

The best known (n, r) -arcs in $\text{PG}(2, 17)$

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After the absurd death of Axel Kohnert in 2013 several hardware errors on the site of the University of Bayreuth have occurred and the database for (n, r) -arcs is no longer available. For that reason we decided to reconstruct the database.

Lower and upper bounds on $m_r(2, 17)$ [1]

$r = 2$	$r = 3$	$r = 4$	$r = 5$	$r = 6$	$r = 7$	$r = 8$	$r = 9$
18	28–33	48–52	61–69	79–86	95–103	114–120	137
$r = 10$	$r = 11$	$r = 12$	$r = 13$	$r = 14$	$r = 15$	$r = 16$	$r = 17$
154	166–171	183–189	205–207	221–225	239–243	256–261	

1. A (28,3)-arc [2]

$(0, 1, 7), (1, 7, 0), (1, 0, 5), (0, 1, 14), (1, 14, 0), (1, 0, 11), (1, 1, 1), (1, 1, 12), (1, 12, 1), (1, 10, 10), (1, 13, 6), (1, 7, 4), (1, 3, 5), (1, 14, 8), (1, 3, 11), (1, 15, 6), (1, 11, 13), (1, 12, 14), (1, 4, 10), (1, 11, 10), (1, 4, 14), (1, 12, 13), (1, 10, 11), (1, 13, 12), (1, 14, 4), (1, 10, 14), (1, 15, 12), (1, 11, 8)$

The secant distribution of the arc is $\tau = (82, 45, 81, 99, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \left(\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \right) \right\rangle.$$

The order of the group is 3.

2. A (48,4)-arc [2]

The points in the arc are as follows

$(1, 1, 3), (1, 3, 1), (1, 2, 7), (1, 6, 8), (1, 7, 2), (1, 4, 5), (1, 8, 6), (1, 12, 13), (1, 14, 16), (1, 5, 4), (1, 16, 14), (1, 13, 12), (1, 11, 9), (1, 10, 15), (1, 15, 10), (1, 9, 11), (1, 1, 4), (1, 4, 1), (1, 2, 15), (1, 8, 8), (1, 15, 2), (1, 16, 13), (1, 13, 16), (1, 9, 9), (1, 1, 13), (1, 13, 1), (1, 2, 2), (1, 9, 8), (1, 4, 16), (1, 8, 9), (1, 16, 4), (1, 15, 15), (1, 1, 14), (1, 14, 1), (1, 2, 10), (1, 11, 8), (1, 10, 2), (1, 4, 12), (1, 8, 11), (1, 5, 13), (1, 3, 16), (1, 12, 4), (1, 16, 3), (1, 13, 5), (1, 6, 9), (1, 7, 15), (1, 15, 7), (1, 9, 6)$

(1, 3, 11), (1, 3, 12), (1, 3, 14), (1, 3, 15), (1, 4, 1), (1, 4, 4), (1, 4, 5), (1, 4, 6), (1, 4, 11), (1, 4, 12), (1, 4, 13), (1, 4, 16), (1, 5, 0), (1, 5, 1), (1, 5, 3), (1, 5, 6), (1, 5, 7), (1, 5, 10), (1, 5, 11), (1, 5, 14), (1, 5, 16), (1, 6, 0), (1, 6, 1), (1, 6, 2), (1, 6, 4), (1, 6, 5), (1, 6, 12), (1, 6, 13), (1, 6, 15), (1, 6, 16), (1, 7, 0), (1, 7, 2), (1, 7, 4), (1, 7, 7), (1, 7, 8), (1, 7, 9), (1, 7, 10), (1, 7, 13), (1, 7, 15), (1, 8, 2), (1, 8, 4), (1, 8, 6), (1, 8, 7), (1, 8, 10), (1, 8, 11), (1, 8, 13), (1, 8, 15), (1, 9, 1), (1, 9, 6), (1, 9, 7), (1, 9, 8), (1, 9, 9), (1, 9, 10), (1, 9, 11), (1, 9, 16), (1, 10, 0), (1, 10, 1), (1, 10, 2), (1, 10, 6), (1, 10, 8), (1, 10, 9), (1, 10, 15), (1, 10, 16), (1, 11, 0), (1, 11, 1), (1, 11, 3), (1, 11, 4), (1, 11, 8), (1, 11, 9), (1, 11, 13), (1, 11, 14), (1, 11, 16), (1, 12, 0), (1, 12, 4), (1, 12, 5), (1, 12, 6), (1, 12, 7), (1, 12, 10), (1, 12, 11), (1, 12, 12), (1, 12, 13), (1, 13, 1), (1, 13, 3), (1, 13, 4), (1, 13, 7), (1, 13, 10), (1, 13, 13), (1, 13, 14), (1, 13, 16), (1, 14, 0), (1, 14, 3), (1, 14, 5), (1, 14, 7), (1, 14, 8), (1, 14, 9), (1, 14, 10), (1, 14, 12), (1, 14, 14), (1, 15, 3), (1, 15, 4), (1, 15, 5), (1, 15, 8), (1, 15, 9), (1, 15, 12), (1, 15, 13), (1, 15, 14), (1, 16, 2), (1, 16, 5), (1, 16, 7), (1, 16, 8), (1, 16, 9), (1, 16, 10), (1, 16, 12), (1, 16, 15), (1, 10, 11), (0, 1, 0), (1, 0, 0), (1, 1, 4), (1, 1, 13), (1, 2, 7), (1, 2, 10), (1, 4, 8), (1, 4, 9), (1, 8, 3), (1, 8, 14), (1, 9, 5), (1, 9, 12), (1, 13, 2), (1, 13, 15), (1, 15, 6), (1, 15, 11), (1, 16, 1)

The secant distribution of the arc is $\tau = (1, 16, 1, 0, 0, 0, 0, 0, 0, 152, 121, 16, 0, 0, 0, 0, 0, 0, 0)$

9. A (166,11)-arc [2]

The points in the arc are

(0, 1, 15), (1, 15, 0), (0, 1, 2), (1, 0, 8), (1, 2, 0), (1, 0, 9), (0, 1, 14), (1, 14, 0), (0, 1, 3), (1, 0, 11), (1, 3, 0), (1, 0, 6), (0, 1, 13), (1, 13, 0), (0, 1, 4), (1, 0, 4), (1, 4, 0), (1, 0, 13), (0, 1, 12), (1, 12, 0), (0, 1, 5), (1, 0, 10), (1, 5, 0), (1, 0, 7), (0, 1, 10), (1, 10, 0), (0, 1, 7), (1, 0, 12), (1, 7, 0), (1, 0, 5), (1, 1, 1), (1, 1, 16), (1, 16, 1), (1, 16, 16), (1, 1, 13), (1, 13, 1), (1, 1, 4), (1, 4, 4), (1, 13, 16), (1, 4, 1), (1, 4, 13), (1, 13, 4), (1, 13, 13), (1, 4, 16), (1, 16, 13), (1, 16, 4), (1, 1, 12), (1, 12, 1), (1, 1, 5), (1, 10, 10), (1, 12, 16), (1, 5, 1), (1, 10, 7), (1, 7, 10), (1, 7, 7), (1, 5, 16), (1, 16, 12), (1, 16, 5), (1, 1, 10), (1, 10, 1), (1, 1, 7), (1, 12, 12), (1, 10, 16), (1, 7, 1), (1, 12, 5), (1, 5, 12), (1, 5, 5), (1, 7, 16), (1, 16, 10), (1, 16, 7), (1, 1, 8), (1, 8, 1), (1, 1, 9), (1, 15, 15), (1, 8, 16), (1, 9, 1), (1, 15, 2), (1, 2, 15), (1, 2, 2), (1, 9, 16), (1, 16, 8), (1, 16, 9), (1, 13, 8), (1, 15, 4), (1, 13, 9), (1, 15, 8), (1, 15, 13), (1, 2, 4), (1, 15, 9), (1, 2, 8), (1, 2, 9), (1, 2, 13), (1, 4, 8), (1, 4, 9), (1, 10, 14), (1, 15, 12), (1, 10, 3), (1, 11, 8), (1, 15, 5), (1, 2, 12), (1, 11, 9), (1, 6, 8), (1, 6, 9), (1, 2, 5), (1, 7, 14), (1, 7, 3), (1, 12, 10), (1, 15, 10), (1, 12, 7), (1, 12, 8), (1, 15, 7), (1, 2, 10), (1, 12, 9), (1, 5, 8), (1, 5, 9), (1, 2, 7), (1, 5, 10), (1, 5, 7), (1, 13, 15), (1, 9, 4), (1, 13, 2), (1, 8, 2), (1, 9, 13), (1, 8, 4), (1, 8, 15), (1, 9, 2), (1, 9, 15), (1, 8, 13), (1, 4, 15), (1, 4, 2), (1, 11, 13), (1, 12, 14), (1, 11, 4), (1, 4, 10), (1, 12, 3), (1, 5, 14), (1, 4, 7), (1, 13, 10), (1, 13, 7), (1, 5, 3), (1, 6, 13), (1, 6, 4), (1, 11, 14), (1, 9, 14), (1, 11, 3), (1, 11, 2), (1, 9, 3), (1, 8, 14), (1, 11, 15), (1, 6, 2), (1, 6, 15), (1, 8, 3), (1, 6, 14), (1, 6, 3), (1, 10, 12), (1, 8, 12), (1, 10, 5), (1, 10, 15), (1, 8, 5), (1, 9, 12), (1, 10, 2), (1, 7, 15), (1, 7, 2), (1, 9, 5), (1, 7, 12), (1, 7, 5)

The secant distribution of the arc is $\tau = (4, 6, 0, 0, 0, 0, 0, 12, 42, 12, 87, 144, 0, 0, 0, 0, 0, 0, 0)$

The arc is found by prescribing the group generated by

$$\left\langle \left(\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 15 \end{pmatrix} \right) \right\rangle.$$

The order of the group is 96.

10. A (183,12)-arc (Daskalov, 2010)

The complement of the (124,6)-blocking set

(0, 0, 1), (0, 1, 1), (0, 1, 2), (0, 1, 7), (0, 1, 10), (0, 1, 16), (1, 0, 1), (1, 0, 5), (1, 0, 7), (1, 0, 10), (1, 0, 16), (1, 1, 6), (1, 1, 7), (1, 1, 8), (1, 1, 9), (1, 1, 11), (1, 2, 5), (1, 2, 8), (1, 2, 9), (1, 2, 12), (1, 2, 13), (1, 2, 15), (1, 3, 3), (1, 3, 4), (1, 3, 5), (1, 3, 7), (1, 3, 10), (1, 3, 11), (1, 3, 13), (1, 3, 14), (1, 4, 0), (1, 4, 1), (1, 4, 3), (1, 4, 6), (1, 4, 7), (1, 4, 8), (1, 4, 10), (1, 4, 11), (1, 4, 12), (1, 4, 13), (1, 4, 14), (1, 5, 0), (1, 5, 2), (1, 5, 5),

(1, 5, 12), (1, 5, 14), (1, 5, 15), (1, 6, 0), (1, 6, 1), (1, 6, 4), (1, 6, 7), (1, 6, 9), (1, 6, 10), (1, 6, 11), (1, 6, 12), (1, 6, 13), (1, 6, 16), (1, 7, 0), (1, 7, 2), (1, 7, 3), (1, 7, 14), (1, 7, 16), (1, 8, 1), (1, 8, 2), (1, 8, 4), (1, 8, 6), (1, 8, 13), (1, 8, 15), (1, 8, 16), (1, 9, 1), (1, 9, 2), (1, 9, 6), (1, 9, 10), (1, 9, 11), (1, 9, 13), (1, 9, 15), (1, 9, 16), (1, 10, 0), (1, 10, 1), (1, 10, 3), (1, 10, 14), (1, 10, 16), (1, 11, 1), (1, 11, 4), (1, 11, 5), (1, 11, 6), (1, 11, 8), (1, 11, 9), (1, 11, 10), (1, 11, 13), (1, 11, 16), (1, 12, 0), (1, 12, 2), (1, 12, 5), (1, 12, 12), (1, 12, 15), (1, 13, 3), (1, 13, 5), (1, 13, 6), (1, 13, 7), (1, 13, 11), (1, 13, 12), (1, 13, 14), (1, 14, 4), (1, 14, 5), (1, 14, 7), (1, 14, 10), (1, 14, 12), (1, 14, 13), (1, 14, 14), (1, 14, 16), (1, 15, 1), (1, 15, 2), (1, 15, 4), (1, 15, 5), (1, 15, 8), (1, 15, 9), (1, 15, 12), (1, 15, 15), (1, 16, 3), (1, 16, 6), (1, 16, 8), (1, 16, 9), (1, 16, 11)

The secant distribution of the blocking set is $\tau = (0, 0, 0, 0, 0, 0, 124, 93, 53, 23, 3, 2, 1, 0, 0, 0, 0, 2, 6)$

11. A (205,13)-arc (Daskalov)

The complement of the (102, 5)-blocking set

(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 1, 4), (0, 1, 13), (0, 1, 16), (1, 0, 0), (1, 0, 1), (1, 0, 4), (1, 0, 13), (1, 0, 16), (1, 1, 0), (1, 1, 1), (1, 1, 4), (1, 1, 13), (1, 2, 4), (1, 2, 8), (1, 2, 9), (1, 2, 13), (1, 3, 4), (1, 3, 5), (1, 3, 12), (1, 3, 13), (1, 4, 0), (1, 4, 1), (1, 4, 2), (1, 4, 3), (1, 4, 4), (1, 4, 5), (1, 4, 6), (1, 4, 7), (1, 4, 8), (1, 4, 9), (1, 4, 10), (1, 4, 11), (1, 4, 12), (1, 4, 13), (1, 4, 14), (1, 4, 15), (1, 4, 16), (1, 5, 3), (1, 5, 4), (1, 5, 13), (1, 5, 14), (1, 6, 4), (1, 6, 7), (1, 6, 10), (1, 6, 13), (1, 7, 4), (1, 7, 6), (1, 7, 11), (1, 7, 13), (1, 8, 2), (1, 8, 4), (1, 8, 13), (1, 8, 15), (1, 9, 2), (1, 9, 4), (1, 9, 13), (1, 9, 15), (1, 10, 4), (1, 10, 6), (1, 10, 11), (1, 10, 13), (1, 11, 4), (1, 11, 7), (1, 11, 10), (1, 11, 13), (1, 12, 3), (1, 12, 4), (1, 12, 13), (1, 12, 14), (1, 13, 0), (1, 13, 1), (1, 13, 2), (1, 13, 3), (1, 13, 4), (1, 13, 5), (1, 13, 6), (1, 13, 7), (1, 13, 8), (1, 13, 9), (1, 13, 10), (1, 13, 11), (1, 13, 12), (1, 13, 13), (1, 13, 14), (1, 13, 15), (1, 13, 16), (1, 14, 4), (1, 14, 5), (1, 14, 12), (1, 14, 13), (1, 15, 4), (1, 15, 8), (1, 15, 9), (1, 15, 13), (1, 16, 0), (1, 16, 1), (1, 16, 4), (1, 16, 13), (1, 16, 16)

The secant distribution of the blocking set is $\tau = (0, 0, 0, 0, 0, 169, 65, 51, 10, 4, 2, 0, 0, 0, 0, 0, 0, 0, 6)$

12. A (221,14)-arc (Daskalov)

The complement of the (86, 4)-blocking set

(0, 1, 5), (0, 1, 12), (1, 0, 12), (1, 1, 0), (1, 1, 7), (1, 1, 10), (1, 2, 2), (1, 2, 5), (1, 2, 15), (1, 3, 3), (1, 3, 10), (1, 3, 14), (1, 4, 8), (1, 4, 9), (1, 4, 15), (1, 5, 3), (1, 5, 4), (1, 5, 13), (1, 6, 1), (1, 6, 8), (1, 6, 16), (1, 7, 6), (1, 7, 11), (1, 7, 13), (1, 8, 1), (1, 8, 6), (1, 8, 11), (1, 9, 1), (1, 9, 6), (1, 9, 16), (1, 10, 4), (1, 10, 11), (1, 11, 8), (1, 11, 9), (1, 11, 16), (1, 12, 3), (1, 12, 4), (1, 12, 14), (1, 13, 9), (1, 13, 15), (1, 14, 7), (1, 14, 10), (1, 14, 14), (1, 15, 0), (1, 15, 1), (1, 15, 2), (1, 15, 3), (1, 15, 5), (1, 15, 6), (1, 15, 9), (1, 15, 10), (1, 15, 11), (1, 15, 12), (1, 15, 13), (1, 15, 15), (1, 15, 16), (1, 16, 0), (1, 16, 7), (1, 15, 8), (1, 15, 14), (1, 15, 4), (1, 0, 15), (1, 15, 7), (1, 10, 13), (1, 13, 2), (1, 0, 5), (0, 0, 1), (1, 14, 1), (1, 14, 8), (1, 14, 16), (1, 14, 5), (1, 14, 11), (1, 14, 12), (1, 14, 13), (1, 14, 6), (1, 14, 9), (1, 14, 3), (1, 14, 4), (1, 14, 0), (1, 14, 15), (1, 5, 9), (1, 16, 3), (1, 4, 2), (1, 4, 12), (1, 6, 13), (0, 1, 9)

The secant distribution of the blocking set is $\tau = (0, 0, 0, 0, 126, 114, 52, 7, 3, 0, 0, 0, 0, 0, 0, 0, 0, 1, 4)$

13. A (239,15)-arc (Daskalov)

The complement of the (68, 3)-blocking set

(0, 0, 1), (0, 1, 5), (0, 1, 12), (1, 0, 5), (1, 0, 12), (1, 1, 0), (1, 1, 6), (1, 1, 7), (1, 1, 10), (1, 2, 2), (1, 2, 5), (1, 2, 10), (1, 2, 15), (1, 3, 3), (1, 3, 10), (1, 3, 13), (1, 3, 14), (1, 4, 8), (1, 4, 9), (1, 4, 15), (1, 5, 3), (1, 5, 4), (1, 5, 13), (1, 6, 1), (1, 6, 8), (1, 6, 16), (1, 7, 6), (1, 7, 11), (1, 7, 13), (1, 8, 1), (1, 8, 6), (1, 8, 11), (1, 9, 1), (1, 9, 6), (1, 9, 16), (1, 10, 4), (1, 10, 11), (1, 10, 12), (1, 10, 13), (1, 11, 8), (1, 11, 9), (1, 11, 16), (1, 12, 3), (1, 12, 4), (1, 12, 14), (1, 13, 2), (1, 13, 4), (1, 13, 9), (1, 13, 15), (1, 14, 7), (1, 14, 10), (1, 14, 14), (1, 15, 0), (1, 15, 1), (1, 15, 2), (1, 15, 3), (1, 15, 4), (1, 15, 5), (1, 15, 6), (1, 15, 9), (1, 15, 10), (1, 15, 11), (1, 15, 12),

(1, 15, 13), (1, 15, 15), (1, 15, 16), (1, 16, 0), (1, 16, 7)

The secant distribution of the blocking set is $\tau = (0, 0, 0, 114, 141, 40, 7, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 3)$

14. A (256,16)-arc (Daskalov)

The complement of the (51, 2)-blocking set

(0, 0, 1), (0, 1, 0), (1, 0, 13), (1, 1, 13), (1, 2, 13), (1, 3, 13), (1, 4, 0), (1, 4, 1), (1, 4, 2), (1, 4, 3), (1, 4, 4),
(1, 4, 5), (1, 4, 6), (1, 4, 7), (1, 4, 8), (1, 4, 9), (1, 4, 10), (1, 4, 11), (1, 4, 12), (1, 4, 13), (1, 4, 14), (1, 4, 15),
(1, 4, 16), (1, 5, 13), (1, 6, 13), (1, 7, 13), (1, 8, 13), (1, 9, 13), (1, 10, 13), (1, 11, 13), (1, 12, 13), (1, 13, 0),
(1, 13, 1), (1, 13, 2), (1, 13, 3), (1, 13, 4), (1, 13, 5), (1, 13, 6), (1, 13, 7), (1, 13, 8), (1, 13, 9), (1, 13, 10),
(1, 13, 11), (1, 13, 12), (1, 13, 13), (1, 13, 14), (1, 13, 15), (1, 13, 16), (1, 14, 13), (1, 15, 13), (1, 16, 13)

The secant distribution of the blocking set is $\tau = (0, 0, 48, 256, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 3)$

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