Since the entrepreneur has no capital of his own, he cannot promise to return to investors more income than what his project returns. He cannot commit to deliver anything in the future, given that all the agents live for one period alone, and physical punishments are not allowed. Hence, the entrepreneur has limited liability.

reducing overload costs. Delegation can be helpful in externally ¯nanced ¯rms in two ways: ¯rst of all, because diversi¯cation, although it strengthen the incentive problem of the owner, it is costly to achieve due to overload costs, delegation can be bene¯cial as the overload costs of the owner are reduced; secondly, because delegation, as we will show in section 4.2, reduces the moral hazard of external ¯nance, it adds a further bene¯t in addition to those already discussed in the section on self-¯nanced ¯rms. Could then be that delegation alone helps alleviating the incentive problem of external ¯nancing? In section 5 we show that both diversi¯cation and delegation can be necessary ingredients for the ¯rm to become viable.

4.1 The scope for diversi¯cation

From Cerasi and Daltung (2000) we know that if overload costs are not too high the entrepreneur can commit to a higher e®ort by increasing the diversi¯cation of the portfolio by adding non-correlated projects, given that the contract with the investors is a debt contract. For this result to hold true the debt contract must be conditioned on the return of the portfolio of projects, allowing thus cross-subsidization among projects. Let us therefore consider the case in which the entrepreneur establishes one ¯rm which issues one-period debt contracts. The entrepreneur is in fact the owner of this ¯rm, since he is entitled to the residual income. This provides him with incentives to exert e®ort.

The pro¯ts of an entrepreneur, who owns and runs a debt-¯nanced ¯rm consisting of n projects can be written as:

$$\sum_{i=1}^{n} p(e_i)R - [nr_D - S_n(\underline{e})] - c(\underline{e}); \tag{20}$$

where r_D is the promised gross return per unit of capital and S_n are the expected shortfalls on the total debt, that is the di®erence in expected terms between the promised amount, nr_D, and the amount recovered by the ¯nanciers when the entrepreneur fails to repay nr_D. The expected shortfalls depend upon the promised return on debt and upon the vector of e®orts by the entrepreneur, $\underline{e}$, through the expected returns of the projects.

For each project, the entrepreneur chooses the e®ort so as to maximize pro¯ts:

$$\phi\, pR + \frac{@S_n}{@e_i} - c'(e_i) \geq 0; \qquad i = 1; ....; n: \tag{21}$$

The moral hazard problem comes through the impact of a change in the e®ort level on the expected shortfalls. This term is negative, as a reduction in the e®ort increases the expected shortfalls. The entrepreneur exploits the fact that investors cannot observe an increase in the expected shortfalls due to lower e®ort, as e®ort levels are not observable.

For the entrepreneur to be able to fund the projects, the promised return to investors per unit of capital must satisfy the individual rationality (IR) constraint:[20]

$$r_D \; ¡ \; \frac{1}{n} S_n(\underline{e}) = y: \tag{22}$$

Hence, if there is an equilibrium, the equilibrium pro¯t function of the owner looks like the pro¯t function of the self-¯nanced entrepreneur. De¯ne the symmetric equilibrium e®ort e^0 for all e: The pro¯ts of the owner can be written as:

$$¦_n(e^0) = n \; \left[p(e^0)R \; ¡ \; y \; ¡ \; \frac{c}{2}(e^0)^2 \right] = n ¢ ¼_n(e^0): \tag{23}$$

According to the Law of Large Numbers, the probability that the average portfolio return is equal to the expected project return approaches one, as n goes to in¯nity. Hence, if the expected project return is larger than the promised return to investors, the probability that the average portfolio return will be smaller than the promised return, per unit invested, approaches zero. That is, the expected shortfalls approach zero as n goes to in¯nity. Intuitively the derivative of the expected shortfalls with respect to the e®ort approaches zero as well. The amount by which an increase in the e®ort level could reduce the probability of default is limited by the size of the probability of default, as the probability cannot be less than zero. Hence, as the probability of default approaches zero, so does the derivative of the expected shortfalls.[21] Therefore, if the average expected portfolio return is larger than the promised return on debt, the IC constraint of a perfectly diversi¯ed owner corresponds to eq.(1). Furthermore, the FOC for optimal e®ort choice of the self-¯nanced entrepreneur, and the promised return, r_D, will be equal to the alternative return, y. By establishing a single perfectly diversi¯ed ¯rm the owner can always ful¯ll his promise to repay the debt. If he instead were to own n independent ¯rms, he would fail to pay his debt in about $(1 ¡ p^0) ¤ n$ ¯rms.

We will now show that in a symmetric equilibrium, the owner of an externally ¯-nanced ¯rm might want to diversify more compared to the self-¯nanced entrepreneur.[22] Taking the derivative of the pro¯t function in eq.(23) with respect to n gives

$$\frac{d¦_n(e^0)}{dn} = ¼_n(e^0) \; ¡ \; n\frac{c_2}{2}(e^0)^2 + n\,(¢ pR \; ¡ \; ce^0)\,\frac{de^0}{dn}: \tag{24}$$

The ¯rst two terms correspond to the terms in eq.(4). However, since the owner, due to moral hazard, is not choosing the e®orts to maximize the pro¯ts in eq.(23), the

[20] We assume that the supply of capital exceeds the demand so that there is no competition for capital. Hence, investors will only earn the alternative return.

[21] This is shown for the case of the normal distribution in Cerasi and Daltung (2000).

[22] It could be that there are other solutions rather than the symmetric one, when the ¯rm is externally ¯nanced and not perfectly diversi¯ed. We are however restricting the analysis to the symmetric equilibrium.

Envelope Theorem does not apply, and there is an additional term. Due to moral hazard the equilibrium e®ort of the externally ¯nanced entrepreneur, e^0, is less than the e®ort of the self-¯nanced entrepreneur, $e^¤$; as long as the entrepreneur is not perfectly diversi¯ed. The derivative of the ¯rst two terms in eq.(19) is equal to

$$¢pR ¡ ce^0 ¡ nc_2e^0;$$

which is negative for nc_2 su±ciently large. In that case, the ¯rst two terms are larger than $\frac{d¦_n(e^¤)}{dn}$, as $e^0 < e^¤$: According to the IC constraint of the owner, eq.(21) $¢pR ¡ ce^0$ is positive, if the ¯rm is not perfectly diversi¯ed. If overload costs are positive, but not too high, also $\frac{de^0}{dn}$ will be larger than zero, since the derivative of the expected shortfalls decreases with n. In this case, the owner of an externally ¯nanced ¯rm has incentive to choose number of projects larger than $n^¤$, as diversi¯cation reduces his moral hazard problem. However, for larger overload costs an increase in the number of projects may actually reduce the e®ort of the owner. In that case diversi¯cation alone cannot resolve the moral hazard problem of the owner.

4.2 Bene¯ts of delegation in externally ¯nanced ¯rms

We have seen that diversi¯cation can be a way to reduce moral hazard of the owner. From section 3 we know that the incentive to delegate increases with the size of the ¯rm. However, we would like to show that there is an additional incentive to delegate when the ¯rm is debt ¯nanced, namely that delegation reduces the moral hazard connected to external ¯nancing. In order to disentangle the two bene¯ts of delegation, we ¯rst study an entrepreneur who tries to raise funds from investors in order to ¯nance just one project.

4.2.1 The focused ¯rm (n=1)

Consider ¯rst the case in which the entrepreneur runs the project himself. The entrepreneur promises to pay r_D to his ¯nanciers when the project succeeds. We can think of r_D as a gross return on the debt issued by the ¯rm. However, for the focused ¯rm, because of the simple distribution of the returns of the project it could as well be the gross return on shares. For the unfocused ¯rm, however, debt and equity contracts are not equivalent.

Investors accept to ¯nance the project if and only if the expected return is larger than the safe return of the alternative technology, that is if the following IR condition is satis¯ed:

$$p(e)r_D = y:$$

Investors are assumed to have rational expectations. Hence, they expect the entrepreneur to choose the e®ort level that maximizes his expected pro¯ts,

$$p(e)(R ¡ r_D) ¡ \frac{c_1}{2}e^2:$$

The FOC is given by

$$¢p(R ¡ r_D) ¡ c_1 e ¸ 0: \tag{25}$$

The moral hazard comes through the term $¢p r_D$. Once r_D is given, the entrepreneur has lower incentives to increase his e®ort in the project as the gains are partly appropriated by the ¯nanciers, while the cost is only on the entrepreneur shoulders. We have that if

$$p_H R ¡ y ¡ \frac{p_H}{¢p} c_1 < 0; \tag{26}$$

the moral hazard problem will lead an externally ¯nanced entrepreneur to choose an e®ort level smaller than 1: In this case, the information problem gives rise to a moral hazard problem.

When delegating the task of running the project to a manager, the pro¯ts of the owner are given by:

$$p^d(e; m)R^N ¡ \frac{c_1}{2} m^2;$$

where $p^d(e; m) = p_H ¡ (1 ¡ e)(1 ¡ m)¢p$ and $R^N ´ (R ¡ w ¡ r_D)$: Given the face value of debt, r_D, the owner chooses his monitoring e®ort according to the FOC:

$$(1 ¡ e)¢pR^N ¡ c_1 m = 0: \tag{27}$$

We can now show that the incentives to delegate are larger in the externally ¯nanced ¯rm compared to the case of self-¯nanced ¯rm.

Proposition 5 *In a focused ¯rm an externally ¯nanced entrepreneur has more incentive to delegate than a self-¯nanced entrepreneur.*

Proof: Consider a self-¯nanced owner of a focused ¯rm. From Assumption 2 it follows that this owner will choose the maximum e®ort level, that is $e^¤(1) = 1$: An owner of an externally ¯nanced focused ¯rm will however, due to moral hazard, choose an e®ort level smaller than 1. Let the owner of the externally ¯nanced ¯rm hire a manager and pay him a salary such that the manager chooses to exert maximum e®ort. If the owner does not monitor the manager the di®erence in per project equilibrium pro¯ts can be written as

$$¼_1^d(e^¤; 0) ¡ ¼_1(e^0) = (p_H ¡ p^0)R ¡ p_H w + \frac{c_1}{2}(e^0)^2;$$

where $p^0 = p_L + e^0 ¢p$; and e^0 is the e®ort level chosen by the externally ¯nanced entrepreneur when there is a moral hazard problem. The right hand side can be rewritten as

$$¡ p_H w + \frac{c_1}{2} + (p_H ¡ p^0)R + \frac{c_1}{2}(e^0)^2 ¡ 1 :$$

The ¯rst two terms are the same as in eq.(15), that is in the case of a self-¯nanced focused ¯rm. We will now show that the sum of the last two terms is positive, in which case the incentive to delegate is larger in the externally ¯nanced ¯rm.

We have that

$$(p_H - p^0)R + \frac{c_1}{2}\left[(e^0)^2 - 1\right] = (1 - e^0)\left[pR - \left[1 - (e^0)^2\right]\frac{c_1}{2}\right] =$$
$$(1 - e^0)\left[pR - (1 + e^0)\frac{c_1}{2}\right];$$

that is positive when Assumption 2 holds true. Q.E.D.

The reason why the externally ¯nanced entrepreneur has a larger incentive to delegate is because delegation reduces the moral hazard of the owner. This e®ect can be seen from the IC constraint of the owner. The moral hazard of the owner is captured by the derivative of the expected shortfall with respect to e®ort,

$$-\Delta p r_D$$

in eq. (25). The corresponding derivative with delegation is

$$-(1 - e)\Delta p r_D:$$

Delegation reduces the moral hazard, since the impact of a reduction in e®ort by the owner on the expected shortfall is now mitigated by the fact that the probabiblity of success now also depends on the e®ort of the manager. Notice, however, that when eq.(26) is satisifed, that is when the owner faces a moral hazard problem, R is not large enough for a salary to exist, such that the manager chooses to put maximum e®ort into the project. Hence, the moral hazard problem of external ¯nancing cannot be eliminated through delegation.

4.2.2 The unfocused ¯rm (n>1)

Let us now analyze the bene¯ts of delegation when the ¯rm undertakes $n > 1$ projects. Because of overload costs, it is costly for one entrepreneur alone to run several projects. In section 3 we have shown that delegating the running of projects to other agents can be a way to keep costs down. In section 4.1 we showed that the externally ¯nanced entrepreneur has incentive to diversify more than the self-¯nanced entrepreneur, and hence should have stronger incentive to delegate than the self-¯nanced entrepreneur. Then we showed in the previous section that the externally ¯nanced entrepreneur has additional reasons to delegate, as delegation reduces his moral hazard problem with investors.

The stronger incentive to diversify of the externally ¯nanced entrepreneur only arises if the entrepreneur establishes one diversi¯ed ¯rm. When the entrepreneur is self-¯nanced there are no bene¯ts from setting up a conglomerate. The entrepreneur could equivalently own n independent ¯rms. When the ¯rm is externally ¯nanced, the entrepreneur may indeed have incentive to set up a conglomerate.

Consider an entrepreneur who owns a single ¯rm. The ¯rm hires n managers to run n projects paying each manager a salary w.[23] The owner monitors the managers. As before, the ¯rm is assumed to issue debt. Let us assume that managers are senior with respect to investors.[24] The owner will be residual claimant of the ¯rm's returns, which provides him with the incentive to monitor the managers.

Let us now examine the incentive of the owner to monitor the managers. Given the above assumptions the expected return of the owner can be written as:

$$\sum_{i=1}^{n} p_i^d(e_i; m_i)(R - w) - \left[nr_D - S_n^d(\underline{e}; \underline{m}) \right] - c(\underline{m}); \tag{28}$$

where $p_i^d(e_i; m_i) = p_H - (1 - e_i)(1 - m_i)¢p$ is the probability of success of project i, and $S_n^d(\underline{e}; \underline{m})$ are the expected shortfalls on debt, that is the di®erence in expected terms between the promised amount, nr_D, and the amount recovered by the ¯nanciers when the entrepreneur fails to repay nr_D. The expected shortfalls depend now on both the vector of e®orts by the entrepreneur and the vector of e®orts by managers, $(\underline{e}; \underline{m})$, as both are a®ecting the expected returns on projects.

For a given salary, the owner chooses the monitoring e®ort to maximize pro¯ts in eq.(28) for each project. The FOCs are given by

$$(1 - e_i)¢p(R - w) + \frac{@S_n^d}{@m_i} - \left[c_1 m_i + c_2 \sum_{j \neq i} m_i \right] = 0; \qquad i = 1; ::::; n: \tag{29}$$

As before the derivative of the expected shortfalls with respect to e®orts captures the moral hazard problem of the owner. This term is negative, as a reduction in the e®ort increases the expected shortfalls. The entrepreneur exploits the fact that investors cannot observe an increase in the expected shortfalls, as the e®ort level is not observable. However, as for the focused ¯rm, the e®ect on the shortfall is mitigated by the fact that also the manager puts e®ort into the project, that is the absolute value of $\frac{@S_n^d}{@m_i}$ decreases with e:

From eq.(5) we get the FOCs of the managers:

$$(¢p + m_i p_L)w - c_1 e_i = 0; \qquad i = 1; ::::; n: \tag{30}$$

[23] The contract with the manager implies that the owner pays a salary w whenever the project is succesfull and whenever he, when monitoring, does not ¯nd out that the manager has choosen an e®ort level smaller than 1: When the owner ¯nds out, after having spent some monitoring e®ort, that the manager has shirked, he can ¯re him, without paying him the salary. He will instead hire a new manager, pay him w, and make sure that he chooses the maximum e®ort. This assumption does not require that the level of e®ort is veri¯able to third parties. It only requires to assume that the outcome of the monitoring, whether the e®ort of the manager is smaller than 1; is veri¯able.

[24] Making managers senior to investors means that the wage has to be paid, before any other creditor, out of the return of each succesfull project. Actually one would have to show that seniority of the managers is the optimal contract from the owner's point of view.

For given size of the ¯rm, n, and salary, w, the equilibrium is given by the solution to the above system of equations (29),(30) and by the IR condition for investor:

$$r_D - \frac{1}{n}S_n^d(\underline{m};\underline{e}) = y: \tag{31}$$

Assume that n is su±ciently large for the ¯rm to be perfectly diversi¯ed. This means that there are no shortfalls on debt, if the average return of the ¯rm's portfolio is larger than the alternative return. This requires that overload costs are not too high.[25] If there is an equilibrium in which the overall e®ort levels are high enough for the ¯rm to be able to reimburse its debt, this is determined by[26]

$$(¢p + mp_L)w - c_1e = 0; \tag{32}$$

$$(1 - e)¢p(R - w) - cm = 0; \tag{33}$$

$$r_D = y: \tag{34}$$

The above system of equations is equivalent to that of a self-¯nanced entrepreneur that delegates to managers, that is to eq.(8) and eq.(9). In Proposition 3 we have proved that there are parameter values for which the solution to the above system is preferred by the owner to the optimal solution without delegation. We also showed that, for the same set of parameters, the owner wanted to undertake a larger number of projects when delegating, so that the ¯rm indeed would be approximately perfectly diversi¯ed.[27]Hence, we know that there are parameters for which the owner can do better by delegating than in the non-delegation equilibrium.

However, the bene¯ts of delegation are potentially larger in a non-perfectly diversi¯ed ¯rm. First of all, in a less than perfectly diversi¯ed ¯rm, moral hazard would reduce the e®ort of the entrepreneur. From Proposition 4, we now that if the e®ort of the entrepreneur is smaller than 0:5, delegating will increase the overall e®ort put into each project. Moreover, in the previous section we have proved that a focused externally ¯nanced ¯rm has more incentives to delegate than a self-¯nanced ¯rm as delegation improves the incentives of the owner to exert e®ort. Thus externally ¯-nanced ¯rms might have additional bene¯ts from delegation compared to self-¯nanced ¯rms, when the ¯rm is not perfectly diversi¯ed.

In this section we have shown that, in the limit for a perfectly diversi¯ed ¯rm, we can replicate the result in Proposition 3, namely that there are parameters value for

[25]From Cerasi and Daltung (2000) one can show that there exists a $\hat{c}_2 > 0$; such that for any $c_2 \in [0;\hat{c}_2]$ the incentives of a debt-¯nanced owner are equivalent to that of a self-¯nanced entrepreneur.

[26]As before we focus on symmetric equilibria. The reason is that, when the ¯rm is perfectly diversi¯ed, we are back to the system of equations (5) and (7). Thus it is easy to show that if Assumption 1 holds, there is only a symmetric equilibrium.

[27]We know that for the parameter values in Proposition 3, the entrepreneur would like to carry out 1000 projects when he delegates the running of projects. Applying the Central Limit Theorem one can show that a 1000 projects ¯rm is approximately perfectly diversi¯ed.

which the owner would like to delegate. In the next section, we show that diversi¯cation without delegation can be too costly, so that the ¯rm cannot even raise external ¯nance. Delegation with diversi¯cation however increases the incentive to exert e®ort of the owner without increasing overload costs too much, so that the ¯rm becomes viable.

5 The need for both diversi¯cation and delegation

In this section we want to show that delegation can be so bene¯cial that in fact it can be the only solution for a ¯rm in search of external ¯nance to become viable. We will show that, although a perfectly diversi¯ed ¯rm cannot raise any external ¯nance, delegation makes the ¯rm viable. However, delegation alone is not su±cient for the ¯rm to become viable; the ¯rm must also be diversi¯ed. Hence, building a conglomerate could be the only way to overcome the asymmetry of information in the capital market.

To build this argument, let's ¯rst de¯ne the conditions under which a focused ¯rm is non viable. Then we will show that delegation will not make the focused ¯rm viable. After that we will show that there are cases in which, even though a perfectly diversi¯ed ¯rm remains non-viable, delegating the task of running projects to managers makes this ¯rm viable. Hence both diversi¯cation and delegation are necessary ingredients for the ¯rm to become viable.

First of all, a focused ¯rm cannot raise external ¯nance, when the following condition applies:

Proposition 6 If $¢pR ¡ 2\sqrt{yc_1} + \frac{c_1 p_L}{¢p} < 0$; a focused ¯rm without own capital cannot raise funds from investors.

Proof: A debt ¯nanced entrepreneur running just one project maximizes his pro¯ts under the IR condition for investors

$$p(e)r_D = y;$$

where e is the e®ort that investors believe the entrepreneur will put into the project. Investors expect the entrepreneur to choose the e®ort level by maximizing his expected pro¯ts,

$$p(e)(R ¡ r_D) ¡ \frac{c_1}{2}e^2:$$

where $p(e) ´ p_H ¡ (1 ¡ e)¢p$. For a given face value of the debt contract, r_D; the entrepreneur chooses the e®ort level such that:

$$¢p(R ¡ r_D) ¡ c_1 e ¸ 0: \tag{35}$$

Substituting r_D from the IR of investors, we have that the entrepreneur cannot get funding if

$$¢p(R ¡ \frac{y}{p_H ¡ (1 ¡ e)¢p}) ¡ c_1 e < 0; \tag{36}$$

for any $e \in [0;1]$: The left hand side of eq.(36) reaches its maximum for

$$e = \sqrt{\frac{y}{c_1}} ¡ \frac{p_L}{¢p}:$$

By substituting the maximum e®ort into eq.(36) we have that $¢pR ¡ 2\sqrt{yc_1} + \frac{c_1 p_L}{¢p} < 0$: Q.E.D.

In this case the moral hazard of the owner is so severe, that a focused ¯rm is not able to raise external funds. Let us now show that delegation alone, in a focused ¯rm, does not help raising funds:

Proposition 7 Delegation cannot make a non-viable focused ¯rm, viable.

Proof: We will show that there exists no salary w for which the one-project ¯rm with delegation can raise debt when the non-delegated ¯rm cannot. The equilibrium e®ort levels are given by the solution to the system of FOCs:

$$(¢p + mp_L)w ¡ c_1 e \geq 0; \tag{37}$$

$$(1 ¡ e)¢pR^N ¡ c_1 m \geq 0: \tag{38}$$

where $R^N \equiv R ¡ w ¡ r_D$ is the net return to the entrepreneur. For investors to be willing to ¯nance the ¯rm, they must be promised a return r_D which ful¯lls

$$p^d(e;m)r_D = y; \tag{39}$$

where $p^d(e;m) = p_H ¡ (1 ¡ e)(1 ¡ m)¢p$: Notice that for a given r_D, the e®ort levels e and m ful¯ll respectively the FOC for the manager and the FOC for the owner. When $w = 0$; the equilibrium is given by

$$\begin{aligned} e &= 0; \\ ¢p(R ¡ r_D) ¡ c_1 m &\geq 0; \\ [p_H ¡ (1 ¡ m)¢p]\, r_D &= y: \end{aligned}$$

Then we know from the proof of the previous Proposition that there is no equilibrium. When $w = R ¡ \frac{y}{p^d}$, the equilibrium instead is given by

$$\begin{aligned} m &= 0; \\ ¢p\left[R ¡ \frac{y}{p_H ¡ (1 ¡ e)¢p}\right] ¡ c_1 e &\geq 0; \\ [p_H ¡ (1 ¡ e)¢p]\, r_D &= y: \end{aligned}$$

and again it follows directly from the proof of the previous Proposition that there is no equilibrium.

Now we will show that also for all w such that $0 < w < R - \frac{y}{p^d}$, there is no equilibrium. In this case both FOCs bind and the candidate equilibrium is given by

$$(¢p + mp_L)w - c_1 e = 0; \quad (40)$$

$$(1 - e)¢pR^N - c_1 m = 0; \quad (41)$$

$$[p_H - (1 - e)(1 - m)¢p]\, r_D = y \quad (42)$$

From equation (41) we get

$$(1 - e) = \frac{c_1}{¢pR^N} m;$$

and therefore $p^d = p_H - \frac{c_1}{R^N} m(1 - m)$: For given r_D; p^d is minimized for $m = \frac{1}{2}$ and maximized either for $m = 0$ or $m = 1$: We know that there exists no equilibrium in which $m = 0$ or $m = 1$: Since p^d is smaller for $0 < m < 1$; given r_D, and the equilibrium r_D must be higher for lower p^d, there can neither be an equilibrium in which $0 < m < 1$: Q.E.D.

Hence, to conclude, delegation cannot resolve the moral hazard between the owner and investors in the focused ¯rm.

However delegation can resolve the moral hazard problem in the unfocused ¯rm, namely when diversi¯cation comes together with delegation. What we will show is that, although a perfectly diversi¯ed ¯rm without delegation cannot raise external funds, a diversi¯ed ¯rm with delegation can become viable. So, when diversi¯cation is costly to achieve, because overload costs are large, delegation can be the only way to raise external ¯nance.

Consider a perfectly diversi¯ed ¯rm in which the owner delegates the task of running the projects to managers. Thus if there is an equilibrium in which the overall e®ort levels are high enough for the ¯rm to repay its debt, this is determined by the solution to the system of equations (32) to (34). Thus we have the following result:

Proposition 8 There are parameter values for which a su±ciently diversi¯ed ¯rm can raise debt if and only if the running of projects is delegated to managers.

Proof: Consider the following numerical example

$$R = 2{:}4; p_H = 0{:}8; p_L = 0{:}4; c_1 = 0{:}6; c_2 = 0{:}025; y = 1{:}02; n = 1001$$

For these parameter values a perfectly diversi¯ed ¯rm cannot raise debt when the owner runs all the projects by himself. Substituting the parameter values into the

FOC of the owner who runs the projects himself given by[28]:

$$¢pR ¡ ce = 0;$$

gives $e^¤ = 0:0375$ which in turn implies $p^¤ = 0:415$. The portfolio return is $p^¤R = 0:996$; which is less than y, so no investor would lend money to this ¯rm.

However, if the owner delegates the running of projects, the ¯rm could be viable. For instance, this is true for a salary equal to 0:4. Substituting the parameter values into eq.(10) and (11) gives $¢p = 0:022727$ and $pd = 0:27273$ which in turn implies $p^d = 0:5157$: This means that the average portfolio return net of salaries, $p^d(R ¡ w)$; is larger than y: Hence, if the owner delegates the running of projects to managers, a perfectly diversi¯ed ¯rm is able to raise debt.

Applying the central limit theorem we get that for $n = 1001$ the ¯rm is approximately perfectly diversi¯ed. We have that neither n nor c_2, but only c is of importance for the solution. Hence, it is possible to choose any degree of diversi¯cation by correspondingly adjusting c_2. In order for the ¯rm to be perfectly diversi¯ed the number of projects must in principle be in¯nitely large, which means that c_2 must be in¯nitely small for c to be equal to 25:6 as it is in the example. However, since $p^d(R ¡ w)$ is strictly larger than y; the ¯rm would be viable even if there is a small probability of default. Q.E.D.

This Proposition shows that delegation can be crucial in order to exploit the gains from diversi¯cation, that is when, due to diversi¯cation, overload costs are high.

The comparison in the Proposition is made for a given number of projects. It could well be that the e®ort levels would be higher in a less diversi¯ed ¯rm both in the non-delegation case and in the delegation case. In both cases there will be a number of projects n for which, increasing further diversi¯cation, the overload e®ect will dominate the moral hazard e®ect: However, as pointed out, delegation reduces the moral hazard problem of the owner. Hence, the incentive to delegate could be even stronger in a less diversi¯ed ¯rm.

6 Conclusions

We have shown that delegation can be a way to reduce overload costs, although delegation gives rise to agency costs. Hence, separation of ownership and control can arise as a response to overload costs. We also proved that a focused externally ¯-nanced ¯rm has more incentives to delegate than a self-¯nanced ¯rm as dele smDF06D 7E10(c)7.7

improves the owner's incentives to exert e®ort. Thus externally ¯nanced ¯rms might have additional reasons to delegate compared to self-¯nanced ¯rms. Moreover, externally ¯nanced ¯rms may be forced to diversify to reduce moral hazard related to external ¯nancing and we show, under some circumstances, delegation can be the key ingredient to be able to diversify. Hence, conglomerates can arise as a solution to the asymmetry of information in the capital market.

In this paper we have analyzed two extreme cases, one in which the entrepreneur has enough inside capital to fund as many projects as he would like and one in which the entrepreneur has no inside capital at all. If the entrepreneur had some little but positive capital instead, say one unit, he could choose between running just one project or become diversi¯ed enough to be able to raise external funds. His choice would depend on the size of overload costs compared to agency costs. If it takes time to increase the size of the ¯rm, he would undergo troubled time, since neither he would have enough inside capital, nor he would be diversi¯ed enough to raise external ¯nance. In this period time the entrepreneur would need to ¯nd an investor willing to monitor him. If this was too costly, he might prefer not to grow.

In order to focus on the bene¯ts of debt ¯nancing we have not allowed for other types of cross-subsidization among projects, as for instance paying di®erent salaries to di®erent managers. It could be the case that the owner might be willing to pay some of the managers a higher salary than to others and then only monitor the managers with the lower salary. It could also be the case that a better allocation could be reached with the managers having lower seniority than investors in bankruptcy. This however requires further investigation.

Another issue that would require further research is a discussion of the optimal hierarchy in this context. We have shown that delegation might be pro¯table for a given hierarchy, namely the one that minimizes overload costs. However, there might be di®erent types of hierarchies that makes delegation even more pro¯table.

Although the argument in this paper proves to be relevant for conglomerates, we think it may also apply to ¯nancial ¯rms.[29] Just like the owner of the conglomerate, a ¯nancial intermediary monitors projects. As a matter of fact we think that our framework supports the idea of Gertner et al. (1994), that is that the di®erence between conglomerates and banks steams from the di®erence in the control rights associated with the asset used in the production. In a conglomerate ¯rm, the ownership of the asset rests with the conglomerate. Hence the real control on the assets,

[29] The paper is in fact related to the ¯nancial intermediation literature, see for instance Diamond (1984) or Cerasi and Daltung (2000), since the main result can be applied as well to the monitoring of projects. If we were to interpret the running of projects as monitoring of projects, then one could show that delegating the task of monitoring to a single investor, although well diversi¯ed, cannot be always viable when there are diseconomies of scale in monitoring. However delegation of monitoring to managers inside the ¯nancial intermediary can be the optimal solution. In Cerasi and Daltung (1996) we had a preliminary analysis of delegation inside the intermediary although managers were sensible only to private bene¯ts and not to monetary incentives.

used for production, is in the hand of the formal authority, namely the owner of the conglomerate.[30] So, in case of disagreement between the manager and the owner, the decision is taken by the owner. A bank, on the other hand, does not have control rights on the assets of the borrowers, so that in case of disagreement, the banker can only advice the borrower about what to do with the asset, but the real authority on the asset rests with the borrower. Therefore, the banker has no guarantee that his preferred project will be chosen with certainty. As Gertner et al. (1994) show, control rights provide the owner of the conglomerate with higher incentives to monitor the managers. Thus, a conglomerate should have stronger incentives to monitor than a bank. In this paper, however, we have showed that in some cases also diversi¯cation increases the owner's incentive to monitor. This could then explain why banks are typically more diversi¯ed compared to conglomerates: the incentive of the intermediary to monitor borrowers comes from diversi¯cation instead of control rights. This issue however has to do with the di®erence between inside and external capital markets, which constitutes the topic of our future research agenda.

[30] We follow the de¯nition of Aghion and Tirole (1997) for real and formal authority.

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7 Appendix

We will show that there is only a symmetric solution in the benchmark case if $c_1 > c_2$. Consider the self-¯nanced entrepreneur running n projects. His expected pro¯ts are:

$$\Pi = \sum_{i=1} p_i R - ny - \frac{c_1}{2}\sum_{i=1} e_i^2 - \frac{c_2}{2}\sum_{i=1}\sum_{j\neq i} e_i e_j;$$

where $p_i = p_L + e_i \Delta p$ and e_i is the level of e®ort in project i. The FOCs are given by

$$\Delta pR - \left(c_1 e_i + c_2 \sum_{j\neq i} e_j\right) \geq 0; \quad i = 1...n:$$

We will show that e_i must be equal for all i: First of all the FOCs must either bind for all i or for any of the i. Assume that they were binding for some i and not for others. That the FOC binds, it means that the e®ort level is smaller than 1. Since $c_1 > c_2$; reducing the e®ort level from 1 it will have a larger impact on the FOC of that speci¯c project rather than on the FOCs of all the other projects. This means that the left hand side (LHS) of the other FOCs is smaller than the LHS of the project for which the e®ort is less than 1, but this implies that they also must bind. For the same reason the e®ort levels must be equal if all the FOCs bind. Assume instead di®erent e®ort levels. The LHS of the FOC for the project with the lowest e®ort level will then have the highest value of the LHS of the FOC. Hence, not all FOCs can be binding at the same time, unless the e®ort levels are the same.